(Exotic) heavy hadrons on the lattice

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Hadron Spectrum Collaboration

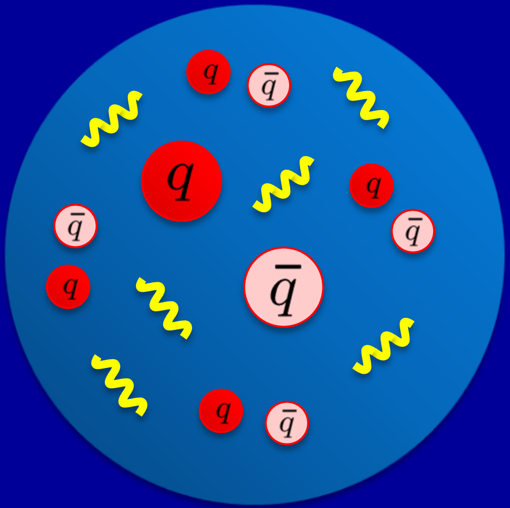
Outline

- Introduction
- Excited charmonium, *D* and D_s spectra
- Exotic meson photocoupling (briefly)
- Excited charmed baryons (briefly)
- Outlook

Heavy meson spectroscopy

Probe non-perturbative regime of QCD

Relevant degrees of freedom?



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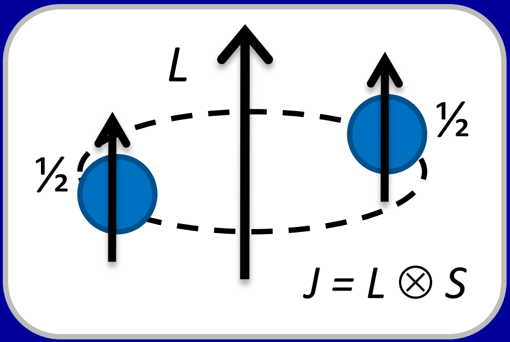

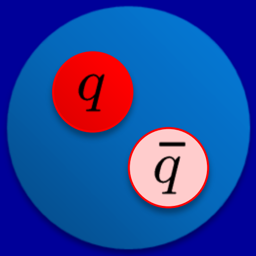

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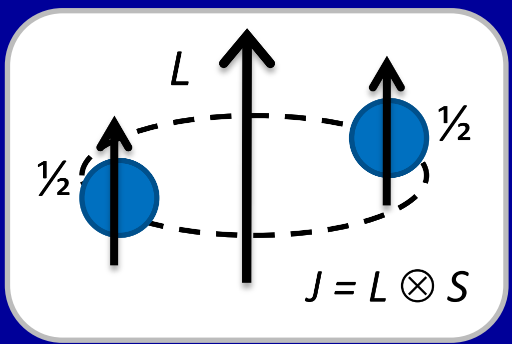
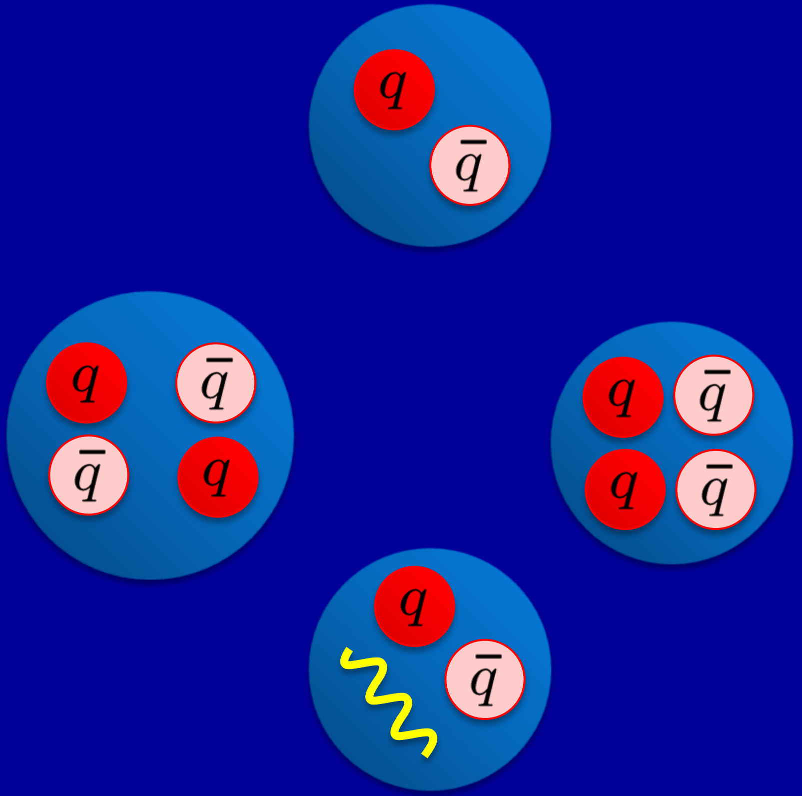


Exotic J^{PC} (**0**$^{--}$, **0**$^{+-}$, **1**$^{-+}$, **2**$^{+-}$, ...)
or flavour quantum numbers
– can't just be a $q\bar{q}$ pair



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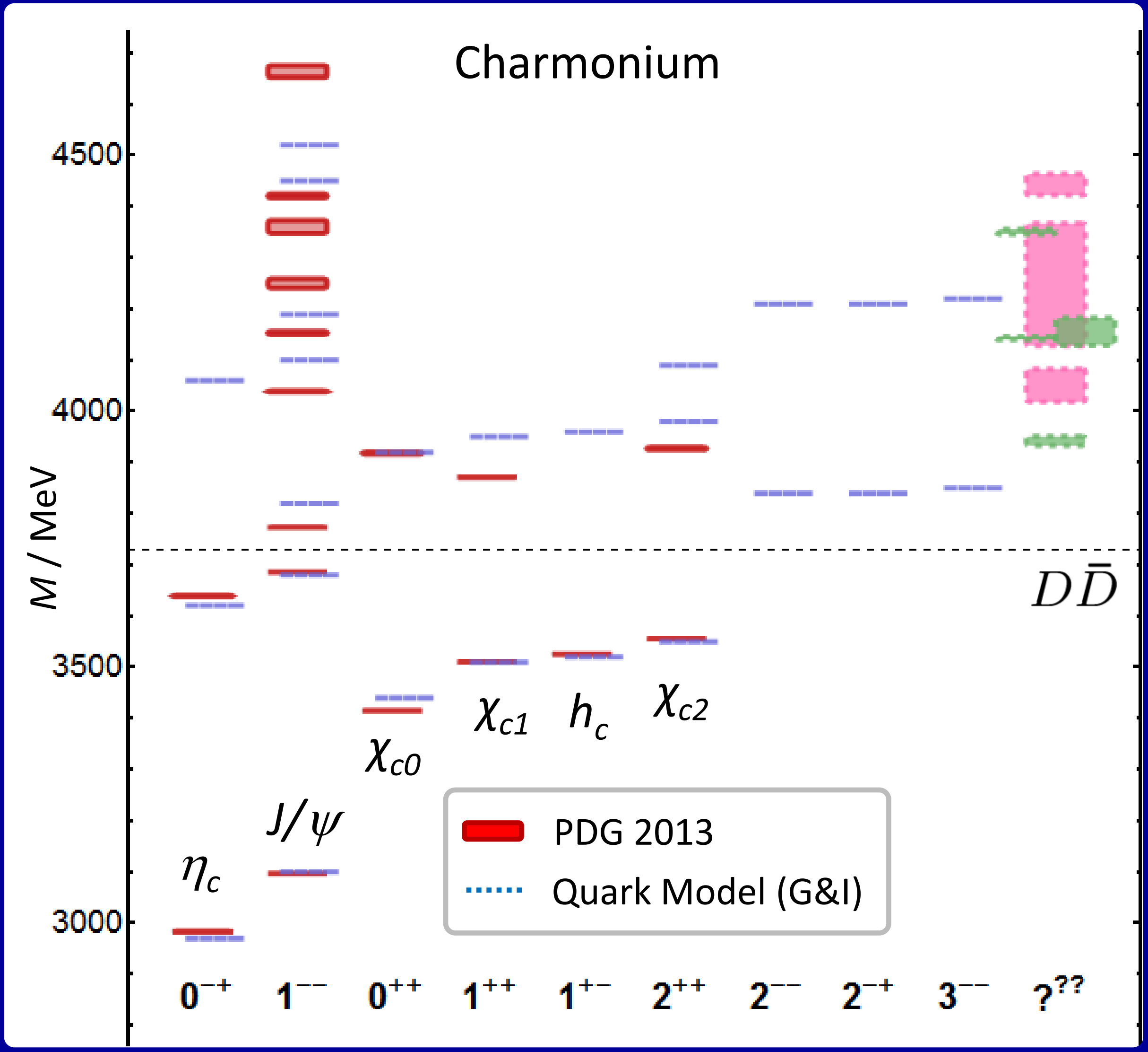

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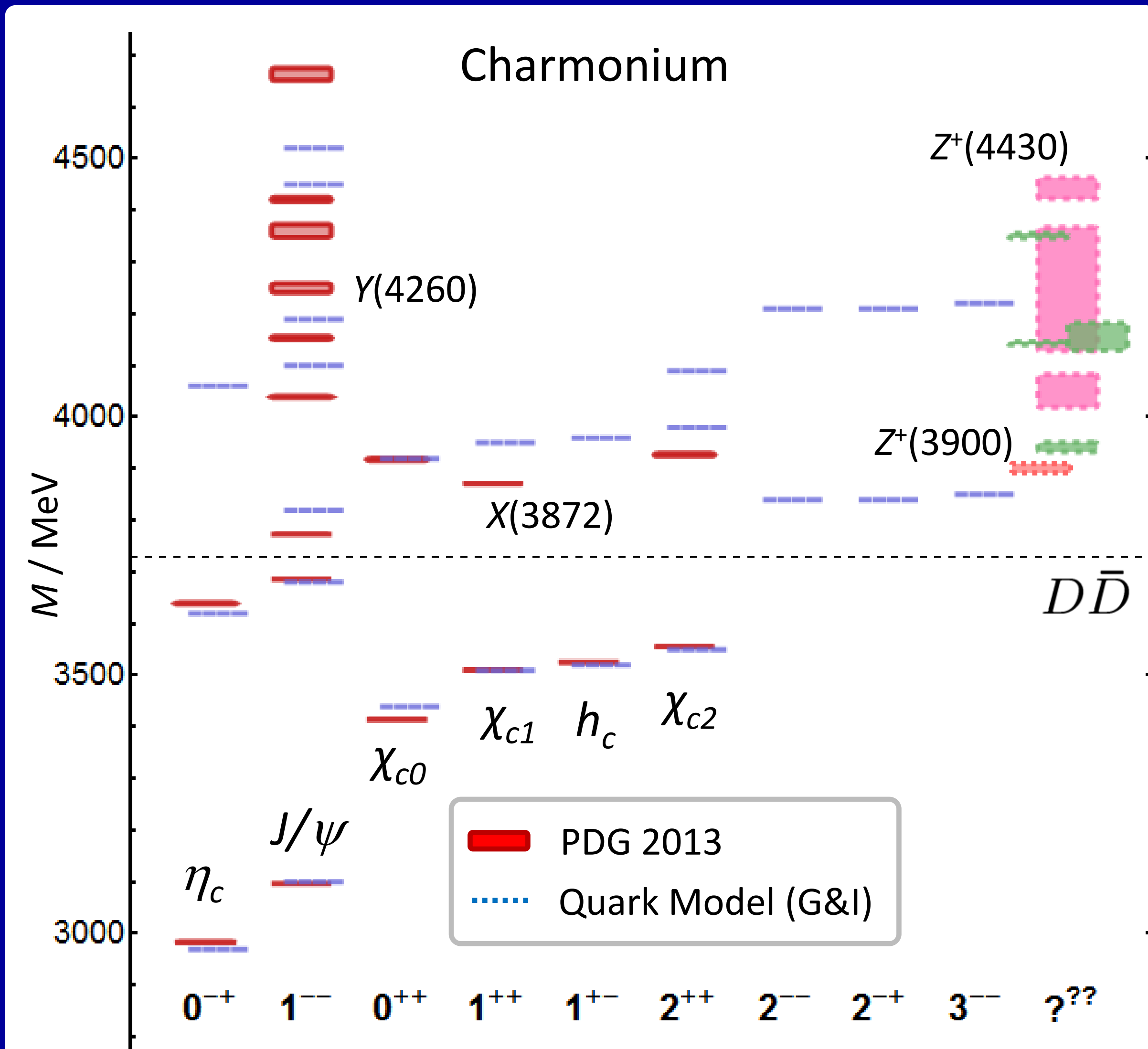
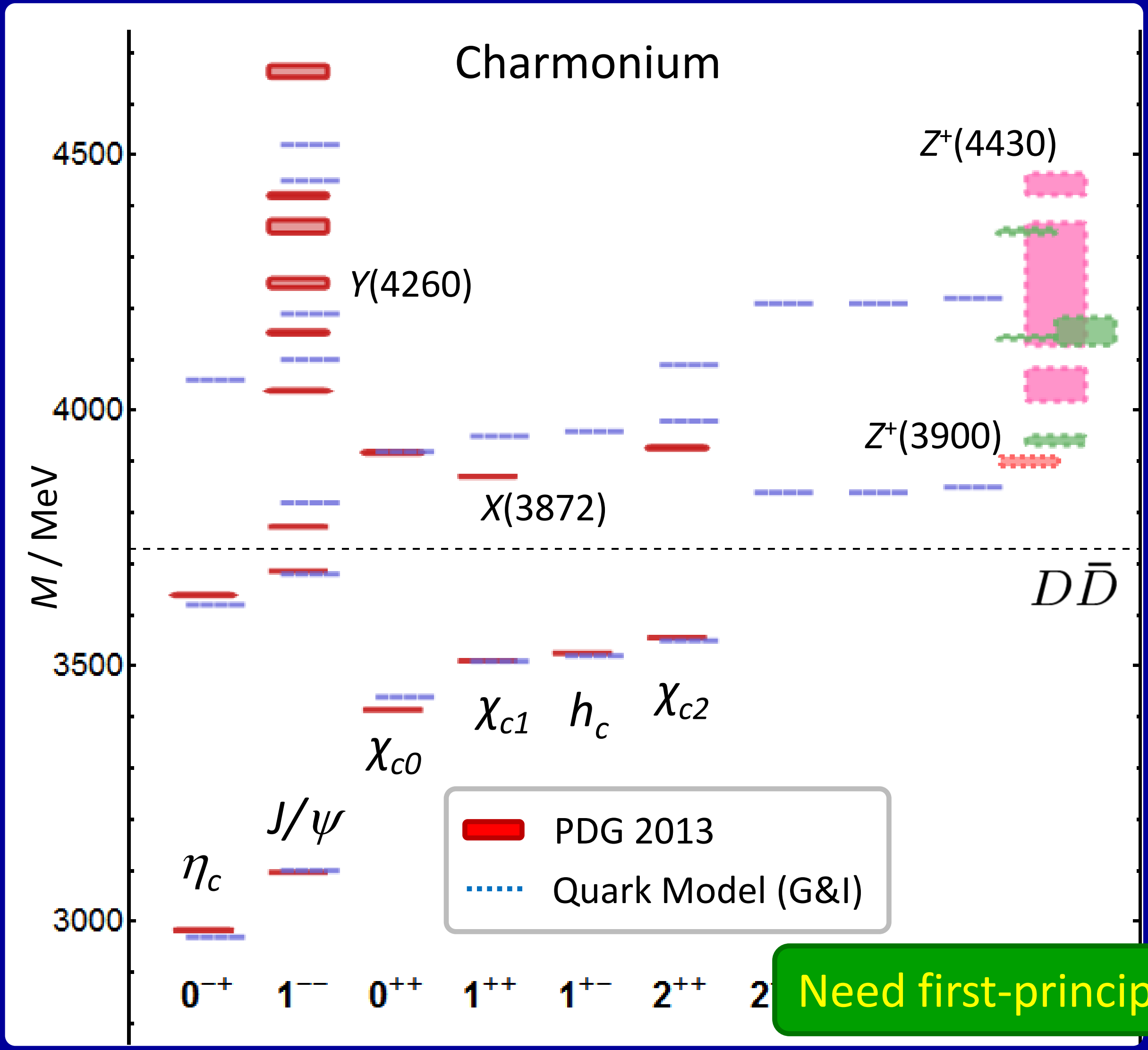


X(3872), *Y*(4260), Z^{+}(4430), Z^{+}(3900), ... ?

Also: D_s(2317), light scalars, ...

Exotic 1^{-+} ?

Heavy meson spectroscopy



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Also: $D_s(2317)$, light scalars, ...

Exotic 1^{-+} ?

BESIII

LHC

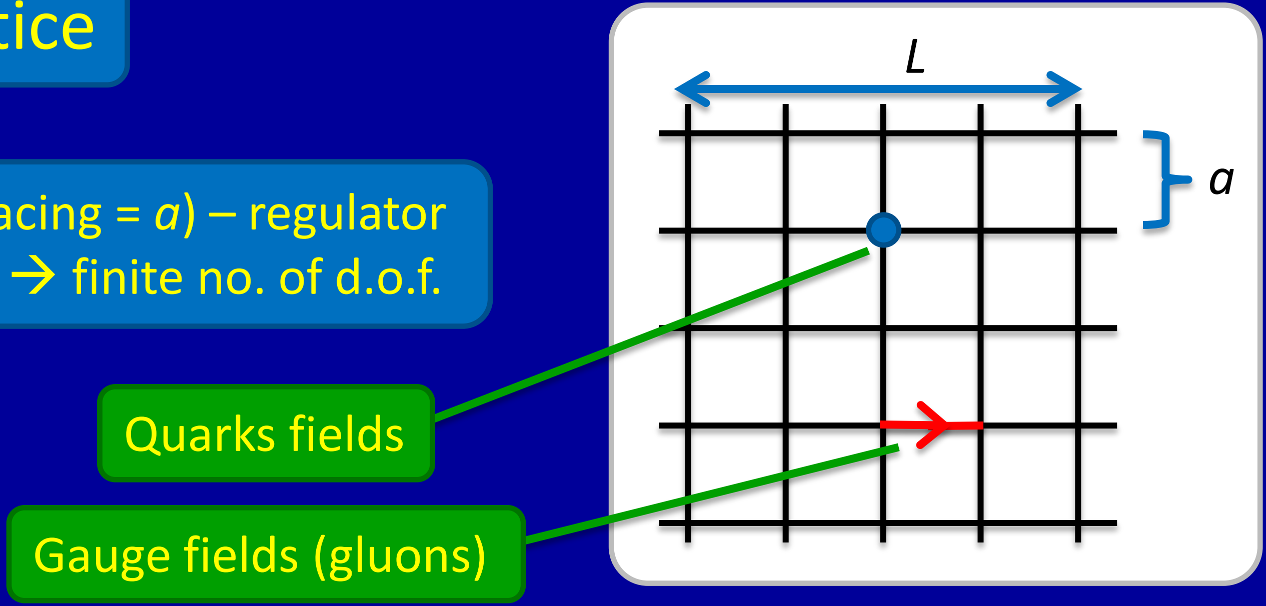
Belle II

...

Need first-principles calculations in QCD

QCD on a lattice

Discretise (spacing = a) – regulator
Finite volume → finite no. of d.o.f.



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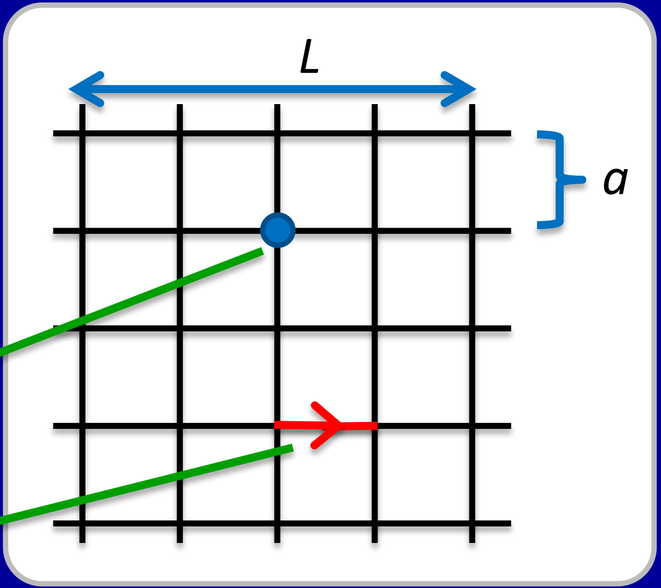

Quarks fields

Gauge fields (gluons)

Path integral formulation

Euclidean time: *t* → i *t*

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U\, f(\psi, \bar{\psi}, U) e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

Numerical methods (Monte Carlo)

- Finite *a* and *L* (and reduced sym.)
- Unphysical m_π

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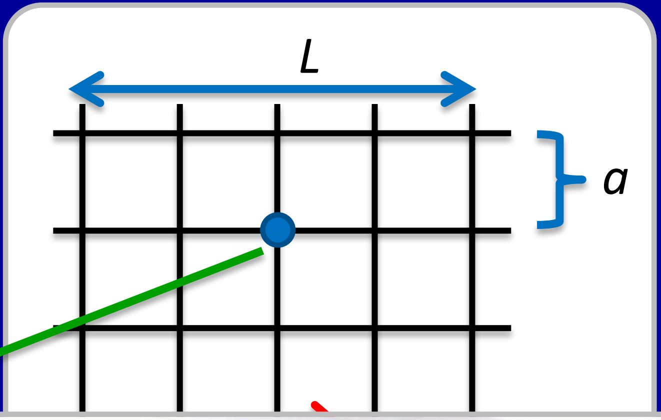

Quar

Gauge fi

Excited spectroscopy on the lattice

Energy eigenstates of Hamiltonian from 2-pt corrs.

$$C_{ij}(t) =< 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2 E_n} < 0|\mathcal{O}_i(0)|n >< n|\mathcal{O}_j^\dagger(0)|0 >$$

Excited spectroscopy on the lattice

(our approach)

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Interpolating operators

$$O(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\ \bar{\psi}(x)\left[\Gamma \overleftrightarrow{D}\overleftrightarrow{D}\ldots\right]\psi(x)$$

Definite $J^{P(C)}$ (**p** = **0**)

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Large basis of ops → matrix of corrs.

Generalised eigenvalue problem

$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$

$$Z_i^{(n)} \equiv < 0|\mathcal{O}_i|n >$$

$$\lambda^{(n)}(t) \to e^{-E_n(t-t_0)}$$

Eigenvectors → $Z^{(n)}$

($t >> t_0$)

Lattice setup

Hadron Spectrum Collaboration

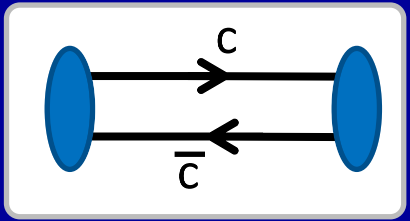
Dynamical (unquenched) *u*, *d* and *s* quarks [$N_f = 2+1$]. Relativistic *c* quark

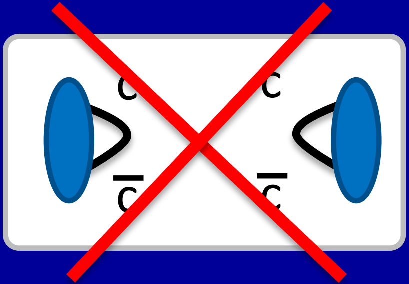
Anisotropic – finer in temporal direction ($a_s/a_t \approx 3.5$), $a_s \approx 0.12$ fm

Two volumes: 16^3, $\mathbf{24^3}$ ($L_s \approx 1.9$, 2.9 fm)

$M_\pi \approx 400$ MeV.

Only connected charm quark contributions





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JHEP 05 (2013) 021 – Graham Moir, Mike Peardon, Sinéad Ryan, CT, Liuming Liu

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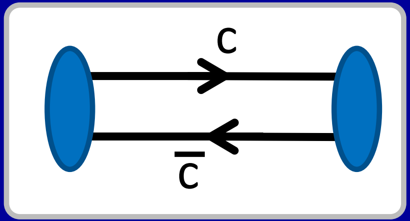

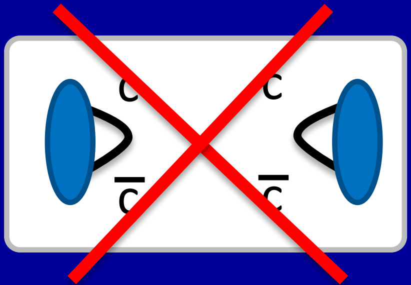

Large no. of 'single-meson' ops. of different structures
Sensitive to gluonic excitations

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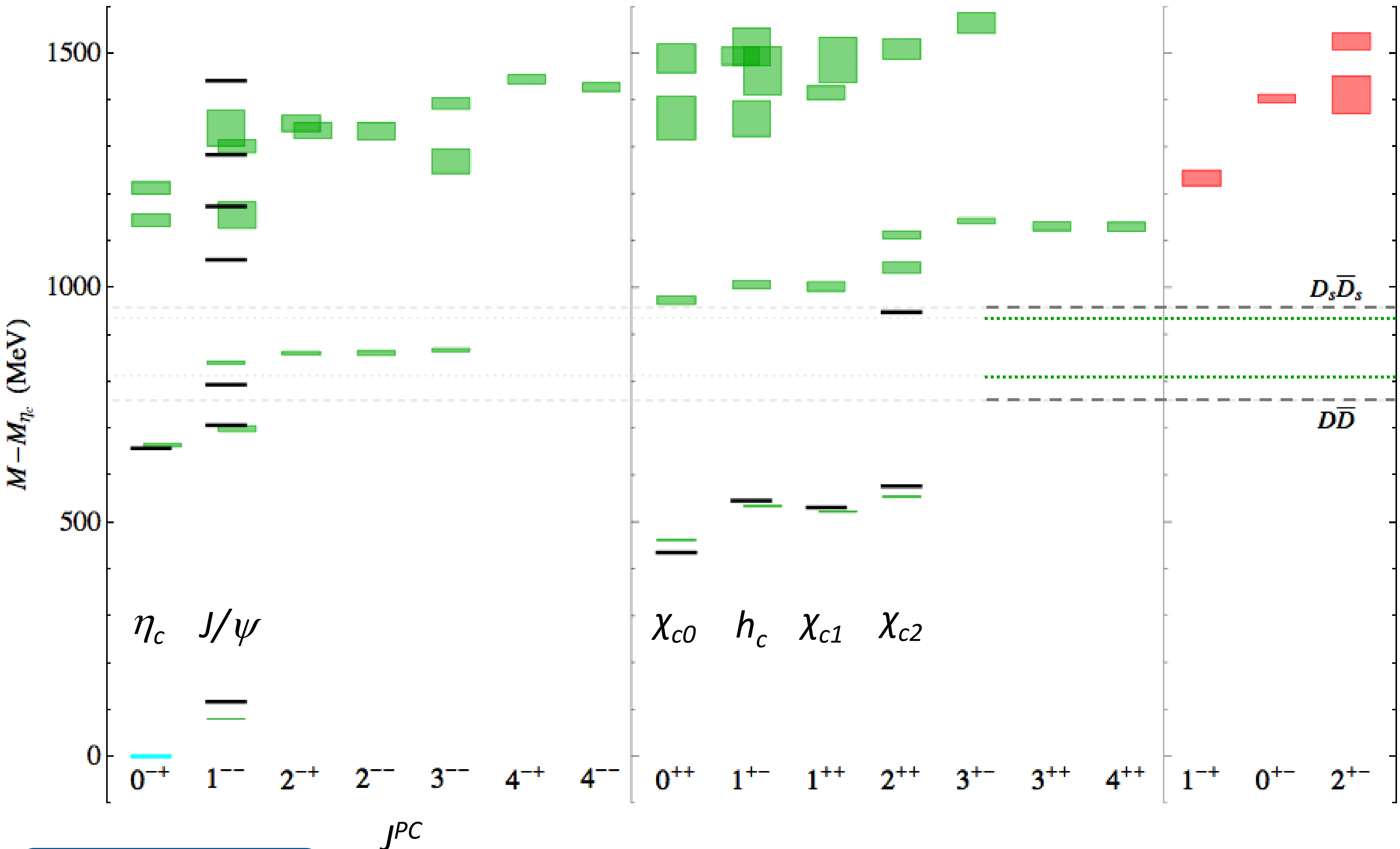

Charmonium

24^3, $M_\pi \approx 400$ MeV

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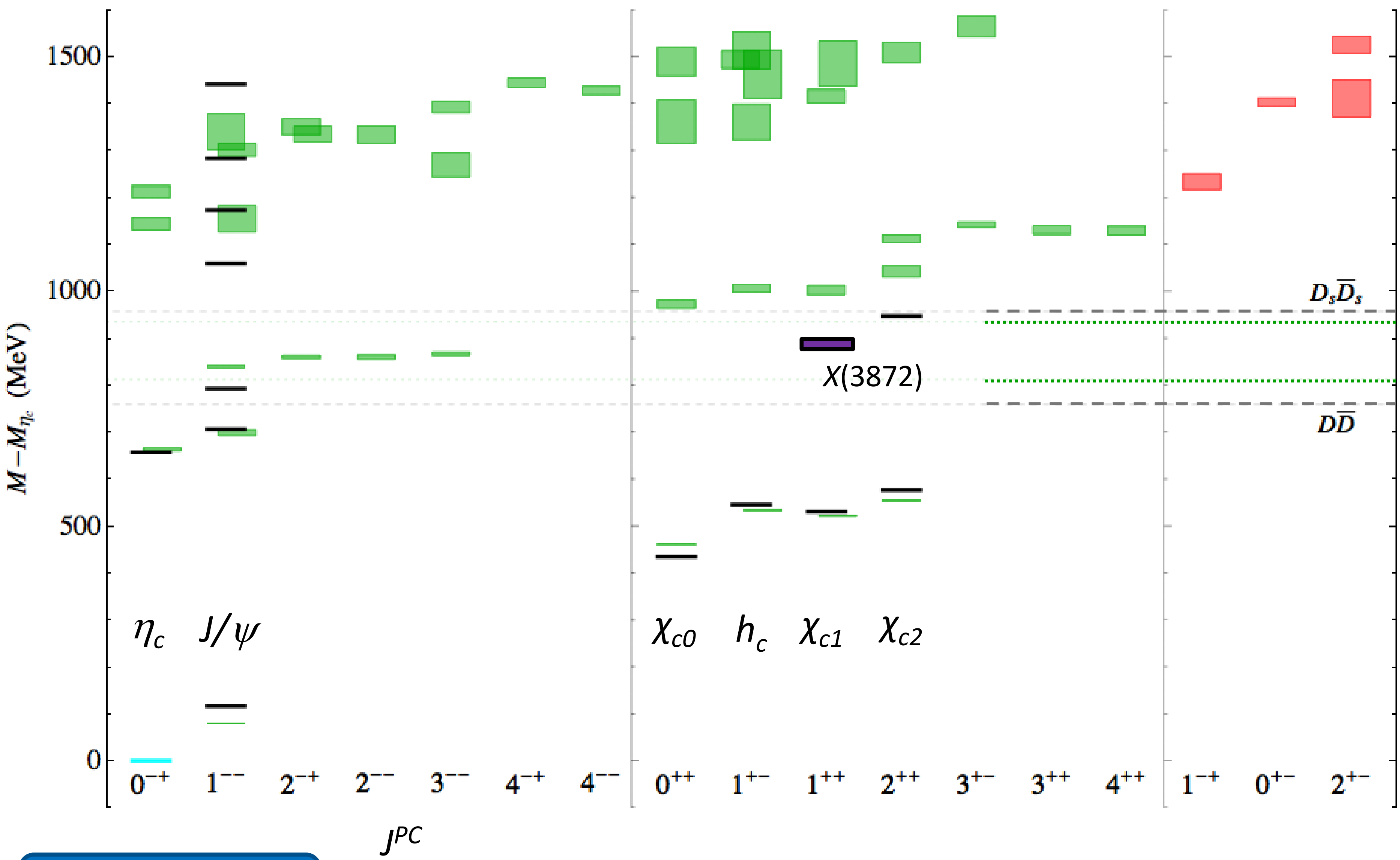

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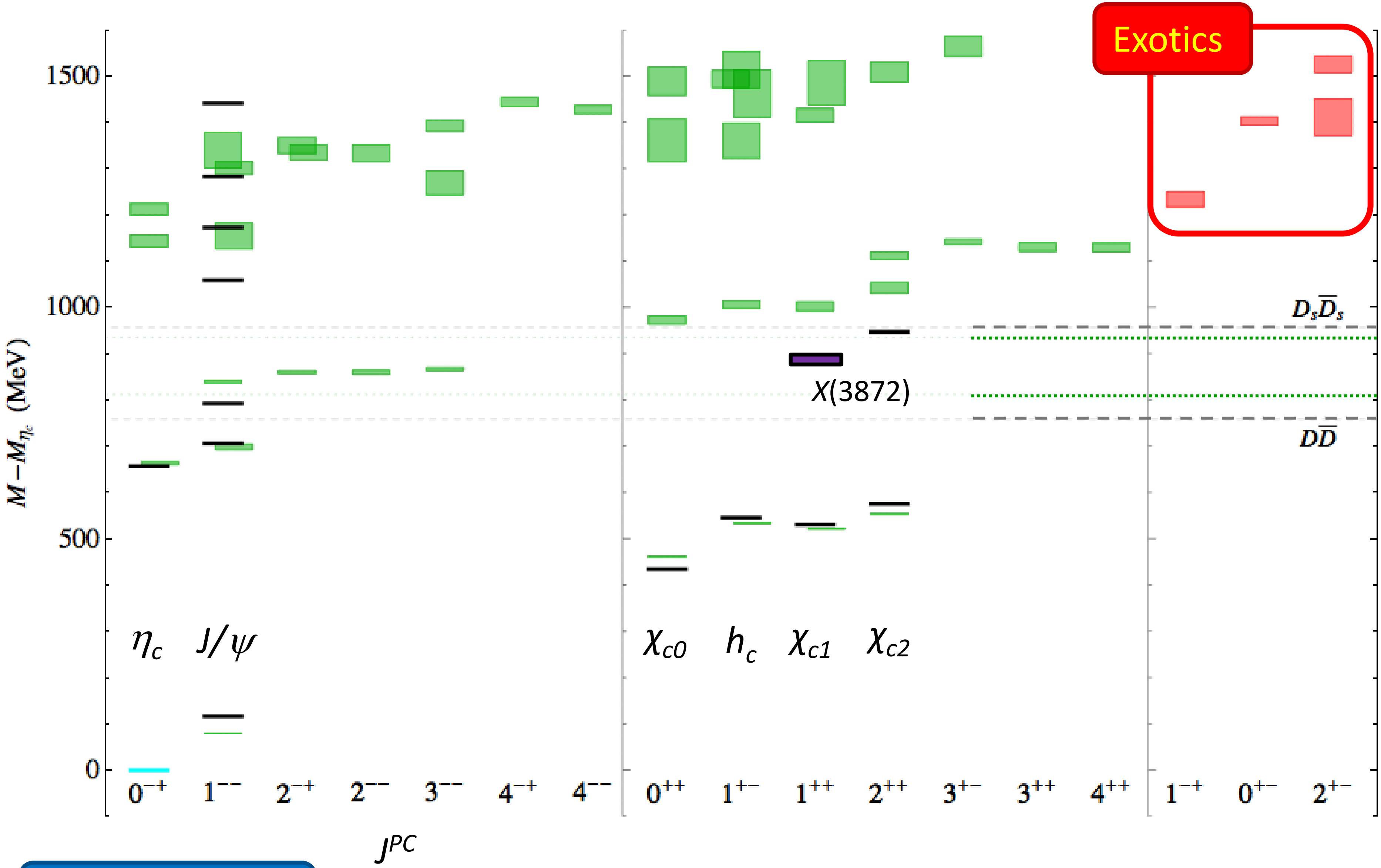

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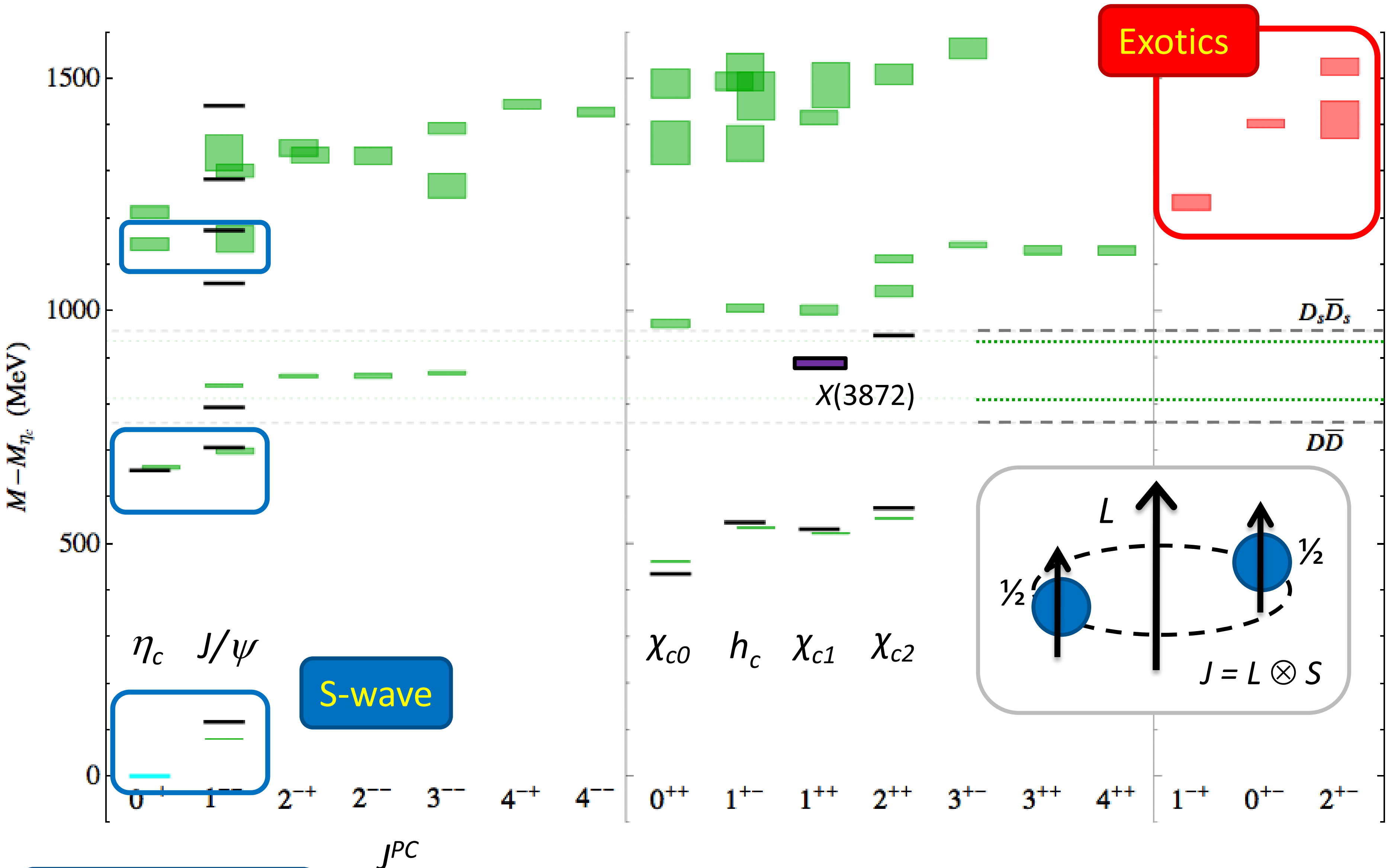

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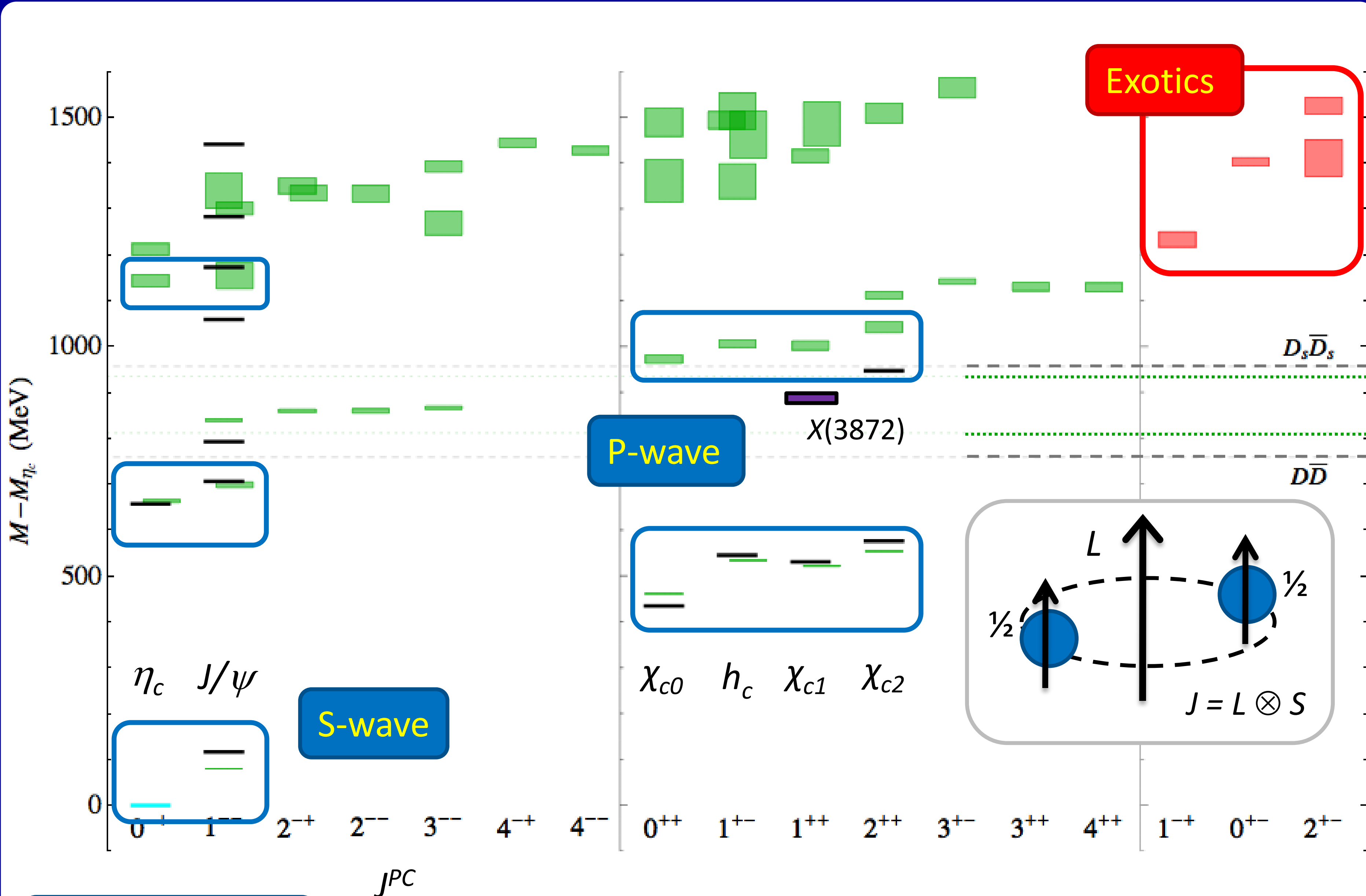

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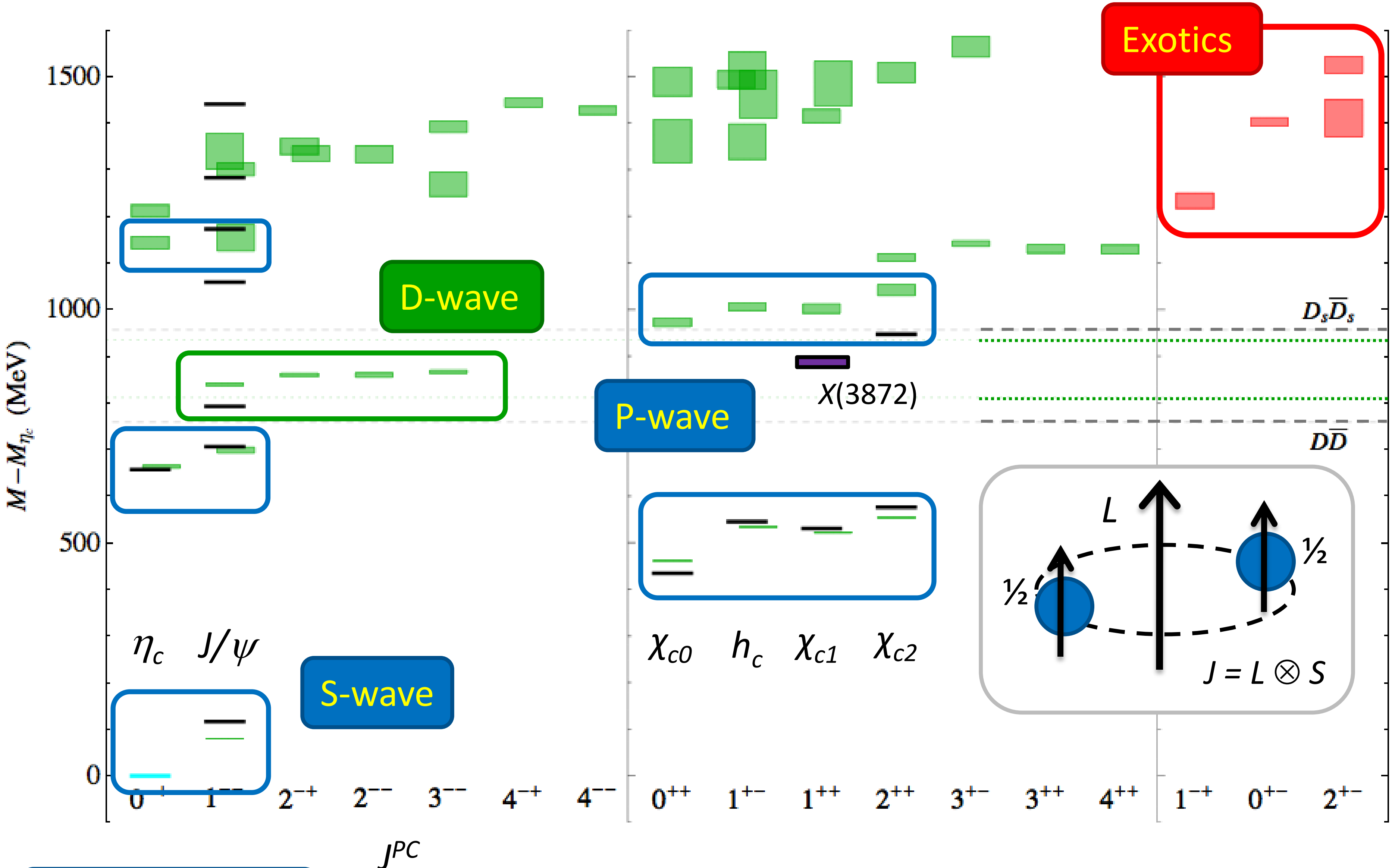

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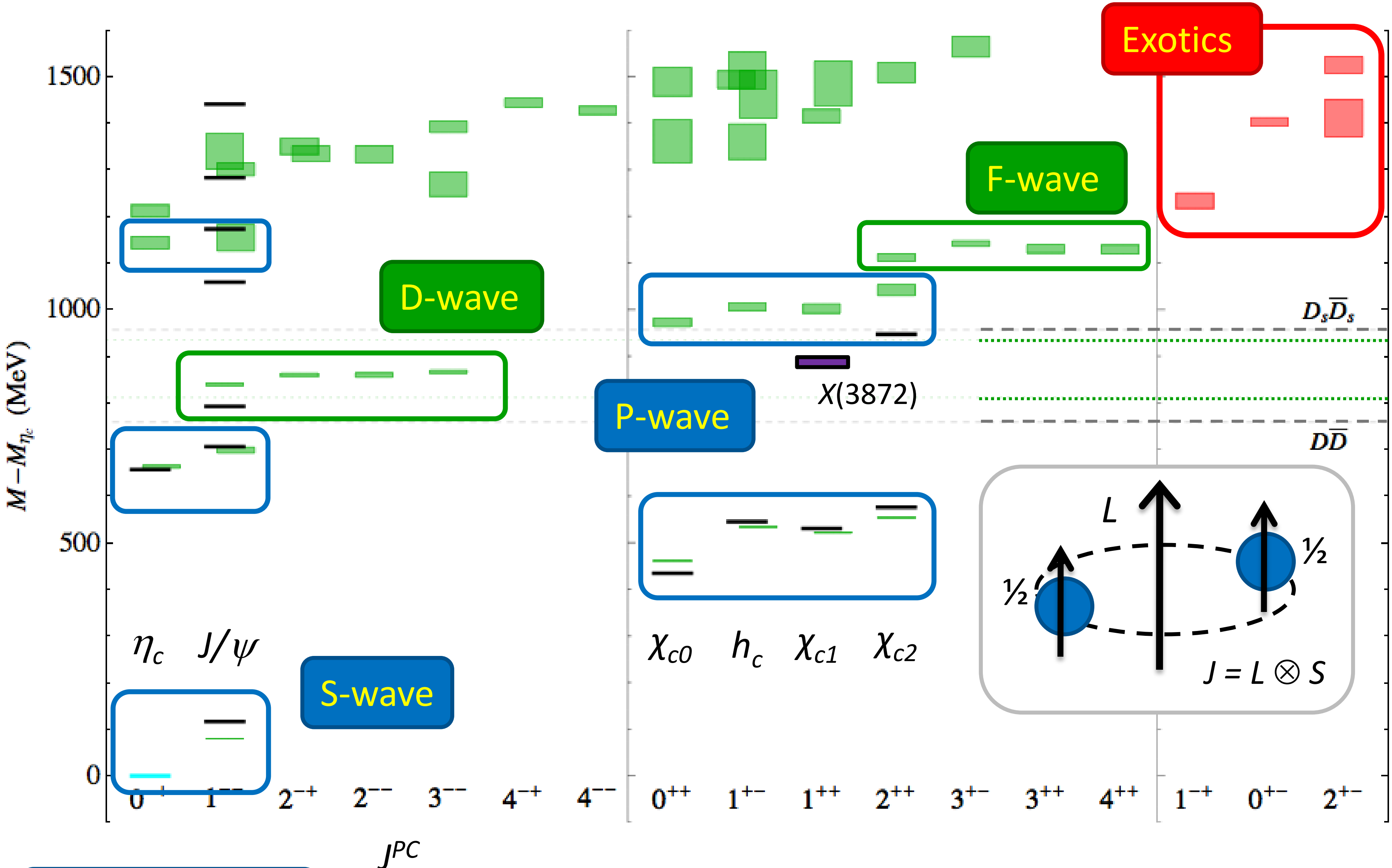

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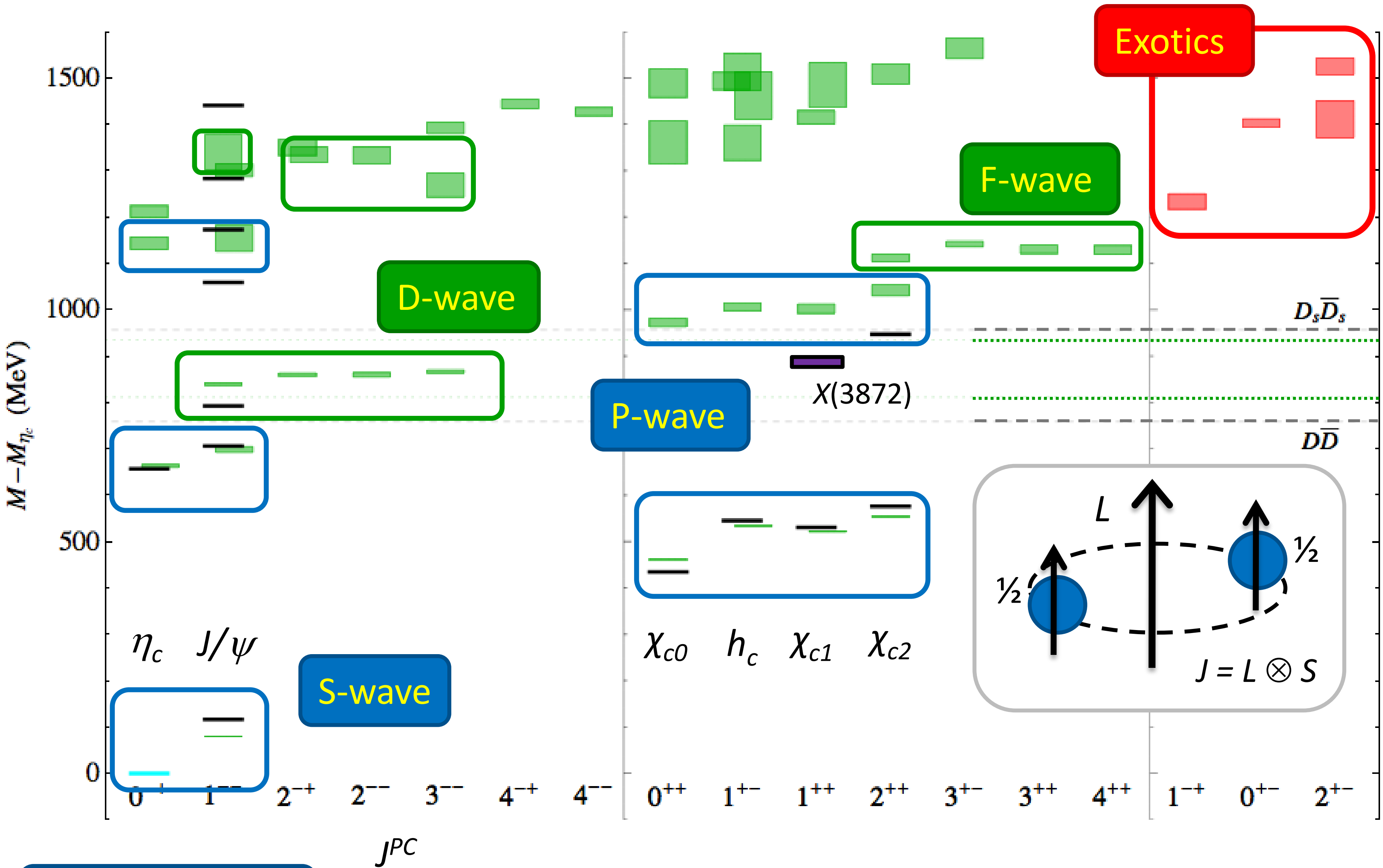

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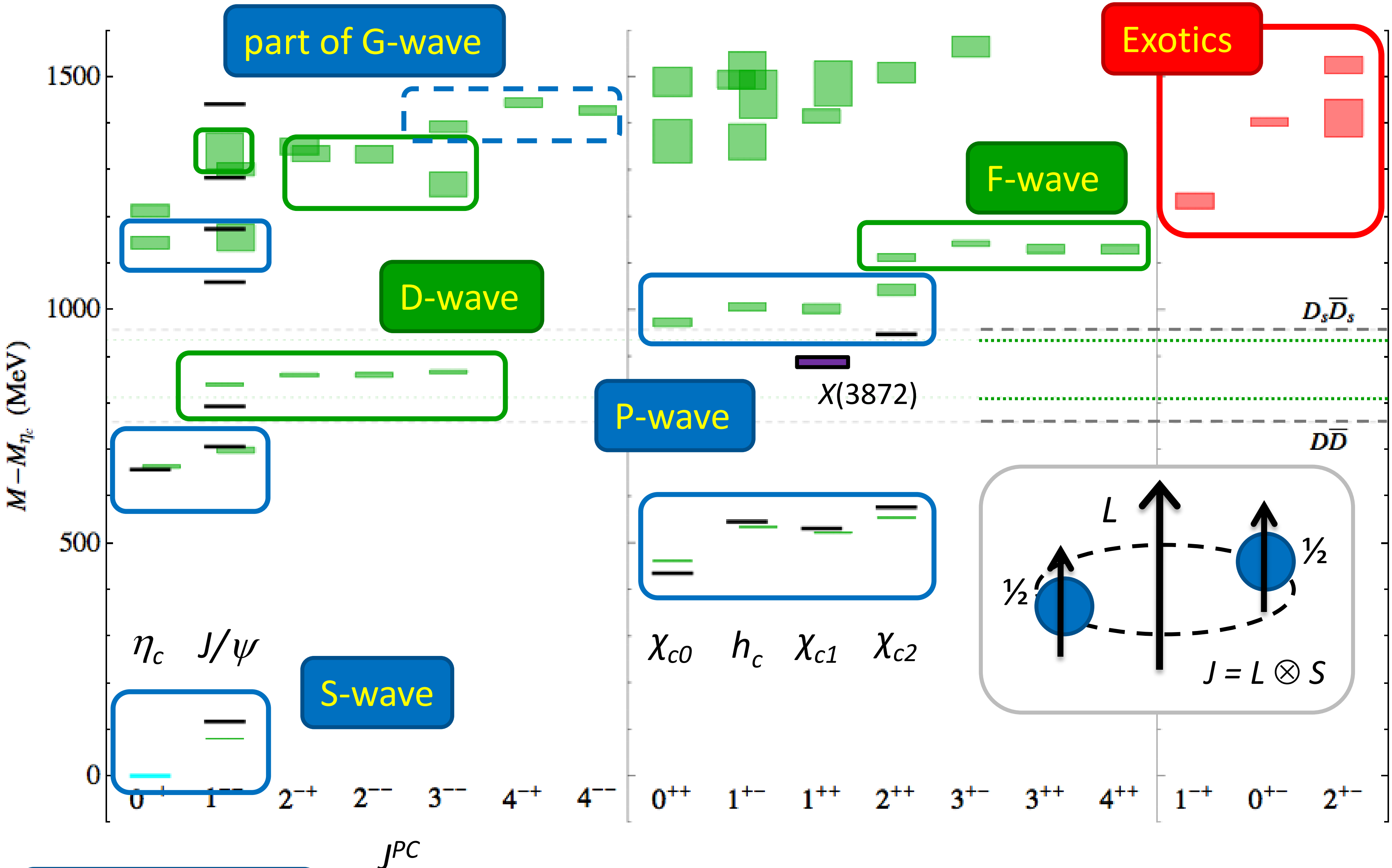

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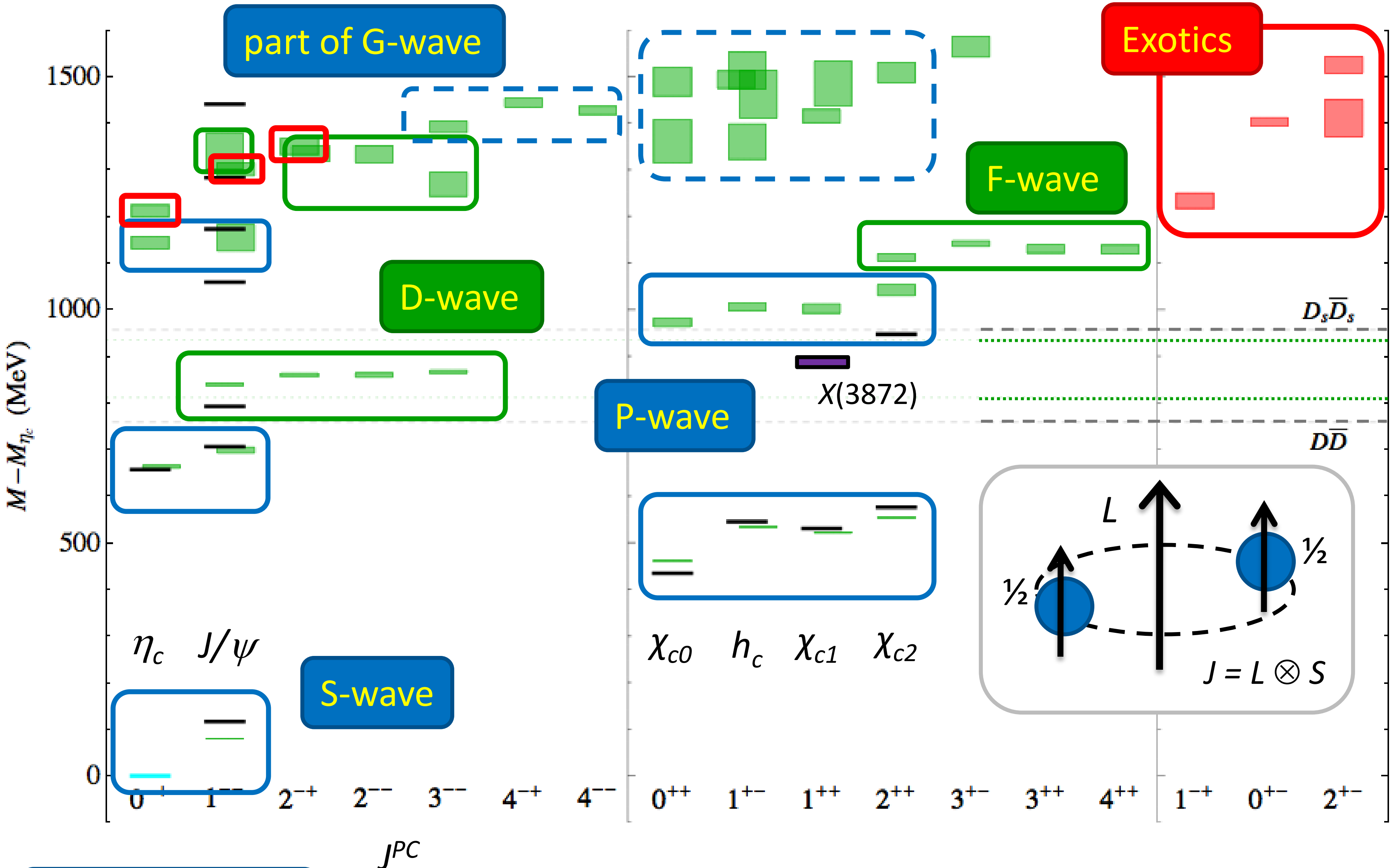

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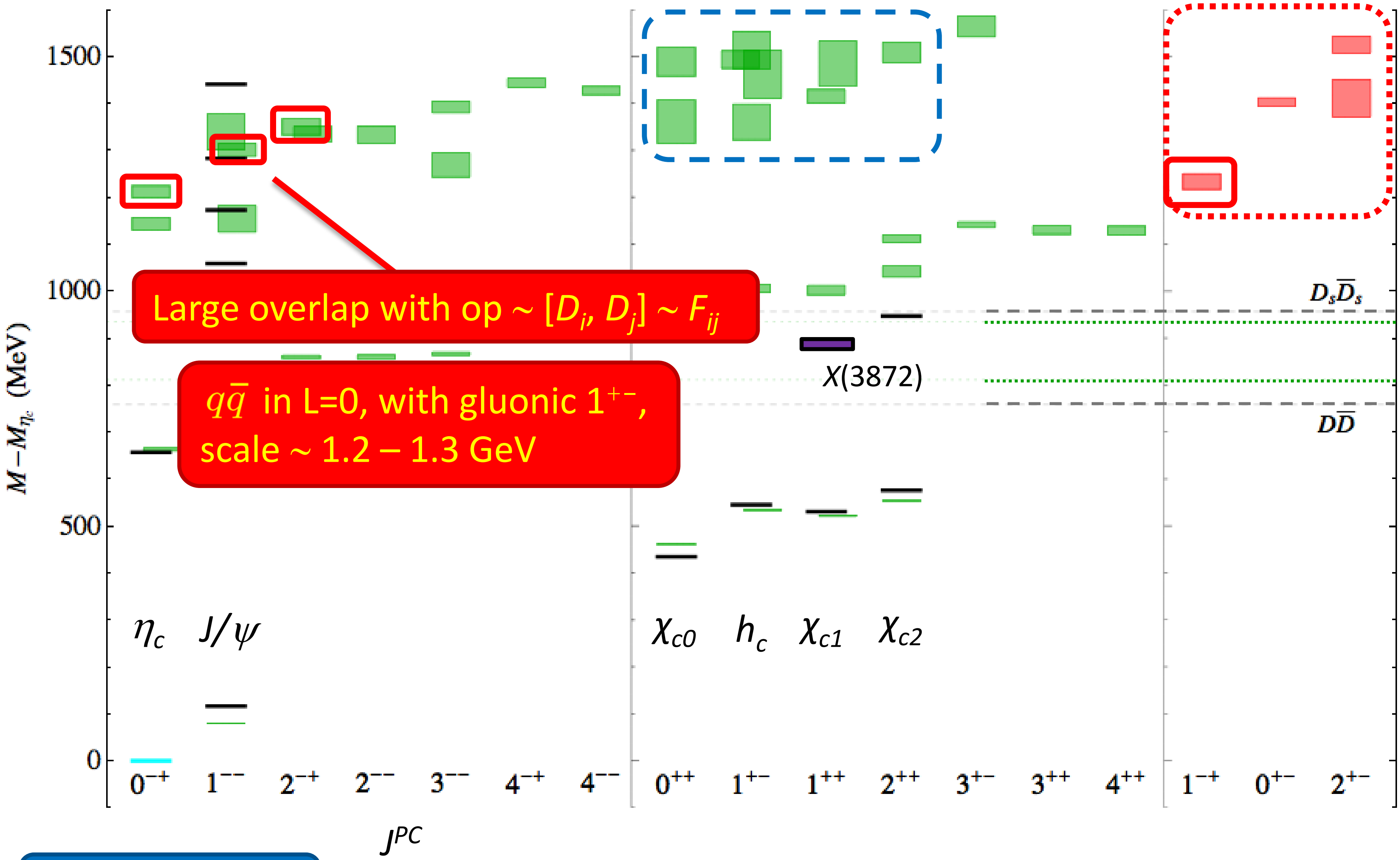

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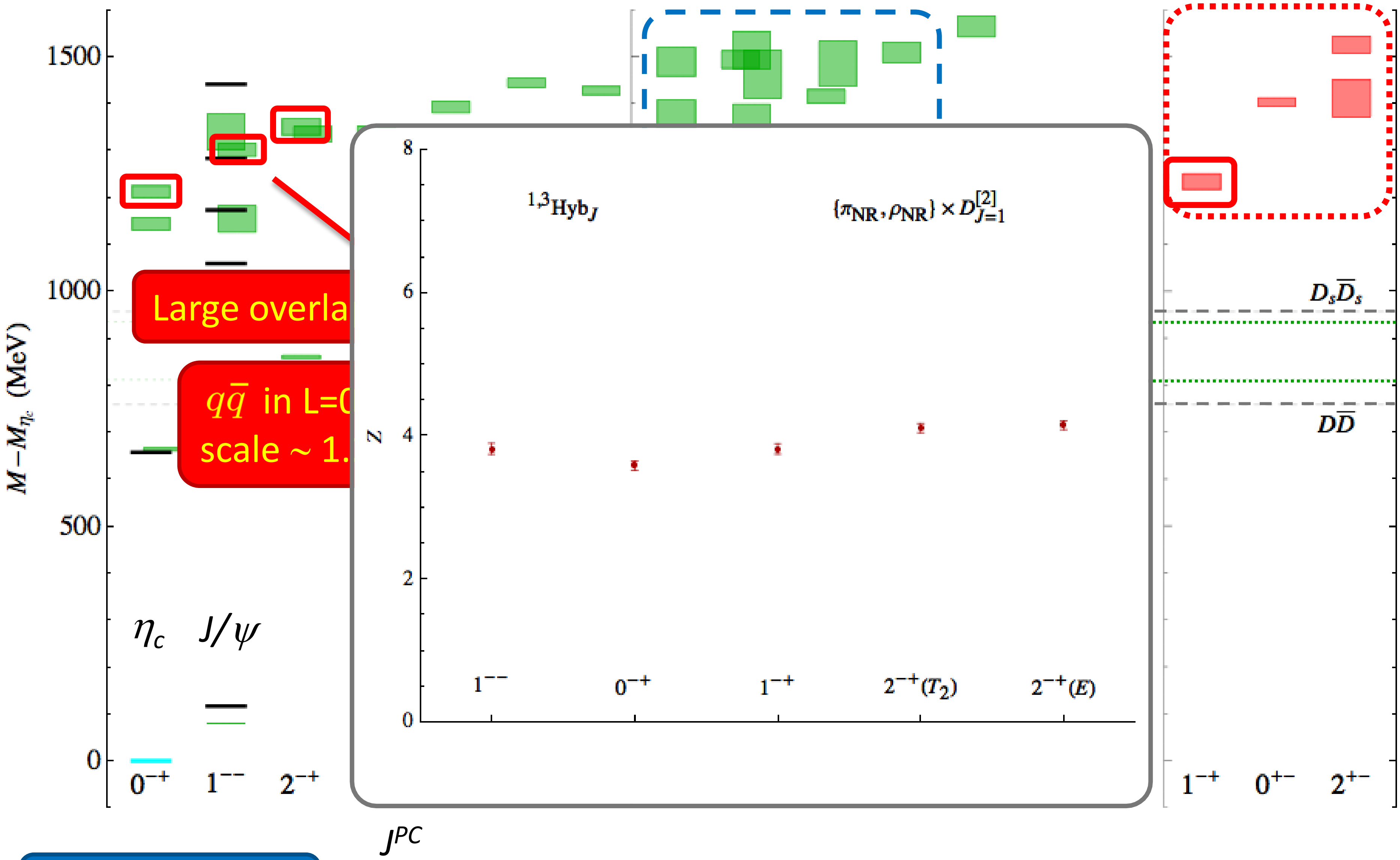

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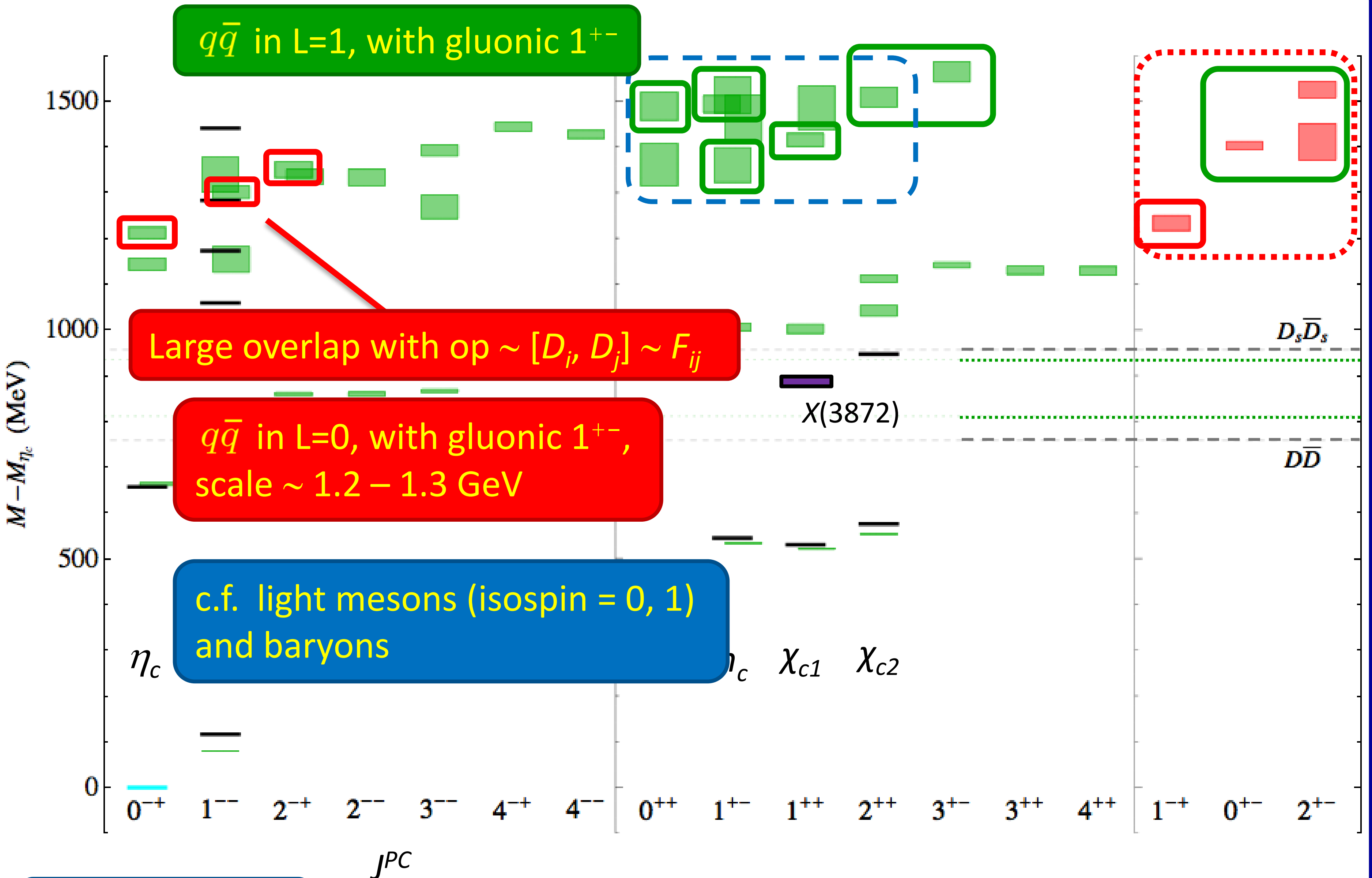

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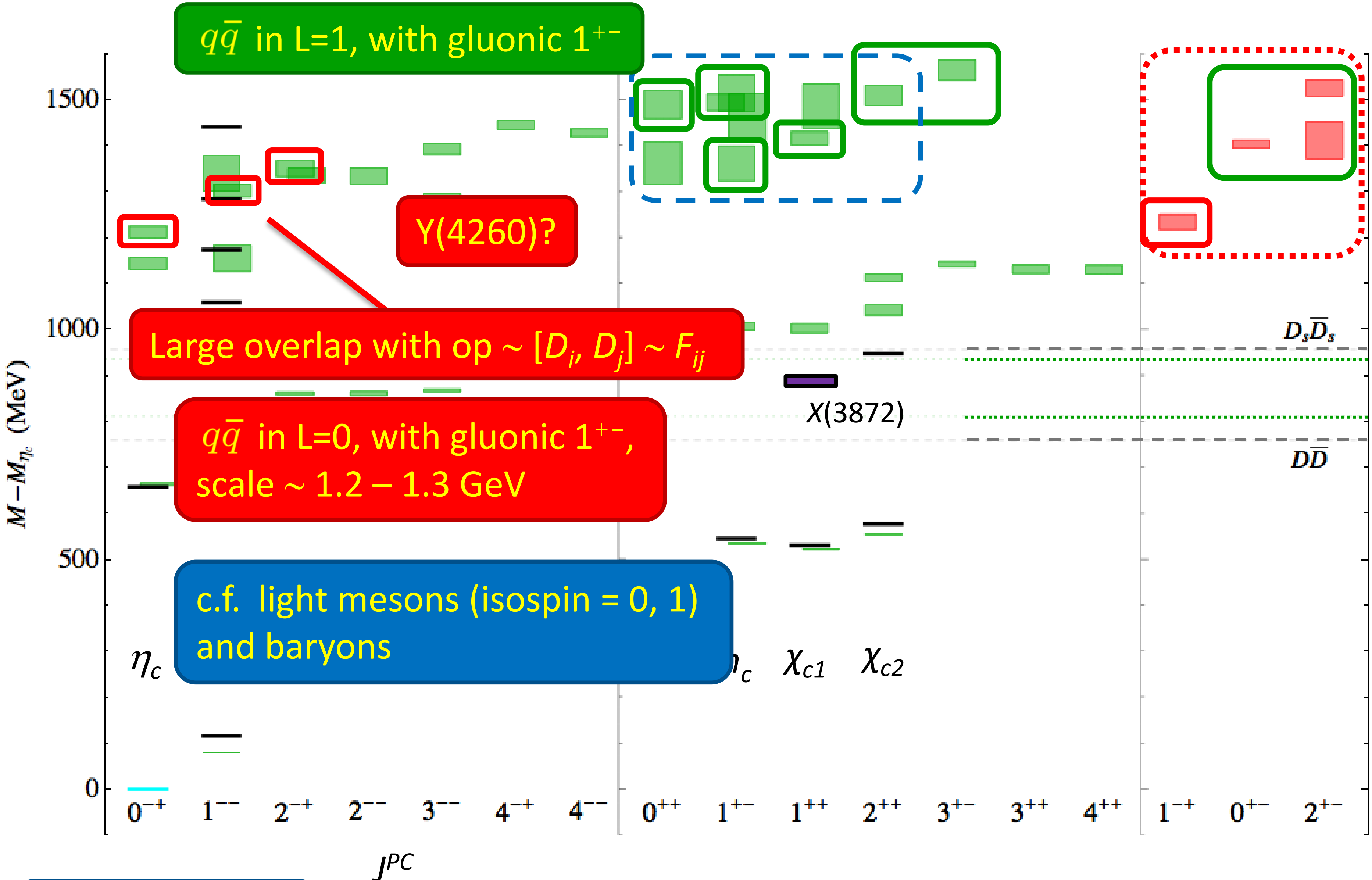

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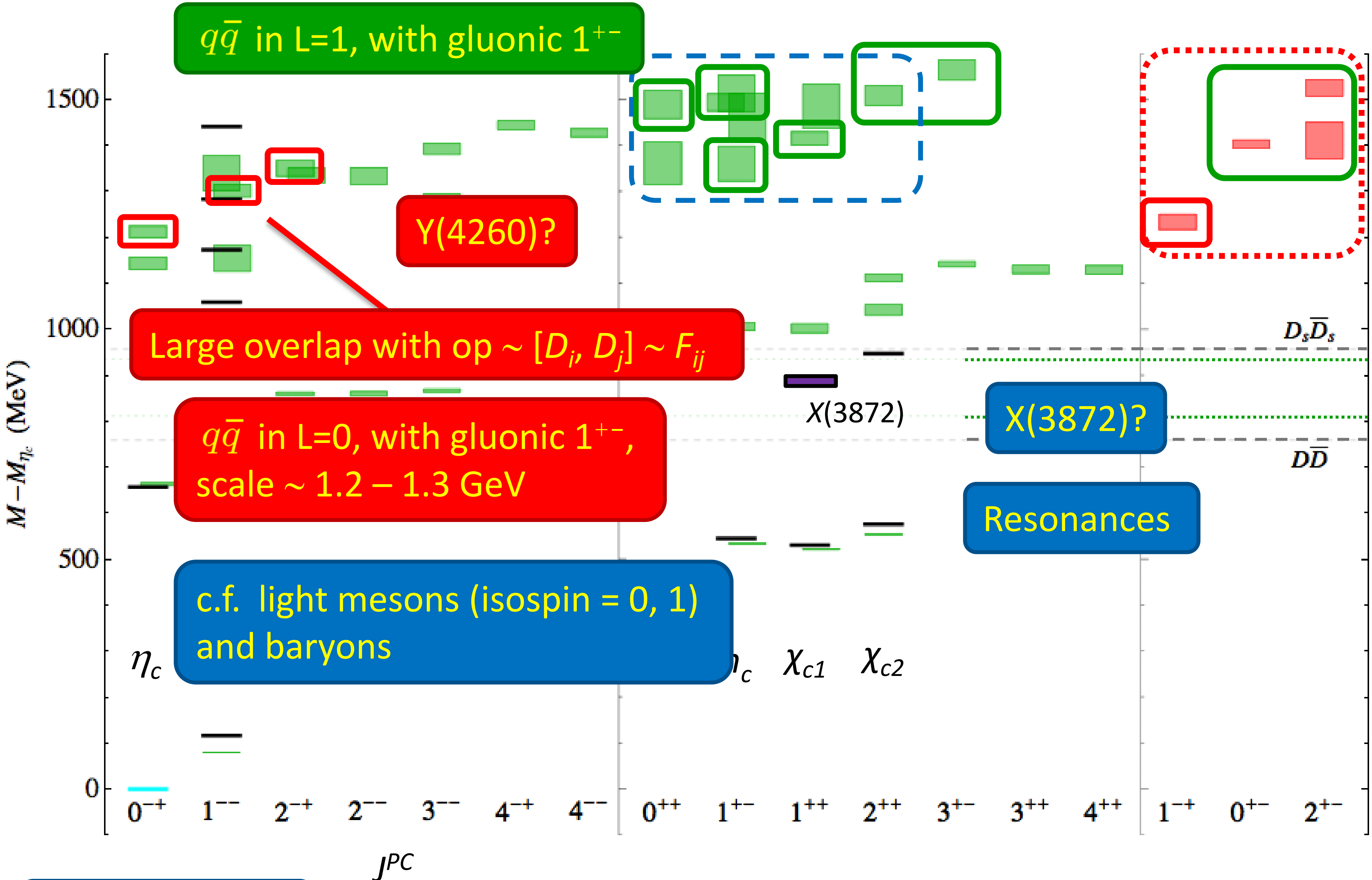

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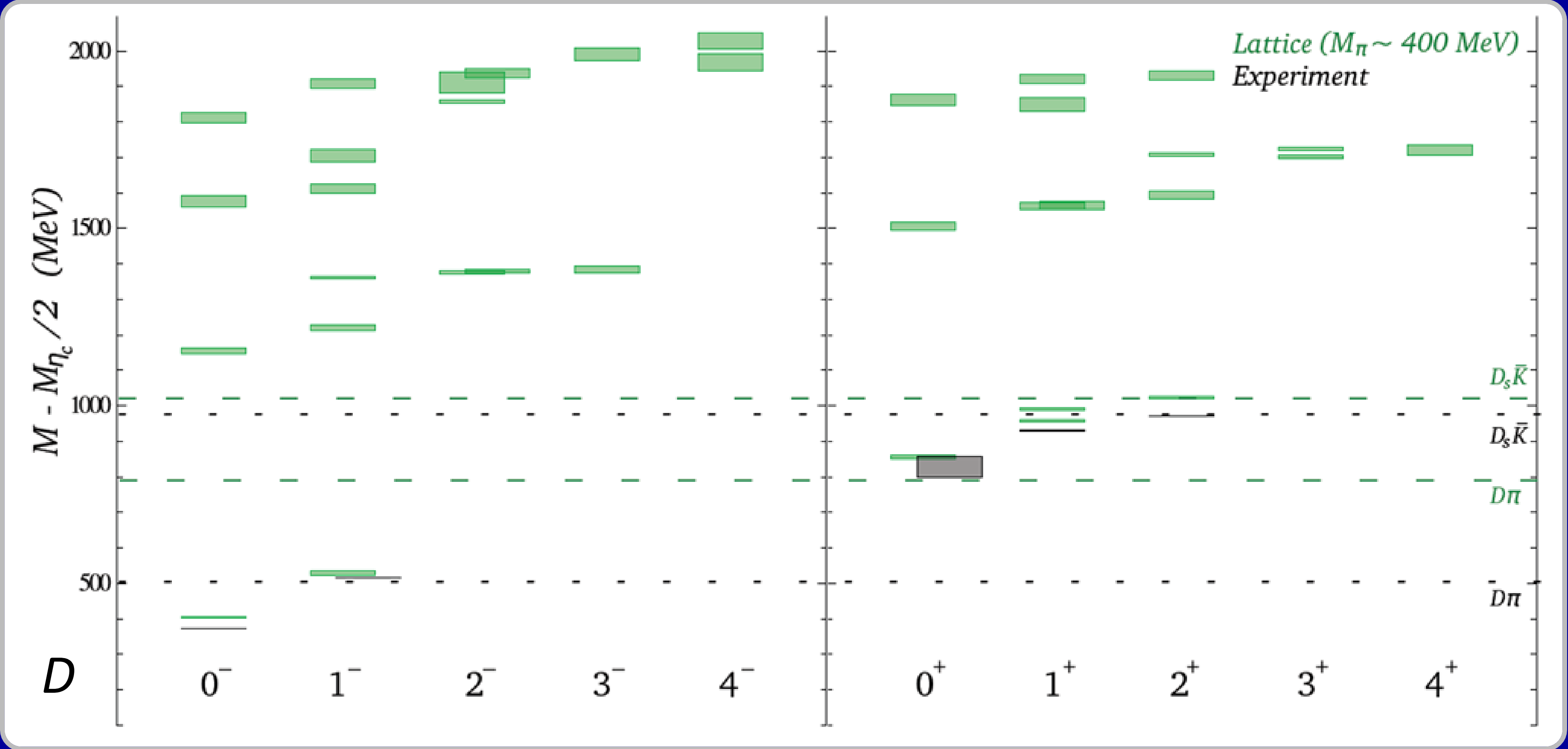
Charmed (D/D_s) mesons



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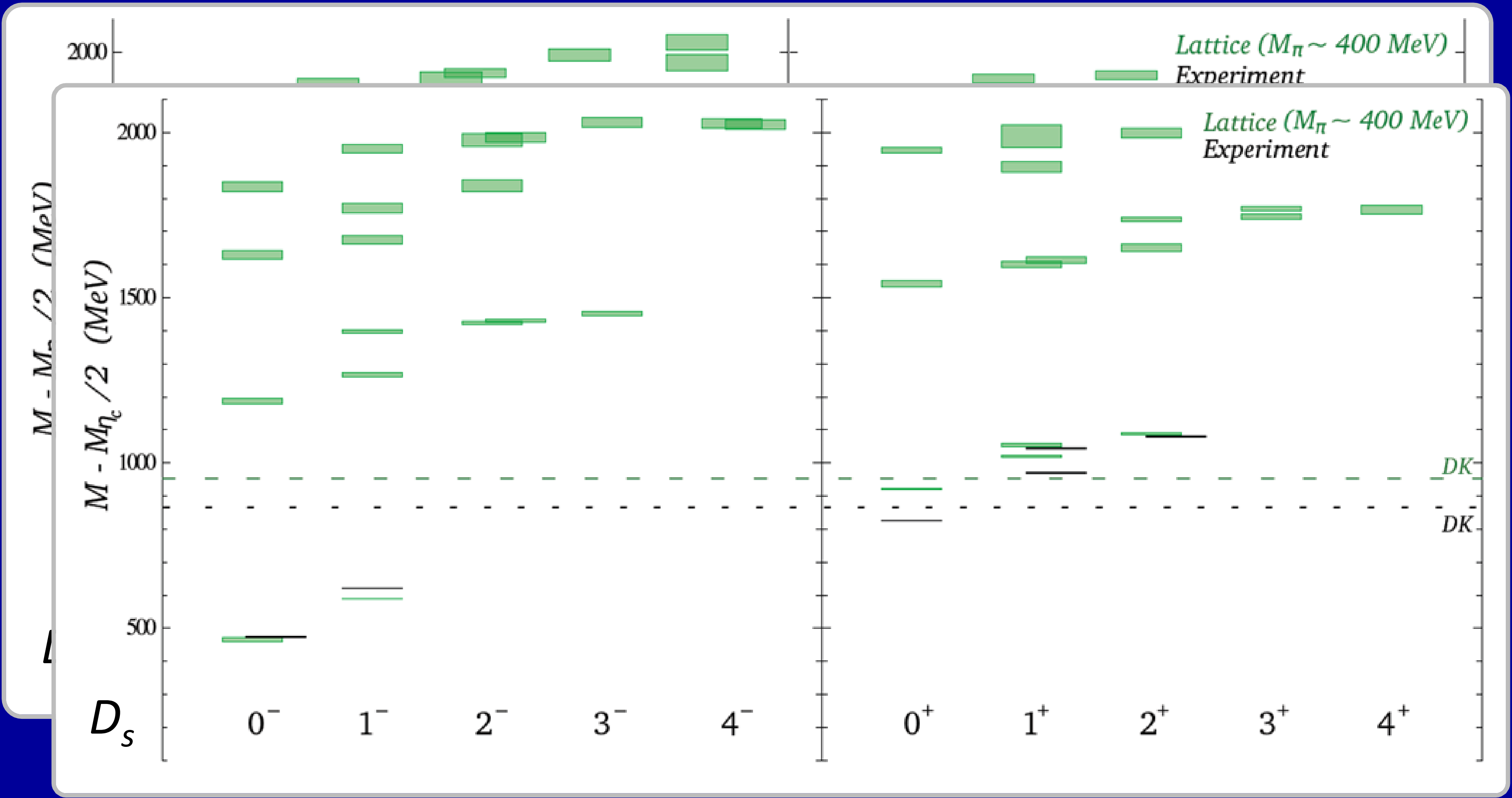

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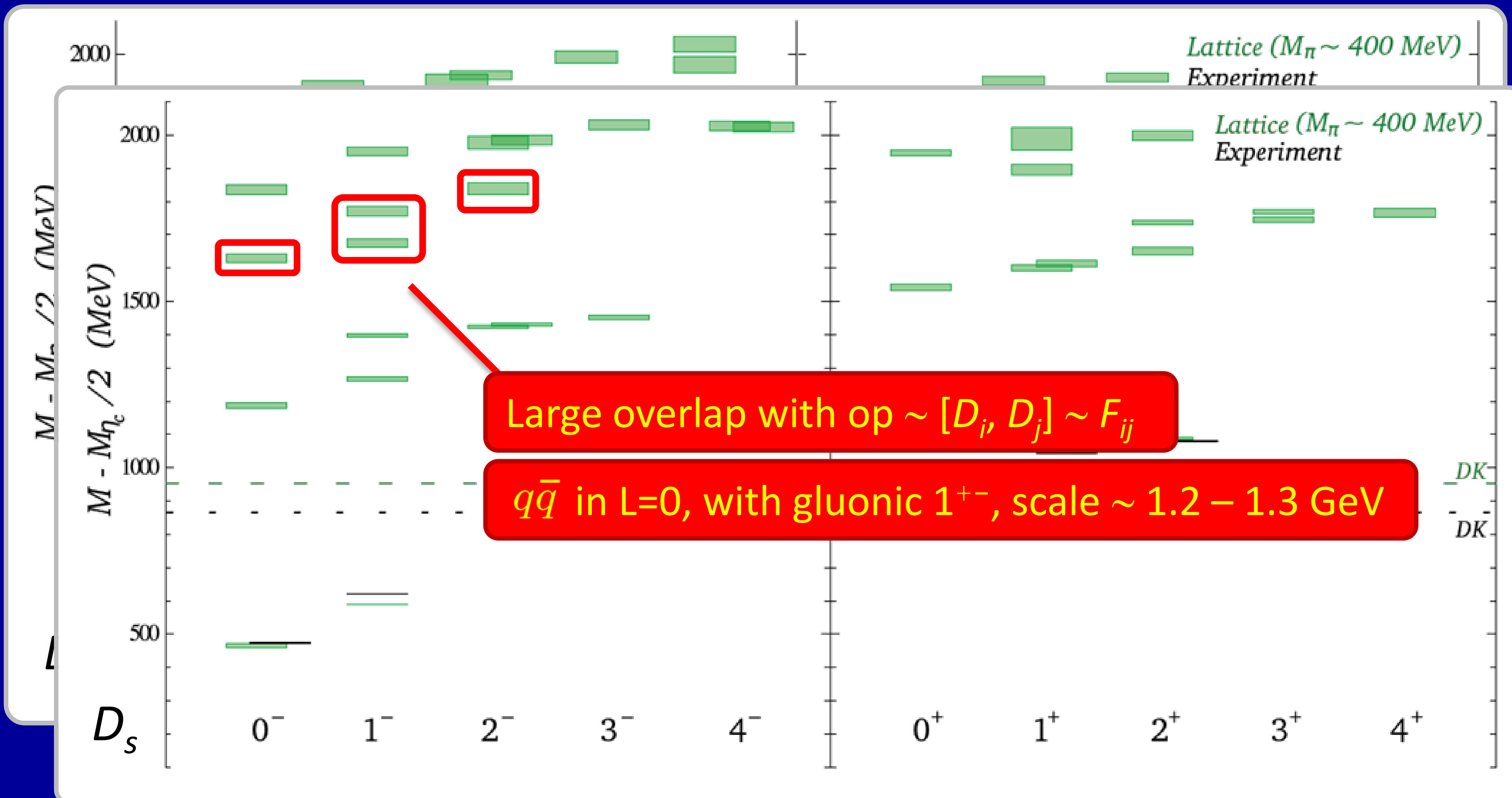

24^3, $M_\pi \approx 400$ MeV

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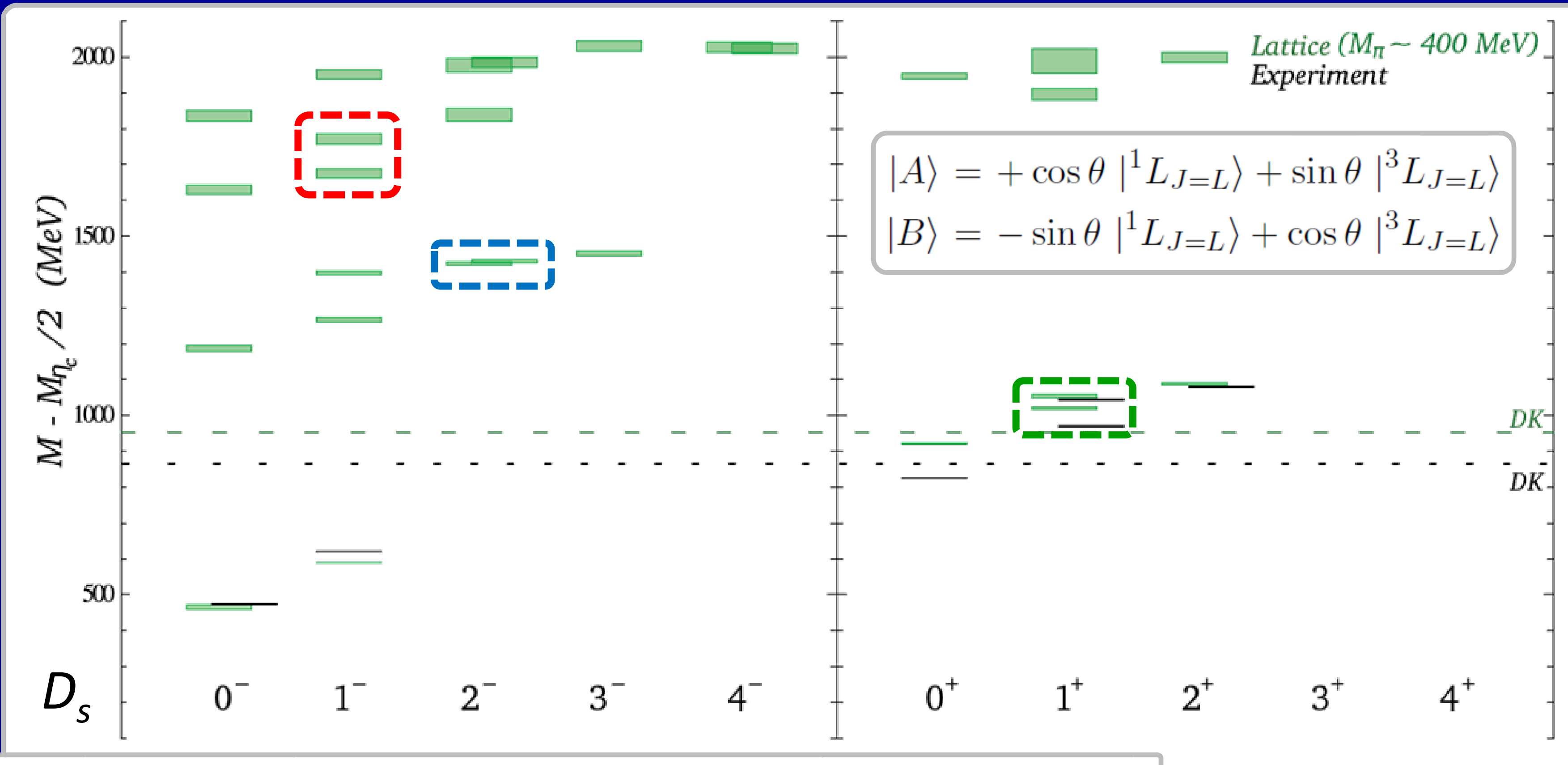
Charmed (D/D_s) mesons – mixing



	J^P	$\lvert\theta\rvert/^\circ$ $\sim(\rho-\rho_2)$	$\sim\pi$	$\sim\pi_2$	Heavy-quark limit
	1^+	60.9(0.6)	64.9(0.2)	66.4(0.4)	54.7 or 35.3
	2^-	64.9(1.9)	68.7(2.0)	70.9(1.8)	50.8 or 39.2
	1^- (hybrid)	59.9(1.7)	67.9(0.9)	67.3(0.9)	

24^3, $M_\pi \approx 400$ MeV

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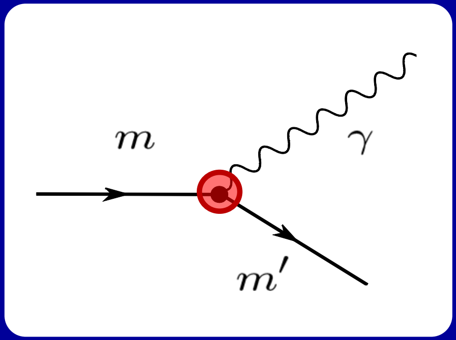
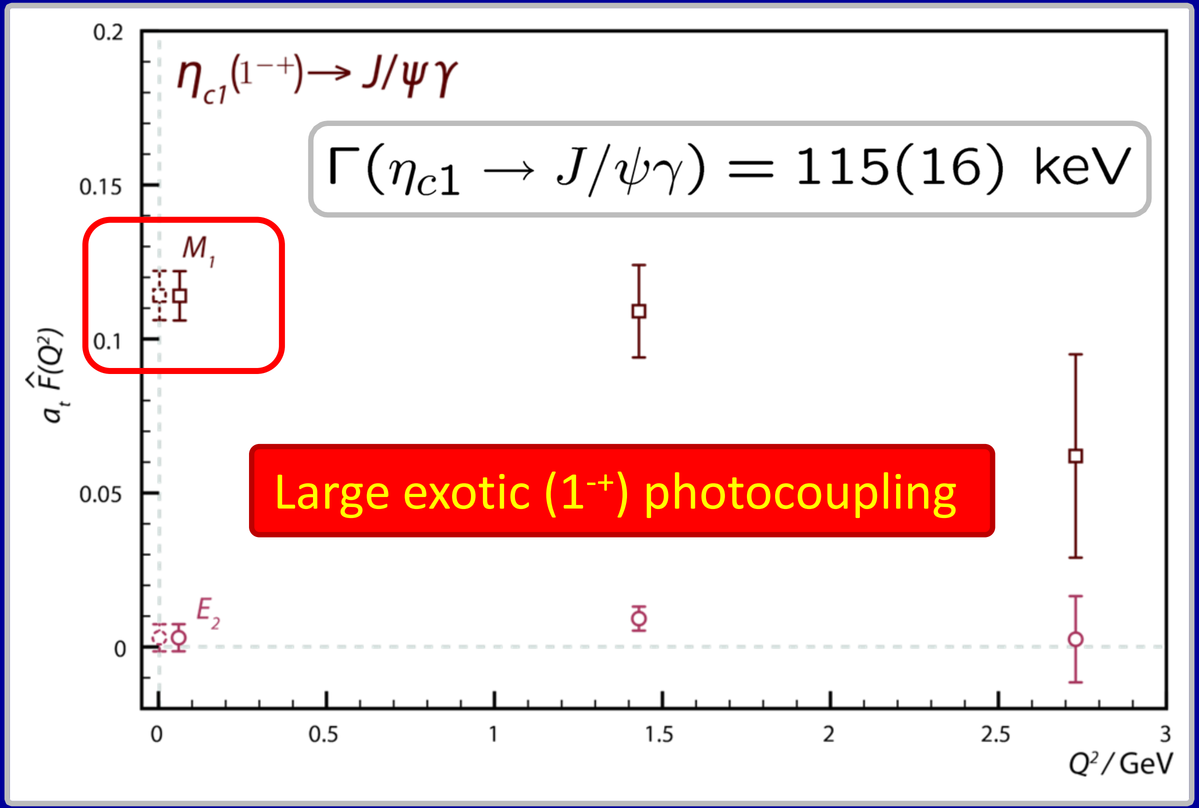
Exotic (1^{-+}) photocoupling

Older quenched calc.
PR D79 094504 (2009)

$$C_{ij}(t_f, t, t_i) = < 0|O_i(t_f)\ \bar{\psi}(t)\gamma^\mu\psi(t)\ O_j(t_i)|0 >$$





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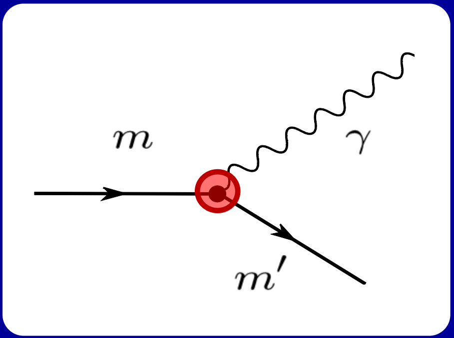
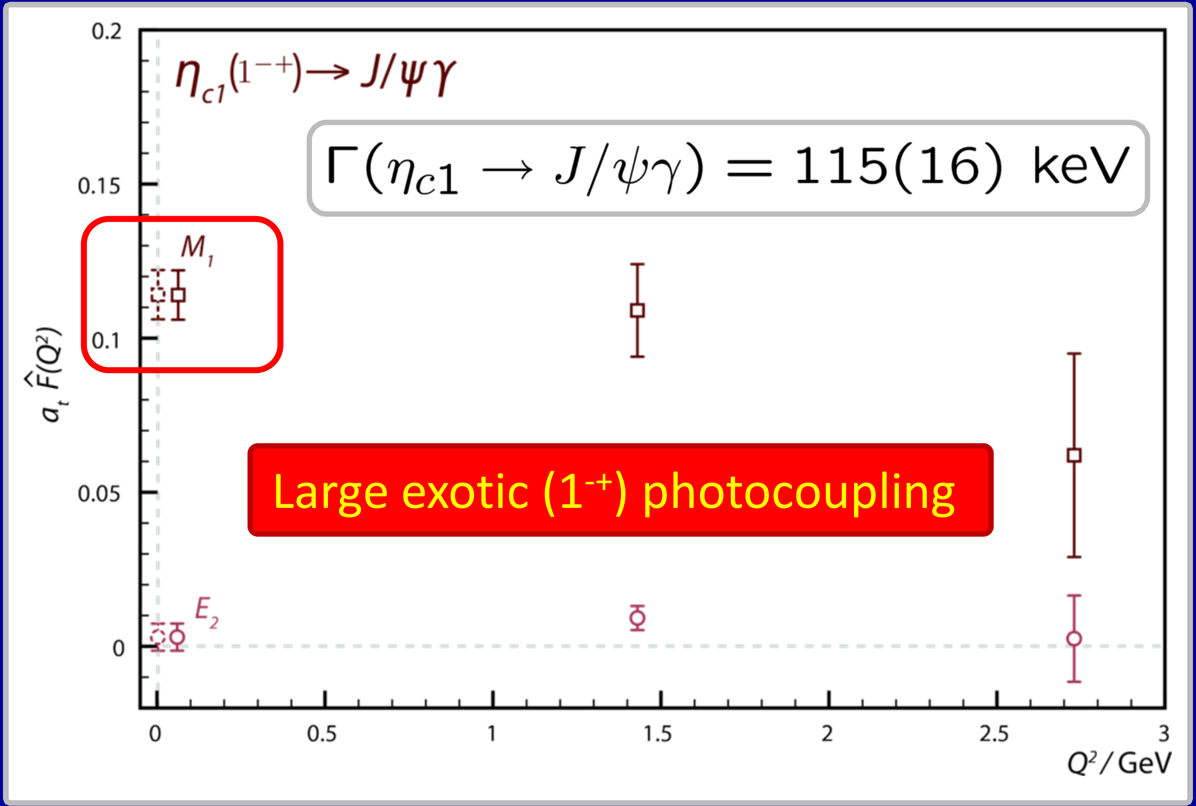




Large for an M_1 transition

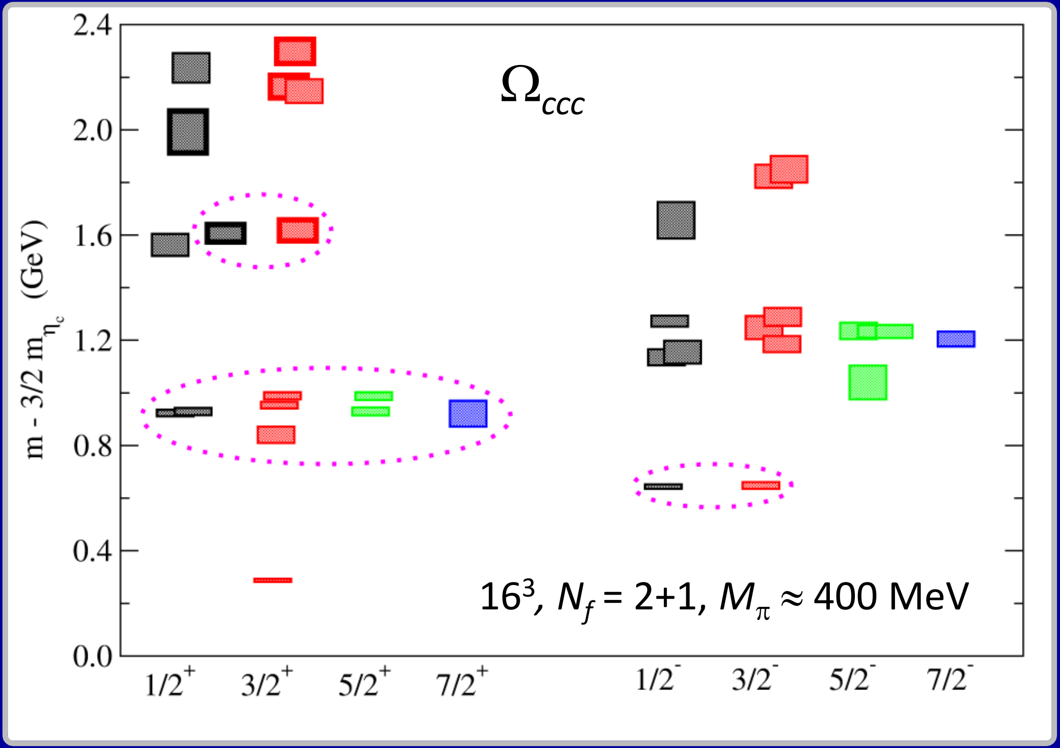
$\Gamma(J/\psi \to \eta_c\gamma) \sim 2$ keV

Spin-triplet hybrid?

Charmed baryons

Padmanath et al (Had Spec Collab) [arXiv:1307.7022]
Excited ccc baryons

Hybrid baryons (non-exotic quantum numbers)



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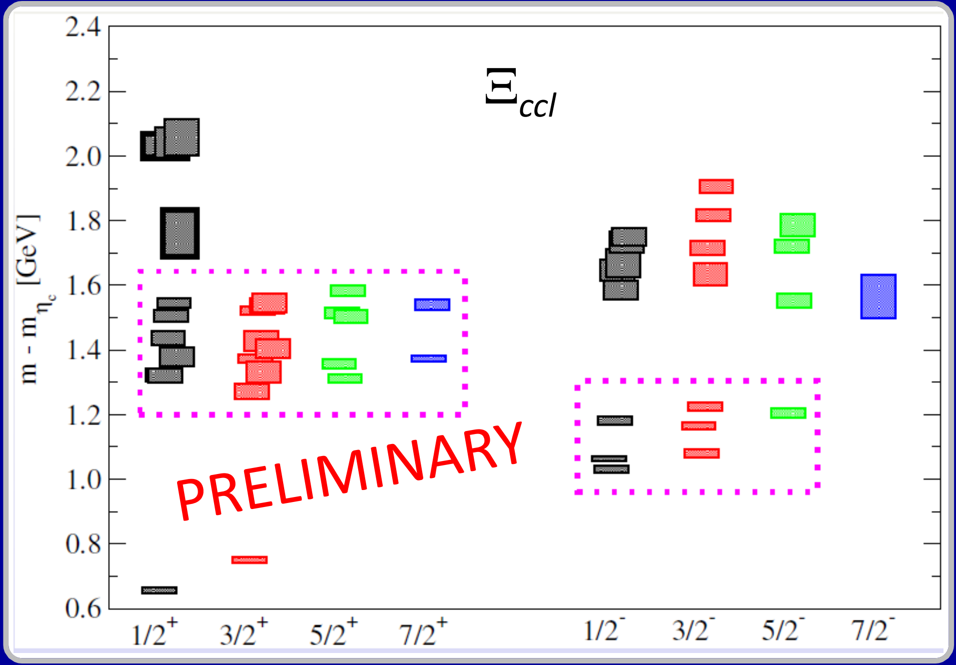
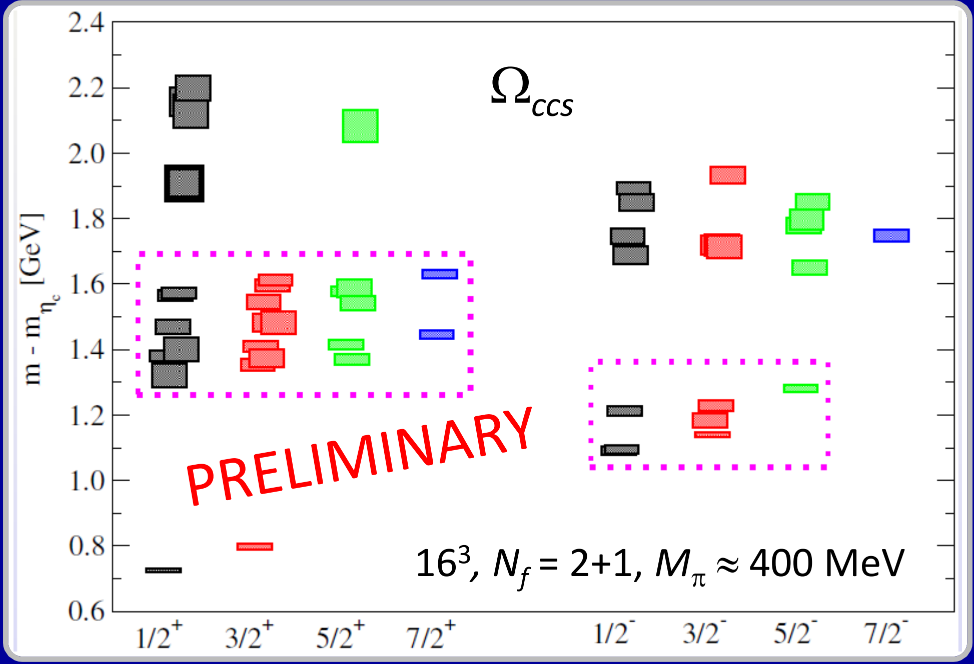


Padmanath et al (Had Spec Collab) PRELIMINARY
Excited cc baryons
(also excited c baryons)



Some other lattice calculations

Excited spectra using single-hadron operators:

- Bali et al (BMW & QCDSF) preliminary [arXiv:1108.6147, 1212.0565] (charmonium, charm mesons and baryons)
- Meinel [PR D85, 114510 (2012)] (bbb baryons)

A first go at including multi-hadron operators in the charm sector:

- X(3872), $Z^+(3900)$ – Prelovsek & Leskovec [arXiv:1307.5172, 1308.2097]
- D_s (0^+) – Mohler et al [arXiv:1308.3175]
- D $(0^+, 1^+)$ – Mohler, Prelovsek, Woloshyn [PR D87, 034501 (2013)]

Summary and outlook

Summary

- Significant progress in studying **excited spectra**
  - **exotics**, **gluonic excitations**, degrees of freedom
- Charmonium, charmed, light (I=0,1) mesons, baryons
- Lots of **experimental interest**

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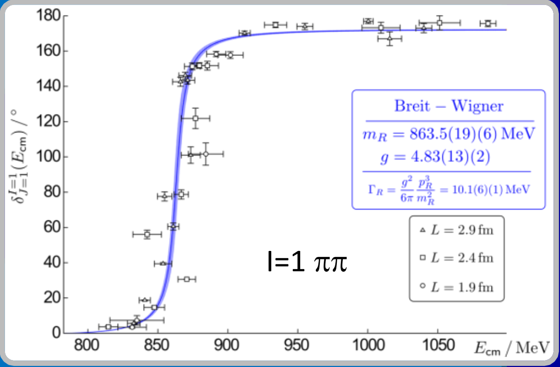


Outlook

- **Scattering** – **resonances**, decays, ...

  [PR D83 071504, D86 034031, D87 034505]
- **Transitions**
- Lighter pion masses, larger volumes, ..., glueballs
- Can **understand puzzles** in the near future?

Extra slides

Resonances on the lattice

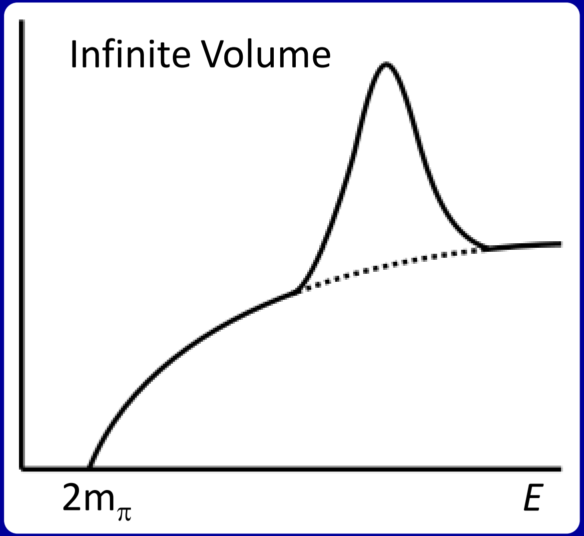
Imaginary time – can't study dynamics (e.g. scattering) directly

Resonances on the lattice

Imaginary time – can't study dynamics (e.g. scattering) directly

Infinite volume

Continuous spectrum



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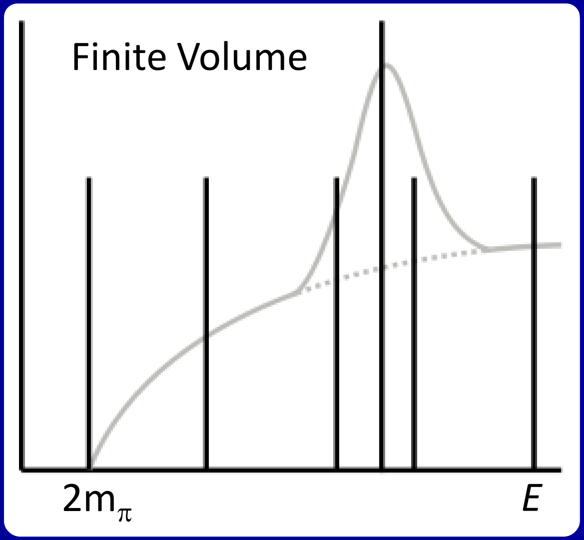
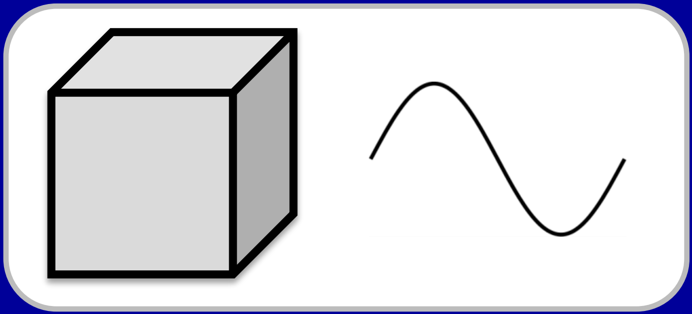


Infinite volume

Continuous spectrum

Finite volume



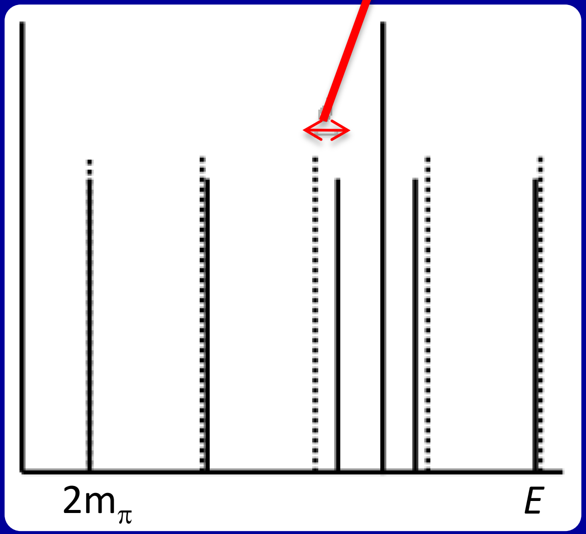
Quantised momenta → discrete spectrum

Resonances on the lattice

Understanding unstable mesons –
need energies of multi-hadron states

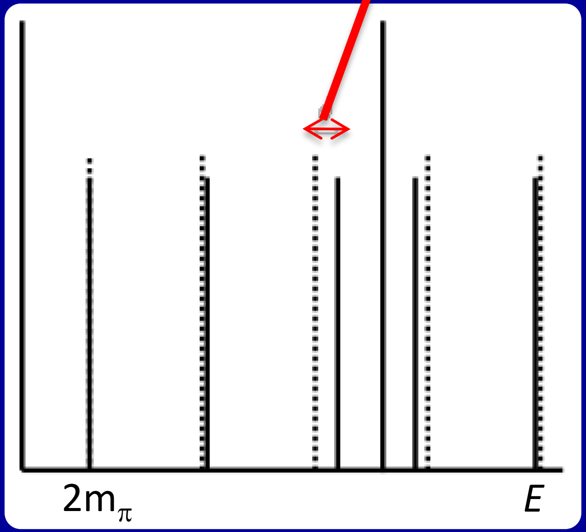
Lüscher: (elastic) energy shifts in **finite vol.**
→ **infinite vol.** scattering phase shift

$\Delta E(L_s) \to \delta(E, L_s)$



Resonances on the lattice

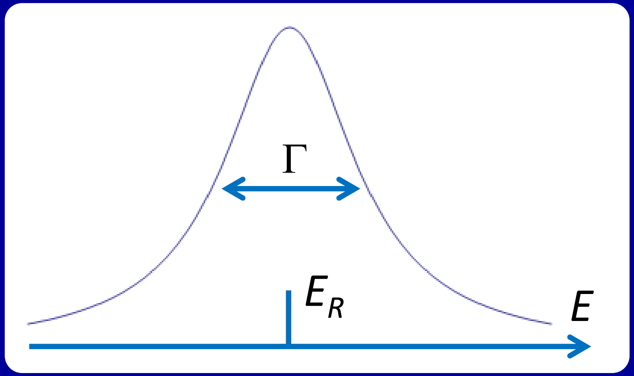
$\Delta E(L_s) \rightarrow \delta(E, L_s)$



Understanding unstable mesons – need energies of multi-hadron states

Lüscher: (elastic) energy shifts in **finite vol.** → **infinite vol.** scattering phase shift

e.g. $\pi\pi \rightarrow \rho \rightarrow \pi\pi$



$$\sigma_l(E) \propto \sin^2 \delta_l(E) = \frac{(\Gamma/2)^2}{(E - E_R)^2 + (\Gamma/2)^2}$$

Extract phase shift at discrete E_{cm}

Map out phase shift → resonance parameters etc

The ρ resonance in ππ scattering

$\pi\pi \rightarrow \rho \rightarrow \pi\pi$

Dudek, Edwards, Thomas, PR D87, 034505 (2013)

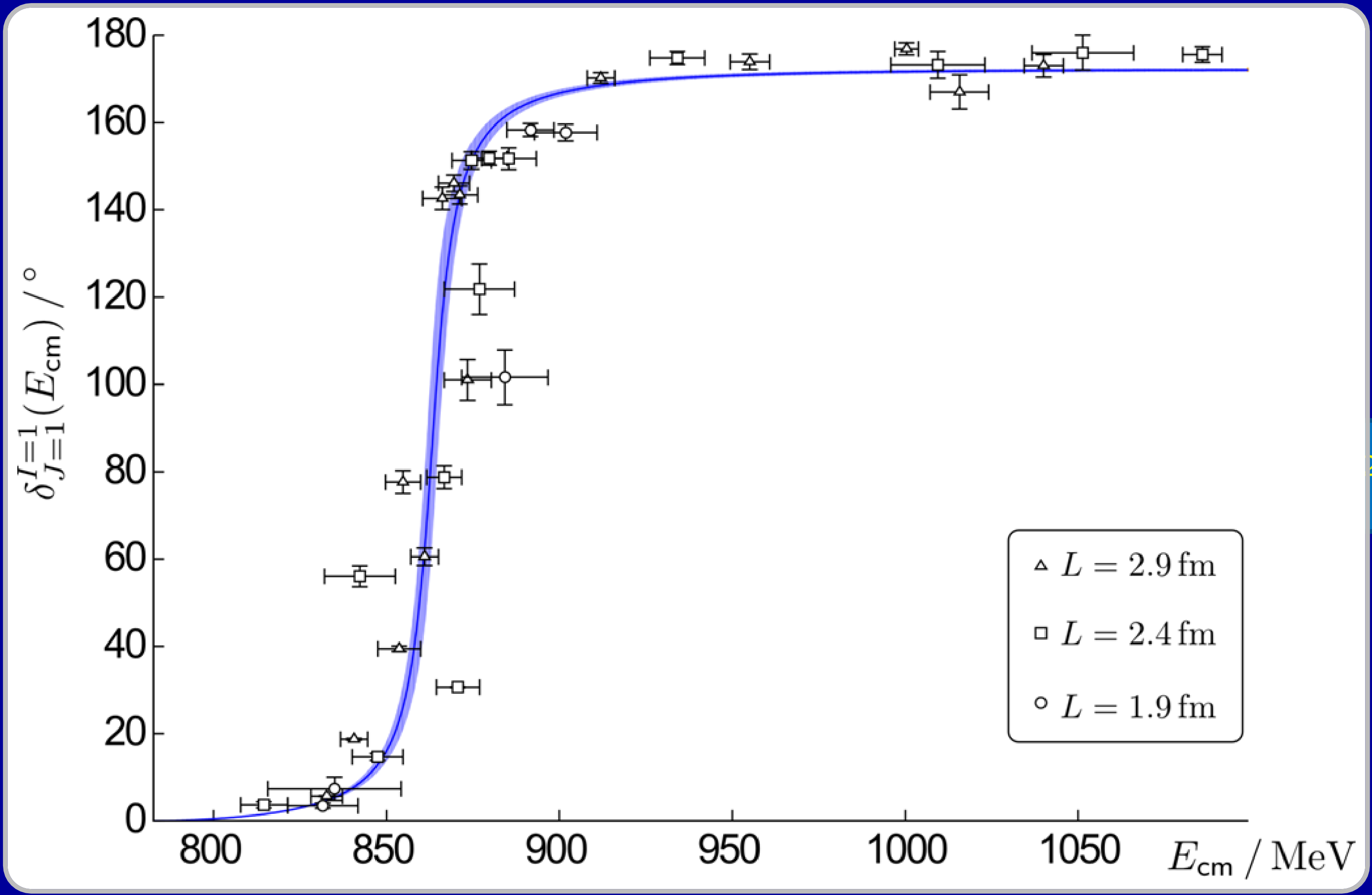
$C_{ij}(t) =< 0|\boxed{\mathcal{O}_i(t)}\boxed{\mathcal{O}_j^\dagger(0)}|0 >$

Operators: single-meson $\sim \bar{\psi}\Gamma D \ldots \psi$

and ππ $\mathcal{O}(\vec{P}) = \sum_{\vec{p}_1,\ \vec{p}_2} \mathcal{C}_\Lambda(\vec{P},\vec{p}_1,\vec{p}_2)\ \mathcal{O}_\pi(\vec{p}_1)\ \mathcal{O}_\pi(\vec{p}_2)$

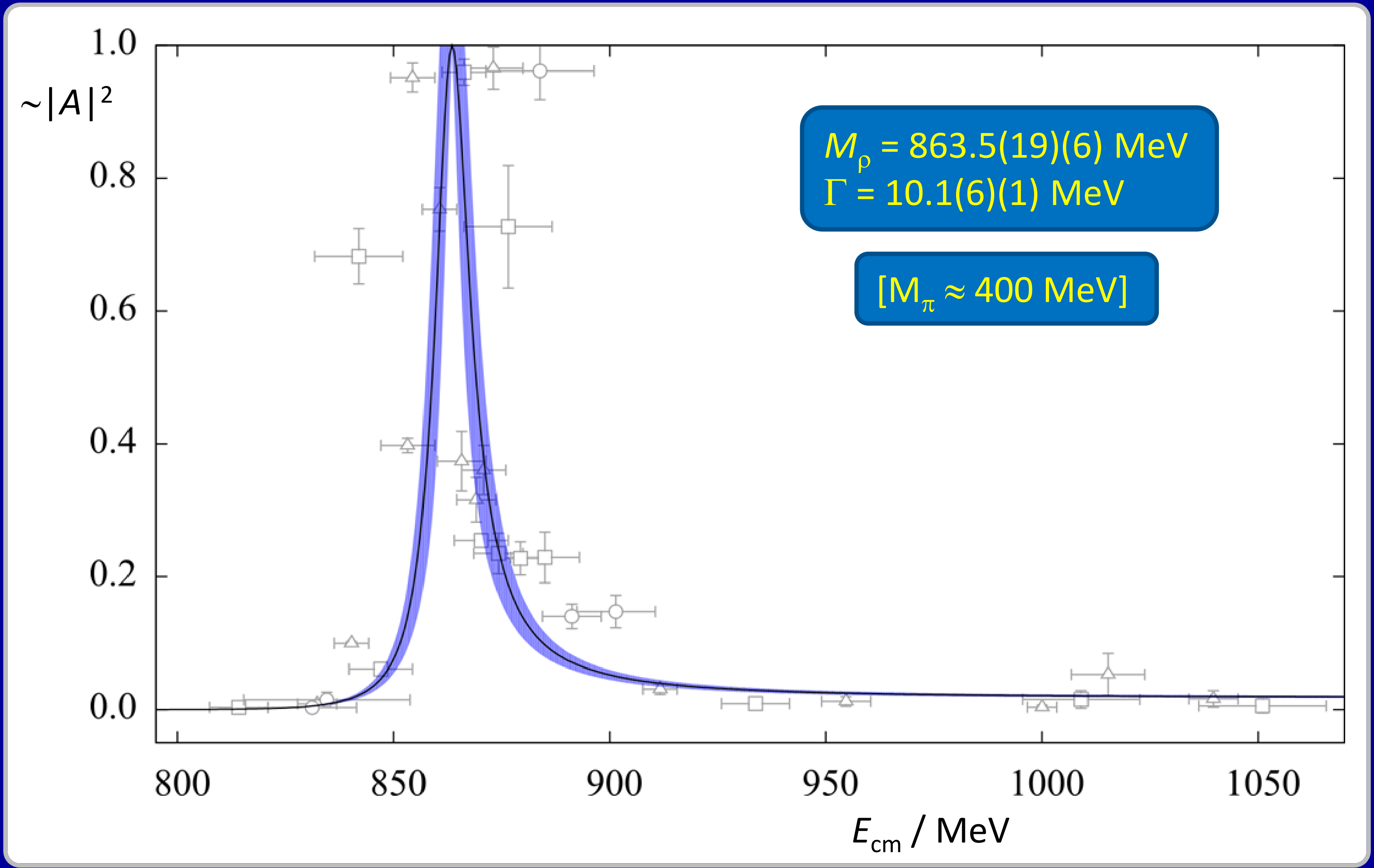
Earlier work on ππ scattering with isospin = 2 (non-resonant):
PR D83, 071504 (2011); PR D86, 034031 (2012)

The ρ resonance in ππ scattering



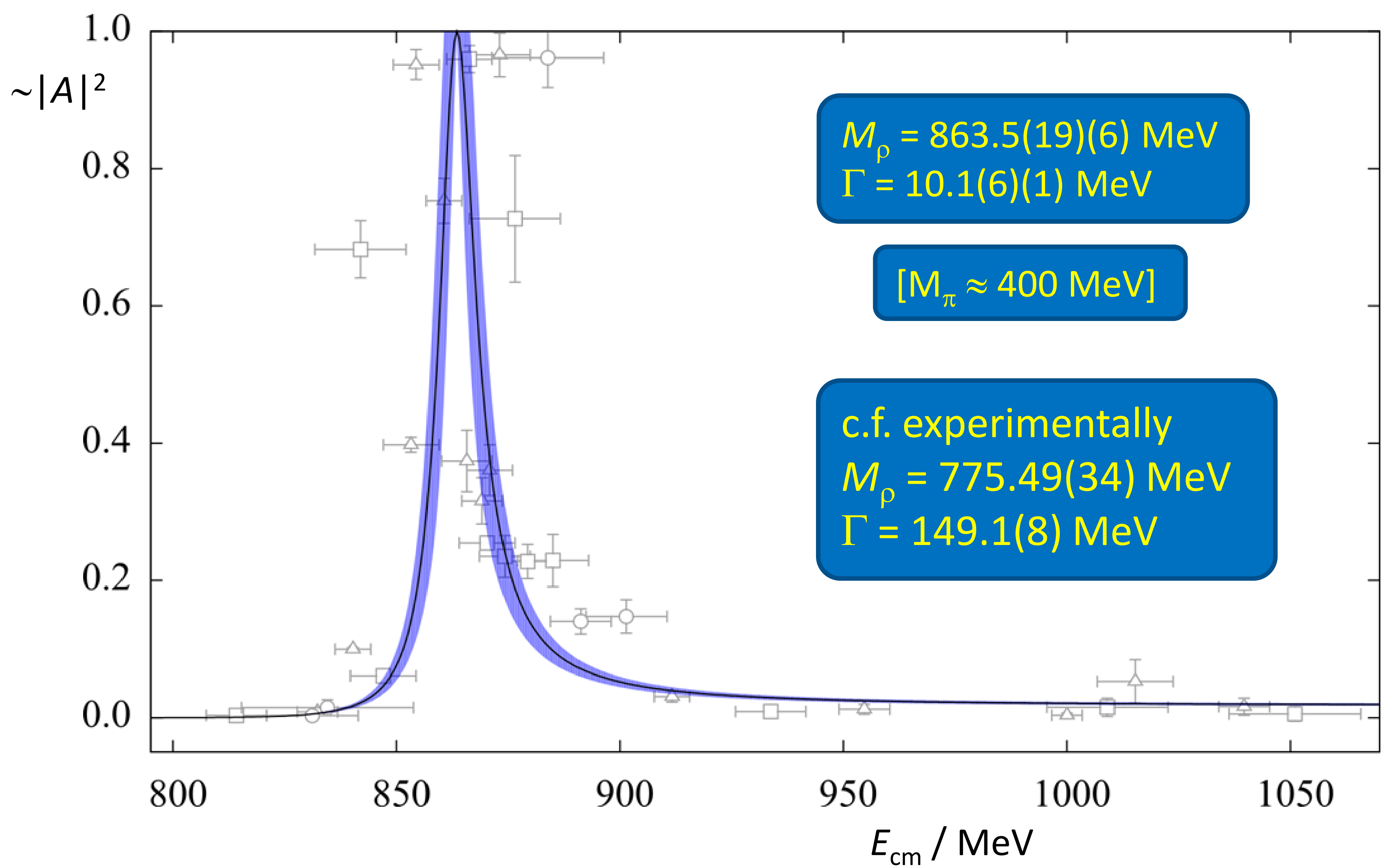
Mapped out in detail

The ρ resonance in ππ scattering



Mapped out in detail

The ρ resonance in ππ scattering



M_ρ = 863.5(19)(6) MeV
Γ = 10.1(6)(1) MeV

[$M_\pi \approx$ 400 MeV]

c.f. experimentally
M_ρ = 775.49(34) MeV
Γ = 149.1(8) MeV

Mapped out in detail