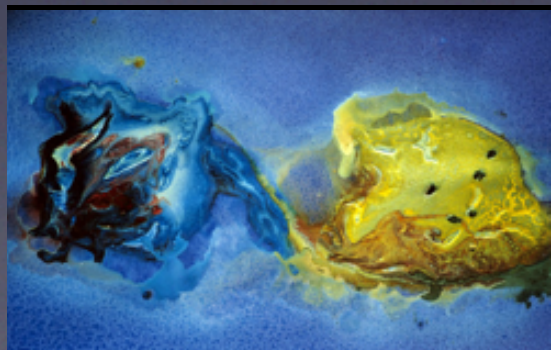


Heavy Quarkonium with Effective Field Theories

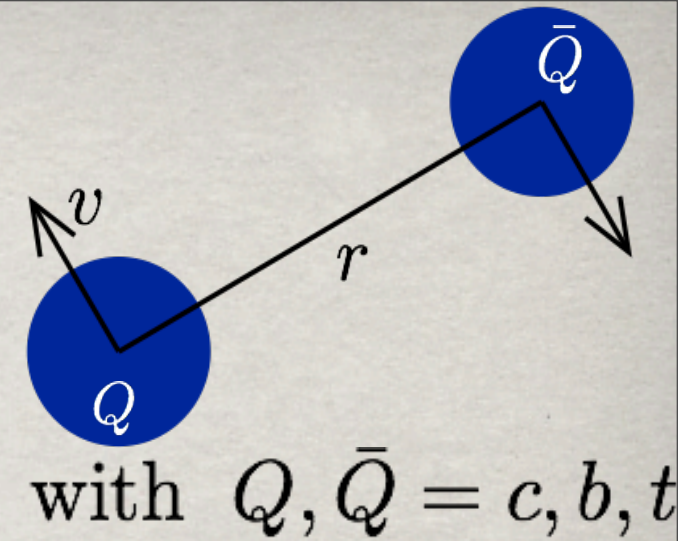
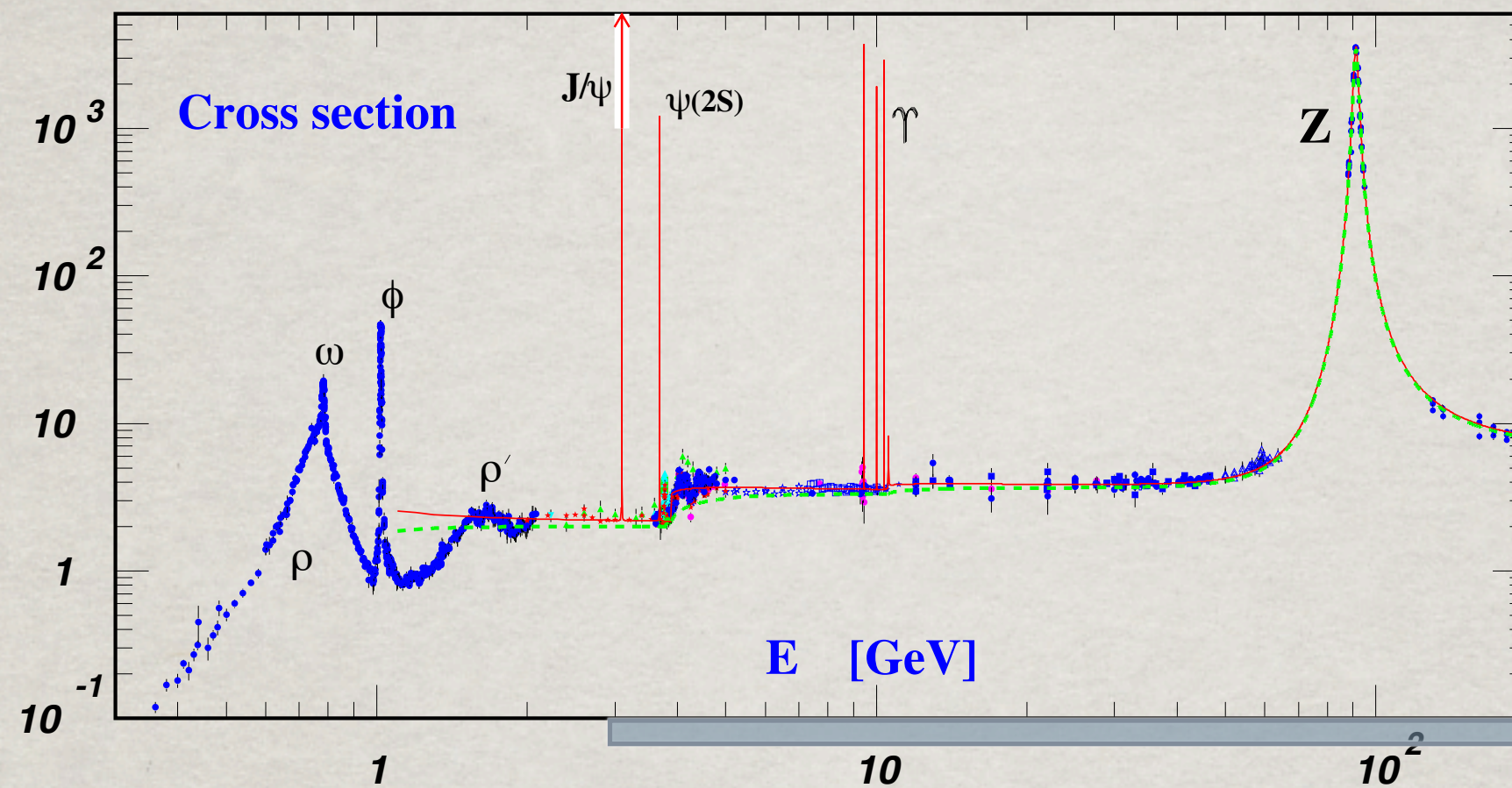


NORA BRAMBILLA

- quarkonium and its relevance in the study of strong interactions
- the state of the art theory tools and their impact on our understanding of strong interactions/sm physics
- experimental/theoretical challenges and opportunities

QUARKONIUM is a privileged
window over the **HADRONIC WORLD**

Heavy quarks offer a privileged access

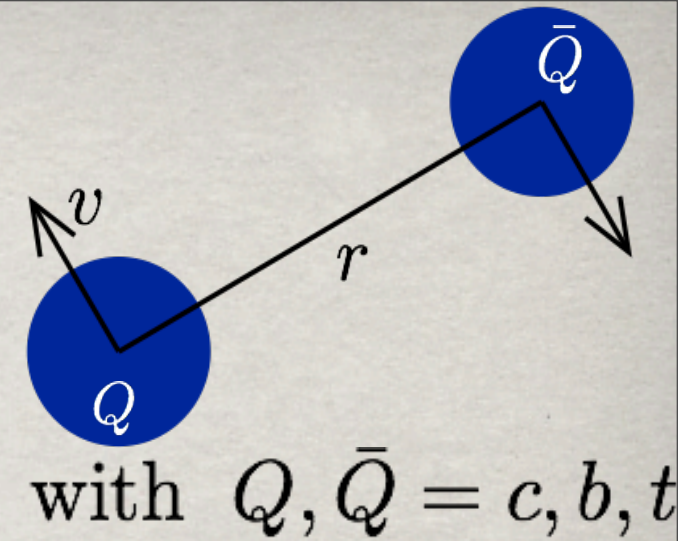
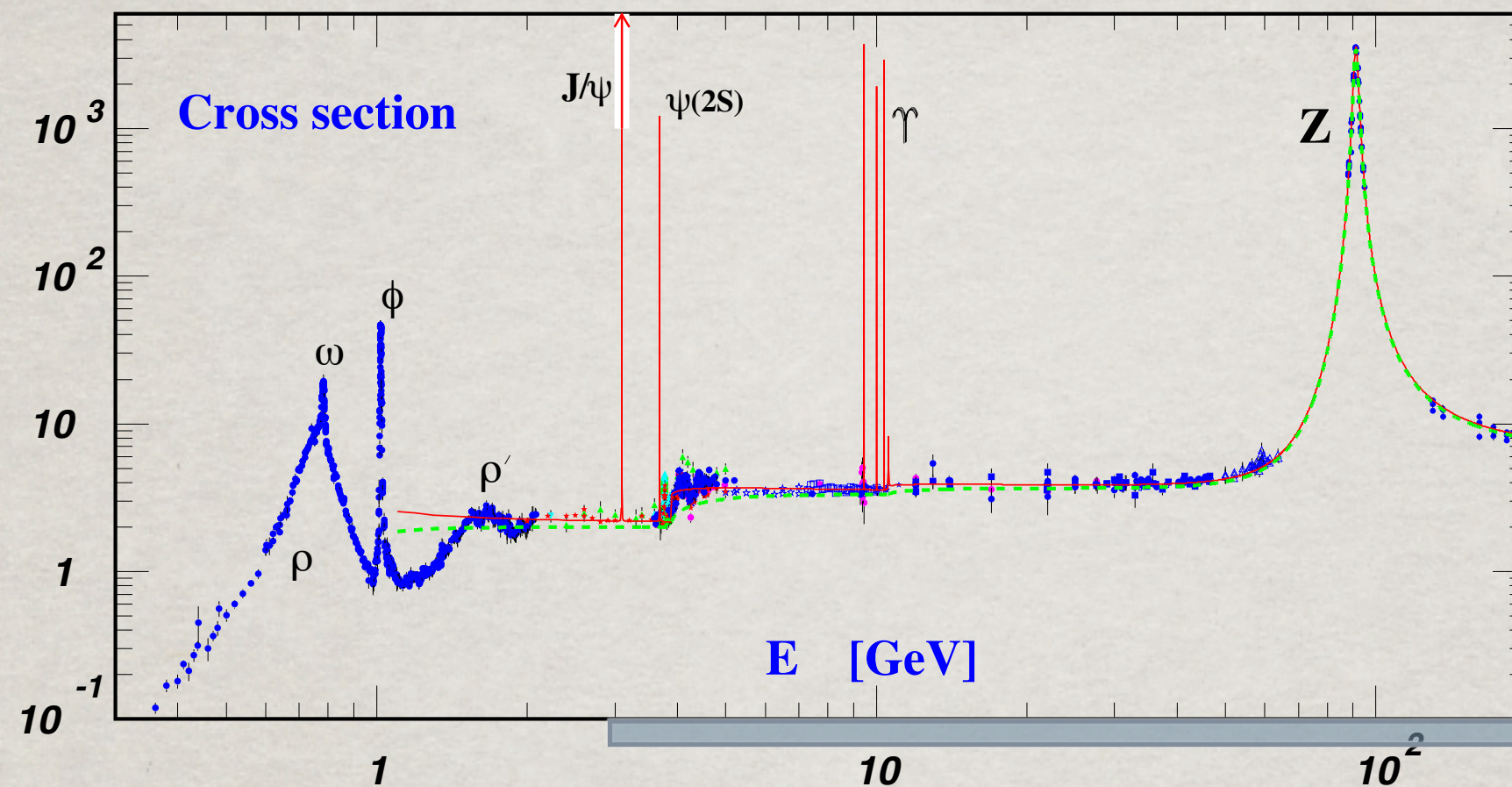


$m_c \sim 1.5 \text{ GeV}$

$m_b \sim 5 \text{ GeV}$

$m_t \sim 170 \text{ GeV}$

Heavy quarks offer a privileged access



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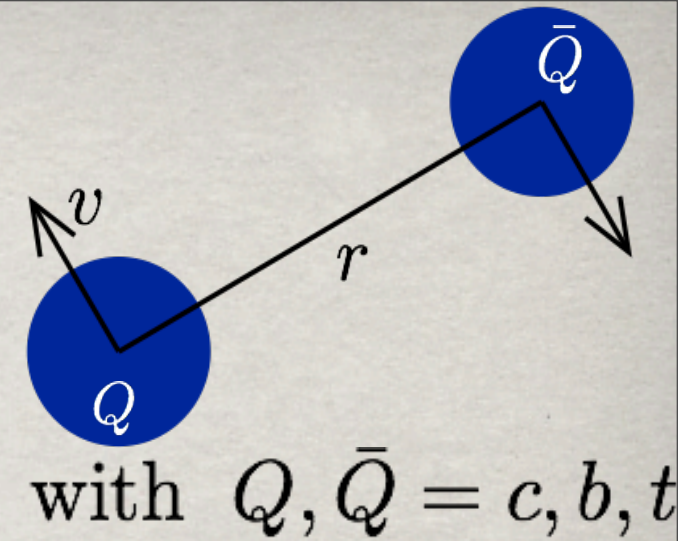
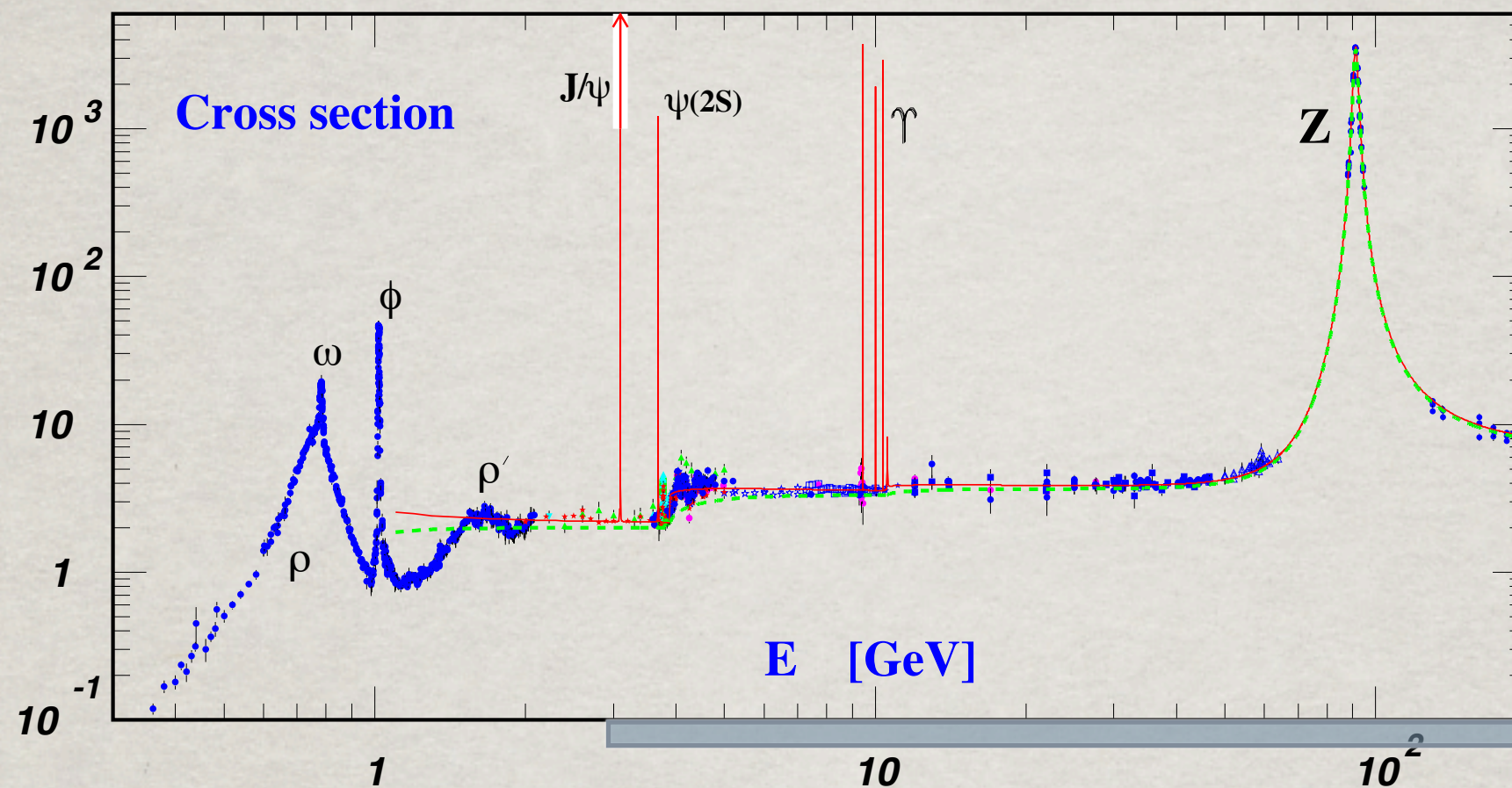
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A large scale

$$m_Q \gg \Lambda_{\text{QCD}}$$

$$\alpha_s(m_Q) \ll 1$$

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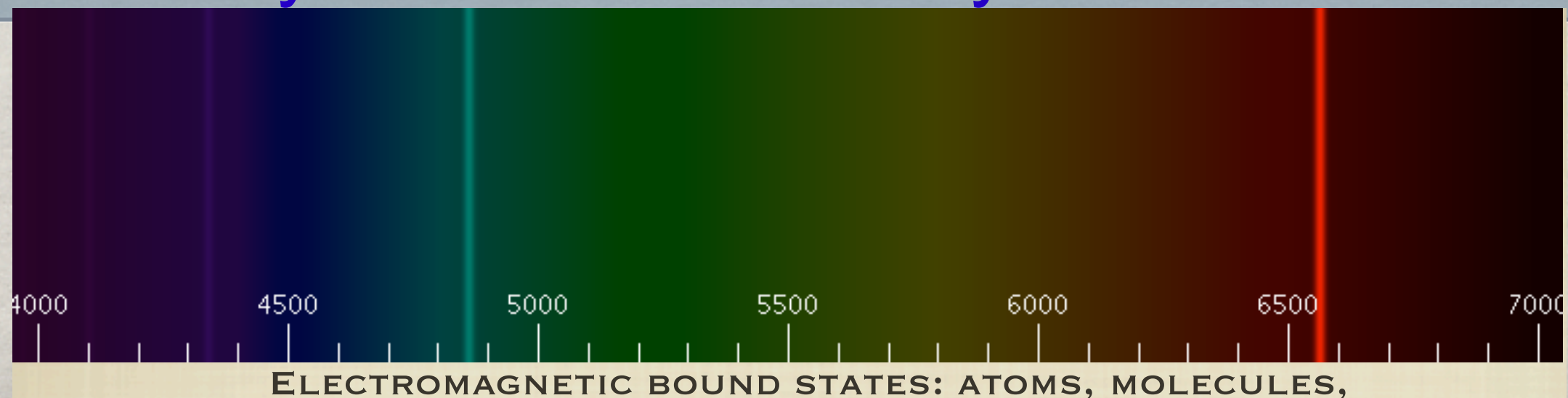
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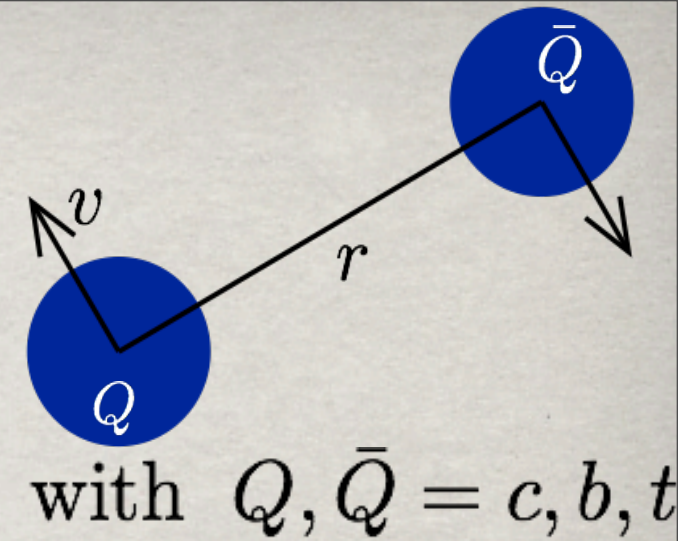
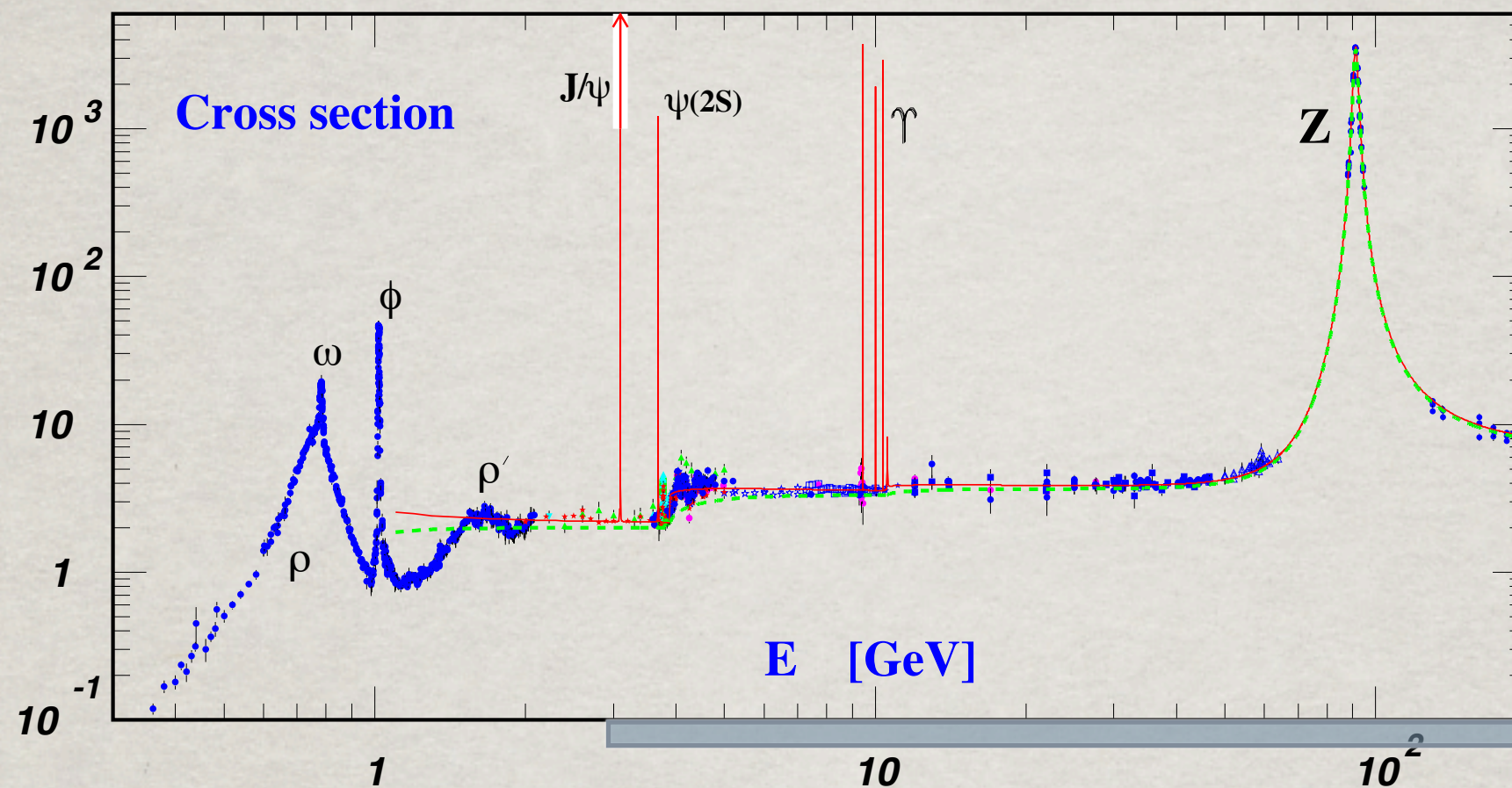
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Heavy quarkonia are nonrelativistic bound systems: multiscale systems



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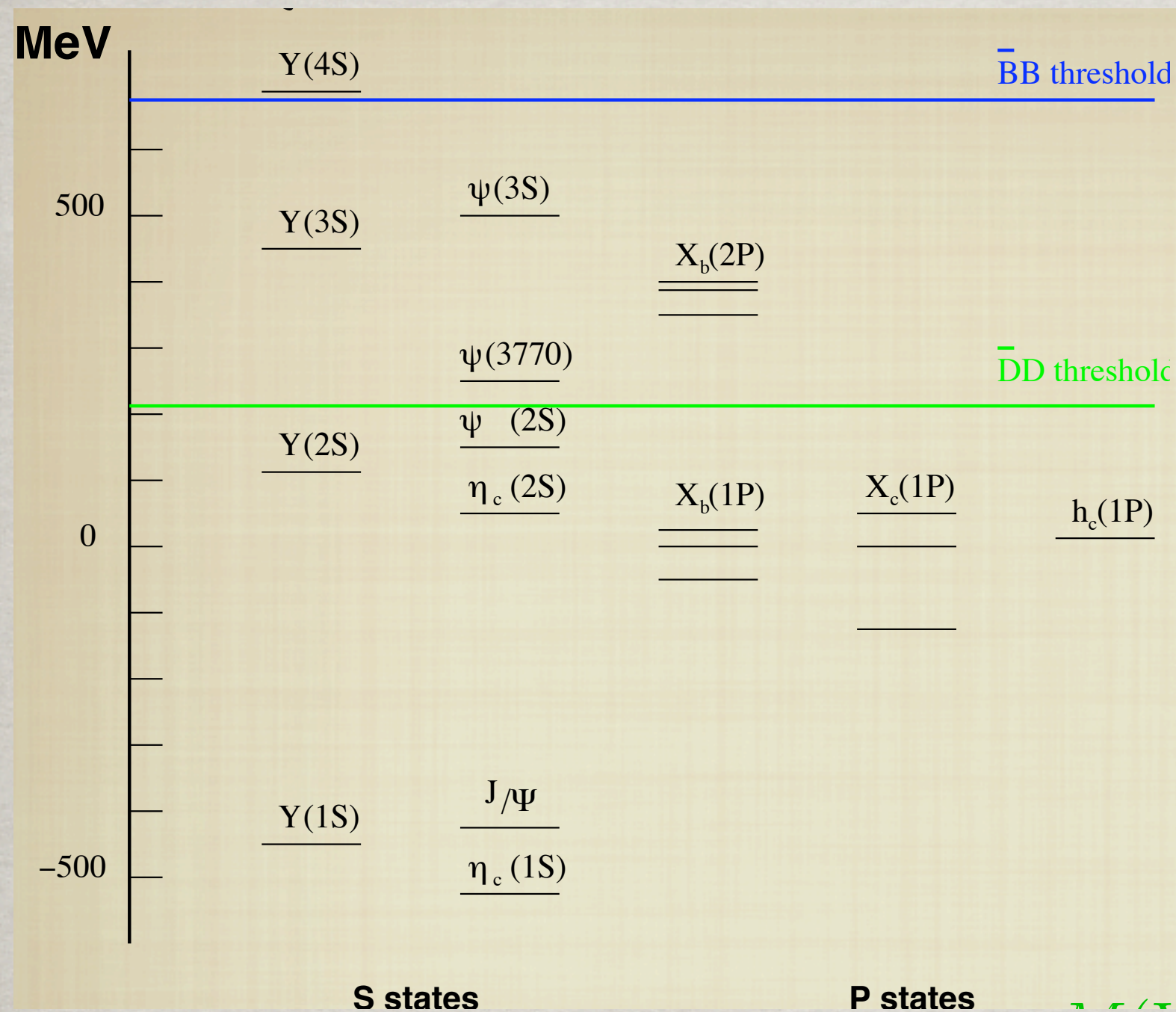
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many scales: a challenge and an opportunity

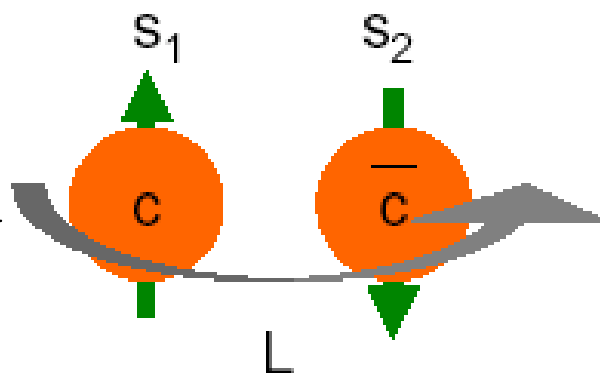


Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$$2S+1 L_J$$

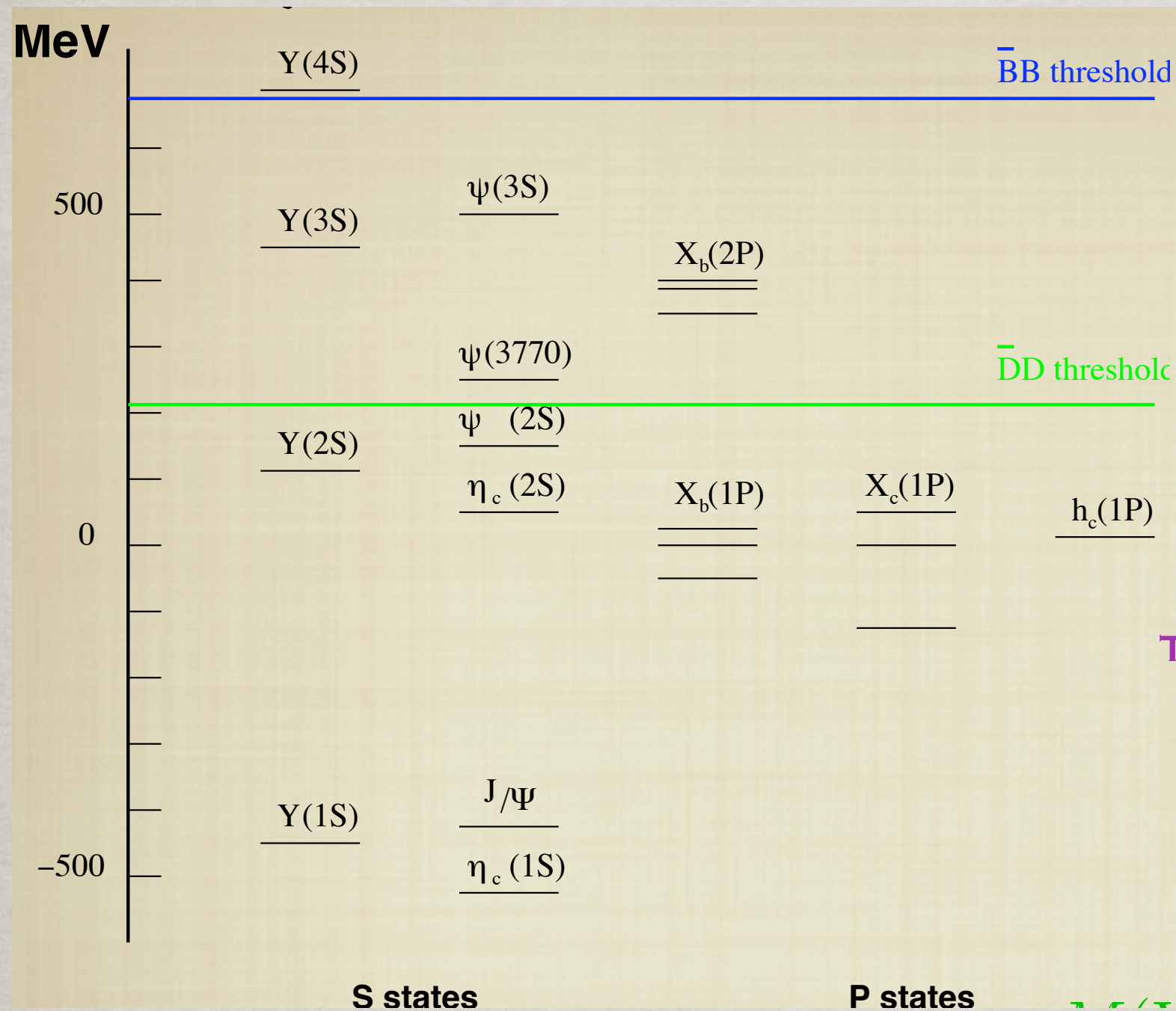


THE MASS SCALE IS PERTURBATIVE

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$$m_b \simeq 5 \text{ GeV}; m_c \simeq 1.5 \text{ GeV}$$

Quarkonium scales



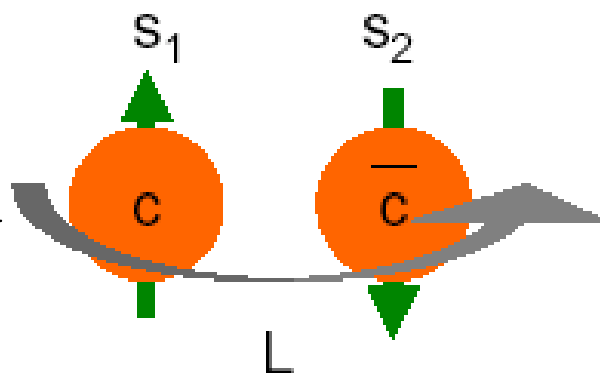
THE SYSTEM IS NONRELATIVISTIC(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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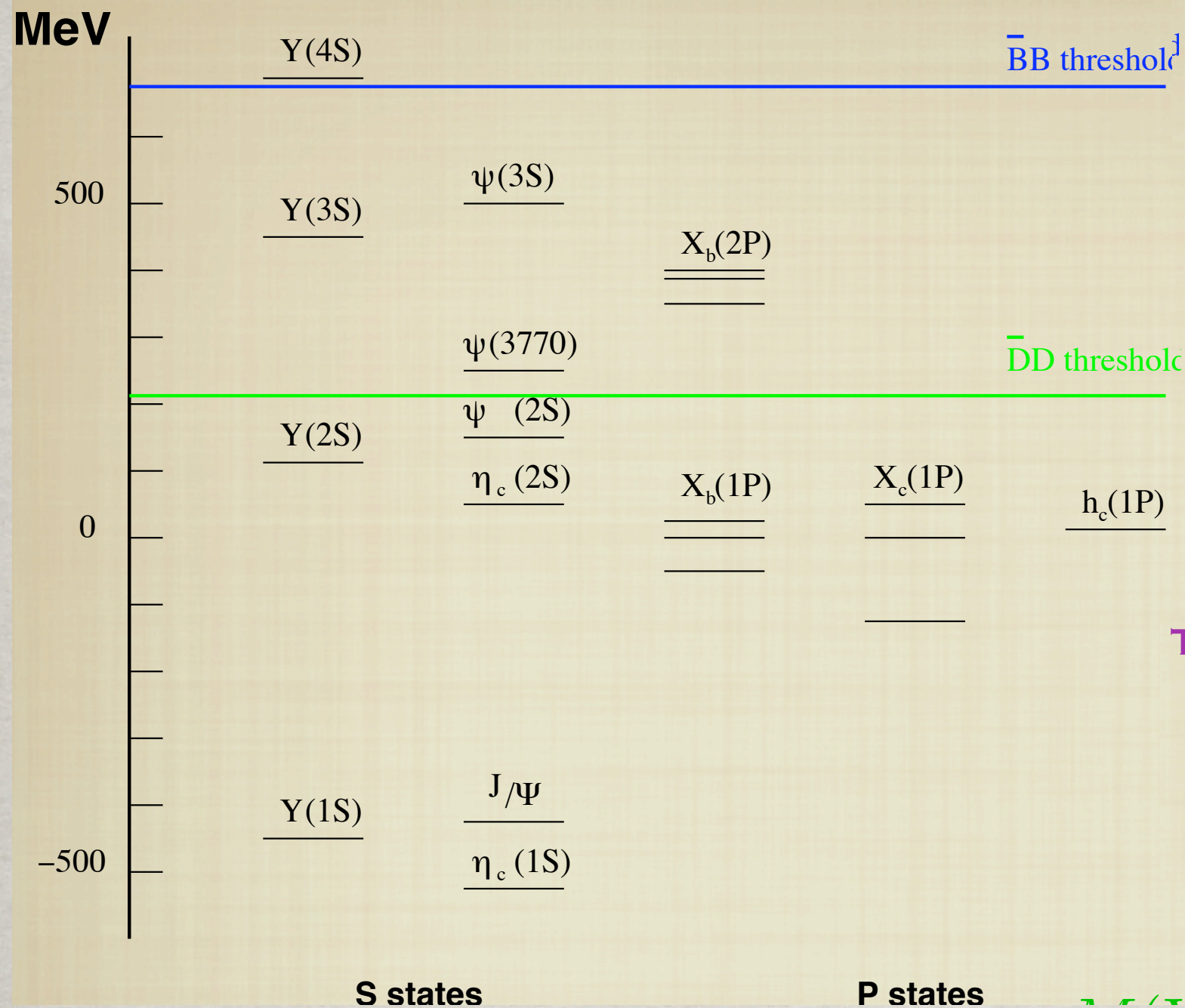


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NR BOUND STATES HAVE AT LEAST
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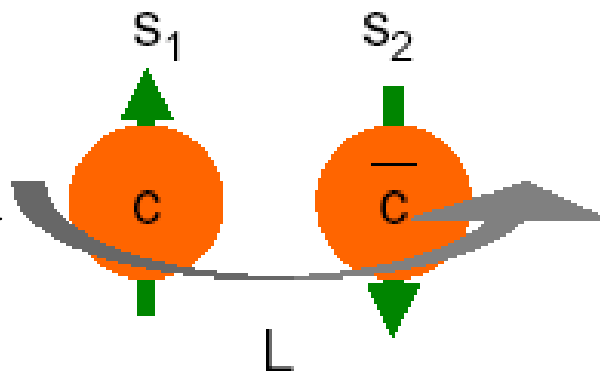
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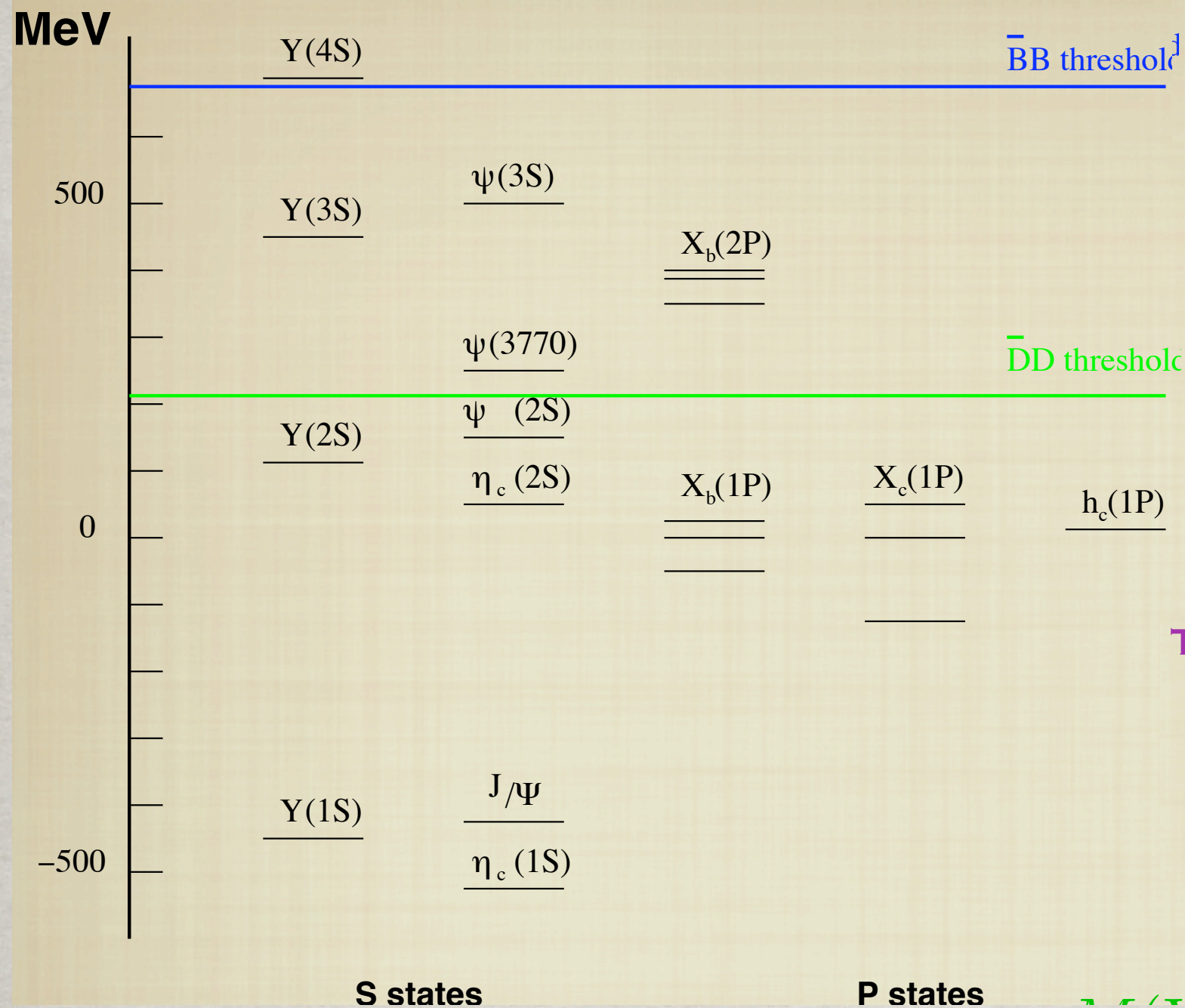


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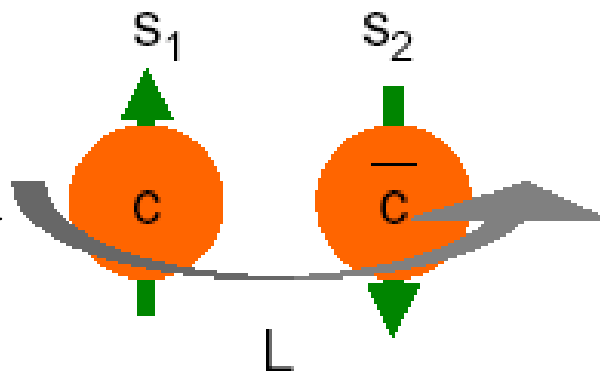
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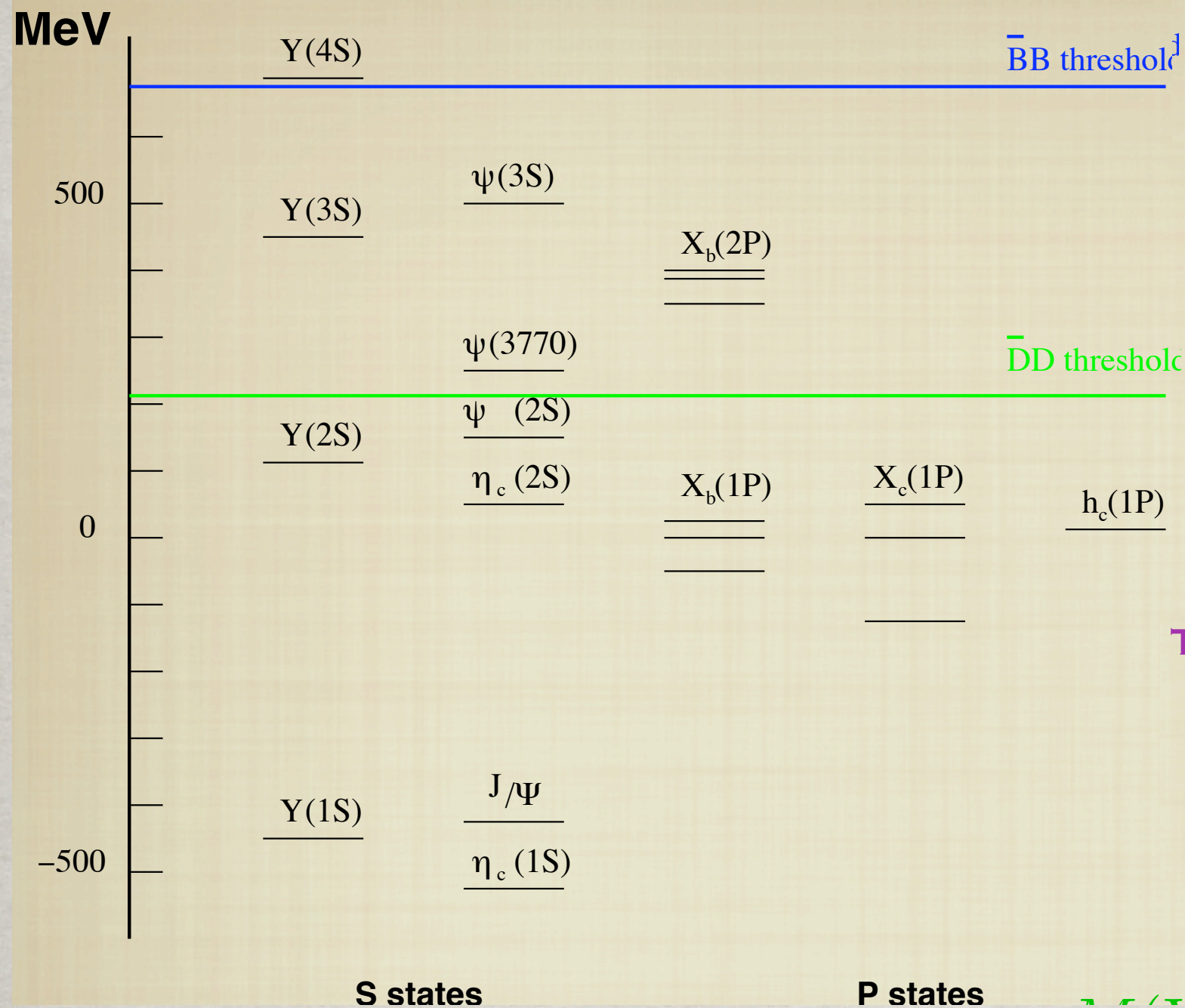


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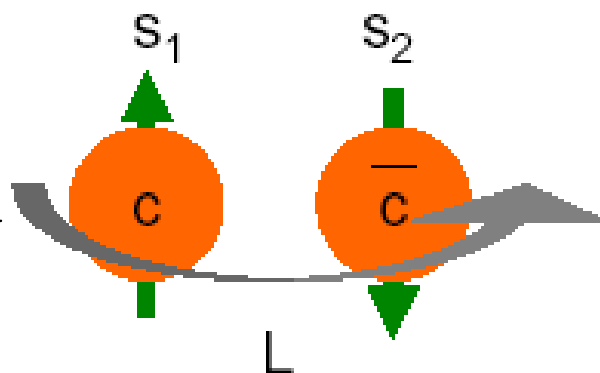
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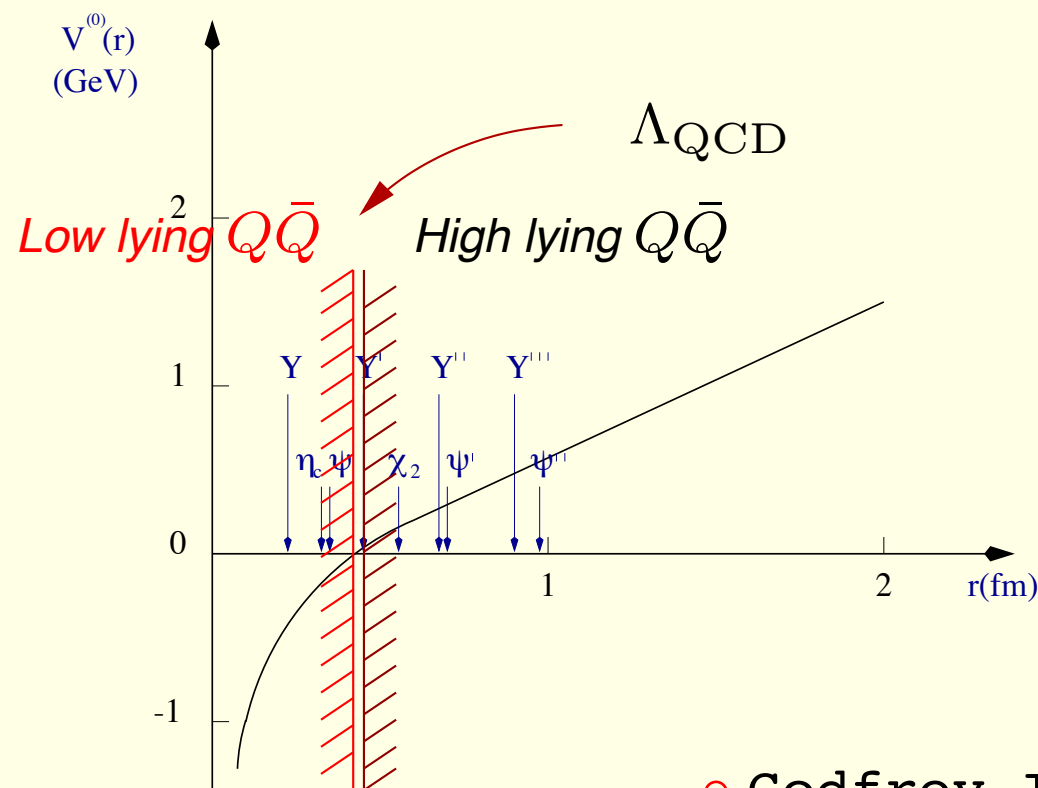
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The rich structure of separated energy scales makes quarkonium an ideal probe of confinement/deconfinement

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At zero temperature

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.

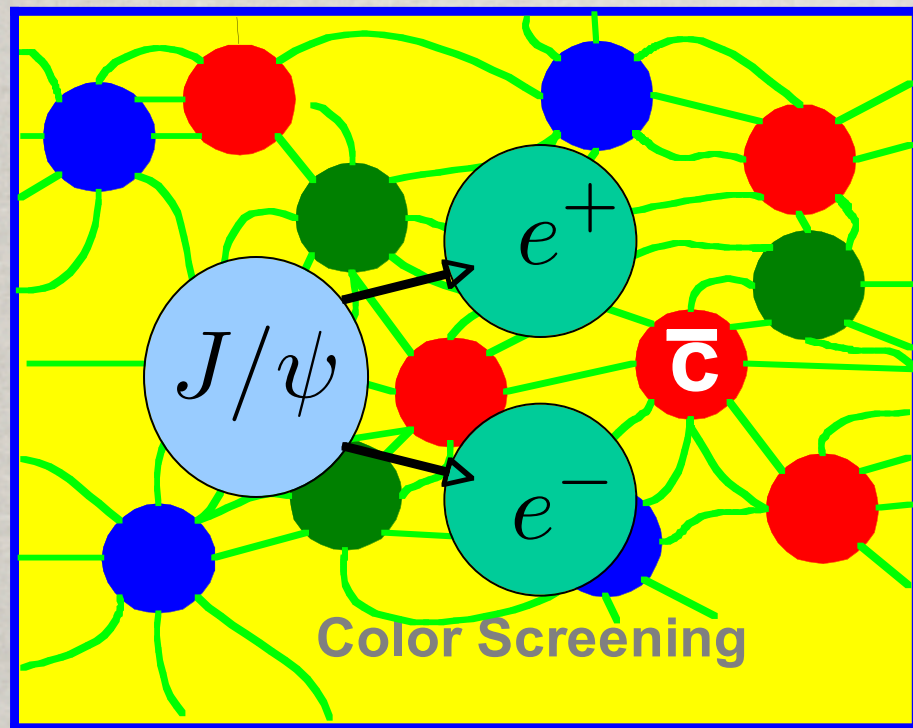


○ Godfrey Isgur PRD 32(85)189

quarkonia probe the perturbative (high energy) and non perturbative region (low energy) as well as the transition region in dependence of their radius r

Quarkonium as a confinement and deconfinement probe

At finite temperature T they are sensitive to the formation of a quark gluon plasma via color screening



Debye charge screening $m_D \sim gT$

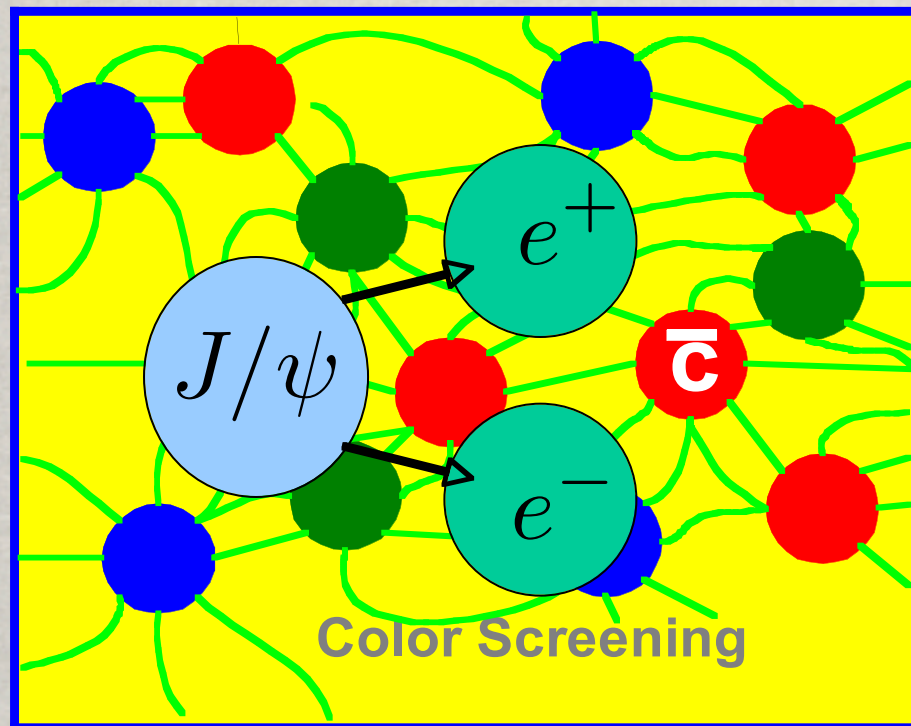
$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$r \sim \frac{1}{m_D} \longrightarrow \text{Bound state dissolves}$$

Matsui Satz 1986

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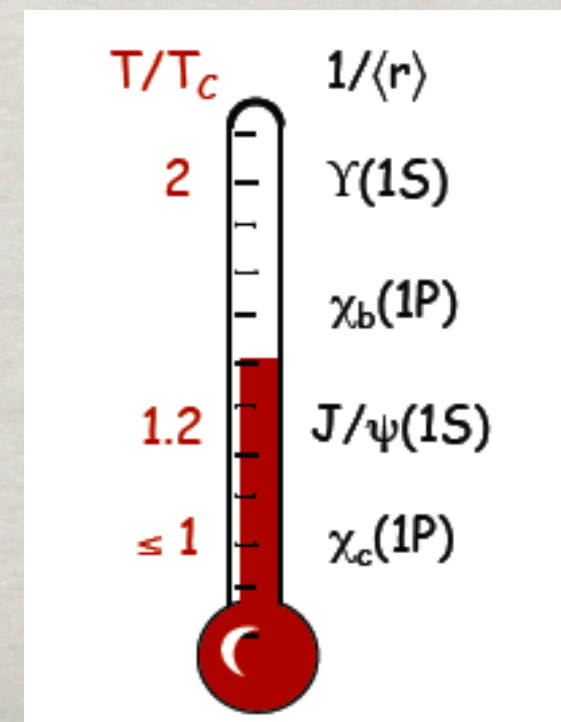
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quarkonia dissociate at different temperature in dependence of their radius: they are a Quark Gluon Plasma thermometer



Quarkonium Today is
a golden system to study strong interactions

Many experimental data and opportunities

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a golden system to study strong interactions

New theoretical tools:
Effective Field Theories (EFTs) of QCD
and progress in lattice QCD

Today: new data

B-FACTORIES: Heavy Mesons Factories

CLEO-c BESII tau charm factories

CLEO-III bottomonium factory

Fermilab CDF, D0, E835

Hera RHIC (Star, Phenix), NA60

Today: new data

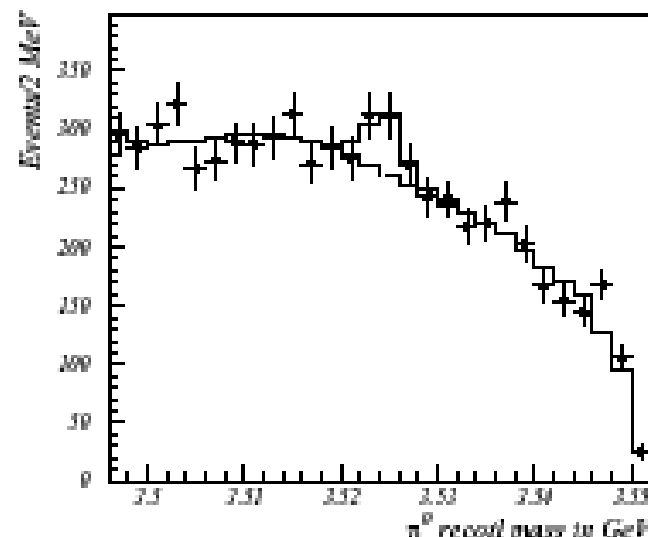
B-FACTORIES: Heavy Mesons Factories

CLEO, BESIII, charm factories

CLEO

Fermilab

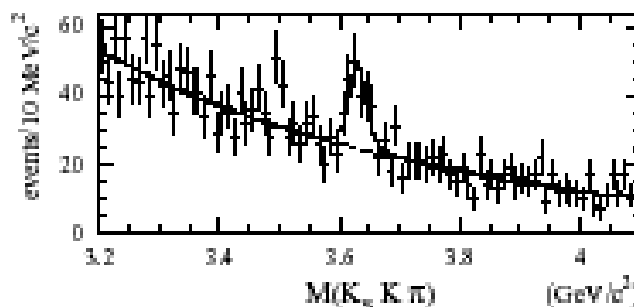
Herndon



$h_c(3523)$

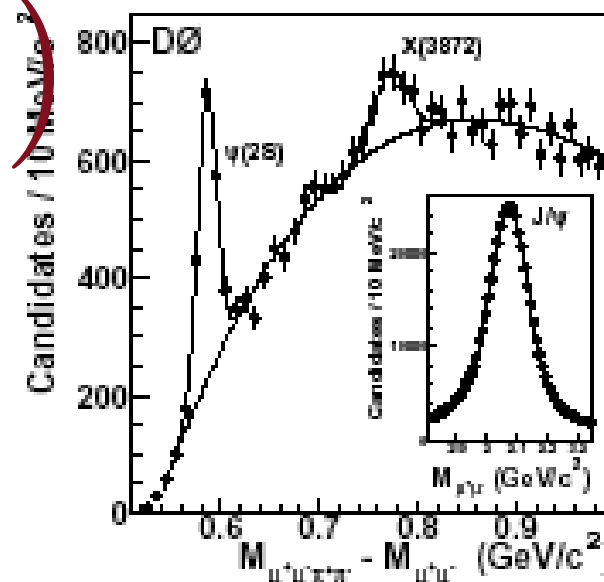
CLEO 05
E835 05

$Z_c(3900)$
BESIII 2013



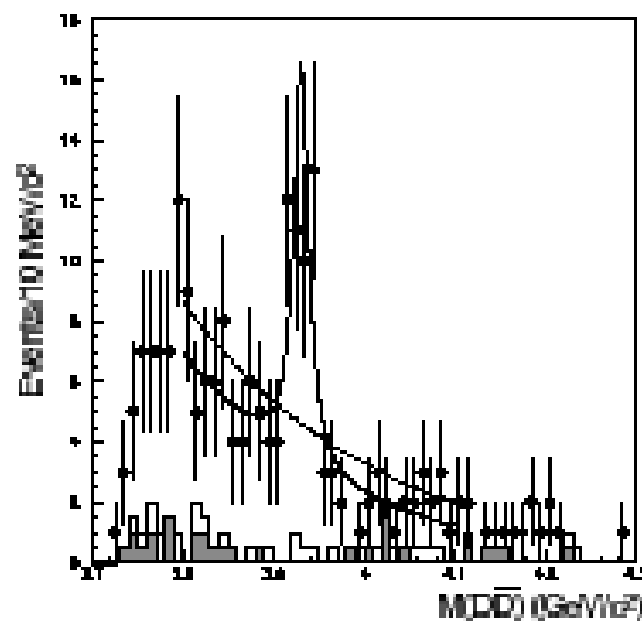
$\eta_c(2S)(3630)$

BaBar 04
CLEO 04
Belle 02



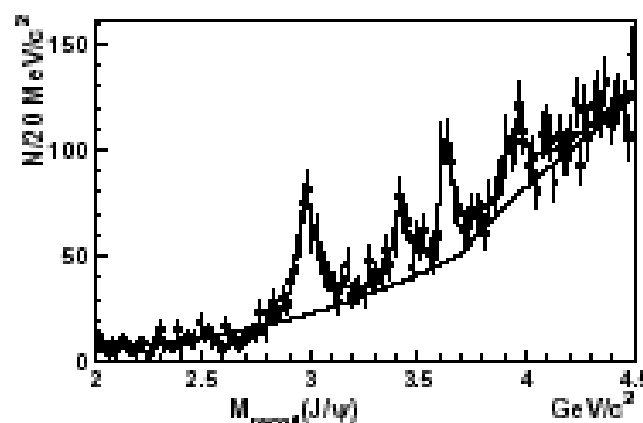
X(3872)

CDF D0/QWG 04
Belle 02
BaBar 05



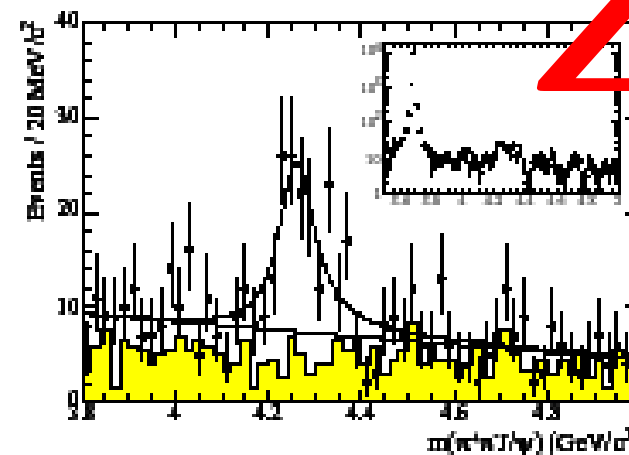
Z(3930)

Belle 05



X(3940)

Belle 05



Y(4260)

BaBar 05

η_b

BABAR
08

Z^+

BELLE
07-08

Today: new data

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Discovery of New States, New
Production Mechanisms, Exotics, New
decays and transitions, Precision and
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BESIII

CMS ATLAS LHC-b

ALICE

and in the future PANDA, Belle2

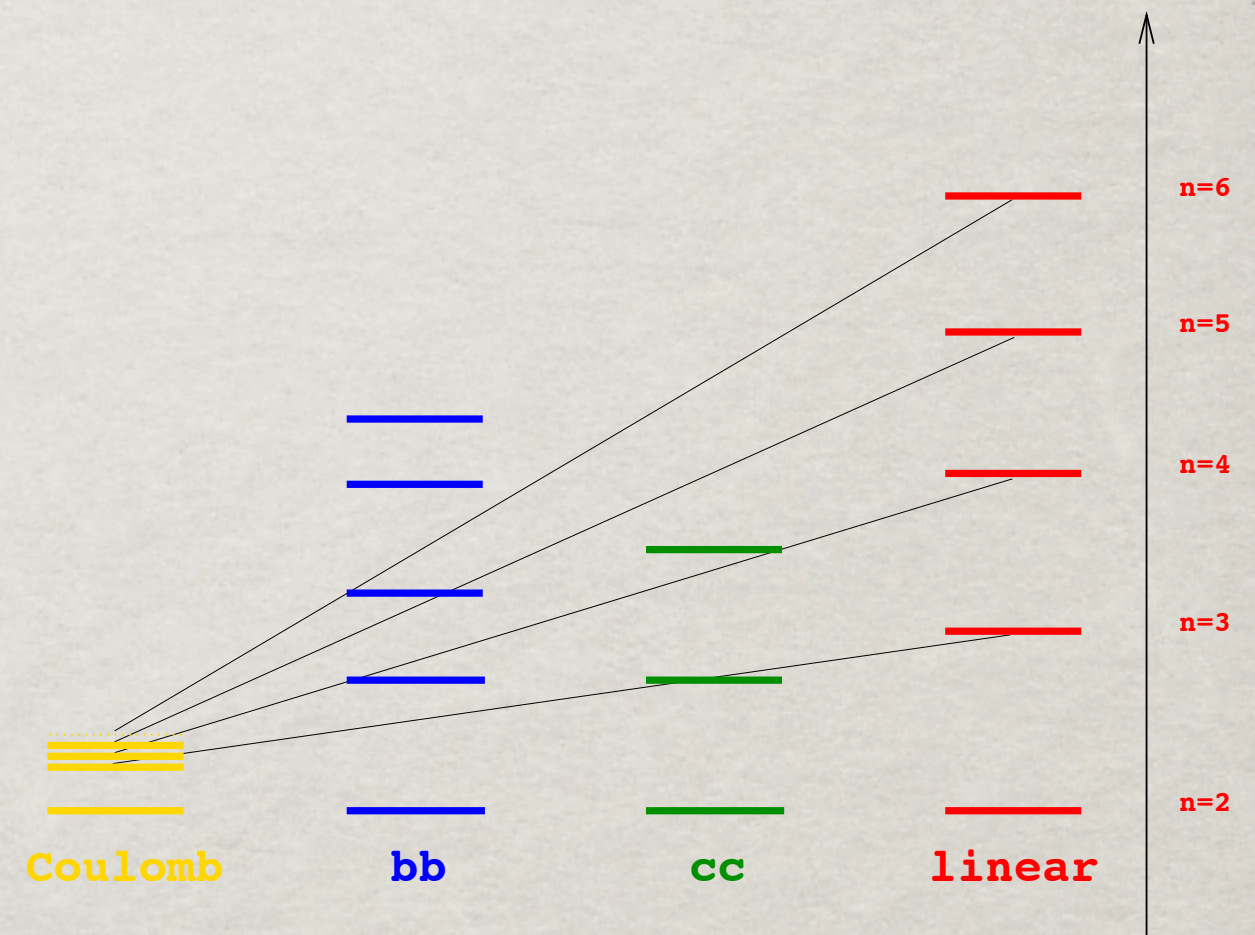
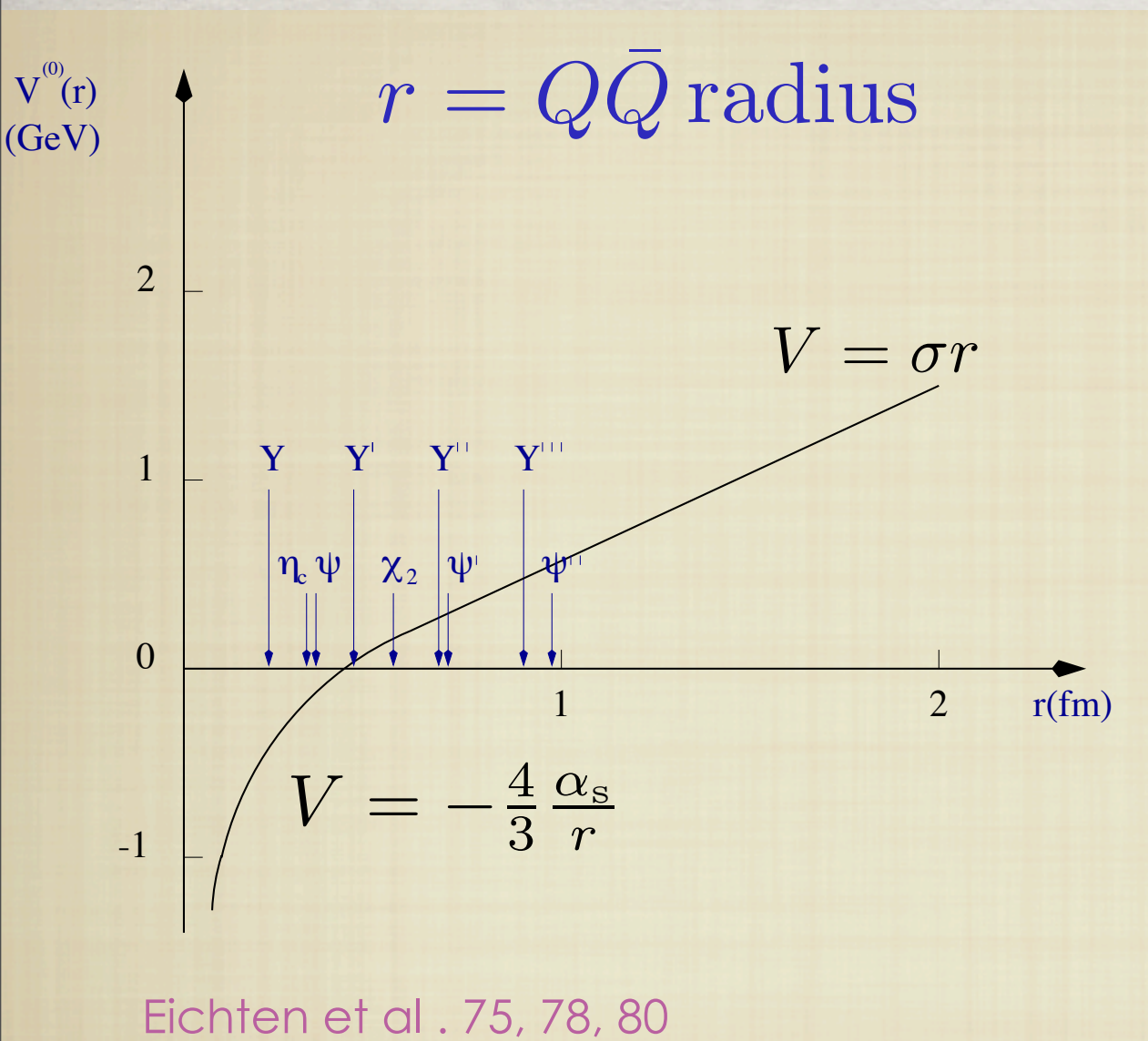
QCD theory of Quarkonium: a very hard problem

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Initial phenomenological/model descriptions of the 70s,80s

QCD theory of Quarkonium: a very hard problem

Initial phenomenological/model descriptions of the 70s,80s



$b\bar{b}$ and $c\bar{c}$ energy levels in comparison to
Coulomb and linear potential energy levels

Variety of potential models used

Confinement and asymptotic freedom--> QCD

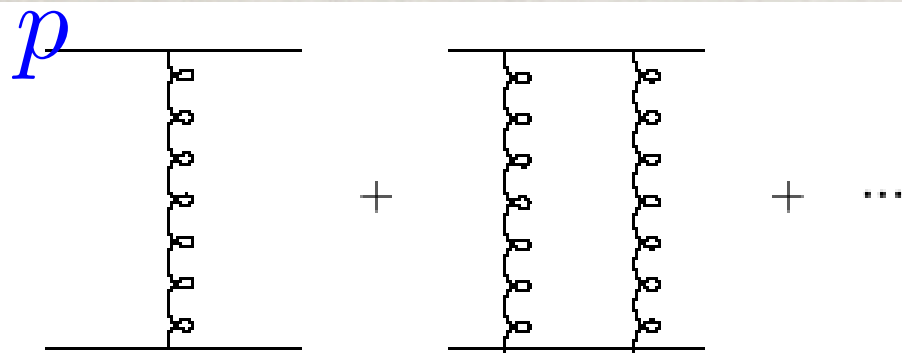
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Close to the bound state $\alpha_s \sim v$

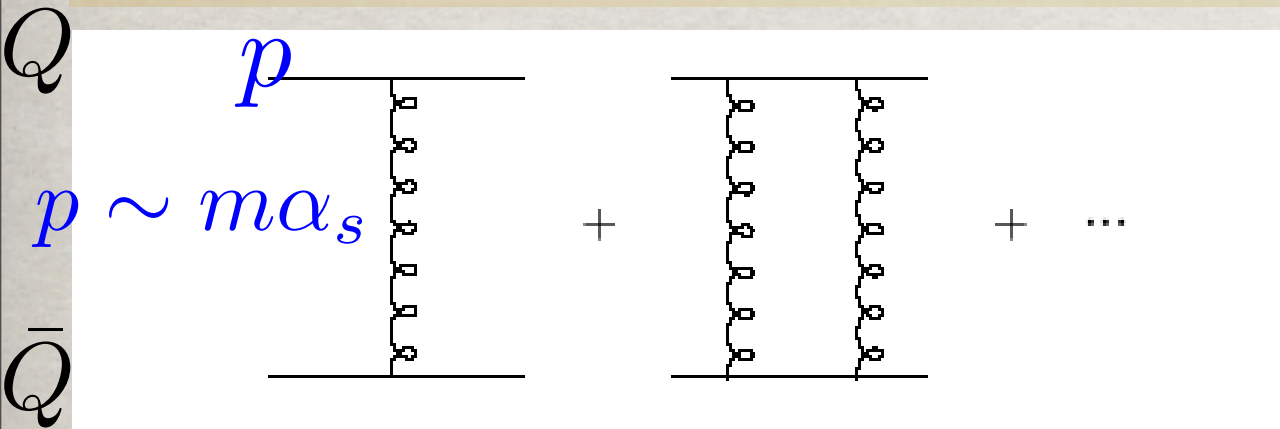
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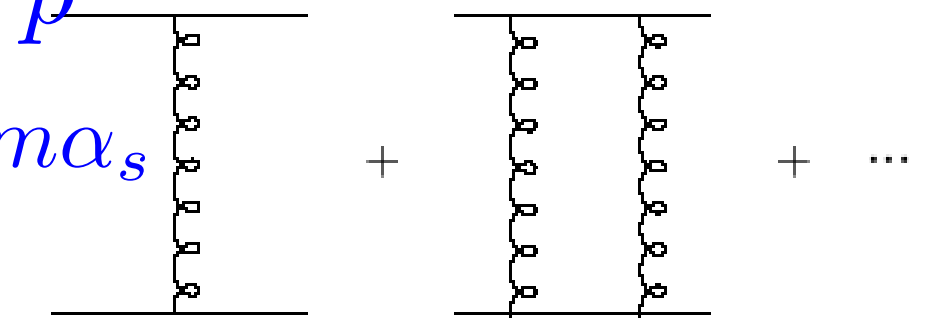
QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$

Q

$p \sim m\alpha_s$

Q



The diagram shows a series of Feynman diagrams representing the self-energy of a quarkonium state. The first diagram shows a quark line with momentum p and a gluon loop. The second diagram shows a quark line with a gluon loop and a gluon exchange between the quark and antiquark lines. The third diagram shows a quark line with a gluon loop and a gluon exchange between the quark and antiquark lines. The diagrams are summed together, indicated by plus signs and an ellipsis.

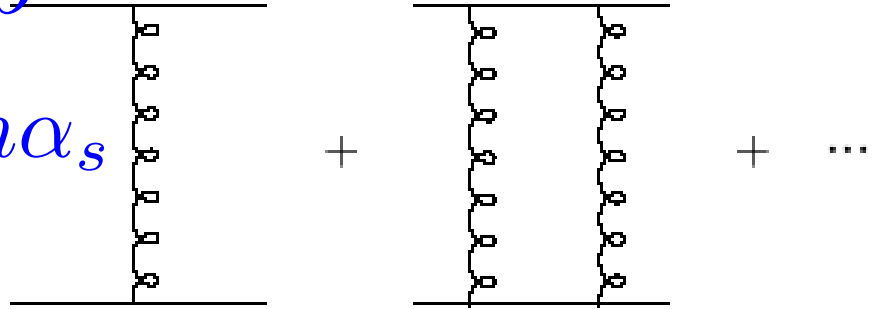
$$\frac{g^2}{p^2} \left(1 + \frac{m\alpha_s}{p} \right)$$

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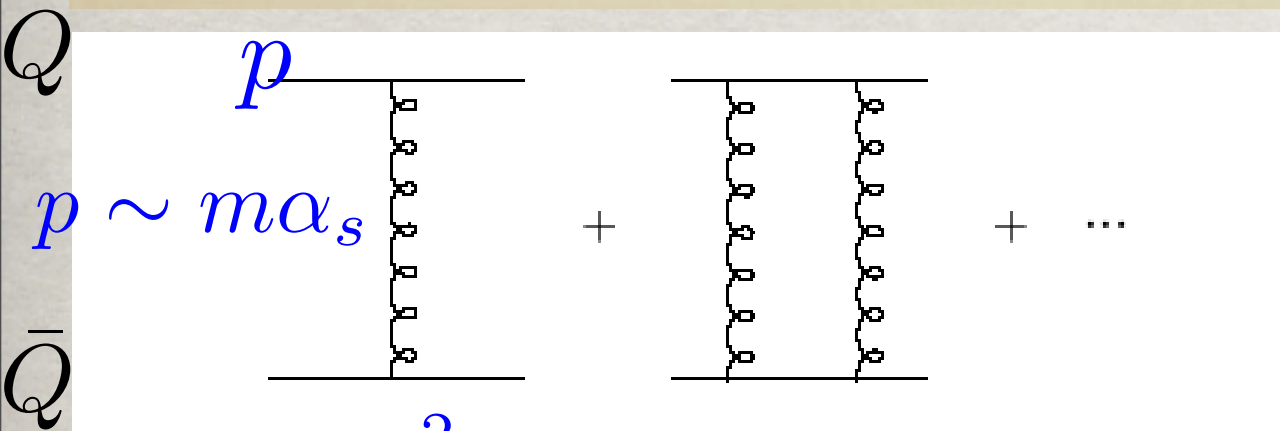
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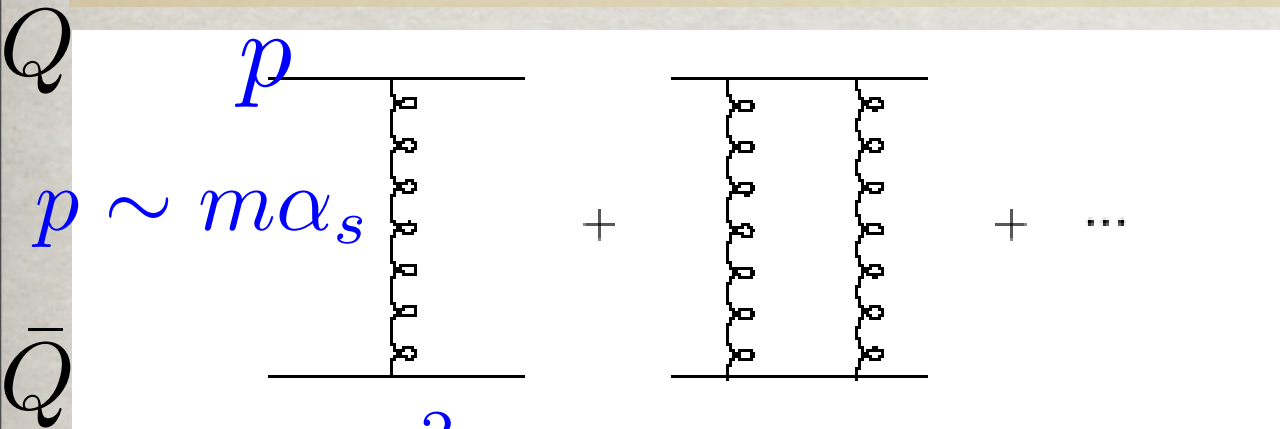
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- From $\left(\frac{p^2}{m} + V \right) \phi = E \phi \rightarrow p \sim m\alpha_s$ and $E = \frac{p^2}{m} + V \sim m\alpha_s^2$.

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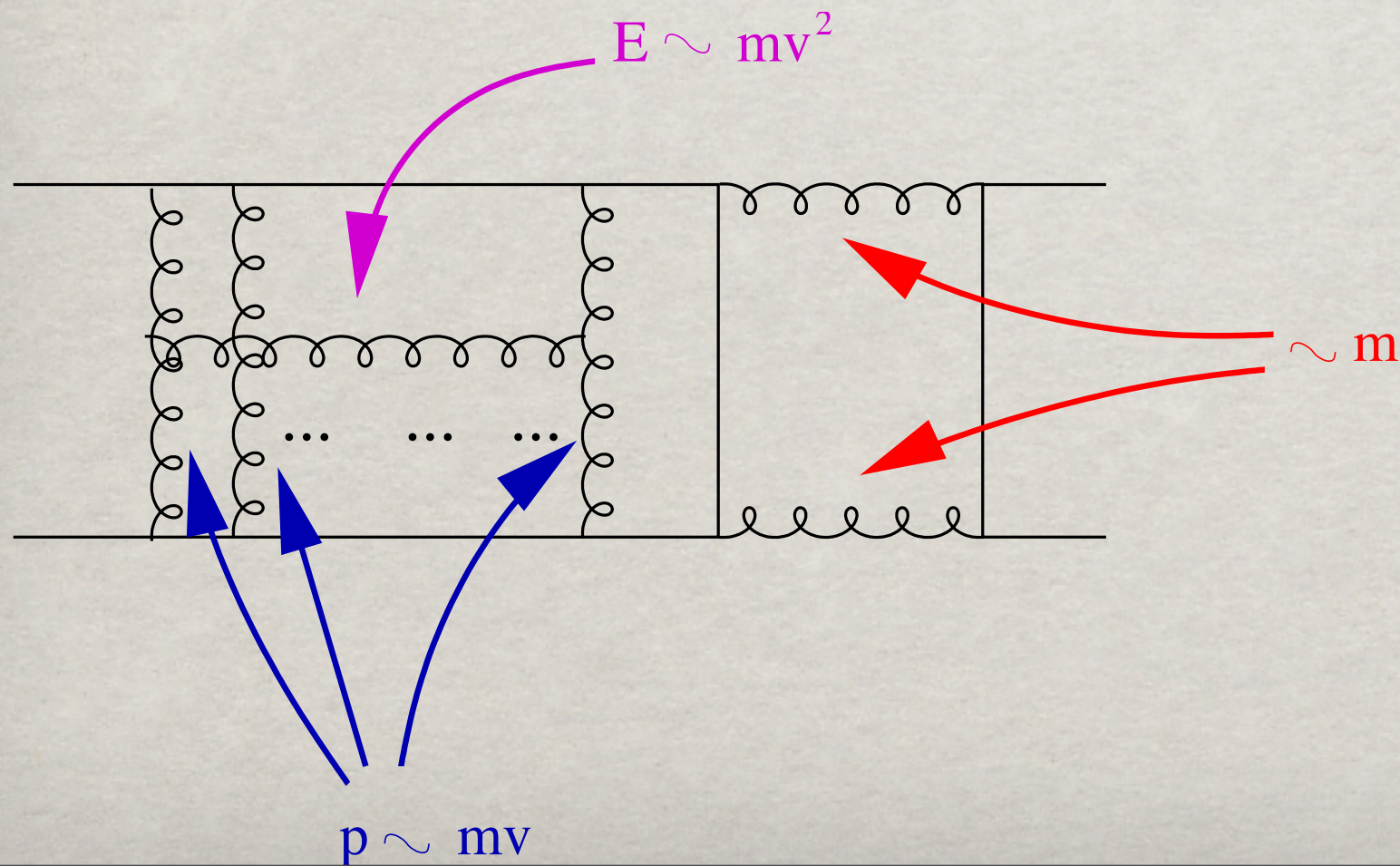
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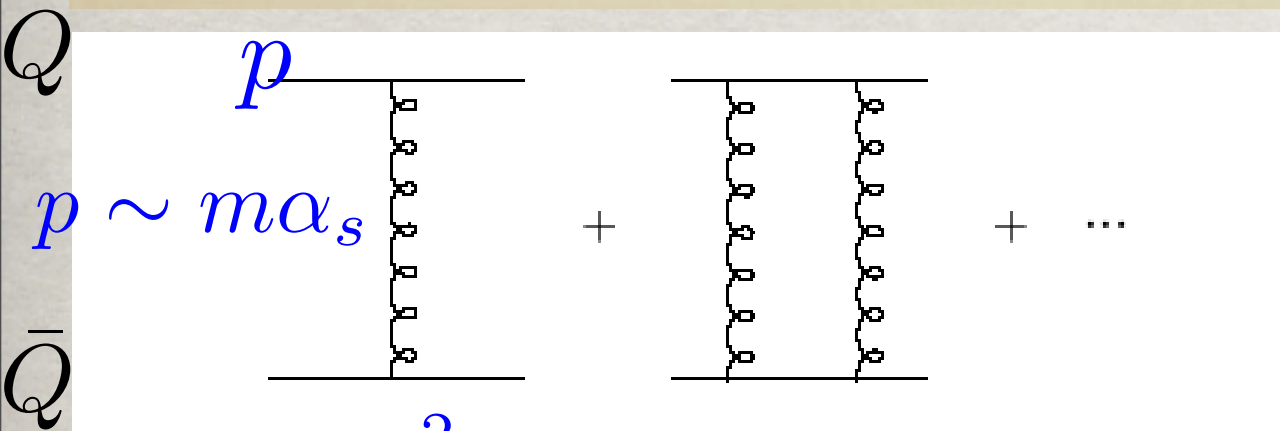
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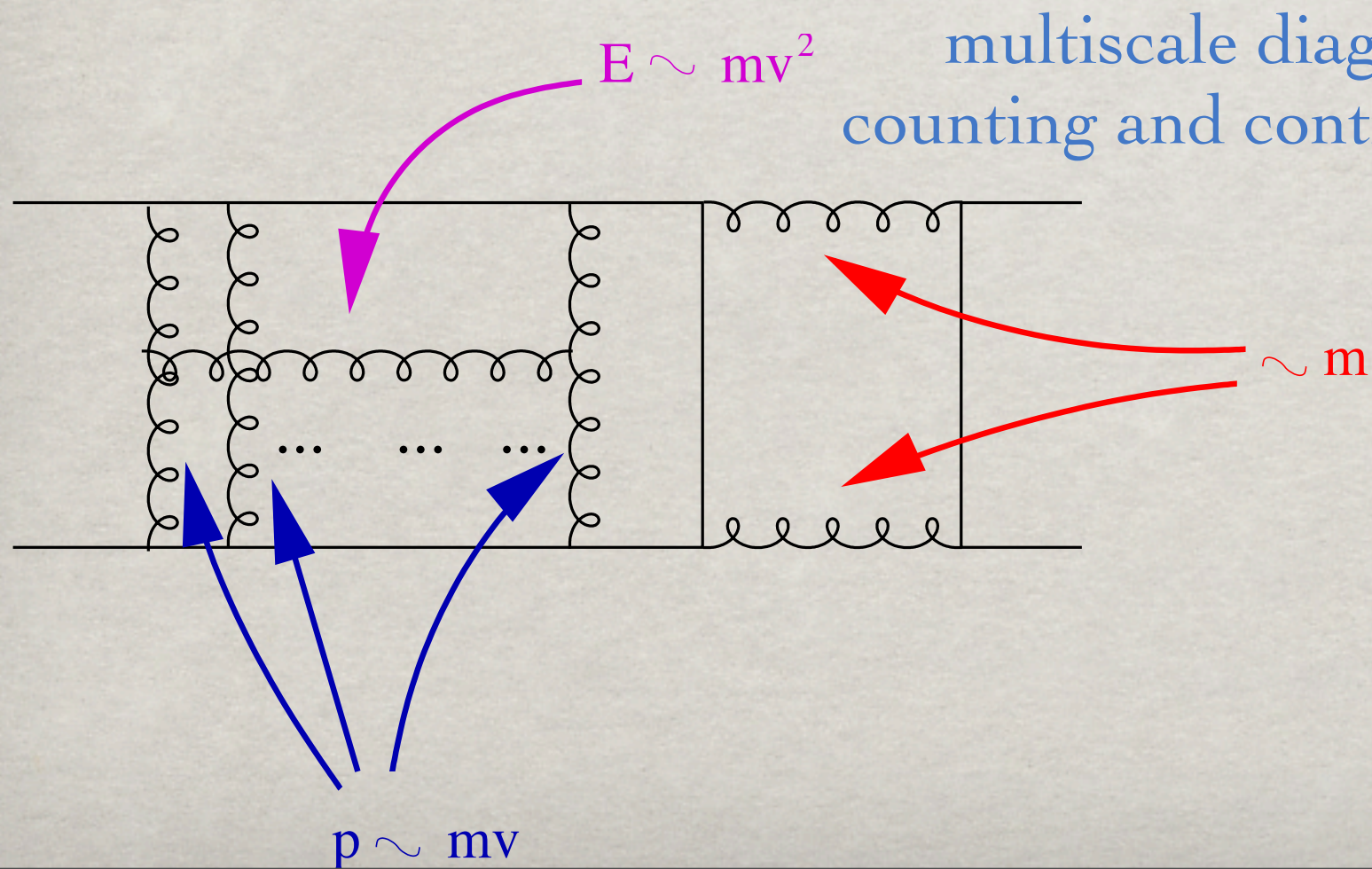
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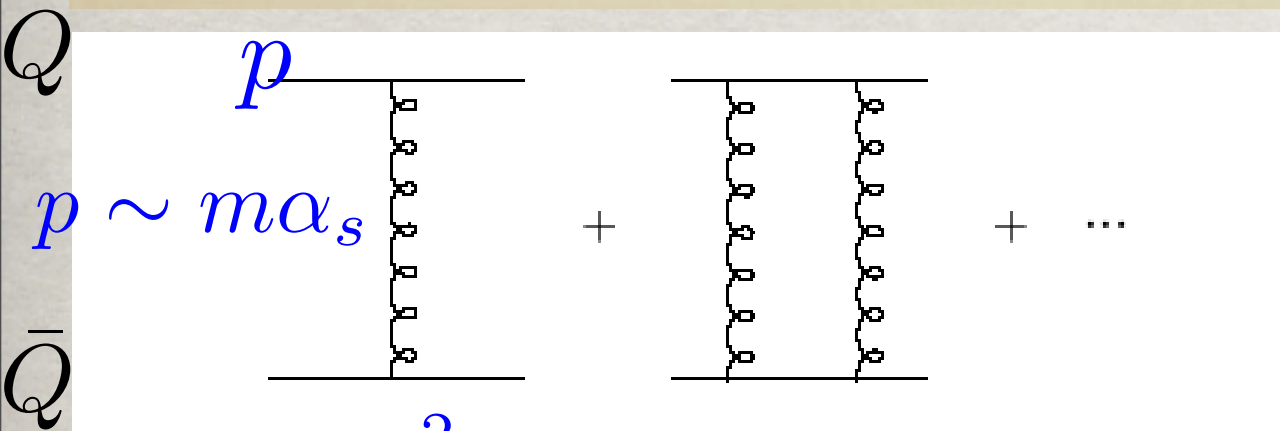
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multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

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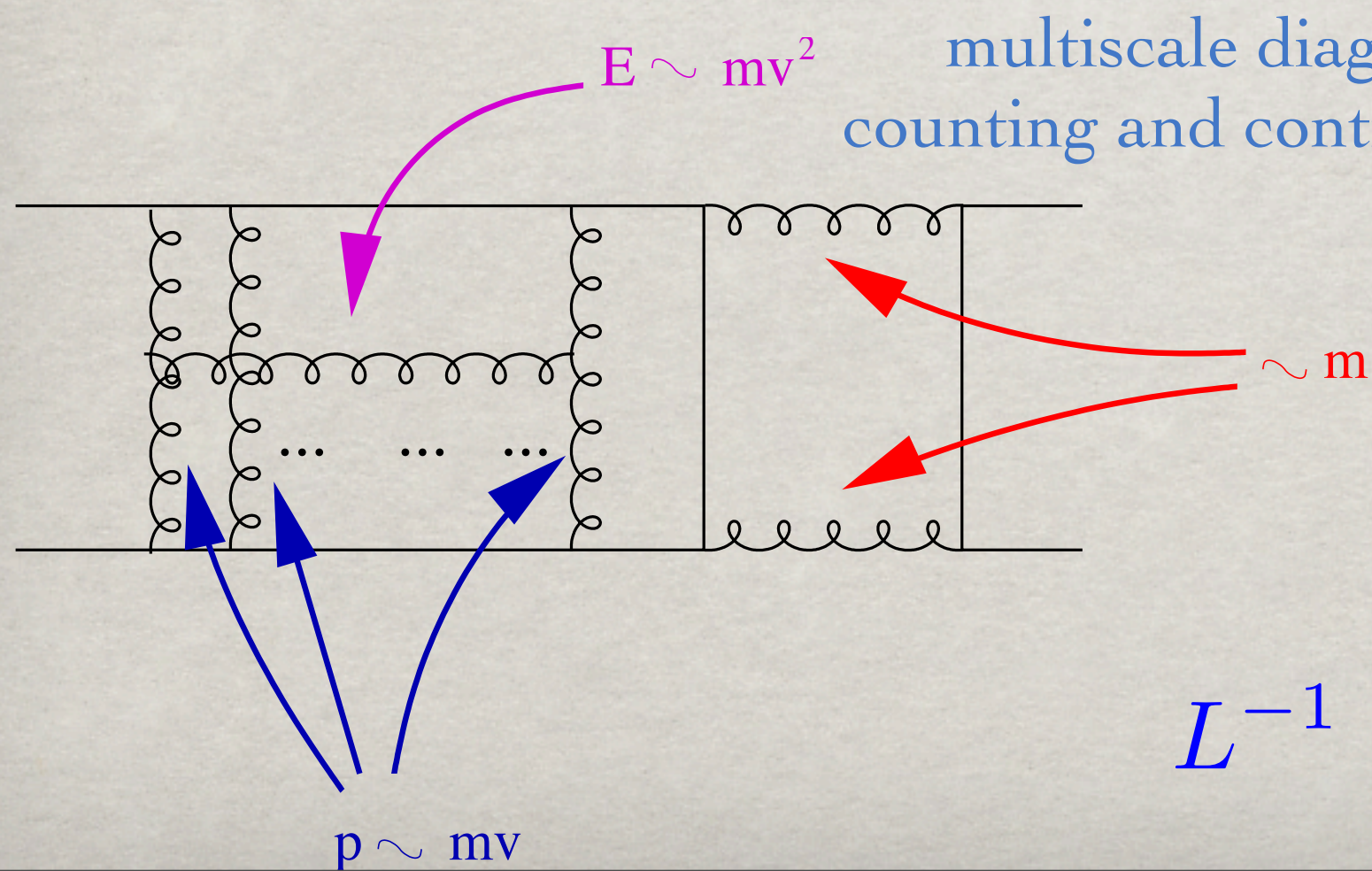
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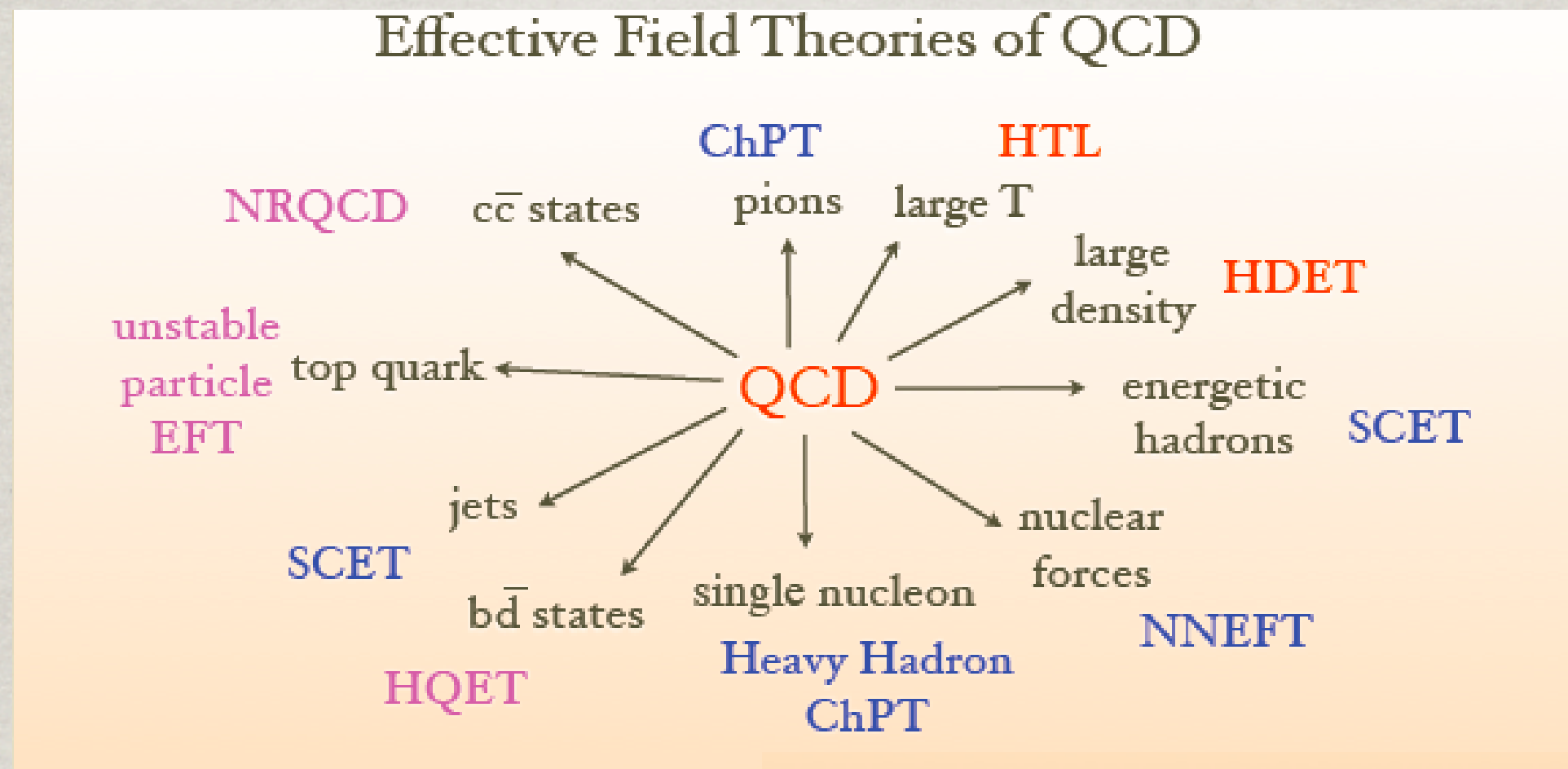


multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

QCD Effective Field Theories to address the research frontier of hadronic physics

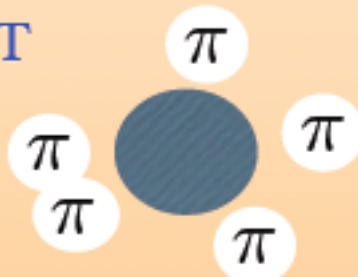


- Heavy quark effective theory (HQET): $\frac{\lambda}{\Lambda} = \frac{\Lambda_{\text{QCD}}}{m}$

Soft-Collinear Effective Theory (SCET)



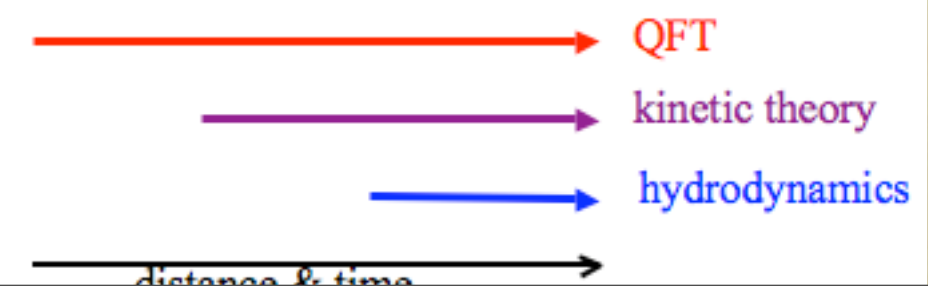
ChPT



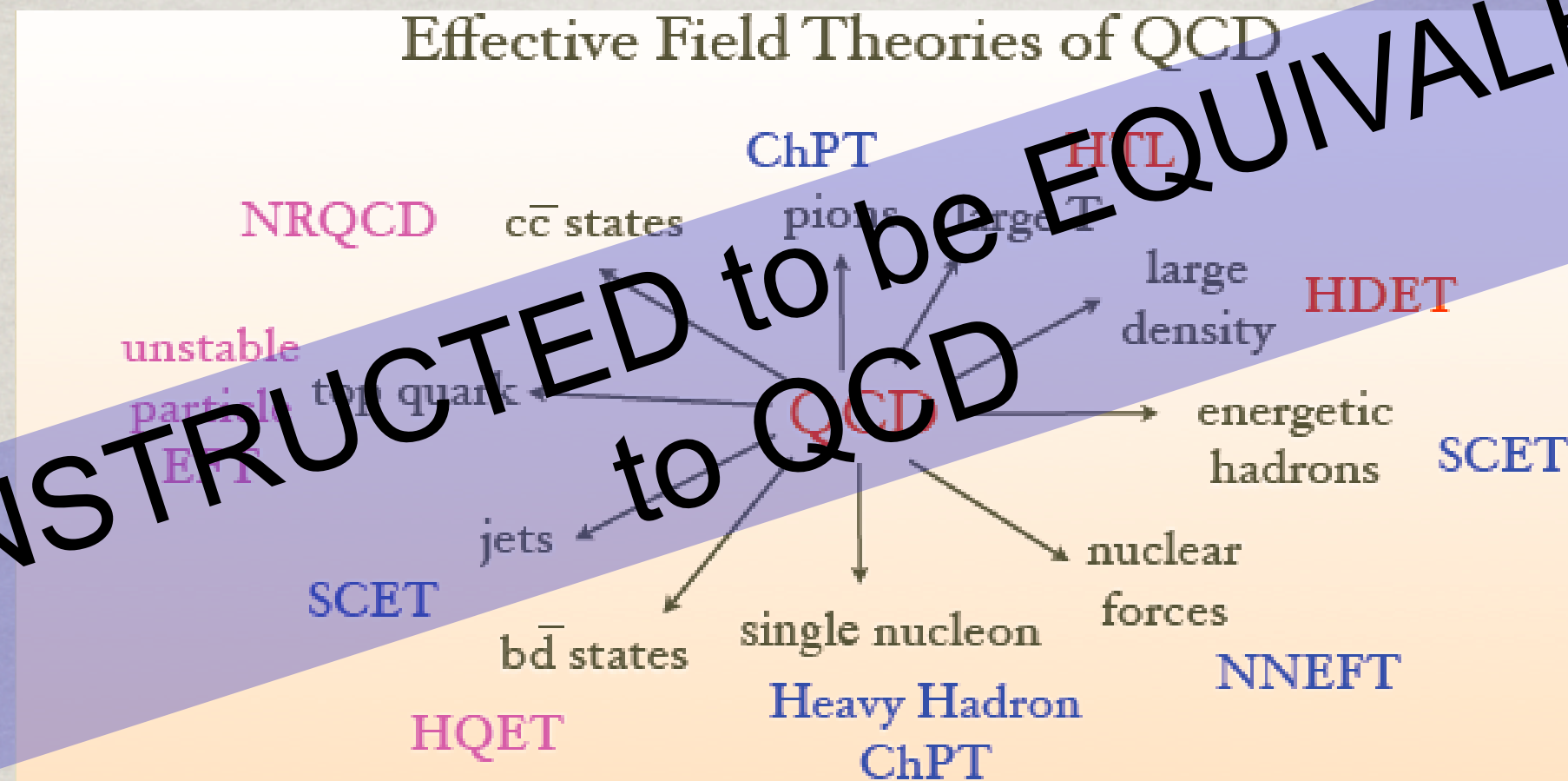
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Lattice QCD \equiv Effective Field Theory ($\Lambda = \pi/a$).



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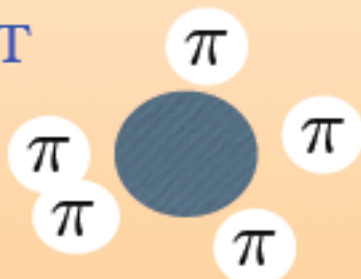


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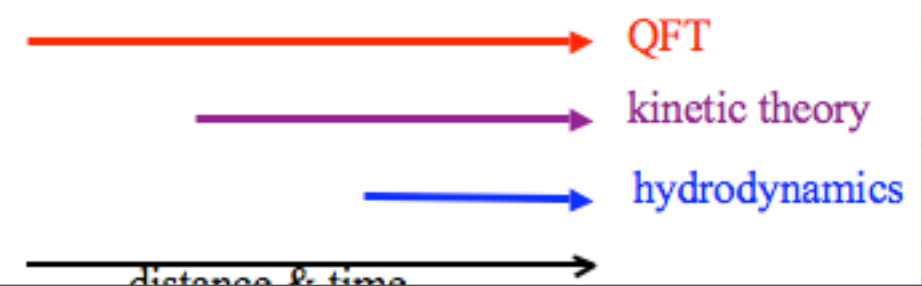
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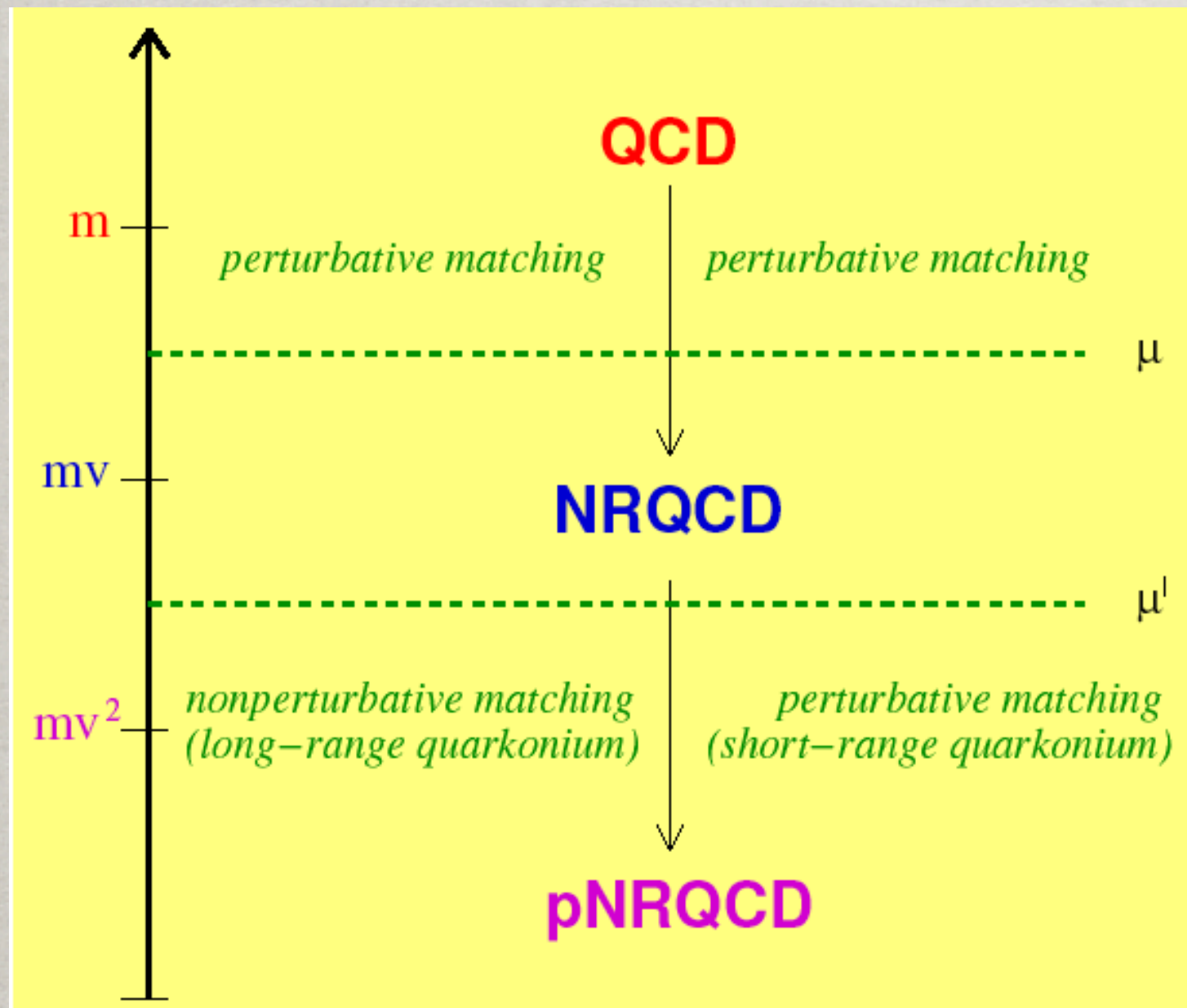
Quarkonium with Non Relativistic EFT

Color degrees of freedom
 $3 \times 3 = 1 + 8$
singlet and octet $Q\bar{Q}$

Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)



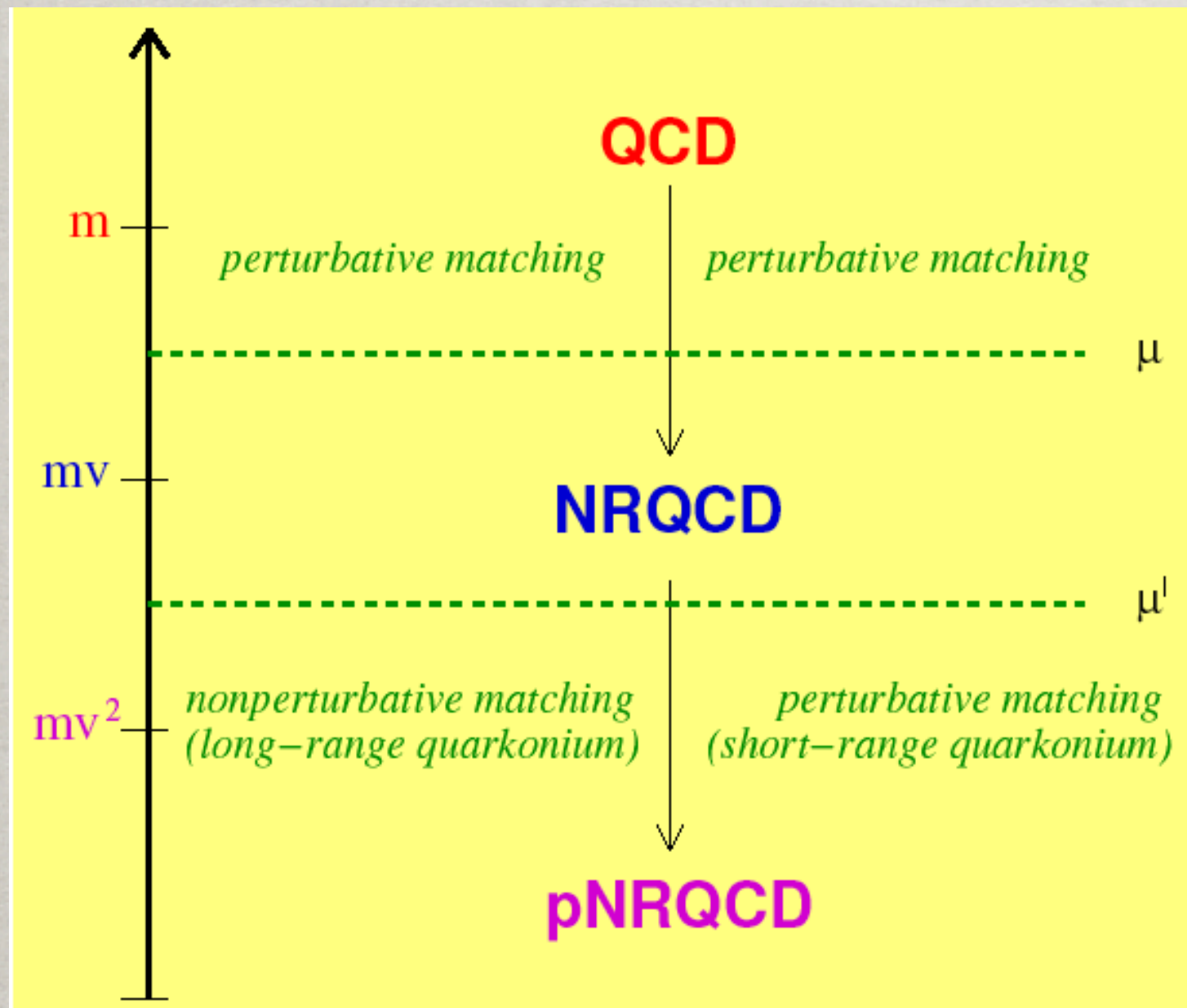
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$$\mathcal{L}_{\text{EFT}} = \sum_n c_n(E_\Lambda/\mu) \frac{O_n(\mu, \lambda)}{E_\Lambda}$$

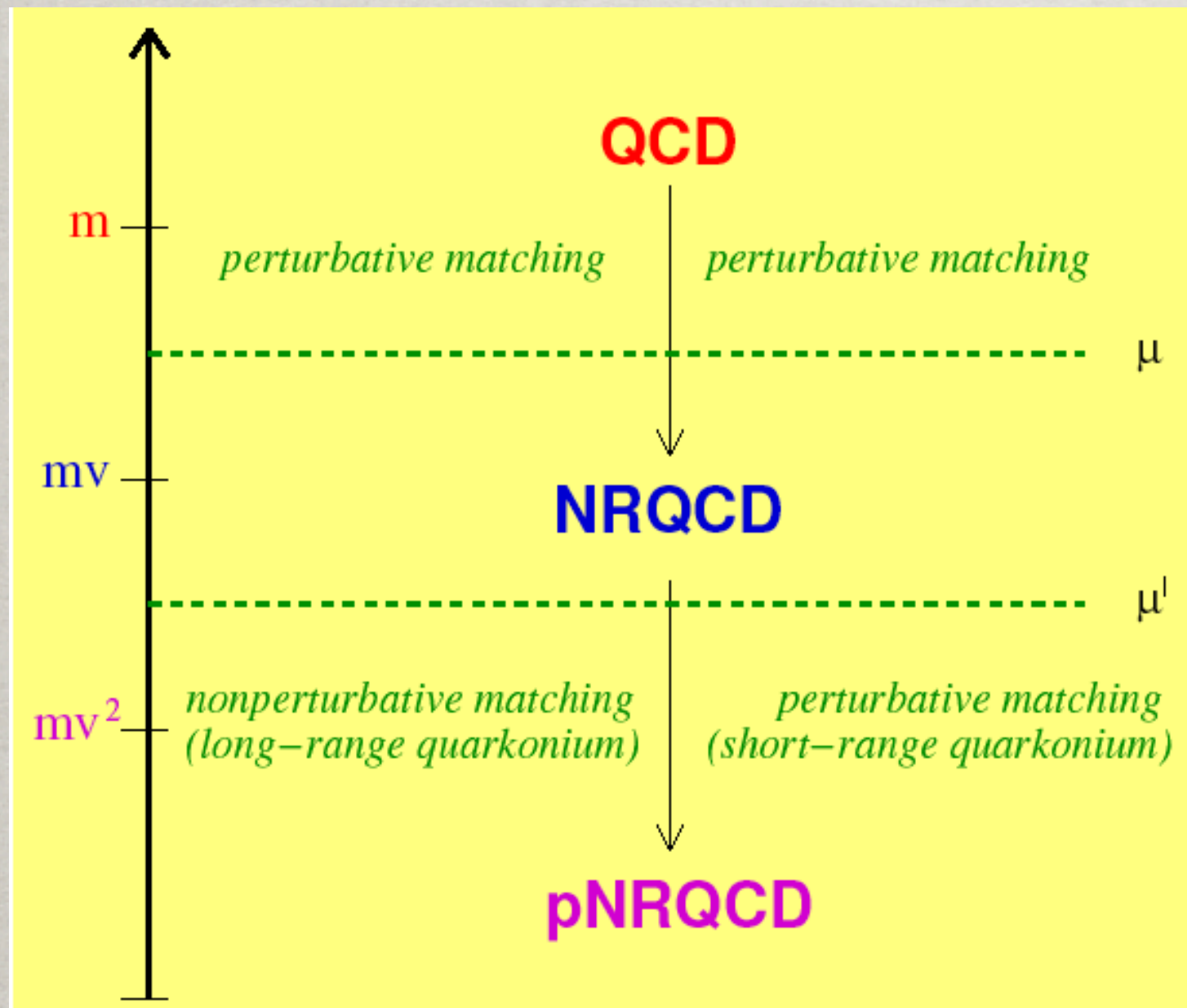
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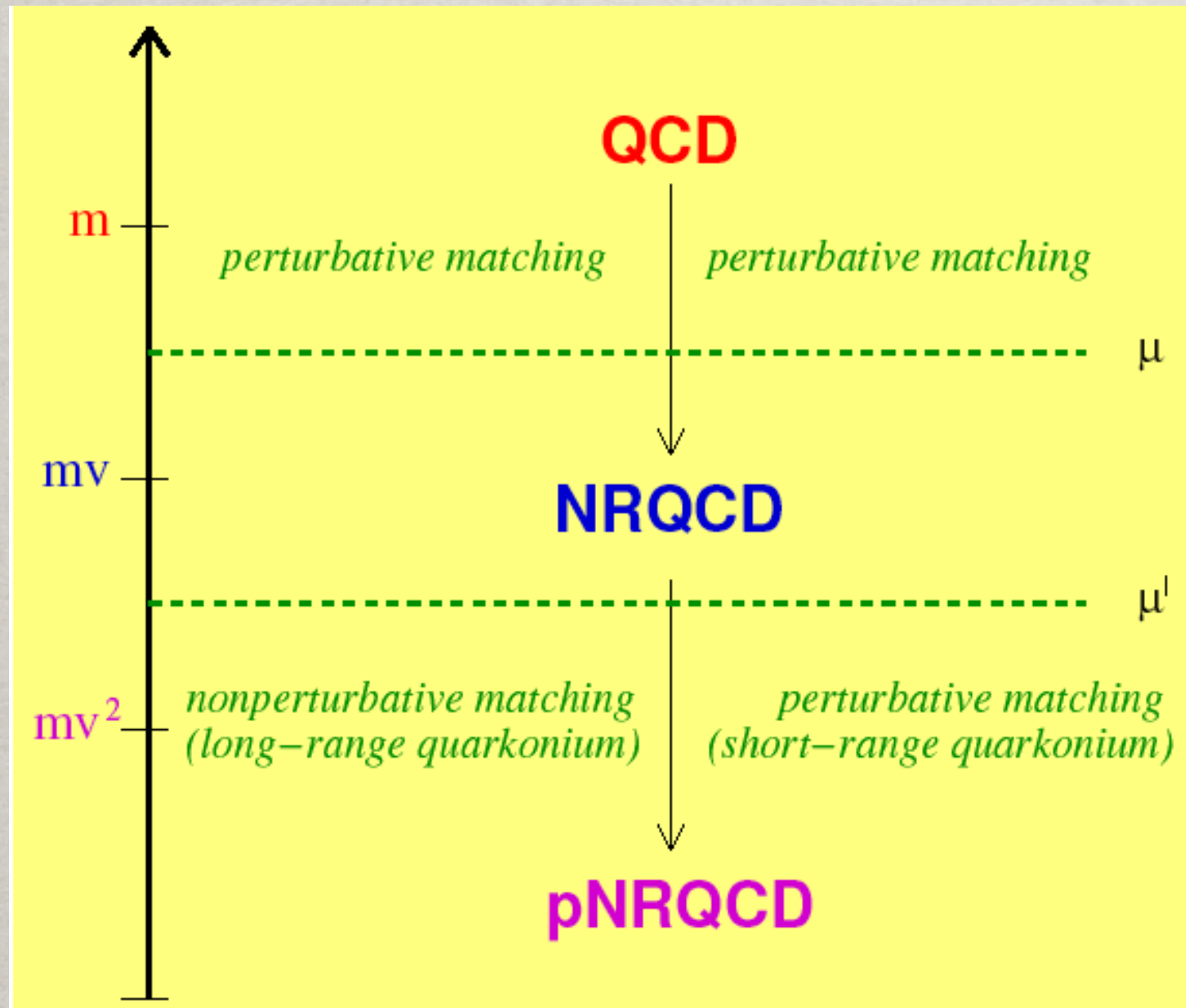
$$\mathcal{L}_{\text{EFT}} = \sum_n c_n(E_\Lambda/\mu) \frac{O_n(\mu, \lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

Quarkonium with Non Relativistic EFT

Color degrees of freedom
 $3 \times 3 = 1 + 8$
 singlet and octet $Q\bar{Q}$

Hard



$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft
 (relative momentum)

Ultrasoft
 (binding energy)

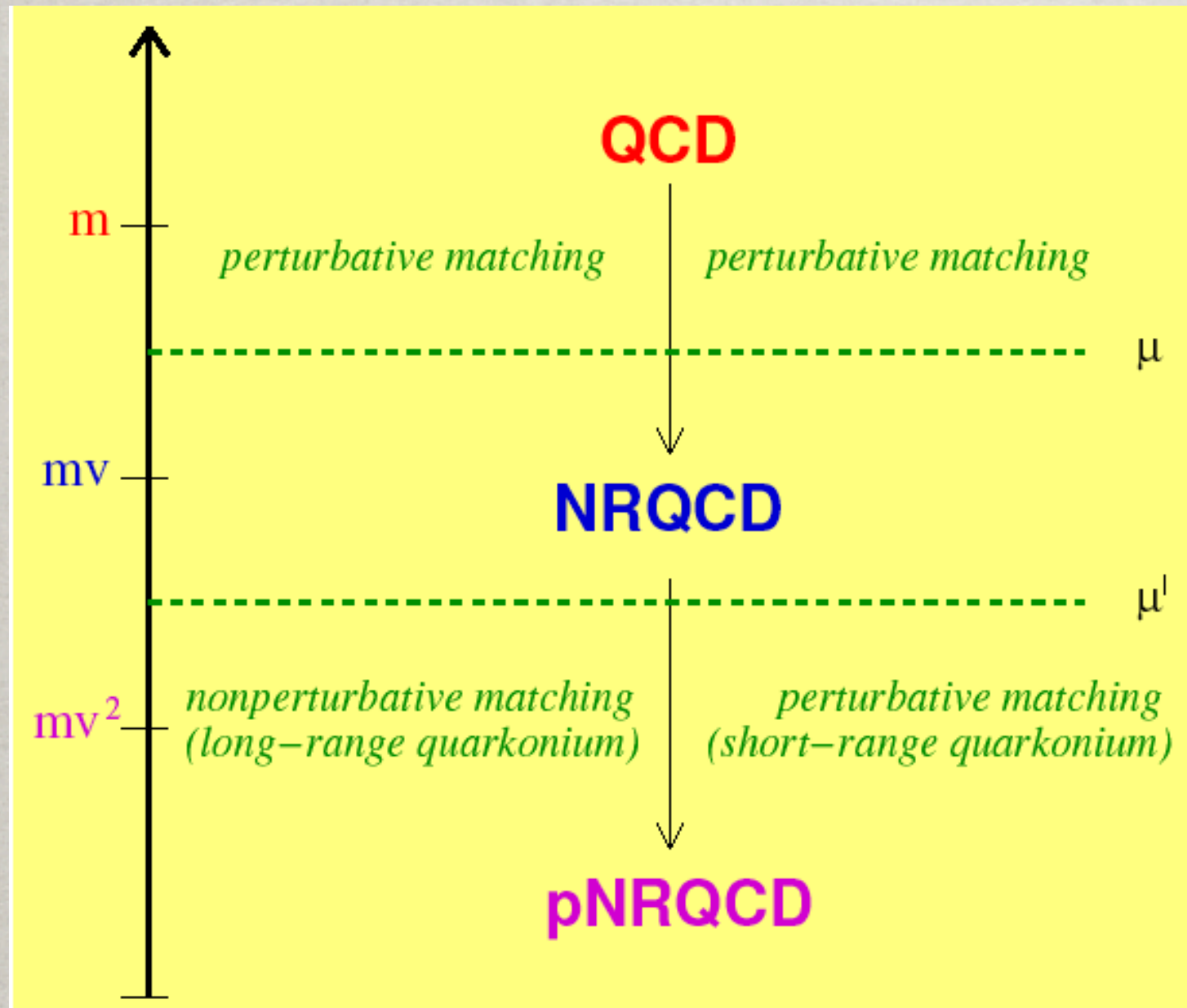
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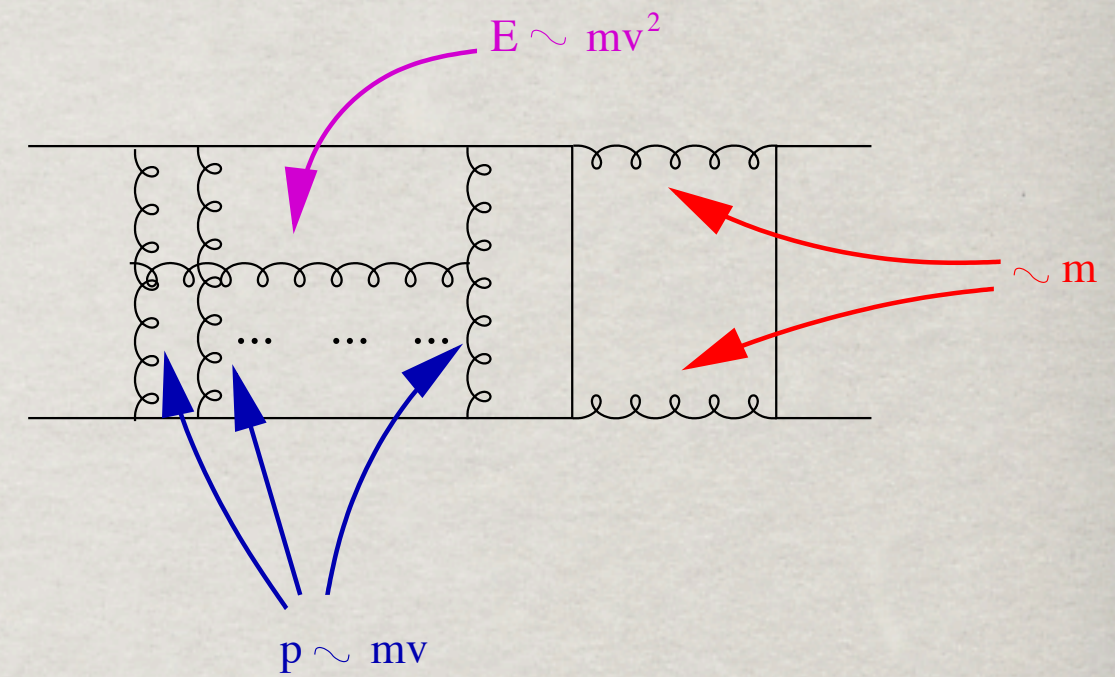
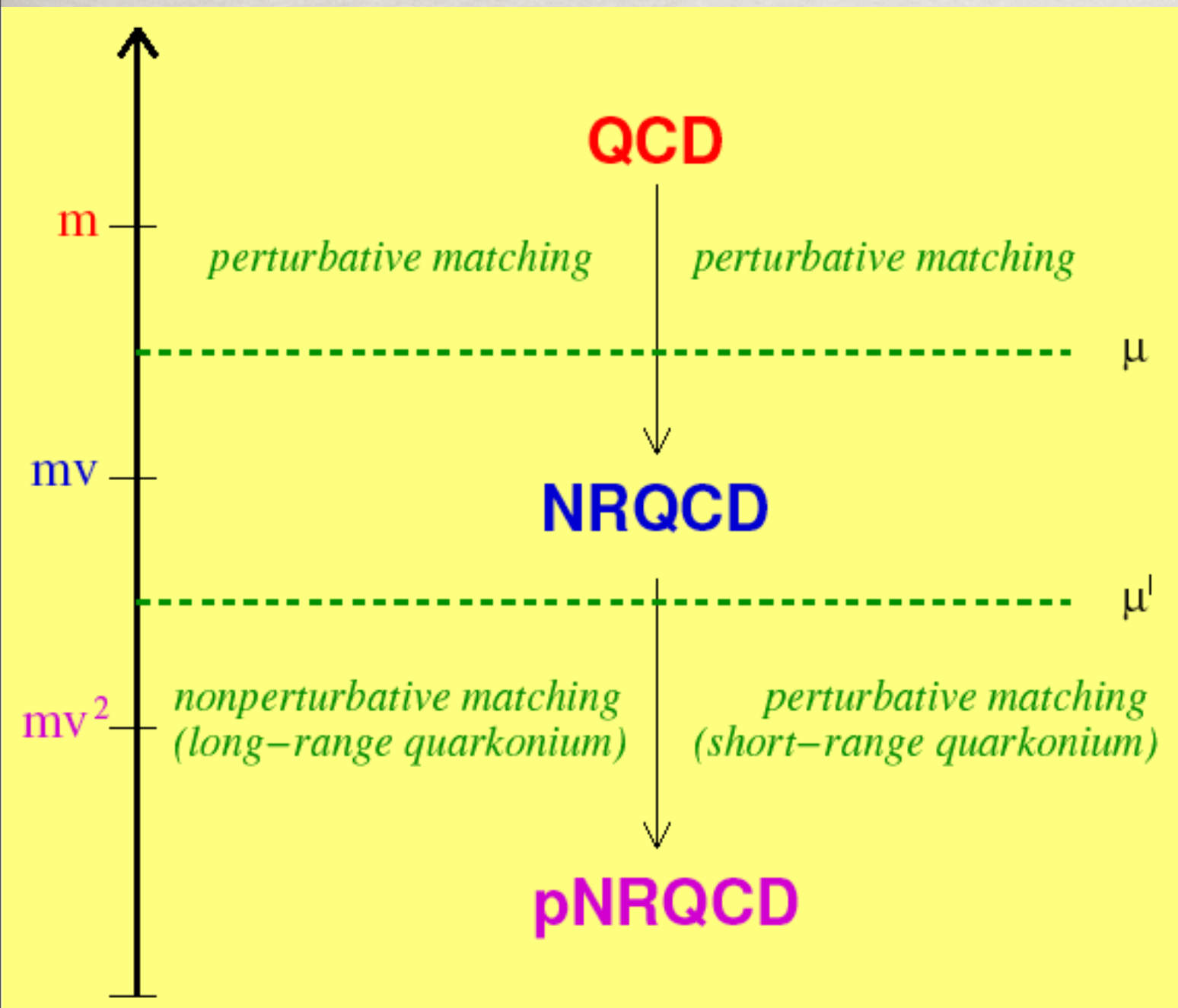
$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

Ultrasoft
 (binding energy)

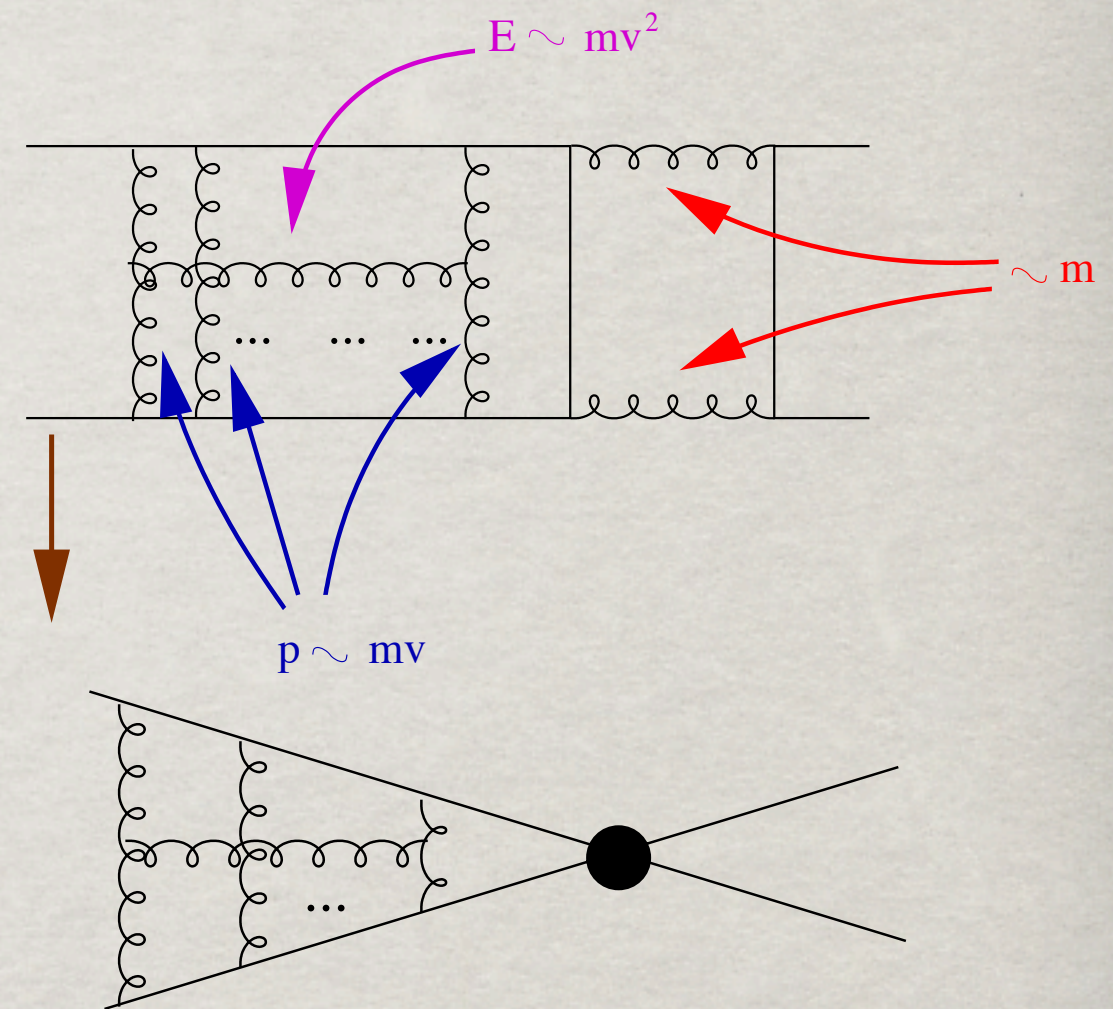
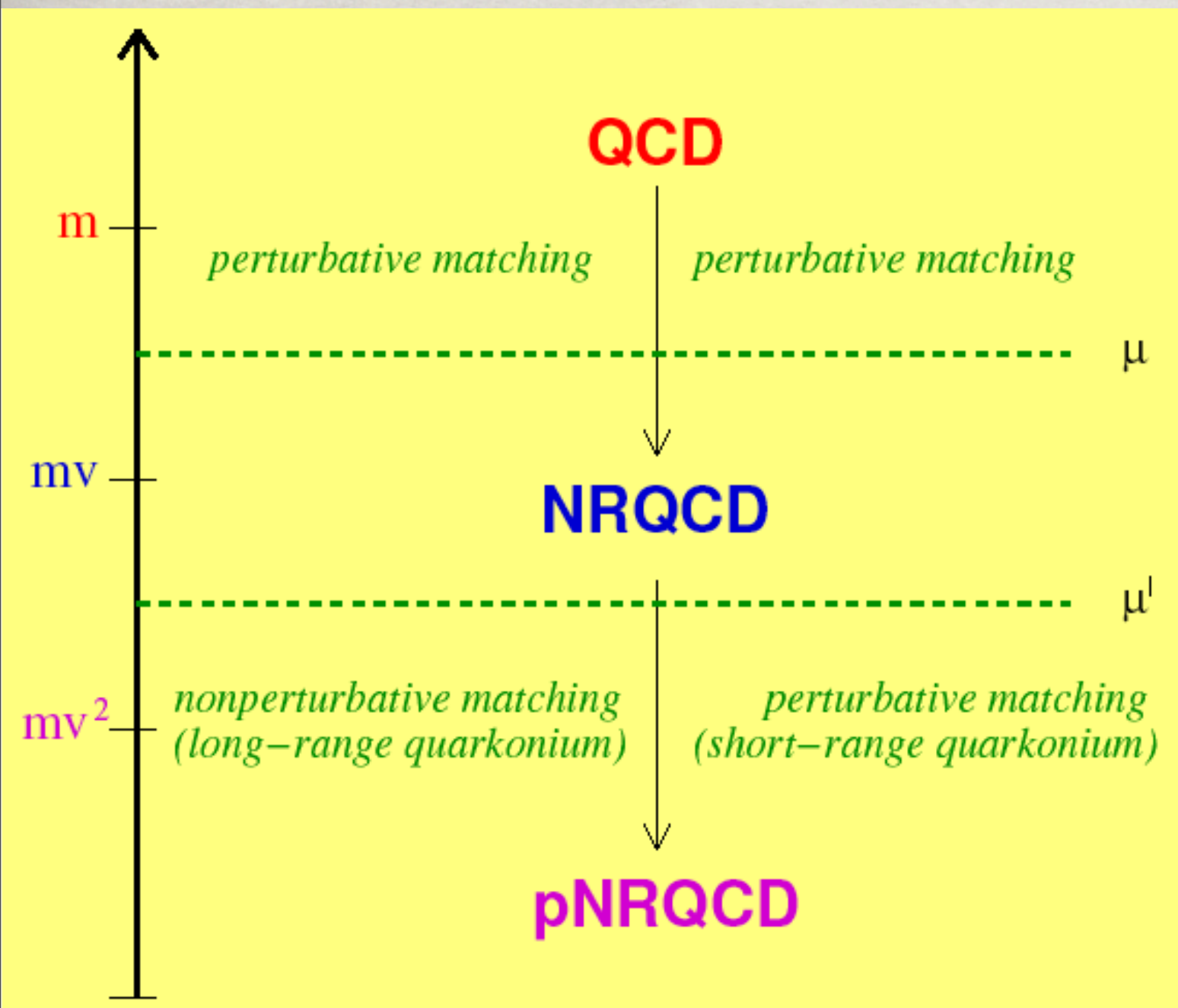
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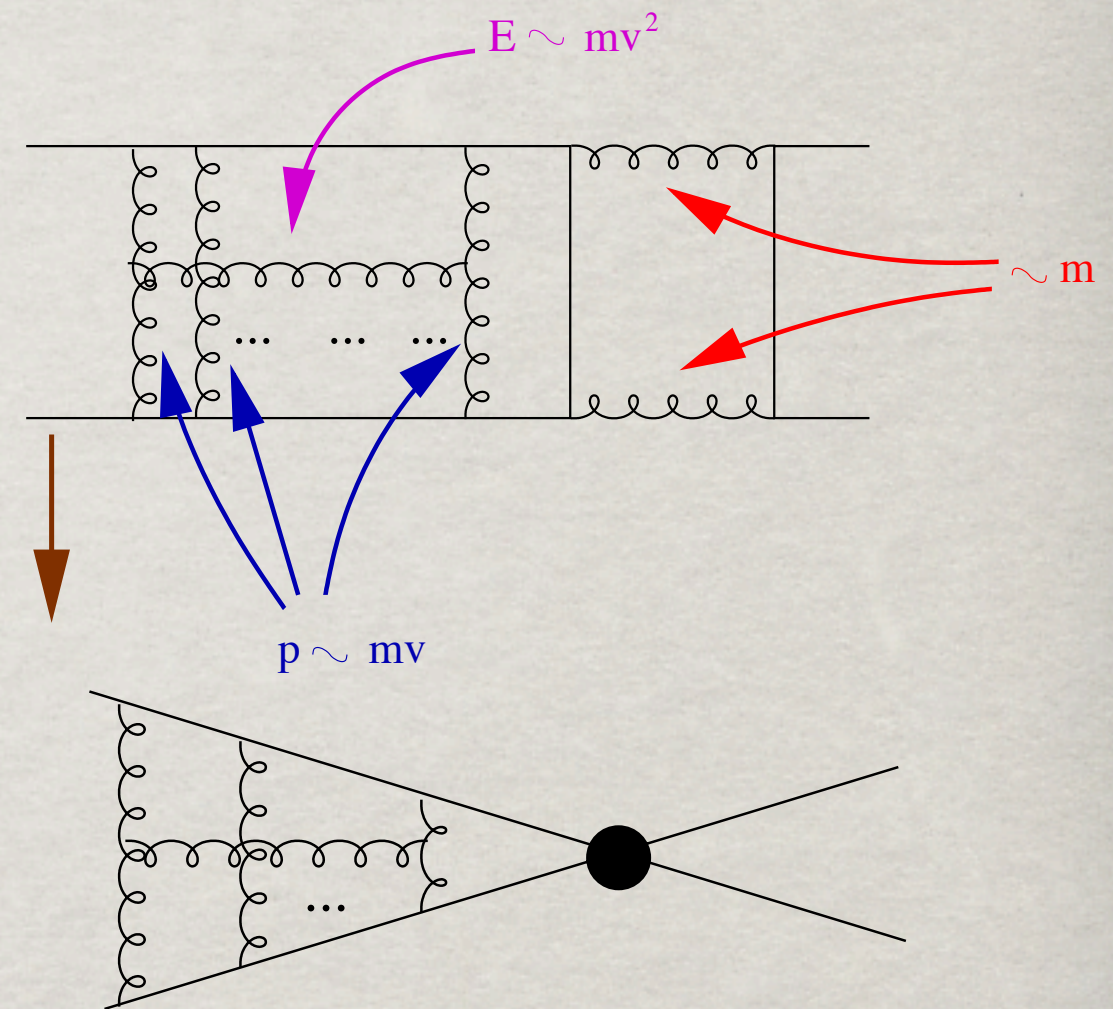
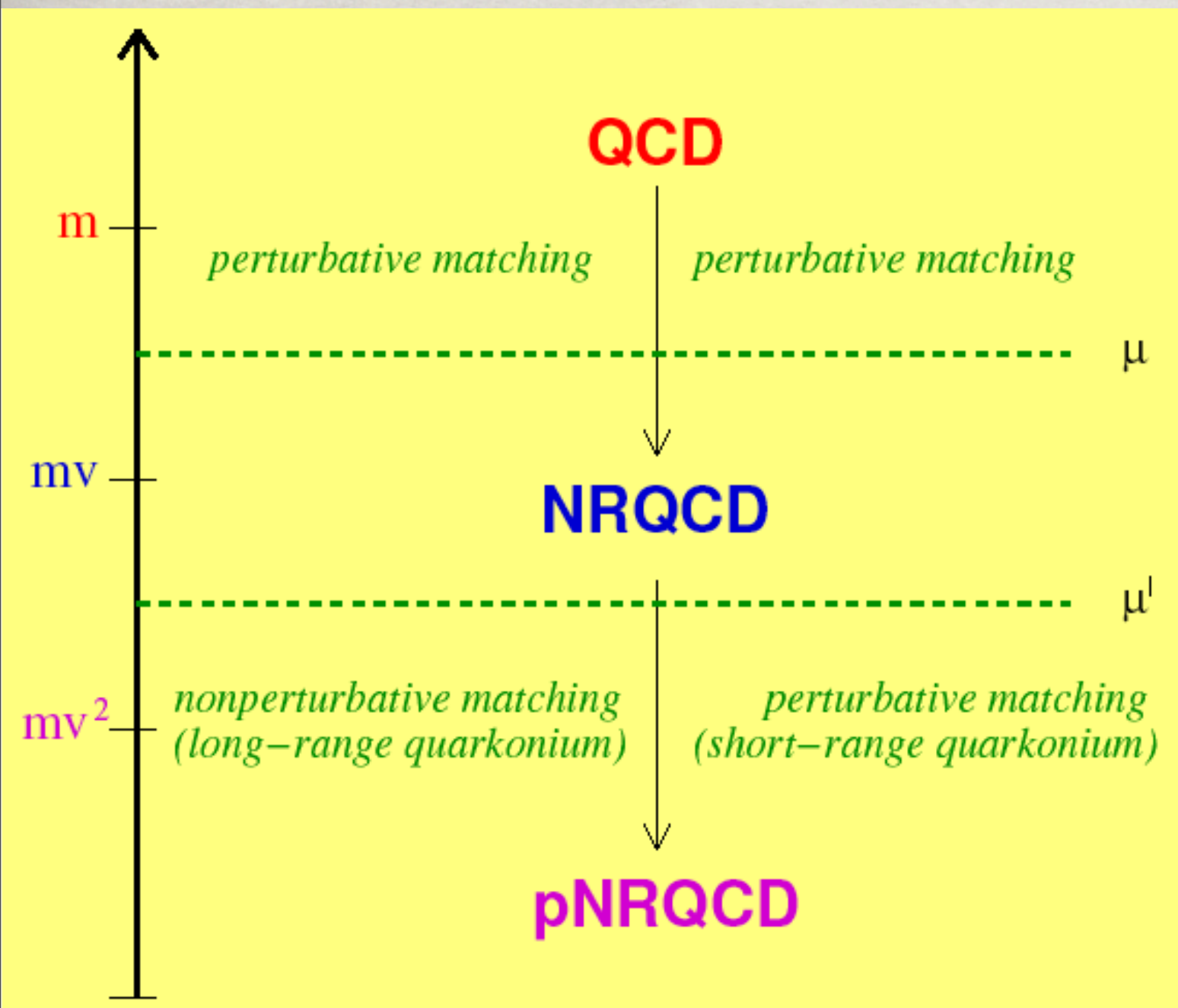
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)



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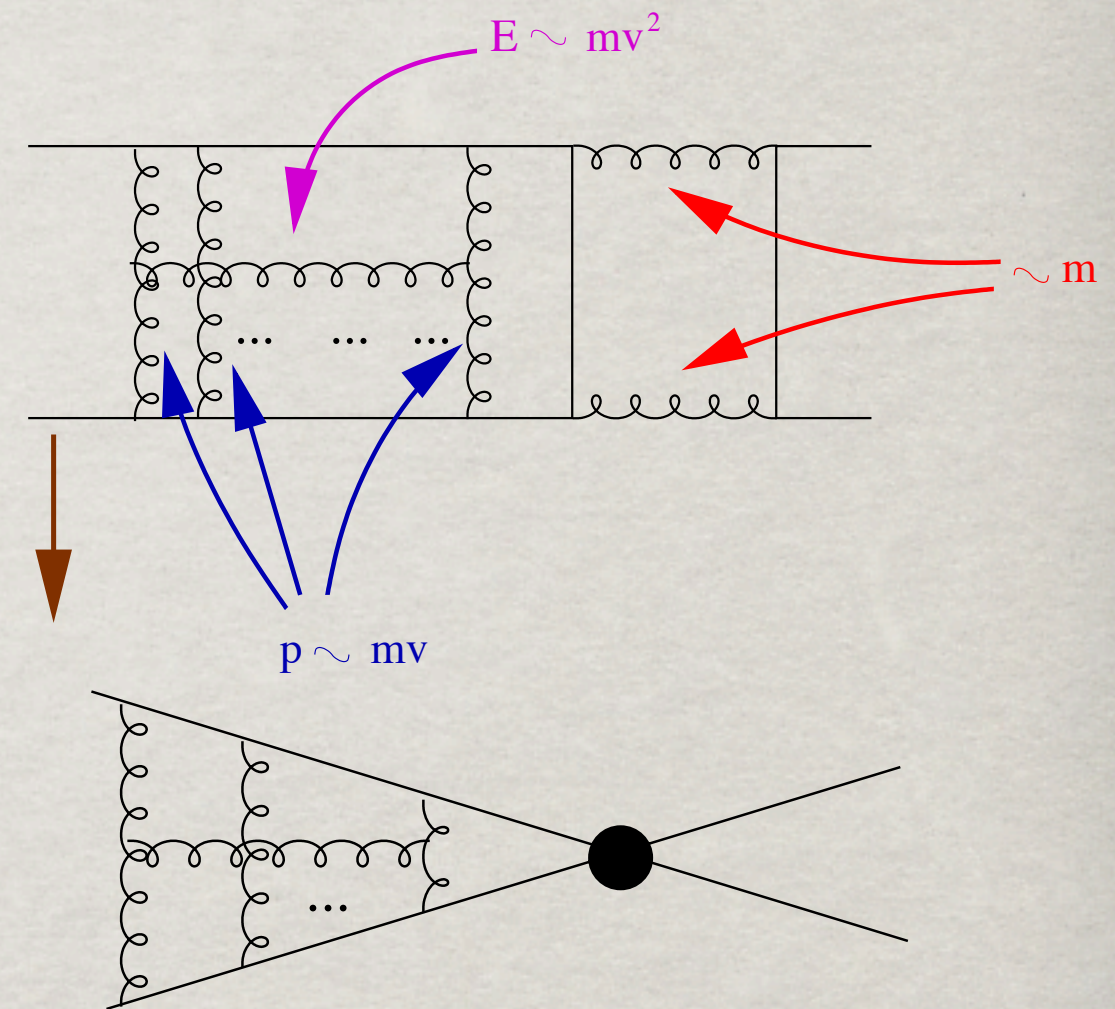
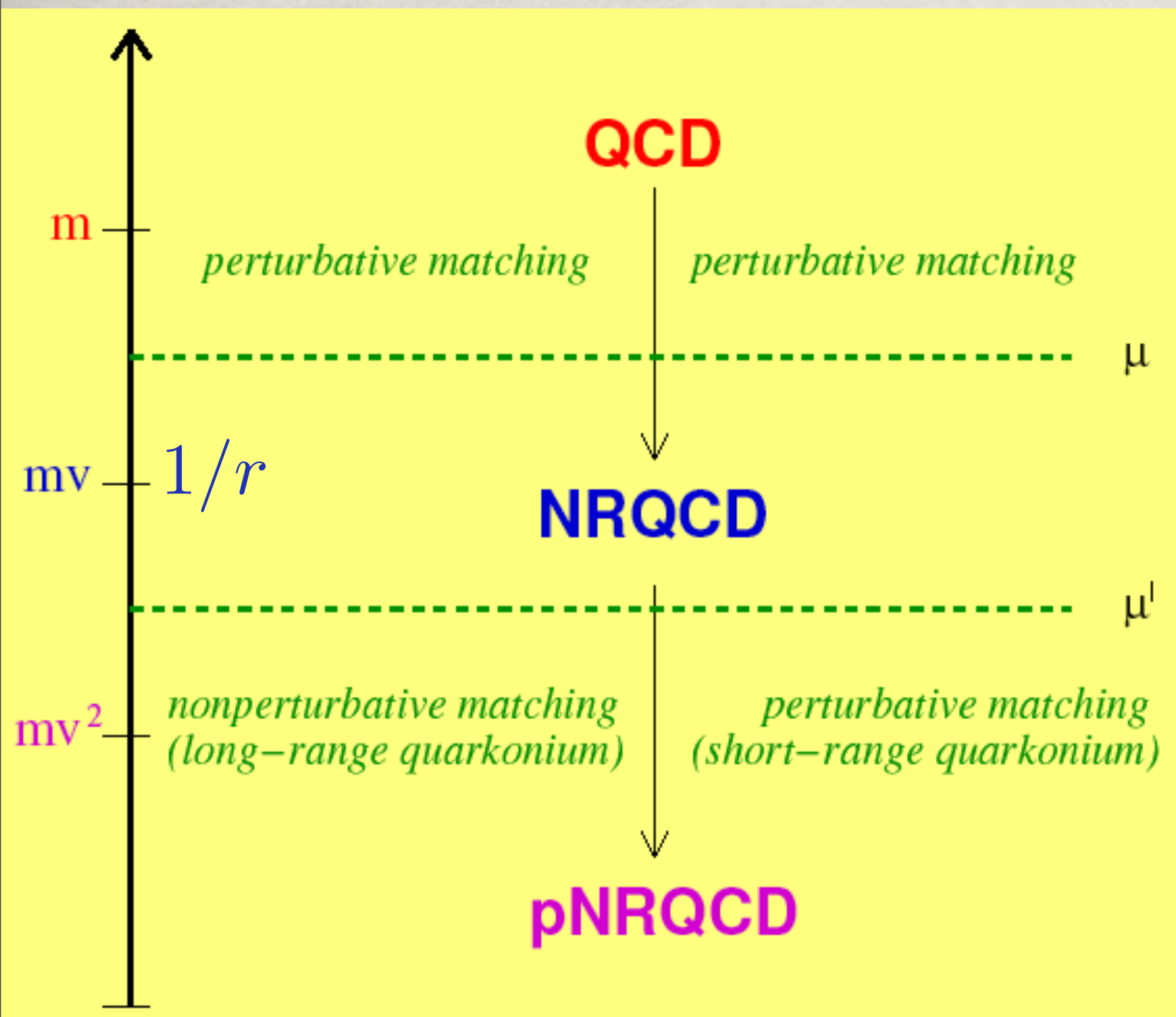


Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)

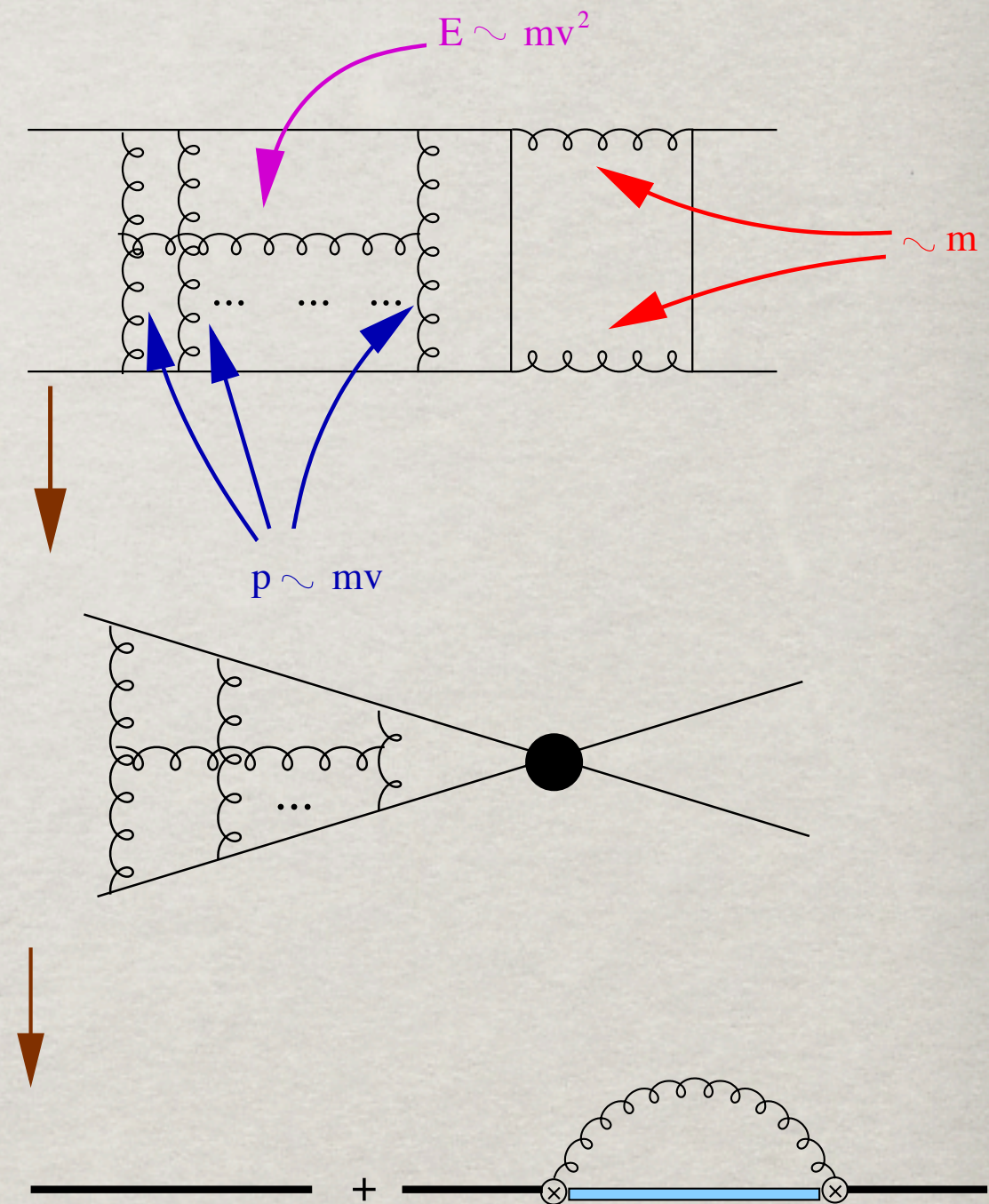
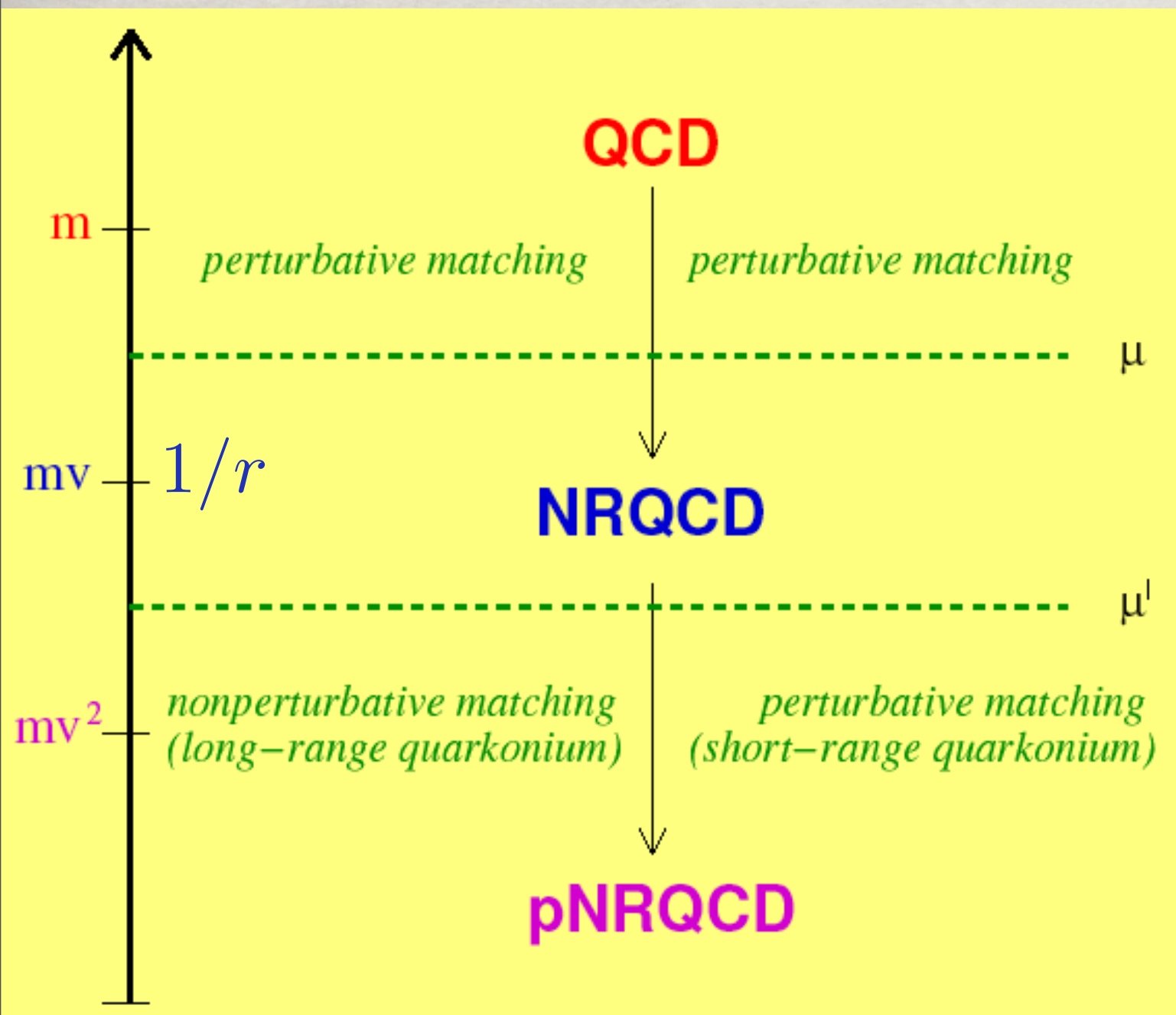


$$\mathcal{L}_{\text{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times \frac{O_n(\mu, \lambda)}{m^n}$$

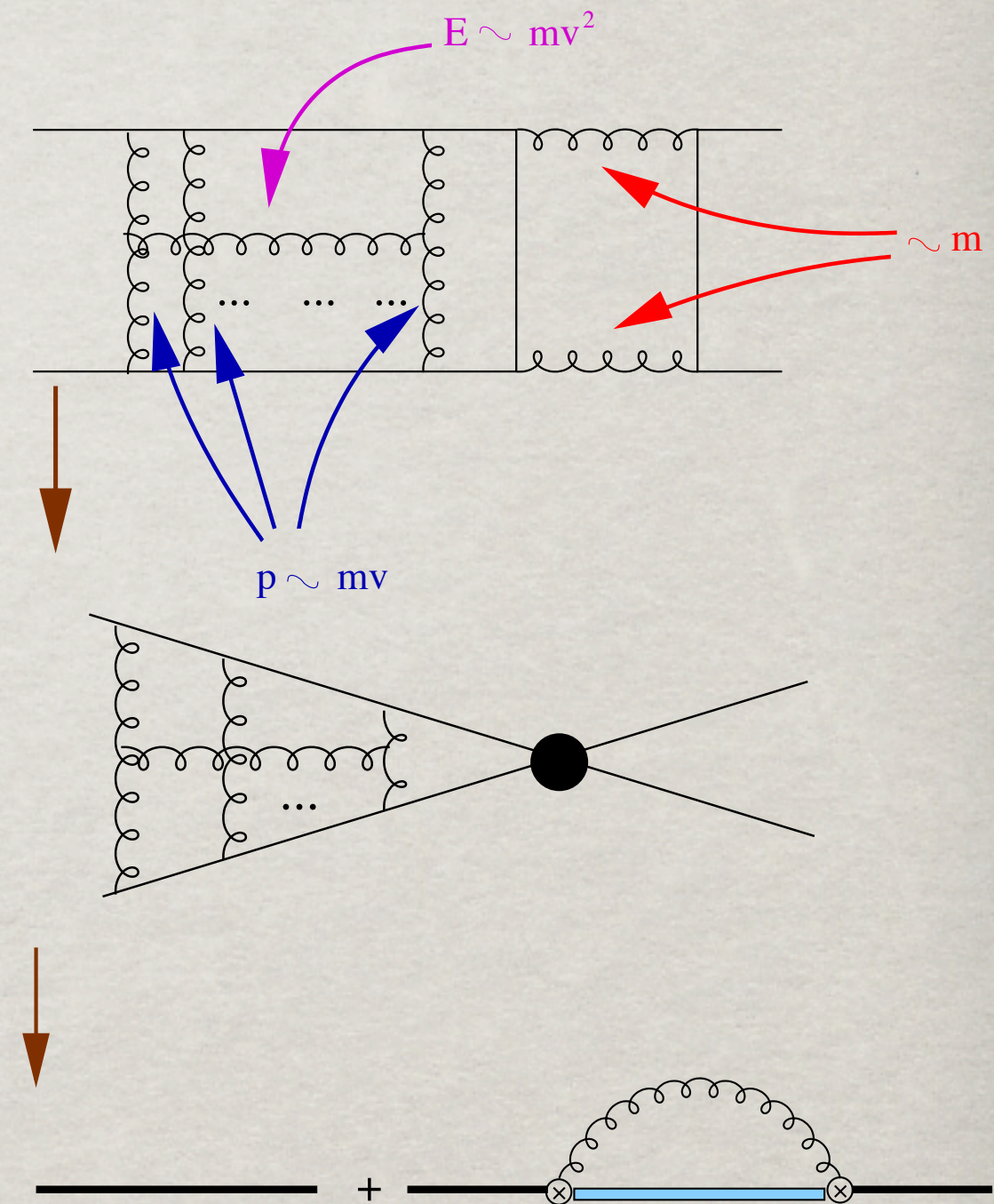
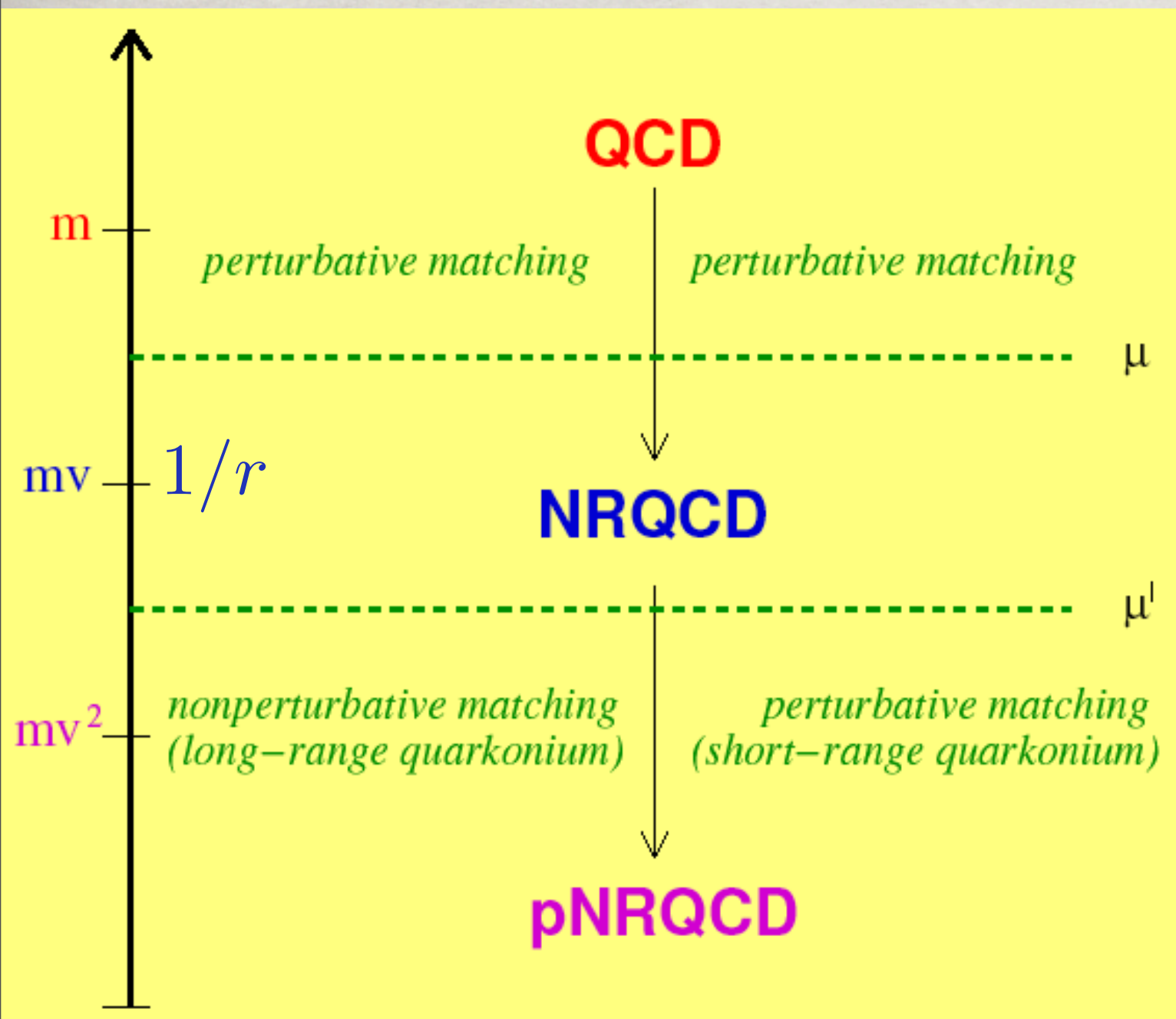
Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)



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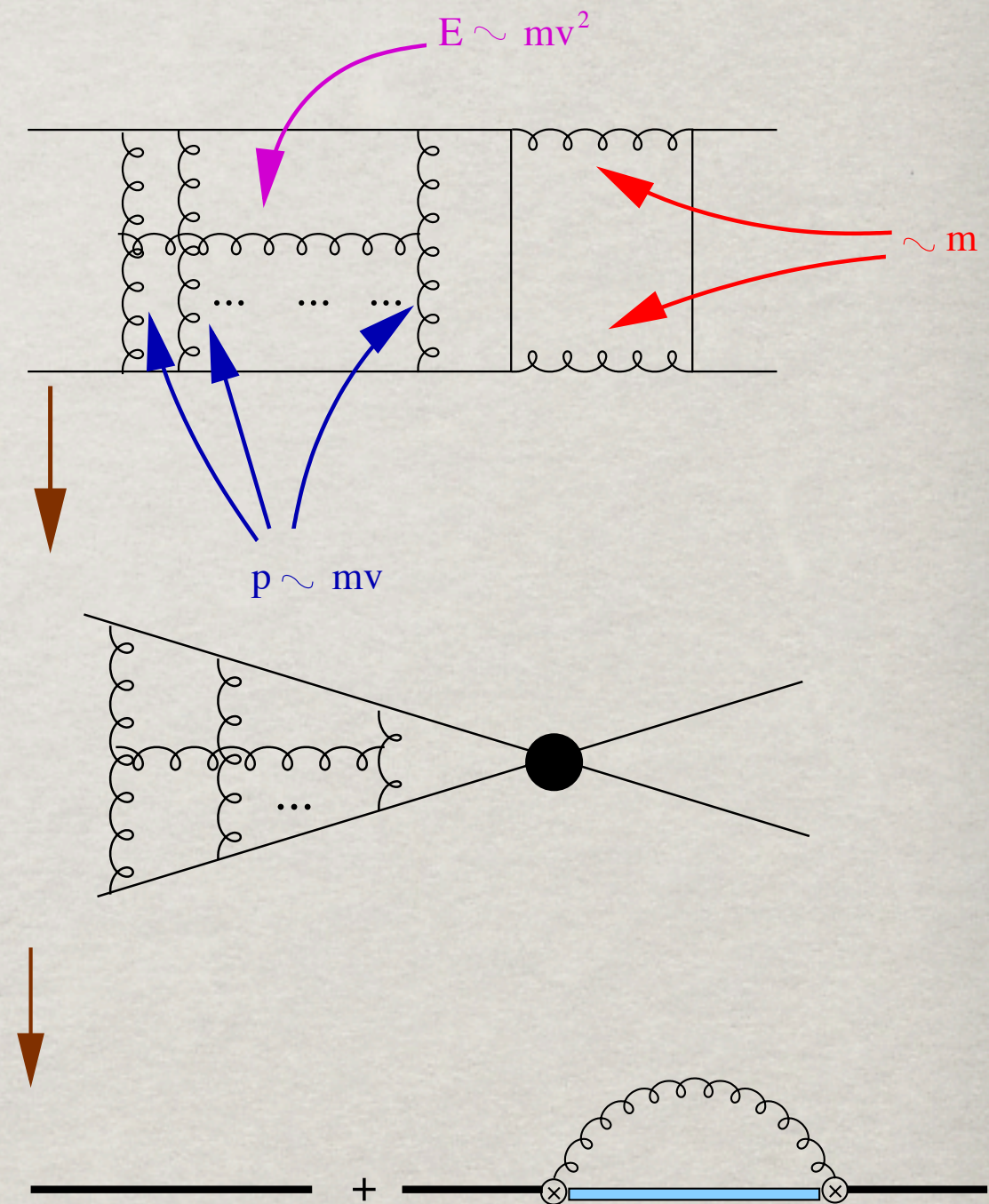
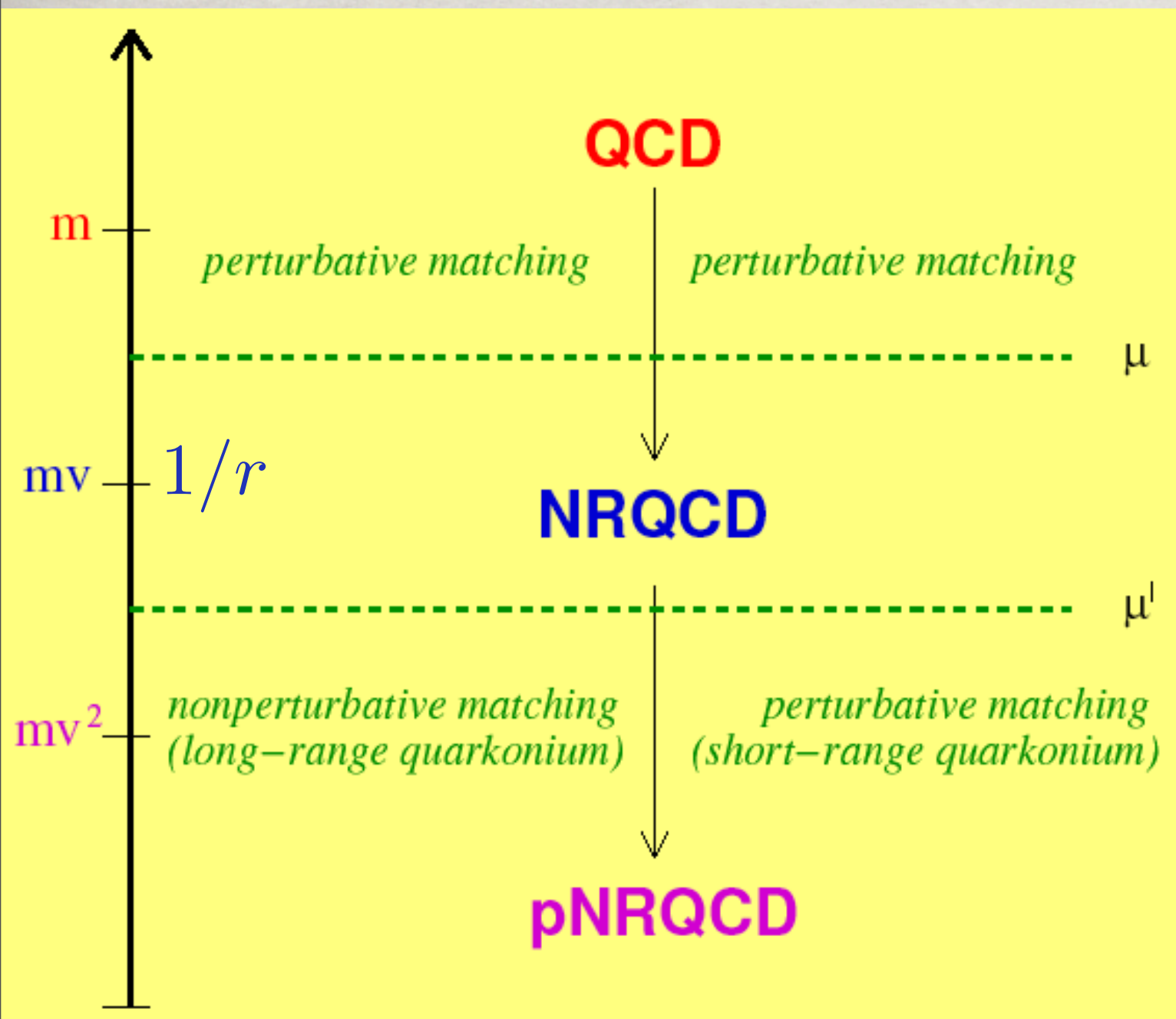


Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)



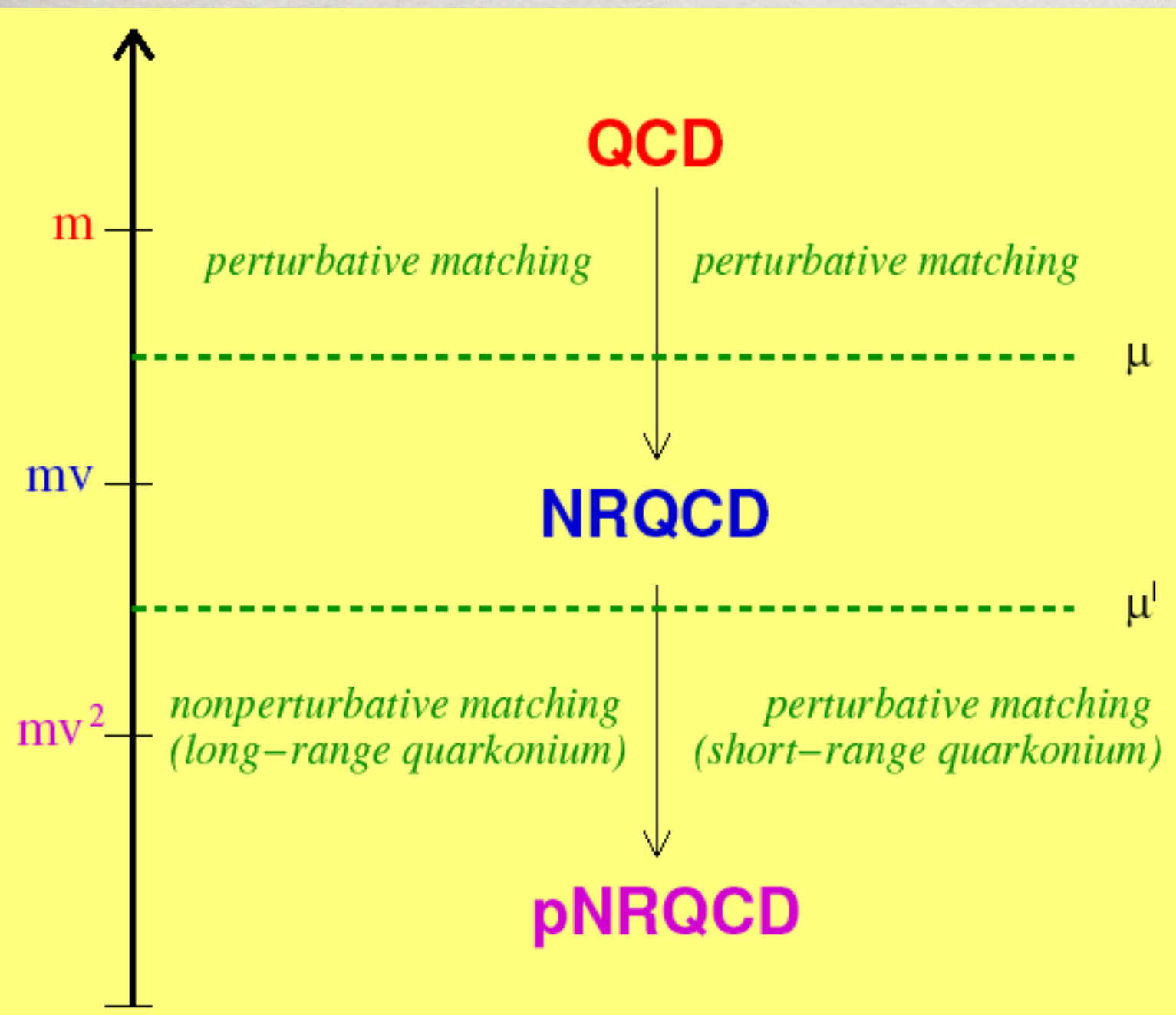
$$\mathcal{L}_{\text{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda) r^n$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)



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Quarkonium with NR EFT: pNRQCD



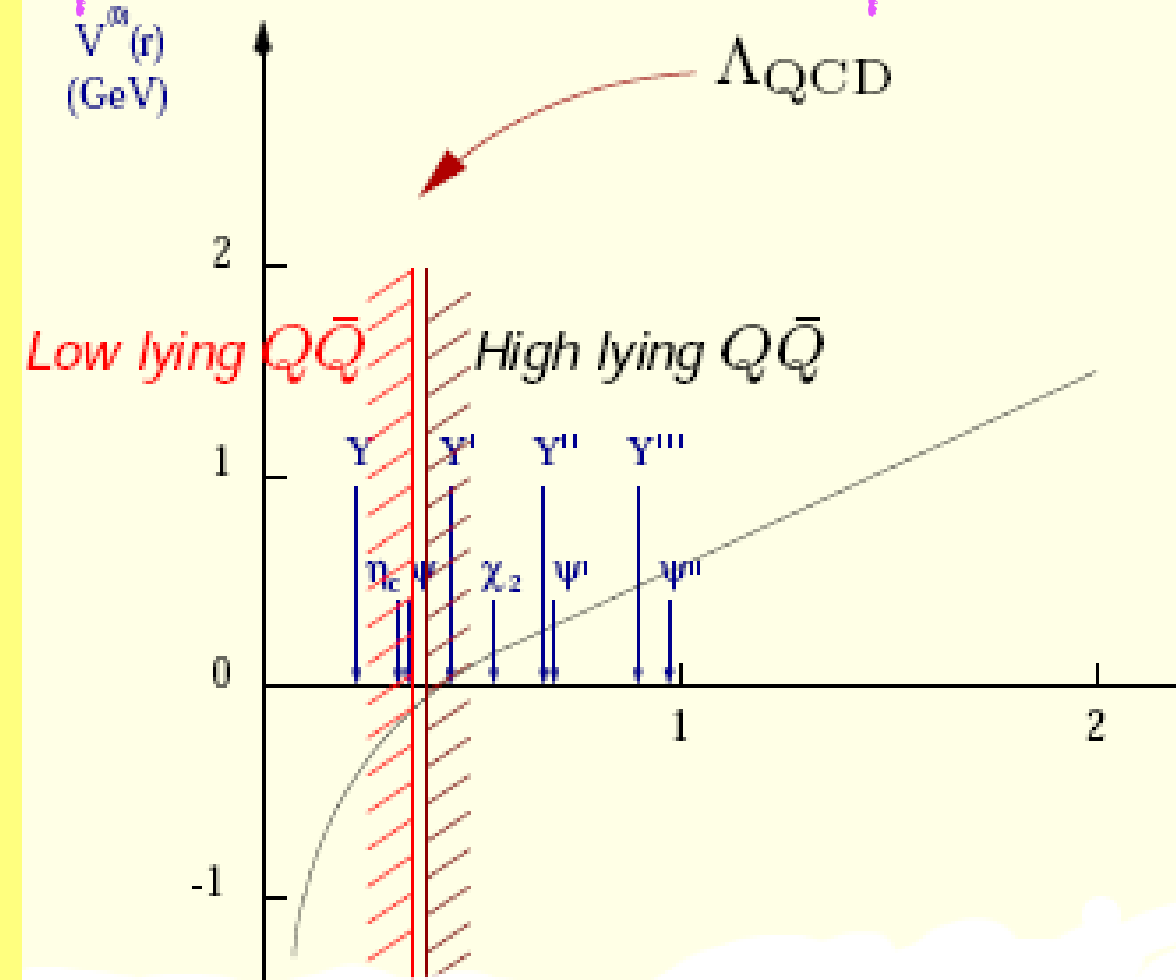
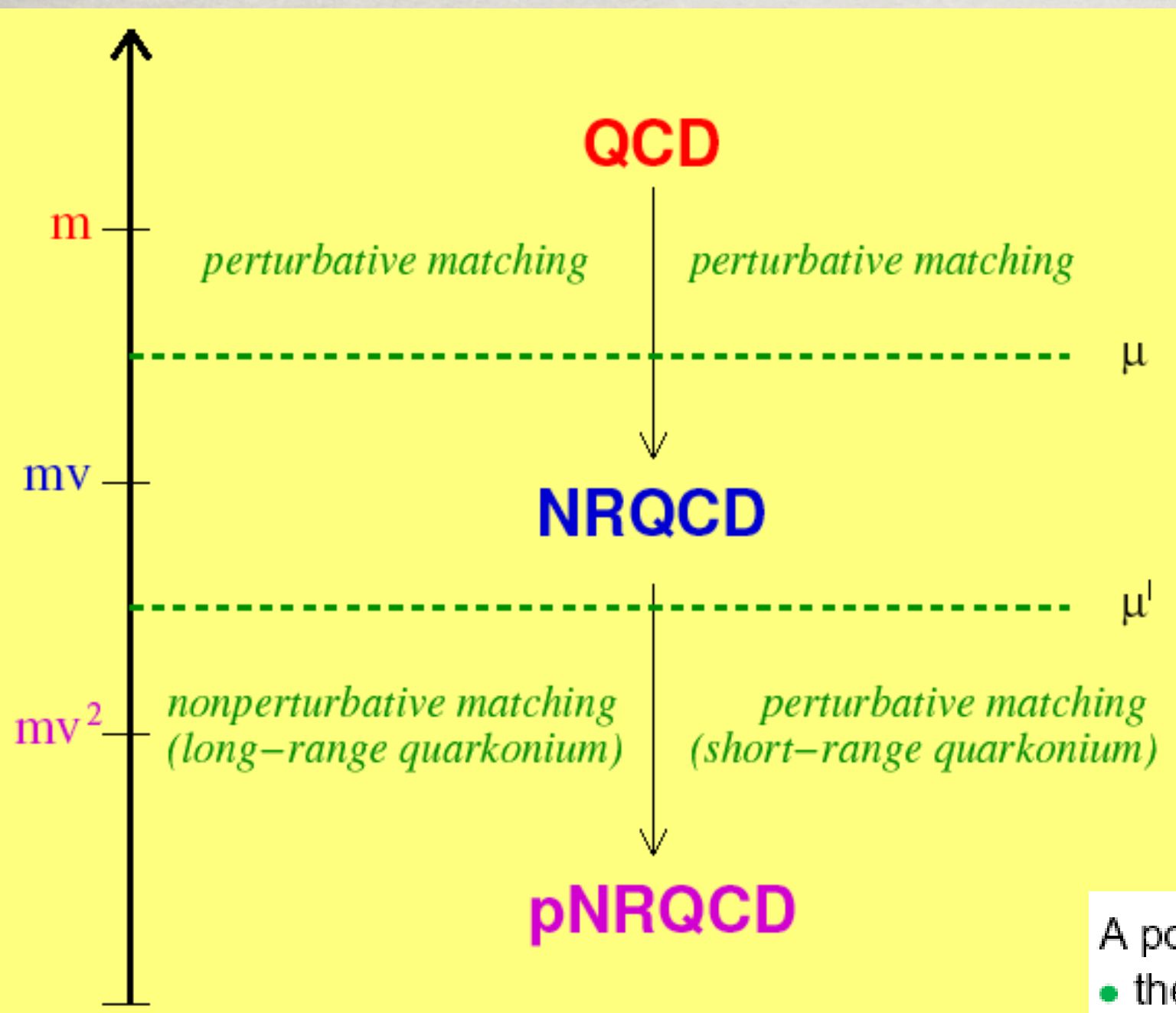
In QCD another scale is relevant

$$\Lambda_{\text{QCD}}$$

Quarkonium with NR EFT: pNRQCD

weakly
coupled
pNRQCD

strongly
coupled
pNRQCD



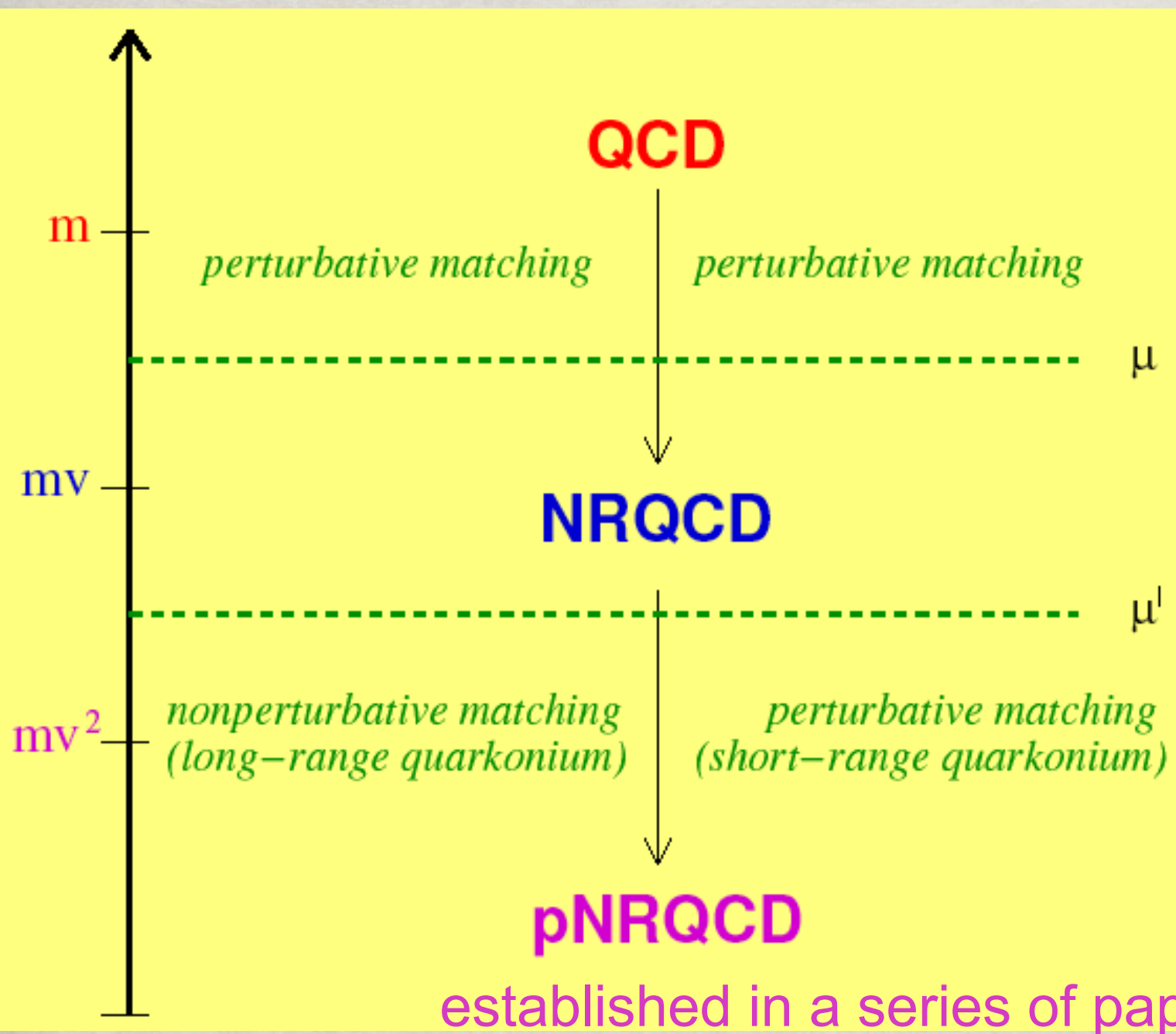
A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{\text{QCD}}$
- the potential is non-perturbative if $mv \sim \Lambda_{\text{QCD}}$

In QCD another scale is relevant

Λ_{QCD}

Quarkonium with EFT



Caswell, Lepage 86,
Lepage, Thacker 88
Bodwin, Braaten, Lepage 95.....

Pineda, Soto 97, N.B. et al, 99,00,
Luke Manohar 97, Luke Savage 98,
Beneke Smirnov 98, Labelle 98
Labelle 98, Grinstein Rothstein 98
Kniehl, Penin 99, Griesshammer 00,
Manohar Stewart 00, Luke et al 00,
Hoang et al 01, 03->

established in a series of papers:

Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. et al 00--013

N.B., Pineda, Soto, Vairo Review of Modern Physics 77(2005)
1423

Physics at the scale m : NRQCD
Quarkonium production and Decays

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Physics at the scale mv and mv^2 : pNRQCD
bound state formation

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Physics at the scale mv and mv^2 : pNRQCD
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pNRQCD is today the theory used to address
quarkonium bound states properties

pNRQCD and quarkonium Several cases for the physics at hand

The EFT has been constructed (away from the strong threshold)

- *Work at calculating higher order perturbative corrections in v and α_s
- *Resumming the log
- *Calculating/extracting nonperturbatively the low energy quantities
- *Extending the theory (electromagnetic effect, 3 bodies)

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lattice calculations of nonlocal condensates/Wilson loops are needed

pNRQCD and quarkonium Several cases for the physics at hand

The EFT is being constructed (Finite T)

Laine et al, 2007, Escobedo, Soto
2007 N. B. et al. 2008

*Results on the static potential hint at a new physical picture of dissociation

*Mass and width of quarkonium at $m \alpha^5(Y(1S) \text{ bbar at LHC})$ N. B. et al. 2010

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- The potential is neither the color singlet free energy nor the internal energy
- The quarkonium dissociation is a consequence of the appearance of a thermal decay width rather than being due to the color screening of the real part of the potential

We have now a coherent and systematical setup to calculate masses and width of quarkonium at finite T for small coupling

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The EFT has not yet been constructed (Exotics close to threshold)

*Degrees of freedom still to be identified

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only in particular cases (X(3872)) a universal treatment is possible

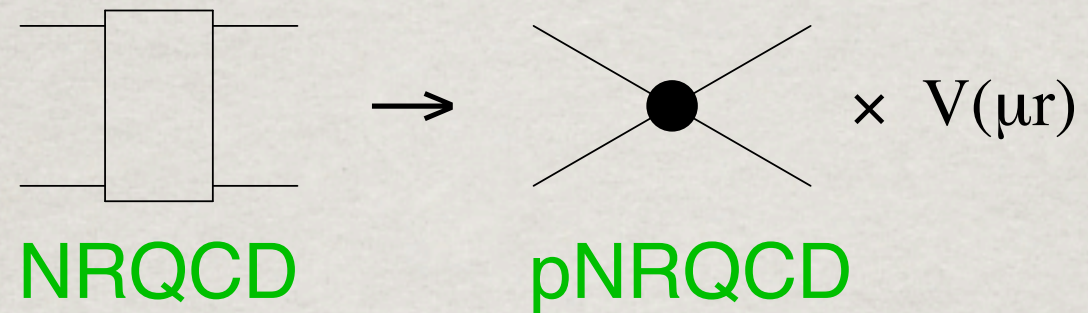
E. Braaten et al

Quarkonium systems with small radius

$$r \ll \Lambda_{\text{QCD}}^{-1}$$

pNRQCD for quarkonia with small radius $r \ll \Lambda_{\text{QCD}}^{-1}$

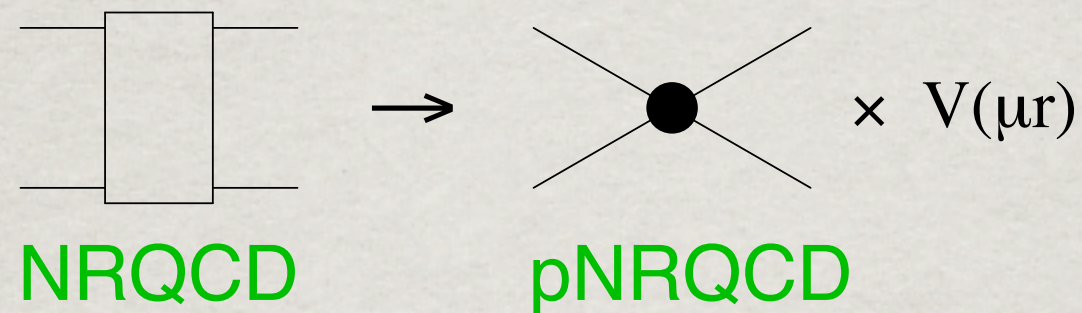
Degrees of freedom that **scale** like mv are integrated out:



pNRQCD for quarkonia with small radius

$$r \ll \Lambda_{\text{QCD}}^{-1}$$

Degrees of freedom that **scale** like mv are integrated out:



- If $mv \gg \Lambda_{\text{QCD}}$, the **matching** is **perturbative**

- Degrees of freedom: **quarks** and **gluons**

Q - \bar{Q} states, with energy $\sim \Lambda_{\text{QCD}}$, mv^2 and momentum $\lesssim mv$

\Rightarrow (i) **singlet S** (ii) **octet O**

Gluons with energy and momentum $\sim \Lambda_{\text{QCD}}$, mv^2

- Definite power counting: $r \sim \frac{1}{mv}$ and $t, R \sim \frac{1}{mv^2}, \frac{1}{\Lambda_{\text{QCD}}}$

The gauge fields are **multipole expanded**:

$$A(R, r, t) = A(R, t) + \mathbf{r} \cdot \nabla A(R, t) + \dots$$

Non-analytic behaviour in $r \rightarrow$ **matching coefficients** V

weakly coupled pNRQCD $r \ll \Lambda_{\text{QCD}}^{-1}$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + \text{Tr} \left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S + O^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) O \right\}$$

LO in r

S singlet field

O octet field

————

=====

singlet propagator

octet propagator

weakly coupled pNRQCD $r \ll \Lambda_{\text{QCD}}^{-1}$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + \text{Tr} \left\{ \mathbf{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathbf{S} \right. \\ \left. + \mathbf{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathbf{O} \right\}$$

LO in r

$$+ V_A \text{Tr} \left\{ \mathbf{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \mathbf{S} + \mathbf{S}^\dagger \mathbf{r} \cdot g\mathbf{E} \mathbf{O} \right\} \\ + \frac{V_B}{2} \text{Tr} \left\{ \mathbf{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \mathbf{O} + \mathbf{O}^\dagger \mathbf{O} \mathbf{r} \cdot g\mathbf{E} \right\} \\ + \dots$$

NLO in r

S singlet field

O octet field

————

=====

singlet propagator

octet propagator

weakly coupled pNRQCD

$$r \ll \Lambda_{\text{QCD}}^{-1}$$

Singlet static potential

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + \text{Tr} \left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S \right. \\ \left. + O^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) O \right\}$$

LO in r

Octet static potential

$$+ V_A \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g\mathbf{E} S + S^\dagger \mathbf{r} \cdot g\mathbf{E} O \right\} \\ + \frac{V_B}{2} \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g\mathbf{E} O + O^\dagger O \mathbf{r} \cdot g\mathbf{E} \right\} \\ + \dots$$

NLO in r

S singlet field

O octet field

—————

=====

singlet propagator

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pNRQCD

- ✱ pNRQCD provides a QM description from field theory: the Schroedinger equation and the potentials appear once all scales above the binding energy have been integrated out
- ✱ The EFT accounts for non-potential terms as well. They provide loop corrections to the leading potential picture. Retardation effects are typically related to the nonperturbative physics
- ✱ The Quantum Mechanical divergences are cancelled by the NRQCD matching coefficients.
- ✱ Poincare' invariance is intact and is realized via exact relations among the matching coefficients (potentials)

QCD singlet static potential and singlet static energy

The diagram illustrates the expansion of the NRQCD static potential into pNRQCD terms. On the left, a rectangular loop with arrows on its top and bottom edges represents the NRQCD potential, labeled $e^{ig \oint dz^\mu A_\mu}$ and NRQCD. This is set equal to a series of terms on the right. The first term is a single horizontal line. The second term is a horizontal line with a gluon loop (represented by a wavy line) attached between two vertices marked with a cross in a circle. This is followed by an ellipsis, indicating further terms in the expansion. The label pNRQCD is placed below the second term.

$$\text{NRQCD} = \text{pNRQCD} + \dots$$

QCD singlet static potential and singlet static energy

$$\begin{array}{c}
 \boxed{e^{ig \oint dz^\mu A_\mu}} \\
 \text{NRQCD}
 \end{array}
 =
 \begin{array}{c}
 \text{---} + \text{---} \otimes \text{---} \otimes \text{---} + \dots \\
 \text{pNRQCD}
 \end{array}$$

$$\lim_{T \rightarrow \infty} \frac{i}{T} \ln \langle \boxed{} \rangle = \overset{\text{potential}}{V_s(r, \mu)} - i \frac{g^2}{N_c} \overset{\text{ultrasoft contribution}}{V_A^2} \int_0^\infty dt e^{-it(V_o - V_s)} \langle \text{Tr}(\overset{\text{static energy}}{r} \cdot \overset{\text{contributes from 3 loops}}{E}(t) \overset{\text{contributes from 3 loops}}{r} \cdot \overset{\text{contributes from 3 loops}}{E}(0)) \rangle(\mu) + \dots$$

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The potential is a Wilson coefficient of an EFT. In general, it undergoes renormalization, develops scale dependence and satisfies renormalization group equations, which allow to resum large logarithms.

Quarkonium singlet static potential at N⁴LO

$$\begin{aligned} V_s(r, \mu) = & -C_F \frac{\alpha_s(1/r)}{r} \left[1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 \right. \\ & + \left(\frac{16\pi^2}{3} C_A^3 \ln r\mu + a_3 \right) \left(\frac{\alpha_s(1/r)}{4\pi} \right)^3 \\ & \left. + \left(a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9} \pi^2 C_A^3 \beta_0 (-5 + 6 \ln 2) \right) \ln r\mu + a_4 \right) \left(\frac{\alpha_s(1/r)}{4\pi} \right)^4 \right] \end{aligned}$$

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 \end{aligned}$$

a_1 Billoire 80

a_2 Schroeder 99, Peter 97

coeff $\ln r\mu$ N.B. Pineda, Soto, Vairo 99

a_4^{L2}, a_4^L N.B., Garcia, Soto, Vairo 06

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

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coeff $\ln r\mu$ N.B. Pineda, Soto, ~~Vainsen 00~~ **3LOOPS REDUCES TO 1 LOOP IN THE EFT**

a_4^{L2}, a_4^L N.B., Garcia, Soto ~~''''''''~~ **4LOOPS REDUCES TO 2LOOPS IN THE EFT**

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

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Two problems:

- 1) Bad convergence of the series due to large beta₀ terms
- 2) Large logs

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 \end{aligned}$$

Two problems:

for long it was believed that such series was not convergent
problem for any phenomenological application

1) Bad convergence of the series due to large beta₀ terms

2) Large logs

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 V_s(r, \mu) = & -C_F \frac{\alpha_s(1/r)}{r} \left[1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 \right. \\
 & + \left(\frac{16\pi^2}{3} C_A^3 \ln r\mu + a_3 \right) \left(\frac{\alpha_s(1/r)}{4\pi} \right)^3 \\
 & \left. + \left(a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9} \pi^2 C_A^3 \beta_0 (-5 + 6 \ln 2) \right) \ln r\mu + a_4 \right) \left(\frac{\alpha_s(1/r)}{4\pi} \right)^4 \right]
 \end{aligned}$$

Two problems: for long it was believed that such series was not convergent
 problem for any phenomenological application

1) Bad convergence of the series due to large beta₀ terms

2) Large logs

The eft cures both:

1) Renormalon subtracted scheme

Beneke 98, Hoang, Lee 99, Pineda 01, n.brambilla et

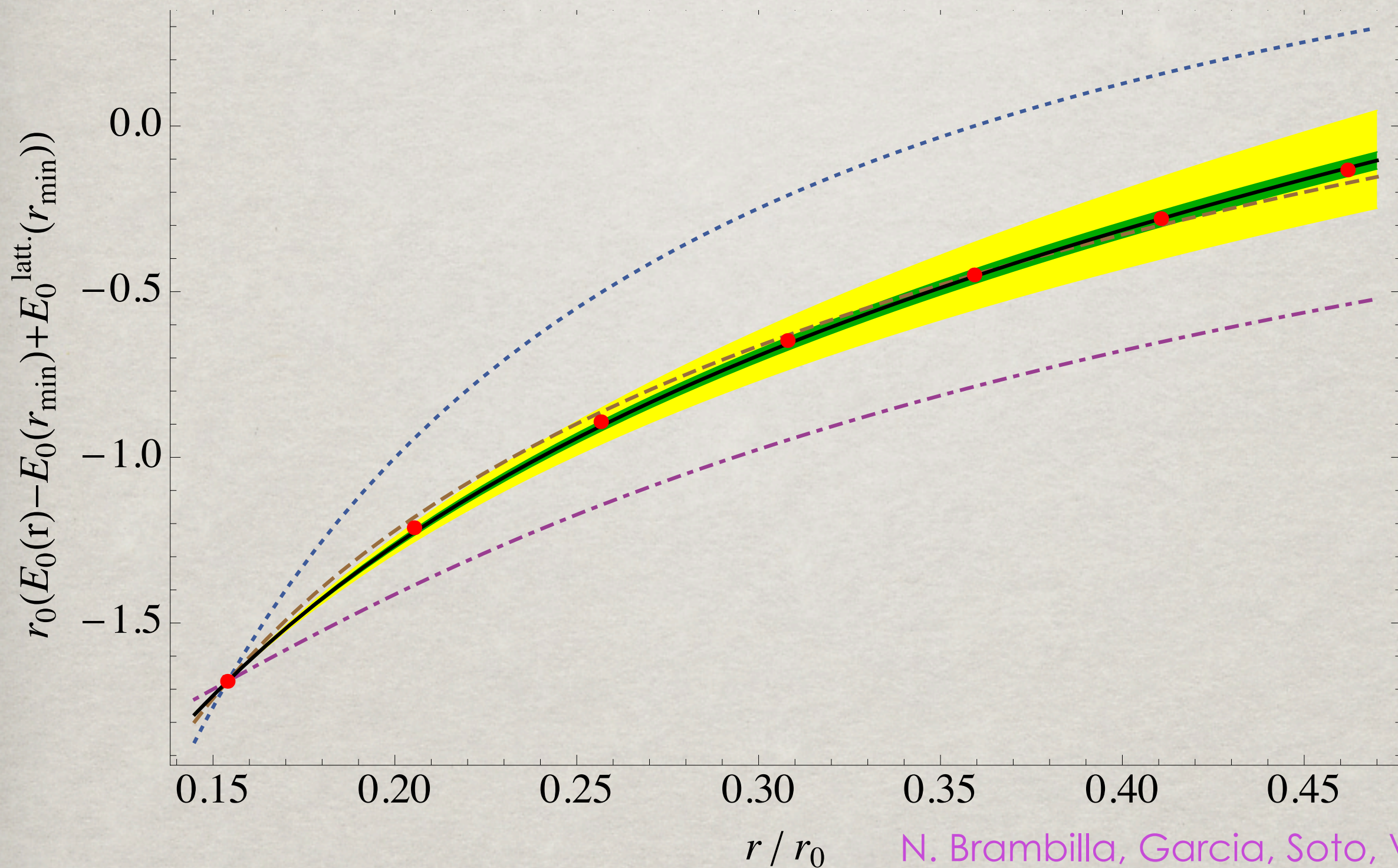
2) Renormalization group summation of the logs^{al 09}

up to N³LL $(\alpha_s^{4+n} \ln^n \alpha_s)$

Pineda, Soto, N. B., X. Garcia, Soto, Vairo . et al
 2007, 2009

Quarkonium singlet static energy at N³LI in comparison with lattice data (red points

Necco Sommer 2002)



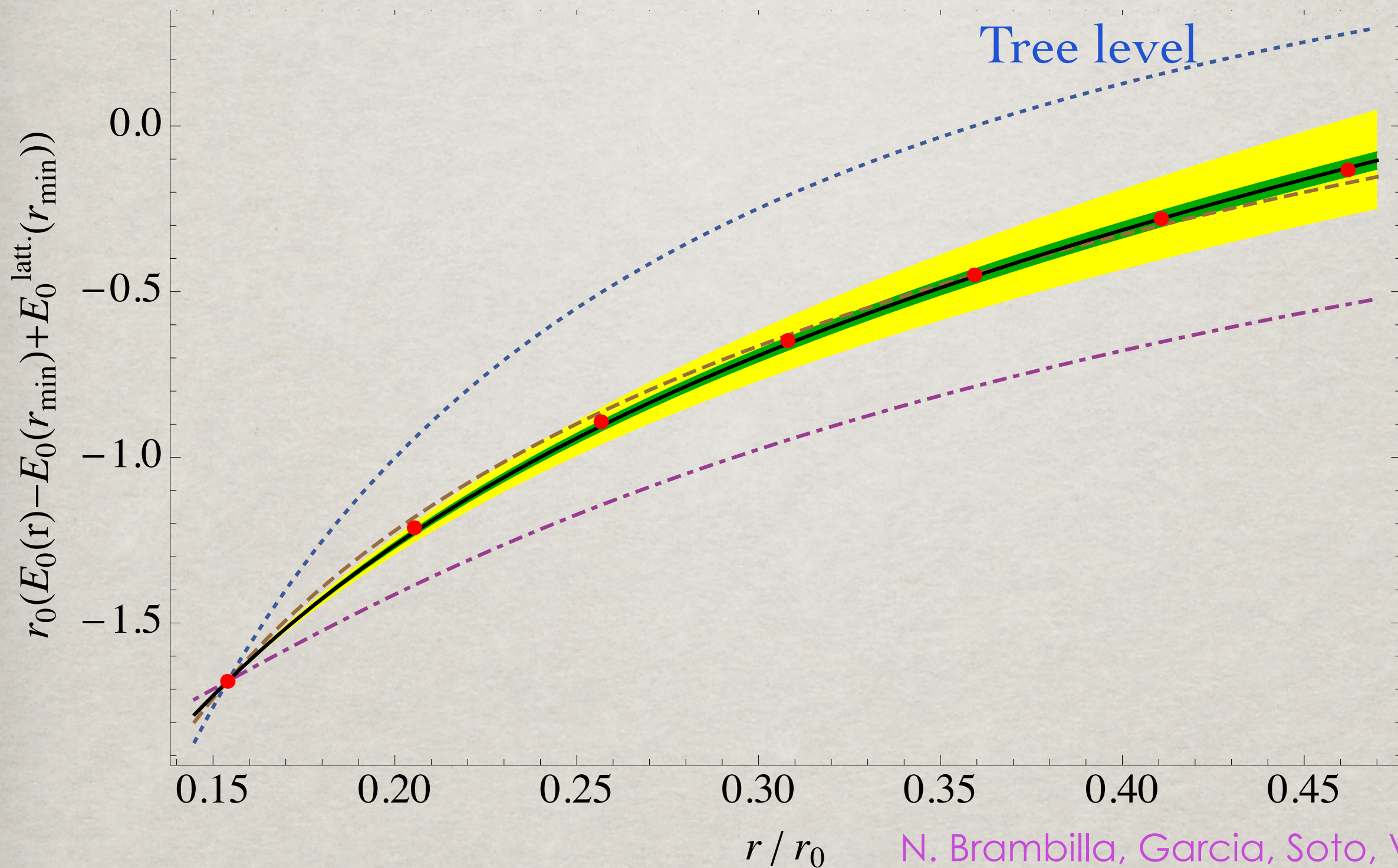
N. Brambilla, Garcia, Soto, Vairo 010

Yellow band : uncertainty in α_s ($r_0 \cdot \Lambda_{\text{QCD_MS}}$)

Green band: uncertainty in higher order terms

Quarkonium singlet static energy at N³LI in comparison with lattice data (red points

Necco Sommer 2002)

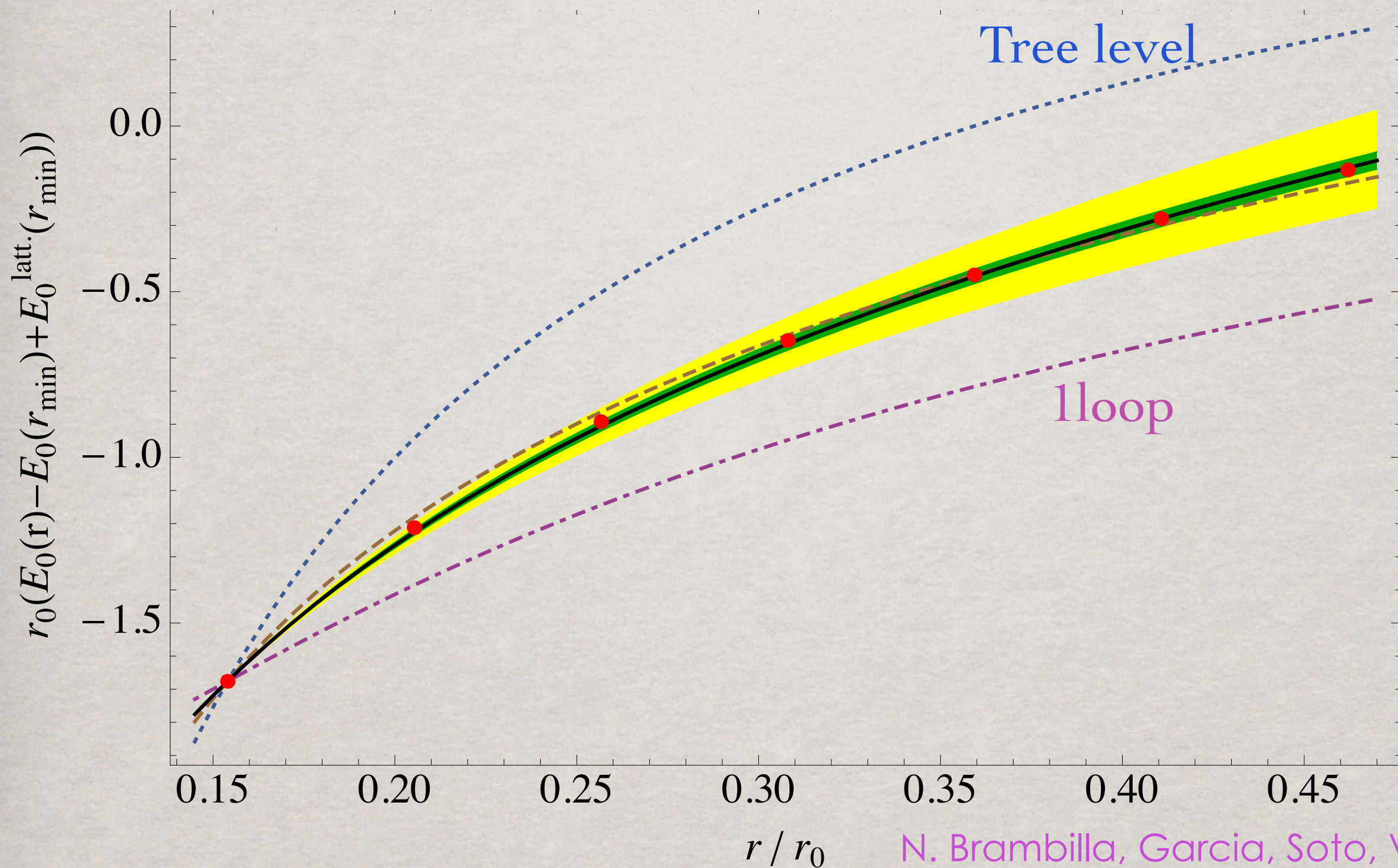


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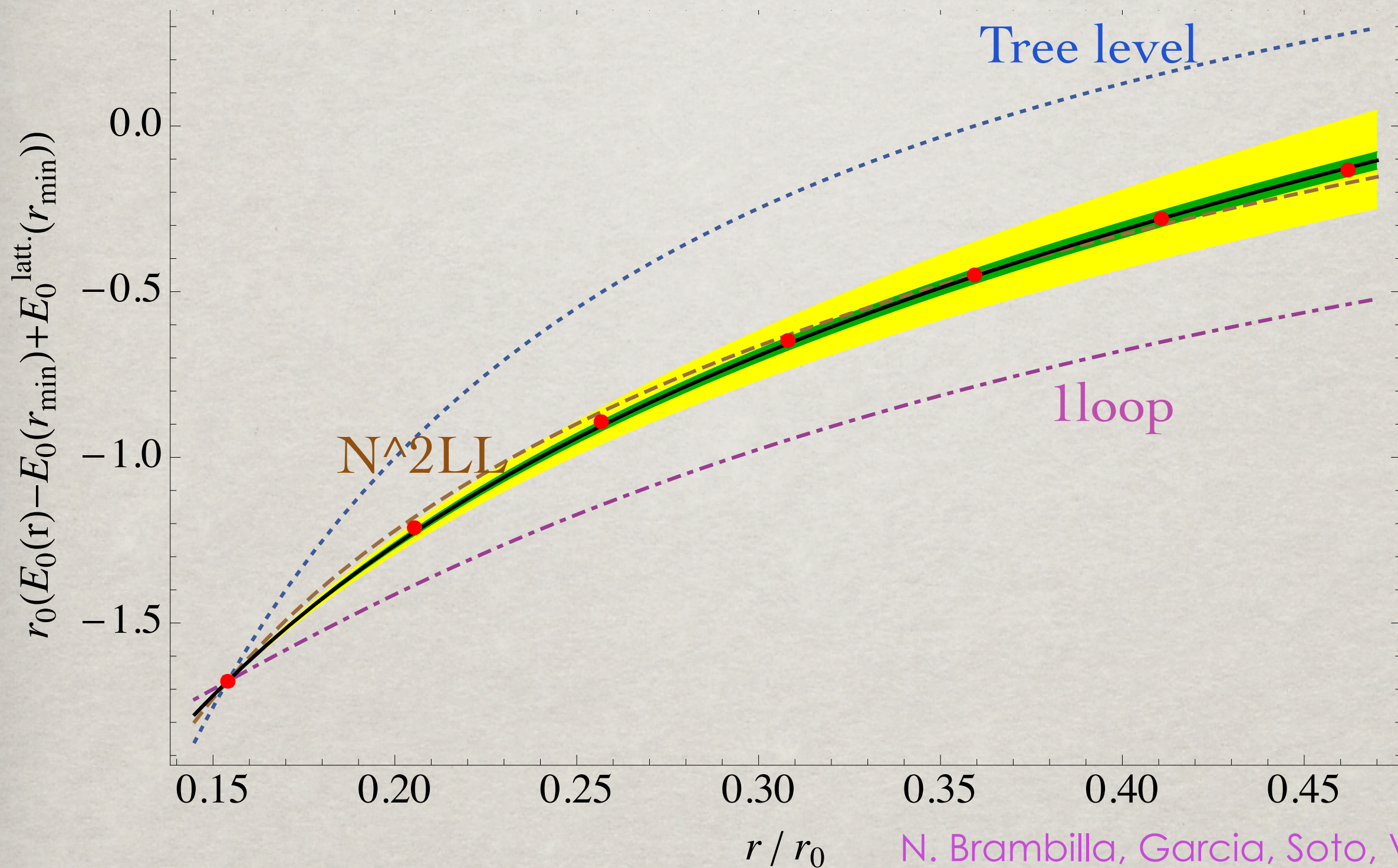
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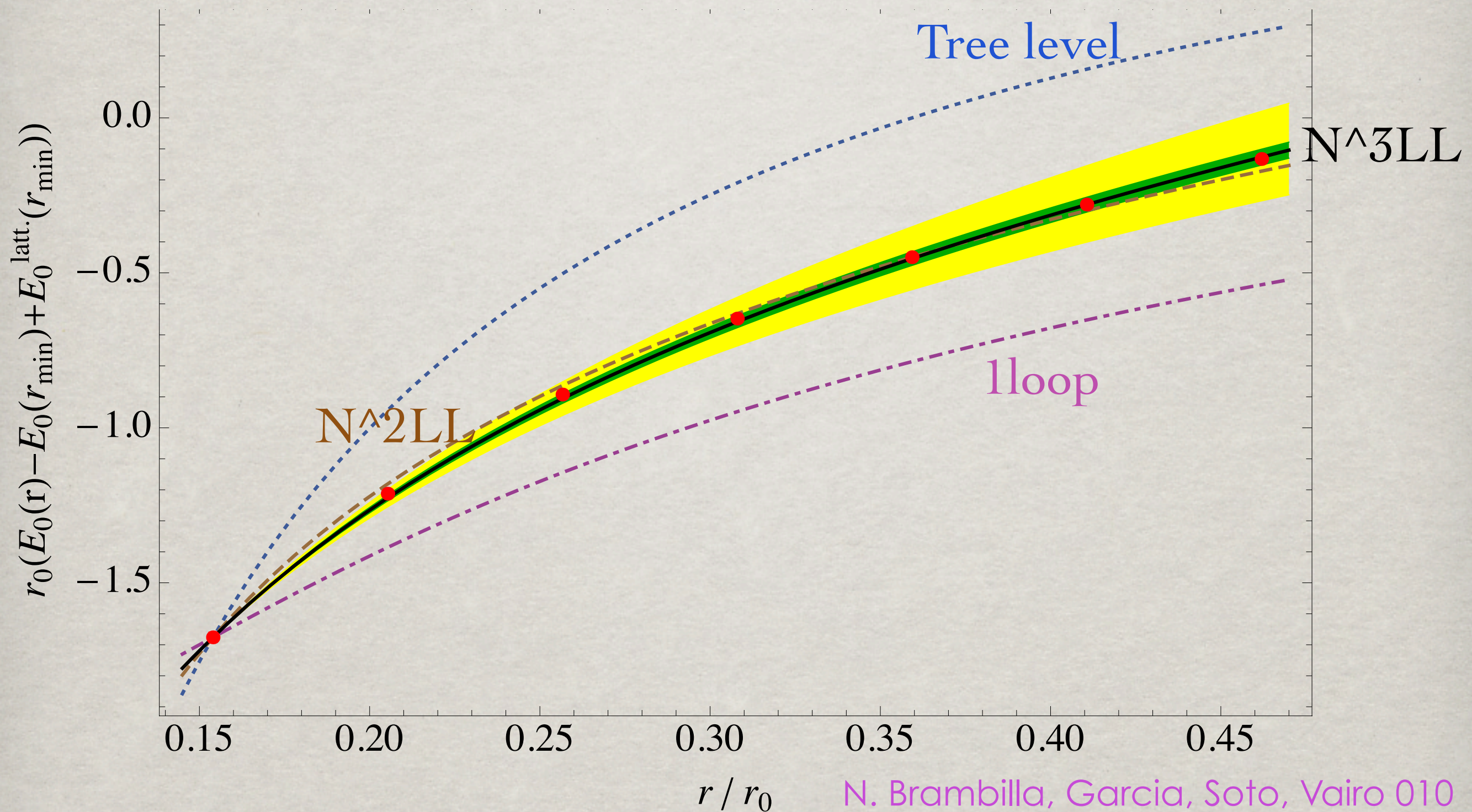
N. Brambilla, Garcia, Soto, Vairo 010

Yellow band : uncertainty in α_s ($r_0 \cdot \Lambda_{\text{QCD}}^{\overline{\text{MS}}}$)

Green band: uncertainty in higher order terms

Quarkonium singlet static energy at N³LL in comparison with lattice data (red points)

Necco Sommer 2002)

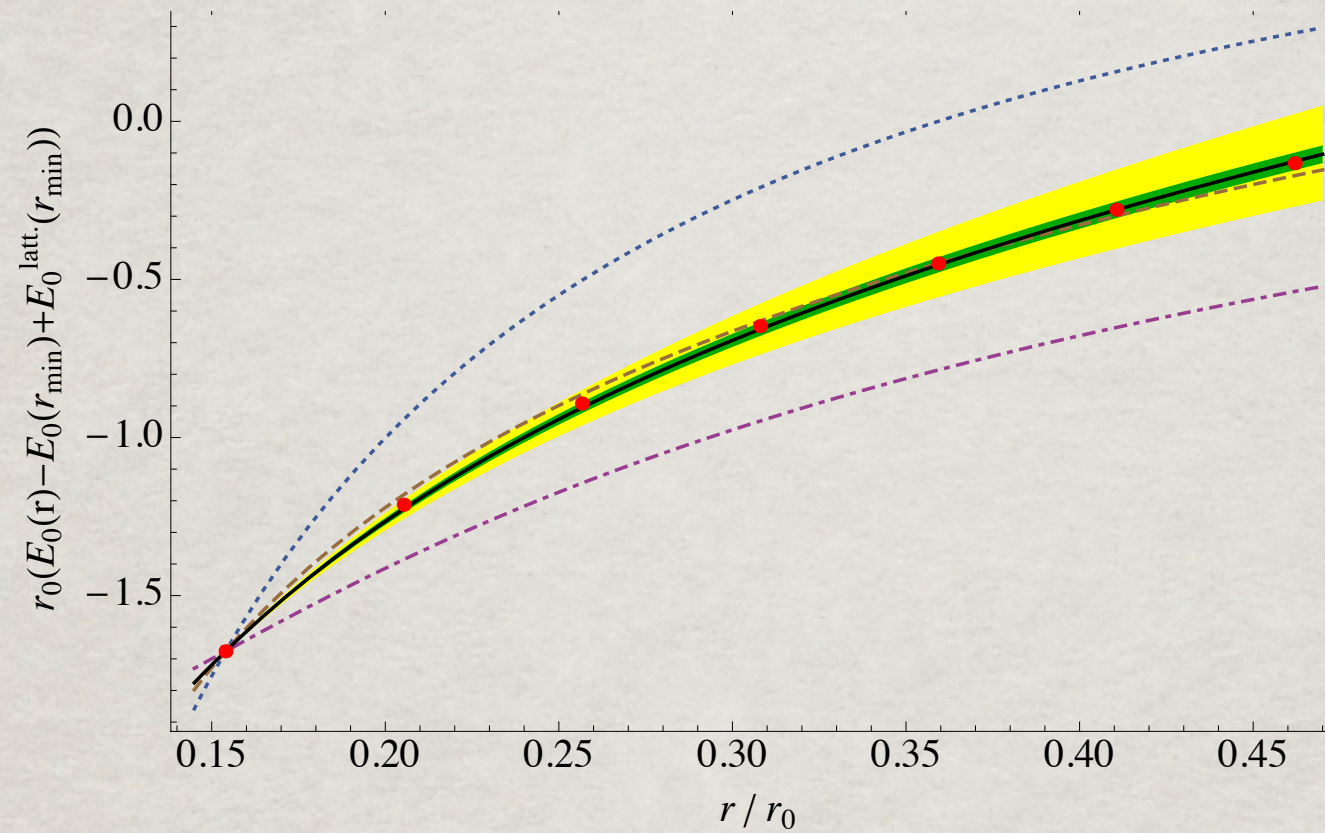


Yellow band : uncertainty in α_s ($r_0 \cdot \Lambda_{\text{QCD_MS}}$)

Green band: uncertainty in higher order terms

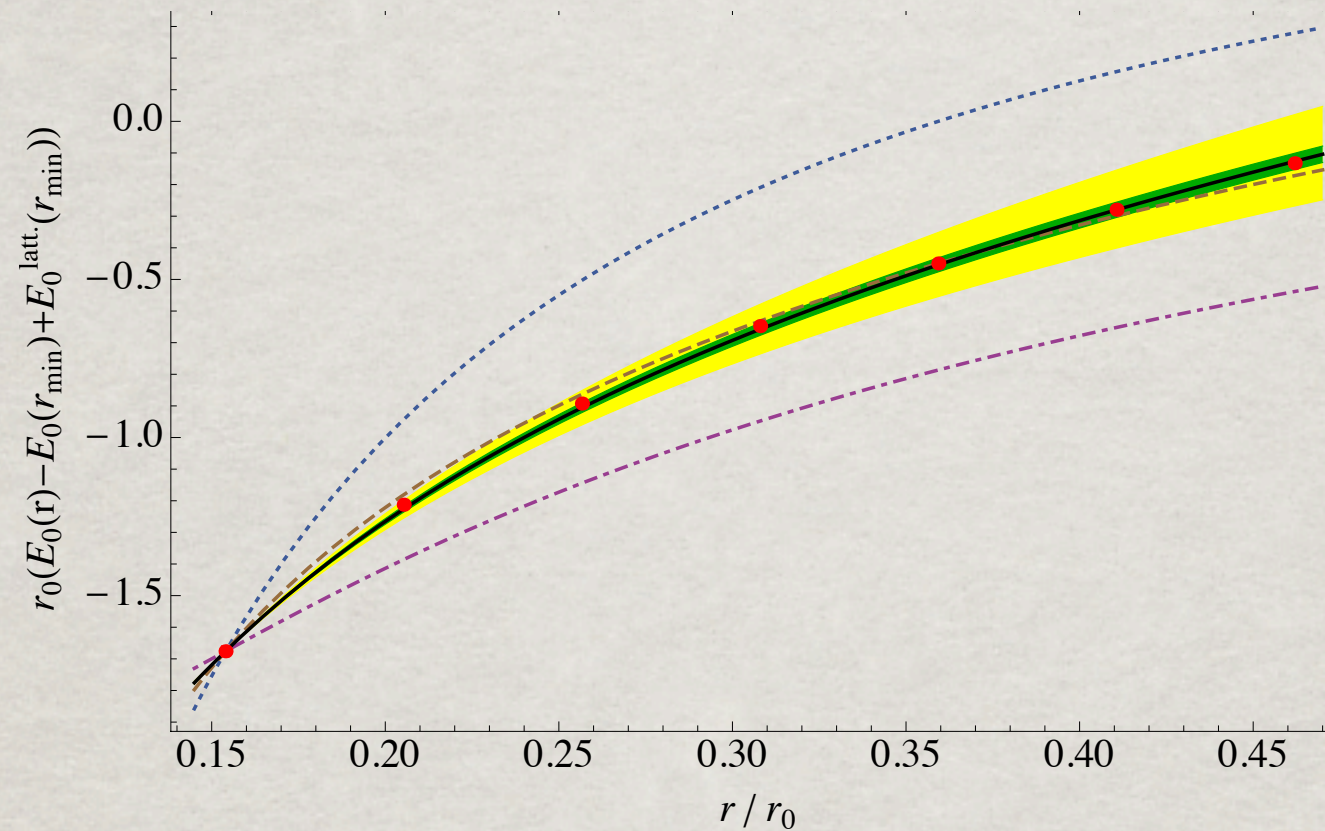
Quarkonium singlet static energy at N³LI in comparison with lattice data (red points)

Necco Sommer 2002)



Quarkonium singlet static energy at N³LI in comparison with lattice data (red points)

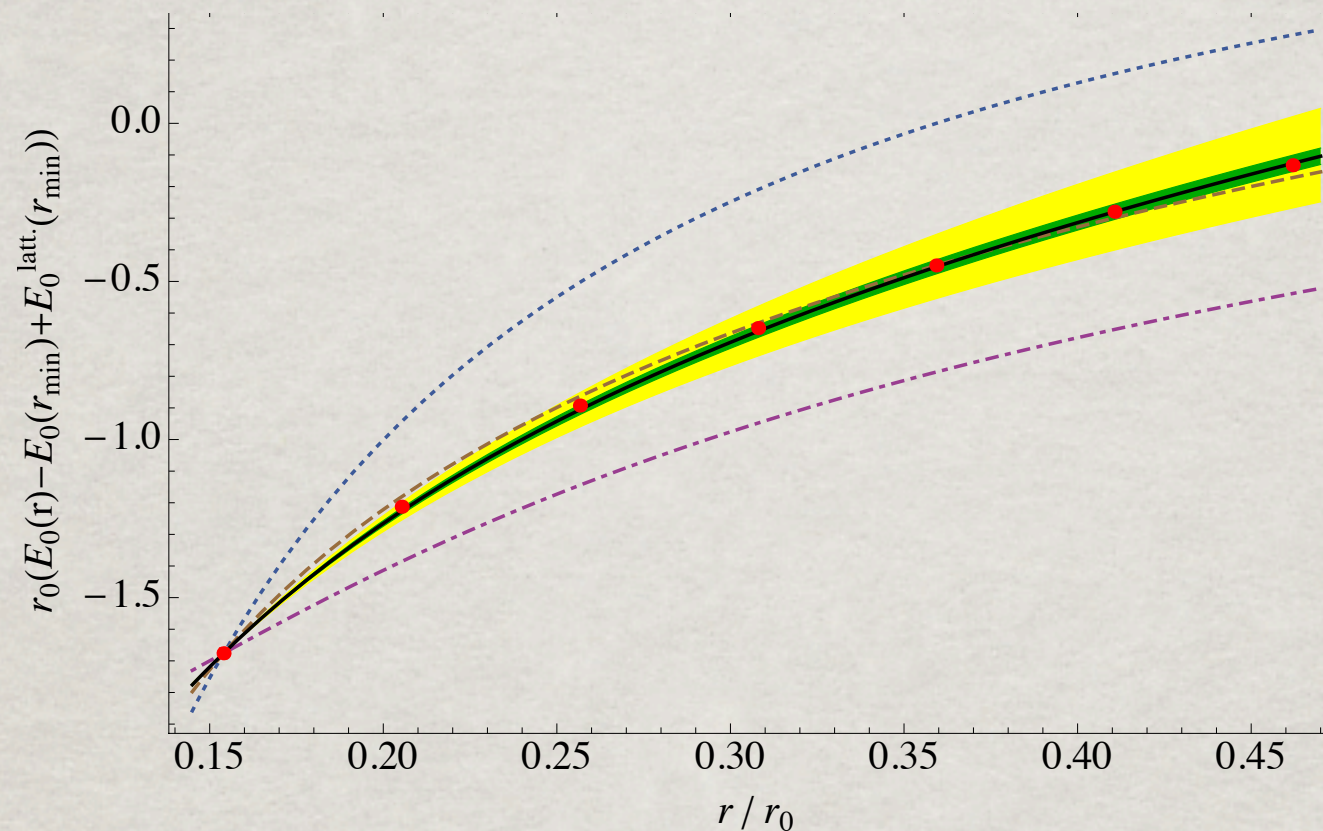
Necco Sommer 2002)



- Very good convergence of the QCD bound state perturbative series

Quarkonium singlet static energy at N³LI in comparison with lattice data (red points)

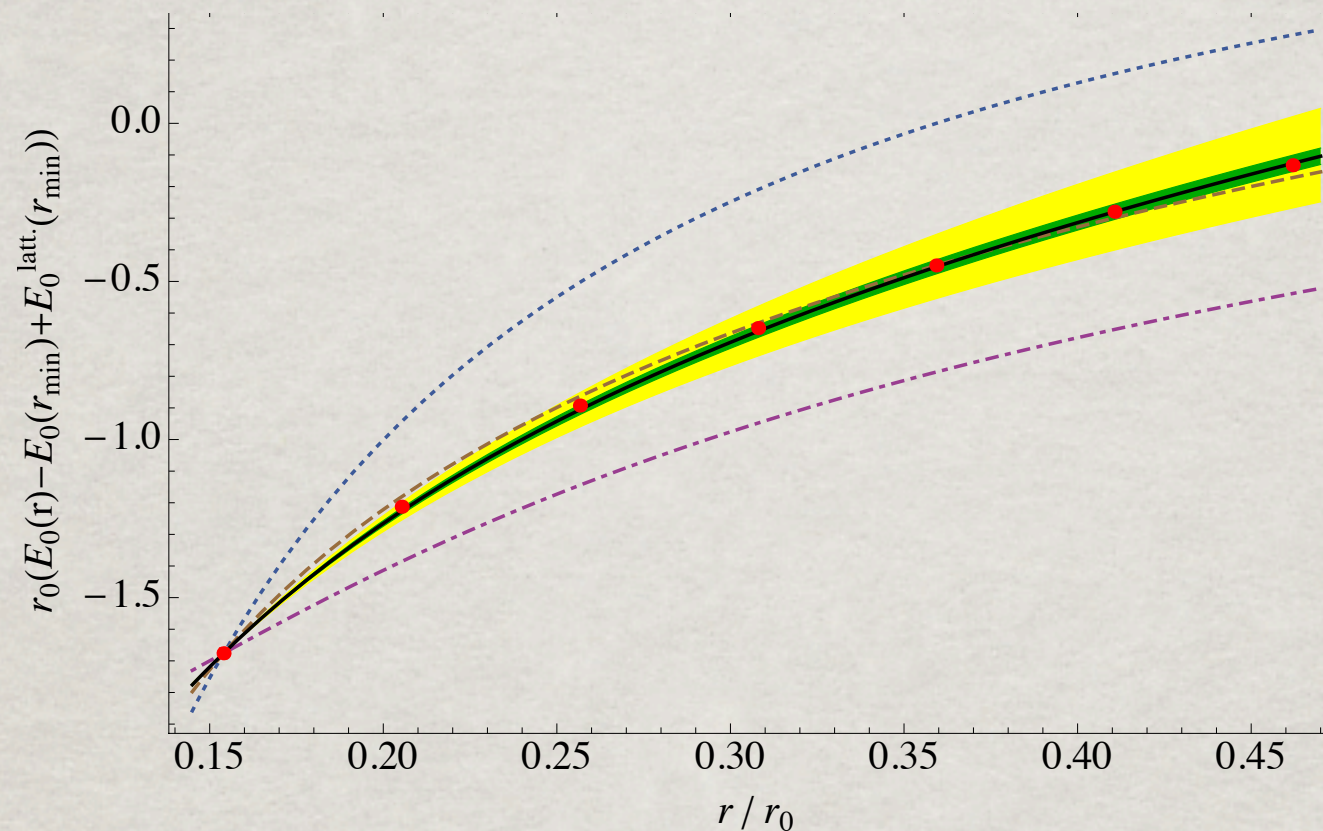
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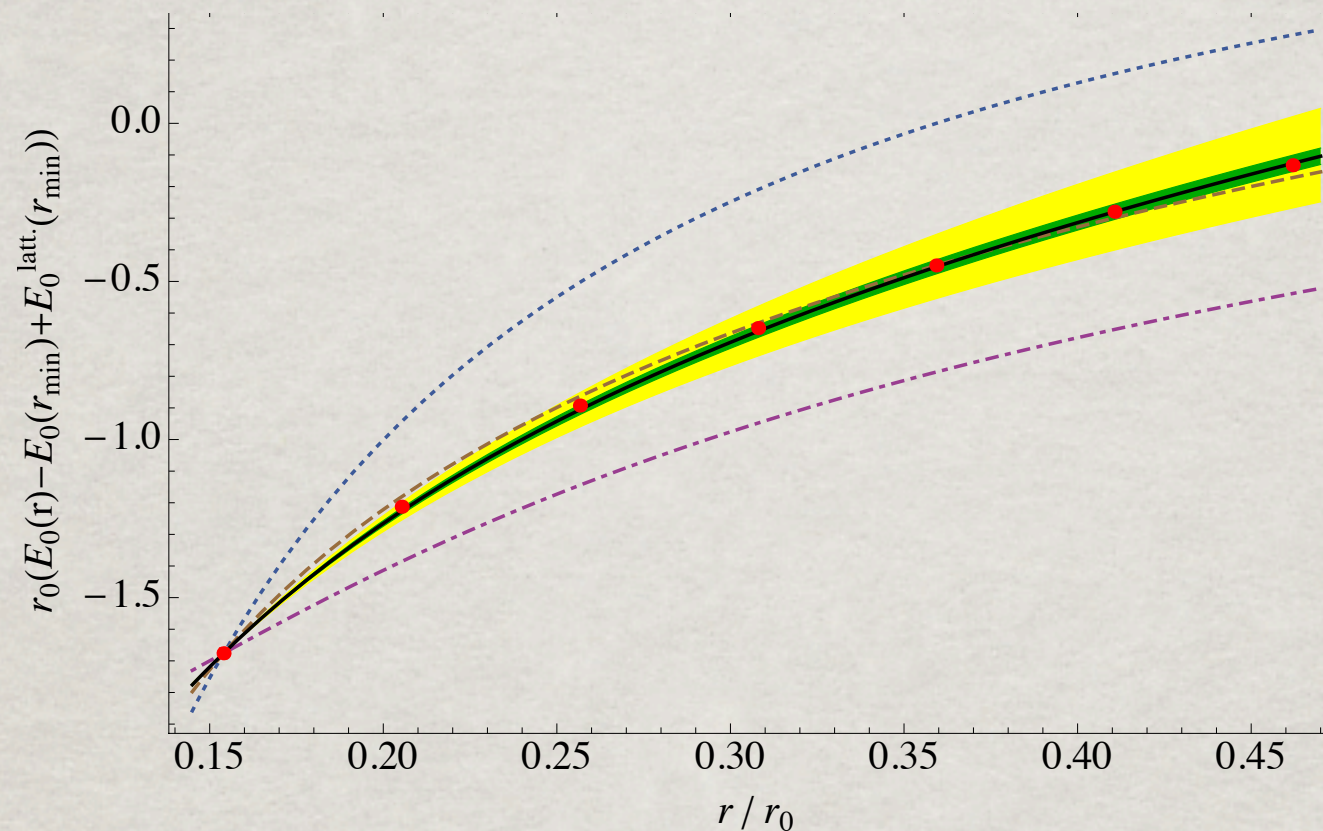
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Quarkonium singlet static energy at N³LI in comparison with lattice data (red points)

Necco Sommer 2002)



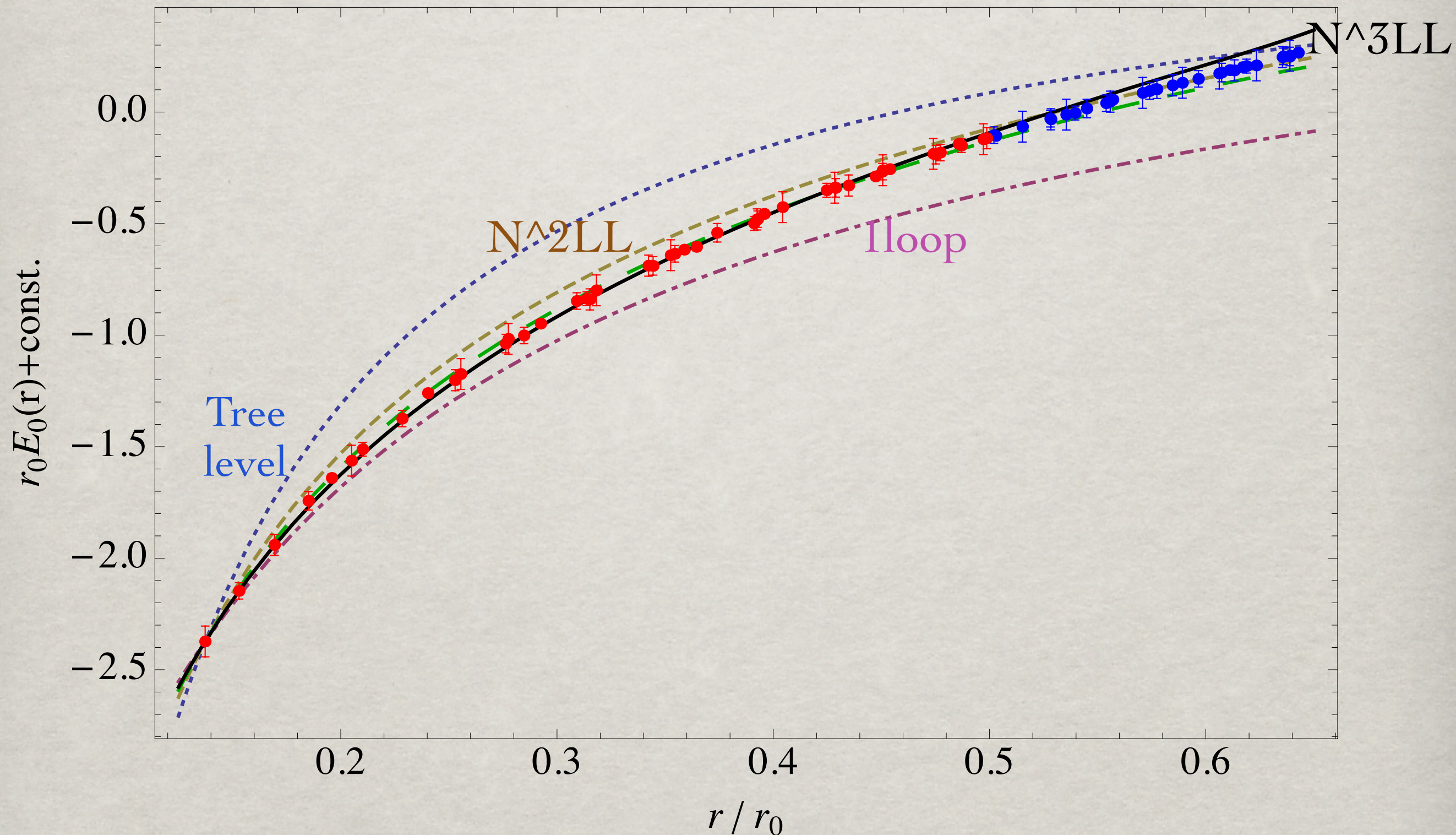
- Very good convergence of the QCD bound state perturbative series
- The lattice data are perfectly described from perturbation theory up to more than 0.2 fm
- Allows to rule out models: no string contribution at small r !
- Allows precise extraction of fundamental parameters of QCD

$$r_0 \Lambda_{\bar{M}S} = 0.622^{+0.019}_{-0.015}$$

N. Brambilla, Garcia, Soto, Vairo 010)

QQbar singlet static energy at N³LL in comparison with unquenched (n_f=2+1) lattice data (red points, blue points)

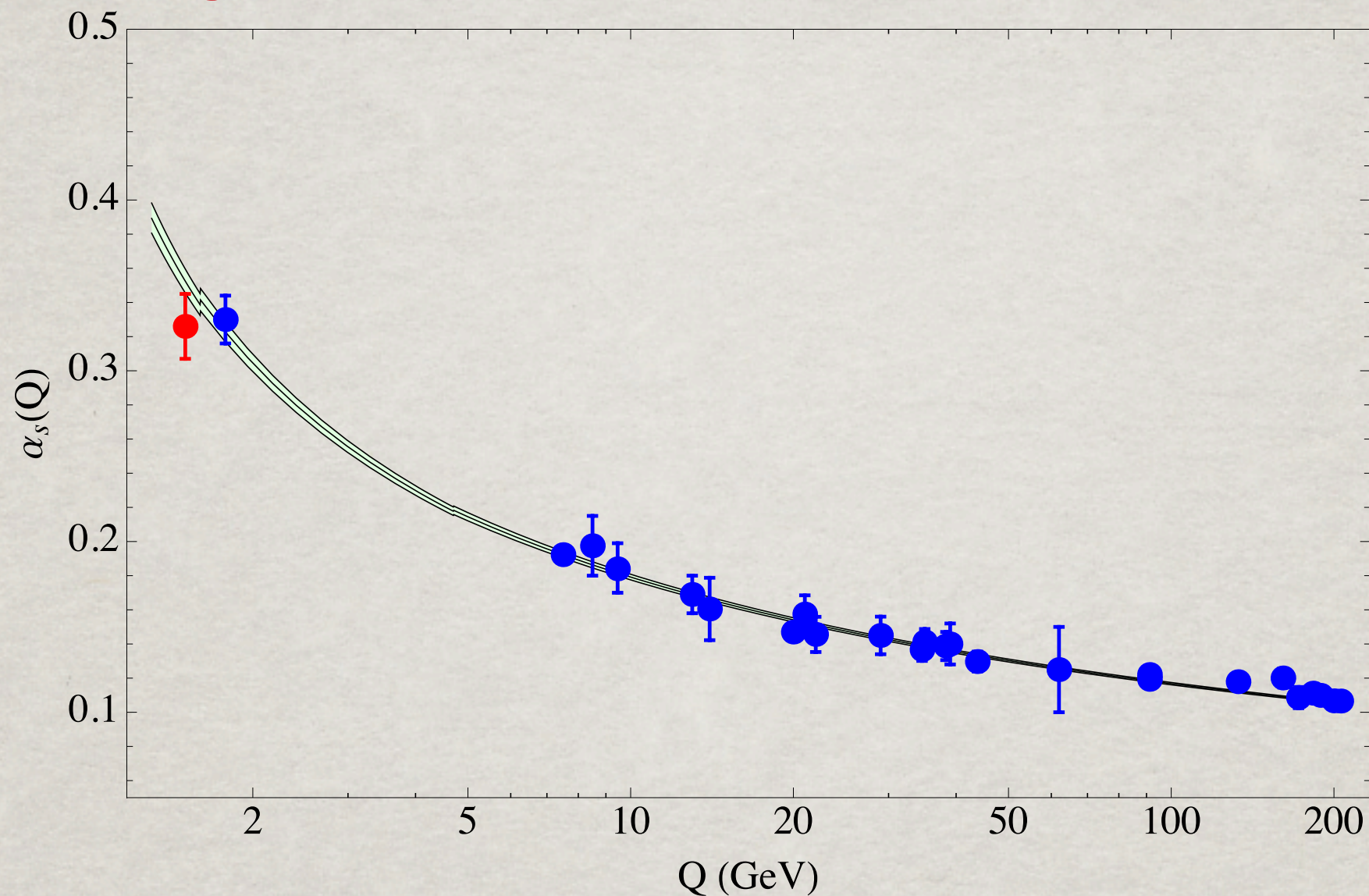
Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012



Good convergence to the lattice data

Lattice data less accurate in the unquenched case

α_s extraction



We obtain an extraction of alphas at **N³LO plus leading log resummation**
at the lowest energy scale (at the m_c mass)!

$$\alpha_s(\rho = 1.5 \text{ GeV}, n_f = 3) = 0.326 \pm 0.019$$

corresponding to

$$\alpha_s(M_z, n_f = 5) = 0.1156^{+0.0021}_{-0.0022}$$

Applications to Quarkonium physics: systems with small radius

for references see the QWG doc
[arXiv:1010.5827](https://arxiv.org/abs/1010.5827)

- c and b masses at NNLO, $N^3\text{LO}^*$, NNLL^* ;
- B_c mass at NNLO; Penin et al 04
- B_c^* , η_c , η_b masses at NLL; Kniehl et al 04
- Quarkonium $1P$ fine splittings at NLO;
- $\Upsilon(1S)$, η_b electromagnetic decays at NNLL;
- $\Upsilon(1S)$ and J/ψ radiative decays at NLO;
- $\Upsilon(1S) \rightarrow \gamma\eta_b$, $J/\psi \rightarrow \gamma\eta_c$ at NNLO;
- $t\bar{t}$ cross section at NNLL;
- QQq and QQQ baryons: potentials at NNLO, masses, hyperfine splitting, ... ; N. B. et al 010
- Thermal effects on quarkonium in medium: potential, masses (at $m\alpha_s^5$), widths, ...;

E1 transitions N. B., Pietrulewicz, Vairo 2012

$$\Gamma(J/\psi \rightarrow \gamma\eta_c) = 2.12(40) \text{ KeV}$$

$$\Gamma(\Upsilon(1S) \rightarrow \gamma\eta_b) = 15.18(51) \text{ eV}$$

$$\Gamma(\eta_b(1S) \rightarrow \gamma\gamma) = 0.54 \pm 0.15 \text{ keV}.$$

$$\Gamma(\eta_b(1S) \rightarrow \text{LH}) = 7\text{-}16 \text{ MeV}$$

N.B., Yu Jia, A. Vairo 2005, many other
M1 transitions Pineda et al 2013

Y. Kiyo, A. Pineda, A. Signer 2010

Magnetic dipole transitions

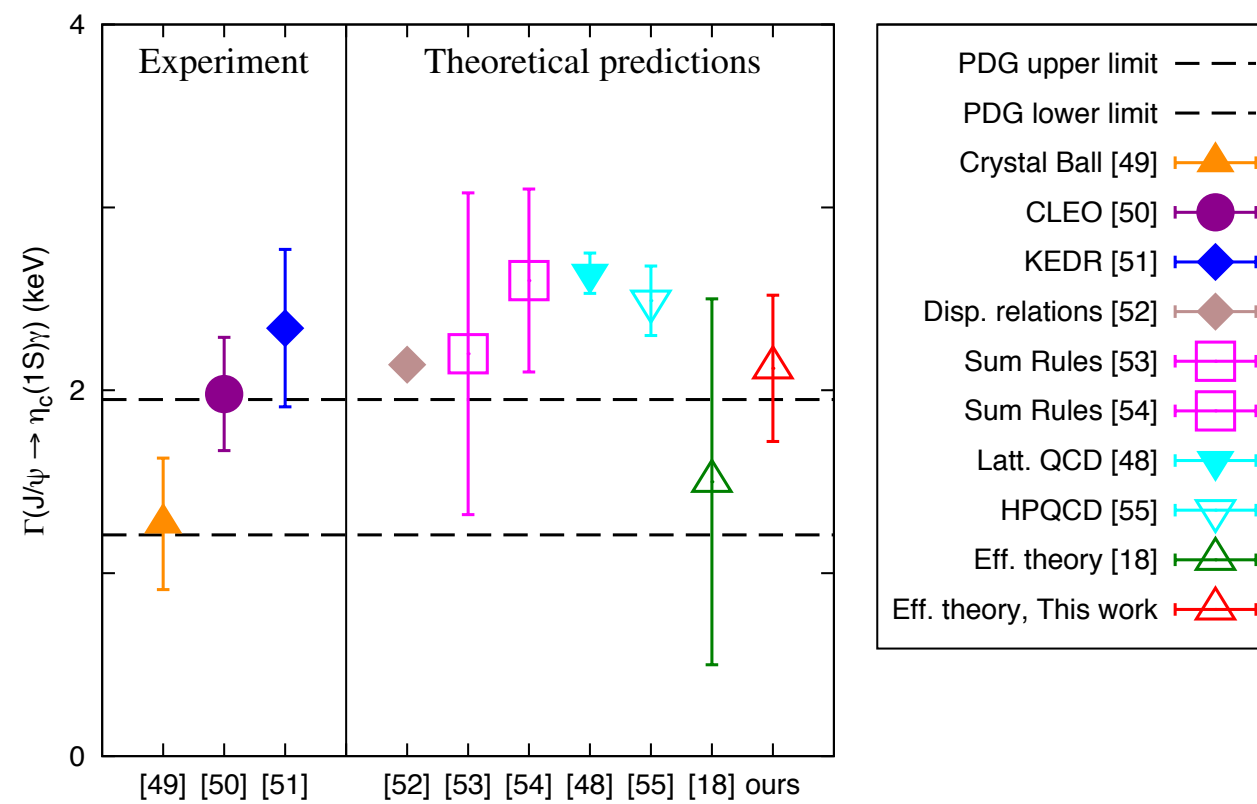


Figure : Comparison of different theoretical and experimental predictions for $\Gamma_{J/\psi \rightarrow \eta_c \gamma}$.

from Pineda at the charm013 conf

- No nonperturbative physics at order v^2
- Exact relations from Poincare invariance-> no scalar interaction
- No large anomalous magnetic moment
--> it receives no soft contribution

$$\kappa_Q = \frac{2\alpha_s(m)}{(3\pi)} + O(\alpha_s^2)$$

N.B., Yu Jia, A. Vairo 2005

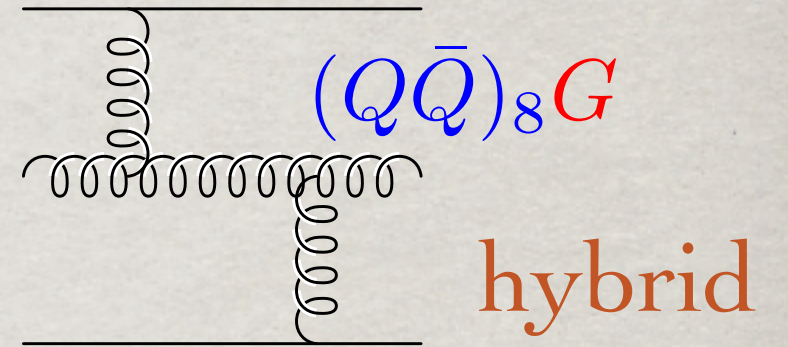
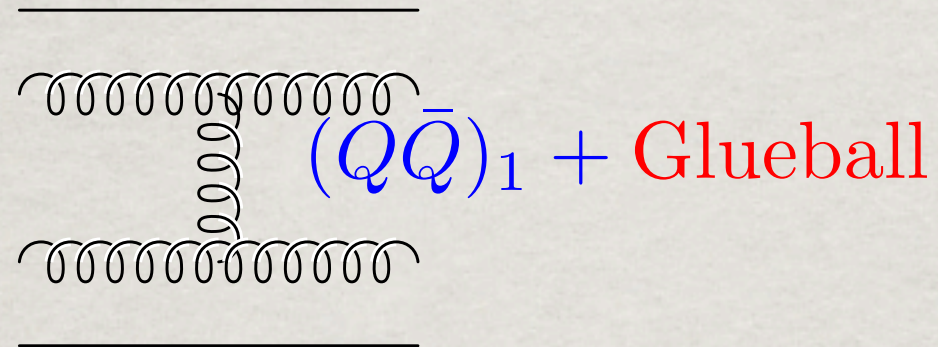
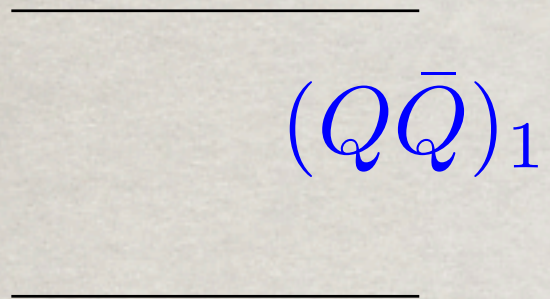
Quarkonium systems with large radius

$$r \sim \Lambda_{QCD}^{-1}$$

— Hitting the scale Λ_{QCD} $r \sim \Lambda_{\text{QCD}}^{-1}$

— Hitting the scale Λ_{QCD}

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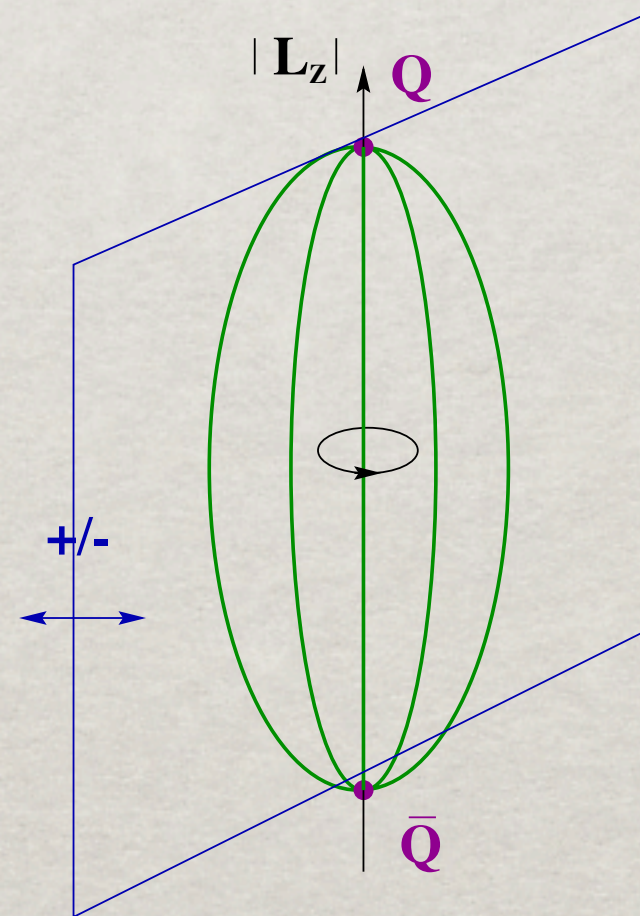
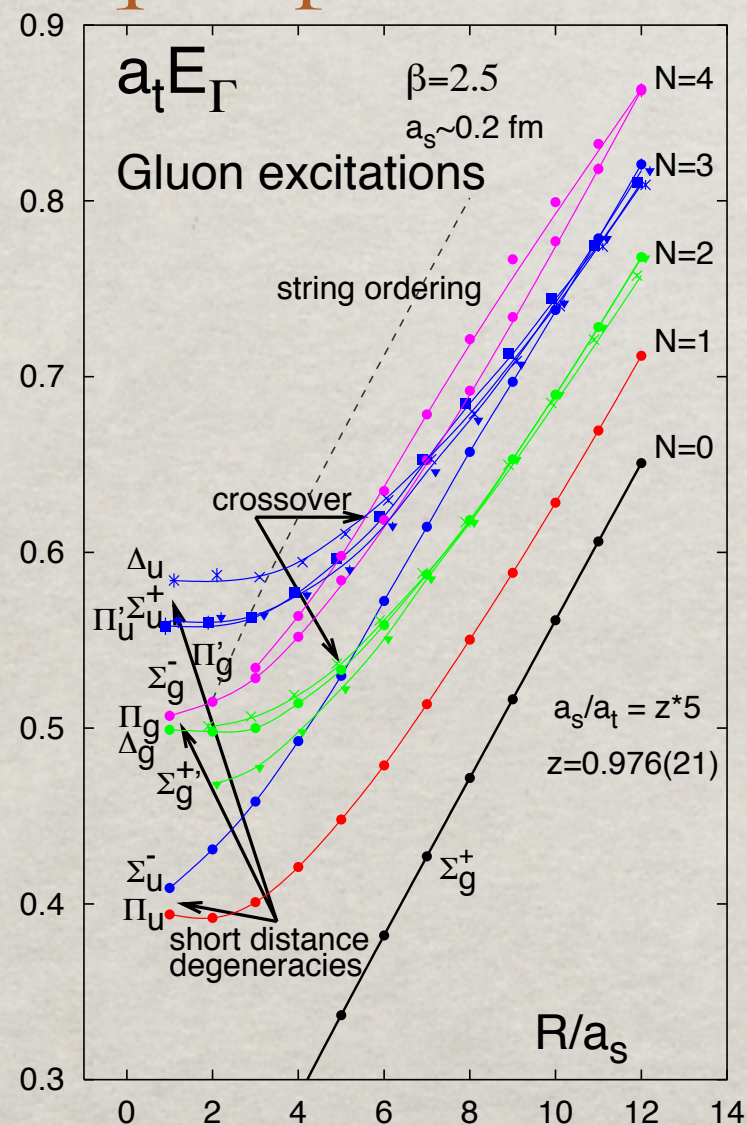
$(Q\bar{Q})_1$

$(Q\bar{Q})_1 + \text{Glueball}$

$(Q\bar{Q})_8 G$
hybrid

Static qcd spectrum

L
a
t
t
i
c
e

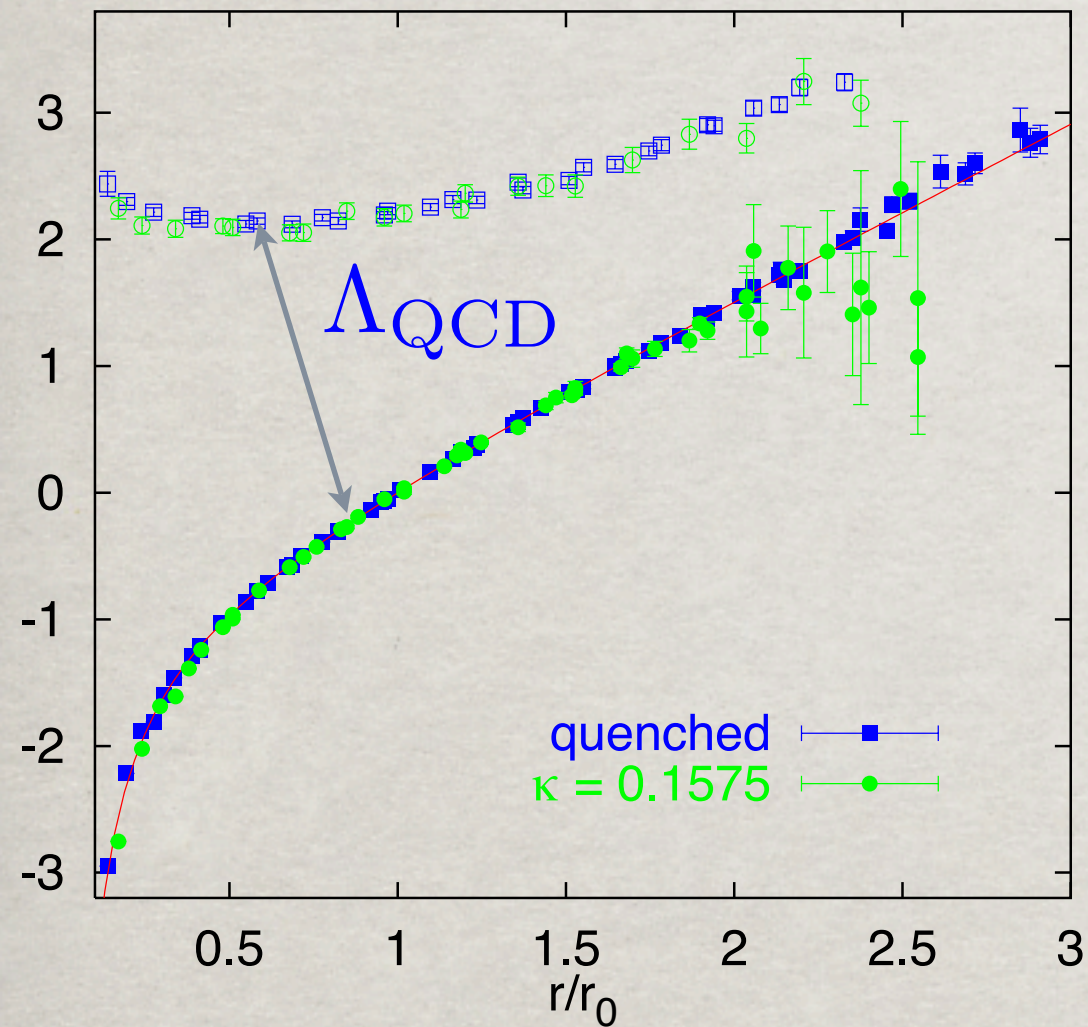


Symmetries of a
diatomic molecule
+ C.C.

- a) $|L_z| = 0, 1, 2, \dots$
 $= \Sigma, \Pi, \Delta \dots$
- b) CP (u/g)
- c) Reflection (+/-)
(for Σ only)

Quarkonium develops a gap to hybrids

Bali et al. 98



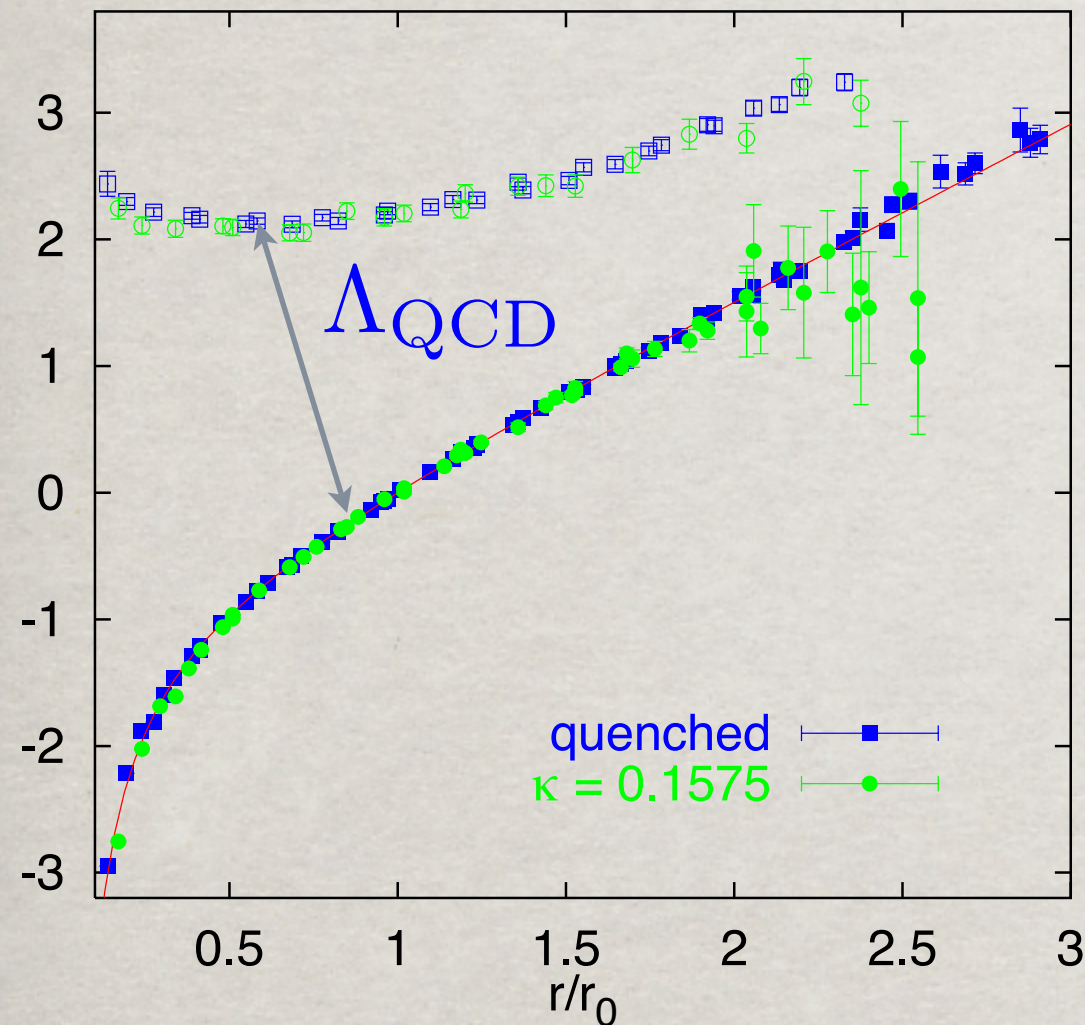
- $mv \sim \Lambda_{QCD}$

- integrate out all scales above mv^2

- gluonic excitations develop a gap Λ_{QCD} and are integrated out

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⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$

\Rightarrow The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \text{Tr} \left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S \right\}$$

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Brambilla Pineda Soto Vairo 00

- A potential description emerges from the EFT

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Brambilla Pineda Soto Vairo 00

- A potential description emerges from the EFT
- The potentials $V = \text{Re}V + i\text{Im}V$ from QCD in the matching:
get spectra and decays
- V to be calculated on the lattice or in QCD vacuum models

Quarkonium singlet static potential

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2} (V_{SD} + V_{VD})$$

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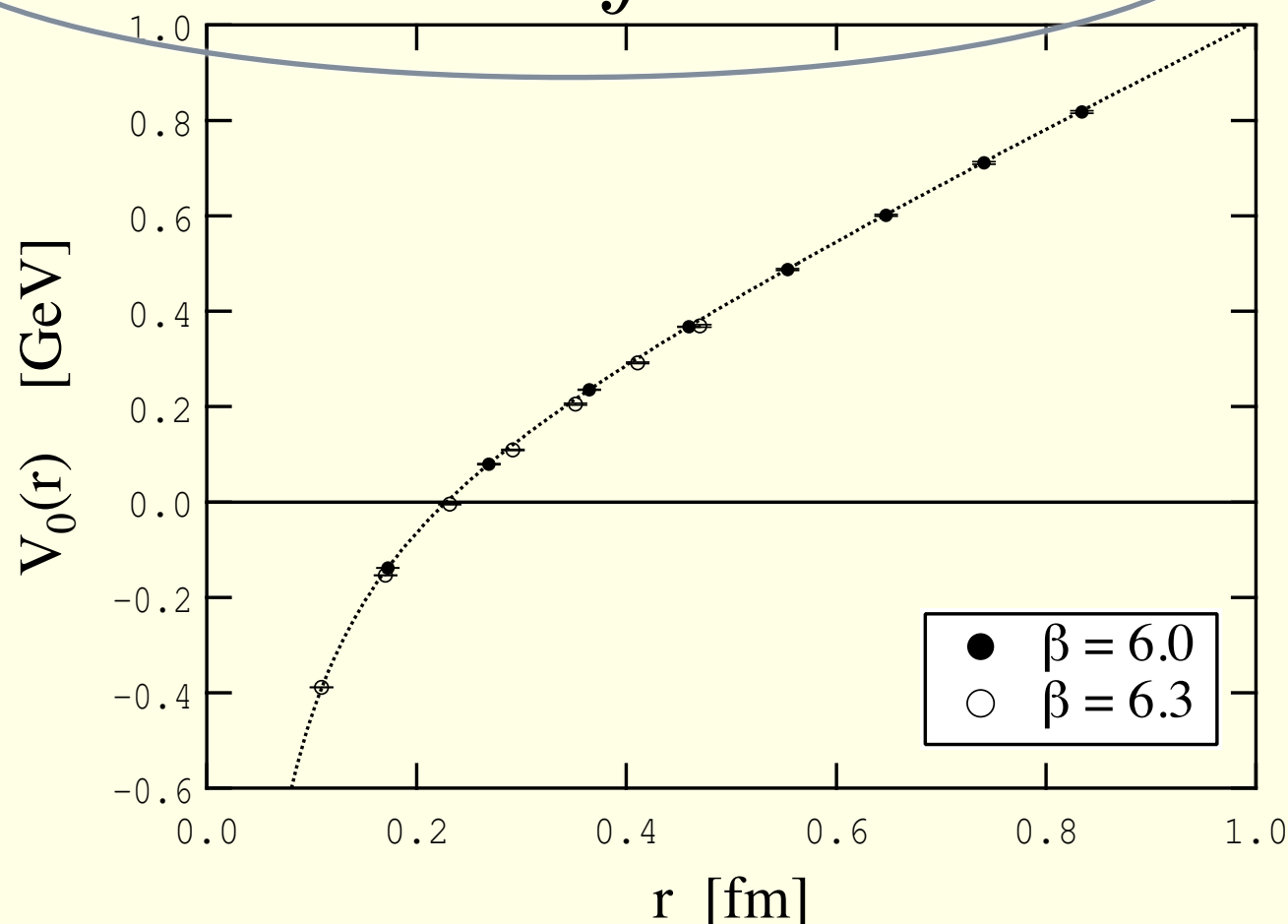
$$W = \langle \exp \{ ig \oint A^\mu dx_\mu \} \rangle$$

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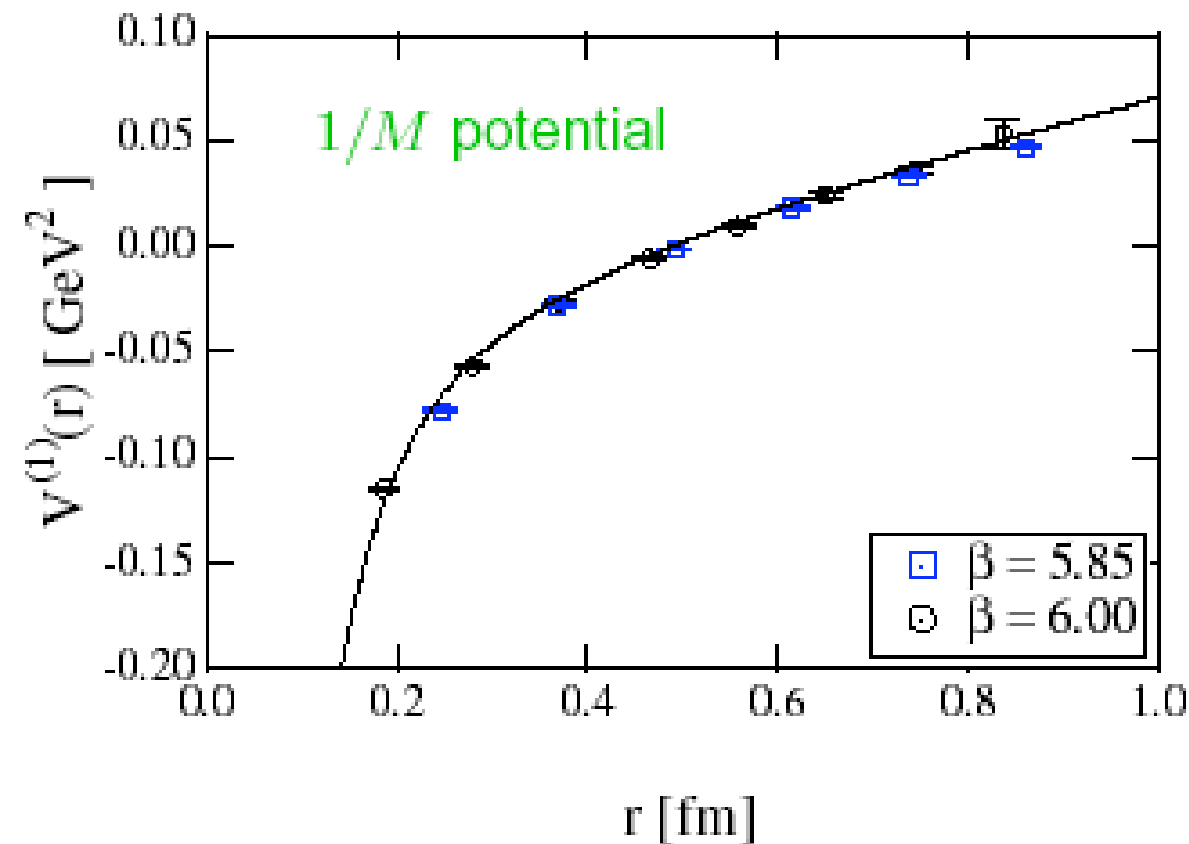
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Quarkonium singlet static potential

Potentials are given in a factorized form as product of NRQCD matching coefficients and low energy terms. These are gauge invariant wilson loop with electric and magnetic insertions



○ Koma Koma Wittig PoS LAT2007(07)111

$$\frac{V_s^{(1)}}{m} = -\frac{1}{2m} \int_0^\infty dt t \langle \text{Wilson Loop with Electric Insertions} \rangle$$

QCD Spin dependent potentials

$$\begin{aligned}
 V_{\text{SD}}^{(2)} = & \frac{1}{r} \left(c_F \epsilon^{kij} \frac{2r^k}{r} i \int_0^\infty dt t \left\langle \begin{array}{c} \text{E} \\ \boxed{1 \quad j} \\ \text{B} \end{array} \right\rangle - \frac{1}{2} V_s^{(0)'} \right) (\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L} \\
 & - c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\left\langle \begin{array}{c} \boxed{1 \quad j} \\ \text{---} \end{array} \right\rangle - \frac{\delta_{ij}}{3} \left\langle \begin{array}{c} \boxed{\quad \quad} \\ \text{---} \end{array} \right\rangle \right) \\
 & \quad \times \left(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}}) \right) \\
 & + \left(\frac{2}{3} c_F^2 i \int_0^\infty dt \left\langle \begin{array}{c} \boxed{\quad \quad} \\ \text{---} \end{array} \right\rangle - 4(d_2 + C_F d_4) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S}_2
 \end{aligned}$$

Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99

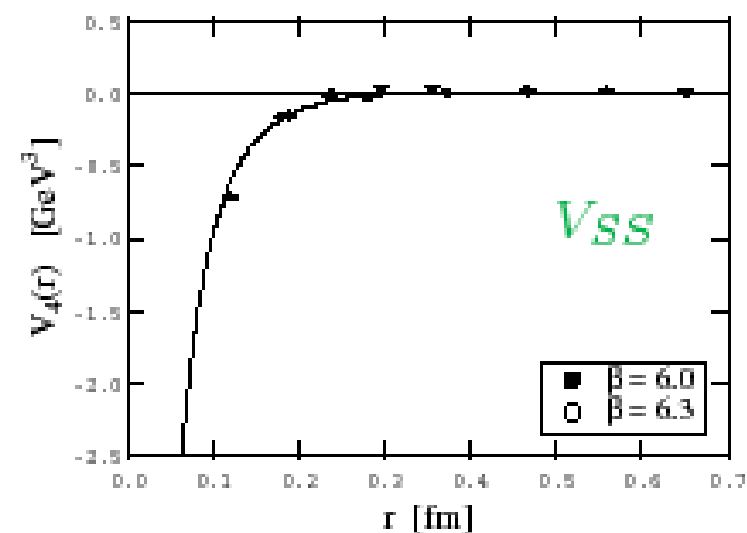
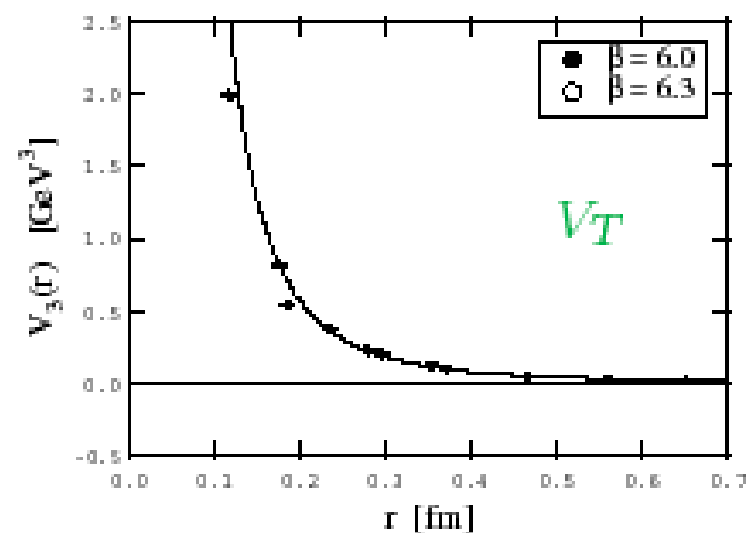
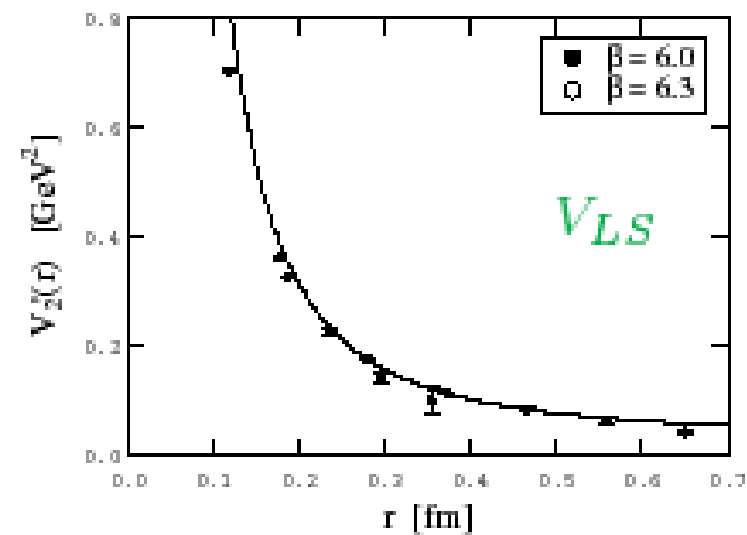
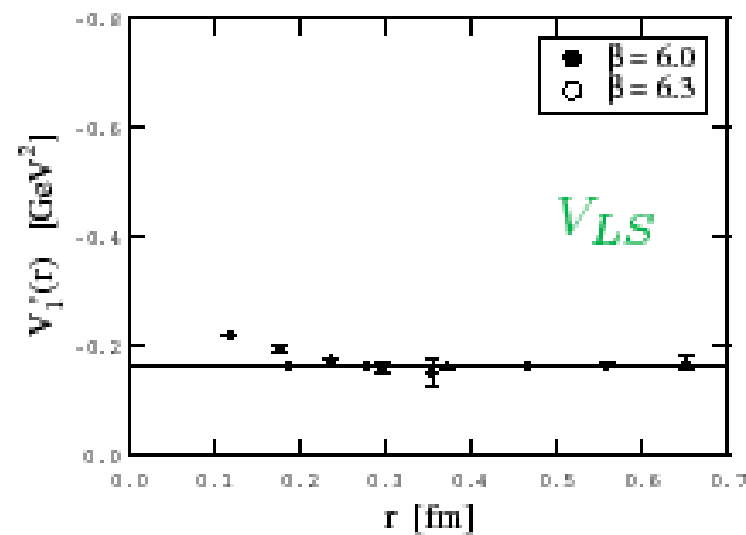
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 & + \left(\frac{2}{3} c_F^2 i \int_0^\infty dt \left\langle \begin{array}{c} \boxed{ } \\ \hline \end{array} \right\rangle - 4(d_2 + C_F d_4) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S}_2
 \end{aligned}$$

Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99

-factorization; power counting;
 QM divergences absorbed by
 NRQCD matching coefficients

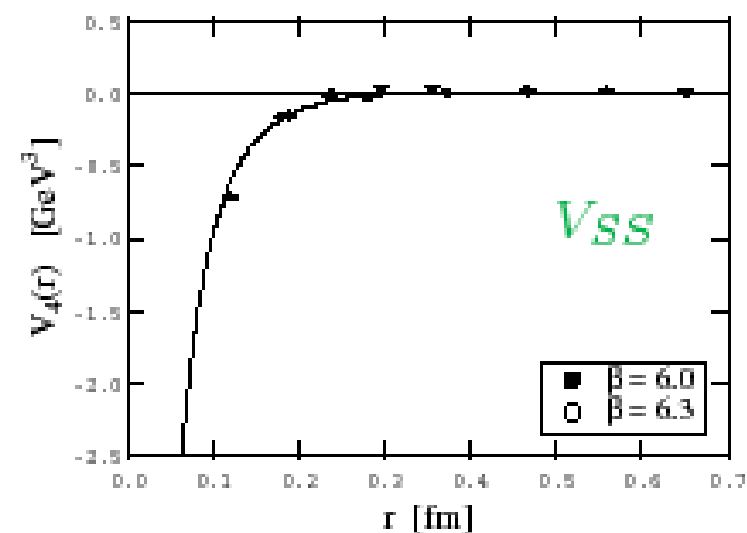
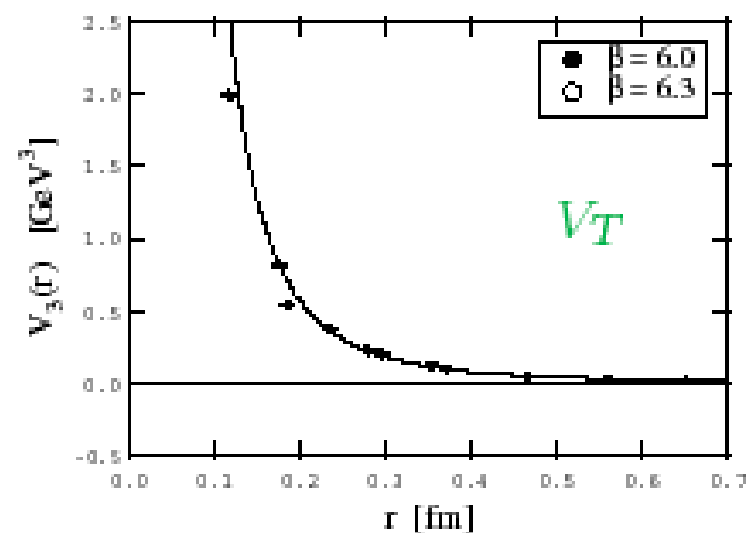
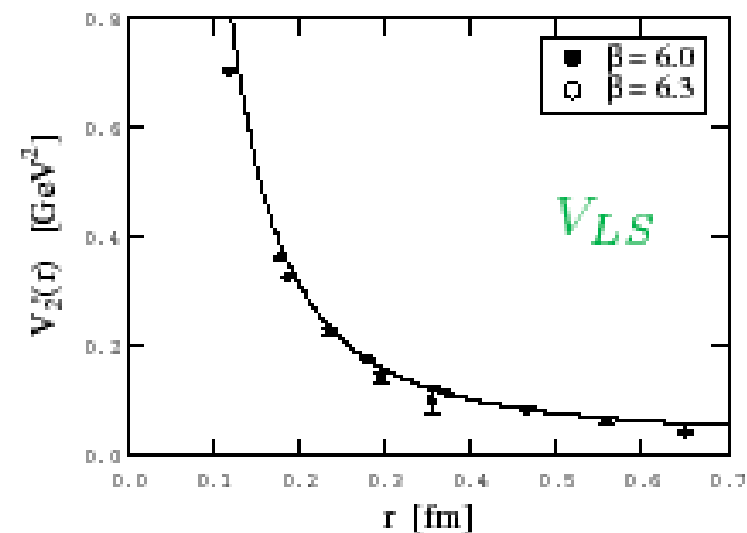
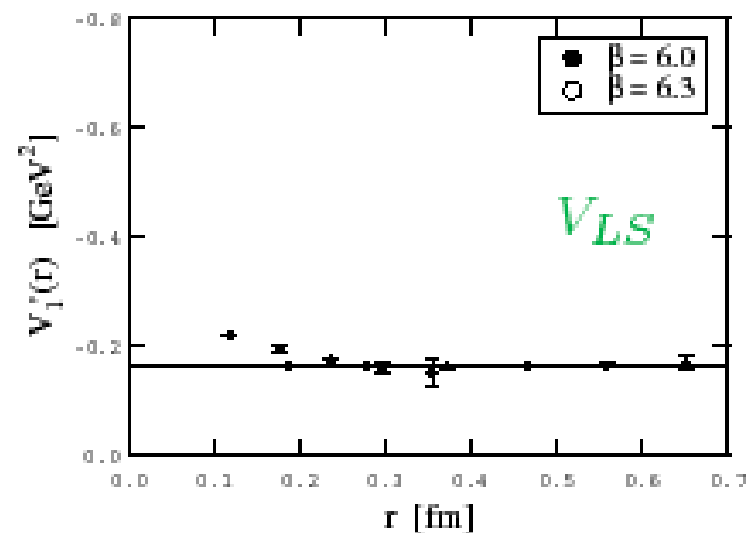
Spin dependent potentials



Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

Spin dependent potentials



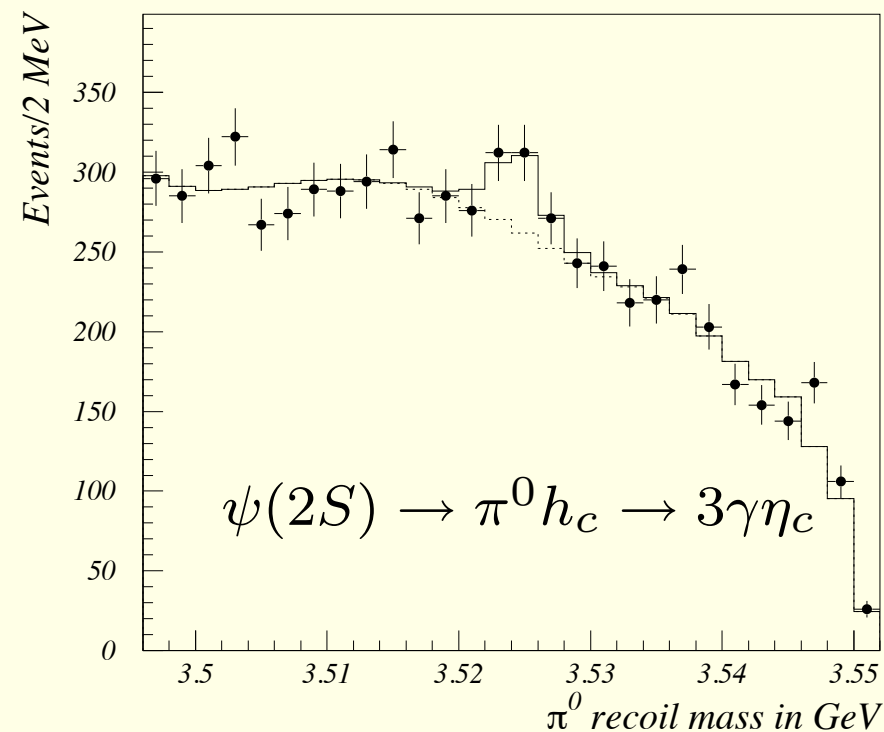
Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

Such data can distinguish different models for the dynamics of low energy QCD

Confirmed in the spectrum, e.g. no long range spin-spin interaction

h_c, h_b



$$M_{h_c} = 3524.4 \pm 0.6 \pm 0.4 \text{ MeV}$$

○ CLEO PRL 95 (2005) 102003

$$M_{h_c} = 3525.8 \pm 0.2 \pm 0.2 \text{ MeV},$$

$$\Gamma < 1 \text{ MeV}$$

○ E835 PRD 72 (2005) 032001

$$M_{h_c} = 3525.40 \pm 0.13 \pm 0.18 \text{ MeV},$$

$$\Gamma < 1.44 \text{ MeV}$$

○ BES PRL 104 (2010) 132002

To be compared with $M_{c.o.g.}(1P) = 3525.36 \pm 0.2 \pm 0.2 \text{ MeV}$.

● Also

$$M_{h_b} = 9902 \pm 4 \pm 1 \text{ MeV}$$

○ BABAR arXiv:1102.4565

To be compared with $M_{c.o.g.}(1P) = 9899.87 \pm 0.28 \pm 0.31 \text{ MeV}$.

Spin-independent $1/m^2$ potentials

$$\begin{aligned}
 V_{\text{SI}}^{(2)} = & \textcolor{violet}{p}^i \left(i \int_0^\infty dt t^2 \left\langle \begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \text{i} & \text{j} \\ \hline \end{array} \right\rangle + \left\langle \begin{array}{|c|c|} \hline & \bullet \\ \hline \text{i} & \text{j} \\ \hline \bullet & \\ \hline \end{array} \right\rangle \right) \textcolor{violet}{p}^j \\
 & - \frac{\textcolor{red}{c}_F^2}{2} i \int_0^\infty dt \left\langle \begin{array}{|c|c|} \hline \textcolor{blue}{B} & \\ \hline & \\ \hline & \\ \hline \end{array} \right\rangle + \left(\textcolor{red}{d}_1 + \frac{4}{3} \textcolor{red}{d}_3 + \frac{4}{3} \pi \alpha_s \textcolor{red}{c}_D \right) \delta^{(3)}(\mathbf{r}) \\
 & - i \int_0^\infty dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 (t_2 - t_3)^2 \left(\left\langle \begin{array}{|c|c|} \hline \bullet & \bullet & \bullet & \bullet \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right\rangle + \left\langle \begin{array}{|c|c|} \hline \bullet & \bullet & & \\ \hline & & & \\ \hline & & \bullet & \bullet \\ \hline \end{array} \right\rangle \right) \\
 & + \int_0^\infty dt_1 \int_0^{t_1} dt_2 (t_1 - t_2)^2 \nabla^i \\
 & \quad \times \left(\left\langle \begin{array}{|c|c|} \hline \bullet & \bullet & \bullet \\ \hline \text{i} & & \\ \hline & & \\ \hline \end{array} \right\rangle + \frac{1}{2} \left\langle \begin{array}{|c|c|} \hline \bullet & & & \\ \hline \text{i} & & & \\ \hline & \bullet & \bullet & \\ \hline \end{array} \right\rangle + \frac{1}{2} \left\langle \begin{array}{|c|c|} \hline & \bullet & \bullet & \\ \hline \text{i} & & & \\ \hline \bullet & & & \\ \hline \end{array} \right\rangle \right) \\
 & - 2 \textcolor{red}{b}_3 f_{abc} \int d^3 \mathbf{x} g \langle\langle G_{\mu\nu}^a(\mathbf{x}) G_{\mu\alpha}^b(\mathbf{x}) G_{\nu\alpha}^c(\mathbf{x}) \rangle\rangle_{\square}^c
 \end{aligned}$$

Spin-independent $1/m^2$ potentials

$$\begin{aligned}
 V_{\text{SI}}^{(2)} = & \textcolor{violet}{p}^i \left(i \int_0^\infty dt t^2 \left\langle \begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \text{i} & \text{j} \\ \hline \end{array} \right\rangle + \left\langle \begin{array}{|c|c|} \hline \bullet & \\ \hline \text{i} & \text{j} \\ \hline \bullet & \\ \hline \end{array} \right\rangle \right) \textcolor{violet}{p}^j \\
 & - \frac{\textcolor{red}{c}_F^2}{2} i \int_0^\infty dt \left\langle \begin{array}{|c|c|} \hline \textcolor{blue}{B} & \\ \hline & \\ \hline \end{array} \right\rangle + \left(\textcolor{red}{d}_1 + \frac{4}{3} \textcolor{red}{d}_3 + \frac{4}{3} \pi \alpha_s \textcolor{red}{c}_D \right) \delta^{(3)}(\mathbf{r}) \\
 & - i \int_0^\infty dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 (t_2 - t_3)^2 \left(\left\langle \begin{array}{|c|c|} \hline \bullet & \bullet & \bullet & \bullet \\ \hline & & & \\ \hline \end{array} \right\rangle + \left\langle \begin{array}{|c|c|} \hline \bullet & \bullet & & \\ \hline & & \bullet & \bullet \\ \hline \end{array} \right\rangle \right) \\
 & + \int_0^\infty dt_1 \int_0^{t_1} dt_2 (t_1 - t_2)^2 \nabla^i \\
 & \quad \times \left(\left\langle \begin{array}{|c|c|} \hline \bullet & \bullet & \bullet \\ \hline \text{i} & & \\ \hline \end{array} \right\rangle + \frac{1}{2} \left\langle \begin{array}{|c|c|} \hline \bullet & & \\ \hline \text{i} & \bullet & \bullet \\ \hline \end{array} \right\rangle + \frac{1}{2} \left\langle \begin{array}{|c|c|} \hline & \bullet & \bullet \\ \hline \bullet & & \\ \hline \end{array} \right\rangle \right) \\
 & - 2 \textcolor{red}{b}_3 f_{abc} \int d^3 \mathbf{x} g \langle\langle \textcolor{blue}{G}_{\mu\nu}^a(\mathbf{x}) \textcolor{blue}{G}_{\mu\alpha}^b(\mathbf{x}) \textcolor{blue}{G}_{\nu\alpha}^c(\mathbf{x}) \rangle\rangle_{\square}^c
 \end{aligned}$$

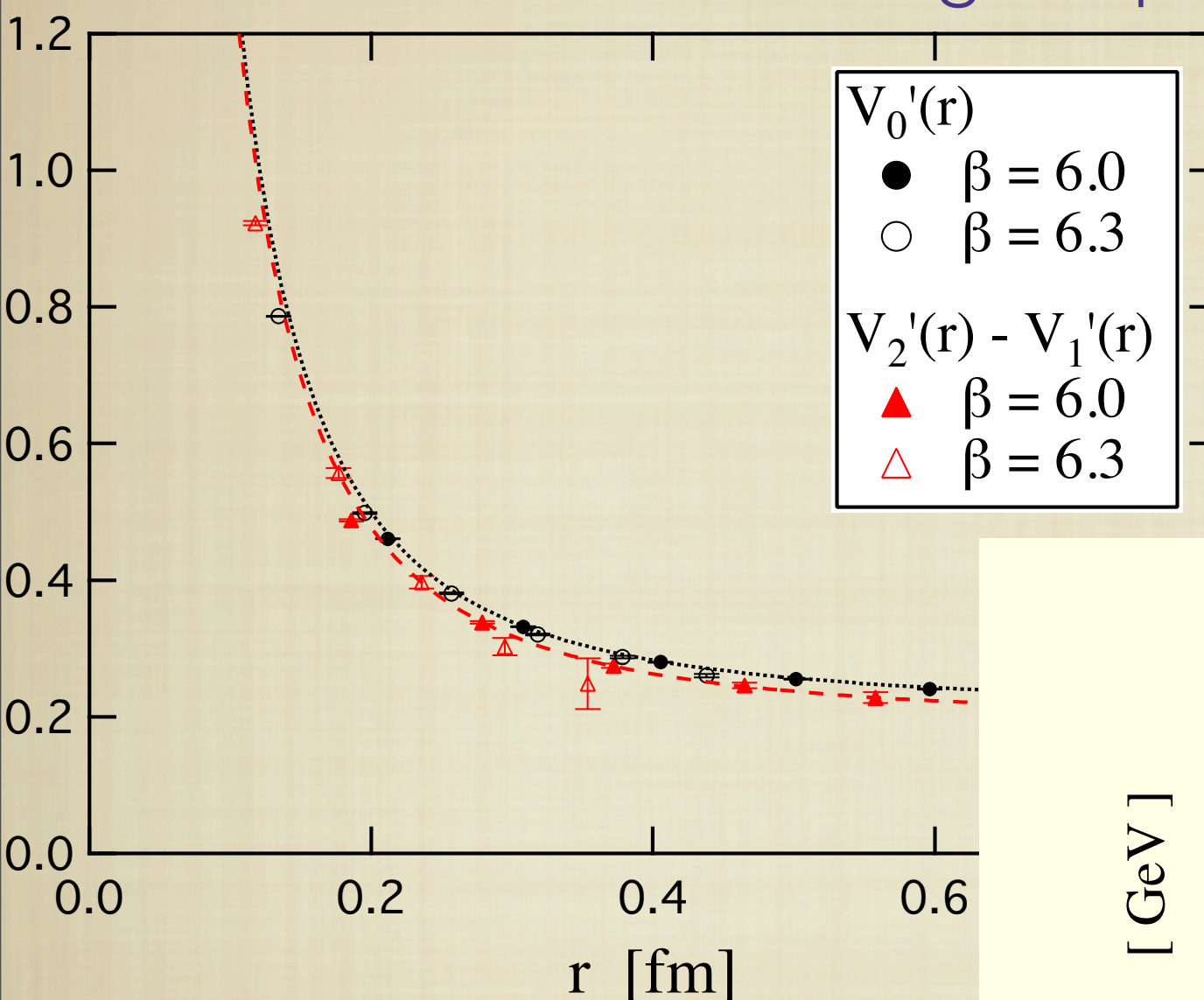
calculated in part on the lattice

calculated all in an effective string model [N.B., H. Martinez, A. Vairo 2013](#)

Exact relations from Poincare' invariance

The EFT is still Poincare' invariant-> this induces relations among the potentials

Koma and Koma 2006



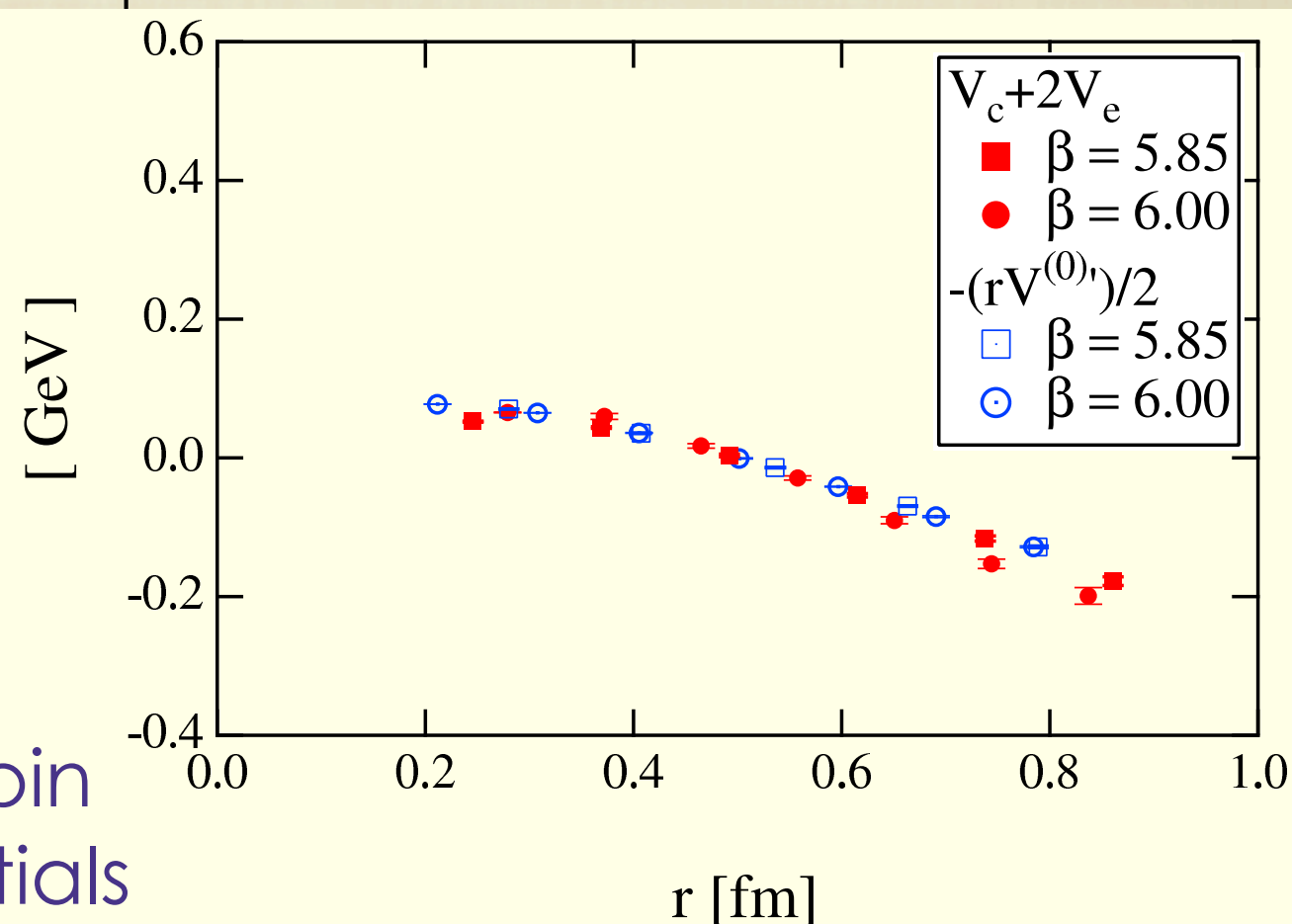
e. g. $V_0'(r) = V_2'(r) - V_1'(r)$

Gromes relation

It is a check of the lattice calculation

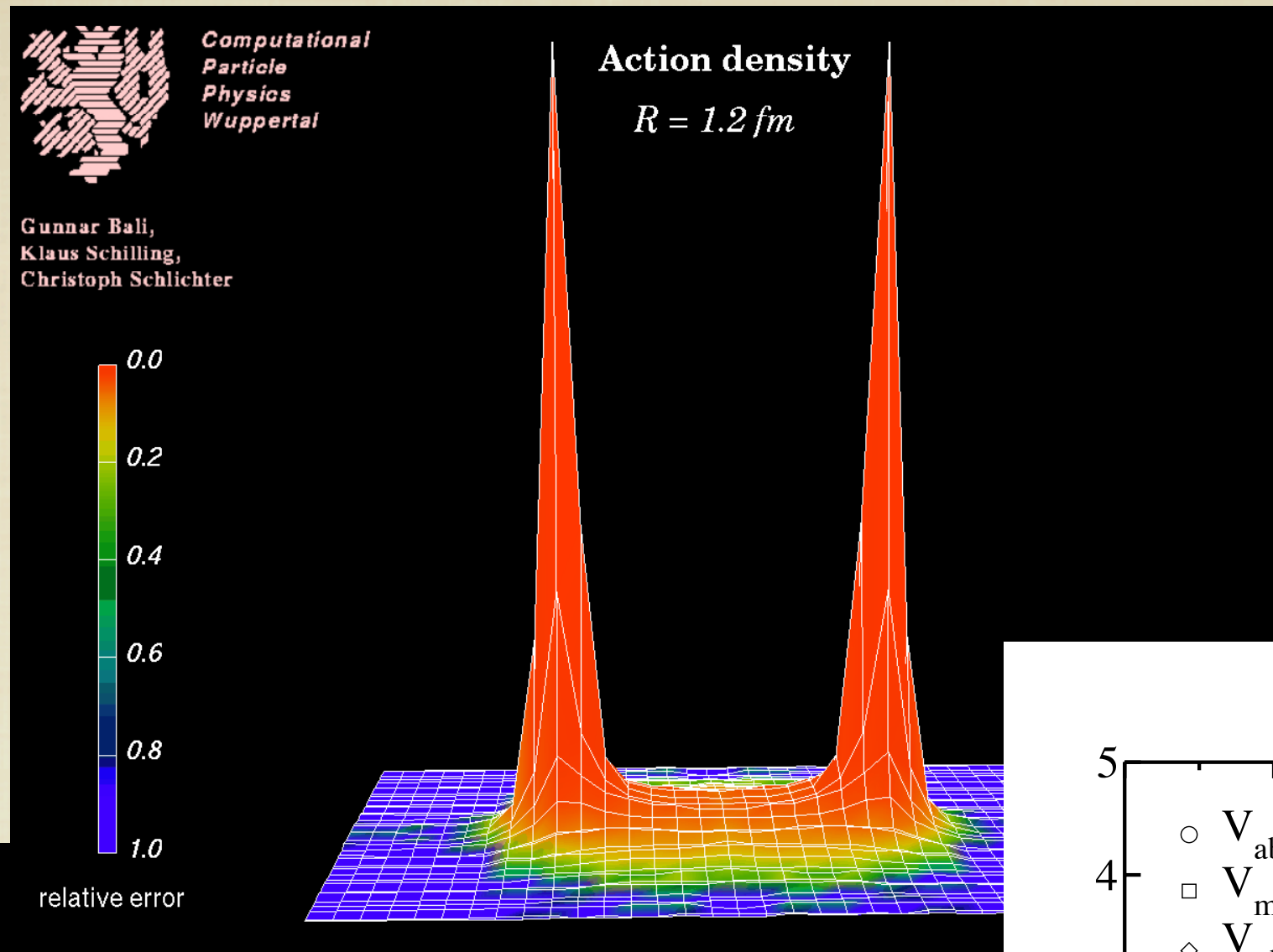
many other relations among potentials in the EFT

relations involving spin independent potentials

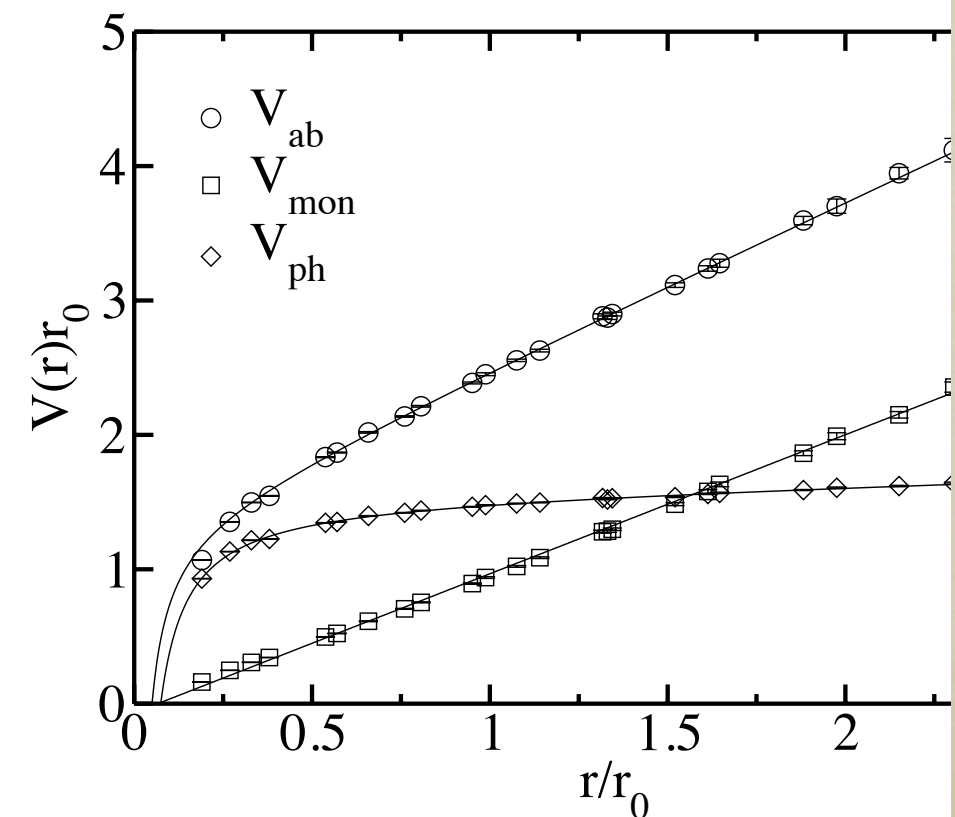


Low energy physics factorized in Wilson loops: can be used to probe the confinement mechanism

Bali et al

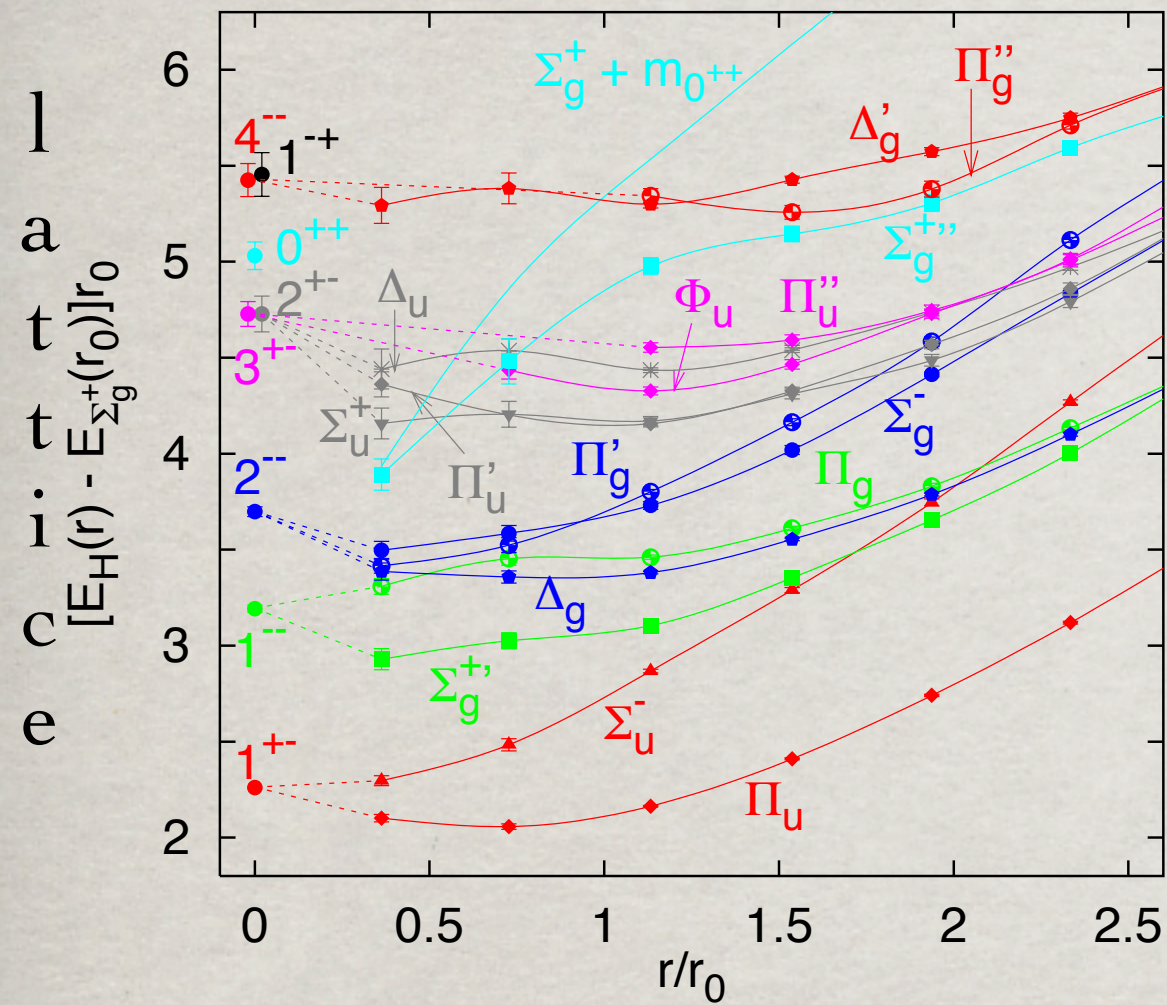


Boryakov et al. 04



Adding degrees of freedom to the bound state:
Quarkonium Hybrids

Gluelumps and hybrids in pNRQCD

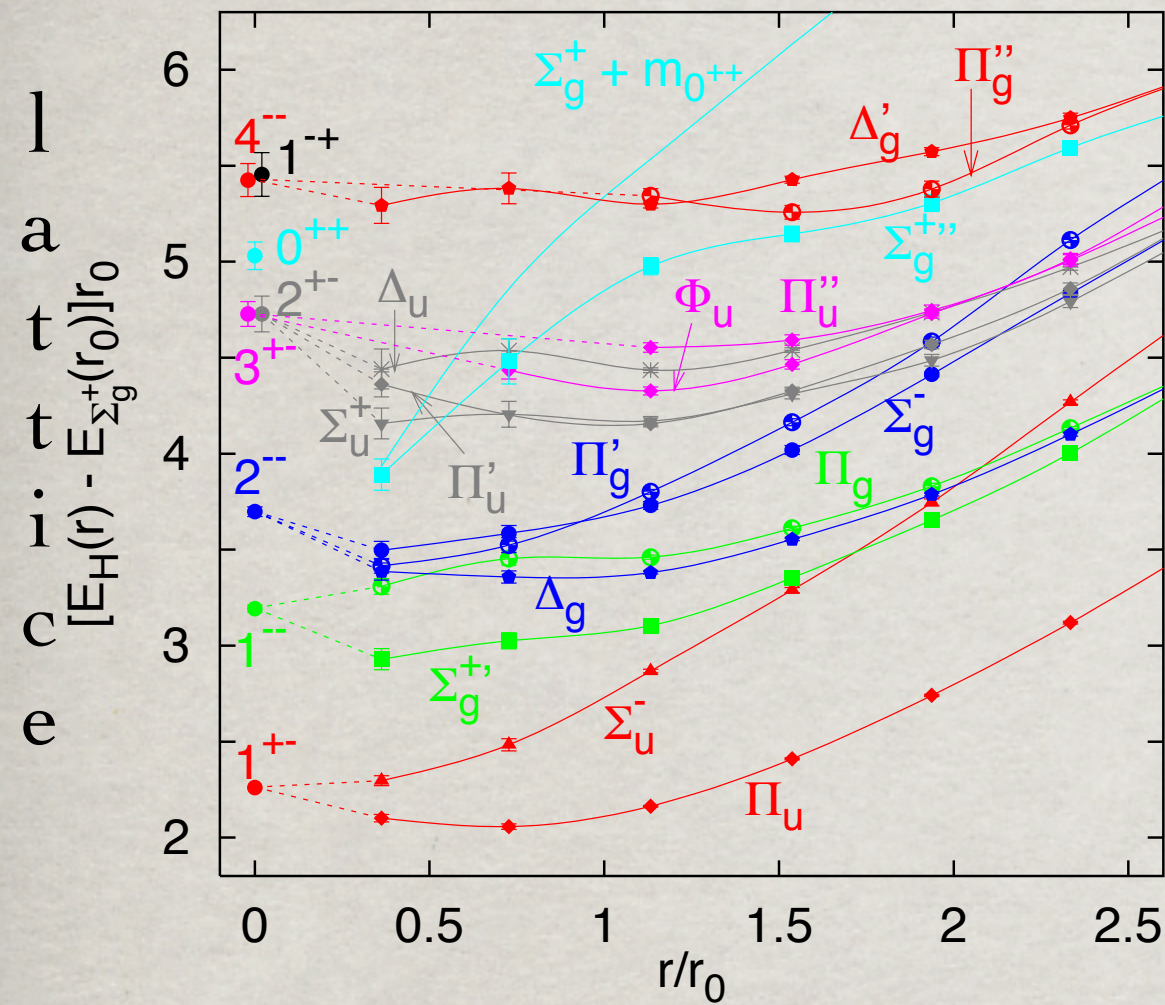


Juge Kuti Morningstar 00 03

for small r pNRQCD applies

Brambilla Pineda Soto Vairo 00

Gluelumps and hybrids in pNRQCD



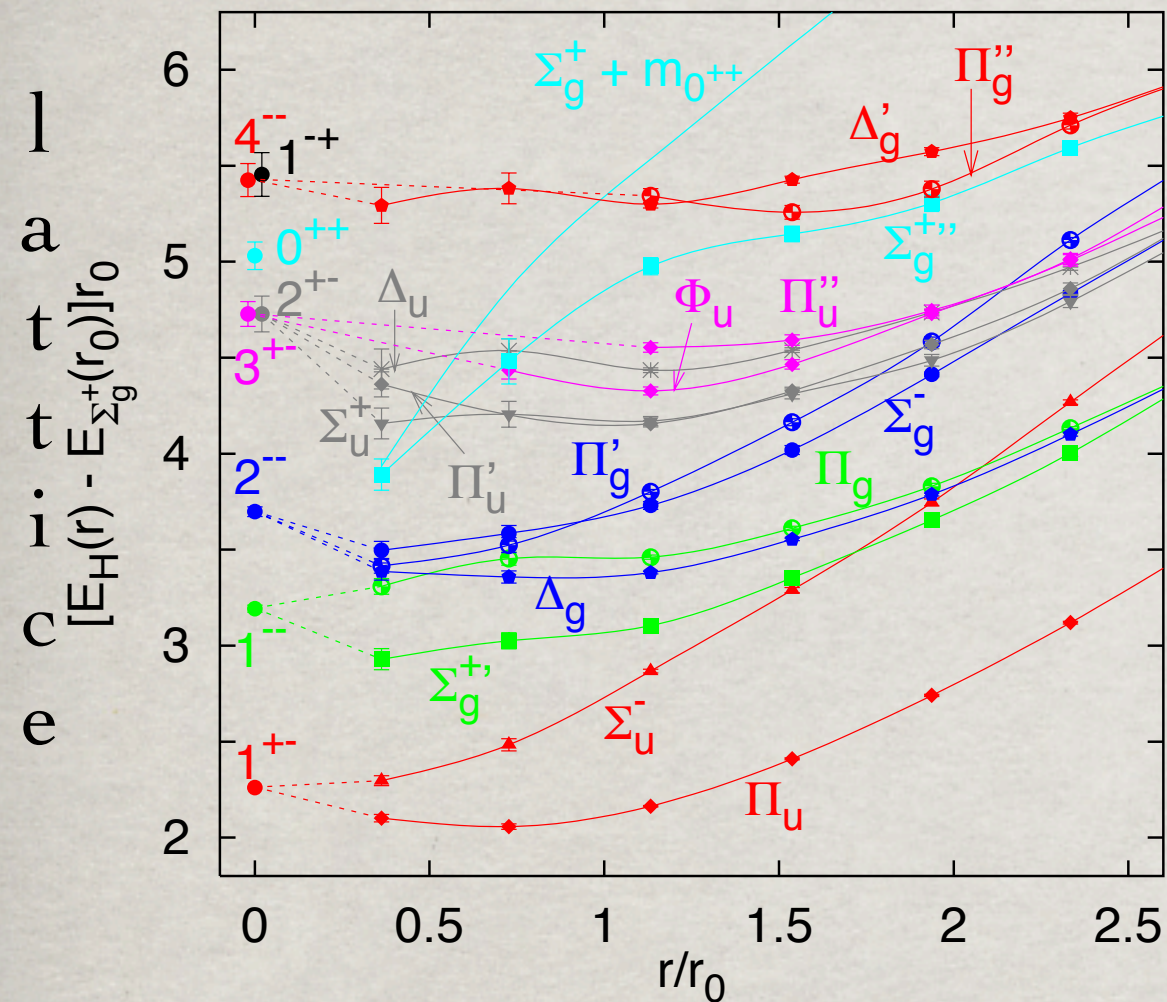
Juge Kuti Morningstar 00 03

for small r pNRQCD applies

Brambilla Pineda Soto Vairo 00

- At lowest order in the multipole expansion, the *singlet decouples* while the *octet is still coupled to gluons*.

Gluelumps and hybrids in pNRQCD



Juge Kuti Morningstar 00 03

for small r pNRQCD applies

Brambilla Pineda Soto Vairo 00

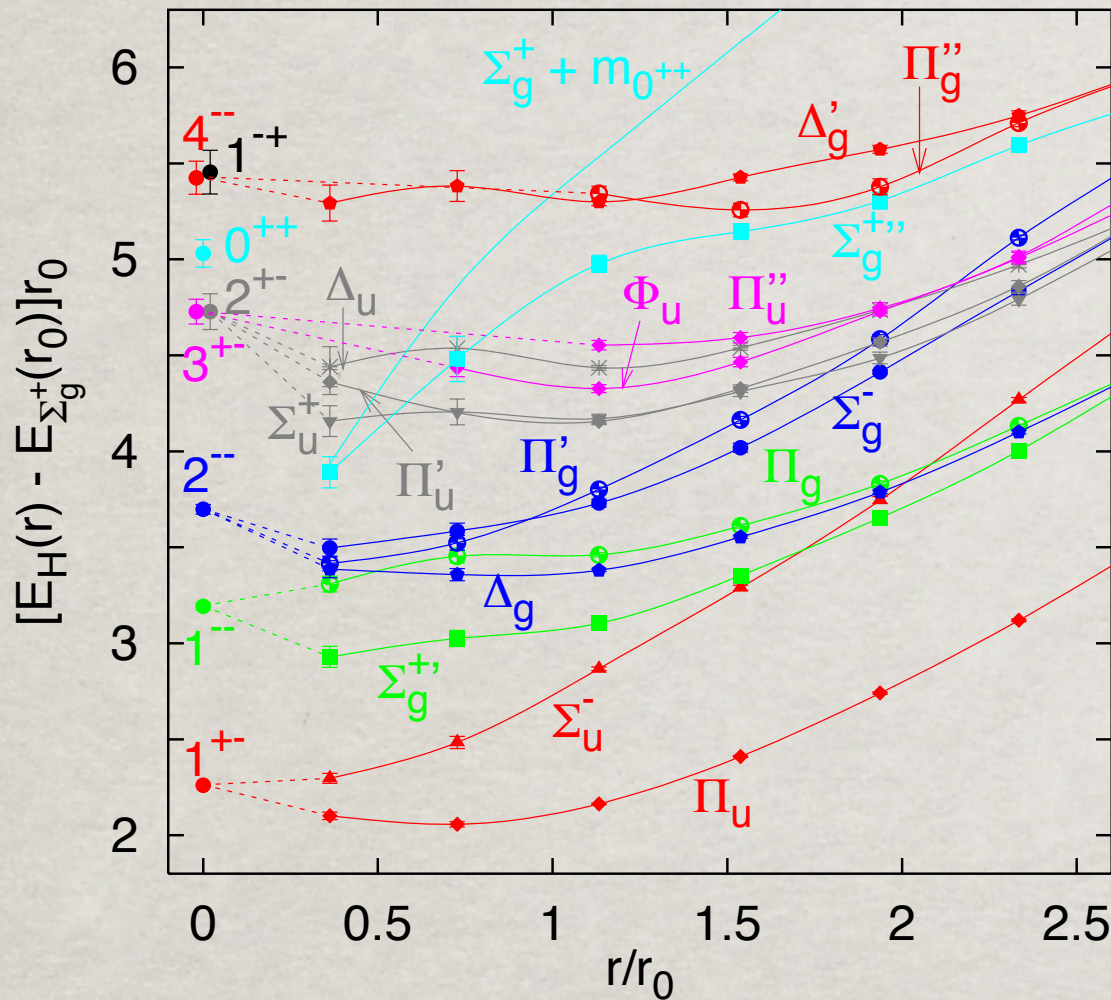
- At lowest order in the multipole expansion, the **singlet decouples** while the **octet is still coupled to gluons**.

- Static hybrids at short distance are called **gluelumps** and are described by a **static adjoint source** (O) in the presence of a **gluonic field** (H):

$$H(R, r, t) = \text{Tr}\{OH\}$$

Gluelumps and hybrids in pNRQCD: more symmetry!

l
a
t
t
i
c
e



Juge Kuti Morningstar 00 03

	$L = 1$	$L = 2$
$\Sigma_g^{+'}$	$\mathbf{r} \cdot (\mathbf{E})$	
Σ_g^-		$(\mathbf{r} \cdot \mathbf{D})(\mathbf{r} \cdot \mathbf{B})$
Π_g	$\mathbf{r} \times (\mathbf{E})$	
Π'_g		$\mathbf{r} \times ((\mathbf{r} \cdot \mathbf{D})\mathbf{B} + \mathbf{D}(\mathbf{r} \cdot \mathbf{B}))$
Δ_g		$(\mathbf{r} \times \mathbf{D})^i(\mathbf{r} \times \mathbf{B})^j + (\mathbf{r} \times \mathbf{D})^j(\mathbf{r} \times \mathbf{B})^i$
Σ_u^+		$(\mathbf{r} \cdot \mathbf{D})(\mathbf{r} \cdot \mathbf{E})$
Σ_u^-	$\mathbf{r} \cdot \mathbf{B}$	
Π_u	$\mathbf{r} \times \mathbf{B}$	
Π'_u		$\mathbf{r} \times ((\mathbf{r} \cdot \mathbf{D})\mathbf{E} + \mathbf{D}(\mathbf{r} \cdot \mathbf{E}))$
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Brambilla Pineda Soto Vairo 00

H

H

$= e^{-iT E_H}$

$$E_H = V_o + \frac{i}{T} \ln \langle H^a(\frac{T}{2}) \phi_{ab}^{\text{adj}} H^b(-\frac{T}{2}) \rangle$$

$$\langle H^a(\frac{T}{2}) \phi_{ab}^{\text{adj}} H^b(-\frac{T}{2}) \rangle_{\text{np}} \sim h e^{-iT \Lambda_H}$$

$$E_H(\boldsymbol{r}) = V_o(\boldsymbol{r}) + \Lambda_H$$

Gluelumps Masses Λ_H

- Foster Michael PRD 59(99)094509
- Bali Pineda PRD 69(04)094001

J^{PC}	H	$\Lambda_H^{\text{RS}} r_0$	$\Lambda_H^{\text{RS}}/\text{GeV}$
1^{+-}	B_i	2.25(39)	0.87(15)
1^{--}	E_i	3.18(41)	1.25(16)
2^{--}	$D_{\{i}B_{j\}}$	3.69(42)	1.45(17)
2^{+-}	$D_{\{i}E_{j\}}$	4.72(48)	1.86(19)
3^{+-}	$D_{\{i}D_jB_{k\}}$	4.72(45)	1.86(18)
0^{++}	\mathbf{B}^2	5.02(46)	1.98(18)
4^{--}	$D_{\{i}D_jD_kB_{l\}}$	5.41(46)	2.13(18)
1^{-+}	$(\mathbf{B} \wedge \mathbf{E})_i$	5.45(51)	2.15(20)

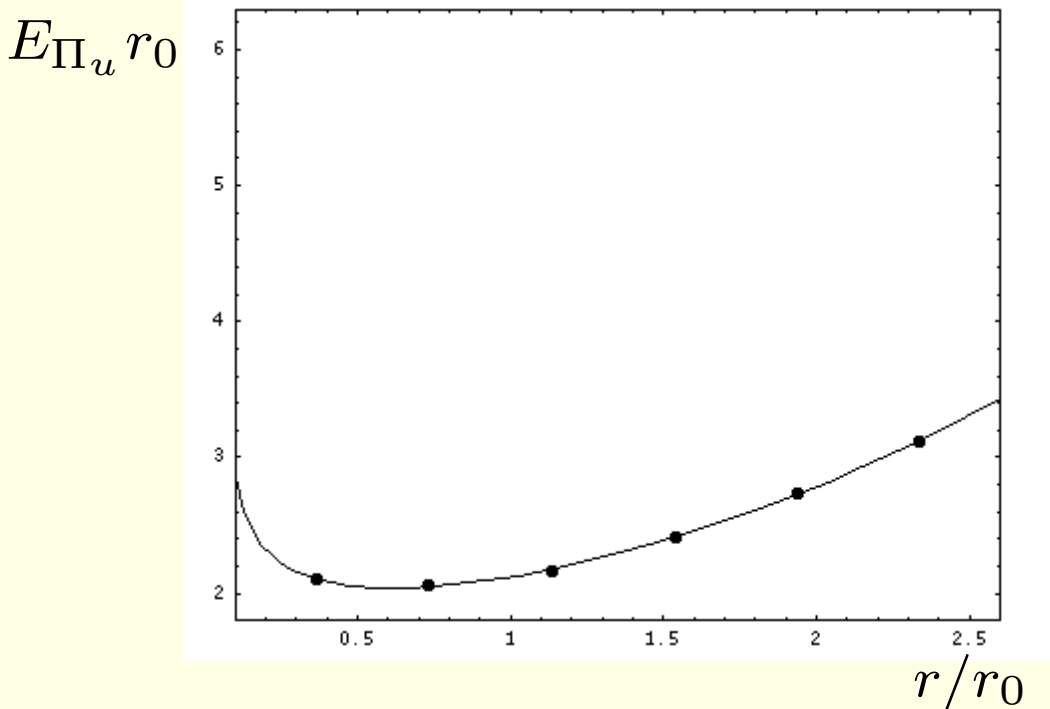
Gluelumps Masses Λ_H

- Foster Michael PRD 59(99)094509
- Bali Pineda PRD 69(04)094001

$Y(4260)$ with pNRQCD

$$|Y\rangle = |\Pi_u\rangle \otimes |\phi\rangle$$

- $|\Pi_u\rangle$ is a 1^{+-} static hybrid state that encodes the glue content.
- $|\phi\rangle$ is a 0^{-+} solution of the Schrödinger equation whose potential is the static energy of $|\Pi_u\rangle$.



Fitting the Π_u curve, $E_{\Pi_u} = (0.87 + 0.11/r + 0.24 r^2)$ GeV and solving the Schrödinger equation, one gets

$$M(Y) = 2 \times 1.48 + 0.87 + 0.53 = 4.36 \text{ GeV}$$

- Vairo IJMP A22(07)5481

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Heating the system:
Quarkonium at finite T

Y(1S) at LHC below T_d

The relative size of non-relativistic and thermal scales depends on the medium and on the quarkonium state.

The **bottomonium ground state**, which is a weakly coupled non-relativistic bound state: $mv \sim m\alpha_s$, $mv^2 \sim m\alpha_s^2 \gtrsim \Lambda_{\text{QCD}}$, produced in the QCD medium of heavy-ion collisions at the LHC may possibly realize the hierarchy

$$m \approx 5 \text{ GeV} > m\alpha_s \approx 1.5 \text{ GeV} > \pi T \approx 1 \text{ GeV} > m\alpha_s^2 \approx 0.5 \text{ GeV} \gtrsim m_D, \Lambda_{\text{QCD}}$$

Vairo AIP CP 1317 (2011) 241

N.B., Escobedo,
Ghiglieri, Soto ,Vairo
010

Y(1S)

m_c (MeV)	T_d (MeV)
∞	480
5000	480
2500	460
1200	440
0	420

Escobedo, Soto
010

The complete mass and width up to $\mathcal{O}(m\alpha_s^5)$

$$\delta E_{1S}^{(\text{thermal})} = \frac{34\pi}{27} \alpha_s^2 T^2 a_0 + \frac{7225}{324} \frac{E_1 \alpha_s^3}{\pi} \left[\ln \left(\frac{2\pi T}{E_1} \right)^2 - 2\gamma_E \right] \\ + \frac{128 E_1 \alpha_s^3}{81\pi} L_{1,0} - 3a_0^2 \left\{ \left[\frac{6}{\pi} \zeta(3) + \frac{4\pi}{3} \right] \alpha_s T m_D^2 - \frac{8}{3} \zeta(3) \alpha_s^2 T^3 \right\}$$

$$\Gamma_{1S}^{(\text{thermal})} = \frac{1156}{81} \alpha_s^3 T + \frac{7225}{162} E_1 \alpha_s^3 + \frac{32}{9} \alpha_s T m_D^2 a_0^2 I_{1,0} \\ - \left[\frac{4}{3} \alpha_s T m_D^2 \left(\ln \frac{E_1^2}{T^2} + 2\gamma_E - 3 - \ln 4 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) + \frac{32\pi}{3} \ln 2 \alpha_s^2 T^3 \right] a_0^2$$

where $E_1 = -\frac{4m\alpha_s^2}{9}$, $a_0 = \frac{3}{2m\alpha_s}$ and $L_{1,0}$ (similar $I_{1,0}$) is the Bethe logarithm.

◦ Brambilla Escobedo Ghiglieri Soto Vairo JHEP 1009 (2010) 038

Consistent with NRQCD lattice calculations of spectral functions

◦ Aarts Allton Kim Lombardo Oktay Ryan Sinclair Skullerud
JHEP 1111 (2011) 103

Conclusions

Nonrelativistic Effective Field Theories provide a systematic tool to investigate a wide range of heavy quarkonium observables in the realm of QCD

Allow us to make calculations with unprecedented precision, where high order perturbative calculations are possible and to systematically factorize short from long range contributions where observables are sensitive to the nonperturbative dynamics of QCD

They allow us to give the appropriate definition and define a calculational scheme for quantities of huge phenomenological interest like the $q\bar{q}$ static energies and the $q\bar{q}$ potential at finite T

in the EFT framework heavy quark bound states become a unique laboratory for the study of strong interaction from the high energy to the low energy scales



arXiv:1010.5827

CONCLUSIONS AND PRIORITIES

Below we present a summary of the most crucial developments in each of the major topics and suggested directions for further advancement.

Spectroscopy: An overview of the last decade in heavy quarkonium spectroscopy.

With regard to experiment

1. New measurements

1. New measurements of inclusive hadronic cross sections (*i.e.*, R) for e^+e^- collisions just above open $c\bar{c}$ and $b\bar{b}$ flavor thresholds have enabled improved determinations of some resonance parameters but more precision and fine-grained studies are needed to resolve puzzles and ambiguities. Likewise, progress has been made studying exclusive n -flavor two-body and multibody composition regions, but further data are needed to fill in the details. Theory has not yet been able to explain the measured exclusive two-body branching ratios.

Effective field theories for heavy quarkonium

Pre-Print: [hep-ph/0410047](https://arxiv.org/abs/hep-ph/0410047)

Pre-Print: [hep-ph/0410047](https://arxiv.org/abs/hep-ph/0410047)

Selected Outlook for future research

Finite T : masses, width of quarkonia states, impact of anisotropy of the medium, transport coefficients of heavy quarks, energy losses, viscosity

Spectra/decays of quarkonia

EFT for states close to thresholds: X , Y , Z

Quarkonium-quarkonium van der Waals interaction; quarkonium on nuclei

Quarkonium production

CMS, Atlas, Alice, LHC-b

Belle, BESIII, Panda, LHC exps

Alice, Rhic

Fair

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Fair

CMS, Atlas, Alice, LHC-b

Invaluable effect of Spin-off of QCD physics to other fields:

An example among many: EFTs developed at finite T and for heavy masses used in cosmology: calculation of thermal production of dark matter

Institute: “Jets, particle production and transport properties in collider and cosmological environments”, MITP Mainz 2014

Backup

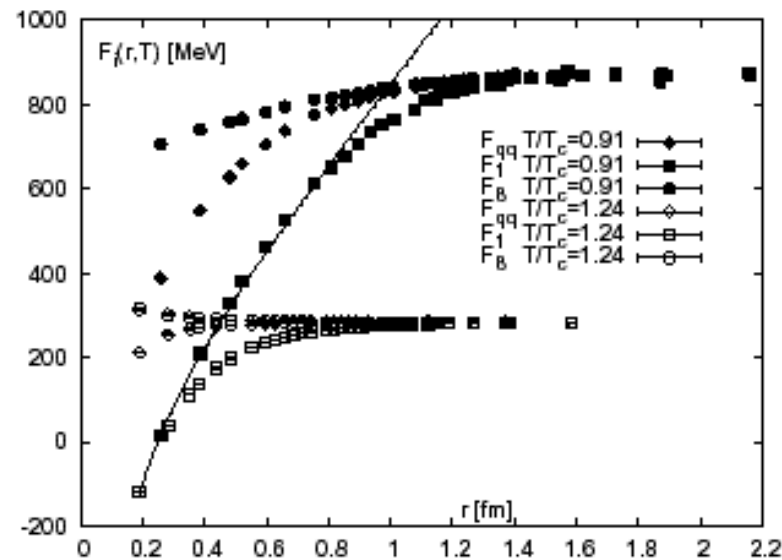
Heating quarkonium systems

$$T > 0$$

Quarkonium in a hot medium: the interaction potential

Free energy vs potential

- Either phenomenological potentials have been used so far or the free energy calculated on the lattice.
- The free energy is not the static potential: the average free energy ($\sim \langle \text{Tr } L^\dagger(r) \text{Tr } L(0) \rangle$) is an overlap of singlet and octet quark-antiquark states, what is called the singlet ($\sim \langle \text{Tr } L^\dagger(r) L(0) \rangle$) and the octet ($\sim \langle \text{Tr } L^\dagger(r) \text{Tr } L(0) \rangle - 1/3 \langle \text{Tr } L^\dagger(r) L(0) \rangle$) free energy are gauge dependent;

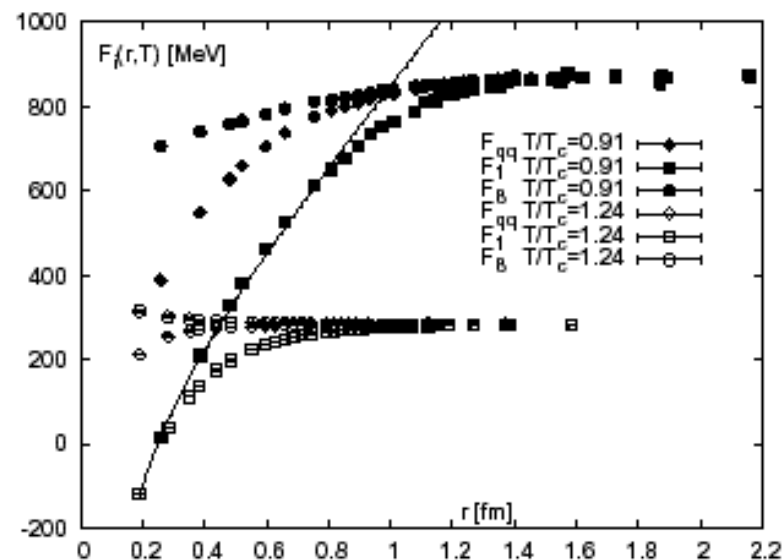


○ Kaczmarek Zantow PRD 71 (2005) 114510

Quarkonium in a hot medium: the interaction potential

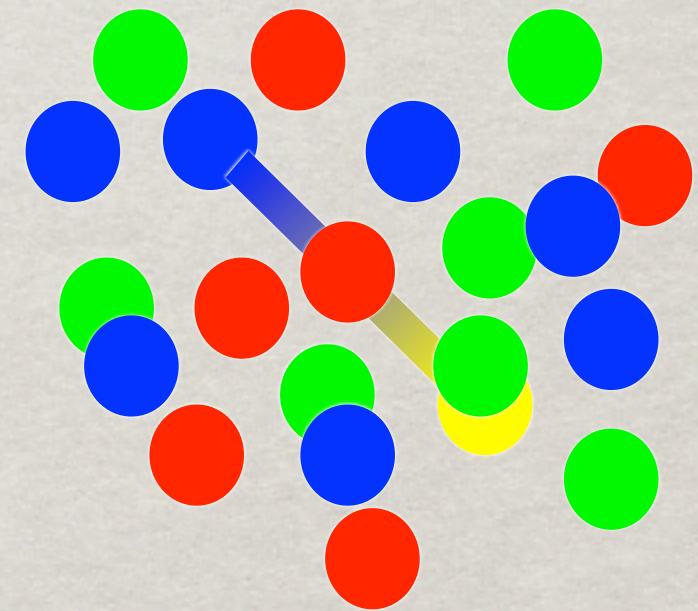
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○ Kaczmarek Zantow PRD 71 (2005) 114510

It was believed
that the color
screening
of the potential
originates quarkonium
dissociation
Matsui Satz 86



Debye charge screening
(electromagnetic plasma)

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$r \sim \frac{1}{m_D} \longrightarrow$ Bound state
dissolves

But, at finite temperature what is the quarkonium potential?

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The potential $V(r,T)$ dictates through the Schrödinger equation the real time evolution of the $Q\bar{Q}$ pair in the medium \rightarrow use the EFT to define and calculate it

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more scales

But, at finite temperature what is the quarkonium potential?

The potential $V(r,T)$ dictates through the Schroedinger equation the real time evolution of the $Q\bar{Q}$ pair in the medium \rightarrow use the EFT to define and calculate it

more scales

$$m \gg mv \gg mv^2$$

?

and Λ_{QCD}

$$T \gg gT \gg g^2 T \dots$$

$m_D \sim gT$
Debye mass
Screening Scale

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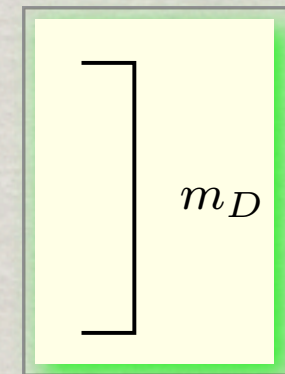
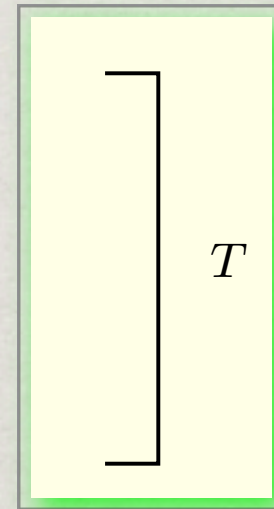
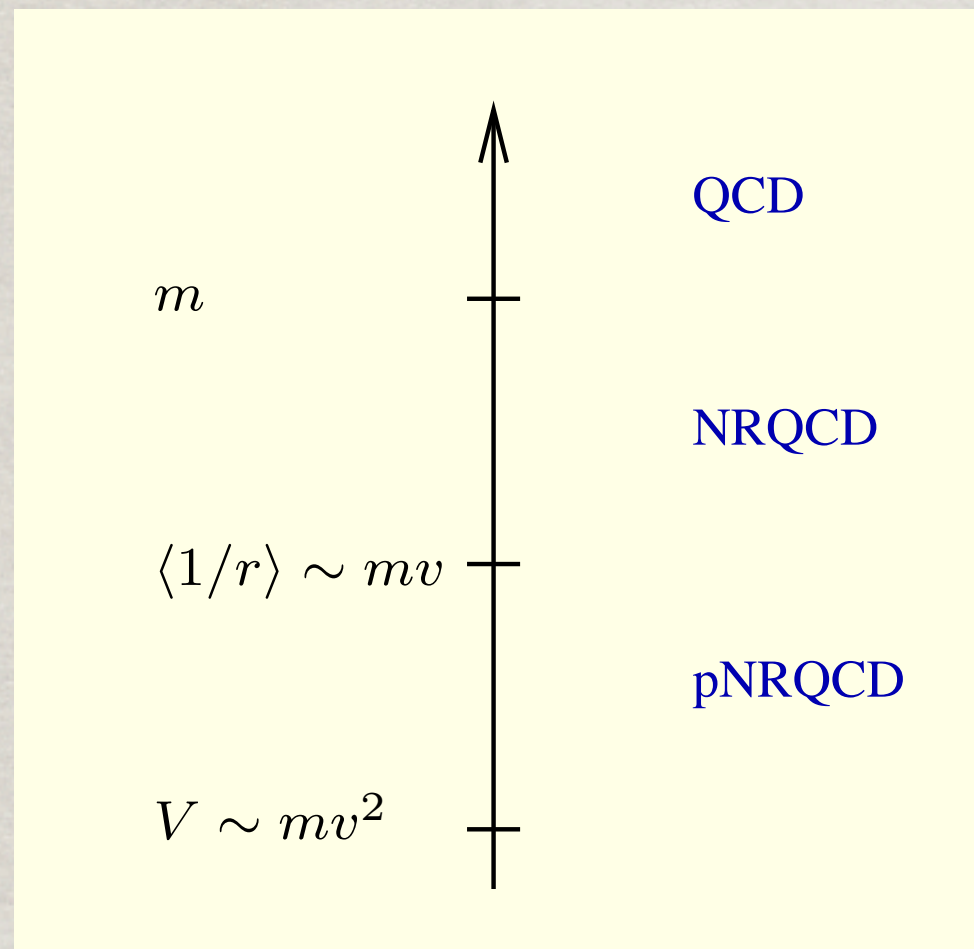
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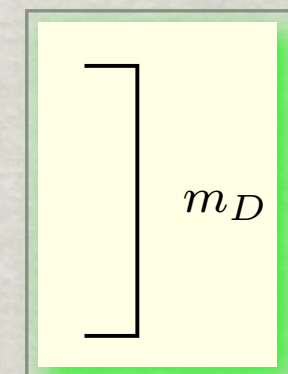
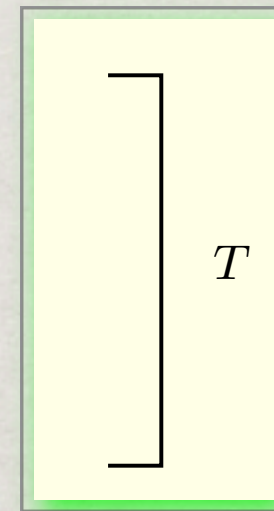
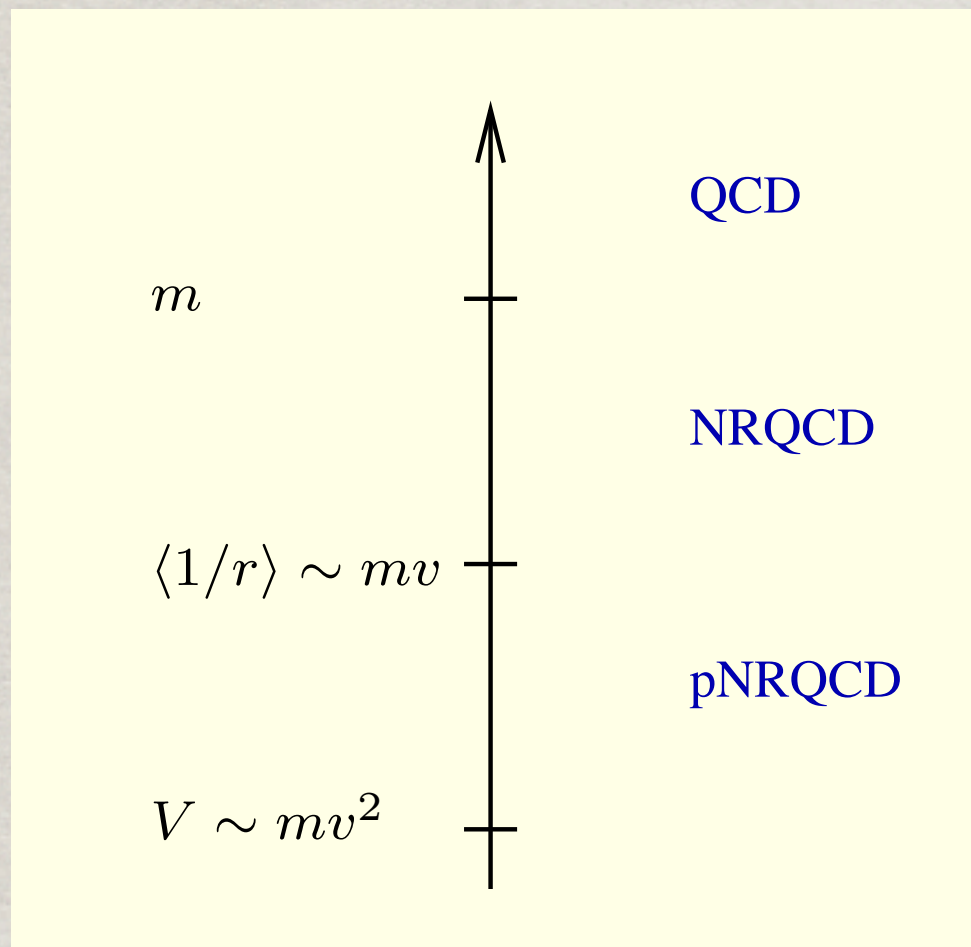
Without heavy quarks an EFT already exists that comes from integrating out hard gluon of $p \sim T$:

Hard Thermal Loop EFT

Quarkonium at finite T with pNRQCD

N. B. J. Ghiglieri, P. Petreczky,
M. Escobedo, A. Vairo 08--013





We work under the conditions:

We assume that bound states exist for

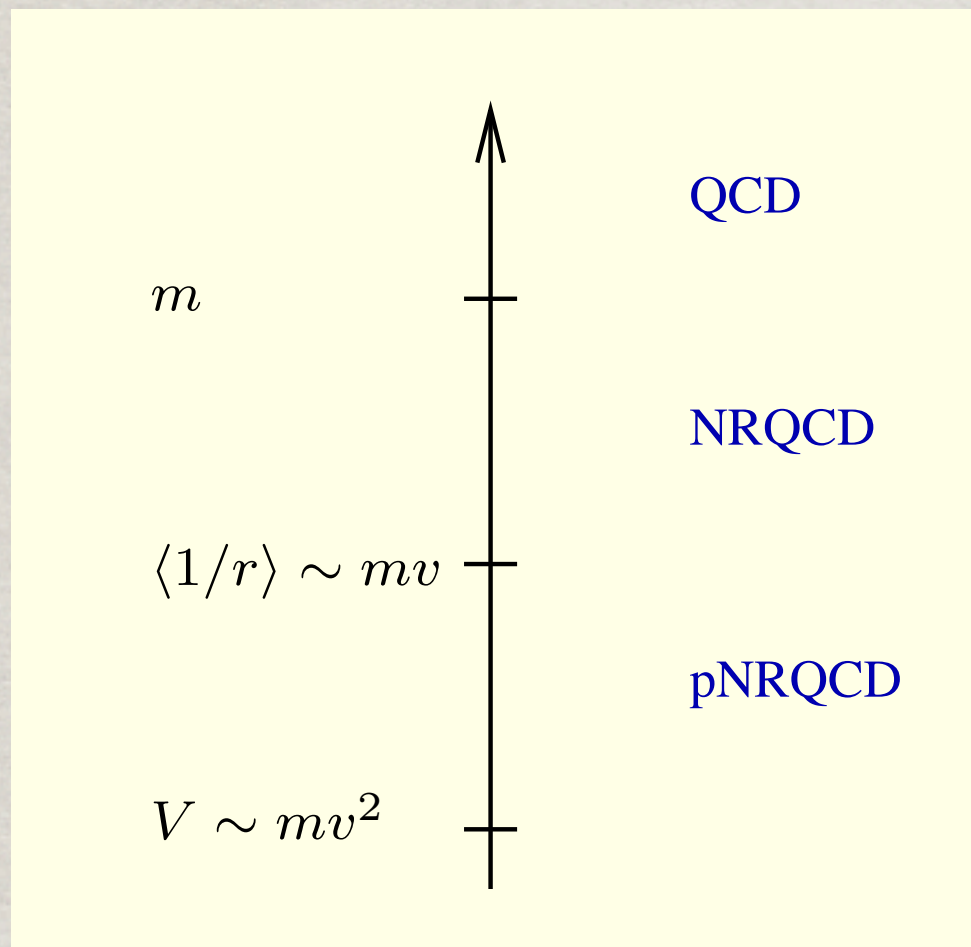
- $T \ll m$
- $\langle 1/r \rangle \sim mv \gtrsim m_D$

We neglect smaller thermodynamical scales.

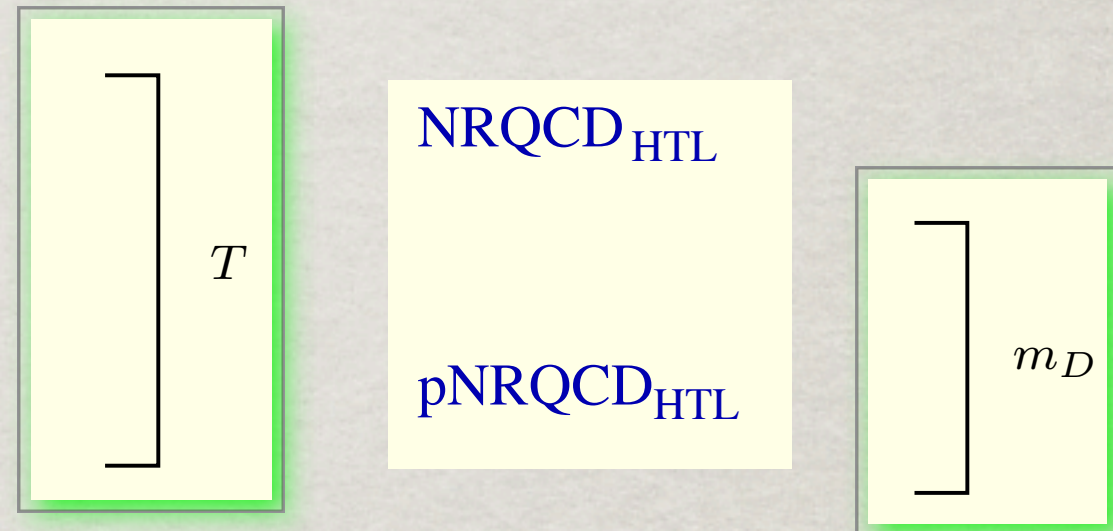
In the weak coupling regime:

- $v \sim \alpha_s \ll 1$; valid for tightly bound states: $\Upsilon(1S)$, J/ψ , ...
- $T \gg gT \sim m_D$.

Effects due to the scale Λ_{QCD} will not be considered.



pNRQCD at finite T allows us to define the static QQbar potential in the medium in real time



We work under the conditions:

We assume that bound states exist for

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The singlet static potential

- The thermal part of the potential has a real and an imaginary part

$\text{Re}V_s(r,T)$

$\text{Im}V_s(r,T)$

thermal width of $Q\bar{Q}$



N. B., Ghiglieri,
Petreczky, Vairo 2008 **Singlet-to-octet**
New effect, specific of QCD
dominates for $E/m_D \gg 1$
(gluo dissociation)

Landau damping Laine et al 2007
Known from QED
dominates for $m_D/E \gg 1$
(dissociation via inelastic
parton scattering)

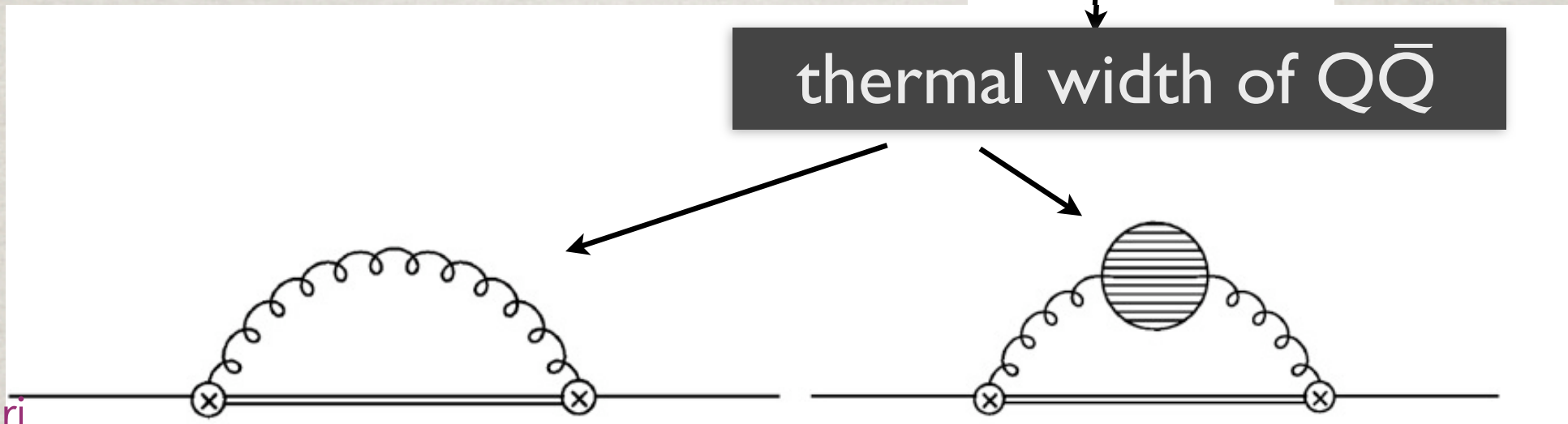
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Landau damping

Laine et al 2007

Known from QED

dominates for $m_D/E \gg 1$

(dissociation via inelastic
parton scattering)

- The imaginary part is bigger than the real part before the screening $\exp\{-m_D r\}$ sets in

->the imaginary part is responsible for $Q\bar{Q}$ dissociation !

$T \gg 1/r \gg m_D \gg V$: Quarkonium melts in the medium

Escobedo Soto arXiv:0804.0691

Laine arXiv:0810.1112

$E_{\text{binding}} \sim \Gamma$

The singlet static potential

- Temperature effects can be other than screening

The singlet static potential

- Temperature effects can be other than screening

$$T > 1/r \text{ and } 1/r \sim m_D \sim gT$$

exponential screening but $\text{Im}V \gg \text{Re}V$

$$T > 1/r \text{ and } 1/r > m_D \sim gT$$

no exponential screening but
power-like T corrections

$$T < E_{\text{bin}}$$

no corrections to the potential,
corrections to the energy

pNRQCD and quarkonium

Several cases for the physics at hand

pNRQCD and quarkonium Several cases for the physics at hand

The EFT has not yet been constructed (Exotics close to threshold)

*Degrees of freedom still to be identified

pNRQCD and quarkonium

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E. Braaten et al

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E. Braaten et al

Gluonic excitations

A plethora of states built on each of the hybrid potentials is expected. These states typically develop a width also without including light quarks, since they may decay into lower states, e.g. like **hybrid** \rightarrow **glueball** + **quark-antiquark**.

CLOSE AND ABOVE THRESHOLD

The QCD spectrum with light quarks

- We still have states just made of heavy quarks and gluons. They may develop a width because of the decay through pion emission. If new states made with heavy and light quarks develop a mass gap of order Λ_{QCD} with respect to the former ones, then these new states may be absorbed into the definition of the potentials or of the (local or non-local) condensates.
 - Brambilla et al. PRD 67(03)034018
- In addition new states built using the light quark quantum numbers may form.
 - Soto NP PS 185(08)107

States made of two heavy and light quarks

- Molecular states, i.e. states built on the pair of heavy-light mesons.
 - Tornqvist PRL 67(91)556
- Tetraquark states.
 - Polosa et al
 - Jaffe PRD 15(77)267
 - Ebert Faustov Galkin PLB 634(06)214
- Pairs of heavy-light mesons: $D\bar{D}, B\bar{B}, \dots$

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

- Pairs of heavy-light baryons.

◦ Qiao PLB 639 (2006) 263

(hadro-quarkonium).

Coupled channels

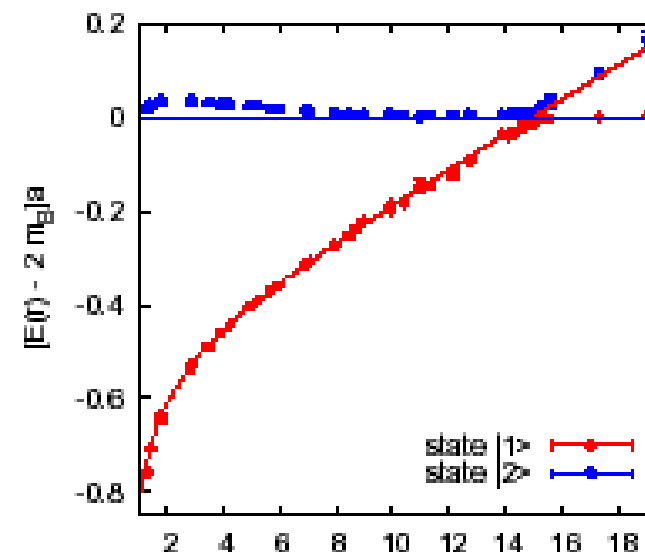
An important (and unsolved) issue is how all the different kind of states (with and without light quarks) interact with each other.

A systematic treatment does not exist so far. For the coupling with two-meson states, most of the existing analyses rely on two models, which are now more than 30 years old:

- the Cornell coupled-channel model;
 - Eichten et al. PRD 17(78)3090, 21(80)313
 - Eichten et al. PRD 69(04)094019, 73(06)014014, 73(06)079903
- and the 3P_0 model.
 - Le Yaouanc et al. PRD 8(73)2223
 - Kalashnikova PRD 72(05)034010

Steps towards a lattice based approach have been undertaken

- SESAM PRD 71(05)114513



States near or above threshold: "exotics" !
hybrids, molecular states, tetraquarks

Many new states from
experiments: Xs, Ys, Zs

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In some cases it is possible to develop an EFT
owing to special dynamical condition

- An example is the $X(3872)$ interpreted as a $D^0 \bar{D}^{*0}$ or $\bar{D}^0 D^{*0}$ molecule. In this case, one may take advantage of the hierarchy of scales:

$$\Lambda_{\text{QCD}} \gg m_\pi \gg m_\pi^2/M_{D^0} \approx 10 \text{ MeV} \gg E_{\text{binding}} \\ \approx M_X - (M_{D^{*0}} + M_{D^0}) = (0.1 \pm 1.0) \text{ MeV}$$

*Systems with a short-range interaction and a large scattering length have universal properties that may be exploited: in particular, production and decay amplitudes factorize in a short-range and a long-range part, where the latter depends only on one single parameter, the scattering length. An universal property that fits well with the observed large branching fraction of the $X(3872)$ decaying into $D^0 \bar{D}^0 \pi^0$ is $\mathcal{B}(X \rightarrow D^0 \bar{D}^0 \pi^0) \approx \mathcal{B}(D^{*0} \rightarrow D^0 \pi^0) \approx 60\%$.*

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