Theory prediction of $B_s \to \mu^+ \mu^-$ and NP in the Z-Penguin

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Based on work with
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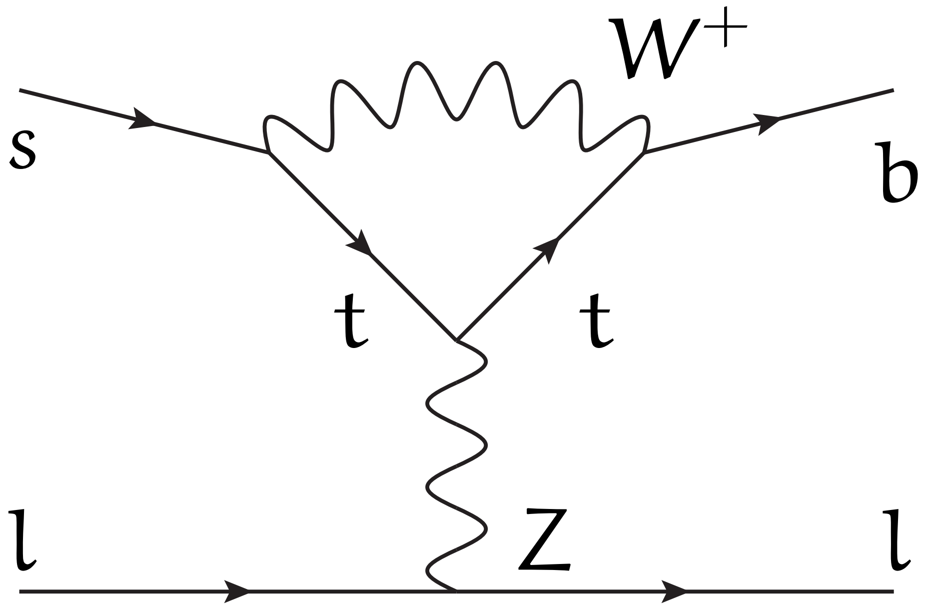
$B_s \to \mu^+ \mu^-$



B_s is pseudoscalar – no photon penguin

$$Q_A = (\bar{b}_L \gamma_\mu q_L)(\bar{l}\gamma_\mu \gamma_5 l)$$

Dominant operator (SM) Wilson

helicity suppression $\left(\propto \frac{m_l^2}{M_B^2}\right)$

Effective Lagrangian in the SM (NP + chirality flipped):

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} (C_S Q_S + C_P Q_P + C_A Q_A) + \text{h.c.}$$

Scalar operators: $Q_S = m_b(\bar{b}_R q_L)(\bar{l}l)$ $\quad Q_P = m_b(\bar{b}_R q_L)(\bar{l}\gamma_5 l)$

Alternative normalisation [Misiak `11]:

$$\mathcal{L}_{\text{eff}} = G_F^2 M_W^2 \, V_{tb}^* V_{ts} \, (C_A Q_A + C_S Q_S + C_P Q_P) + \text{h.c.}$$

Theory Status (SM)

Standard Model: Scalar Operators are highly suppressed

C_A is known at NLO in QCD [Buras, Buchalla; Misiak, Urban `99]

$C_A(m_t\ /\ M_W)^{NLO} = 1.0113\ C_A(m_t\ /\ M_W)^{LO}$
– for QCD MS-bar $m_t = m_t(m_t)$

The matrix-element $<Q_A>$ is given through the precisely known decay constant f_{Bs}
(f_{Bs} =225(5)MeV [Dowdall `13] - average = 225(3)MeV
twisted mass N_f=2: f_{Bs} =228(8)MeV [Carrosa `13])

There will be non-perturbative correction for $\alpha_e \neq 0$

Theory Status (SM)

Soft photon corrections [Buras, Guadagnoli, Isidori `12] :
→ cut on the invariant mass of the lepton pair suppresses the direct photon emission
→ bremsstrahlung affects the branching ratio
→ experimentalists simulate the signal fully inclusive of bremsstrahlung and remove this correction

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The B_s system has a non-zero decay width difference:
→ instantaneous ≠ time integrated branching ratio [de Bruyn, Fleischer et. al. `12]
→ correction factor can be extracted from experiment
→ B_s mixing allows for additional observables beyond the branching ratio [de Bruyn, Fleischer et. al. `12, Buras et. al. `13]

Electroweak Corrections

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} C_A Q_A + \text{h.c.}$$

$G_F\,\alpha/\sin^2\theta_W$ does not renormalise under QCD: can be factored out for QCD calculation

Only $G_F\,\alpha/\sin^2\theta_W\,C_A(m_t/M_W)$ invariant under electroweak scheme change

This combination should always give the same result if we use the same input (G_F, α, M_Z, M_t, M_H) up to higher order corrections

Renormalisation of G_F

We identify G_F with the measured muon lifetime and its theory prediction $G_\mu = G_\mu^{(0)} + G_\mu^{(1)} + \ldots$

G_F can be combined or factored out of the Wilson coefficient

$$\mathcal{H}_{\text{eff}} = \tilde{C}\, Q = G_F C\, Q$$

Since G_F is now an observable and C dimensionless the vacuum expectation dependence cancels in C:

$$C^{(0)} = \frac{\tilde{C}^{(0)}}{G_\mu^{(0)}}, \quad C^{(EW)} = \frac{\tilde{C}^{(EW)}}{G_\mu^{(0)}} - \frac{\tilde{C}^{(0)} G_\mu^{(EW)}}{\left(G_\mu^{(0)}\right)^2}$$

but other parameters are not so easily fixed from experiment

Electroweak Scheme Uncertainties

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V^*_{tb} V_{ts}}{\sin^2 \theta_W} C_A(\frac{m_t}{M_W}) Q_A + h.c.$$

	MS-bar	OS	unct. $B_s\ \mu^+\ \mu^-$
$\sin \theta_W$	0.231	0.223	4 %
m_t(QCD-MS-bar)	163,5 GeV	164,8 GeV	1 %

Electroweak scheme shift larger then present pure theory error

[Buras, Guadagnoli, Isidori `12]: Follow [Brod, MG, Stamou] and use MS-bar θ_W plus renormalise the masses on-shell (hybrid)

Box contributions contribute differently to $l^+ l^-$ modes than to νν. Are both box and penguin corrections tiny in the hybrid scheme?

Renormalisation Schemes

Calculate in the MS-bar scheme
using tadpole counterterms to produce gauge
independence for intermediate results
fit g_1, g_2, v, λ, m_t from data (G_F, α, M_Z, M_t, M_H)

Use OS scheme: Determine M_W including loop
corrections from input – then $\sin^2\theta_W = 1 - M_W^2 / M_Z^2$

Tree level $M_W^2 = M_Z^2 \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi \alpha_e^{OS}}{\sqrt{2} G_F M_Z^2}} \right) = 80.94^2 GeV^2$

Add finite $\sin\theta_W$, m_t and M_W counterterms to $C_A^{(EW)}$

NLO predictions should agree up to residual scheme
uncertainties if we use the same input data.

Matching Correction for C_A

There are sizeable shifts and reduction of scale dependence
if we go from 1-loop to 2-loop

Note: α(nq=6) used for plot

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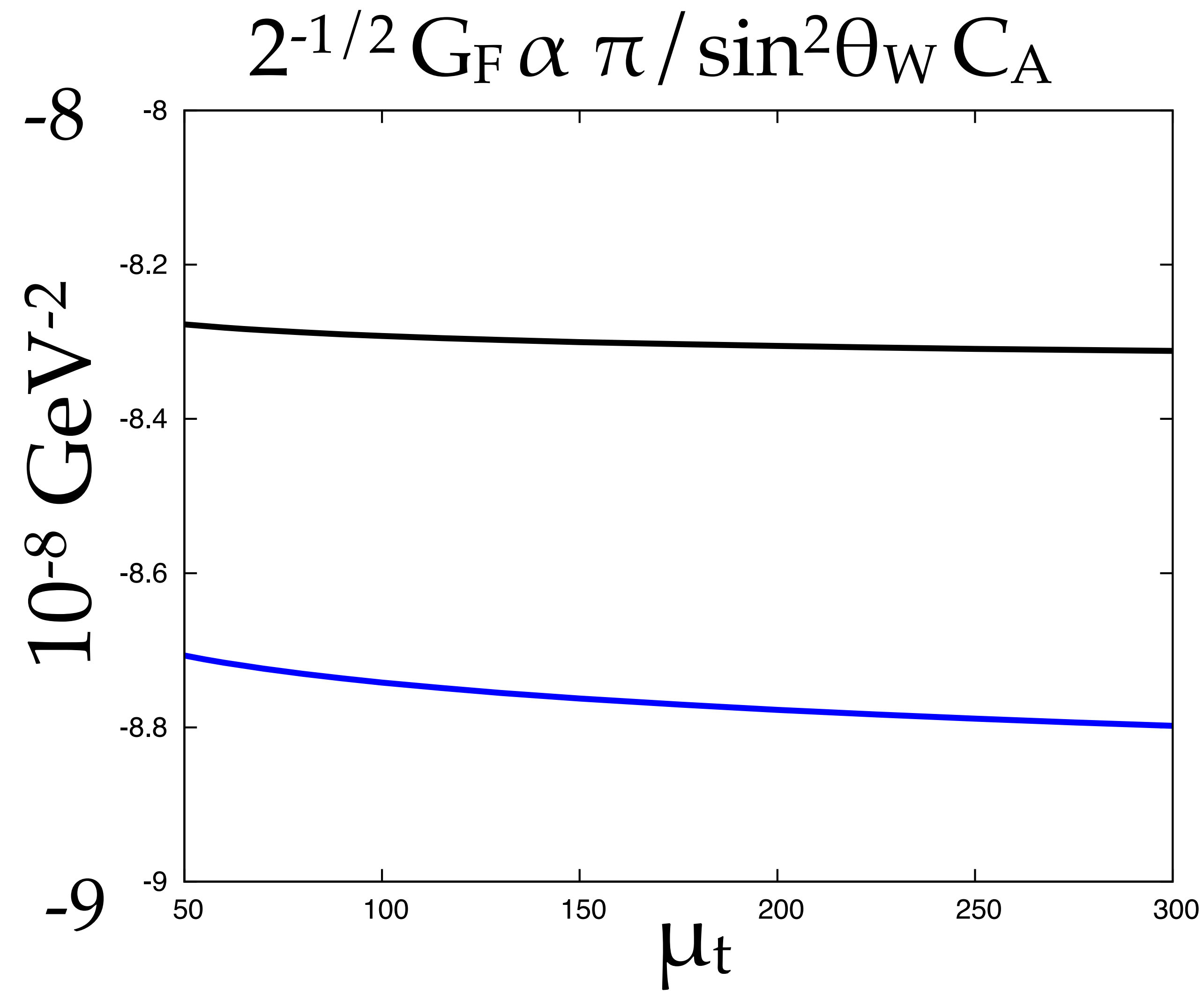
largest shift in the on-shell scheme



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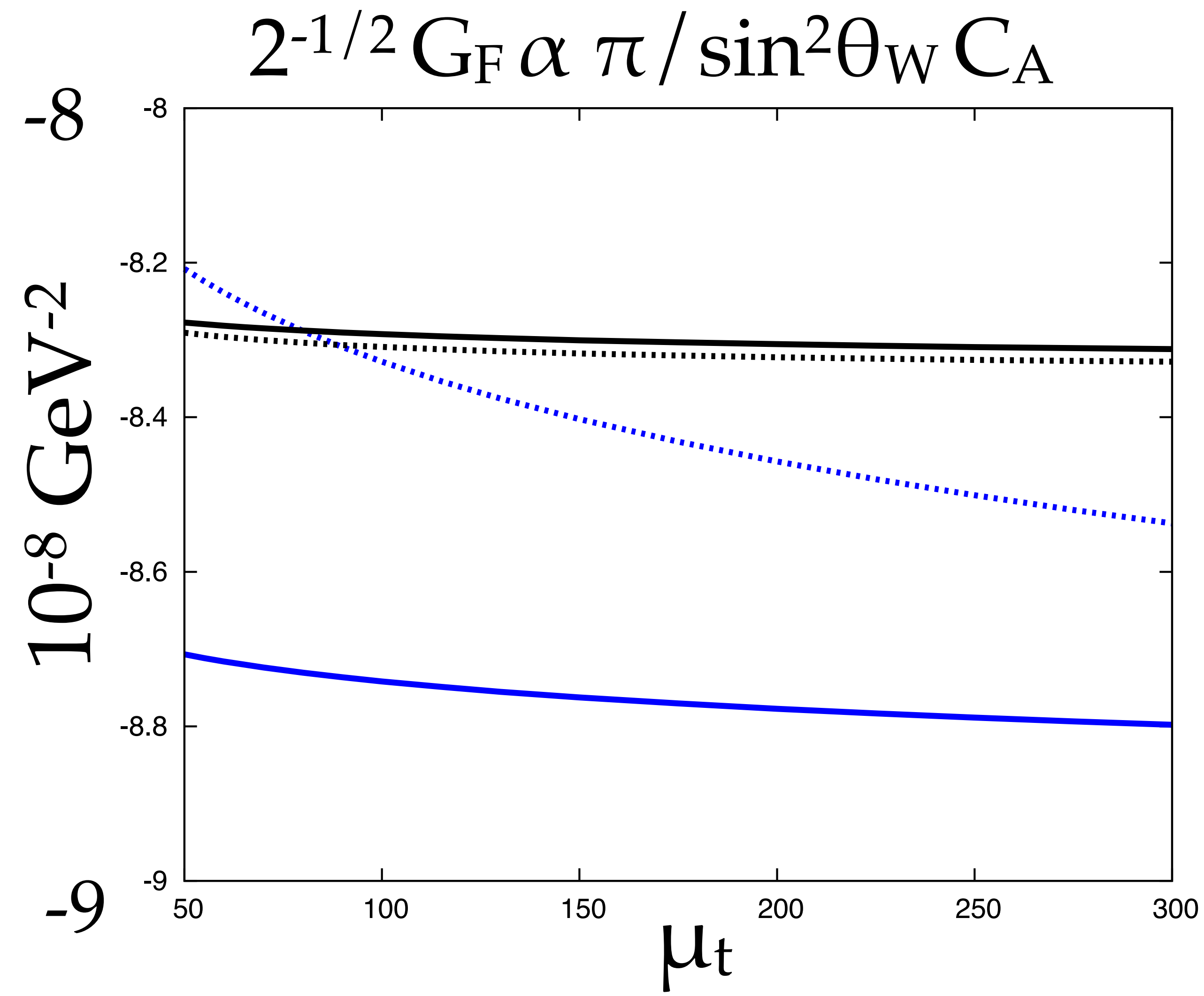

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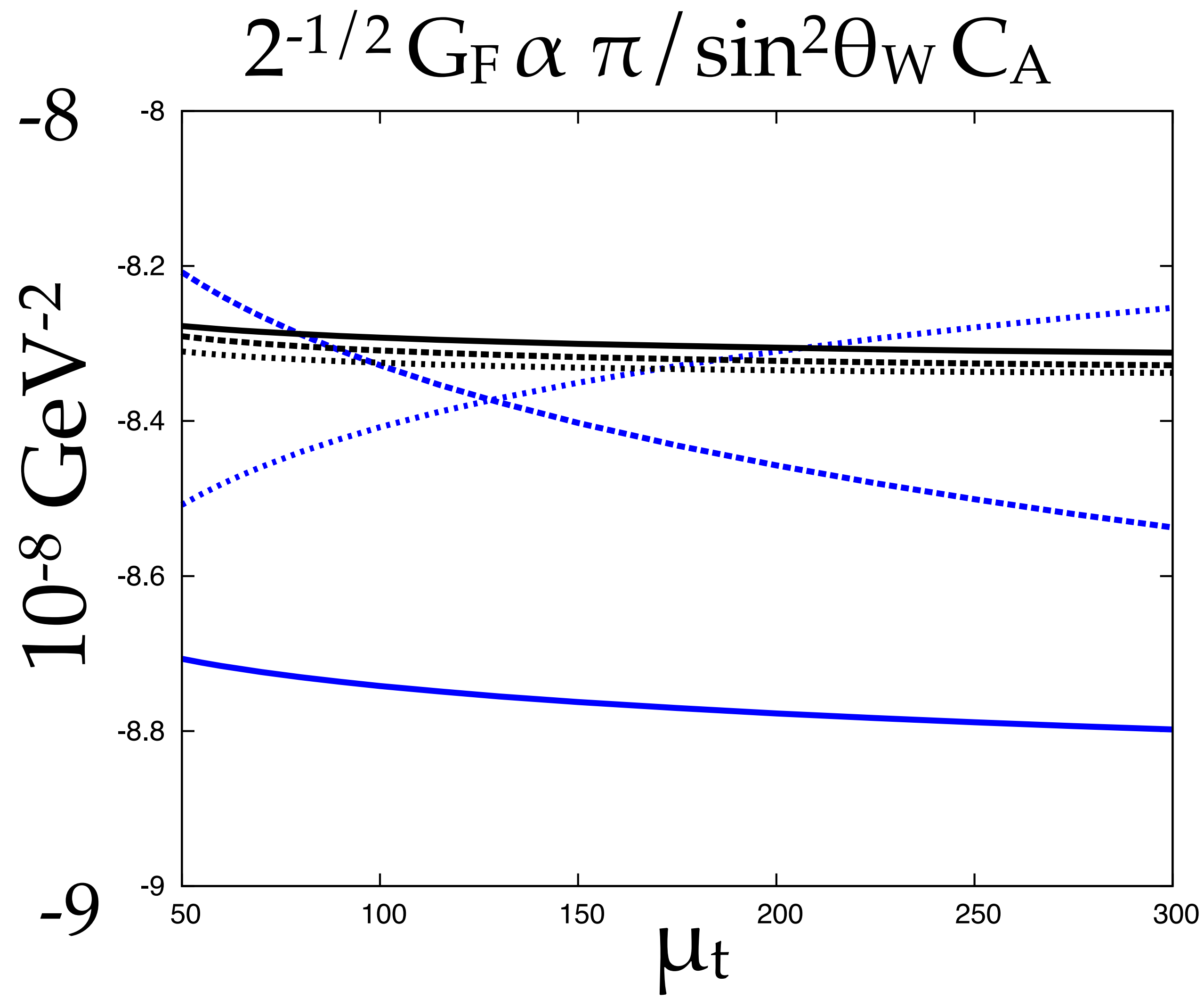
and the hybrid scheme



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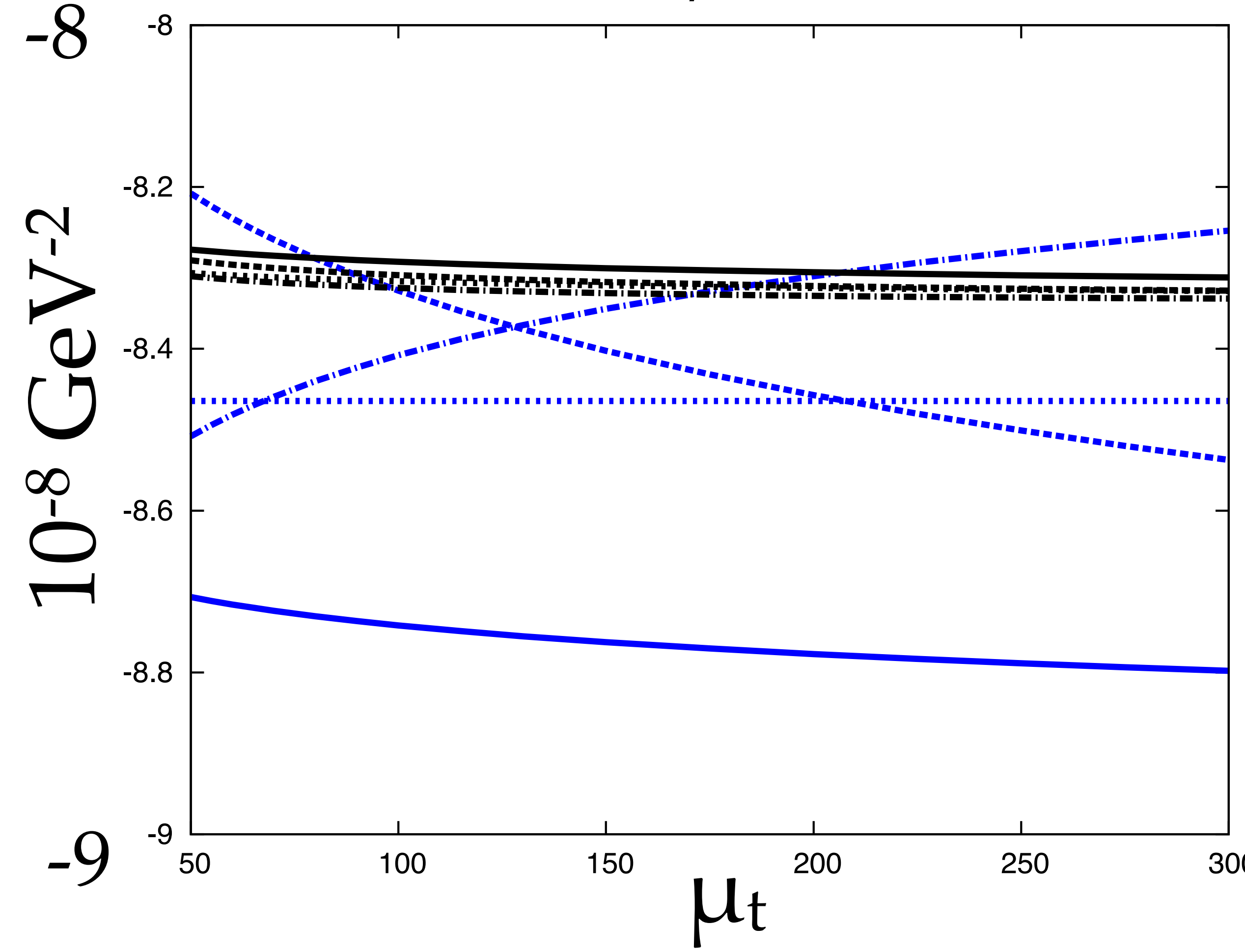
and the hybrid scheme

$G_F^2 M_W^2$ removes `artificial´ scale (and parameter

Note: α(nq=6) used for plot

$2^{-1/2} G_F \alpha \pi / \sin^2\theta_W C_A$



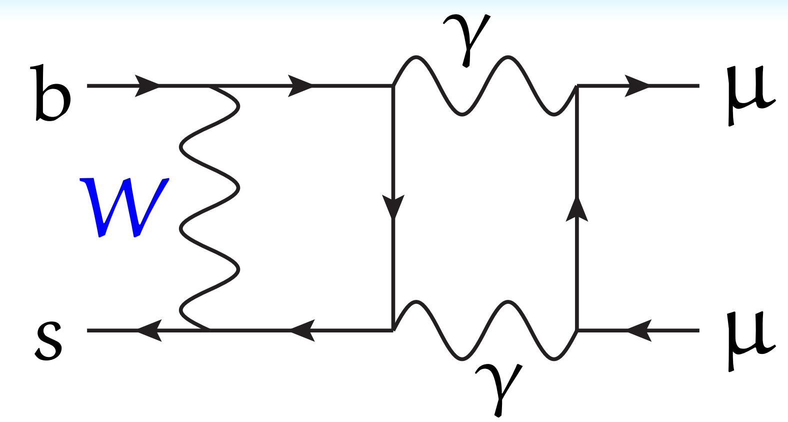
EW corrections reduce modulus of Wilson Coefficient and remove 7 % scale uncertainty in the BR

Renormalisation Group Equation

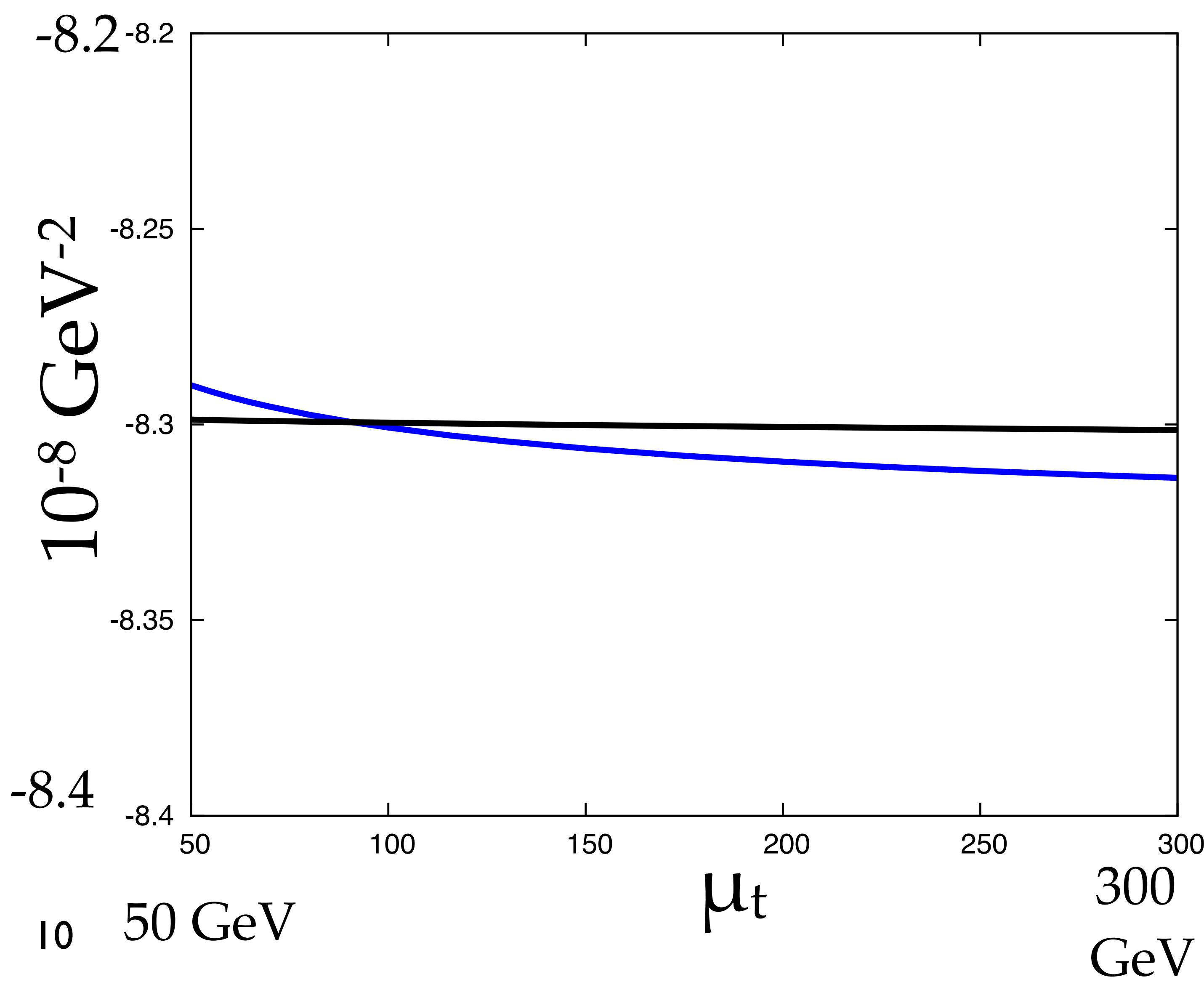
Log enhanced QED corrections known [Bobeth, Gambino, MG, Haisch `03; Huber et. al. `05, Misiak `11]

Study residual scale dependence for the G_F^2 M_W^2 normalised results

$G_F^2 M_W^2 C(\mu_0)$ is scale dependent, while $U(M_Z, \mu_0)\, G_F^2 M_W^2 C(\mu_0)$ is only residually scale dependent.



$G^2_F M^2_W C_A(M_Z)$



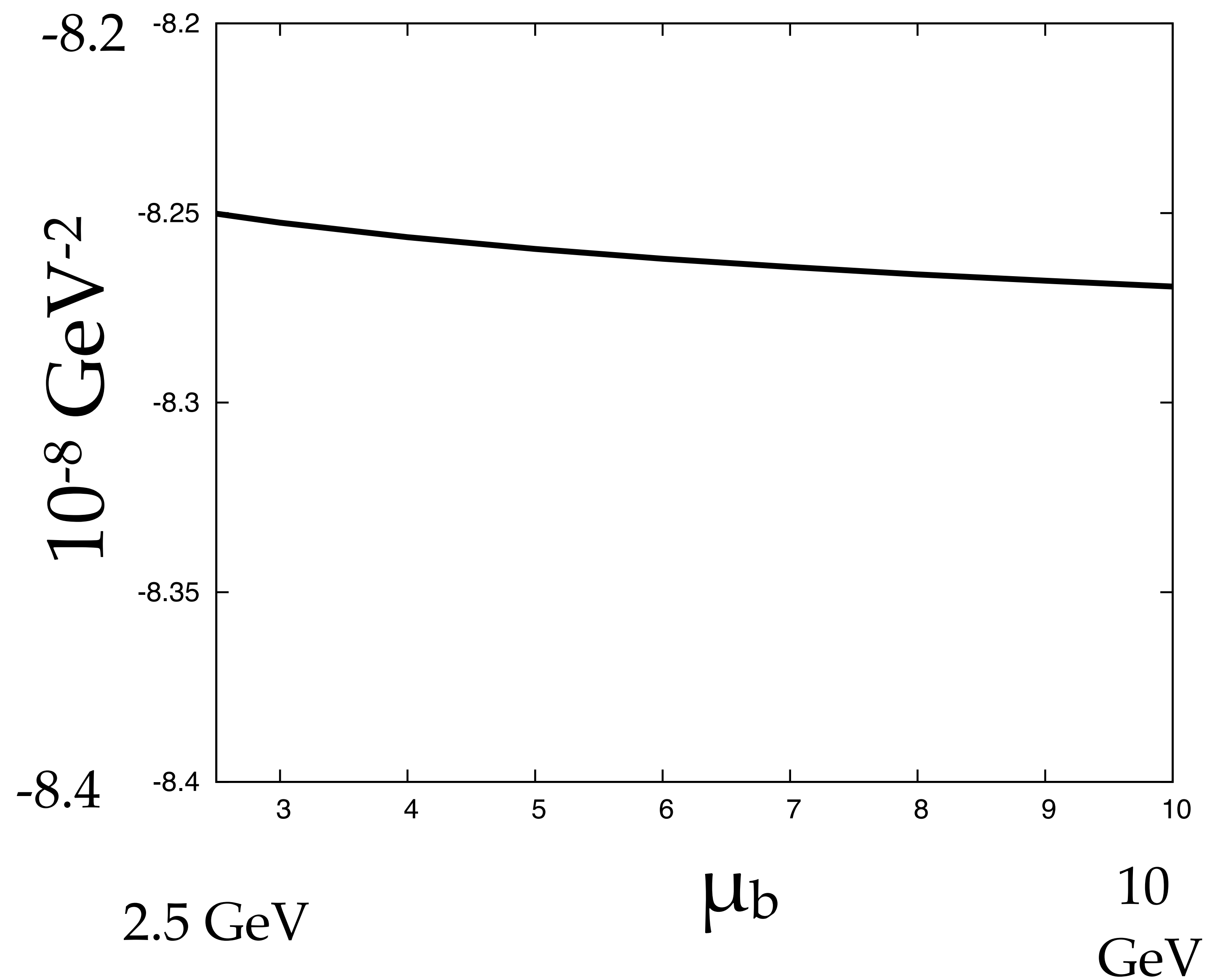
Wilson Coefficient at m_b

The log enhanced QED corrections further reduce the modulus of the Wilson coefficient

Varying μ_b in $U(\mu_b, m_t)\, G_F^2 M_W^2\, C(m_t)$ gives a measure of uncertainty regarding the contributions of virtual QED corrections at m_b.

LO analysis: Multiply μ_b uncertainty by 2

$G^2_F M^2_W C_A(m_b)$



there are also uncertainties from non-perturbative QED corrections

Theory Prediction

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O(3.X%) reduction of BR

for $s_W(M_Z)$ and masses OS

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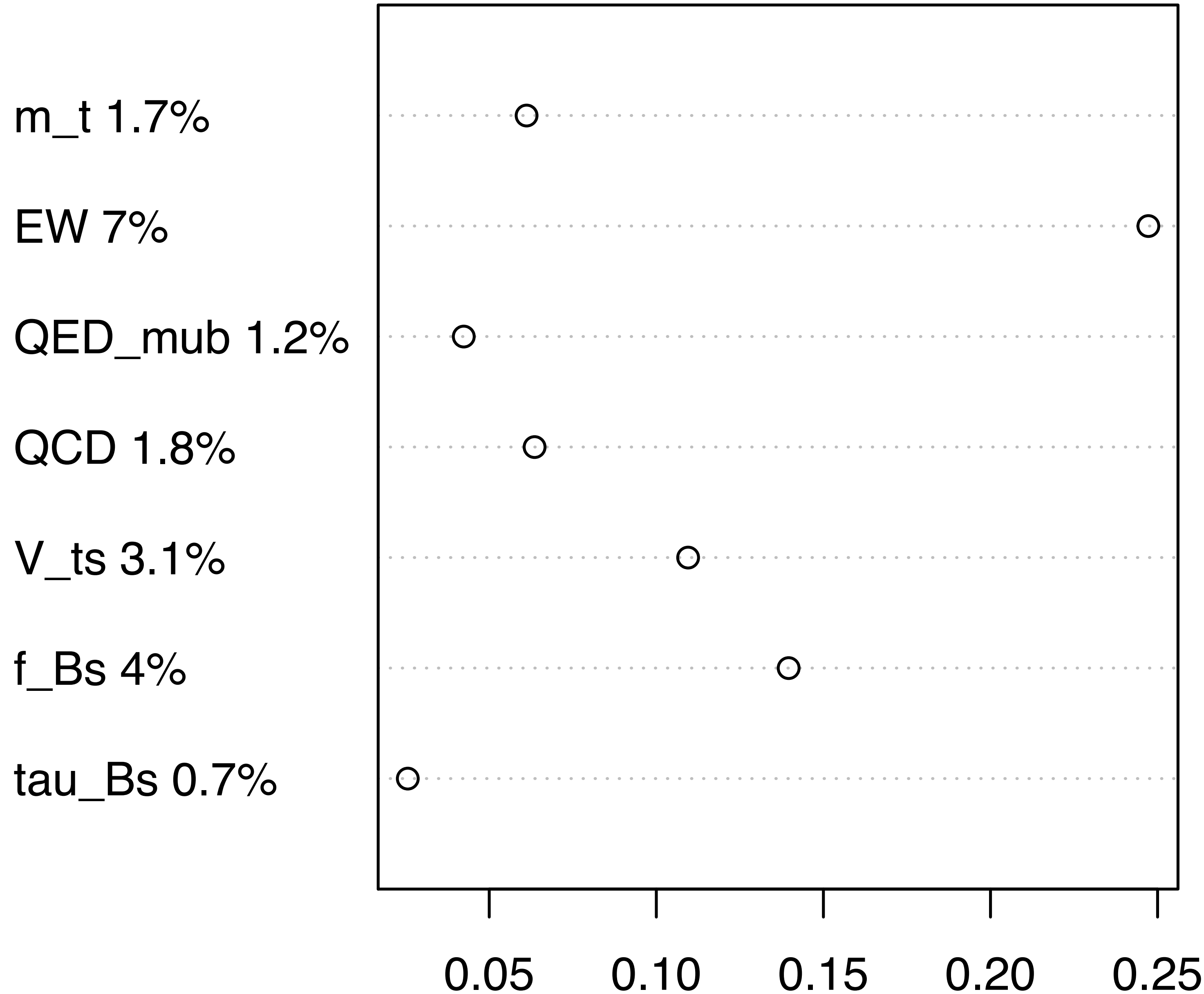
For $G_F^2 M_W^2$ normalisation:
$B(B_s \to \mu^+ \mu^-)_{NLO} = 3.53(32)$



f_{Bs} [MeV]	τ_{Bs} [ps^{-1}]	V_{ts}	M_t [GeV]
227.7(45)	1.516(11)	0.04218(64)	173.2(10)

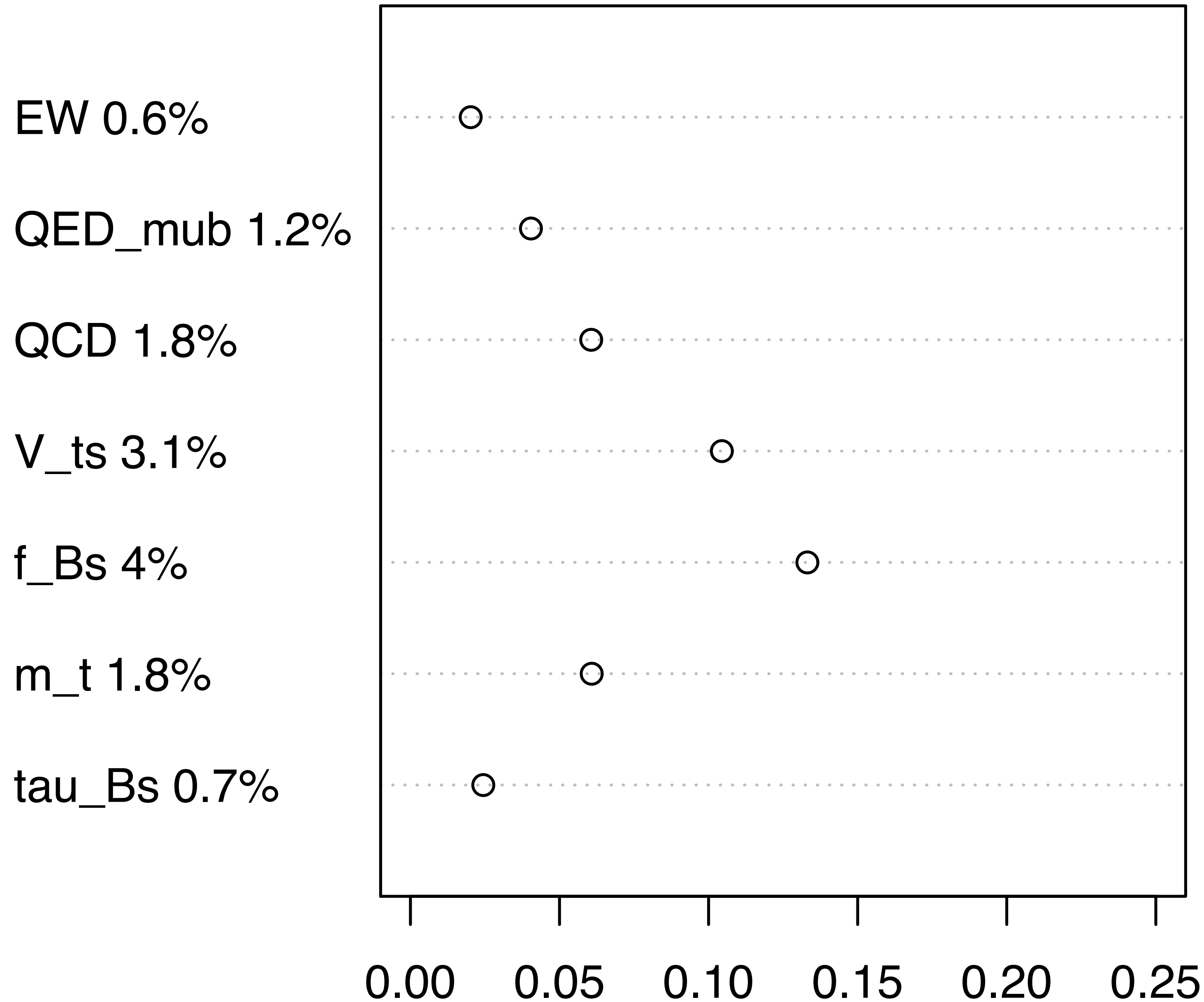
Theory Prediction

O(3.X%) reduction of BR for $s_W(M_Z)$ and masses OS

For $G_F^2 M_W^2$ normalisation:

$B(B_s \to \mu^+ \mu^-)_{NLO} = 3.53(32)$

$B(B_s \to \mu^+ \mu^-)_{+EW} = 3.37(20)$



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Comparison with Experiment

Time integration results in another shift

$$\overline{\mathcal{B}}_{B_s} = 1.096(17) \times \mathcal{B}_{B_s}^{(0)}$$

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Introduces experimental uncertainty in theory prediction

$B(B_s \rightarrow \mu^+ \mu^-)_{t\,int} = 3.69(22)$

NNLO QCD matching will reduce perturbative uncertainty

Comparison with Experiment

Time integration results in another shift

$$\overline{\mathcal{B}}_{B_s} = 1.096(17) \times \mathcal{B}^{(0)}_{B_s}$$

Introduces experimental uncertainty in theory prediction

$B(B_s \to \mu^+ \mu^-)_{\text{t int}} = 3.69(22)$

NNLO QCD matching will reduce perturbative uncertainty

	$Br(B_s \to \mu^+ \mu^-)$
CMS	$3.0\ ^{+0.9}_{-0.8}$ (stat) $^{+0.6}_{-0.4}$ (syst) 10^{-9}
LHCb	$2.9\ ^{+1.1}_{-1.0}$ (stat) $^{+0.3}_{-0.1}$ (syst) 10^{-9}
Combi-nation	$2.9(7)\ 10^{-9}$ [EPS 13 Hansmann-Menzemer]

New Physics

Sensitive to new scalar flavour changing interactions

Precision probe of the Z-Penguin

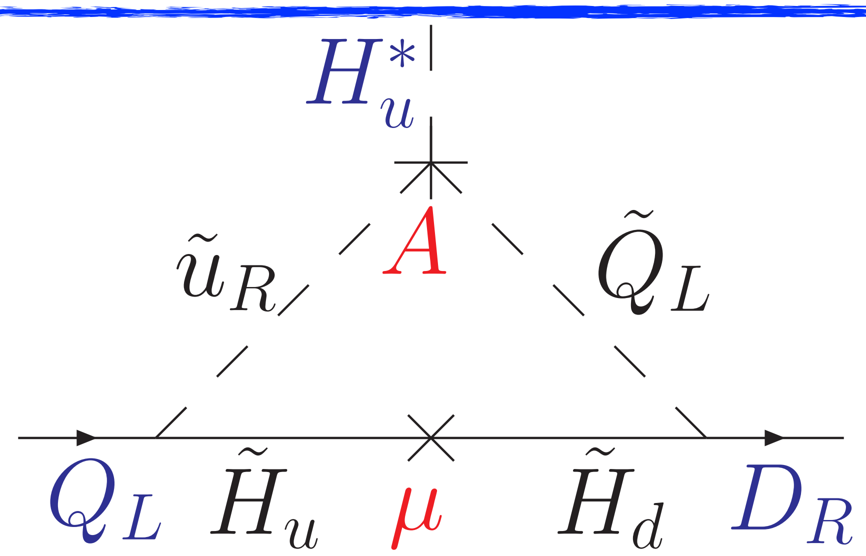
MSSM: MFV and Large tan β

MSSM at tree level:

$H_d \leftrightarrow d_R \qquad H_u \leftrightarrow u_R$

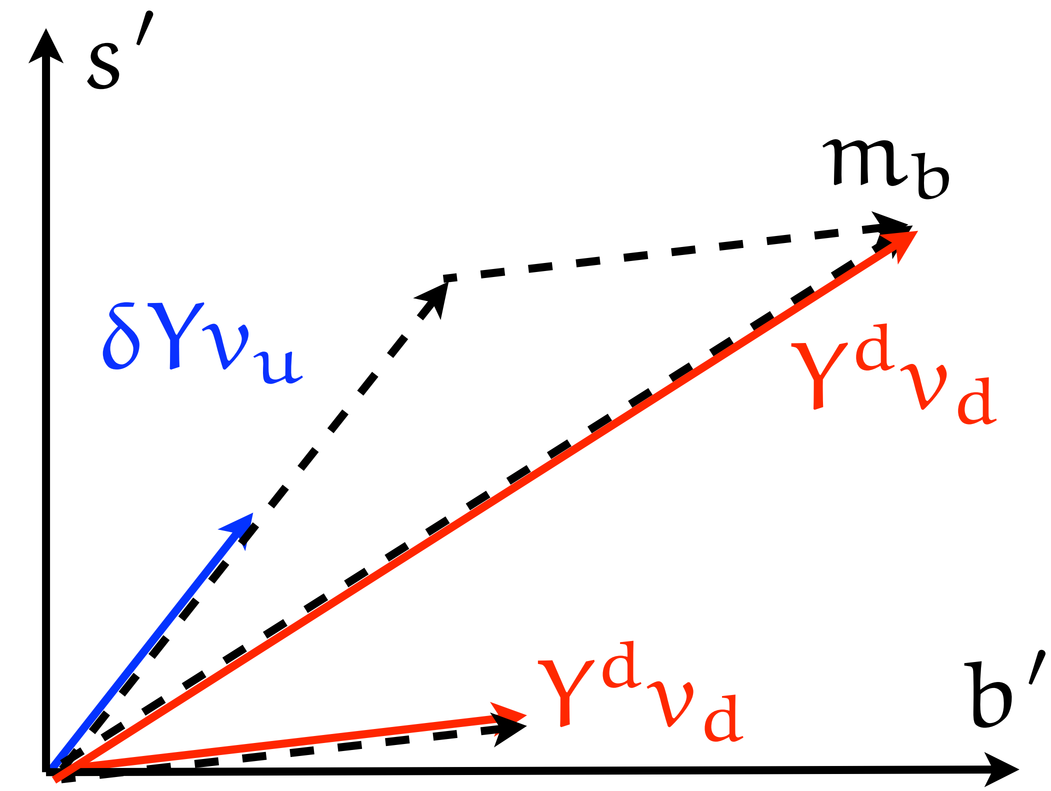
$$-\mathcal{L} = Y^d_{ij} H_d \bar{d}^i_R q^j + Y^u_{ij} H_u \bar{u}^i_R q^j + \text{h.c.}$$

MSSM at one-loop:



One loop: 2HDM of type 3

$$\Delta\mathcal{L}^Y_{\text{eff}} = \epsilon_Y \bar{d}_R Y^d Y^{u\dagger} Y^u H_u^* \cdot Q_L$$

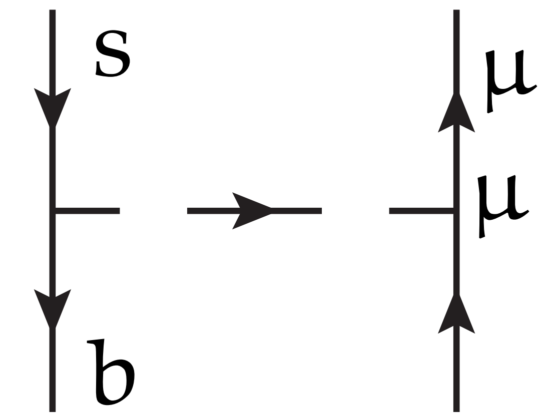


Masses and Yukawas not aligned

Flavour Violation at large tan β

Large FC scalar interactions: $\kappa_b \bar{b}_R s_L h_d^{0*} \propto Y_b$

[Babu, Kolda '00; ...]



$$\mathrm{Br}(B_s \to \mu^+ \mu^-) \propto (\tan\beta)^6 \,/\, (M_A)^4$$

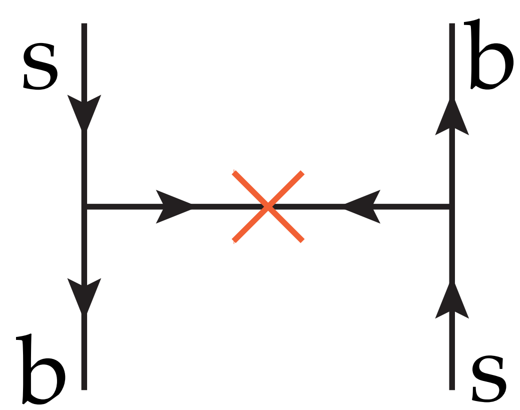
Note, that the tan β sensitivity of the MSSM is unique:

MSSM Higgs sector at $v_d = 0$: a symmetry

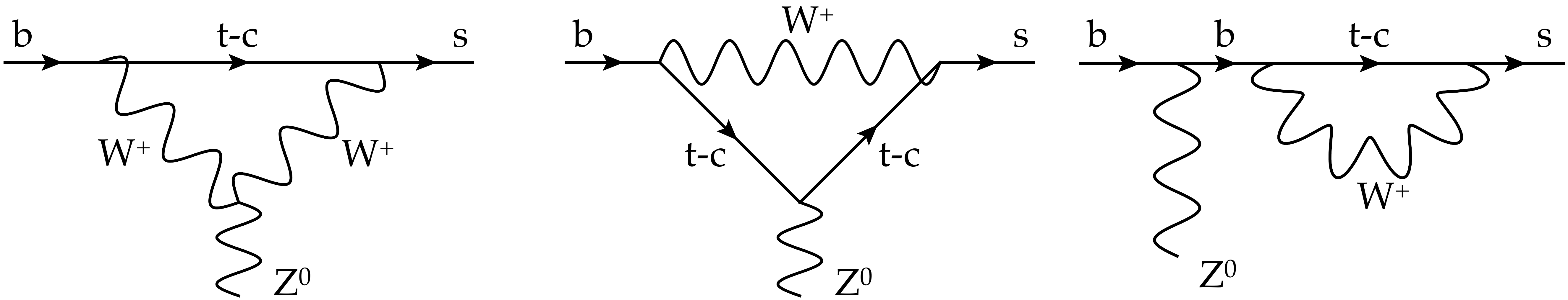
$Q(H_d) = 1,\ Q(b_R) = 1$ forbids the operator $(\bar{b}_R s_L)(\bar{b}_R s_L)$

This protects ΔM_s. Contribution of symmetry-breaking terms small

[MG, Jäger, Nierste, Trine '09]



Z-Penguin and Beyond



$$\sum_i V_{is}^* V_{id} F(x_i) = V_{tb}^* V_{ts} (F(x_t) - F(x_c)) \overset{x_t \to \infty}{=} V_{tb}^* V_{ts} x_t$$

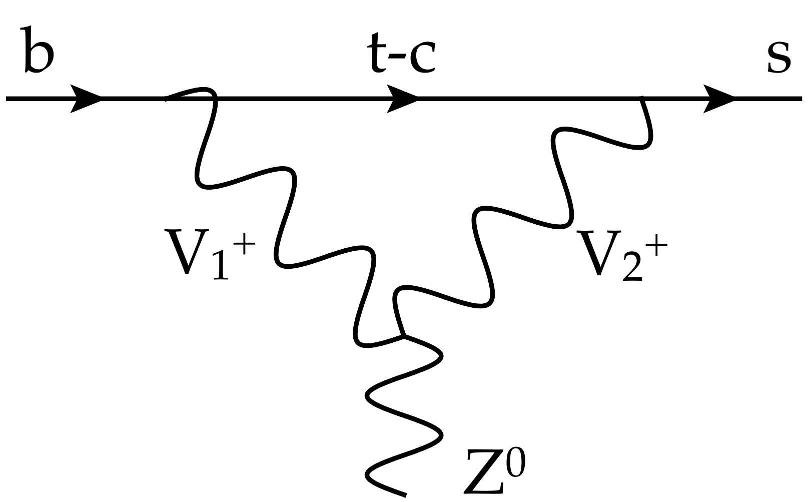
To get a finite result $M_W = M_Z \cos(\theta_w)$ has to hold

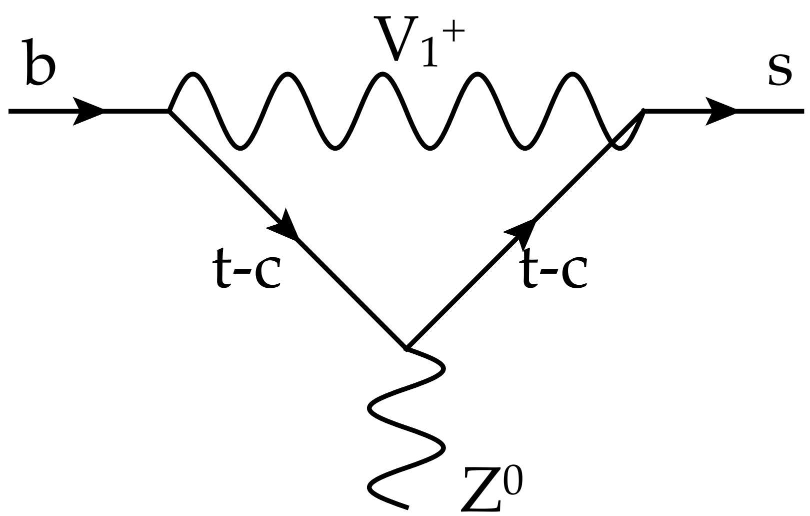
Involves the Z-Boson coupling to
1, up and 2, down-quarks and 3 W-Bosons
as well as 4, W-Boson couplings to quarks

Can we study all standard model extensions at once?

Generic Extensions

Let us consider a theories with arbitrary number of $W^{\pm}$





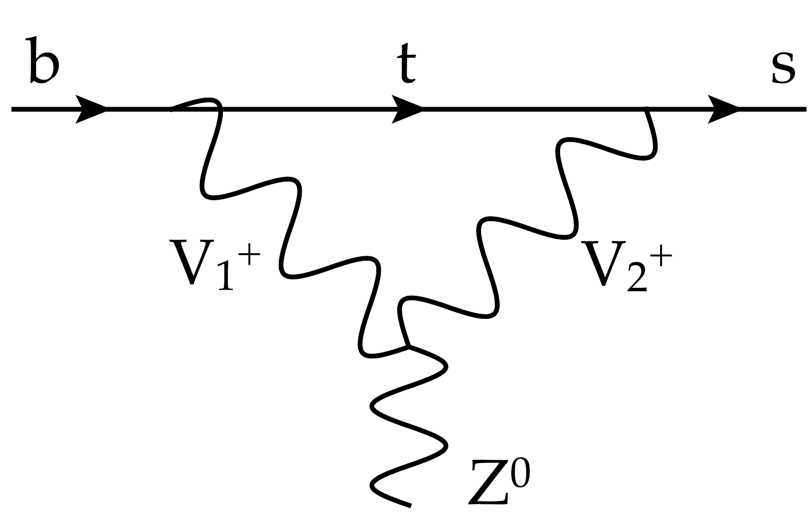
$$\mathcal{L}_3 = \sum_{f_1 f_2 v_1 \sigma} g^{\sigma}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{f}_1 \gamma^{\mu} P_{\sigma} f_2 + \sum_{v_1 v_2 v_3} g_{v_1 v_2 v_3} \Big[V_1, V_2, V_3\Big]$$

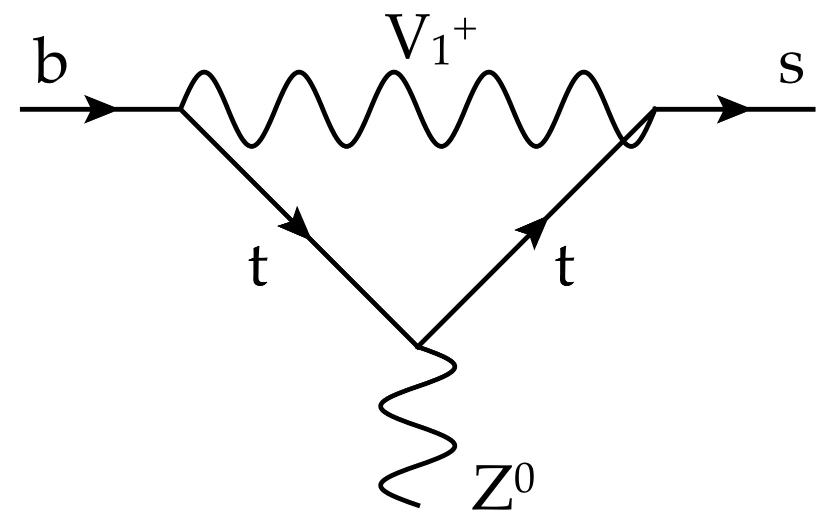
$$\Big[V_1, V_2, V_3\Big] = \tfrac{i}{6}\Big(V_{1,\mu} V_{2,\nu}\, \partial^{[\mu} V_3^{\nu]} + V_{3,\mu} V_{1,\nu}\, \partial^{[\mu} V_2^{\nu]} + V_{2,\mu} V_{3,\nu}\, \partial^{[\mu} V_1^{\nu]}\Big)$$

We assume a perturbative unitary theory
(vector-bosons are R_{ξ}-gauge fixed)

Fixes Goldstone-Boson interactions

Generic Result





$$\sum_{f_1 v_1 v_2} k^{\sigma}_{f_1 v_1 v_2} C_0\left(m_{f_1}, M_{v_1}, M_{v_2}\right) + \tilde{k}^{\sigma}_{f_1 v_1 v_2}\left(\tilde{C}_0\left(m_{f_1}, M_{v_1}, M_{v_2}\right) + \tfrac{1}{2}\right) + k'^{\sigma}_{f_1 v_1 v_2} +$$

$$\sum_{f_1 f_2 v_1} k^{\sigma}_{f_1 f_2 v_1} C_0\left(m_{f_1}, m_{f_2}, M_{v_1}\right) + \tilde{k}^{\sigma}_{f_1 f_2 v_1}\left(\tilde{C}_0\left(m_{f_1}, m_{f_2}, M_{v_1}\right) - \tfrac{1}{2}\right) + k'^{\sigma}_{f_1 f_2 v_1}$$

C_0 is a finite loop function – only $\widetilde{C}_0$ is divergent $\displaystyle\sum_{f_1 v_1 v_2} \tilde{k}^{\sigma}_{f_1 v_1 v_2} + \sum_{f_1 f_2 v_1} \tilde{k}^{\sigma}_{f_1 f_2 v_1} = 0$

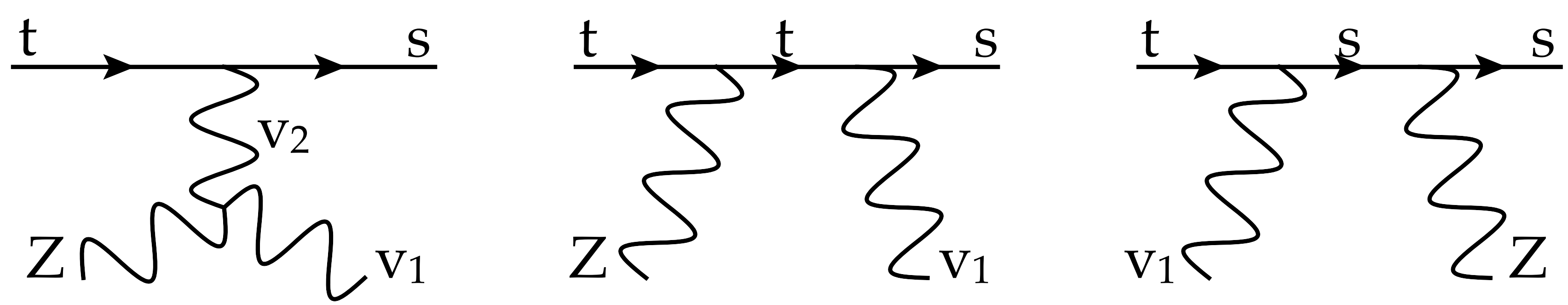
$$\sum_{v_1}\left(-\sum_{v_2}\frac{M^2_{v_1} + M^2_{v_2} - M^2_Z}{4M^2_{v_1}M^2_{v_2}} g_{Z v_1^+ v_2^-} g^{\sigma}_{v_1^- \bar{b} t} g^{\sigma}_{v_2^+ \bar{t} s} - \frac{1}{2M^2_{v_1}} g^{\sigma}_{Z\bar{s}s} g^{\sigma}_{v_1^- \bar{b} t} g^{\sigma}_{v_1^+ \bar{t} s}\right.$$

$$\left. + \frac{1}{2M^2_{v_1}} g^{\bar{\sigma}}_{Z\bar{t}t} g^{\sigma}_{v_1^- \bar{b} t} g^{\sigma}_{v_1^+ \bar{t} s}\right) = 0$$

(using GIM & Universality)

Renormalisation

Let us consider a theories with arbitrary number of $W^{\pm}$



Unitarity or STIs lead to the following constraints on the couplings:

$$g^{\sigma}_{v_1^+\bar{t}s}\, g^{\sigma}_{Z\bar{t}t} \to \sum_{v_2} g_{Z v_1^+ v_2^-}\, g^{\sigma}_{v_2^+\bar{t}s} + g^{\sigma}_{v_1^+\bar{t}s}\, g^{\sigma}_{Z\bar{s}s}$$

plus the one $\propto m_t$:

$$g^{\sigma}_{v_1^+\bar{t}s}\, g^{\bar{\sigma}}_{Z\bar{t}t} \to \sum_{v_2} \frac{M^2_{v_1} - M^2_Z}{2M^2_{v_2}}\, g_{Z v_1^+ v_2^-}\, g^{\sigma}_{v_2^+\bar{t}s} + \frac{1}{2} g^{\sigma}_{v_1^+\bar{t}s} \left(g^{\sigma}_{Z\bar{s}s} + g^{\sigma}_{Z\bar{t}t} \right)$$

Which imply the finiteness of the Z-Penguin through:

$$g^{\sigma}_{v_1^+\bar{t}s}\, g^{\bar{\sigma}}_{Z\bar{t}t} \to \sum_{v_2} \frac{M^2_{v_1} + M^2_{v_2} - M^2_Z}{2M^2_{v_2}}\, g_{Z v_1^+ v_2^-}\, g^{\sigma}_{v_2^+\bar{t}s} + g^{\sigma}_{v_1^+\bar{t}s}\, g^{\sigma}_{Z\bar{s}s}$$

generalisation of the SM $M_W = M_Z \cos(\theta_w)$ renormalisation

Renormalised Result

Applying the unitarity constraints on the full result yields

$$\sum_{\nu_1 \nu_2} g_{Z\nu_1^+\nu_2^-} g^L_{\nu_1^- \bar{b} t} g^L_{\nu_2^+ \bar{t} s} F_1(m_t, M_{\nu_1}, M_{\nu_2}) + \sum_{\nu_1} g^L_{Z\bar{s}s} g^L_{\nu_1^- \bar{b} t} g^L_{\nu_2^+ \bar{t} s} F_0(m_t, M_{\nu_1})$$

a finite result for the (left-handed) Z-Penguin

The result can be extended to include an arbitrary number of new scalar-bosons and fermions
[Brod, MG, Casagrande]

Can be used to classify and study new physics contributions to Wilson Coefficients

Could be useful to combine indirect and direct search results

Conclusions

Electroweak scheme ambiguity removed

Corrections small w.r.t. experimental error, significant w.r.t. to theory uncertainty

$B_s \rightarrow \mu^+ \mu^-$ can provide a precision probe of scalar, but also axial vector coupling interactions