Rare decays on the lattice

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Outline

- Brief introduction
- $B \to K\, l\, l$
- $B \to K^*\, l\, l$ and $B_s \to \varphi\, l\, l$
- $\Lambda_b \to \Lambda\, l\, l$ (and $\Lambda \to p\, l\, \nu$)

Brief introduction

Rare b decays

$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i (C_i \mathcal{O}_i + C_i' \mathcal{O}_i')$$

Most important short-distance effects in $b \to s\, ll$ come from:

$$\mathcal{O}_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b\, \bar{\ell}\gamma_\mu \ell \qquad \mathcal{O}_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b\, \bar{\ell}\gamma_\mu \gamma_5 \ell$$

$$\mathcal{O}_7^{(\prime)} = \frac{m_b e^2}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b\, F_{\mu\nu}$$

Long-distance effects arise from

$$\mathcal{O}_1 = \bar{c}^\alpha \gamma^\mu P_L b^\beta\, \bar{s}^\beta \gamma^\mu P_L c^\alpha \qquad \mathcal{O}_2 = \bar{c}^\alpha \gamma^\mu P_L b^\alpha\, \bar{s}^\beta \gamma^\mu P_L c^\beta$$

SM Wilson coefficients (NNLL order):

$$C_9^{\text{eff}}(m_b) = 4.211 + Y(q^2) \qquad C_{10}^{\text{eff}}(m_b) = -4.103 \qquad C_7^{\text{eff}}(m_b) = -0.304$$

$$C_9' = C_{10}' = C_7' = 0 \qquad C_1(m_b) = -0.257 \qquad C_2(m_b) = 1.009$$

W Altmannshofer *et al.*, JHEP01(2009)019.

Lattice QCD and $b \to s$

- Compute hadronic matrix elements of local operators
- Exclusive modes, with (at most) 1 hadron in final state
- Require lattice momenta to be small compared to lattice scale

Thus we can contribute by calculating

$$B \to K\, l\, l$$

$$B \to K^*\, l\, l \;\; \& \;\; B_s \to \varphi\, l\, l$$

$$\Lambda_b \to \Lambda\, l\, l$$

form factors in the low recoil (large q^2) regime

Caveat: long-distance effects (resonant contributions) not included

$B \to K$ *form factors*

C Bouchard *et al.*, (HPQCD)

$B \to K$ form factors

$$\langle K(k)|\bar{s}\gamma^\mu b|B(p)\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_K^2}{q^2}\, q^\mu\right] f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2}\, q^\mu f_0(q^2)$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu} q_\nu b|B(p)\rangle = \frac{i f_T(q^2)}{m_B + m_K}\left[q^2 (p+k)^\mu - (m_B^2 - m_K^2) q^\mu\right]$$

- “Gold-plated” matrix elements: QCD-stable $|i\rangle$ and $|f\rangle$ states
- Observables: differential branching fraction $d\Gamma/dq^2$, forward/backward asymmetry A_{FB} (zero in SM), and “flat term” F_H

Lattice actions & parameters

MILC lattices (2+1 asqtad staggered)

HISQ light & strange quarks

NRQCD bottom quarks

ens	$L^3 \times N_t$	r_1/a	$au_0 m_{\rm sea}$	u_0	$N_{\rm conf}$	$N_{\rm tsrc}$	$am_l^{\rm val}$	$am_s^{\rm val}$	am_b	$aE_{b\bar{b}}^{\rm sim}$	T
C1	$24^3 \times 64$	2.647(3)	0.005/0.05	0.8678	1200	2	0.0070	0.0489	2.650	0.28356(15)	12 – 15
C2	$20^3 \times 64$	2.618(3)	0.01/0.05	0.8677	1200	2	0.0123	0.0492	2.688	0.28323(18)	12 – 15
C3	$20^3 \times 64$	2.644(3)	0.02/0.05	0.8688	600	2	0.0246	0.0491	2.650	0.27897(20)	12 – 15
F1	$28^3 \times 96$	3.699(3)	0.0062/0.031	0.8782	1200	4	0.00674	0.0337	1.832	0.25653(14)	21 – 24
F2	$28^3 \times 96$	3.712(4)	0.0124/0.031	0.8788	600	4	0.01350	0.0336	1.826	0.25558(28)	21 – 24

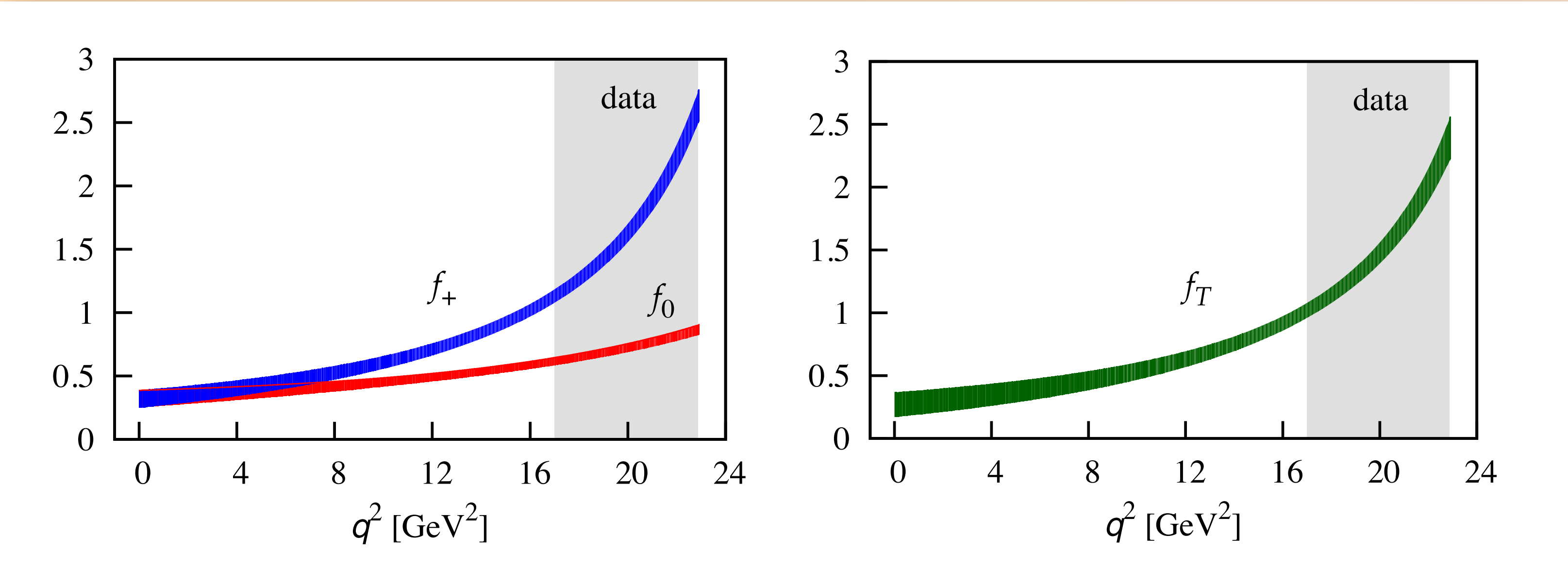
r_1 = 0.3133(23) fm

ens	$1/a$ (GeV)	m_π(MeV)
C1	1.667(12)	267(2)
C2	1.649(12)	348(3)
C3	1.665(12)	488(4)
F1	2.330(17)	313(2)
F2	2.338(17)	438(3)

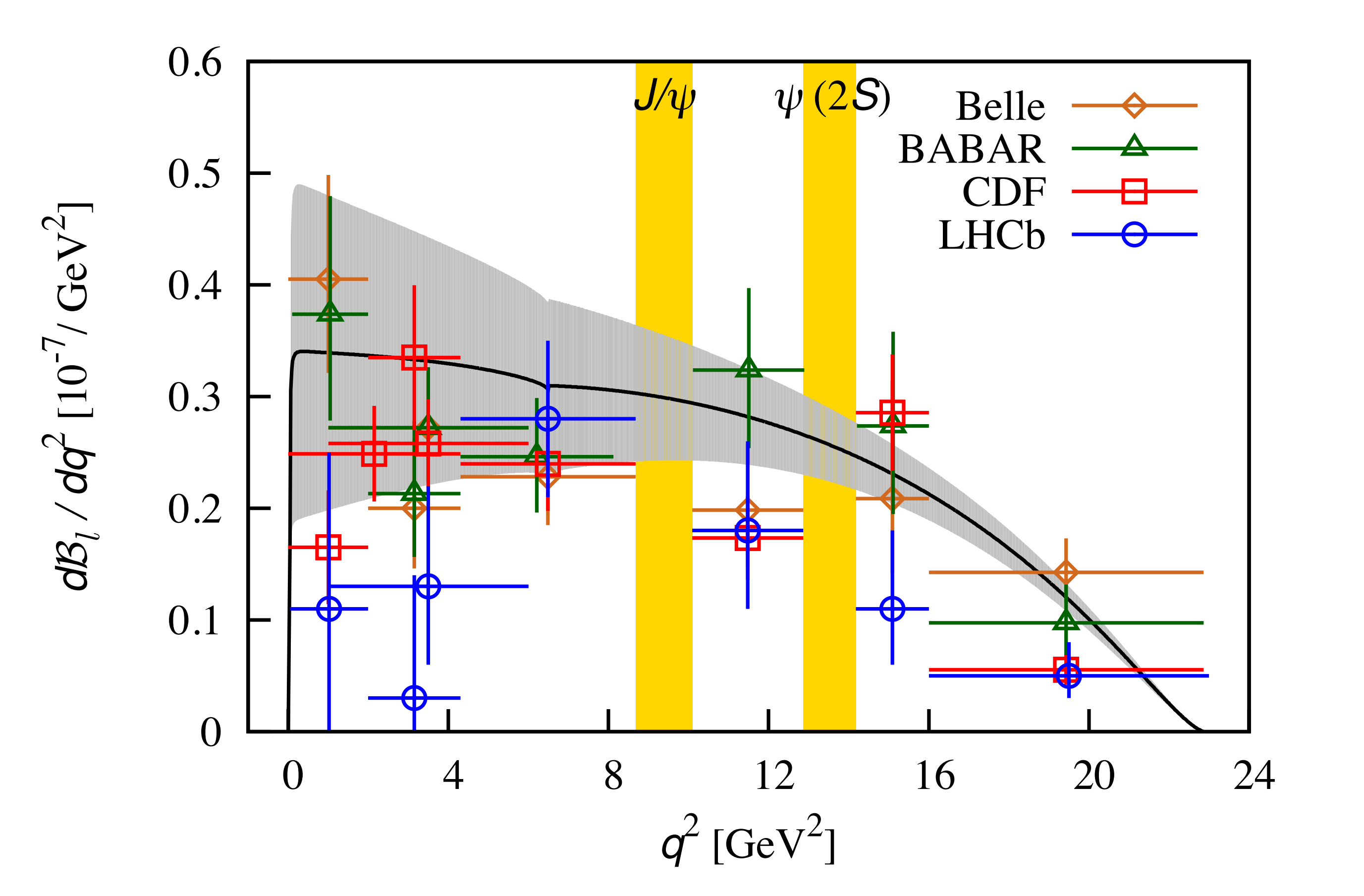
C. Bouchard *et al.*, arXiv:1306.0434, arXiv:1306:2384

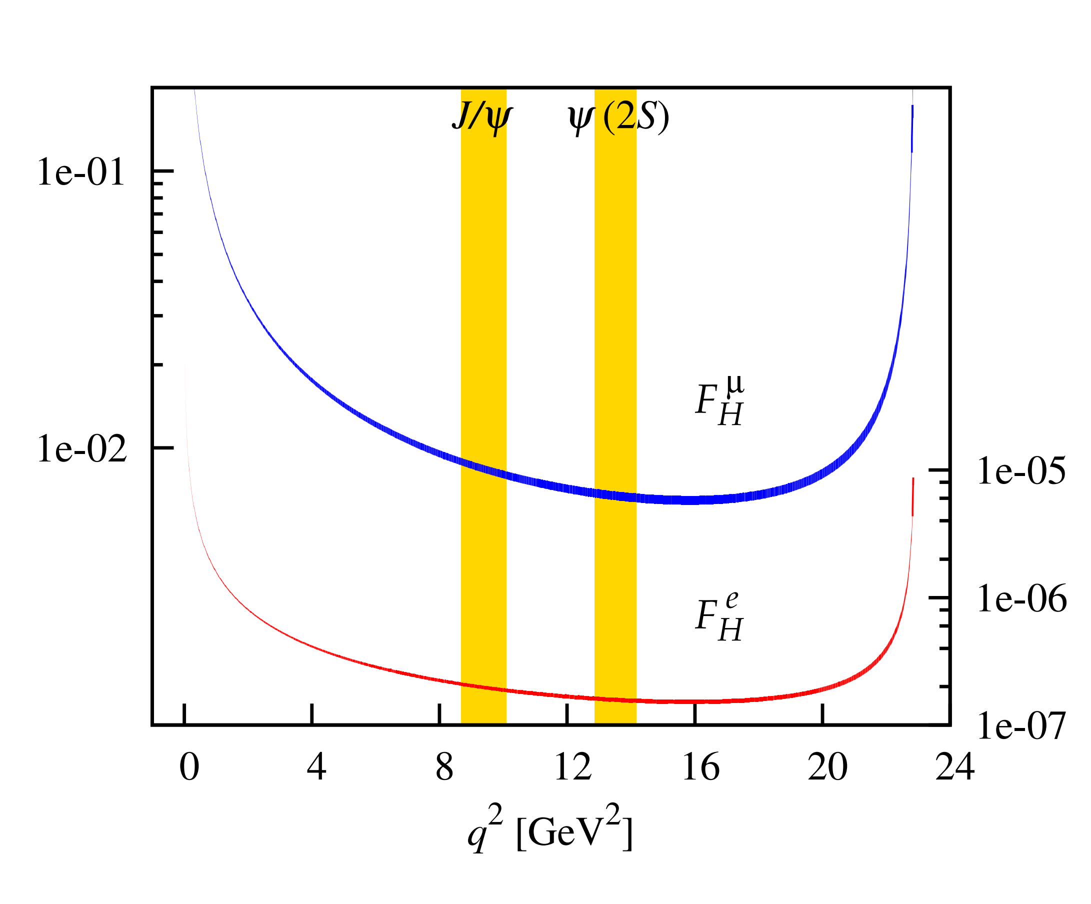
1-loop operator matching: C Monahan, J Shigemitsu, RR Horgan, PRD87 (2013)

$B \to K\, l^+ l^-$



HPQCD Collaboration (using NRQCD+HISQ valence on MILC n_f=2+1 asqtad)





C. Bouchard *et al.*, arXiv:1306.0434, arXiv:1306:2384

Resonant contribution

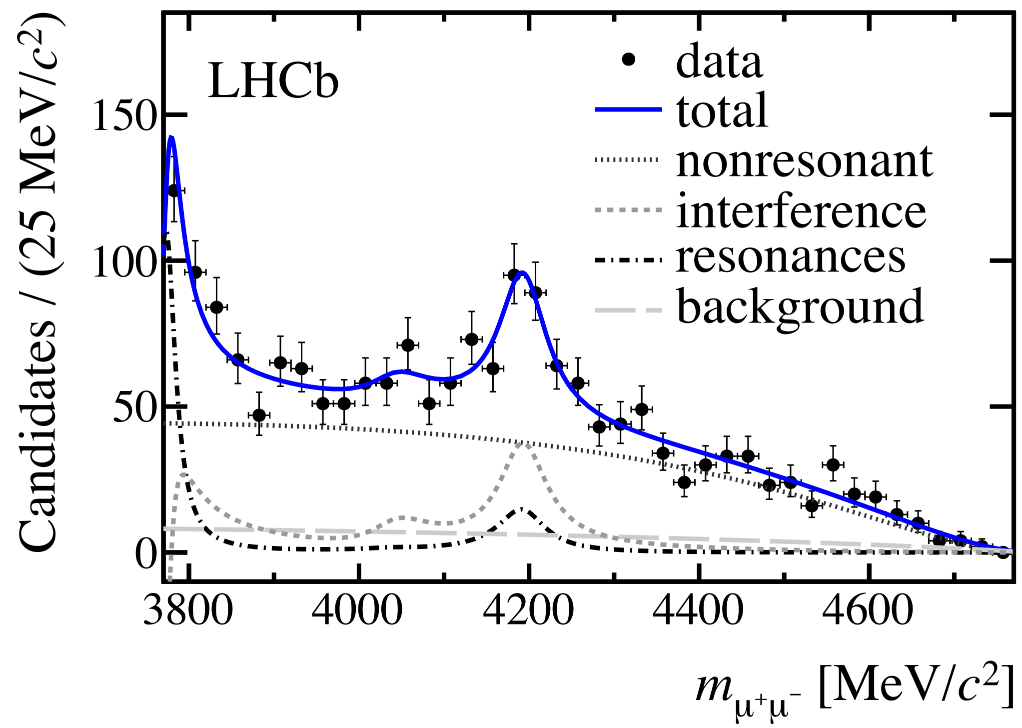


Figure 1: Dimuon mass distribution of data with fit results overlaid for the fit that includes contributions from the non-resonant vector and axial vector components, and the $\psi(3770)$, $\psi(4040)$, and $\psi(4160)$ resonances. Interference terms are included and the relative strong phases are left free in the fit.

R. Aaij *et al.*, (LHC*b*) arXiv:1307.7595

$B \to K^*$ & $B_s \to \varphi$ *form factors*

with RR Horgan, Z Liu, S Meinel

Traditional form factor basis

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma$$

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu\hat{\gamma}^5 b|B(p)\rangle = 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu\right) - A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+k)^\mu - \frac{m_B^2-m_V^2}{q^2}q^\mu\right)$$

$$q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle = 2T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau k^\sigma$$

$$q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle = iT_2(q^2)[\varepsilon^*_\mu(m_B^2-m_V^2) - (\varepsilon^*\cdot q)(p+k)_\mu] + iT_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]$$

Helicity basis

$$V_\pm(q^2) = \frac{1}{2}\left[\left(1+\frac{m_V}{m_B}\right)A_1(q^2) \mp \frac{\sqrt{\lambda}}{m_B(m_B+m_V)}V(q^2)\right]$$

$$T_\pm(q^2) = \frac{1}{2m_B^2}\left[(m_B^2-m_V^2)T_2(q^2) \mp \sqrt{\lambda}T_1(q^2)\right]$$

$$A_{12}(q^2) = \frac{(m_B+m_V)^2(m_B^2-m_V^2-q^2)A_1(q^2)-\lambda A_2(q^2)}{16 m_B m_V^2(m_B+m_V)}$$

$$T_{23}(q^2) = \frac{m_B+m_V}{8 m_B m_V^2}\left[\left(m_B^2+3m_V^2-q^2\right)T_2(q^2) - \frac{\lambda T_3(q^2)}{m_B^2-m_V^2}\right]$$

with $\lambda = (t_+-t)(t_--t) \qquad t = q^2 \qquad t_\pm = (m_B \pm m_V)^2$

Most convenient basis for us: $V, A_0, A_1, A_{12}, T_1, T_2, T_{23}$

Lattice actions & parameters

MILC lattices (2+1 asqtad staggered)

asqtad light & strange quarks

NRQCD bottom quarks

label	#	$N_x^3 \times N_t$	$am_\ell^{\rm sea}/am_s^{\rm sea}$	r_1/a	$1/a$ (GeV)
c007	2109	$20^3 \times 64$	0.007/0.05	2.625(3)	1.660(12)
c02	2052	$20^3 \times 64$	0.02/0.05	2.644(3)	1.665(12)
f0062	1910	$28^3 \times 96$	0.0062/0.031	3.699(3)	2.330(17)

ensemble	m_B (GeV)	m_{B_s} (GeV)	m_π (MeV)	m_K (MeV)	m_{η_s} (MeV)	m_ρ (MeV)	m_{K^*} (MeV)	m_ϕ (MeV)
c007	5.5439(32)	5.6233(7)	313.4(1)	563.1(1)	731.9(1)	892(28)	1045(6)	1142(3)
c02	5.5903(44)	5.6344(15)	519.2(1)	633.4(1)	730.6(1)	1050(7)	1106(4)	1162(3)
f0062	5.5785(22)	5.6629(13)	344.3(1)	589.3(2)	762.0(1)	971(7)	1035(4)	1134(2)
"physical"	5.279	5.366	140	495	686	775	892	1020

Operator matching

- Effective field theory, cutoff by lattice
- HQET power counting: requires working with low recoil
- Current matching

$$(\bar{q}\Gamma_\mu^{V,A} b)|_{\mathrm{cont}} \doteq (1+\alpha_s \rho^{(\mu)})(\bar{c}\Gamma_\mu^{V,A} b)|_{\mathrm{latt}}$$

$$(\bar{q}\hat{\sigma}_{\mu\nu} b)|_{\mathrm{cont}} \doteq (1+\alpha_s c^{(T\nu)})(\bar{q}\hat{\sigma}_{\mu\nu} b)|_{\mathrm{latt}}$$

ensemble	C_v	$\rho^{(0)}$	$\rho^{(k)}$	$c^{(T0)}$	$c^{(Tj)}$
c	2.825	0.043	0.270	0.076	0.076
f	1.996	−0.058	0.332	0.320	0.320

Gulez *et al.*, PRD69 (2003), PRD73 (2006); Mueller *et al.*, PRD83 (2011)

Operator matching

$$(\bar{q}\Gamma_\mu^{V,A} b)|_{\rm cont} \doteq (1+\alpha_s \rho^{(\mu)})(\bar{c}\Gamma_\mu^{V,A} b)|_{\rm latt} + J_\Gamma^{(1)\rm sub}$$

$$(\bar{q}\hat{\sigma}_{\mu\nu} b)|_{\rm cont} \doteq (1+\alpha_s c^{(T\nu)})(\bar{q}\hat{\sigma}_{\mu\nu} b)|_{\rm latt} + J_\Gamma^{(1)\rm sub}$$

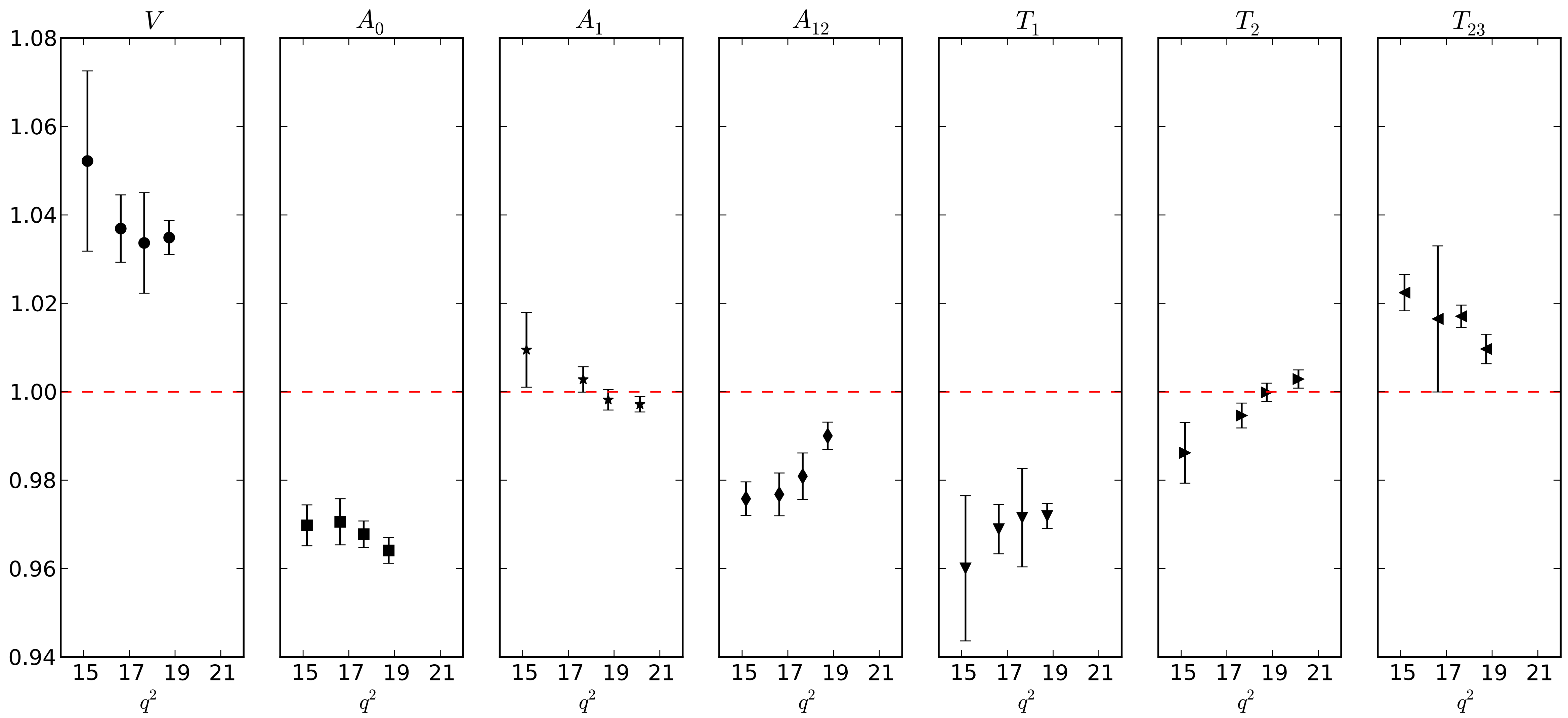
LO $J_\Gamma^{(0)}$ NLO

$$J_\Gamma^{(1)\rm sub} = J_\Gamma^{(1)} - \alpha_s \zeta_{10}^\Gamma J_\Gamma^{(0)}$$

Truncation errors $O(\alpha_s \Lambda_{\rm QCD}/m_b)$

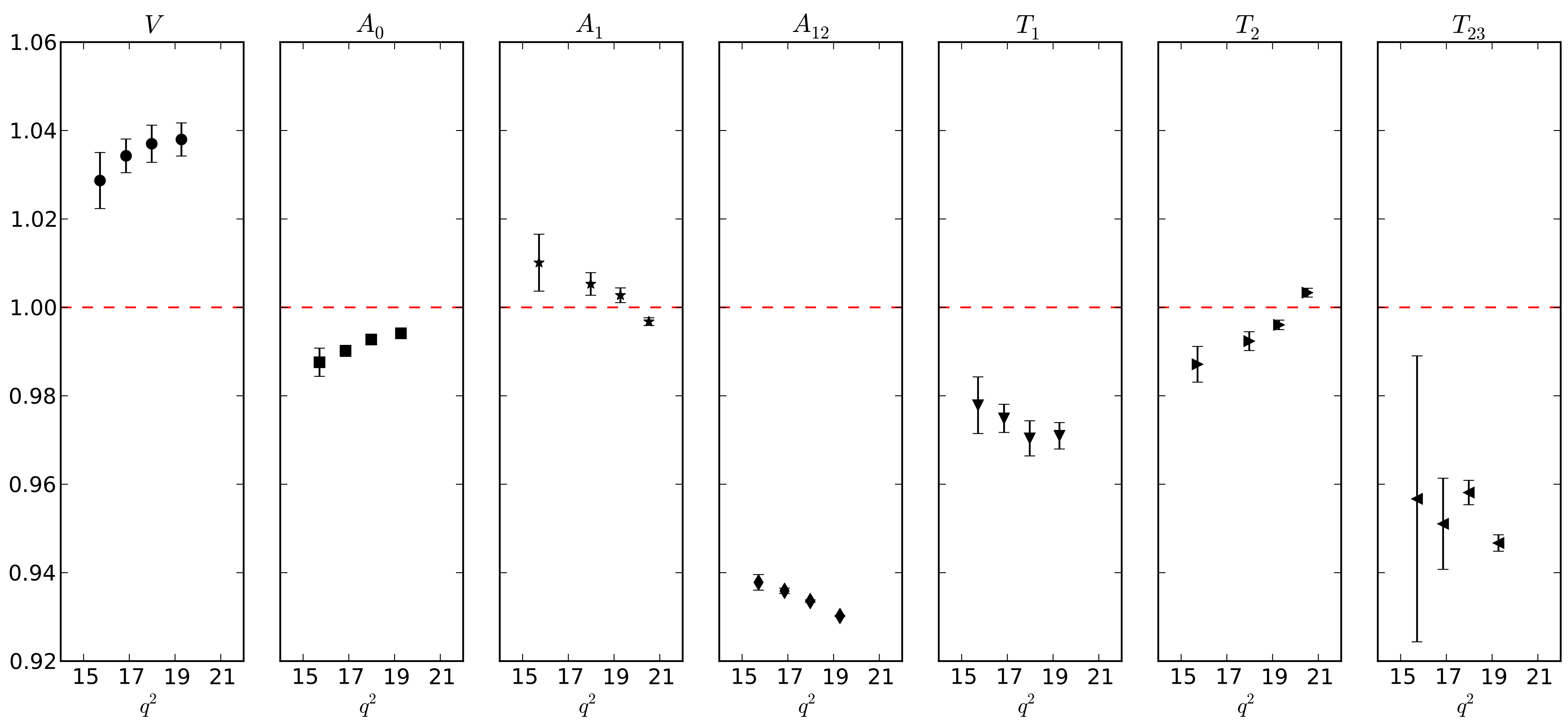
Gulez *et al.*, PRD69 (2003), PRD73 (2006); Mueller, private communication

Ratio: (LO+NLO)/LO



c007

Ratio: (LO+NLO)/LO



f0062

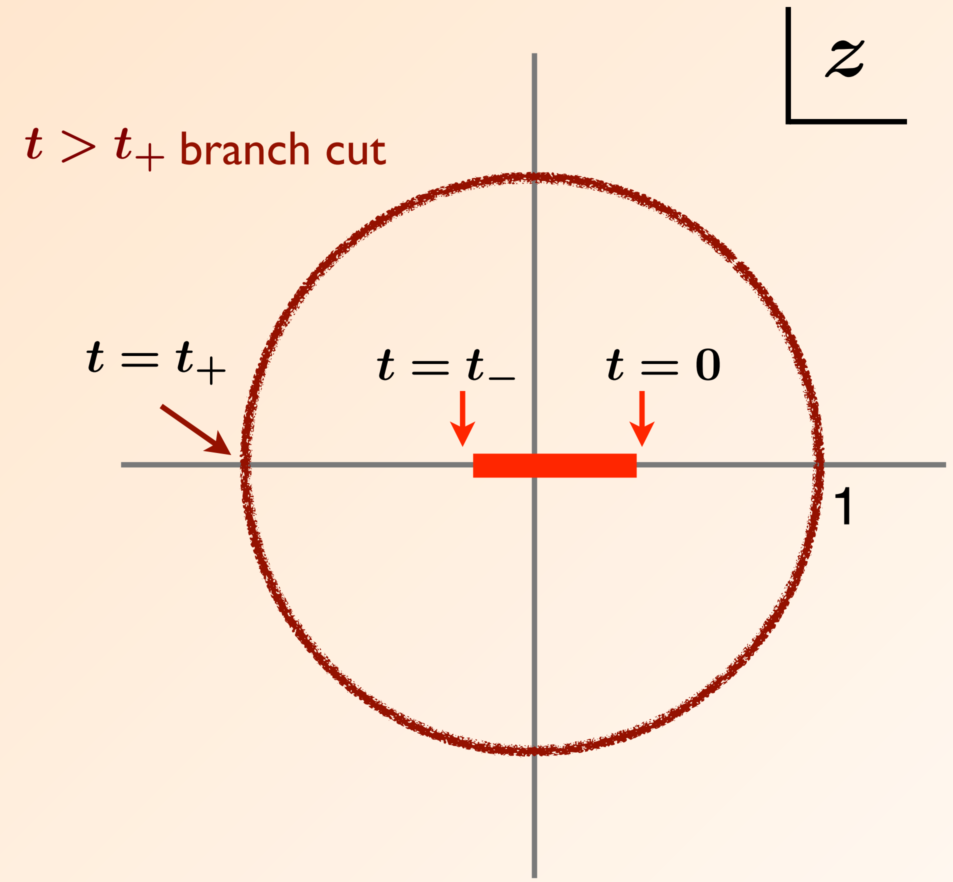
Form factor shape

Series (z) expansion

$$t = q^2 \qquad t_\pm = (m_B \pm m_F)^2$$

Choose, *e.g.* $t_0 = 12\ \mathrm{GeV}^2$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$



Simplified series expansion

$$F(t) = \frac{1}{1 - t/m^2_{\mathrm{res}}} \sum_n a_n z^n$$

Bourrely, Caprini, Lellouch PRD **79** (2009) following Okubo; Bourrely, Machet, de Rafael; Boyd, Grinstein, Lebed; Boyd & Savage; Arneson *et al.*; FNAL/MILC lattice collab; ...

Kinematic-continuum-mass fits

HPQCD

$$\equiv 1/P(t)$$

$$F(t) = \boxed{\frac{1}{1 - t/m_{\rm res}^2}} [1 + b_1 (aE_F)^2 + \ldots] \sum_n a_n d_n z^n$$

discretization errors

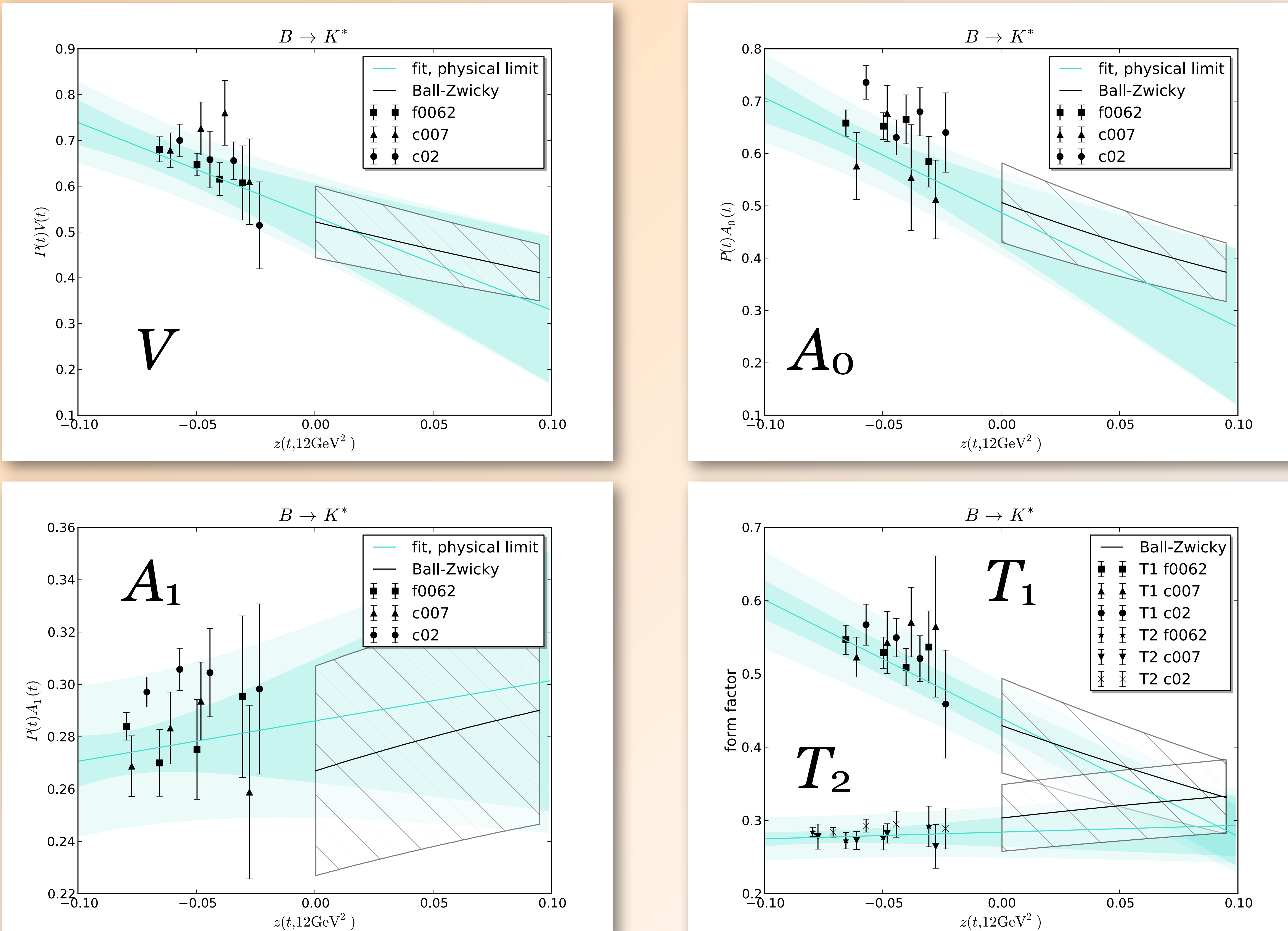
$$d_n = 1 + c_{n1} \frac{m_P^2}{(4\pi f)^2} + \ldots$$

quark mass dependence

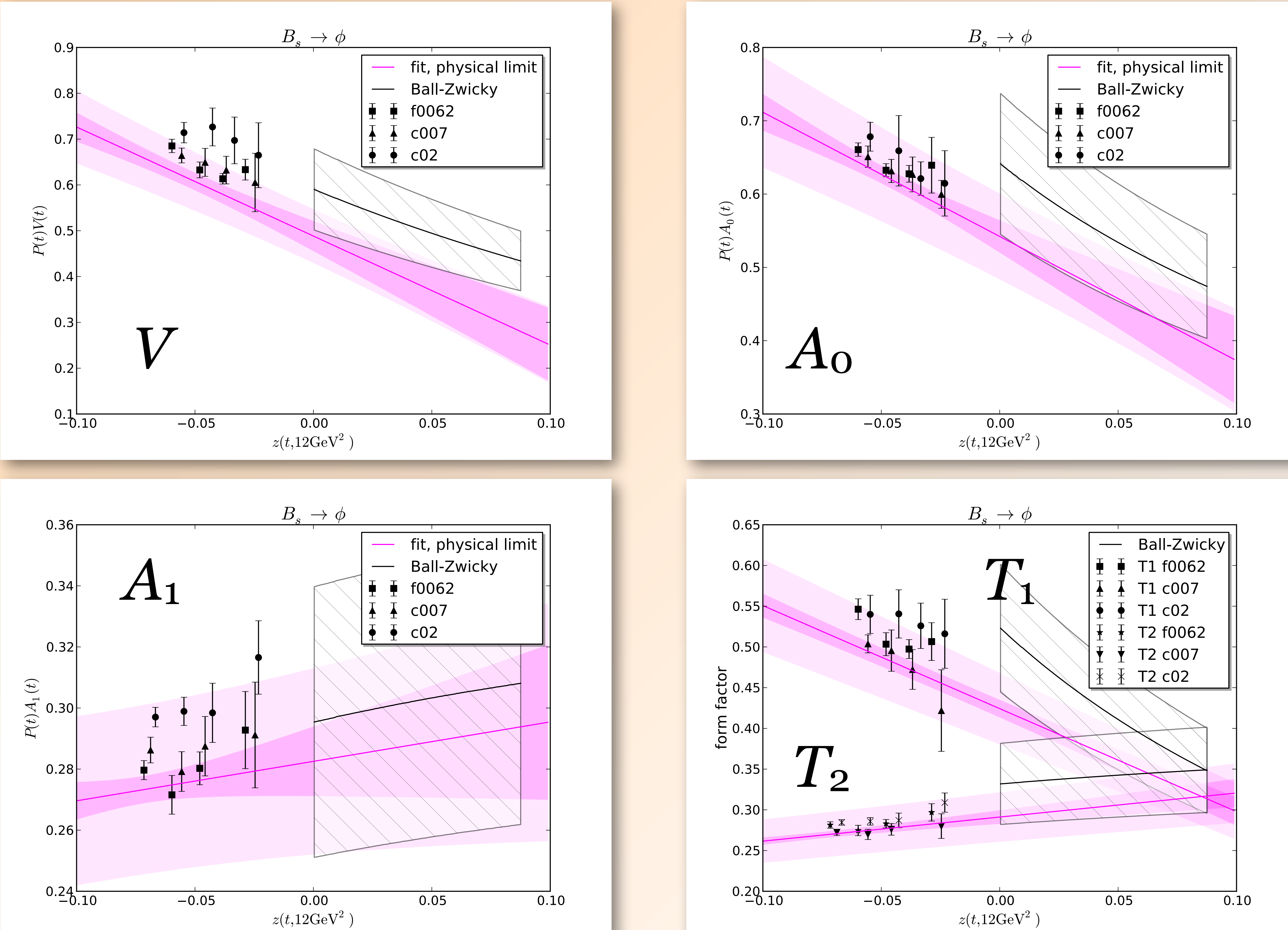
$$m_{\rm res} = m_{\rm comp} + \Delta m_{\rm phys}$$

In practice we need a 3 parameter fit: a_0, a_1, c_{01}

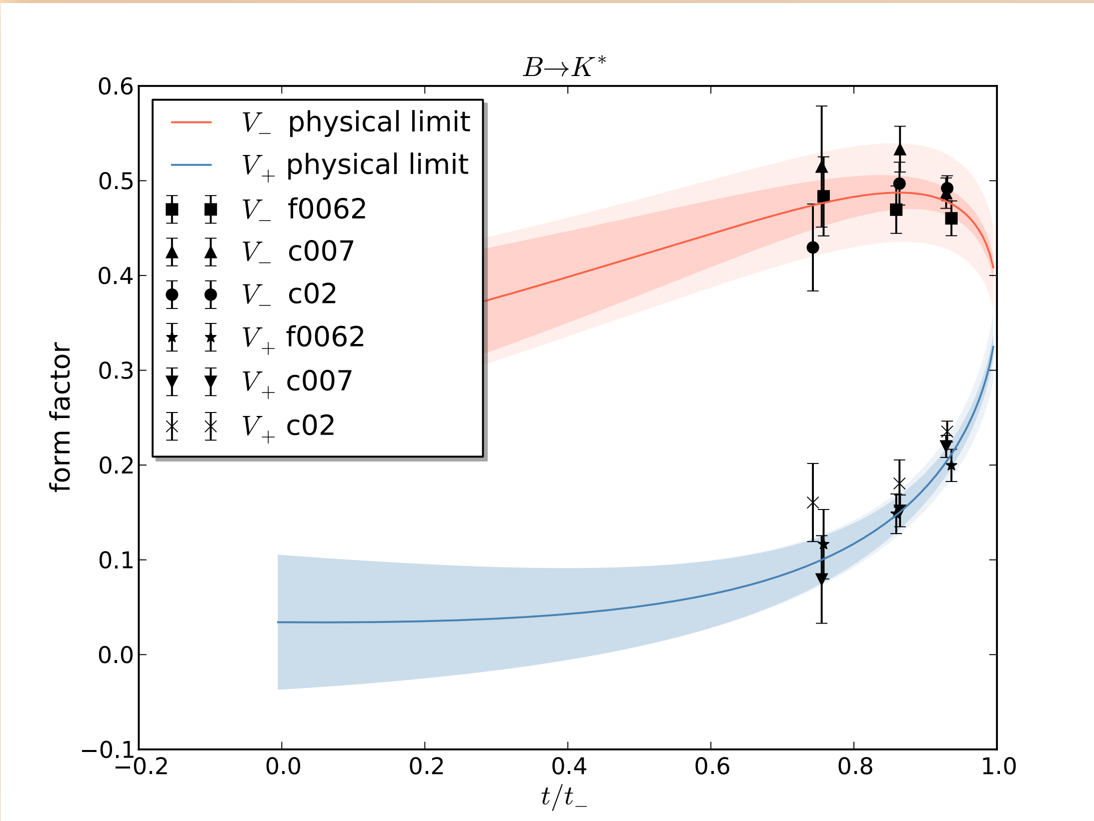
$B \to K^*$ form factors



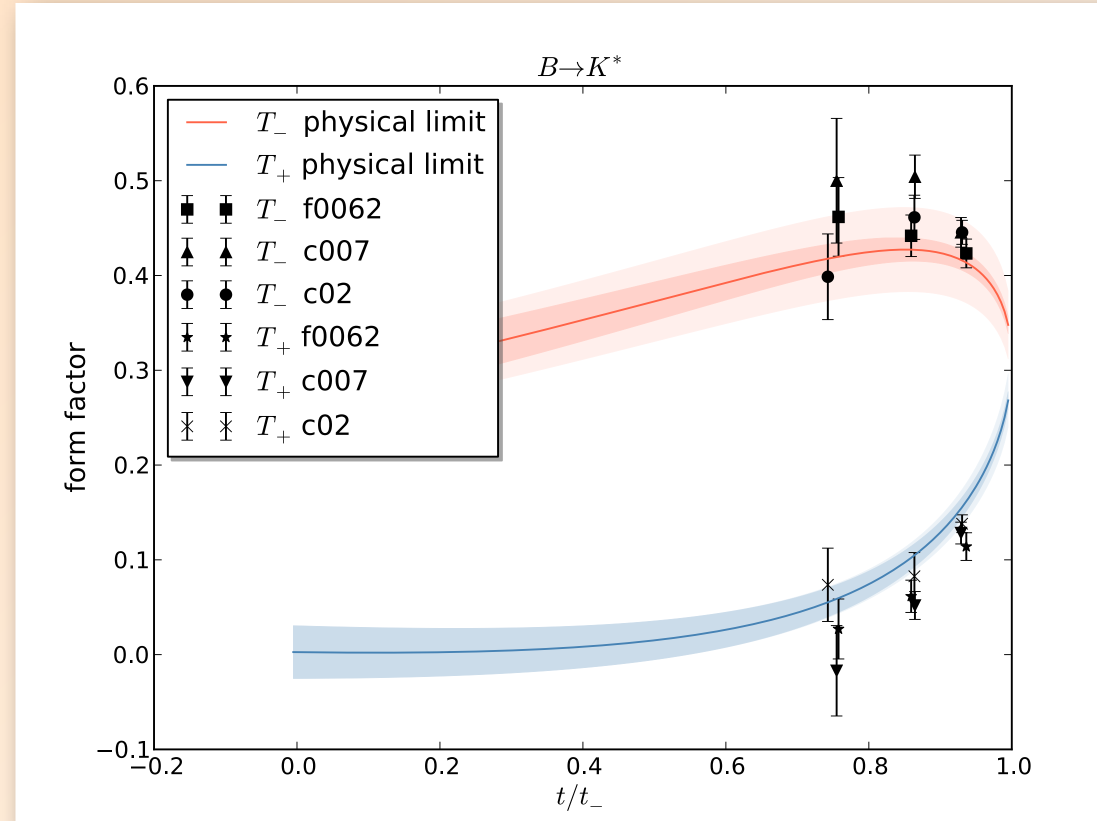
$B_s \to \varphi$ form factors



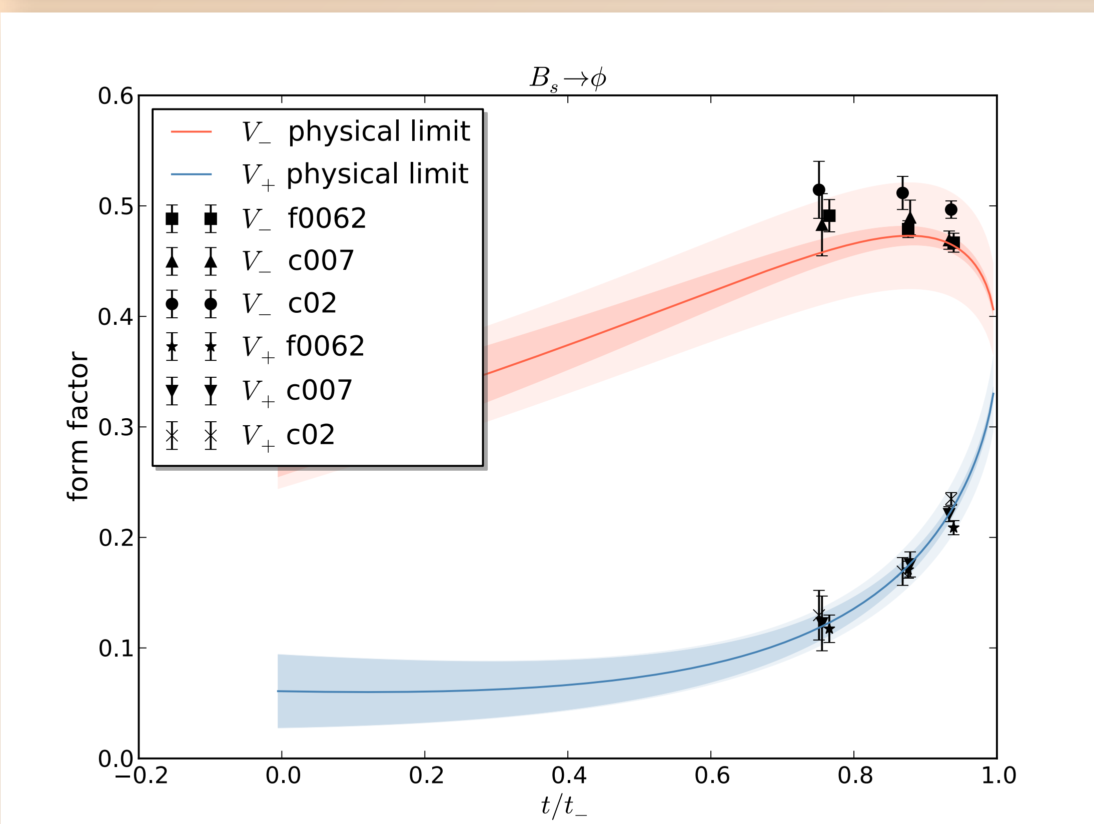
Helicity f.f.



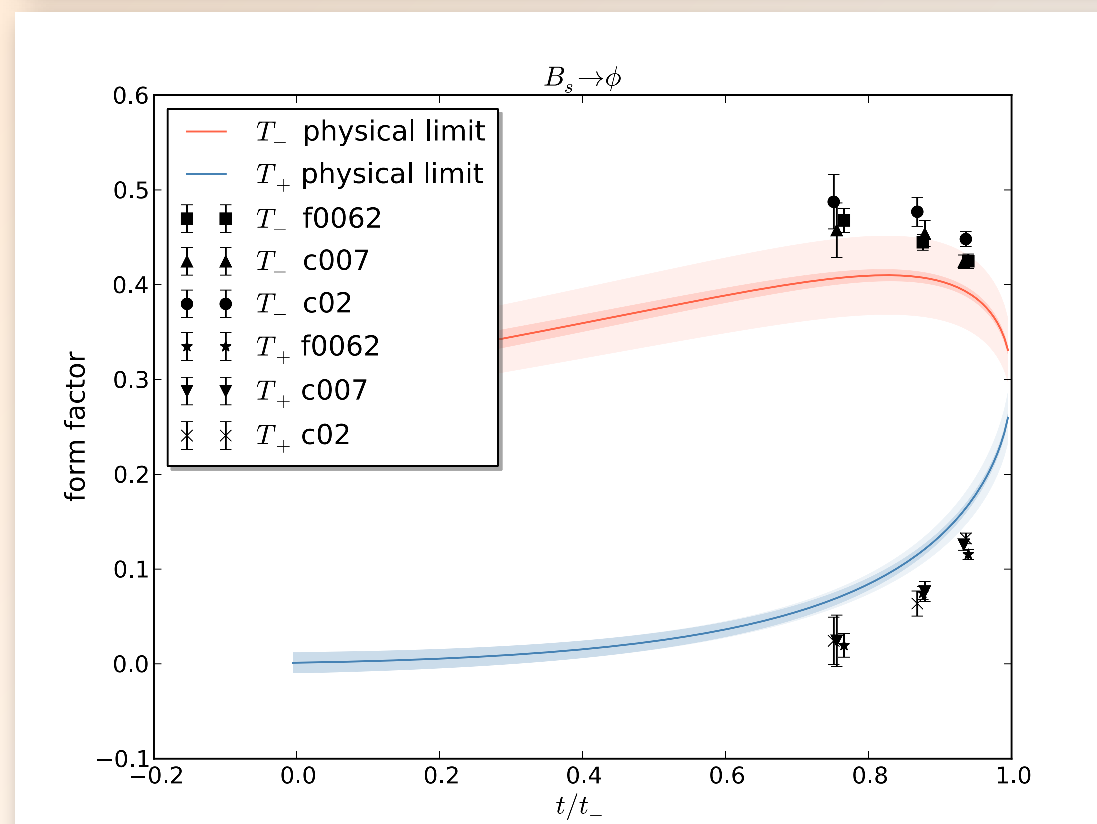
$B \to K^*$
V_- physical limit
V_+ physical limit
V_- f0062
V_- c007
V_- c02
V_+ f0062
V_+ c007
V_+ c02
form factor
t/t_-



$B \to K^*$
T_- physical limit
T_+ physical limit
T_- f0062
T_- c007
T_- c02
T_+ f0062
T_+ c007
T_+ c02
form factor
t/t_-



$B_s \to \phi$
V_- physical limit
V_+ physical limit
V_- f0062
V_- c007
V_- c02
V_+ f0062
V_+ c007
V_+ c02
form factor
t/t_-



$B_s \to \phi$
T_- physical limit
T_+ physical limit
T_- f0062
T_- c007
T_- c02
T_+ f0062
T_+ c007
T_+ c02
form factor
t/t_-

Status

- Results preliminary: final checks & estimates of systematic uncertainties underway
- Will publish form factors, along with SM predictions for observables using LQCD form factors

Caveat

- Lattice calculations done with kinematics such that K^* and φ are stable. Physical K^* is broad, φ less so.
- Not gold-plated. Is consistency between LQCD & LCSR reassuring?

$\Lambda_b \to \Lambda$ (& $\Lambda \to p$) *form factors*

with W Detmold, C-J D Lin, S Meinel

$\Lambda_b \to \Lambda$ form factors

In general

$$\langle \Lambda | \bar{s}\gamma^\mu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^V \gamma^\mu - f_2^V \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^V \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s}\gamma^\mu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^A \gamma^\mu - f_2^A \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^A \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TV} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TV} \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TA} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TA} \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

In the $m_b \to \infty$ limit

$$\langle \Lambda(p', s') | \, \bar{s}\Gamma Q \, | \Lambda_Q(v, 0, s) \rangle = \bar{u}(p', s') [F_1(p' \cdot v) + \not{v} F_2(p' \cdot v)] \Gamma \, \mathcal{U}(v, s)$$

$\Lambda_b \to \Lambda$ form factors

In general

$$\langle \Lambda | \bar{s}\gamma^\mu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^V \gamma^\mu - f_2^V \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^V \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s}\gamma^\mu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^A \gamma^\mu - f_2^A \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^A \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TV} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TV} \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TA} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TA} \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

In the $m_b \to \infty$ limit

$$\langle \Lambda(p', s') | \, \bar{s}\Gamma Q \, | \Lambda_Q(v, 0, s) \rangle = \bar{u}(p', s') [F_1(p' \cdot v) + \not{v} F_2(p' \cdot v)] \Gamma \mathcal{U}(v, s)$$

Lattice actions & parameters

RBC/UKQCD lattices (2+1 domain wall)

Static ($m_b = \infty$) heavy quarks

Set	$N_s^3 \times N_t \times N_5$	am_5	$am_s^{(sea)}$	$am_{u,d}^{(sea)}$	a (fm)	$am_s^{(val)}$	$am_{u,d}^{(val)}$	$m_\pi^{(vv)}$ (MeV)	$m_{\eta_s}^{(vv)}$ (MeV)	N_{meas}
C14	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.04	0.001	245(4)	761(12)	2705
C24	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.04	0.002	270(4)	761(12)	2683
C54	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.04	0.005	336(5)	761(12)	2780
C53	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.03	0.005	336(5)	665(10)	1192
F23	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.03	0.002	227(3)	747(10)	1918
F43	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.03	0.004	295(4)	747(10)	1919
F63	$32^3 \times 64 \times 16$	1.8	0.03	0.006	0.0848(17)	0.03	0.006	352(7)	749(14)	2785

1-loop operator matching: T Ishikawa *et al.*, JHEP 1105, 040 (2011)

Form factor shape

- In static limit, *z*-expansion is not applicable
- Instead, try monopole, dipole, etc. (Latter is a better fit to the data)
- Incorporate discretization and quark mass effects

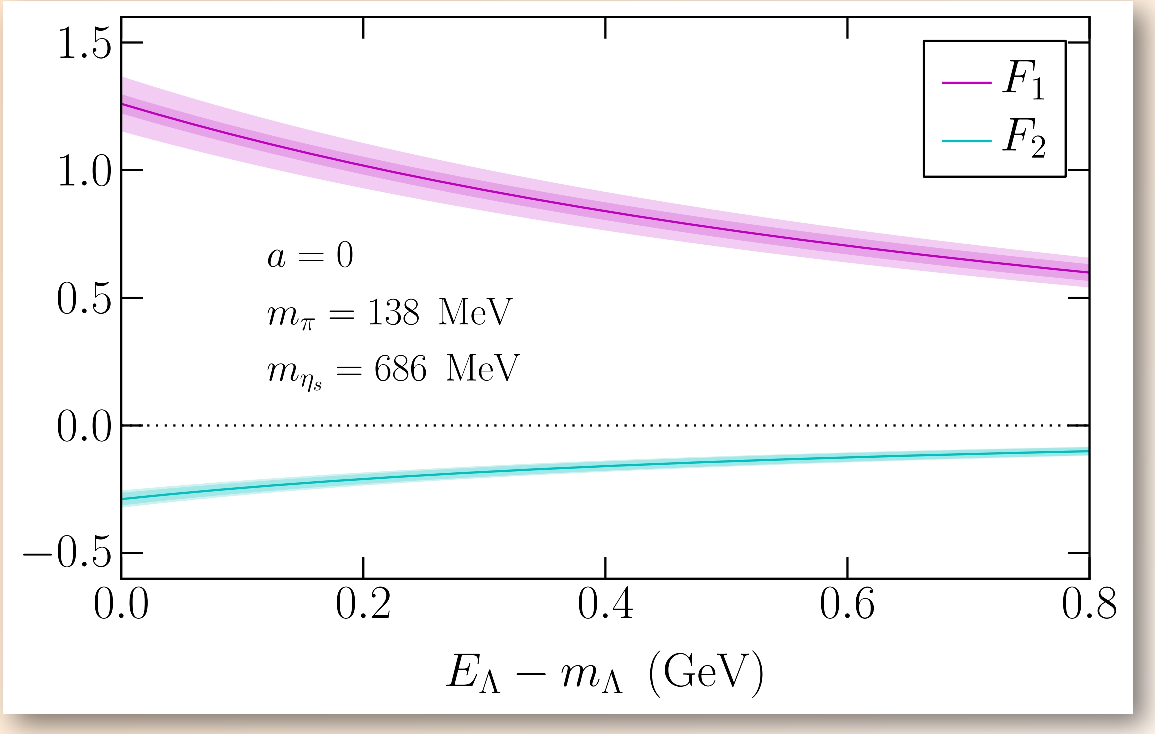
$$F = \frac{N}{(X + E_\Lambda - m_\Lambda)^2}\left[1 + d(aE_\Lambda)^2\right]$$

$$X = X_0 + c_\ell[m_\pi^2 - (m_\pi^{\rm phys})^2] + c_s[m_{\eta_s}^2 - (m_{\eta_s}^{\rm phys})^2]$$

- In practice, *c*'s & *d*'s small, consistent with zero [except $c_{l,+} = 0.094(32)$]

$\Lambda_b \to \Lambda\, l^+ l^-$

(using Static+DWF on n_f=2+1 RBC-UKQCD)

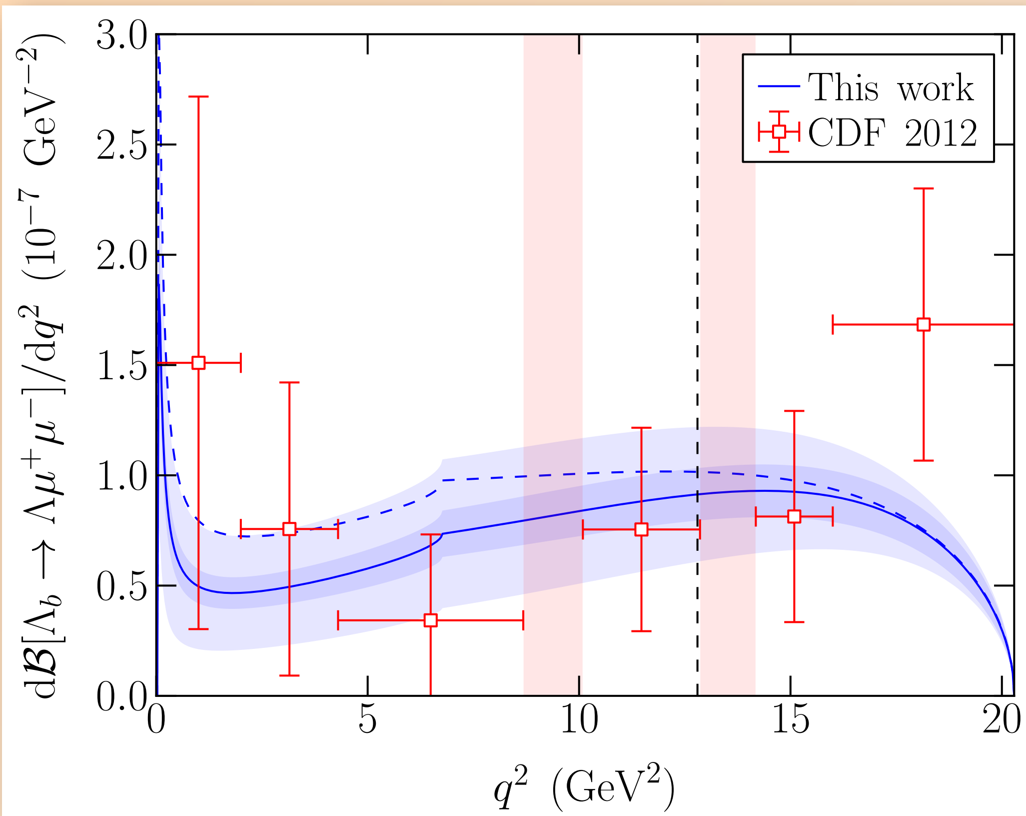


In the static limit, 10 form factors reduce to 2

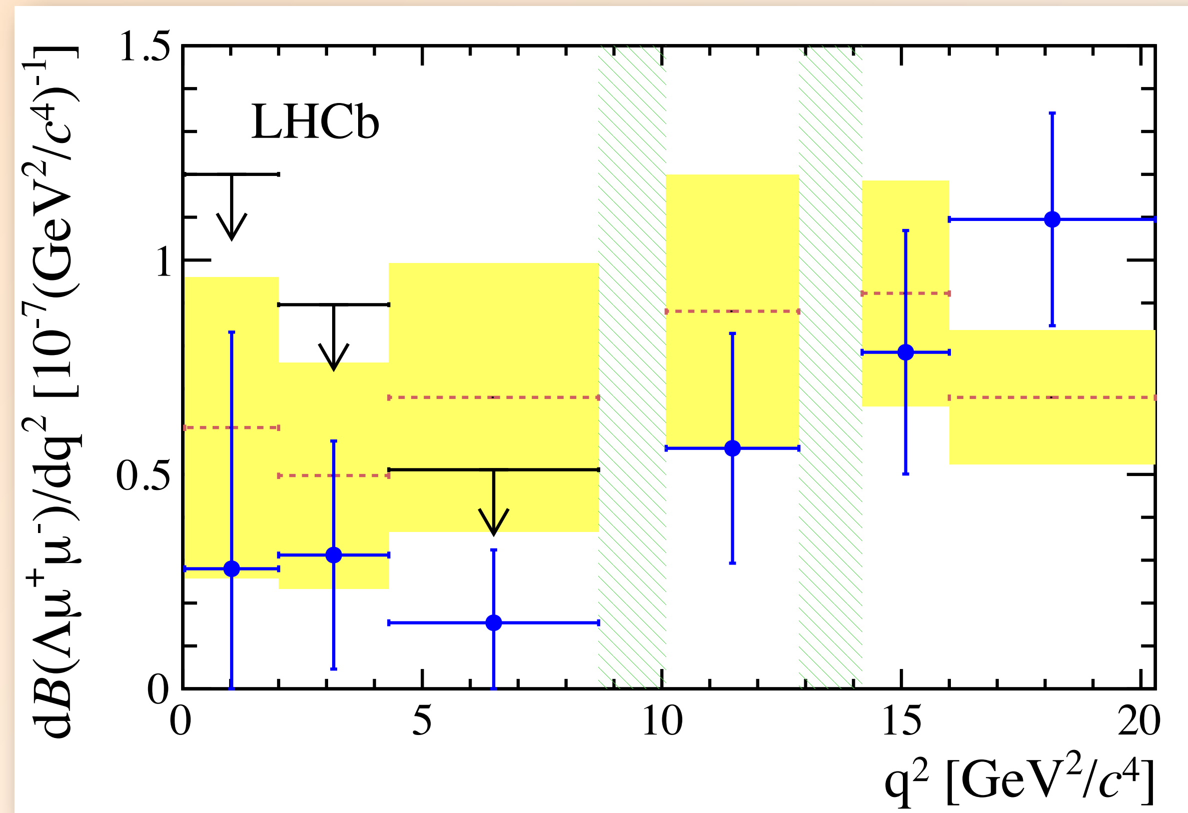
$$\langle \Lambda(p',s')|\,\bar{s}\Gamma Q\,|\Lambda_Q(v,0,s)\rangle = \bar{u}(p',s')[F_1(p'\cdot v) + \not{v} F_2(p'\cdot v)]\Gamma\,\mathcal{U}(v,s)$$

W Detmold *et al.*, Phys Rev D87 (2013)

$\Lambda_b \to \Lambda\, l^+ l^-$



CDF: red; LQCD: blue



LHCb: blue; binned LQCD: red/yellow

W Detmold *et al.*, Phys Rev D87 (2013)
CDF, public note 108xx, v0.1, http://www-cdf.fnal.gov/physics/new/bottom/bottom.html
LHC*b*, R Aaij, arXiv:1306.2577

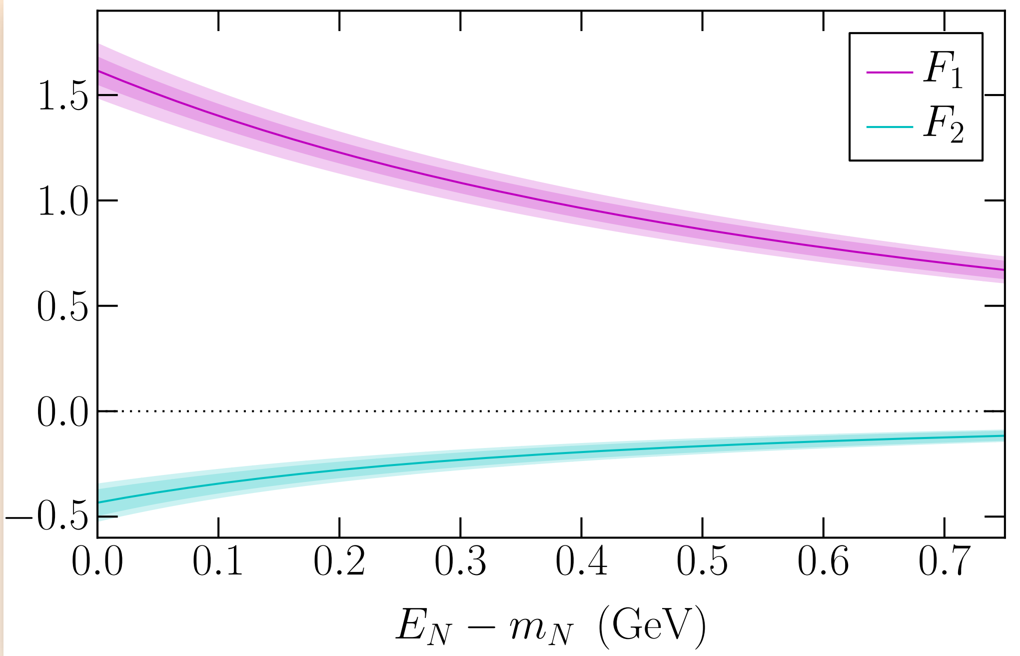
$\Lambda_b \to p\, l^- \nu$

In the static limit, 10 form factors reduce to 2

$$\langle p(p',s')|\,\bar{s}\Gamma Q\,|\Lambda_Q(v,0,s)\rangle = \bar{u}(p',s')[F_1(p'\cdot v) + \not{v} F_2(p'\cdot v)]\Gamma\,\mathcal{U}(v,s)$$

(using Static+DWF on n_f=2+1 RBC-UKQCD)

With expt data, could lead to $|V_{ub}|$with 15% theory error



$$\frac{1}{|V_{ub}|^2}\int_{14\ \mathrm{GeV}^2}^{q^2_{\max}} \frac{\mathrm{d}\Gamma(\Lambda_b \to p\,\ell^-\bar{\nu}_\ell)}{\mathrm{d}q^2}\mathrm{d}q^2 = \begin{cases} 15.3 \pm 2.4 \pm 3.4\ \ \mathrm{ps}^{-1} & \text{for} \quad \ell = e, \\ 15.3 \pm 2.4 \pm 3.4\ \ \mathrm{ps}^{-1} & \text{for} \quad \ell = \mu, \\ 12.5 \pm 1.9 \pm 2.7\ \ \mathrm{ps}^{-1} & \text{for} \quad \ell = \tau. \end{cases}$$

$\Lambda_b \to p\, l\, \nu$: shed light on $|V_{ub}|$ from $B \to X\, l\, \nu$ vs. $B \to \pi\, l\, \nu$???

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Summary

- First unquenched LQCD calculations of $b \to s$ form factors
- These reduce uncertainties in f.f., especially at large q^2
- Pseudoscalar mesons: Precise short distance, LHC*b* observation of resonance
- Vector mesons: complement sum rule calculations, many observables, final results soon
- Baryons: systematically improvable uncertainties, hint of high q^2 resonance