Non-leptonic and rare kaon decays in lattice QCD

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(RBC-UKQCD Collaboration)

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1. Introduction

- In recent years, as the result of improved algorithms and increased computing resources, the precision of lattice QCD calculations of many phenomenologically important quantities has improved beyond all recognition.
- For example, in kaon physics quantities such as V_{us} and B_K are known with impressive precision, e.g.
  - FLAG-2* quote from simulations with $N_f = 2+1$:

    $$\hat{B}_K = 0.766(10) \text{ corresponding to } B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.560(7)\,.$$
  - The FLAG-1 result was $\hat{B}_K = 0.738(20)$ and in EPS-1993 I quoted a summary $\hat{B}_K = 0.8(2)$. M. Lusignoli, L. Maiani, G. Martinelli and L. Reina, Nucl.Phys. B369 (1992) 139
  - The dominant contribution to $\varepsilon_K \propto |V_{cb}|^4$ and PDG(2012) quote $|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$ error on B_K is no longer the dominant one.
- In this talk I will talk about quantities which have not been calculated before, or even yet!

* Flavour Physics Lattice Averaging Group
Review of lattice results concerning low energy particle physics, S.Aoki + 28 authors,
http://itpwiki.unibe.ch/flag/index.php/
updating and extending G.Colangelo et al., arXiv:1011.4408

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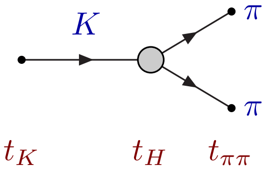
- As the precision of lattice calculations continues to improve, it becomes both possible and necessary to extend the range of physical quantities being studied.
- Outline of Talk:

1. Introduction
2. $K \to (\pi\pi)_{I=2}$ decay amplitudes (benchmark calculation completed)
   T. Blum, P.A. Boyle, N.H. Christ, N. Garron, E. Goode, T. Izubuchi, C. Jung, C. Kelly, C. Lehner, M. Lightman, Q. Liu, A.T. Lytle, R.D. Mawhinney, C.T. Sachrajda, A. Soni, C. Sturm,
   arXiv:1111.1699, arXiv1206.5142.
3. $K \to (\pi\pi)_{I=0}$ decay amplitudes (advanced exploratory work done)
   T. Blum, P.A. Boyle, N.H. Christ, N. Garron, E. Goode, T. Izubuchi, C. Lehner, Q. Liu, R.D. Mawhinney, C.T. Sachrajda, A. Soni, C. Sturm, H. Yin, R. Zhou arXiv:1106.2714.
   P.A. Boyle, N.H. Christ, N. Garron, E. Goode, T. Janowski, C. Lehner, Q. Liu, A.T. Lytle, C.T. Sachrajda, A. Soni, D. Zhang arXiv:1212.1474.
4. (Long-distance contributions to) $\Delta m_K \equiv m_{K_L} - m_{K_S}$
   (significant exploratory work done)
   N.H.Christ, T.Izubuchi, C.T.Sachrajda, A.Soni, J.Yu arXiv:1212.5931.
5. Rare kaon decays (exploratory work being done)

2. $K \to (\pi\pi)_{I=2}$ Decays



- In lattice simulations, where time separations are used to filter away excited states, energy is not automatically conserved.
- The time behaviour of the above correlation function is proportional to

$$e^{-m_K(t_H-t_K)}\,e^{-E_{\pi\pi}(t_{\pi\pi}-t_H)}$$

- To evaluate a physical $K \to \pi\pi$ amplitude, we need to have a volume such that there is a two-pion state with $E_{\pi\pi} = m_K$ and, in practice, we would like this to be the ground state.
- This cannot be achieved with periodic boundary conditions, for which the ground-state corresponds to each pion at rest (up to finite-volume effects).

- The operators whose matrix elements have to be calculated are:

$$\begin{aligned}
O^{3/2}_{(27,1)} &= (\bar{s}^i d^i)_L \left\{ (\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L \right\} + (\bar{s}^i u^i)_L\,(\bar{u}^j d^j)_L \\
O^{3/2}_7 &= (\bar{s}^i d^i)_L \left\{ (\bar{u}^j u^j)_R - (\bar{d}^j d^j)_R \right\} + (\bar{s}^i u^i)_L\,(\bar{u}^j d^j)_R \\
O^{3/2}_8 &= (\bar{s}^i d^j)_L \left\{ (\bar{u}^j u^i)_R - (\bar{d}^j d^i)_R \right\} + (\bar{s}^i u^j)_L\,(\bar{u}^j d^i)_R
\end{aligned}$$

- It is convenient to use the Wigner-Eckart Theorem: (Notation - $O^{\Delta I}_{\Delta I_z}$)

$$_{I=2}\langle \pi^+(p_1)\pi^0(p_2)\,|O^{3/2}_{1/2}|K^+\rangle = \frac{\sqrt{3}}{2}\langle \pi^+(p_1)\pi^+(p_2)\,|O^{3/2}_{3/2}|K^+\rangle\,,$$

  where
  - $O^{3/2}_{3/2}$ has the flavour structure $(\bar{s}d)\,(\bar{u}d)$.
  - $O^{3/2}_{1/2}$ has the flavour structure $(\bar{s}d)\,(\,(\bar{u}u)-(\bar{d}d)\,)+(\bar{s}u)\,(\bar{u}d)$.

- We can then use antiperiodic boundary conditions for the u-quark say, so that the $\pi\pi$ ground-state is $\langle\pi^+(\pi/L)\,\pi^+(-\pi/L)\,|$. C-h Kim, Ph.D. Thesis
  - • Do not have to isolate an excited state. •

For $K \to \pi\pi$ decays we require pions with a physical mass and hence a large volume $\Rightarrow$ coarse lattice. Our published results are obtained on the following lattice:

arXiv:1208.4412

- $a \simeq 0.14\,\mathrm{fm}$, (DWF+IDSDR) $\left\{32^3 \times 64 \times 32\ (L \simeq 4.58\,\mathrm{fm})\right\}$
  - Two light-quark masses corresponding to pions with $m_\pi \simeq 170$ and 250 MeV.
  - The lightest partially quenched pion has a mass of about 142 MeV.
  - The goal was to have a physical $K \to \pi\pi$ decay, with $|p_\pi| = \sqrt{2}\pi/L$.
  - With this coarse lattice, it is not surprising that lattice artefacts are the largest source of systematic error. We mitigate against this in a number of ways and now have preliminary results at two finer lattices.

units	m_π	m_K	$E_{\pi,2}$	$E_{\pi\pi,0}$	$E_{\pi\pi,2}$	$m_K - E_{\pi\pi,2}$
lattice	0.1042(2)	0.3707(7)	0.1739(9)	0.2100(4)	0.356(2)	0.015(2)
MeV	142.1(9)	505.5(3.4)	237(2)	286(2)	486(4)	20.0(3.1)

- The subscripts 0 and 2 refer to $|p_\pi| = 0$ and $\sqrt{2}\pi/L$ respectively.

- The RBC-UKQCD collaboration has performed the first calculation of the $K \to (\pi\pi)_{I=2}$ amplitude A_2: arXiv:1111.1699, arXiv:1206.5142

$$\begin{aligned}\mathrm{Re}A_2 &= (1.381 \pm 0.046_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}})\,10^{-8}\,\mathrm{GeV}\\ \mathrm{Im}A_2 &= -(6.54 \pm 0.46_{\mathrm{stat}} \pm 1.20_{\mathrm{syst}})\,10^{-13}\,\mathrm{GeV.}\end{aligned}$$

- The result for Re A_2 agrees well with the experimental value of $1.479(4)\times 10^{-8}$ GeV obtained from K^+ decays .
- Im A_2 is unknown so that our result provides its first direct determination.
- Combining our result for Im A_2 with the experimental results for Re A_2, Re $A_0 = 3.3201(18)\cdot 10^{-7}$ GeV and ε'/ε we obtain:

$$\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = -1.61(19)_{\mathrm{stat}}(20)_{\mathrm{syst}}\times 10^{-4}\,.$$

(Of course, we wish to confirm this directly.)

$$\begin{aligned}\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} &= \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\sqrt{2}\,|\varepsilon|}{\omega}\frac{\varepsilon'}{\varepsilon}\\ -1.61(19)_{\mathrm{stat}}(20)_{\mathrm{syst}}\times 10^{-4} &= -4.42(31)_{\mathrm{stat}}(89)_{\mathrm{syst}}\times 10^{-5} - 1.16(18)\times 10^{-4}\,.\end{aligned}$$

$$
\begin{aligned}
\mathrm{Re}A_2 &= (1.381 \pm 0.046_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}})\, 10^{-8}\,\mathrm{GeV} \\
\mathrm{Im}A_2 &= -(6.54 \pm 0.46_{\mathrm{stat}} \pm 1.20_{\mathrm{syst}})\, 10^{-13}\,\mathrm{GeV}.
\end{aligned}
$$

arXiv:1111.1699, arXiv:1206.5142

- Currently the error is dominated by *lattice artefacts*, since the calculation was performed at a single, rather coarse, lattice spacing.
  - Preliminary results at two finer lattice spacings were presented at Lattice 2013 on August 1st,

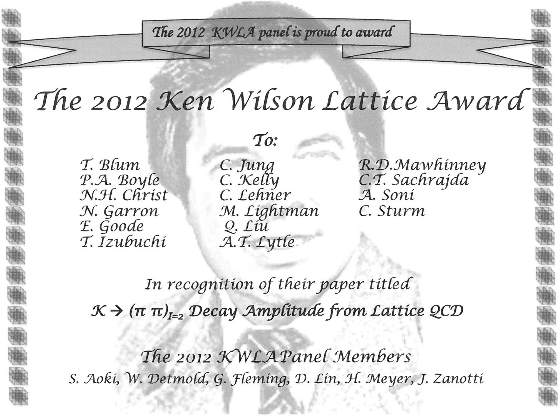
    http://www.lattice2013.uni-mainz.de/presentations/8C/Janowski.pdf,

    $\Rightarrow$ this uncertainty is being reduced very significantly.
- For two-pion states, control of finite-volume and rescattering effects becomes particularly important. M.Lüscher, $\cdots$
  - For hadronic B-decays, for which inelastic intermediate states are important, we do not even know how to formulate a possible computation.

	48^3 $a \simeq 0.114$ fm	64^3 $a \simeq 0.086$ fm
Dispersion relation (c^2)	0.999(9)	1.008(10)
Pion mass [MeV]	139.4(3)	136.0(3)
Kaon mass [MeV]	498.9(4)	495.6(5)
$(\pi\pi)_{I2}$ energy [MeV]	497.8(43)	503.7(38)
Re(A_2)[GeV]	$1.368(41) \times 10^{-8}$	$1.358(28) \times 10^{-8}$
Im(A_2)[GeV]	$-6.30(12) \times 10^{-13}$	$-6.31(10) \times 10^{-13}$

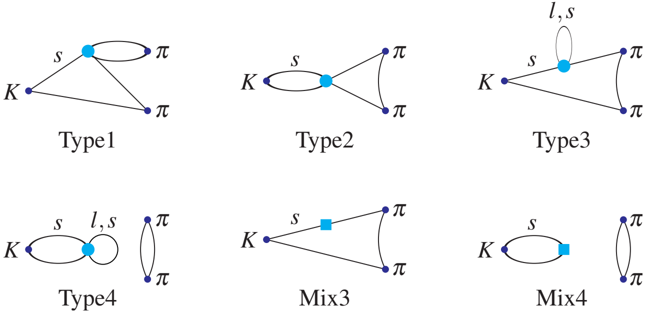
c.f. 32^3 ($a \simeq 0.14$ fm) result:
Re(A_2) = $1.381(41) \times 10^{-8}$ GeV
Im(A_2) = $-6.54(46) \times 10^{-13}$ GeV

Statistical errors only.



- For this work we received the 2012 Ken Wilson Lattice award at Lattice 2012.
- Criteria: The paper must be important research beyond the existing state of the art. $\cdots$

- The calculation is much more difficult for the $K \to (\pi\pi)_{I=0}$ amplitude A_0:
  - The presence of disconnected diagrams:



  - The efficient evaluation of disconnected diagrams is a major area of research in the lattice community.
  - Breaking I-invariance by different boundary conditions for u and d quarks fatal. Even without interactions, $|\pi^+\pi^-\rangle$ and $|\pi^0\pi^0\rangle$ have different energies.

- RBC-UKQCD have computed A_0 with the two pions at rest and with unphysical masses, finding e.g. arXiv:1106.2714, Qi Liu Columbia Un.Thesis

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 9.1 \pm 2.1 \qquad 877\,\mathrm{MeV}\text{ kaon decaying into two }422\,\mathrm{MeV}\text{ pions}$$
$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 12.0 \pm 1.7 \qquad 662\,\mathrm{MeV}\text{ kaon decaying into two }329\,\mathrm{MeV}\text{ pions}$$

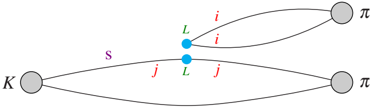
- Whilst both these results are obtained at unphysical kinematics and are different from the physical value of 22.5, it is nevertheless interesting to understand the origin of these enhancements.
- 99% of the contribution to the real part of A_0 and A_2 come from the matrix elements of the current-current operators.
- For a calculation of ε'/ε at physical kinematics, RBC-UKQCD are developing G-parity boundary conditions (estimate timescale $\sim$ 2 years) to avoid having to consider excited states.

RBC-UKQCD Collaboration, arXiv:1212.1474

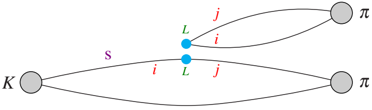
- Re A_2 is dominated by a simple operator:

$$O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \left\{ (\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L \right\} + (\bar{s}^i u^i)_L \, (\bar{u}^j d^j)_L$$

and two diagrams:



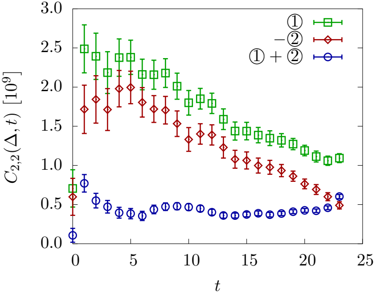
C_1



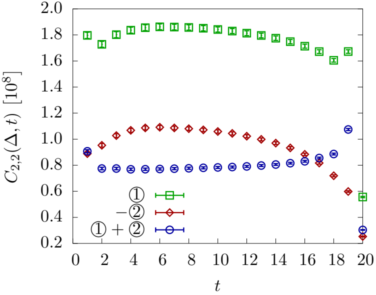
C_2

- Re A_2 is proportional to $C_1 + C_2$.
- The contribution to Re A_0 from Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same overall sign.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3} C_1$.
- We find instead that $C_2 \approx -C_1$ so that A_2 is significantly suppressed!
- **We believe that the strong suppression of Re A_2 and the (less-strong) enhancement of Re A_0 is a major factor in the $\Delta I = 1/2$ rule.**

Evidence for the Suppression of Re A_2



Physical Kinematics



$m_\pi \simeq 330$ MeV at threshold.

- Notation ⓘ $\equiv C_i,\ i = 1, 2$.
- Of course before claiming a quantitative understanding of the $\Delta I = 1/2$ rule we need to compute Re A_0 at physical kinematics and reproduce the experimental value of 22.5.
- Much early phenomenology was based on the vacuum insertion approach; although the qualitative picture we find had been suggested by Bardeen, Buras and Gerard in 1987.

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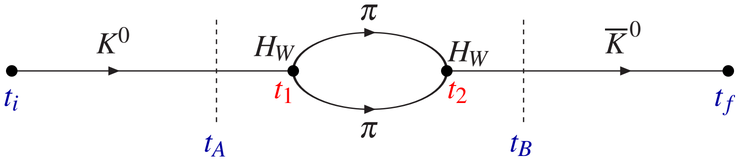
Long-distance contributions to the K_L - K_S mass difference

N.H.Christ, T.Izubuchi, C.T.Sachrajda, A.Soni, J.Yu arXiv:1212.5931.



- We wish to compute the amplitude

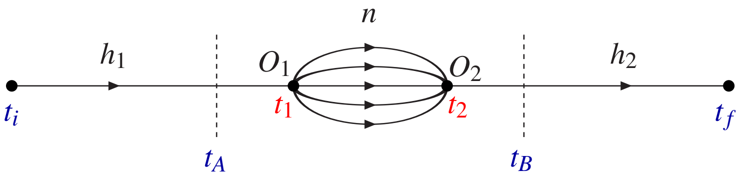
$$\mathscr{A} = \frac{1}{2}\int_{-\infty}^{\infty} dt_1\, dt_2\; T\,\langle \bar{K}^0\,|\,H_W(t_2)\,H_W(t_1)\,|\,K^0\rangle$$

  and to determine the K_L-K_S mass difference:

$$\Delta M_K = 2\mathscr{P}\sum_{\alpha}\frac{\langle \bar{K}^0\,|\,H_W\,|\alpha\rangle\,\langle\alpha\,|H_W\,|\,K^0\rangle}{m_K - E_\alpha}$$

  where the sum over $|\alpha\rangle$ includes an energy-momentum integral.
- This is a significant extension of the standard calculations, where the matrix elements are of local operators.

The fiducial volume

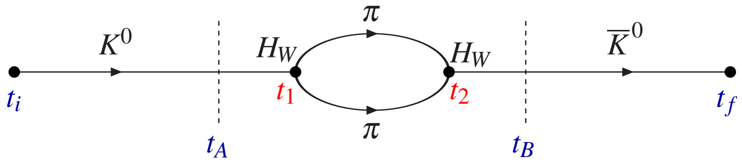


- How do you prepare the states $h_{1,2}$ in

$$\int d^4x \int d^4y \, \langle h_2 \, | \, T\{O_1(x) \, O_2(y)\} \, | \, h_1 \rangle \, ,$$

  when the time of the operators is integrated?
- The practical solution is to integrate over a large subinterval in time $t_A \le t_{x,y} \le t_B$, but to create h_1 and to annihilate h_2 well outside of this region:
- This is the natural modification of standard field theory for which the asymptotic states are prepared at $t \to \pm\infty$ and then the operators are integrated over all time.
- This approach has been successfully implemented in the ΔM_K project.

arXiv:1212.5931



- Δm_K is given by

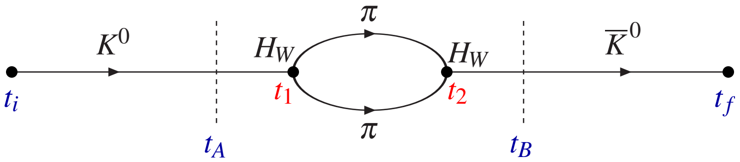
$$\Delta m_K \equiv m_{K_L} - m_{K_S} = \frac{1}{2m_K} 2\mathscr{P} \sum_\alpha \frac{\langle \bar{K}^0 \,|\, \mathscr{H}_W \,|\, \alpha \rangle \, \langle \alpha \,|\, \mathscr{H}_W \,|\, K^0 \rangle}{m_K - E_\alpha} = 3.483(6) \times 10^{-12}\,\mathrm{MeV}.$$

- The above correlation function gives $(T = t_B - t_A + 1)$

$$C_4(t_A, t_B; t_i, t_f) = |Z_K|^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 \,|\, \mathscr{H}_W \,|\, n \rangle \, \langle n \,|\, \mathscr{H}_W \,|\, K^0 \rangle}{(m_K - E_n)^2} \times \left\{ e^{(M_K - E_n)T} - (m_K - E_n)T - 1 \right\}.$$

- From the coefficient of T we can therefore obtain

$$\Delta m_K^{\mathrm{FV}} \equiv 2 \sum_n \frac{\langle \bar{K}^0 \,|\, \mathscr{H}_W \,|\, n \rangle \, \langle n \,|\, \mathscr{H}_W \,|\, K^0 \rangle}{(m_K - E_n)} .$$



In order to evaluate Δm_K we need to be able to:

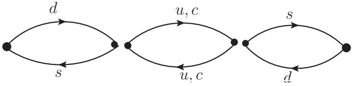
- Relate Δm_K and $\Delta m_K^{\rm FV}$. ✓ RBC-UKQCD; N.H.Christ, G.Martinelli, CTS (in preparation)
  This is a significant extension of the theory of finite-volume effects for two-pion states: the Lüscher quantization condition, Lellouch-Lüscher factor, $\cdots$.
- Control the additional ultraviolet divergences when the weak Hamiltonians are close together. ✓ arXiv:1212.5931
  This is facilitated by the GIM mechanism which requires the presence of charm quarks.
- $\Delta S = 1$ effective weak Hamiltonian including four flavours:

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'})$$

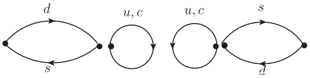
  where

$$Q_1^{qq'} = (\bar{s}_i d_i)_{V-A} (\bar{q}_j q'_j)_{V-A} \quad \text{and} \quad Q_2^{qq'} = (\bar{s}_i d_j)_{V-A} (\bar{q}_j q'_i)_{V-A} \,.$$

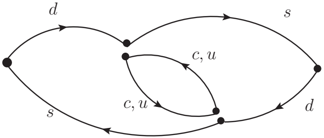
Evaluating Δm_K (cont.)



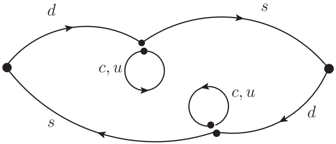
Type 1



Type 4



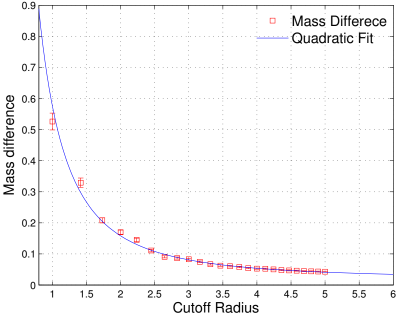
Type 2



Type 3

- In our exploratory study on 16^3 ensembles with $m_\pi = 420$ MeV, we only evaluate Type 1 and Type 2 graphs. arXiv:1212.5931
- We obtain Δm_K in the range {5.81(28) – 10.58(75)}$\times 10^{-12}$ MeV as m_K is varied from 563 to 839 MeV. (The physical value is $3.483(6) \times 10^{-12}$ MeV.)
- Preliminary results from a full study (all diagrams) on a 24^3 lattice with $m_\pi \simeq 330$ MeV were presented by Jianglei Yu at Lattice 2013, http://www.lattice2013.uni-mainz.de/presentations/7C/Yu.pdf

- As an example of our investigations consider the behaviour of the integrated $Q_1 - Q_1$ correlation function without GIM subtraction but with an artificial cut-off, $R = \sqrt{\{(t_2-t_1)^2 + (\vec{x}_2-\vec{x}_1)^2\}}$ on the coordinates of the two Q_1 insertions.



- The plot exhibits the quadratic divergence as the two operators come together.
- The quadratic divergence is cancelled by the GIM mechanism.

F.Mescia, C.Smith, S.Trine hep-ph/0606081

- Rare kaon decays which are dominated by short-distance FCNC processes, $K \to \pi \nu \bar{\nu}$ in particular, provide a potentially valuable window on new physics at high-energy scales.
- The decays $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ are also considered promising because the long-distance effects are reasonably under control using ChPT.
  - They are sensitive to different combinations of short-distance FCNC effects and hence in principle provide additional discrimination to the neutrino modes.
  - A challenge for the lattice community is therefore either to calculate the long-distance effects reliably or at least to determine the Low Energy Constants of ChPT.
- We are now in a position to attempt to meet this challenge.
  - We need help from the experimental and non-lattice theory communities to focus on the key issues.

There are three main contributions to the amplitude:

1. Short distance contributions: F.Mescia, C,Smith, S.Trine hep-ph/0606081

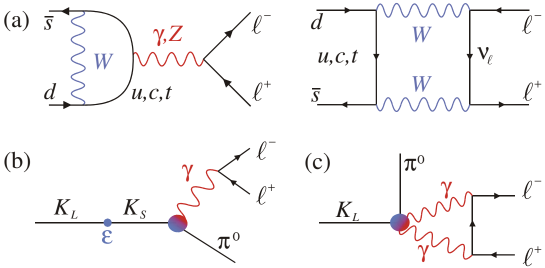
$$H_{\rm eff} = -\frac{G_F\alpha}{\sqrt{2}}\, V_{ts}^* V_{td} \{ y_{7V} (\bar{s}\gamma_\mu d)\, (\bar{\ell}\gamma^\mu \ell) + y_{7A} (\bar{s}\gamma_\mu d)\, (\bar{\ell}\gamma^\mu \gamma_5 \ell) \} + \text{h.c.}$$

   - Direct CP-violating contribution.
   - In BSM theories other effective interactions are possible.

2. Long-distance indirect CP-violating contribution

$$A_{ICPV}(K_L \to \pi^0 \ell^+ \ell^-) = \varepsilon A(K_1 \to \pi^0 \ell^+ \ell^-)\,.$$

3. The two-photon CP-conserving contribution $K_L \to \pi^0(\gamma^*\gamma^* \to \ell^+\ell^-)$.



- The current phenomenological status for the SM predictions is nicely summarised by: V.Cirigliano et al., arXiv1107.6001

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} = 10^{-12} \times \left\{ 15.7|a_S|^2 \pm 6.2|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 2.4 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 \right\}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPV}} = 10^{-12} \times \left\{ 3.7|a_S|^2 \pm 1.6|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 1.0 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 \right\}$$

  - $\lambda_t = V_{td} V_{ts}^*$ and $\mathrm{Im}\,\lambda_t \simeq 1.35 \times 10^{-4}$.
  - $|a_S|$, the amplitude for $K_S \to \pi^0 \ell^+ \ell^-$ at $q^2 = 0$ as defined below, is expected to be $O(1)$ but the sign of a_S is unknown. $|a_S| = 1.06^{+0.26}_{-0.21}$.
  - For $\ell = e$ the two-photon contribution is negligible.
  - Taking the positive sign (?) the prediction is

$$\begin{aligned} \mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} &= (3.1 \pm 0.9) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPV}} &= (1.4 \pm 0.5) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPC}} &= (5.2 \pm 1.6) \times 10^{-12}\,. \end{aligned}$$

  - The current experimental limits (KTeV) are:

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10} \quad \text{and} \quad \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}\,.$$

G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026

- We now turn to the CPC decays $K_S \to \pi^0 \ell^+ \ell^-$ and $K^+ \to \pi^+ \ell^+ \ell^-$ and consider

$$T_i^\mu = \int d^4x\, e^{-iq\cdot x} \langle \pi(p) \,|\, \mathrm{T}\{ J^\mu_{\mathrm{em}}(x)\, Q_i(0)\,\} \,|\, K(k) \rangle\,,$$

  where Q_i is an operator from the effective Hamiltonian.

- Gauge invariance implies that

$$T_i^\mu = \frac{\omega_i(q^2)}{(4\pi)^2} \left\{ q^2 (p+k)^\mu - (m_K^2 - m_\pi^2)\, q^\mu \right\}.$$

- Within ChPT the Low energy constants a_+ and a_S are defined by

$$a = \frac{1}{\sqrt{2}} V_{us}^* V_{ud} \left\{ C_1 \omega_1(0) + C_2 \omega_2(0) + \frac{2N}{\sin^2\theta_W} f_+(0) C_{7V} \right\}$$

  where $Q_{1,2}$ are the two current-current GIM subtracted operators and the C_i are the Wilson coefficients. (C_{7V} is proportional to y_{7V} above).

G.D'Ambrosio, G.Ecker, G.Isidori and J.Portoles, hep-ph/9808289

- Phenomenological values: $a_+ = -0.578 \pm 0.016$ and $|a_S| = 1.06^{+0.26}_{-0.21}$.

Can we do better in lattice simulations?

- The generic non-local matrix elements which we wish to evaluate is

$$
\begin{aligned}
X &\equiv \int_{-\infty}^{\infty} dt_x\, d^3x\, \langle \pi(p)\,|\,\mathrm{T}\,[J(0)\,H(x)]\,|K\rangle \\
&= i\sum_n \frac{\langle \pi(p)\,|\,J(0)\,|n\rangle\,\langle n\,|H(0)\,|\,K\rangle}{m_K - E_n + i\varepsilon} - i\sum_{n_s} \frac{\langle \pi(p)\,|\,H(0)\,|n_s\rangle\,\langle n_s\,|J(0)\,|\,K\rangle}{E_{n_s} - E_\pi + i\varepsilon}\,,
\end{aligned}
$$

  - $\{|n\rangle\}$ and $\{|n_s\rangle\}$ represent complete sets of non-strange and strange sets.
- In Euclidean space we envisage calculating correlation functions of the form

$$
C \equiv \int_{-T_a}^{T_b} dt_x\, \langle \phi_\pi(\vec{p},t_\pi)\,\mathrm{T}\,[J(0)\,H(t_x)]\,\phi_K^\dagger(t_K)\,\rangle \equiv \sqrt{Z_K}\,\frac{e^{-m_K|t_K|}}{2m_K}\,X_E\,\sqrt{Z_\pi}\,\frac{e^{-E_\pi t_\pi}}{2E_\pi}\,,
$$

where

$$
\begin{aligned}
X_{E_-} &= -\sum_n \frac{\langle \pi(p)\,|\,J(0)\,|n\rangle\,\langle n\,|H(0)\,|\,K\rangle}{m_K - E_n}\left(1 - e^{(m_K - E_n)T_a}\right) \quad \text{and} \\
X_{E_+} &= \sum_{n_s} \frac{\langle \pi(p)\,|\,H(0)\,|n_s\rangle\,\langle n_s\,|J(0)\,|\,K\rangle}{E_{n_s} - E_\pi}\left(1 - e^{-(E_{n_s} - E_\pi)T_b}\right).
\end{aligned}
$$

- We can remove the single pion intermediate state.
- Which intermediate states contribute?
    - Are there any states below M_K?
    - We can control q^2 and stay below the two-pion threshold.

    - Do the symmetries protect us completely from two-pion intermediate states at low q^2?
    - Are the contributions from three-pion intermediate states negligible?
    - Answers to the above questions will affect what the finite-volume corrections are?
    - The ChPT-based phemomenology community neglect such possibilities as they are higher order in the chiral expansion.

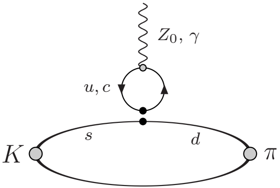
All to be investigated further!

- It looks as though the FV corrections are much simpler than for ΔM_K and may be exponentially small?

Short Distance Effects

$$T_i^\mu = \int d^4x\, e^{-iq\cdot x}\, \langle \pi(p)\,|\,\mathrm{T}\{J^\mu(x)\,Q_i(0)\,\}\,|\,K(k)\rangle\,,$$

- Each of the two local Q_i operators can be normalized in the standard way and for J we imagine taking the conserved vector current.
- We must treat additional divergences as $x \to 0$.

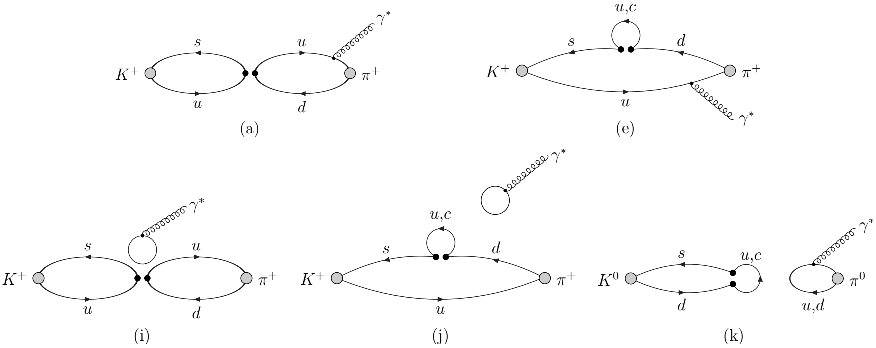


- Quadratic divergence is absent by gauge invariance $\Rightarrow$ Logarithmic divergence.
  - Checked explicitly for Wilson and Clover at one-loop order.

    G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026
  - Absence of power divergences does not require GIM.
  - Logarithmic divergence cancelled by GIM.
  - For DWF the same applies for the axial current.
    - To be investigated further!

Many diagrams to evaluate!

- Sample diagrams:



- The top two diagrams are each representatives of four diagrams, corresponding to the four positions at which the photon can be inserted.
- The last diagram is only present for K^0 decays. The remaining diagrams are present for both K^+ and K^0 decays.
- The ChPT LECs are obtained at $q^2 = 0$, but we are not limited to this choice.

Goal is wide-ranging precision flavour physics

- Our *ab initio* calculation of A_2 builds upon substantial theoretical advances, achieved over many years.
  - The agreement we find for $\mathrm{Re}\,A_2$ with the experimental result is very satisfying and we are also able to determine $\mathrm{Im}\,A_2$ for the first time.
  - The cancellation we find between the two contributions to $\mathrm{Re}\,A_2$ is an important ingredient in the $\Delta I = 1/2$ rule.
  - We are repeating this calculation using two finer lattice spacings so that a continuum extrapolation can be performed thus eliminating the dominant contribution to the error, reducing the total uncertainty to about 5%.
  - We expect that the dominant remaining errors in A_2 will then come from the omission of electromagnetic and other isospin breaking mixing between the large amplitude A_0 and A_2.
- Although significant technical problems remain, we are progressing towards calculating A_0.
- We are now in a position to begin computing long-distance effects in weak processes, including ΔM_K and rare kaon decay amplitudes.
  - If we succeed, then perhaps we will also help to influence the setting of priorities for experimental particle physics in our joint exploration of the limits of the standard model and in searches for new physics.