Comprehensive Analysis of $B \to K^* \ell^+ \ell^-$ and further $|\Delta B| = |\Delta S| = 1$ Decays

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Theor. Physik 1

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DFG FOR 1873

Effective Field Theory for $b \to s\ell^+\ell^-$ FCNCs

Flavor Changing Neutral Current (FCNC)

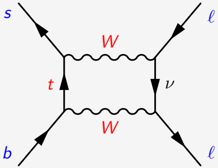
- expand amplitudes in $G_\mathrm{F} \sim 1/M_W^2$ (OPE)
- operators (matrix elem. below $\mu_b \simeq m_b$)

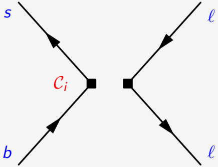
$$\mathcal{O}_i \equiv \left[\bar{s}\Gamma_i b\right]\left[\bar{\ell}\Gamma'_i \ell\right]$$

- Wilson coefficients (above $\mu_b \simeq m_b$)

$$\mathcal{C}_i \equiv \mathcal{C}_i(M_W, M_Z, m_t, \dots)$$

- use $\mathcal{C}_i = \mathcal{C}_i(\mu_b = 4.2\mathrm{GeV})$





Effective Hamiltonian

$$\mathcal{H} = -\frac{4G_\mathrm{F}}{\sqrt{2}}\frac{\alpha_e}{4\pi}\left[V_{tb}V_{ts}^*\sum_i \mathcal{C}_i\mathcal{O}_i + O\left(V_{ub}V_{us}^*\right)\right] + \mathrm{h.c.}$$

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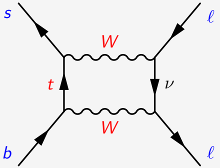

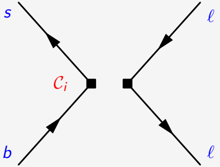

Decay Modes

$B \to K^*\ell^+\ell^-$	$B_s \to \mu^+\mu^-$	$B \to K\ell^+\ell^-$
$B \to K^*\gamma$	$B \to X_s\ell^+\ell^-$	$B \to X_s\gamma$

Model Independent Framework

Basis of Operators $\mathcal{O}_i$

- include as many $\mathcal{O}_i$ beyond SM as needed/as few as possible
- balancing act, test statistically if choice of basis describes data well!

Wilson Coefficients $\mathcal{C}_i$

- treat $\mathcal{C}_i$ as uncorrelated, generalized couplings
- constrain their values from data
- confront new physics models with constraints

Model Independent Framework

Operators/Wilson Coefficients

SM:

$$\mathcal{O}_7 = \frac{m_b}{e}[\bar{s}\sigma_{\mu\nu}P_R b]F^{\mu\nu} \qquad \mathcal{O}_{9(10)} = [\bar{s}\gamma_\mu P_L b]\,[\bar{\ell}\gamma^\mu(\gamma_5)\ell]$$

chirality flipped (beyond SM)

$$\mathcal{O}_{7'} = \frac{m_b}{e}[\bar{s}\sigma_{\mu\nu}P_L b]F^{\mu\nu} \qquad \mathcal{O}_{9'(10')} = [\bar{s}\gamma_\mu P_R b]\,[\bar{\ell}\gamma^\mu(\gamma_5)\ell]$$

Model Independent Framework

Operators/Wilson Coefficients

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Scenario — only real-valued SM-like $\mathcal{C}_i \Rightarrow$ no BSM CPV

fit $\mathcal{C}_7$, $\mathcal{C}_9$, $\mathcal{C}_{10}$ and

- hadronic parameters
- CKM Wolfenstein parameters
- quark masses

$\mathcal{C}_1, \ldots \mathcal{C}_6, \mathcal{C}_8$ as in the SM

Sensitivity to Fit Parameters

Wilson Coefficients

	$\mathcal{C}_7$	$\mathcal{C}_9$	$\mathcal{C}_{10}$	
$B_s \to \mu^+\mu^-$	–	–	✓	
$B \to X_s\gamma$	✓	–	–	
$B \to X_s\ell^+\ell^-$	✓	✓	✓	
$B \to K^*\gamma$	✓	–	–	
$B \to K^*\ell^+\ell^-$	✓	✓	✓	12 CP-avg. angular observables
$B \to K\ell^+\ell^-$	✓	✓	✓	3 CP-avg. angular observables

Form Factors

- interplay between $B \to X_s\{\gamma, \ell^+\ell^-\}$ and $B \to K^*\{\gamma, \ell^+\ell^-\}$
- some $B \to K^*\ell^+\ell^-$ obs. form-factor insensitive by construction
- some $B \to K^*\ell^+\ell^-$ obs. dominantly sensitive to form factor ratios

$B \to K^*\ell^+\ell^-$ $q^2 \in [1,6]\,\mathbf{GeV}^2$, $q^2 \geq M^2_{\psi'}$

- $\mathcal{B}$, A_{FB}, F_{L}, A_{T}^2 (S_3)
- **new**: A_{T}^{re}, P'_4, P'_5, P'_6
- ATLAS, BaBar, Belle, CDF, CMS, LHCb

$B \to K^*\gamma$

- $\mathcal{B}$, $S_{K^*\gamma}$, $C_{K^*\gamma}$
- BaBar, Belle, CLEO

$B \to K\ell^+\ell^-$ $q^2 \in [1,6]\,\mathbf{GeV}^2$, $q^2 \geq M^2_{\psi'}$

- $\mathcal{B}$
- BaBar, Belle, CDF, LHCb

$B \to X_s\gamma$ $E^{\gamma}_{\text{min}} = 1.8\,\mathbf{GeV}$

- $\mathcal{B}$
- BaBar, Belle, CLEO

$B_s \to \mu^+\mu^-$

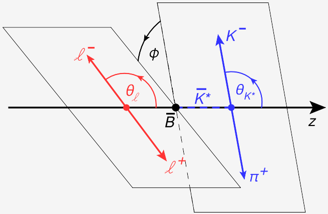
- time-int. $\mathcal{B}$
- CMS, LHCb

$B \to X_s\ell^+\ell^-$ $q^2 \in [1,6]\,\mathbf{GeV}^2$

- $\mathcal{B}$
- BaBar, Belle

(Angular) Observables in $B \to K^* \ell^+ \ell^-$

- kinematics
  - dilepton mass squared q^2
  - three angles
- complicated diff. decay width
  - 12(+) angular observables J_n
  - compose observ. from J_n with specific benefits
  - express observables through J_n



Definitions

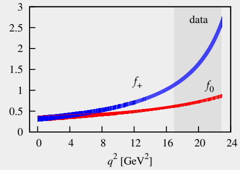
$$\Gamma \sim 3J_{1c} + 6J_{1s} J_{2c} 2J_{2s} \qquad A_{\mathrm{FB}} \sim \frac{J_{6s}}{\Gamma} \qquad F_L \sim \frac{3J_{1c} - J_{2c}}{\Gamma}$$

$$P'_4 \sim \frac{+J_4}{\sqrt{-J_{2s} J_{2c}}} \qquad P'_5 \sim \frac{+J_5}{2\sqrt{-J_{2s} J_{2c}}} \qquad P'_6 \sim \frac{-J_7}{2\sqrt{-J_{2s} J_{2c}}}$$

Further Theory Constraints

Form Factors from Lattice QCD (LQCD) [HPQCD arxiv:1306.2384]

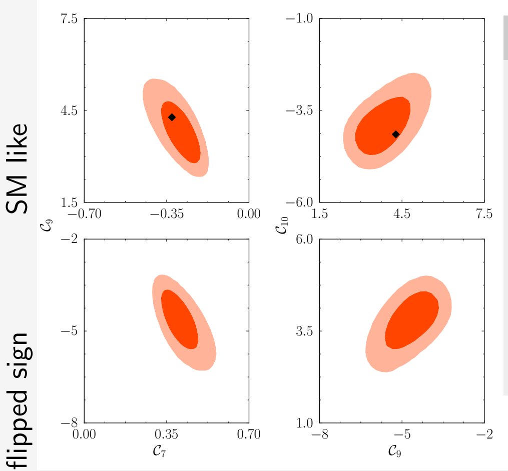
- $B \to K$ form factors available from LQCD
  - data only at high q^2: $17 - 23\,\mathrm{GeV}^2$
  - no data points given
- reproduce 3 data points from z-parametrization
  - $q^2 = 17\,\mathrm{GeV}^2$, $20\,\mathrm{GeV}^2$, $23\,\mathrm{GeV}^2$
  - use as constraint, incl. covariance matrix



$B \to K^*$ Form Factor (FF) Relation at $q^2 = 0$

- FF $V, A_1 \propto \xi_\perp + \ldots$ [Charles et al. hep-ph/9901378]
  - no α_s corrections [Burdmann/Hiller hep-ph/0011266, Beneke/Feldmann hep-ph/0008255]
  - Large Energy Limit: $V(0) = (1.33 \pm 0.4) \times A_1(0)$
- LCSR constraint: $\xi_\parallel(0) = 0.10^{+0.03}_{-0.02}$, to avoid $\xi_\parallel(q^2) \propto A_0(q^2) < 0$.
- see also FF fits by [Hambrock/Hiller/Schacht/Zwicky 1308.4379]

Results (SM Basis)



early 2012

- figure from [1205.1838]
- no $B \to X_s\{\gamma, \ell^+\ell^-\}$
- only LHCb bound on $B_s \to \mu^+\mu^-$
- $B \to K^{(*)}\ell^+\ell^-$: $\mathcal{B}, A_{\rm FB}, F_L, A_T^{(2)}, S_3$
- $B \to K^*\gamma$: $\mathcal{B}, S_{K^*\gamma}, C_{K^*\gamma}$

♦: Standard Model

The $B \to K^* \ell^+ \ell^-$ "Anomaly"

- deviation from SM prediction in form factor-free obs. P'_5 (LHCb)

  [LHCb 1308.1707]

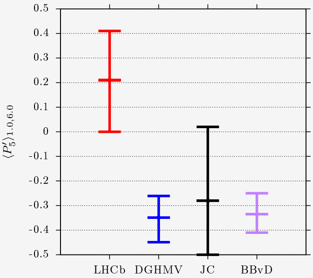
- LHCb uses one SM prediction (DGHMV)

  [Descotes-Genon/Hurth/Matias/Virto 1303.5794]

- however: further SM prediction exist much larger uncertainty (JC)

  [Jäger/Camalich 1212.2263]

- our take on SM prediction for $P'_{4,5,6}$ (BBvD, see also following slide)



difference: treatment of unknown power corrections
(form factor corrections, $\bar{c}c$ resonances)

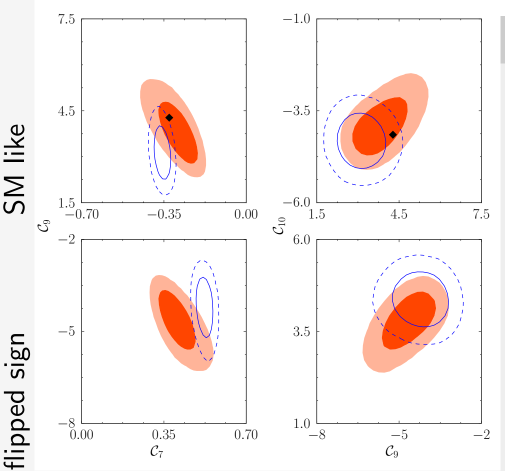
Standard Model Predictions for $P'_{4,5,6}$

- toy Monte Carlo using priors + theory constraints (FFs)
- calculate observable for 10^5 samples
- find minimal 68% CL intervals

	q^2 [GeV^2]	$\langle P'_4 \rangle$	$\langle P'_5 \rangle$	$10^2 \times \langle P'_6 \rangle$
BBvD	$[1, 6]$	$+0.46^{+0.12}_{-0.11}$	$-0.335^{+0.085}_{-0.075}$	-6.4 ± 1.7
	$[2, 4.3]$	$+0.48^{+0.11}_{-0.10}$	$-0.315^{+0.074}_{-0.090}$	$-7.2^{+1.5}_{-2.2}$
LHCb†	$[1, 6]$	$+0.58^{+0.33}_{-0.36}$	$+0.21^{+0.20}_{-0.21}$	$+18 \pm 21$
	$[2, 4.3]$	$+0.74^{+0.11}_{-0.53}$	$+0.29^{+0.40}_{-0.39}$	$+15^{+38}_{-36}$

†: [LHCb 1308.1707], adjusted to theory convention

Results (SM Basis) Preliminary!



♦: Standard Model, (light-) red: 68% CL (95% CL) for early 2012

solid (dashed): 68% CL (95% CL) for post HEP'13 (selection)

post HEP'13 (selection)

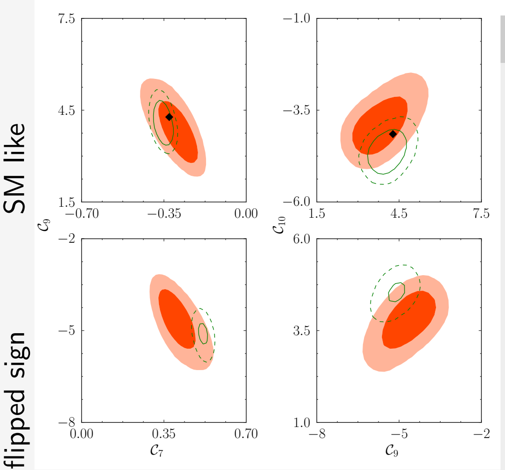
- with $B \to X_s\{\gamma, \ell^+\ell^-\}$
- $B_s \to \mu^+\mu^-$ from LHCb and CMS
- same data as

  [Descotes-Genon/Matias/Virto 1307.5683]

  exclusive decays limited:
  - only $B \to K^*\ell^+\ell^-$!
  - only LHCb data!
  - only $q^2 \in [1, 6]\text{GeV}^2$
- best-fit point similar to

  [Descotes-Genon et al. 1307.5683]
  - less tension, only $\lesssim 2\sigma$
  - $\mathcal{C}_9 - \mathcal{C}_9^{\text{SM}} \simeq -1.2$

Results (SM Basis) Preliminary!



◆: Standard Model, (light-) red: 68% CL (95% CL) for early 2012

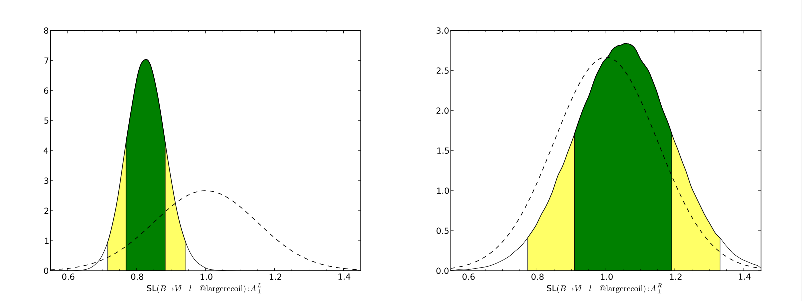
solid (dashed): 68% CL (95% CL) for post HEP'13 (all data)

post HEP'13 (all data)

- SM-like uncertainty reduced by ~ 2 compared to 2012
- SM at the border of 1σ
- flipped-sign barely allowed at 1σ
- good agreement with SM ($\simeq 1\sigma$)
- cannot confirm NP findings
  - in $(\mathcal{C}_7, \mathcal{C}_9)$
    [Descotes-Genon et al. 1307.5683]
  - in $(\mathcal{C}_9, \mathcal{C}_{9'})$
    [Altmannshofer/Straub 1308.1501]

Effects of the Power Corrections

- tension diluted by parameters for unknown power corrections
- effects in parameters for $B \to K^* \ell^+ \ell^-$ power corr. @ large recoil
  - shift by -20% for amplitude $A_\perp^L$
  - shift by 7% for amplitude $A_\perp^R$
- improvement in treating power corrections desirable



dashed: prior, solid: posterior, green: 68% CL, yellow: 95% CL

Goodness of Fit

Pull Values at Best-Fit Point

- largest pulls

  -3.4σ F_L, [1, 6], BaBar 2012

  -2.6σ F_L, [1, 6], ATLAS 2013

  -2.4σ P'_4, [14.18,16], LHCb 2013

  $+2.5\sigma$ $\mathcal{B}$, [16,19.21], Belle 2009

  $+2.2\sigma$ A_{FB}, [16,19], ATLAS 2013
- rest below 2σ, including the anomaly

  $+1.4\sigma$ $\langle P'_5 \rangle$, [1, 6], LHCb 2013

p Values

- pull-based p value at SM-like mode decreased
  - p value early 2012: 0.75
  - p value post HEP'13 (all): 0.15
- still a decent fit
- p value for $\mathcal{C}_{7,9,10} = \mathcal{C}_{7,9,10}^{\mathrm{SM}}$: 0.16

Numeric Results Preliminary

		$\mathcal{C}_7$	$\mathcal{C}_9$	$\mathcal{C}_{10}$
2012	68 %	$[-0.34, -0.23] \cup [0.35, 0.45]$	$[-5.2, -4.0] \cup [3.1, 4.4]$	$[-4.4, -3.4] \cup [3.3, 4.3]$
	95 %	$[-0.41, -0.19] \cup [0.31, 0.52]$	$[-5.9, -3.5] \cup [2.6, 5.2]$	$[-4.8, -2.8] \cup [2.7, 4.7]$
	modes	$\{-0.28\} \cup \{0.40\}$	$\{-4.56\} \cup \{3.64\}$	$\{-3.92\} \cup \{3.86\}$
post HEP '13	68 %	$[-0.38, -0.32] \cup \emptyset^\dagger$	$\emptyset^\dagger \cup [3.5, 4.6]$	$[-5.1, -4.1] \cup \emptyset$
	95 %	$[-0.40, -0.30] \cup [0.47, 0.55]$	$[-5.7, -4.5] \cup [3.3, 4.9]$	$[-5.2, -3.9] \cup [4.0, 5.0]$
	modes	$\{-0.350\} \cup \{0.513\}$	$\{-4.99\} \cup \{4.01\}$	$\{-4.61\} \cup \{4.52\}$
SM	central	$\{-0.327\} \cup \emptyset$	$\emptyset \cup \{4.28\}$	$\{-4.15\} \cup \emptyset$

†: the marginalized 1D distributions barely exclude the flipped-sign solutions @ 68%CL.

post HEP'13

- flipped-sign solution drastically smaller than in previous analysis
- posterior mass ratios SM-like : flipped-sign solutions 77% : 23%

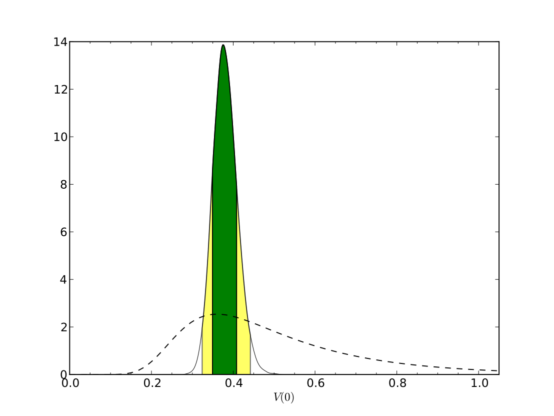
Model Comparison

- full fit with free floating $\mathcal{C}_{7,9,10}$: posterior $P(\mathcal{C}_{7,9,10}, \nu|D)$
- repeat fit with $\mathcal{C}_{7,9,10}$ set to SM values: posterior $P(\mathcal{C}^{\mathrm{SM}}_{7,9,10}, \nu|D)$
- ν: nuisance parameter (CKM, quark masses, hadronic quantities)
- Bayes factor B compares models
  - ratio of posterior masses

$$B = \frac{P(D|\mathcal{C}^{\mathrm{SM}}_{7,9,10}, \nu)}{P(D|\mathcal{C}_{7,9,10}, \nu)} = \frac{(2.04 \pm 0.005)}{(7.04 \pm 0.01)} \times 10^5 = 2.76 \times 10^4$$

  - specific prior volume: $P(\mathcal{C}_{7,9,10}) = 3.6 \times 10^3$
  - $B/P(\mathcal{C}_{7,9,10}) = 27.6/3.6 = 7.67$
  - Occam's Razor: SM explains data more economically than $\mathcal{C}_{7,9,10} \neq \mathcal{C}^{\mathrm{SM}}_{7,9,10}$ by one order of magnitude

Results for $B \to K^*$ Form Factors



dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]

solid: posterior, green: 68% CL, yellow: 95% CL

- more precise than prior
- $B \to K^*$: $\xi_\perp$ from
  - $B \to X_s\gamma$
  - $B \to K^*\gamma$
  - $B \to K^*\ell^+\ell^-$
  - theory input
- results @ 68% CL
  - $V(0) = 0.37^{+0.03}_{-0.02}$

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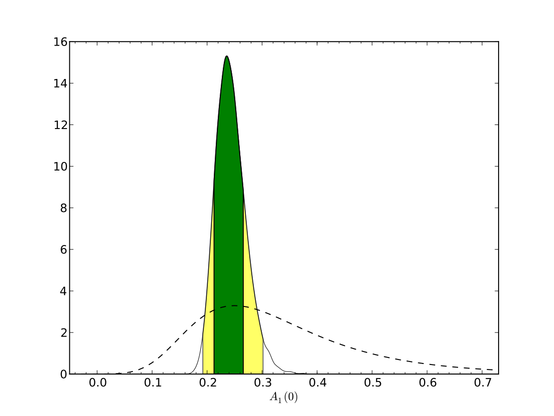

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  - $V(0) = 0.37^{+0.03}_{-0.02}$
  - $A_1(0) = 0.24 \pm 0.03$

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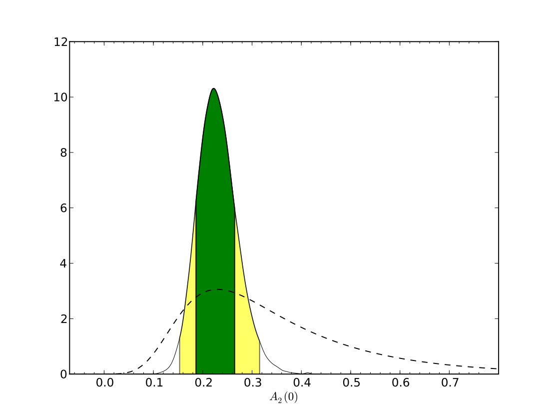

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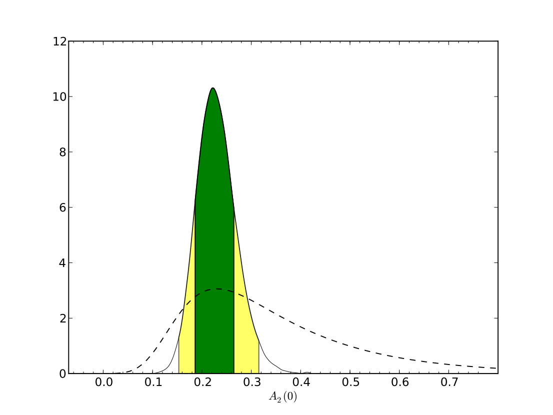
  - $A_2(0) = 0.22 \pm 0.04$

Results for $B \to K^*$ Form Factors



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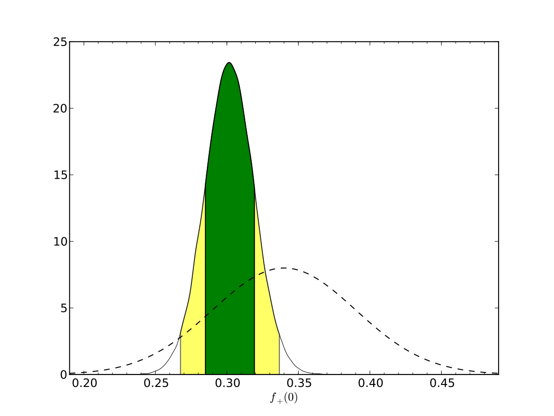
  - $B \to X_s\gamma$
  - $B \to K^*\gamma$
  - $B \to K^*\ell^+\ell^-$
  - theory input
- results @ 68% CL
  - ratio of central values

$$V(0)/A_1(0) \simeq 1.5$$

$$A_2(0)/A_1(0) \simeq 0.9$$

agree w/ (SE2 full) [Hambrock/Hiller/Schacht/Zwicky 1308.4379]

Results for $B \to K$ Form Factors



dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945],
modified to accomodate [Ball/Zwicky hep-ph/0406232]

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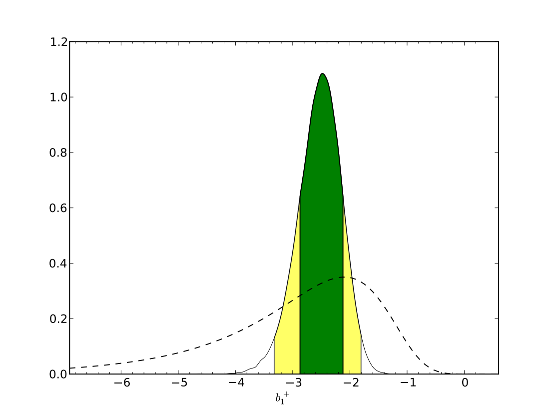
- more precise than prior
- $B \to K$:
  - $B \to K\ell^+\ell^-$
  - Lattice
- results @ 68% CL
  - $f_+(0) = 0.30 \pm 0.02$

Results for $B \to K$ Form Factors



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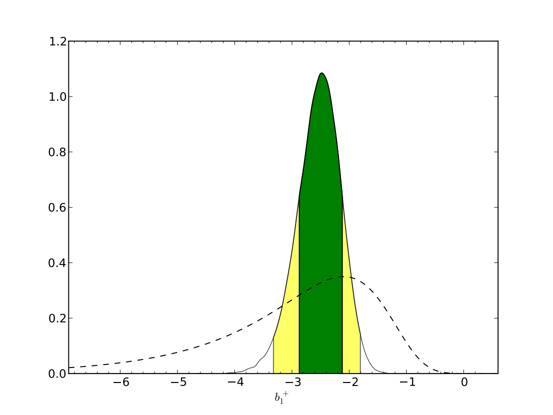

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- more precise than prior
- $B \to K$:
  - $B \to K\ell^+\ell^-$
  - Lattice
- results @ 68% CL
  - $f_+(0) = 0.30 \pm 0.02$
  - $b_1^+ = -2.5 \pm 0.4$
- small tension
  - LHCb lo q^2: -1.4σ
  - LHCb hi q^2: $+1.1\sigma$
  - Lattice: $+0.5\sigma$

Conclusion

Summary

- global analyses of all available $b \to s\{\gamma, \ell^+\ell^-\}$ data
- preliminary SM basis with all data
  - SM-like solution preferred over flipped signs with 77%
  - p-value 0.15
- $B \to K^*$ "anomaly"
  - vanishing effect in fit due to theory uncertainty (power corrections)
- new physics signal only with (limited!) subset of data
  - less tension ($\leq 2\sigma$) than Descotes-Genon,Matias,Virto (3.2σ)
  - reduced ($< 1\sigma$) by data beyond LHCb and $B \to K\ell^+\ell^-$ data
- data also allows inference of hadronic quantities (FF, power corr.)

Conclusion

Outlook

- SM' basis work in progress
- looking forward to further LHC analyses (LHCb 3fb^{-1}) and the prospects of Belle-II

Backup Slides

Priors and Parametrizations (I)

Form Factors [Khodjamirian et al. 1006.4945]

- values @ $q^2 = 0$ and slope: two parameters per FF
- z-parametrization
- asymmetric priors, use LogGamma function

CKM [update of hep-ph/0012308]

- Wolfenstein parametrization
- UTfit pre-Moriond2013, tree-level data only

Quark Masses [PDG]

Priors and Parametrizations (I) - Subleading

parametrize unknown subleading contributions

$B \to K^* \ell^+ \ell^-$

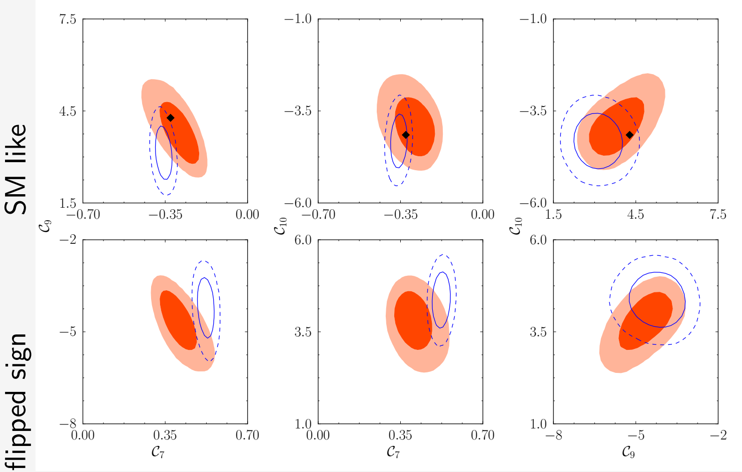
- lo q^2: 6 parameters, one scaling factor per amplitude
- hi q^2: 3 parameters

$B \to K \ell^+ \ell^-$

- lo q^2: 1 parameter
- hi q^2: 1 parameter

for all: Gaussian with mode at $\Lambda_{\text{QCD}}/m_b \simeq 0.15$

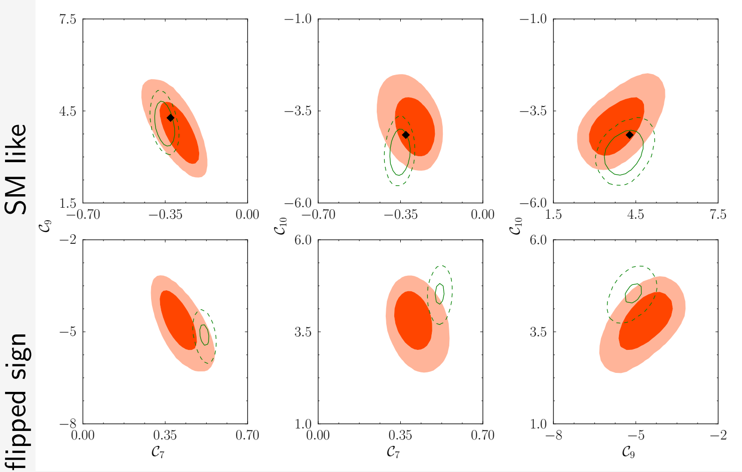
Results (SM Basis) Preliminary!



♦: Standard Model, (light-) red: 68% CL (95% CL) for early 2012

solid (dashed): 68% CL (95% CL) for post HEP'13 (selection)

Results (SM Basis) Preliminary!



♦: Standard Model, (light-) red: 68% CL (95% CL) for early 2012

solid (dashed): 68% CL (95% CL) for post HEP'13 (all data)

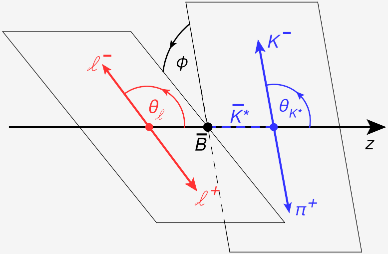
A Note on p Values

- test statistic: function of data and model (parameters) $\chi^2 = \chi^2(D, \vec{\theta})$
- only one data set observed $D_{obs} \Rightarrow \chi^2_{obs}$
- $p \equiv P(\chi^2 > \chi^2_{obs})$

But how to fix $\vec{\theta}$?

1. this work
   $\vec{\theta} = (\vec{\mathcal{C}}, \vec{\nu})$ at (local) mode of posterior, $\chi^2 \sim \frac{(x-\mu)^2}{\sigma^2_{exp}}$
2. Descotes-Genon et al. [1307.5683]
   $\vec{\theta} = \vec{\mathcal{C}}$ at (local) mode of likelihood, $\chi^2 \sim \frac{(x-\mu)^2}{\sigma^2_{exp}+\sigma^2_{theo}}$

Kinematics of $\bar{B} \to \bar{K}\pi\ell^+\ell^-$



Kinematic Variables

$$4m_\ell^2 \leq q^2 \leq (M_B - M_{K^*})^2$$

$$-1 \leq \cos\theta_\ell \leq 1$$

$$-1 \leq \cos\theta_{K^*} \leq 1$$

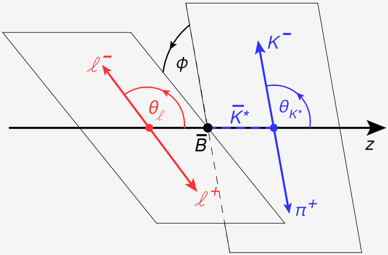
$$0 \leq \phi \leq 2\pi$$

$$\left[(M_K + M_\pi)^2 \leq k^2 \leq (M_B - \sqrt{q^2})^2\right]$$

On-shell and S-Wave

- one usually assumes on-shell decay of P-wave K^* ($\sim \sin\theta_{K^*}, \cos\theta_{K^*}$)
- for high precision: consider width of K^*, and $J = 0$ (S-wave) ($\not\sim \theta_{K^*}$) $K\pi$-final-state from K_0^* and *non-resonant background*

Kinematics of $\bar{B} \to \bar{K}\pi\ell^+\ell^-$



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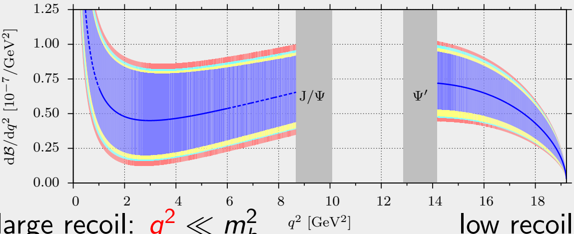
$$\left[(M_K + M_\pi)^2 \leq k^2 \leq (M_B - \sqrt{q^2})^2\right]$$

Large vs. Low Recoil (for illustration)



large recoil: $q^2 \ll m_b^2$ low recoil: $q^2 \simeq m_b^2$

Angular Distribution [Krüger/Matias '05, Altmannshofer et al. '08]

Differential Decay Rate for pure P-wave state

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} \sim J_{1s}\sin^2\theta_{K^*} + J_{1c}\cos^2\theta_{K^*}$$
$$+ \left(J_{2s}\sin^2\theta_{K^*} + J_{2c}\cos^2\theta_{K^*}\right)\cos 2\theta_\ell$$
$$+ \left(J_3\cos 2\phi + J_9\sin 2\phi\right)\sin^2\theta_{K^*}\sin^2\theta_\ell$$
$$+ \left(J_4\sin 2\theta_{K^*}\right)\sin 2\theta_\ell\cos\phi$$
$$+ \left(J_5\sin 2\theta_{K^*}\right)\sin\theta_\ell\cos\phi$$
$$+ \left(J_{6s}\sin^2\theta_{K^*} + J_{6c}\cos^2\theta_{K^*}\right)\cos\theta_\ell$$
$$+ \left(J_7\sin 2\theta_{K^*}\right)\sin\theta_\ell\sin\phi$$
$$+ \left(J_8\sin 2\theta_{K^*}\right)\sin 2\theta_\ell\sin\phi\,,$$

$J_i \equiv J_i(q^2)$: 12 angular observables

Angular Distribution [Krüger/Matias '05, Altmannshofer et al. '08, Blake/Egede/Shires '12]

Differential Decay Rate for mixed P- and S-wave state

$$
\begin{aligned}
\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} \sim\ & J_{1s}\sin^2\theta_{K^*} + J_{1c}\cos^2\theta_{K^*} + J_{1i}\cos\theta_{K^*} \\
& + \left(J_{2s}\sin^2\theta_{K^*} + J_{2c}\cos^2\theta_{K^*} + J_{2i}\cos\theta_{K^*}\right)\cos 2\theta_\ell \\
& + \left(J_3\cos 2\phi + J_9\sin 2\phi\right)\sin^2\theta_{K^*}\sin^2\theta_\ell \\
& + \left(J_4\sin 2\theta_{K^*} + J_{4i}\cos\theta_{K^*}\right)\sin 2\theta_\ell\cos\phi \\
& + \left(J_5\sin 2\theta_{K^*} + J_{5i}\cos\theta_{K^*}\right)\sin\theta_\ell\cos\phi \\
& + \left(J_{6s}\sin^2\theta_{K^*} + J_{6c}\cos^2\theta_{K^*}\right)\cos\theta_\ell \\
& + \left(J_7\sin 2\theta_{K^*} + J_{7i}\cos\theta_{K^*}\right)\sin\theta_\ell\sin\phi \\
& + \left(J_8\sin 2\theta_{K^*} + J_{8i}\cos\theta_{K^*}\right)\sin 2\theta_\ell\sin\phi\,,
\end{aligned}
$$

$J_i \equiv J_i(q^2, k^2)$: 12 angular observables, no further needed [Bobeth/Hiller/DvD '12]

Conclusion: remove S-wave in exp. analysis

- angular analysis [Egede/Blake/Shires '12]
- side-band analysis (for $J_{1s,1c,2s,2c}$) [Bobeth/Hiller/DvD '12]

Building Blocks of the Angular Observables (I)

Form Factors (P-Wave)

- hadronic matrix elements $\langle \bar{K}^* | \bar{s} \Gamma b | \bar{B} \rangle$ parametrized through 7 form factors:

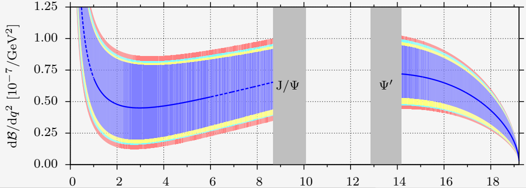
$$\langle \bar{K}^* | \bar{s}\gamma^\mu b | \bar{B} \rangle \sim V \quad \langle \bar{K}^* | \bar{s}\gamma^\mu \gamma_5 b | \bar{B} \rangle \sim A_{0,1,2} \quad \langle \bar{K}^* | \bar{s}\sigma^{\mu\nu} b | \bar{B} \rangle \sim T_{1,2,3}$$

- form factors largest source of theory uncertainty
  amplitude $\sim 10\% - 15\% \Rightarrow$ observables: $\sim 20\% - 50\%$
  - available from Light Cone Sum Rules [Ball/Zwicky '04, Khodjamirian et al. '11]
  - Lattice QCD: work in progress [e.g. Liu et al. '11, Wingate '11]
  - extract ratios from low recoil data

    [Hambrock/Hiller '12, Beaujean/Bobeth/DvD/Wacker '12]

blue band:

form factor uncertainty



Building Blocks of the Angular Observables (II)

Transversity amplitudes A_i

- SM-like + chirality flipped: essentially four amplitudes $A_{\perp,\|,0,t}$
  [Krüger/Matias '05]
- $\mathcal{O}_{S(')}$ give rise to A_S, $\mathcal{O}_{P(')}$ absorbed by A_t [Altmannshofer et al. '08]
- $\mathcal{O}_{T(5)}$ give rise to *6 new* amplitudes A_{ab},
  (ab)=$(0t)$,$(\|\perp)$,$(0\perp)$,$(t\perp)$,$(0\|)$,$(t\|)$ [Bobeth/Hiller/DvD '12]
- altogether: 11 complex-valued amplitudes

Angular Observables

- J_i functionals of A_S, A_a, A_{ab}, $a, b = t, 0, \|, \perp$ e.g.

$$J_3(q^2) = \frac{3\beta_\ell}{4}\left[|A_\perp|^2 - |A_\||^2 + 16\left(|A_{t\|}|^2 + |A_{0\|}|^2 - |A_{t\perp}|^2 - |A_{0\perp}|^2\right)\right]$$

β_ℓ: lepton velocity in dilepton rest frame $m_\ell^2/q^2 \to 0 \Rightarrow \beta_\ell \to 1$

"Standard" Observables

considerable theory uncertainty due to form factors

Batch #1, to be extracted from CP average

$$\langle\Gamma\rangle = \langle 2J_{1s} + J_{1c} - \frac{2}{3}J_{2s} - \frac{1}{3}J_2c\rangle \qquad \langle A_{\mathrm{FB}}\rangle = \frac{\langle 2J_{6s} + J_{6c}\rangle}{2\langle\Gamma\rangle}$$

$$\langle F_L\rangle = \frac{\langle 3J_{1c} - J_2c\rangle}{\langle 3\Gamma\rangle} \qquad \langle F_T\rangle = \frac{\langle 6J_{1s} - 2J_{2s}\rangle}{\langle 3\Gamma\rangle}$$

Γ: decay width A_{FB}: forward-backward asymm. $F_L = 1 - F_T$: long./trans. pol.

Batch #2, CP (a)symmetries [Bobeth/Hiller/Piranishvili '08, Altmannshofer et al. '08]

$$\langle A_i\rangle \sim \frac{\langle J_i - \overline{J_i}\rangle}{\langle\Gamma + \overline{\Gamma}\rangle} \qquad \langle S_i\rangle \sim \frac{\langle J_i + \overline{J_i}\rangle}{\langle\Gamma + \overline{\Gamma}\rangle}$$

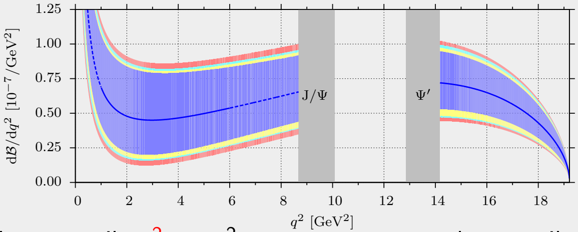
overline: CP conjugated mode, also: mixing-induced CP asymm in $B_s \to \phi\ell^+\ell^-$

$\langle X\rangle \equiv \int \mathrm{d}q^2\, X(q^2)$

Pollution due to Charm Resonances

Narrow Resonances: J/ψ and $\psi(2s)$

- experiments veto q^2-region of narrow charmonia J/ψ and $\psi(2s)$
- however: resonance affects observables outside the veto!



large recoil: $q^2 \ll m_b^2$ low recoil: $q^2 \simeq m_b^2$

Approach by Theorists: Divide and Conquer

- treat region below J/ψ (aka *large recoil*) differently than above $\psi(2s)$
- design combinations of J_i which have reduced theory uncertainty in only one kinematic region

Large Recoil (I)

QCD Factorization (QCDF) + Soft Collinear Effective Theory (SCET)

- calculate $\bar{q}q$ loops perturbatively, expand in $1/m_b$, $1/E_{K^*}$
- relate matrix elements to universal hadronic quantities:
  - Light Cone Distribution Amplitudes (LCDAs)
  - form factors
  - decay constants

[Beneke/Feldmann '00, Beneke/Feldmann/Seidel '01 & '04]

Light Cone Sum Rules (LCSR)

- calculate $\langle \bar{c}c \rangle$, $\langle \bar{c}cG \rangle$ on the light cone for $q^2 \ll 4m_c^2$
- achieves resummation of soft gluon effects
- use analycity of amplitude to relate results to $q^2 < M_{\psi'}^2$
- uses many of the same inputs as QCDF+SCET
- includes parts of QCDF+SCET results

Large Recoil (I)

QCD Factorization (QCDF) + Soft Collinear Effective Theory (SCET)

- calculate $\bar{q}q$ loops perturbatively, expand in $1/m_b$, $1/E_{K^*}$
- relate matrix elements to universal hadronic quantities:
  - Light Cone Distribution Amplitudes (LCDAs)
  - form factors
  - decay constants

[Beneke/Feldmann '00, Beneke/Feldmann/Seidel '01 & '04]

Combination of QCDF+SCET and LCSR Results

- not yet!
  - no studies yet to find impact on optimized observables at large recoil!
  - LCSR results are not included in following discussion

Large Recoil (II)

SM + chirality flipped

- transversity amplitudes factorize up to power supressed terms

$$A_\perp^{L,R} \sim X_\perp^{L,R} \times \xi_\perp \qquad A_\parallel^{L,R} \sim X_\parallel^{L,R} \times \xi_\perp \qquad A_0^{L,R} \sim X_0^{L,R} \times \xi_\parallel$$

$\xi_{\perp,\parallel}$: soft form factors $\qquad$ $X_i^{L,R}$: combinations of Wilson coefficients

[Beneke/Feldmann/Seidel '01 & '04, Bobeth/Hiller/Piranishvili '08, Altmannshofer et al. '08]

Optimized Observables

enhanced sensitivity to right-handed currents, reduced form factor dependence

[Krüger/Matias '05, Egede et al. '08 & '10]

$$A_T^{(2)} = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2} \sim J_3 \qquad A_T^{(3)} = \frac{|A_0^L A_\parallel^{L*} + A_0^{R*} A_\parallel^R|}{\sqrt{|A_0|^2 |A_\parallel|^2}} \sim J_4, J_7$$

$$A_T^{(4)} = \frac{|A_0^L A_\perp^{L*} - A_0^{R*} A_\perp^R|}{\sqrt{|A_0|^2 |A_\perp|^2}} \sim J_5, J_8 \qquad A_T^{(5)} = \frac{|A_\perp^L A_\parallel^{R*} + A_\perp^{R*} A_\parallel^L|}{|A_\perp|^2 + |A_\parallel|^2}$$

Large Recoil (II)

SM + chirality flipped

- transversity amplitudes factorize up to power supressed terms

$$A_\perp^{L,R} \sim X_\perp^{L,R} \times \xi_\perp \qquad A_\parallel^{L,R} \sim X_\parallel^{L,R} \times \xi_\perp \qquad A_0^{L,R} \sim X_0^{L,R} \times \xi_\parallel$$

$\xi_{\perp,\parallel}$: soft form factors $\qquad$ $X_i^{L,R}$: combinations of Wilson coefficients

[Beneke/Feldmann/Seidel '01 & '04, Bobeth/Hiller/Piranishvili '08, Altmannshofer et al. '08]

Further Optimized Observables

enhanced sensitivity to right-handed currents, reduced form factor dependence

[Becirevic/Schneider '11]

$$A_T^{(\mathrm{re})} \propto \frac{J_{6s}}{J_{2s}} \qquad\qquad A_T^{(\mathrm{im})} \propto \frac{J_9}{J_{2s}}$$

Low Recoil

SM basis [Bobeth/Hiller/DvD '10] + chirality flipped [Bobeth/Hiller/DvD '12]

- transversity amplitudes factorize

$$A^{L,R}_{\perp,\parallel,0} \sim C^{L,R}_{\pm} \times f_{\perp,\parallel,0} + O\left(\frac{\alpha_s \Lambda}{m_b}, \frac{\mathcal{C}_7 \Lambda}{\mathcal{C}_9 m_b}\right) \quad \text{SM:} \quad C^{L,R}_{+} = C^{L,R}_{-}$$

f_i: helicity form factors $\qquad$ $C^{L,R}_{\pm}$: combinations of Wilson coeff.

- 4 combinations of Wilson coefficients enter observables:

$$\rho_1^{\pm} \sim |C^R_{\pm}|^2 + |C^L_{\pm}|^2$$

$$\mathrm{Re}\,(\rho_2) \sim \mathrm{Re}\left(C^R_+ C^{R*}_- - C^L_- C^{L*}_+\right) \quad \text{and } \mathrm{Re}\,(\cdot) \leftrightarrow \mathrm{Im}\,(\cdot)$$

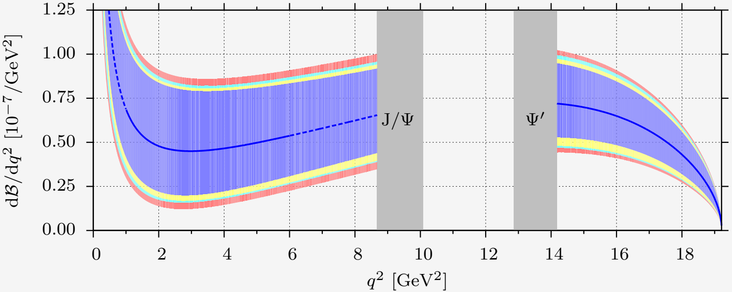
Tensor operators [Bobeth/Hiller/DvD '12]

- 6 new transversity amplitudes, still factorize!

$$A_{ab} \sim \mathcal{C}_{T(T5)} \times f_{\perp,\parallel,0} + O\left(\frac{\Lambda}{m_b}\right)$$

- 3 new combinations of Wilson coefficients

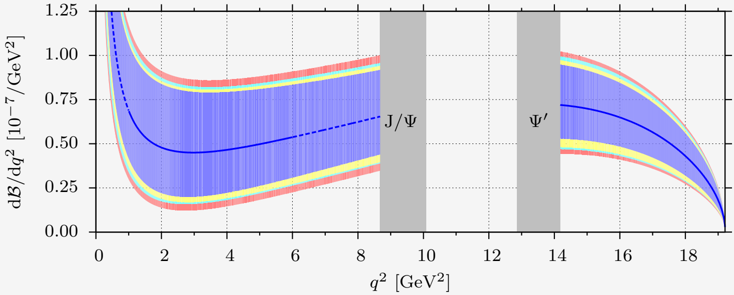
q^2 Spectrum of the Branching Ratio $\mathcal{B} = \tau_B \Gamma$



$\bar{q}q$ Pollution

- 4-quark operators like $\mathcal{O}_{1c,2c}$ induce $b \to s\bar{c}c(\to \ell^+\ell^-)$ via loops
- hadronically $B \to K^* J/\psi(\to \ell^+\ell^-)$ or higher charmonia
- experiment: cut narrow resonances $J/\psi \equiv \psi(1S)$ and $\psi' = \psi(2S)$
- theory: handle non-resonant quark loops/broad resonances $> 2S$

q^2 Spectrum of the Branching Ratio $\mathcal{B} = \tau_B \Gamma$



Large Recoil $E_{K^*} \sim m_b$ QCDF,SCET

- expand in $1/m_b$, $1/E_{K^*}$, α_s
- symmetry: $7 \to 2$ form factors

[Beneke/Feldmann/Seidel '01 & '04]

[Egede et al. '08 & '10]

Low Recoil $q^2 \sim m_b^2$ OPE,HQET

- expand in $1/m_b$, $1/\sqrt{q^2}$, α_s
- symmetry: $7 \to 4$ form factors

[Grinstein/Pirjol '04], [Beylich/Buchalla/Feldmann '11]

[Bobeth/Hiller/DvD '10 & '11]