B hadron lifetimes and mixing

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Outline

- Tests of Standard Model Predictions for Mixing

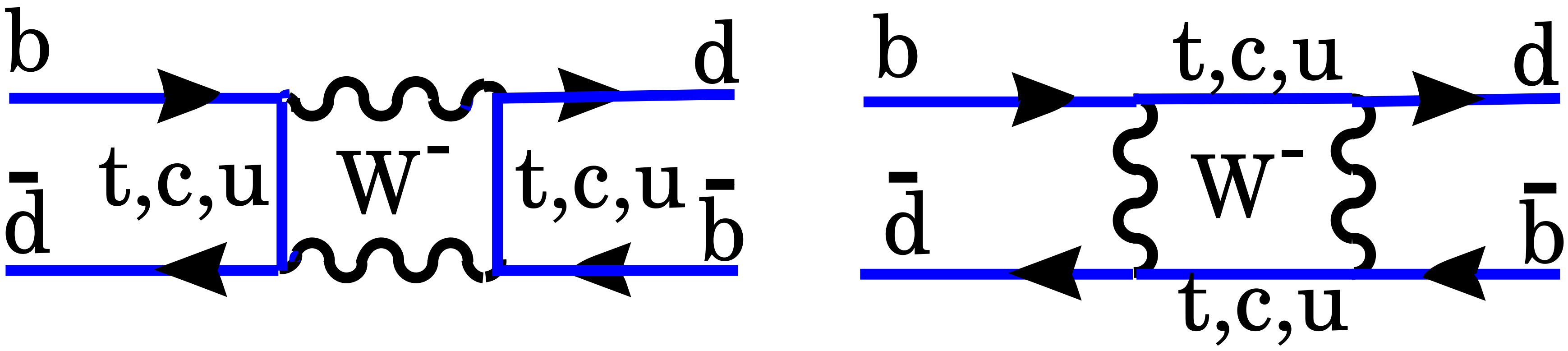
- Search for NP in B-mixing

- Possible roads to follow: More precise tests of the HQE
    - $\tau(B^+)/\tau(B_d)$
    - $\tau(\Lambda_b)/\tau(B_d)$
    - $\tau(B_s)/\tau(B_d)$
    - $\tau(\Xi_b^0)/\tau(\Xi_b^+)$
    - $\tau(B_c^+)$
    - D meson lifetimes
    - $\Delta\Gamma_s$

- Conclusion

see also R. Jones, G. Borissov, G. Cowan, R. Dowdall

Test of our theoretical Understanding



$|M_{12}|$, $|\Gamma_{12}|$ and $\phi = \text{arg}(-M_{12}/\Gamma_{12})$ can be related to three observables:

- **Mass difference:** $\Delta M := M_H - M_L \approx 2|M_{12}|$ **(off-shell)**
  $|M_{12}|$: heavy internal particles: t, SUSY, ...

- **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos\phi$ **(on-shell)**
  $|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!

- **Flavor specific/semi-leptonic CP asymmetries:** e.g. $B_q \to Xl\nu$ (*semi-leptonic*)

$$a_{sl} \quad \equiv \quad a_{fs} = \frac{\Gamma(\overline{B}_q(t) \to f) - \Gamma(B_q(t) \to \overline{f})}{\Gamma(\overline{B}_q(t) \to f) + \Gamma(B_q(t) \to \overline{f})} = \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin\phi$$

Test of our theoretical Understanding

- **Mass difference:** One Operator Product Expansion (OPE)

  Theory **A.L., Nierste 1102.4274** vs. Experiment : **HFAG 13**

  $$\Delta M_d = 0.543 \pm 0.091 \text{ ps}^{-1} \qquad \Delta M_d = 0.510 \pm 0.004 \text{ ps}^{-1}$$
  $$\Delta M_s = 17.30 \pm 2.6 \text{ ps}^{-1} \qquad \Delta M_s = 17.69 \pm 0.08 \text{ ps}^{-1}$$

  - Perfect agreement, still room for NP
  - Important bounds on the unitarity triangle and NP
  - **Dominant uncertainty = Lattice**

- **Decay rate difference:** Second OPE = Heavy Quark Expansion (HQE)

$$\Gamma_{12} = \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^5 \left(\Gamma_5^{(0)} + \ldots\right) + \ldots$$

’96: Beneke, Buchalla; ’98: Beneke, Buchalla, Greub, A.L., Nierste;
’03: Beneke, Buchalla, A.L., Nierste; ’03: Ciuchini, Franco, Lubicz, Mescia, Tarantino;
’06; ’11: A.L., Nierste; ’07 Badin, Gabianni,Petrov

Test of our theoretical Understanding

HQE might be questionable - relies on quark hadron duality

Energy release is small $\Rightarrow$ naive dim. estimate: series might not converge

- Mid 90's: Missing Charm puzzle $n_c^{\text{Exp.}} < n_c^{\text{SM}}$, semi leptonic branching ratio
- Mid 90's: Λ_b lifetime is too short, i.e. $\tau(\Lambda_b) \ll \tau(B_d) = 1.519$ ps
- before 2003: $\tau_{B_s}/\tau_{B_d} \approx 0.94 \neq 1$
- 2010/2011: Di-muon asymmetry too large

Theory arguments for HQE

$\Rightarrow$ calculate corrections in all possible “directions”, to test convergence

$\Rightarrow$ test reliability of HQE via lifetimes (no NP effects expected)

Test of our theoretical Understanding

(Almost) all discrepancies disappeared:

- '12: $n_c^{2011\mathrm{PDG}} = 1.20 \pm 0.06$ vs. $n_c^{\mathrm{SM}} = 1.23 \pm 0.08$ **Krinner, A.L., 1305.5390**
- HFAG '03 $\tau_{\Lambda_b} = 1.229 \pm 0.080$ ps^{-1} $\longrightarrow$ HFAG '13 $\tau_{\Lambda_b} = 1.429 \pm 0.024$ ps^{-1}
  Shift by 2.5σ!; (ATLAS: 1.45 ± 0.04 ps/CMS: 1.50 ± 0.06 ps/LHCb: 1.482 ± 0.022 ps)
- **HFAG 2013:** $\tau_{B_s}/\tau_{B_d} = 0.998 \pm 0.009$
- 2010/2011: Di-muon asymmetry too large — Test Γ_{12} with $\Delta\Gamma_s$!

Theory arguments for HQE

$\Rightarrow$ calculate corrections in all possible "directions", to test convergence

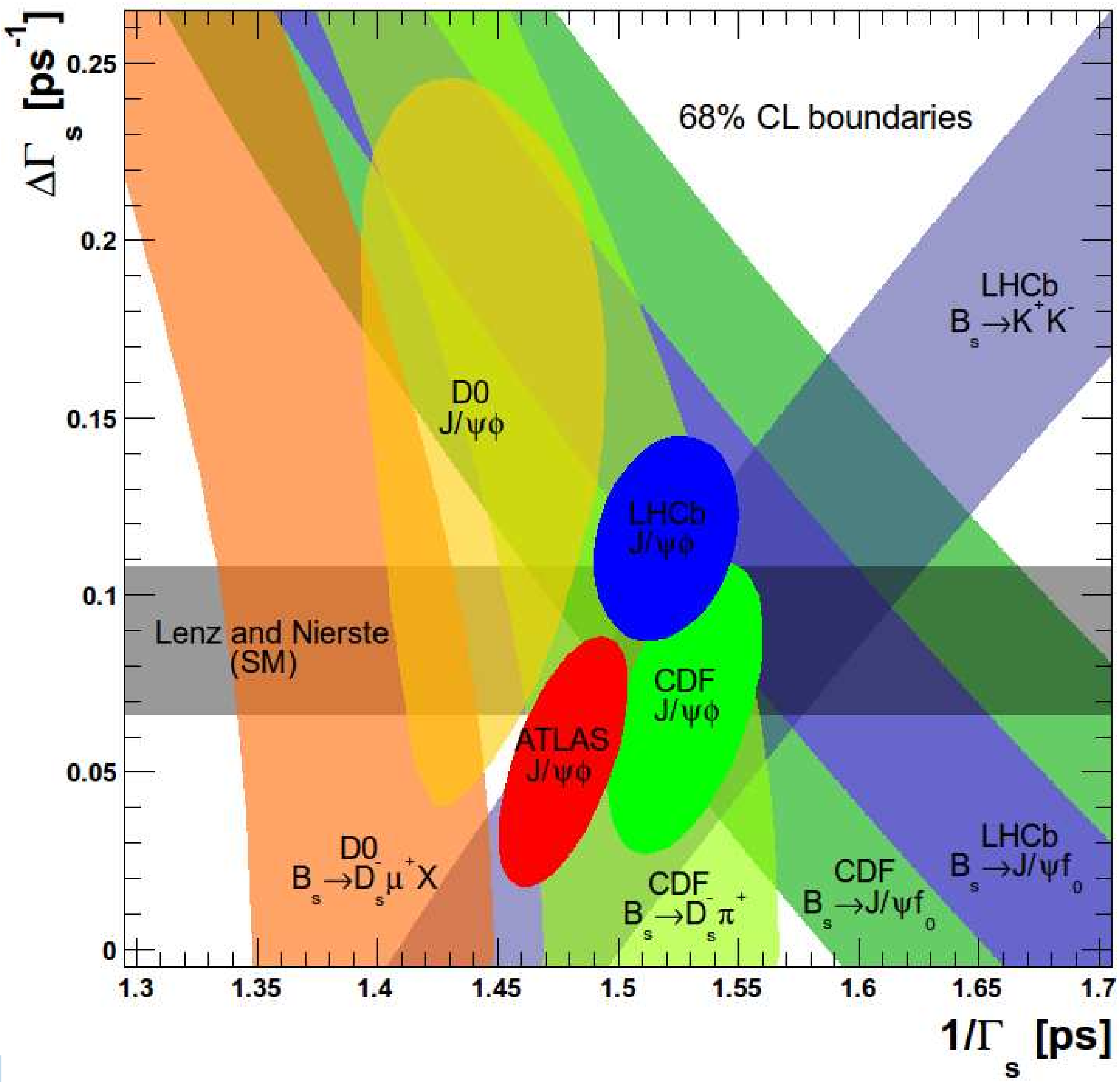
$$\boxed{\begin{aligned} \Delta\Gamma_s &= \Delta\Gamma_s^0 \left(1 + \delta^{\mathrm{Lattice}} + \delta^{\mathrm{QCD}} + \delta^{\mathrm{HQE}}\right) \\ &= 0.142\ \mathrm{ps}^{-1} \left(1 - 0.14 - 0.06 - 0.19\right) \end{aligned}}$$

$\Rightarrow$ looks ok!

$\Rightarrow$ test reliability of HQE via lifetimes (no NP effects expected)
$\Rightarrow$ $\tau(B^+)/\tau(B_d)$ experiment and theory agree within hadronic uncertainties

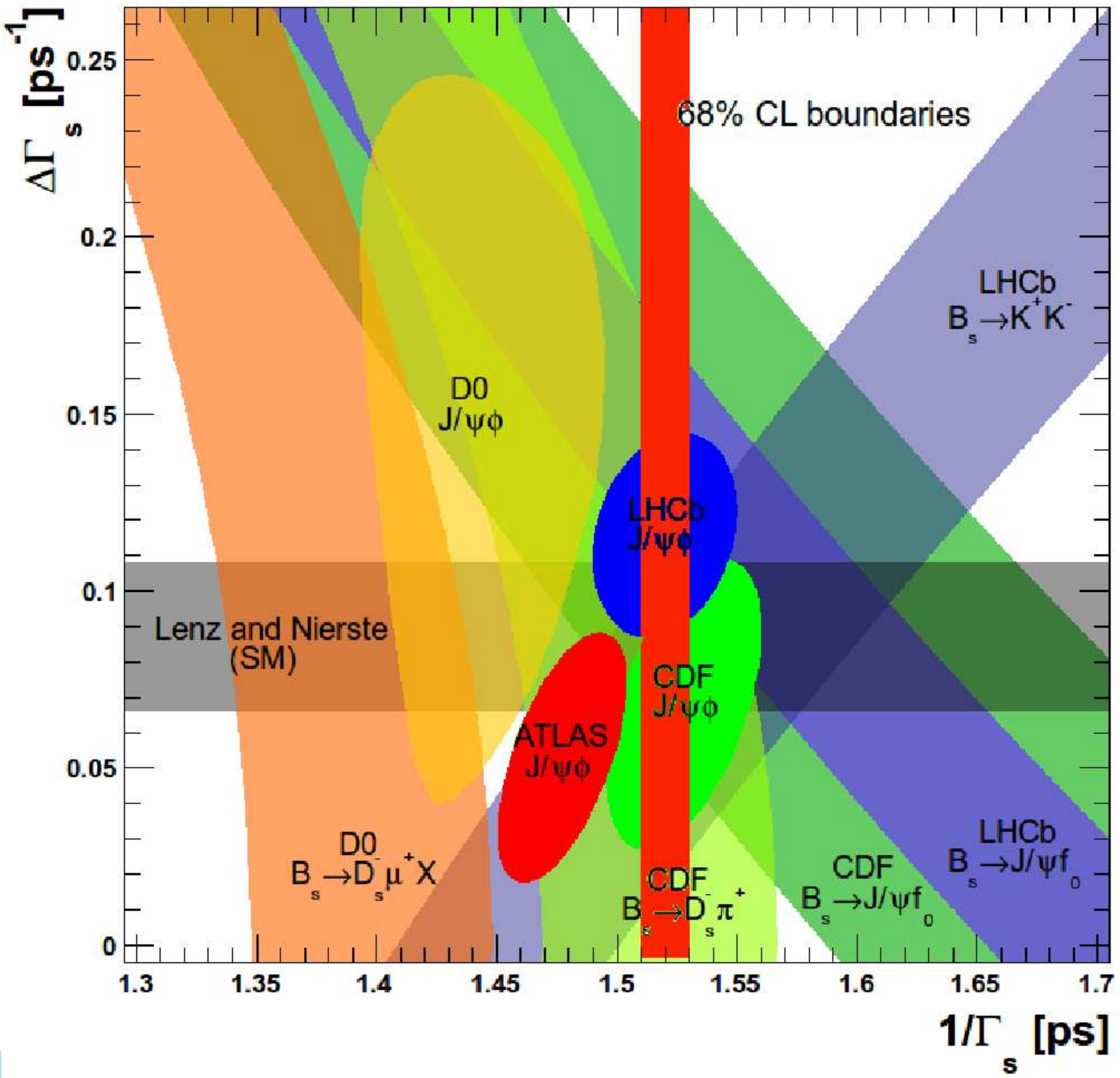
Dominant uncertainties: NLO-QCD + Lattice

Test of our theoretical Understanding



Thanks to Roger

Test of our theoretical Understanding



Thanks to Roger

Test of our theoretical Understanding

Finally $\Delta\Gamma_s$ is measured! E.g. from $B_s \to J/\psi\phi$

LHCb Moriond 2012, 2013; ATLAS; CDF; DO

$$\boxed{\begin{aligned} \Delta\Gamma_s^{\mathrm{Exp}} &= (0.081 \pm 0.011)\,\mathrm{ps}^{-1} \\ \Delta\Gamma_s^{\mathrm{SM}} &= (0.087 \pm 0.021)\,\mathrm{ps}^{-1} \end{aligned}}$$

HFAG 2013

A.L.,Nierste 1102.4274

Cancellation of non-perturbative uncertainties in ratios

$$\boxed{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\mathrm{Exp}} / \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\mathrm{SM}} = 0.92 \pm 0.12 \pm 0.20}$$

Dominant uncertainty = NNLO-QCD + Lattice

Test of our theoretical Understanding

Most important lesson?: HQE works also for Γ_{12}!

- HQE works for the decay $b \to c\bar{c}s$
- Energy release $M_{B_s} - 2M_{D_s} \approx 1.4$ GeV (momentum release: 3.5 GeV)
- Violation quark hadron duality: Theoreticians were fighting for 35 years

How precise does it work? 30%? 10%?

Still more accurate data needed!

LHCb, ATLAS, CMS?, TeVatron, Super-Belle

1. Apply HQE also to $b \to c\bar{c}s$ transitions
2. Apply HQE to quantities that are sensitive to NP
3. Apply HQE also to quantities in the charm system?

Search for New Physics in B-mixing

HQE works! SM predictions: **A.L., U. Nierste, 1102.4274; A.L. 1108.1218**

$$\begin{aligned} a_{fs}^s &= (1.9 \pm 0.3) \cdot 10^{-5} & \phi_s &= 0.22^\circ \pm 0.06^\circ \\ a_{fs}^d &= -(4.1 \pm 0.6) \cdot 10^{-4} & \phi_d &= -4.3^\circ \pm 1.4^\circ \\ A_{sl}^b = 0.406 a_{sl}^s + 0.594 a_{sl}^d &= & &(-2.3 \pm 0.4) \cdot 10^{-4} \\ \left| \frac{\Delta\Gamma_d}{\Gamma_d} \right| &= & &(4.2 \pm 0.8) \cdot 10^{-3} \end{aligned}$$

Older experimental bounds:

$$\phi_s = -51.6^\circ \pm 12^\circ$$ **(A.L., Nierste, CKMfitter, 1008.1593)**

$$\left| \frac{\Delta\Gamma_d}{\Gamma_d} \right| = (15 \pm 18) \cdot 10^{-3}$$ **(HFAG 13)**

$$A_{sl}^b = -(7.87 \pm 1.72 \pm 0.93) \cdot 10^{-3}$$ **(D0,1106.6308)**

$A_{sl}^b(Exp.)/A_{sl}^b(Theory) = \mathbf{34}$ $3.9 - \sigma$**-effect**

Search for New Physics in B-Mixing

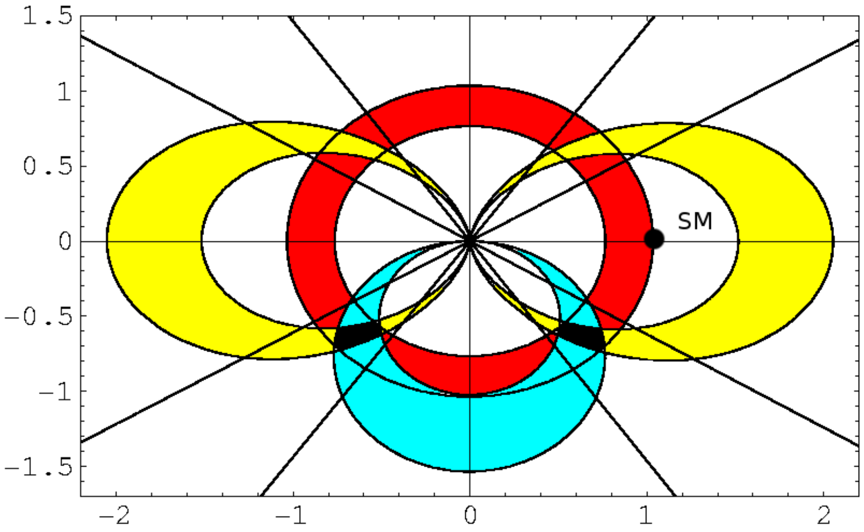
Model independent analysis: **A.L., Nierste, '06**

$$\Gamma_{12,s} = \Gamma_{12,s}^{\rm SM}\,, \qquad M_{12,s} = M_{12,s}^{\rm SM} \cdot \Delta_s\,; \qquad \Delta_s = |\Delta_s| e^{i\phi_s^\Delta}$$

$$\begin{aligned}
\Delta M_s &= 2|M_{12,s}^{\rm SM}| \cdot |\Delta_s| \\
\Delta\Gamma_s &= 2|\Gamma_{12,s}| \cdot \cos\left(\phi_s^{\rm SM} + \phi_s^\Delta\right) \\
\frac{\Delta\Gamma_s}{\Delta M_s} &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\rm SM}|} \cdot \frac{\cos\left(\phi_s^{\rm SM} + \phi_s^\Delta\right)}{|\Delta_s|} \\
a_{fs}^s &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\rm SM}|} \cdot \frac{\sin\left(\phi_s^{\rm SM} + \phi_s^\Delta\right)}{|\Delta_s|}
\end{aligned}$$

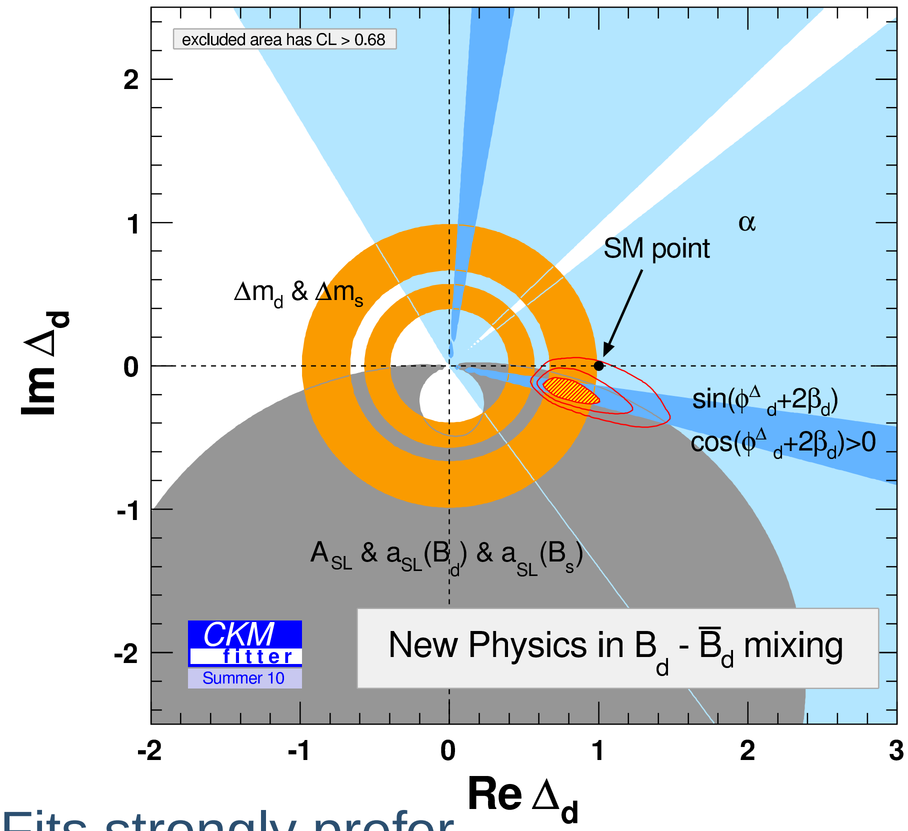
$$\sin(\phi_s^{\rm SM}) \approx 1/240$$

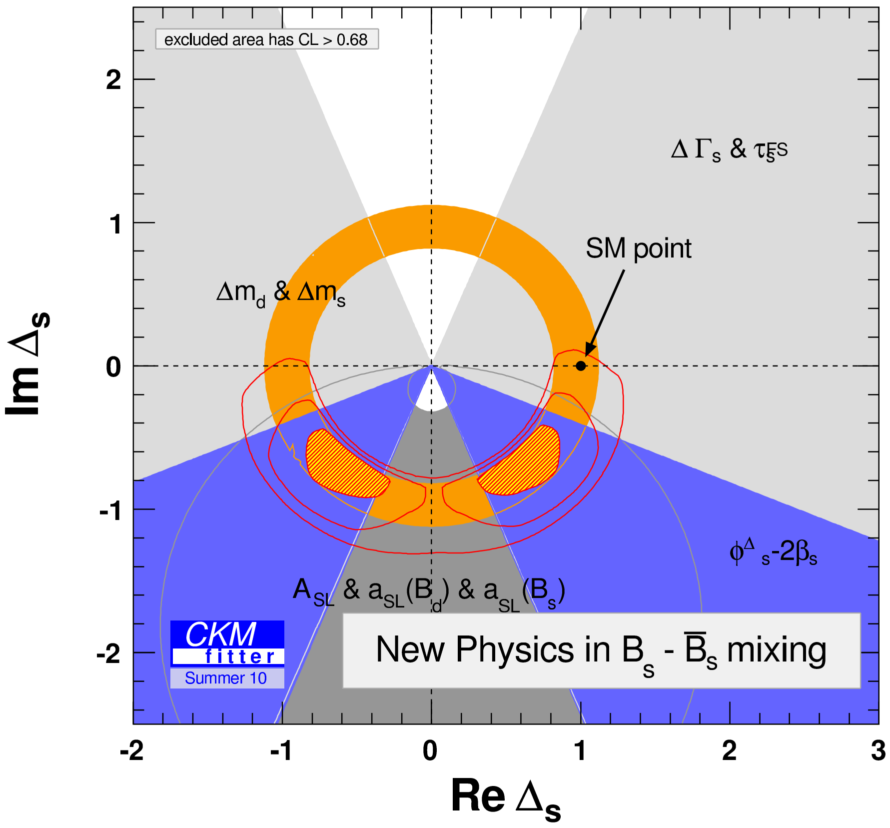
For $|\Delta_s| = 0.9$ and $\phi_s^\Delta = -\pi/4$ one gets the following bounds in the **complex Δ-plane**:



Search for New Physics in B-Mixing

Combine all data before summer 2010 and neglect penguins
fit of Δ_d and Δ_s **A.L.. Nierste. CKMfitter 1008.1593**





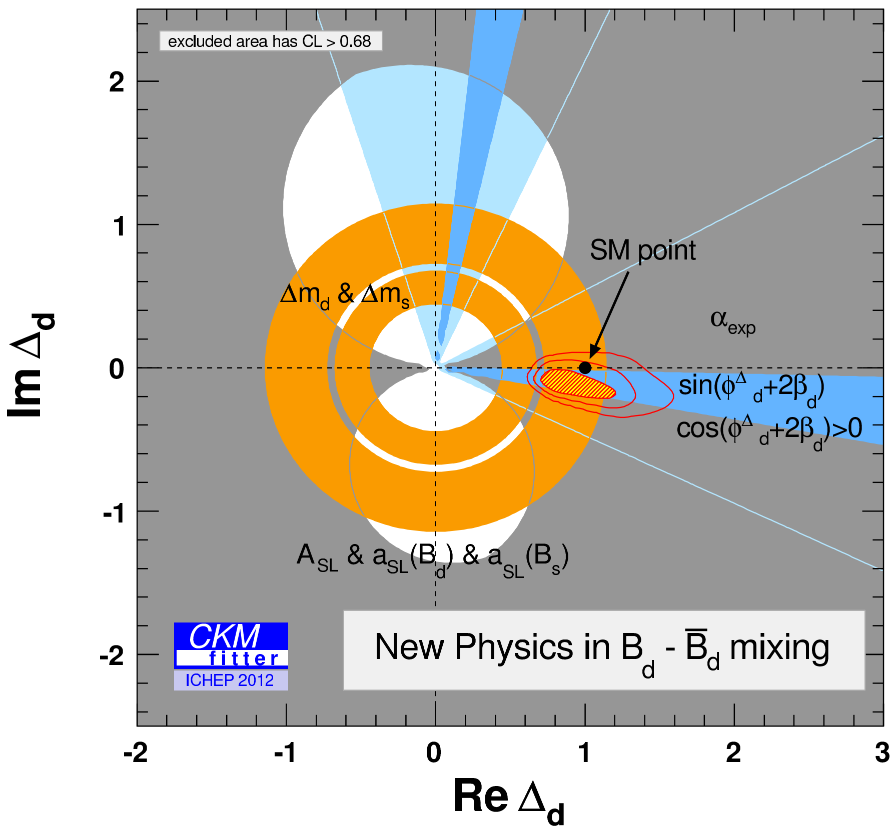
Fits strongly prefer

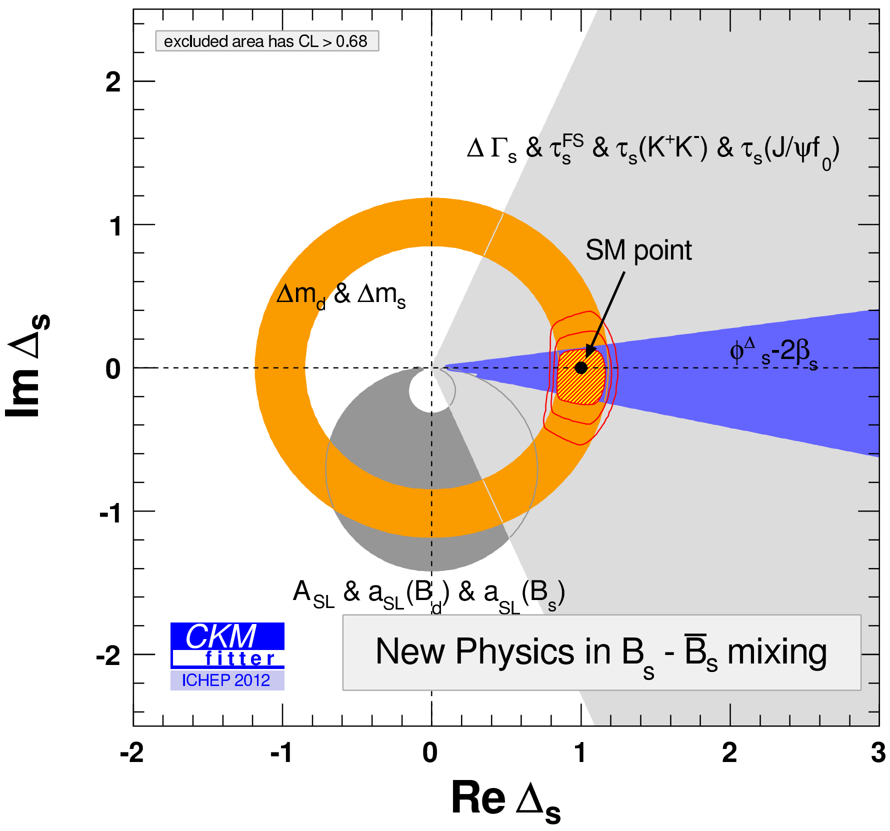
- **large new physics effects in the B_s-system**
- **some new physics effects in the B_d-system**

Search for New Physics in B-Mixing

unpublished: Combine all data till end of 2012 and neglect penguins

fit of Δ_d and Δ_s; update of A.L., Nierste, CKMfitter 1203.0238v2





- **SM seems to be perfect**
- **Still quite some room for NP**

Thanks to CKMfitter!

Search for NP in B-Mixing: A_{sl}^b?

$$\boxed{A_{sl}^b \approx \frac{1}{2}\frac{|\Gamma_{12,d}|}{|M_{12,d}^{\mathrm{SM}}|} \cdot \frac{\sin\left(\phi_d^{\mathrm{SM}} + \phi_d^{\Delta}\right)}{|\Delta_d|} + \frac{1}{2}\frac{|\Gamma_{12,s}|}{|M_{12,s}^{\mathrm{SM}}|} \cdot \frac{\sin\left(\phi_s^{\mathrm{SM}} + \phi_s^{\Delta}\right)}{|\Delta_s|}}$$

BUT: The experimental number is larger than “possible”! **A.L. 1205.1444, 1106.3200**

1. Huge (= several 100 $\%$) duality violations in Γ_{12}? $\rightarrow$ NO! see $\Delta\Gamma_s$
2. Huge NP in Γ_{12}? $\rightarrow$ NO! this also affects observables like $\tau_{B_s}/\tau_{B_d}, n_c, ...$
   But still some sizable NP possible - investigate e.g. n_c **Bobeth, Haisch 1109.1826**
3. Look at experimental side
   - Statistical fluctuation - **soon update from D0**
   - Cross-check via individual asymmetries - **LHCb, D0, BaBar**
     $\Rightarrow$ **consistent with SM, but not yet in conflict with** A_{sl}^b
   - Some systematics neglected - **Borissov, Hoeneisen 1303.0175**
     Discrepancy less than 3σ - **also dependence on** $\Delta\Gamma_d$

$\Rightarrow$ A_{sl}^b: **less promising candidate for** ***the Clue*** - **look also somewhere else**

Search for NP in B-Mixing: A_{sl}^b?

- New measurements for the individual semi leptonic CP asymmetries

$a_{sl}^s = -0.06 \pm 0.50 \pm 0.36\%$	**LHCb 1308.1048**
$a_{sl}^s = -1.12 \pm 0.74 \pm 0.17\%$	**D0 1207.1769**
$a_{sl}^d = 0.68 \pm 0.45 \pm 0.14\%$	**D0 1208.5813**
$a_{sl}^d = 0.06 \pm 0.17^{+0.38}_{-0.32}\%$	**BaBar 1305.1575**

All numbers are consistent with the SM
(no confirmation of large new physics effects)
but also consistent with the value of the dimuon asymmetry

more data urgently needed

- New interpretation of the di-muon asymmetry **Borissov, Hoeneisen 1303.0175**

$$A_{sl}^b = C_d a_{sl}^d + C_s a_{sl}^s + C_\Gamma \frac{\Delta\Gamma_d}{\Gamma_d}$$

There is still sizable space for NP in $\Delta\Gamma_d$, AL, Pecjak, Tetlalmatzi-Xolocotz, to appear

New physics in flavour observables:

What did we learn from current NP searches?

1. Almost everything looks very much **SM like**
   Some remaining discrepancies - below 3 σ
2. There are **no huge NP effects** :-(
3. Still **sizable NP effects possible** :-)

$\Rightarrow$ Life is not as easy as hoped for

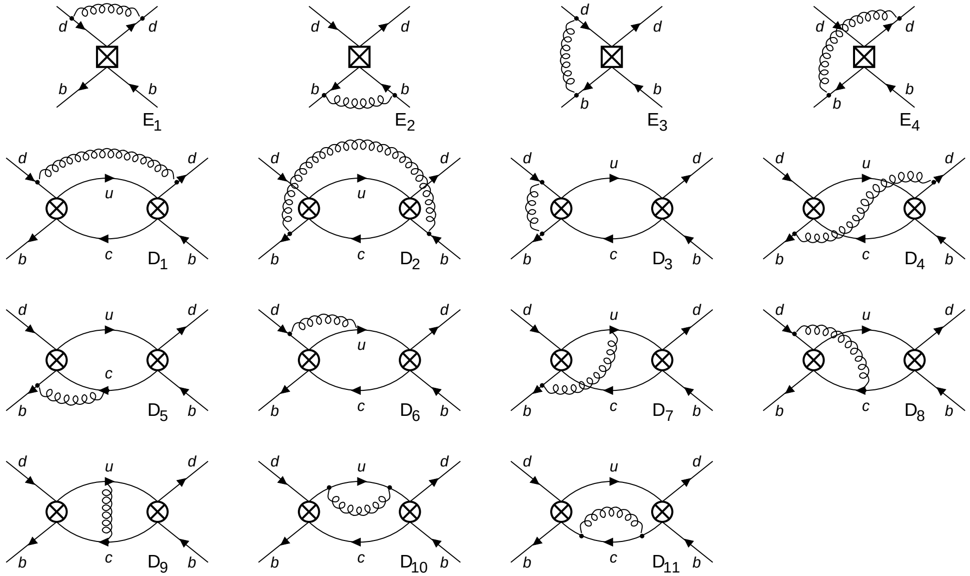
higher precision in experiment and theory needed

Lifetimes: τ_{B^+}/τ_{B_d} in NLO-QCD

$$\frac{\tau_1}{\tau_2} = 1 + \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \ldots\right) + \ldots$$

2002: Beneke, Buchalla, Greub, A.L., Nierste; Franco, Lubicz, Mescia, Tarantino

2004: Greub, A.L., Nierste; 2008 A.L.



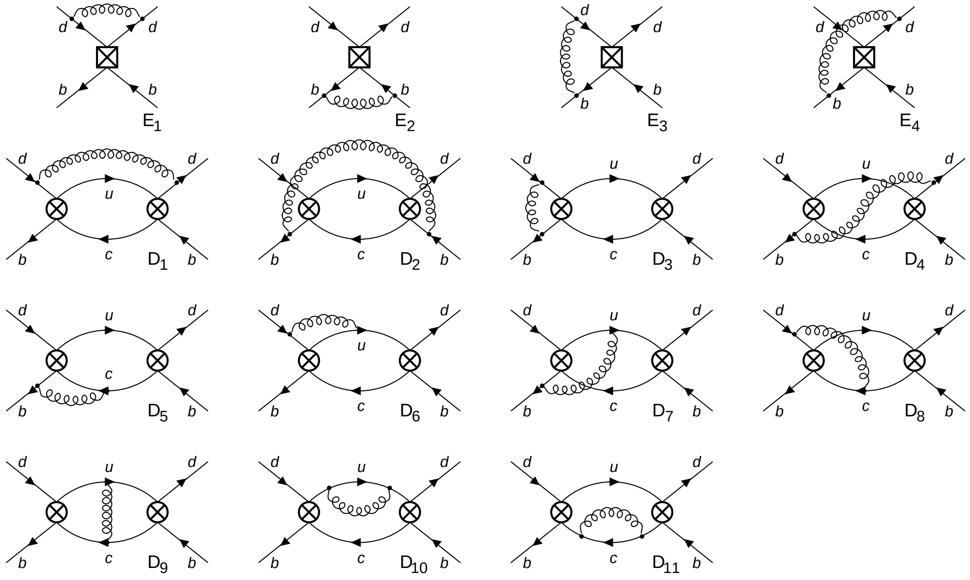
$$\left[\frac{\tau(B^+)}{\tau(B_d^0)}\right]_{\mathrm{LO,NLO,HFAG10}} = 1.047 \pm 0.049 \leftrightarrow 1.063 \pm 0.027 \leftrightarrow 1.071 \pm 0.009$$

Lifetimes: τ_{B^+}/τ_{B_d} in NLO-QCD

$$\frac{\tau_1}{\tau_2} = 1 + \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \ldots\right) + \ldots$$

2002: Beneke, Buchalla, Greub, A.L., Nierste;Franco, Lubicz, Mescia, Tarantino

2004: Greub, A.L., Nierste; 2008 A.L.



$$\left[\frac{\tau(B^+)}{\tau(B_d^0)}\right]_{\mathrm{LO,NLO,HFAG13}} = 1.047 \pm 0.049 \leftrightarrow 1.044 \pm 0.024 \leftrightarrow 1.079 \pm 0.007$$

Lifetimes: τ_{B^+}/τ_{B_d} in NLO-QCD

$$
\begin{aligned}
\frac{\tau_{B^+}}{\tau_{B_d}} - 1 = 0.0324 \left(\frac{f_B}{200\text{MeV}}\right)^2 & \quad [(1.0 \pm 0.2) B_1 + (0.1 \pm 0.1) B_2 \\
- & \quad (17.8 \pm 0.9)\epsilon_1 + (3.9 \pm 0.2)\epsilon_2 - 0.26]
\end{aligned}
$$

with non-perturbative input from **Becirevic hep-ph/0110124** - **12 years old!!!**

$$
\begin{aligned}
B_1 &= 1.10 \pm 0.20 \\
B_2 &= 0.79 \pm 0.10 \\
\epsilon_1 &= -0.02 \pm 0.02 \\
\epsilon_2 &= 0.03 \pm 0.01
\end{aligned}
$$

Update urgently needed!

Lifetimes: $\tau_{\Lambda_b}/\tau_{B_d}$ - Experiment

Year	Exp	Decay	$\tau(\Lambda_b)$ [ps]	$\tau(\Lambda_b)/\tau(B_d)$
2013	HFAG	average	1.429 ± 0.024	0.941 ± 0.016
2013	LHCb	$J/\psi p K^-$	1.482 ± 0.022	0.976 ± 0.012
2013	CMS	$J/\psi \Lambda$	1.503 ± 0.061	$0.989 \pm 0.040*$
2012	ATLAS	$J/\psi \Lambda$	1.449 ± 0.040	$0.954 \pm 0.026*$
2010	CDF	$J/\psi \Lambda$	1.537 ± 0.047	1.020 ± 0.031
2009	CDF	$\Lambda_c + \pi^-$	1.401 ± 0.058	0.922 ± 0.038
2007	D0	$\Lambda_c \mu \nu X$	1.290 ± 0.150	$0.849 \pm 0.099*$
2007	D0	$J/\psi \Lambda$	1.218 ± 0.137	$0.802 \pm 0.090*$
2006	CDF	$J/\psi \Lambda$	1.593 ± 0.089	1.049 ± 0.059
2004	D0	$J/\psi \Lambda$	1.22 ± 0.22	0.87 ± 0.17
2003	HFAG	average	1.212 ± 0.052	0.798 ± 0.034
1998	OPAL	$\Lambda_c l$	1.29 ± 0.25	$0.85 \pm 0.16*$
1998	ALEPH	$\Lambda_c l$	1.21 ± 0.11	$0.80 \pm 0.07*$
1995	ALEPH	$\Lambda_c l$	1.02 ± 0.24	$0.67 \pm 0.16*$
1992	ALEPH	$\Lambda_c l$	1.12 ± 0.37	$0.74 \pm 0.24*$

Lifetimes: $\tau_{\Lambda_b}/\tau_{B_d}$ - Theory

Year	Author	$\tau(\Lambda_b)/\tau(B_d)$
2007	Tarantino	0.88 ± 0.05
2004	Petrov et al.	0.86 ± 0.05
2003	Tarantino	0.88 ± 0.05
2002	Rome	0.90 ± 0.05
2000	Körner,Melic	$0.81...0.92$
1999	Guberina,Melic,Stefanic	0.90
1999	diPierro, Sachrajda, Michael	0.92 ± 0.02
1999	Huang, Liu, Zhu	0.83 ± 0.04
1996	Colangelo, deFazio	> 0.94
1996	Neubert,Sachrajda	$" > 0.90"$
1992	Bigi, Blok, Shifman, Uraltsev, Vainshtein	$> 0.85...0.90$
x	only$1/m_b^2$	0.98

Lifetimes: $\tau_{\Lambda_b}/\tau_{B_d}$ at order $1/m_b^2$

$$
\begin{aligned}
\frac{\tau(\Lambda_b)}{\tau(B_d)} = 1 \quad & + \quad \frac{\Lambda^2}{m_b^2}\left(\Gamma_2^{(0)} + \ldots\right) \\
& + \quad \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) \\
& + \quad \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + \ldots\right) + \frac{\Lambda^5}{m_b^5}\left(\Gamma_5^{(0)} + \ldots\right) + \ldots
\end{aligned}
$$

Leading Term

$$
\begin{aligned}
\frac{\Lambda^2}{m_b^2}\Gamma_2 \quad & = \quad \frac{\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B_d)}{2m_b^2} + c_5 \frac{\mu_G^2(\Lambda_b) - \mu_G^2(B_d)}{m_b^2} \\
& = \quad \frac{(0.1 \pm 0.1)\text{GeV}^2}{2m_b^2} + 1.2 \frac{0 - 0.33\text{GeV}^2}{2m_b^2} \\
& \approx \quad 0.002 - 0.017 = -0.015
\end{aligned}
$$

Numbers from **Bigi, Mannel Uraltsev**, 2011

Lifetimes: $\tau_{\Lambda_b}/\tau_{B_d}$ at order $1/m_b^3$

$$
\begin{aligned}
\frac{\tau(\Lambda_b)}{\tau(B_d)} = 1 \quad & - \quad 0.015 \\
& + \quad \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) \\
& + \quad \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + \ldots\right) + \frac{\Lambda^5}{m_b^5}\left(\Gamma_5^{(0)} + \ldots\right) + \ldots
\end{aligned}
$$

Γ_3 is a linear combination of perturbative Wilson coefficients and non-perturbative matrix elements

- Wilson coefficient of $\Gamma_3^{(0)}$, e.g. 1996 **Uraltsev/ Neubert and Sachrajda**

  Part of $\Gamma_3^{(1)}$ 2002 **Franco, Lubicz, Mescia, Tarantino**
- Matrix element

  HQET: only two different matrix elements (instead of four)

$$
\frac{1}{2m_{\Lambda_b}}\langle \Lambda_b | \bar{b}_L \gamma_\mu q_L \cdot \bar{q}_L \gamma^\mu b_L | \Lambda_b \rangle =: -\frac{f_B^2 m_B}{48} r
$$

$\tau_{\Lambda_b}/\tau_{B_d}$: matrix elements of 4-quark operators

Values for r:

$r \approx 0.2$	$Bag\ model$	**Guberina, Nussinov, Peccei, Rückl, 1979**
$r \approx 0.5$	$NR\ quark\ model$	**–"–**
$r = 0.9 \pm 0.1$	$spectroscopy$	**Rosner, 1996**
$r = 1.8 \pm 0.5$	$spectroscopy$	**–"–**
$r = 0.2 \pm 0.1$	$QCD\ sum\ rules$	**Colangelo, de Fazio, 1996**

Neubert, Sachrajda: $\frac{\tau(\Lambda_b)}{\tau(B_d^0)}$ "> 0.9"

$r = 1.2 \pm 0.2 \pm ?$	$lattice$	**di Pierro, Sachrajda, Michael 1999**
$r = 2.3 \pm 0.6$	$QCD\ sum\ rules$	**Huang, Liu, Zhu, 2000**
$r = 6.2 \pm 1.6$	$QCD\ sum\ rules$	**–"–**

$$!!! \quad \frac{\tau(\Lambda_b)}{\tau(B_d^0)} - 1 \propto r \quad !!!$$

$\tau_{\Lambda_b}/\tau_{B_d}$: matrix elements of 4-quark operators

1996 **Rosner**

$$r = \frac{4}{3}\frac{m^2_{\Sigma_b^*} - m^2_{\Sigma_b}}{m^2_{B^*} - m^2_B}$$

In 1996 b-baryon masses were hardly known

- $m^2_{\Sigma_b^*} - m^2_{\Sigma_b} \approx m^2_{\Sigma_c^*} - m^2_{\Sigma_c} = (0.384 \pm 0.035)$GeV^2

$$\Rightarrow r = 0.9 \pm 0.10$$

- $m_{\Sigma_b^*} - m_{\Sigma_b} = (56 \pm 16)$ MeV

$$\Rightarrow r = 1.8 \pm 0.5$$

- Use the values from PDG 2011: $\tau_{\Lambda_b}/\tau_{B_d} > 0.9$

$$\Rightarrow r = 0.68 \pm 0.08$$

AL 1205.1444

$\tau_{\Lambda_b}/\tau_{B_d}$: matrix elements of 4-quark operators

1999 **DiPierro, Sachrajda, Michael**:

currently the only lattice determination!

- **14 years old!**
- The authors call their study *exploratory*:
  - Larger lattice should be used
  - Larger sample of gluon configurations should be used
  - Matching to continuum only at leading order
  - No chiral extrapolation attempted
  - Penguin contractions are missing and difficult (=disconnected)

2013 **Sachrajda** during Coffee break of eminent workshop in Durham:

We understand disconnected contributions much better now

1999 **Huang, Liu, Zhu**:

QCD sum rule result, which is up to a factor of 31 larger than the one by Colangelo and DeFazio and by accident fitted the low experimental number of that time...

Clean ratio: $\tau(\Xi_b^0)/\tau(\Xi_b^+)$

- Disconnected contributions cancel in $\tau(\Xi_b^0)/\tau(\Xi_b^+)$ as in $\tau(B^+)/\tau(B_d)$
- No matrix elements for Ξ_b available - assume they are equal to the Λ_b
- Get rid of unwanted $s \to u$-transitions

$$\frac{1}{\bar{\tau}(\Xi_b)} = \bar{\Gamma}(\Xi_b) = \Gamma(\Xi_b) - \Gamma(\Xi_b \to \Lambda_b + X)\,.$$

Analytic result given in **Beneke, Buchalla, Greub, AL, Nierste 2002**

$$\frac{\bar{\tau}(\Xi_b^0)}{\bar{\tau}(\Xi_b^+)} = 1 - 0.12 \pm 0.02 \pm ???\,,$$

AL 0802.0977

$???$ unknown systematic hadronic errors.

Further assume $\bar{\tau}(\Xi_b^0) = \tau(\Lambda_b)$ - similar cancellations as in τ_{B_s}/τ_{B_d}

$$\frac{\tau(\Lambda_b)}{\bar{\tau}(\Xi_b^+)} = 0.88 \pm 0.02 \pm ???\,.$$

Further Lifetimes

- $\tau(B_s)/\tau(B_d)$:
  Almost perfect cancellation of spectator quark contributions in the HQE

$$\tau_{B_s}^{\text{Exp}} = \begin{cases} 1.477 \pm 0.018\,\text{ps} & \text{ATLAS} \\ 1.520 \pm 0.020\,\text{ps} & \text{LHCb} \\ 1.528 \pm 0.021\,\text{ps} & \text{CDF} \\ 1.443 \pm 0.037\,\text{ps} & \text{D0} \end{cases} \qquad \tau_{B_s}^{\text{SM}} = (0.996 ... 1.000) \cdot (1.519 \pm 0.007)\text{ps}$$

- $\tau(B_c)$: three contributions
  1. c-quark decays
  2. b-quark decays
  3. $b - c$-annihiliation

$$\begin{aligned} \tau(B_c) &= 0.452 \pm 0.033\,\text{ps} && \textbf{HFAG 2013} \\ \tau(B_c)_{\text{LO}} &= 0.52^{+0.18}_{-0.12}\,\text{ps} && \textbf{Beneke, Buchalla 1996} \end{aligned}$$

HQE at its or beyond its limits?

From a theory point the most "simple" quantities are the lifetimes

In the Charm-system huge lifetimes ratios appear, e.g.

$$\frac{\tau(D+)}{\tau(D^0)} = 2.536 \pm 0.019 \qquad \text{PDG 12}$$

Can theory cope with this?

Be aware:

- Λ/m_c might be too large ($\Lambda \neq \Lambda_{QCD}$!)
- $\alpha_s(m_c)$ might be too large

HQE at its or beyond its limits?

- ’75-’78: Naive expectations (**before first data**):

$$\tau(D+)/\tau(D^0) \approx 1$$

- ’79-’82: Naive expectations (**after first data hinting for a large difference**)

$$\tau(D+)/\tau(D^0) \approx 6...10$$

- Systematic HQE estimates **Voloshin, Shifman (’81,’85)**
  - LO-QCD, 1/N_c: $\tau(D+)/\tau(D^0) \approx 2$ **Bigi, Uraltsev (’92-...)**
  - up-to-date estimate; NLO QCD **A.L., Rauh; 1305.3588**

$$\boxed{\frac{\tau(D+)}{\tau(D^0)} = 2.2 \pm 1.7(0.4)(\text{hadronic ME})^{+0.3}_{-0.7}(\text{scale}) \pm 0.1(\text{parametric})}$$

- Looks promising: huge lifetime difference might be explainable by the HQE
- **Hadronic matrix elements of the 4-quark operators urgently needed**

Dominant uncertainty: NNLO-QCD + Lattice

What did we really learn?

- Search for NP - **Missing CPV for the origin of matter in the universe still not identified**
  - No huge effects, but still some sizable space (mixing, rare decays,...)
  - Some remaining discrepancies - e.g. A_{sl}
- Current theoretical Understanding
  - SM and CKM work perfectly
  - Theoretical tools (HQE) work also perfectly (at least to about 30% for most dangerous modes $\Delta\Gamma_s^{\mathrm{SM}} = \Delta\Gamma_s^{\mathrm{Exp.}}$) - this was unclear for a long time

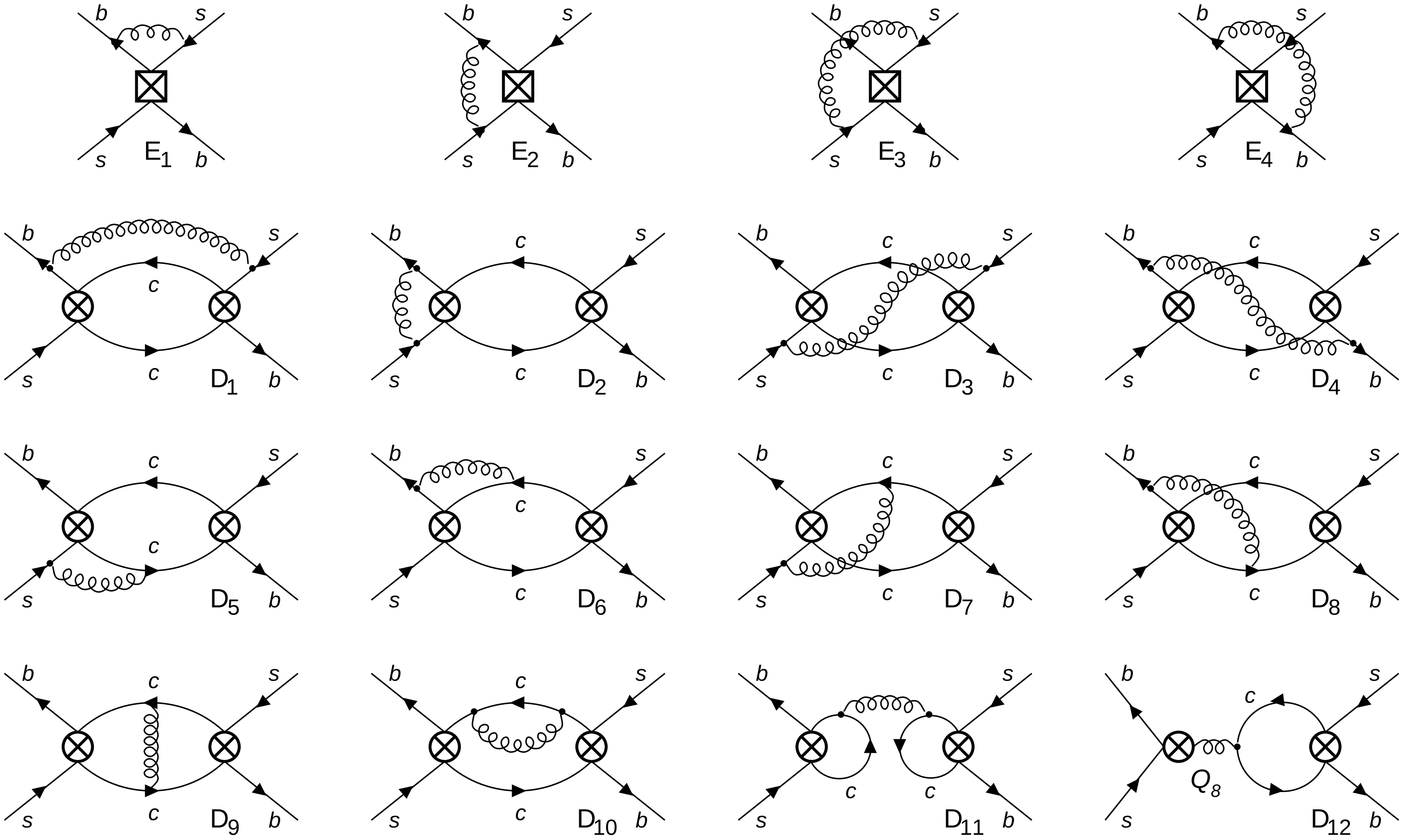
What comes next?

- Further Tests of our theoretical Understanding
  - $\tau(B^+)/\tau(B_d)$: seems ok
    HME urgently needed, NNLO?
  - $\tau(\Lambda_b)$: previous tension seems to have been an experimental problem
    HME urgently needed; part of NLO; more data needed; NNLO?
  - $\tau(B_s)/\tau(B_d)$: seems ok
    More data and HME needed
  - $\tau(B_c^+)$: seems to agree roughly
    Understanding of double expansion in $1/m_c$ and $1/m_b$
  - $\tau(\Xi_b)/\tau(\Xi_b)$: yet not measured
    First measurement needed, HME urgently needed
  - $\tau(D^+)/\tau(D^0)$: HQE might be not completely off
    HME urgently needed, NNLO?
  - $\Delta\Gamma_s$: seems ok
    Power suppressed HME urgently needed, NNLO

HME = non perturbative evaluation of hadronic matrix elements

Theory Prediction for $\Delta\Gamma_s$

Calculating the following diagrams



Theory Prediction for $\Delta\Gamma_s$

one gets Wilson coefficients of the following operators

$$\begin{aligned}
Q &= (\bar{b}_i s_i)_{V-A} \cdot (\bar{b}_j s_j)_{V-A} \\
\tilde{Q}_s &= (\bar{b}_i s_j)_{S-P} \cdot (\bar{b}_i s_j)_{S-P} \\
\langle \bar{B}_s | Q | B_s \rangle &= \frac{8}{3} f_{B_S}^2 M_{B_S}^2 B \\
\langle \bar{B}_s | \tilde{Q}_S | B_s \rangle &= \frac{1}{3} f_{B_S}^2 M_{B_S}^2 \tilde{B}'_S = \frac{1}{3} f_{B_S}^2 M_{B_S}^2 \frac{M_{B_S}^2}{(\bar{m}_b + \bar{m}_s)^2} \tilde{B}_S
\end{aligned}$$

f_{B_s}, B and $\tilde{B}_S$ have to be determined non-perturbatively!

Theory Prediction for $\Delta\Gamma_s$

Expanding also in the small s momenta one get contributions of dimension 7

$$
\begin{aligned}
R_0 &= Q_s + \tilde{Q}_S + \frac{1}{2} Q \\
R_1 &= \frac{m_s}{m_b} (\bar{b}_i s_i)_{S-P} (\bar{b}_j s_j)_{S+P} \\
R_2 &= \frac{1}{m_b^2} (\bar{b}_i \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho s_i)(\bar{b}_j \gamma_\mu (1-\gamma_5) s_j) \\
R_3 &= \frac{1}{m_b^2} (\bar{b}_i \overleftarrow{D}_\rho (1-\gamma_5) D^\rho s_i)(\bar{b}_j (1-\gamma_5) s_j) \\
\tilde{R}_i &= \tilde{R}_i(R_j)
\end{aligned}
$$

There exist no non-perturbative determinations of these operators

A first estimate with QCD sum rules was made by **Mannel, Pecjak, Pivovarov**

Current estimates rely on vacuum insertion approximation

Theory Prediction for $\Delta\Gamma_s$

Improvement in theoretical accuracy

$\Delta\Gamma_s^{\rm SM}$	2011	2006
Central Value	$0.087\,\text{ps}^{-1}$	$0.096\,\text{ps}^{-1}$
$\delta(\mathcal{B}_{\widetilde{R}_2})$	17.2%	15.7%
$\delta(f_{B_s})$	13.2%	33.4%
$\delta(\mu)$	7.8%	13.7%
$\delta(\widetilde{\mathcal{B}}_{S,B_s})$	4.8%	3.1%
$\delta(\mathcal{B}_{R_0})$	3.4%	3.0%
$\delta(V_{cb})$	3.4%	4.9%
$\delta(\mathcal{B}_{B_s})$	2.7%	6.6%
...	....	...
$\sum\delta$	24.5%	40.5%