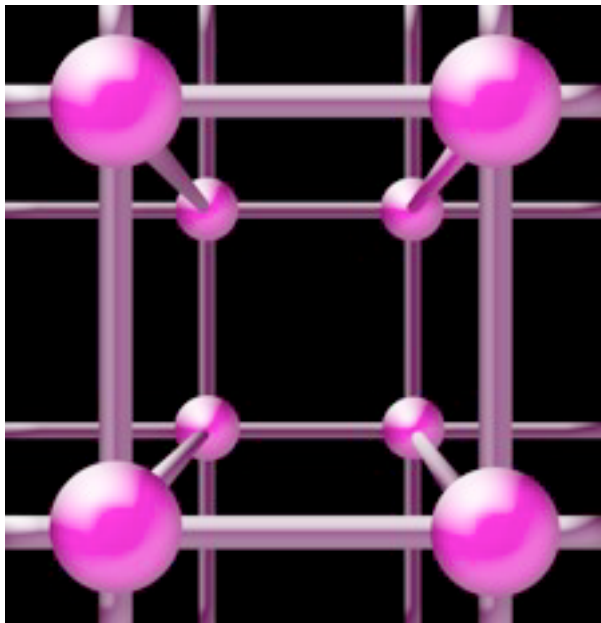


Lattice results for D and D_s leptonic and semileptonic decays

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University of Glasgow



UK Flavour 2013
Durham, UK
September 2013



Outline

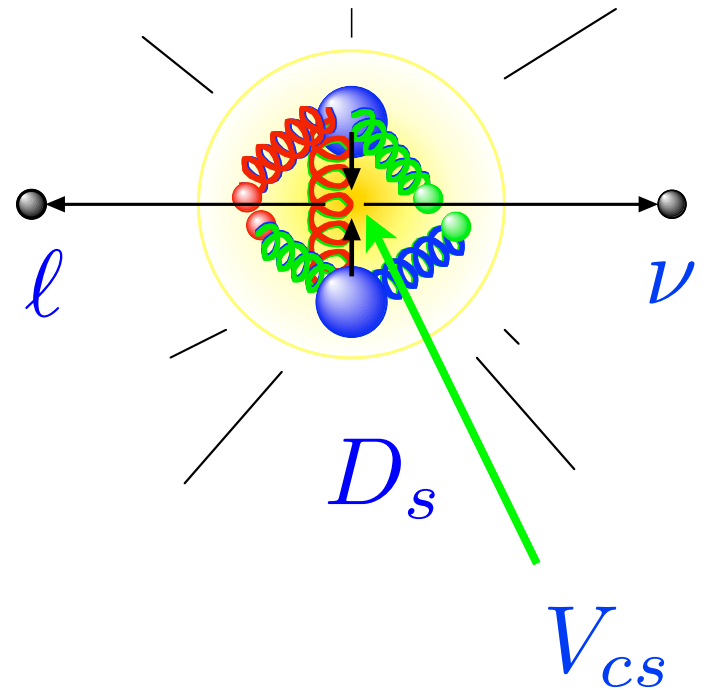
- D and D_s meson leptonic and semileptonic decays
 - motivation, CKM matrix elements
- Very brief introduction to lattice calculations
- Results
 - decay constants
 - semileptonic form factors
 - V_{cd} and V_{cs}
- Summary

Leptonic decays

- D or D_s meson decays to a lepton and its neutrino via a virtual W boson
- The branching fraction is given by

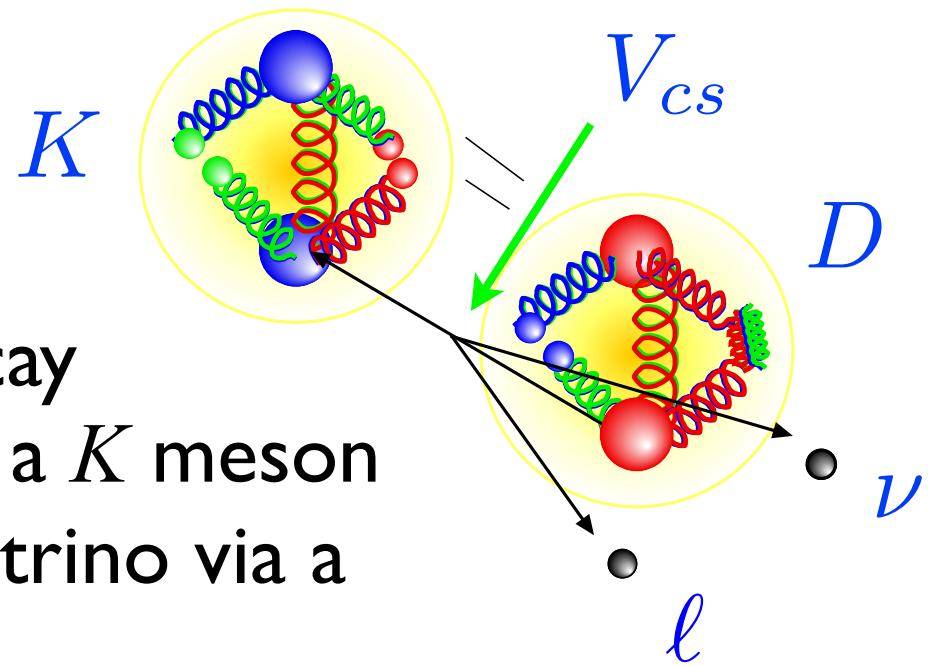
$$\mathcal{B}(D_s \rightarrow \ell \nu) = \frac{G_F^2}{8\pi} m_\ell^2 M_{D_s} \left(1 - \frac{m_\ell^2}{M_{D_s}^2} \right)^2 f_{D_s}^2 |V_{cs}|^2$$

- Decay constant f_{D_s} (or f_D) depends on the hadronic degrees of freedom only and can be calculated in Lattice QCD $\rightarrow$ can determine $|V_{cs}|$ (or $|V_{cd}|$)



Semileptonic decays

- Consider a semileptonic decay where a D meson decays to a K meson (or a π), a lepton and its neutrino via a virtual W boson



- The partial decay rate is (both mesons are pseudoscalars)

$$\frac{d\Gamma^{D \rightarrow K}}{dq^2} = \frac{G_F^2 p^3}{24\pi^3} |V_{cs}|^2 |f_+^{D \rightarrow K}(q^2)|^2$$

q^2 =four-momentum transfer

p =momentum of daughter meson

- Calculating the form factor f_+ in lattice QCD allows to determine the CKM matrix element $|V_{cs}|$ (or $|V_{cd}|$)

CKM matrix elements

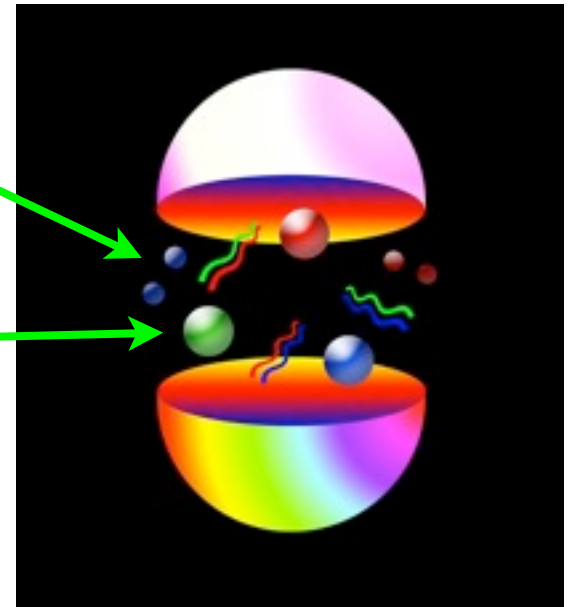
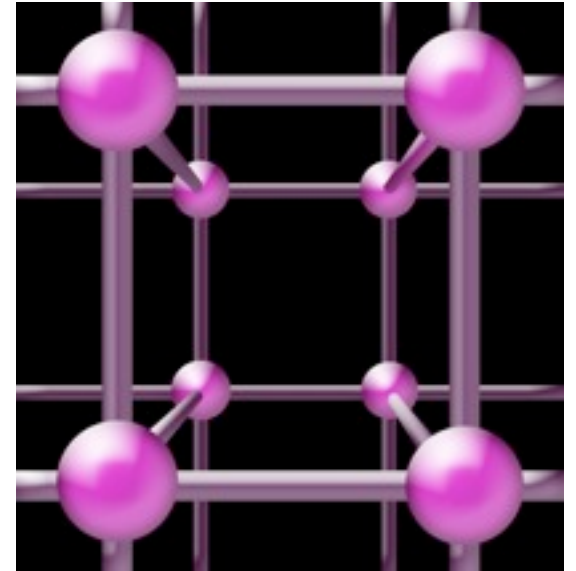
- Several processes can be used to extract the same CKM matrix element involving different experimental and theory input
 - leptonic decays
 - semileptonic decays
 - neutrino experiments
- Allows for cross checks and consistency tests of the Standard Model, constraints/test for new physics (similar methods can be used to study B decays)
- Need non-perturbative theoretical calculation of form factors and decay constants → Lattice QCD

Lattice QCD

= fully nonperturbative QCD calculation

RECIPE

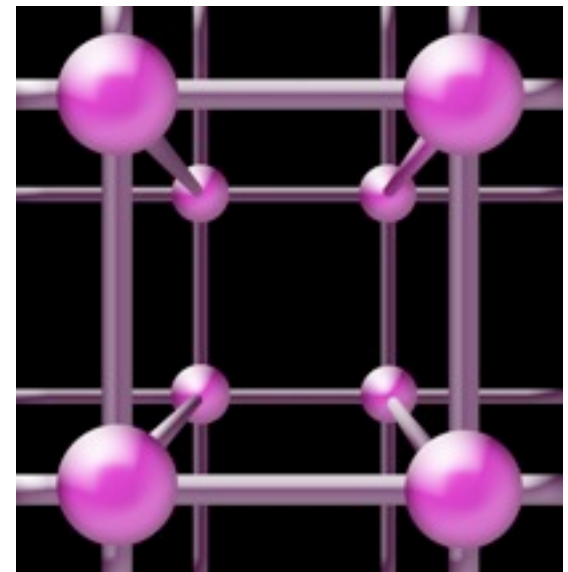
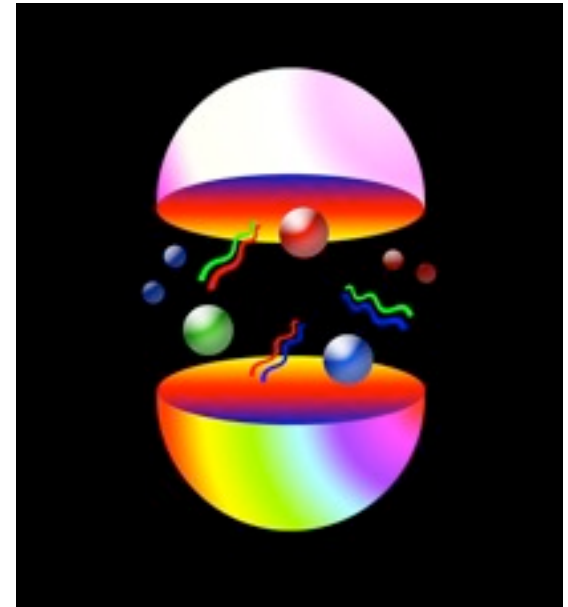
- Generate sets of gluon fields for Monte Carlo integration of path integral (including effects of u, d, s and c sea quarks)
- Calculate averaged “hadron correlators” from valence quark propagators
- Fit as a function of time to obtain masses and simple matrix elements



Lattice QCD **RECIPE**

continued

- Determine lattice spacing a and fix m_q using experimental information (often meson masses) to get results in physical units
- extrapolate to $a=0$, physical u/d quark mass for real world
 - lattices with physical $m_{u,d}$ now appearing: chiral extrapolation becoming a small correction

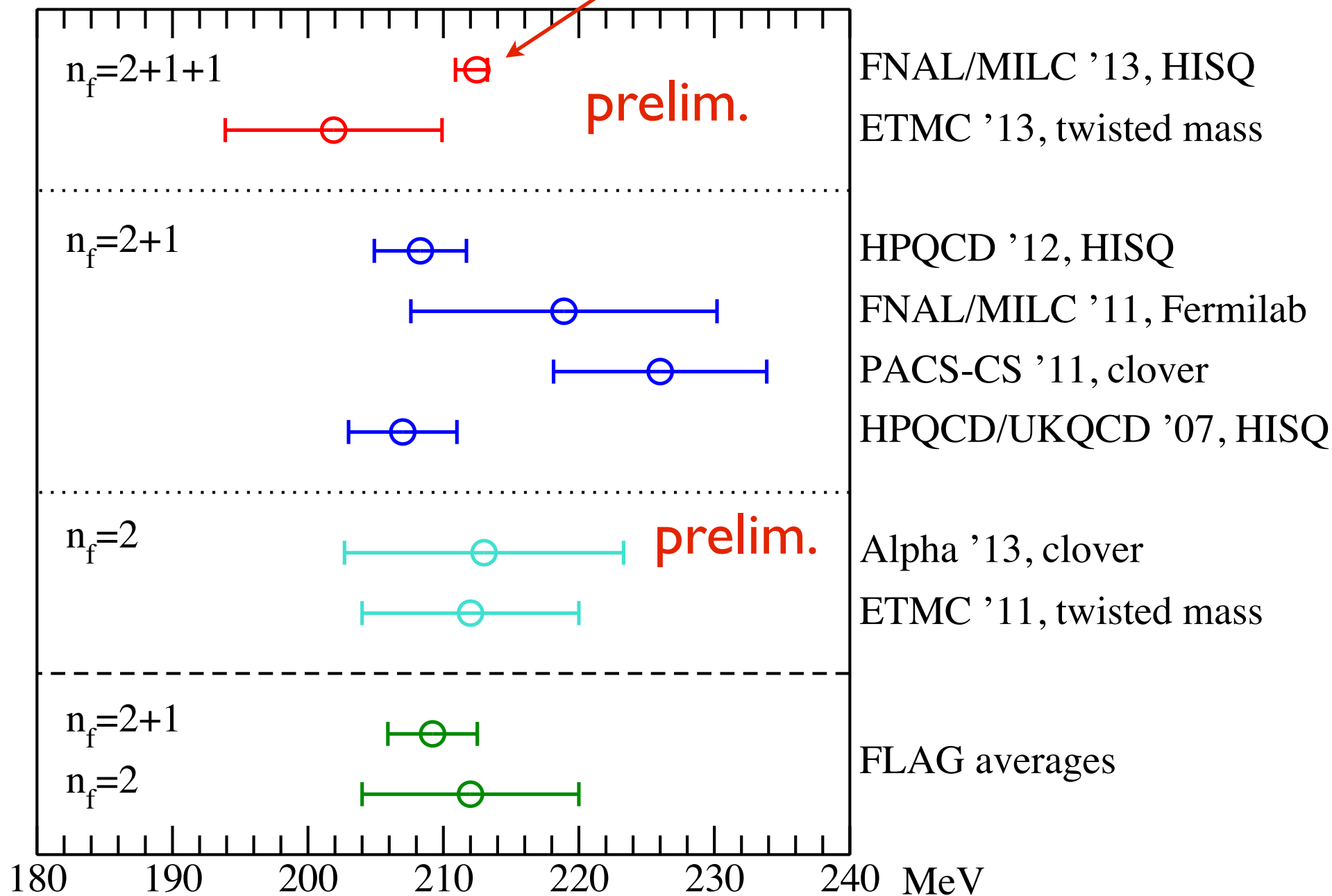


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 a

Leptonic decays, decay constants

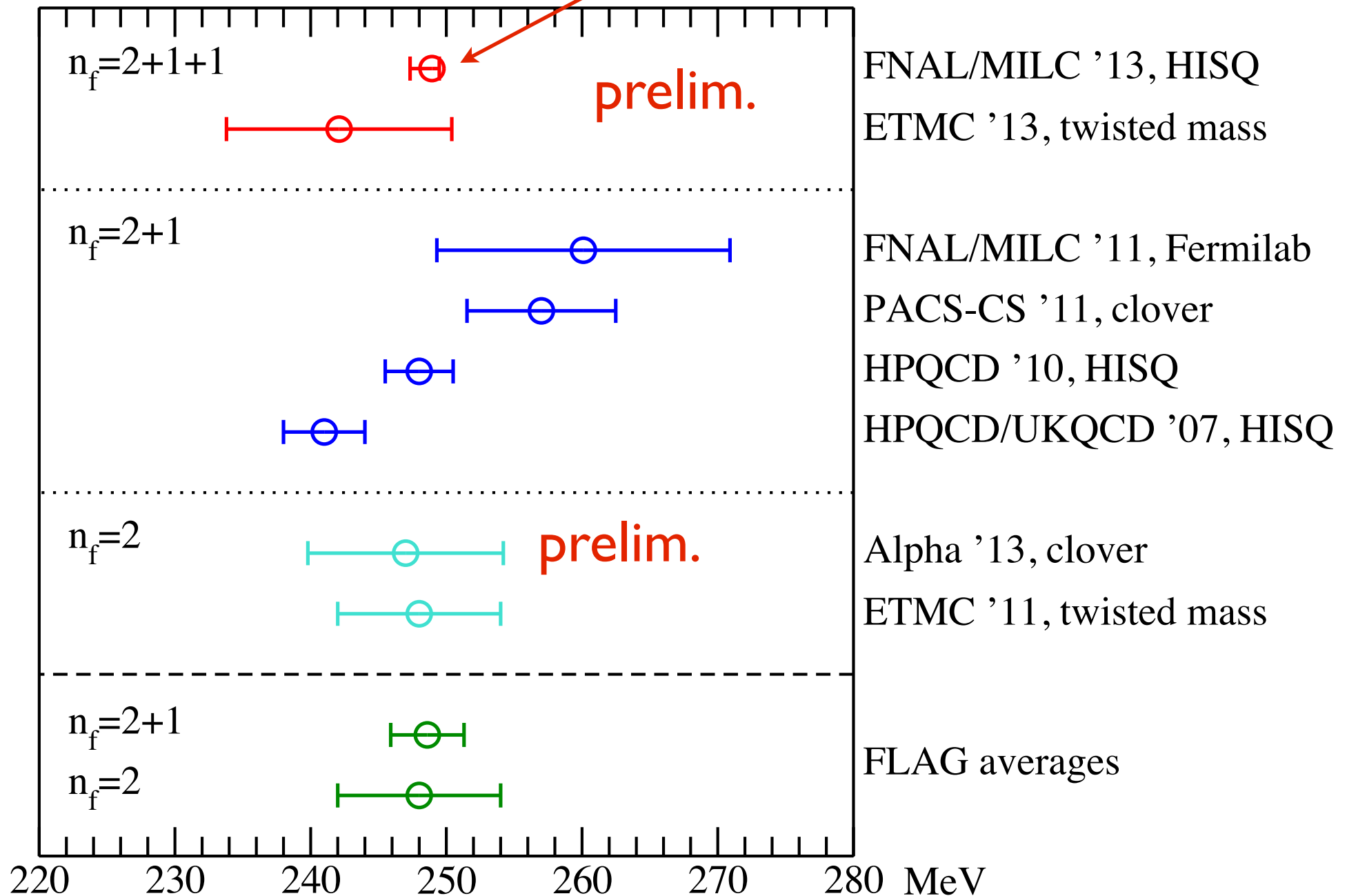
Current status: f_D

$$f_D = 212.5 \pm 0.5_{\text{stat}} \pm 1.5_{\text{syst}} \text{ MeV}$$



Current status: f_{D_s}

$$f_{D_s} = 248.9 \pm 0.2_{\text{stat}} \pm 1.6_{\text{syst}} \text{ MeV}$$



History of D_s decay constant

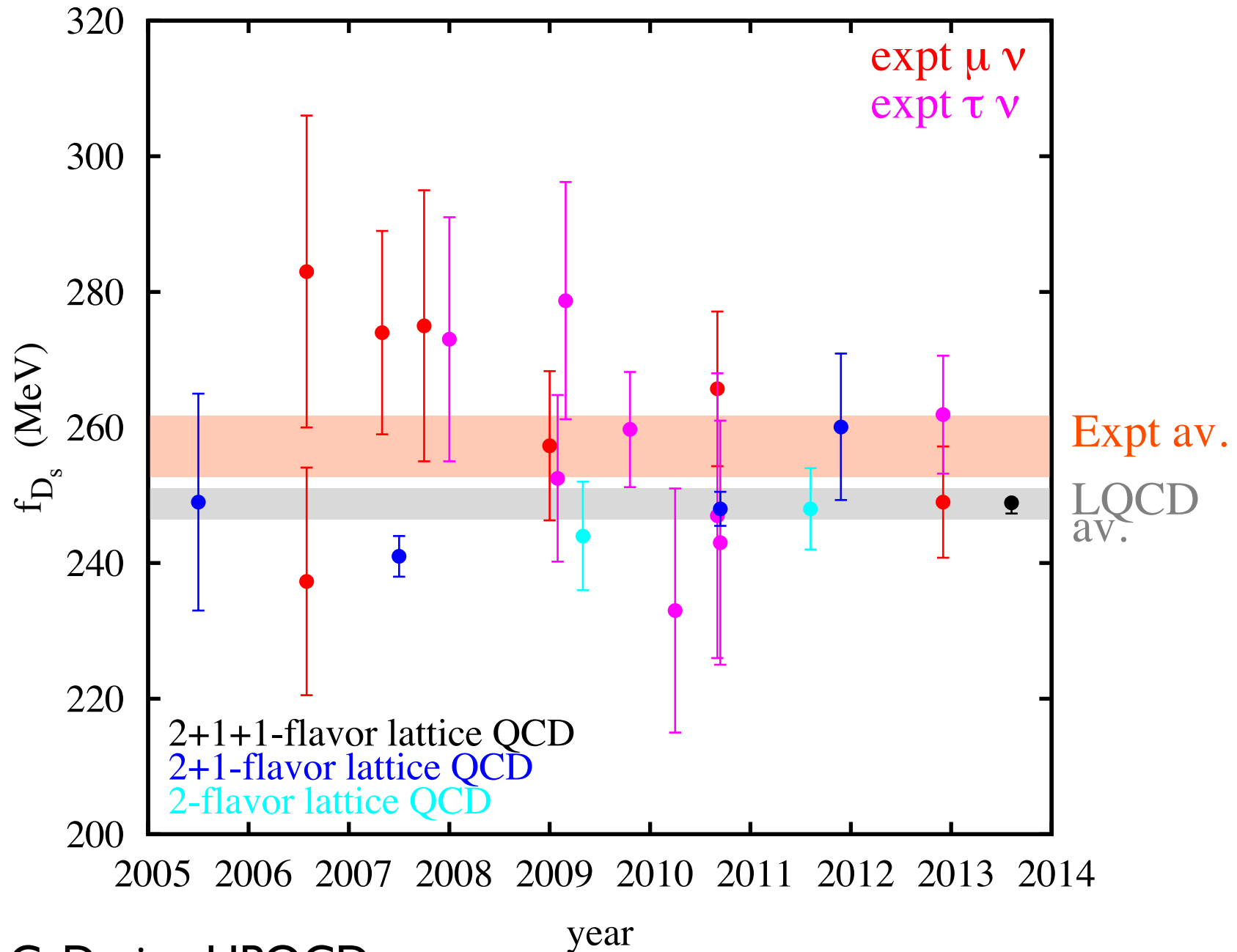
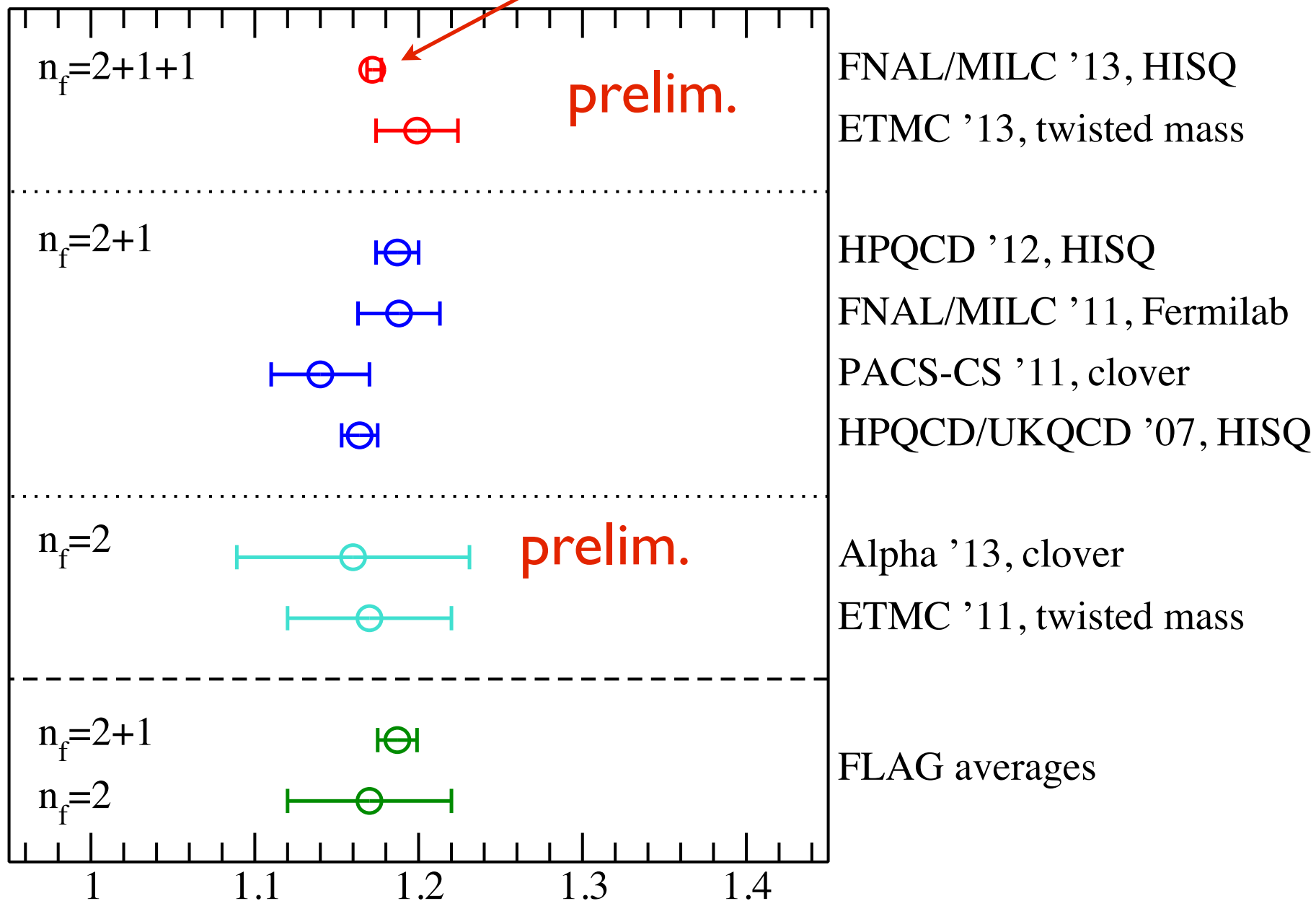


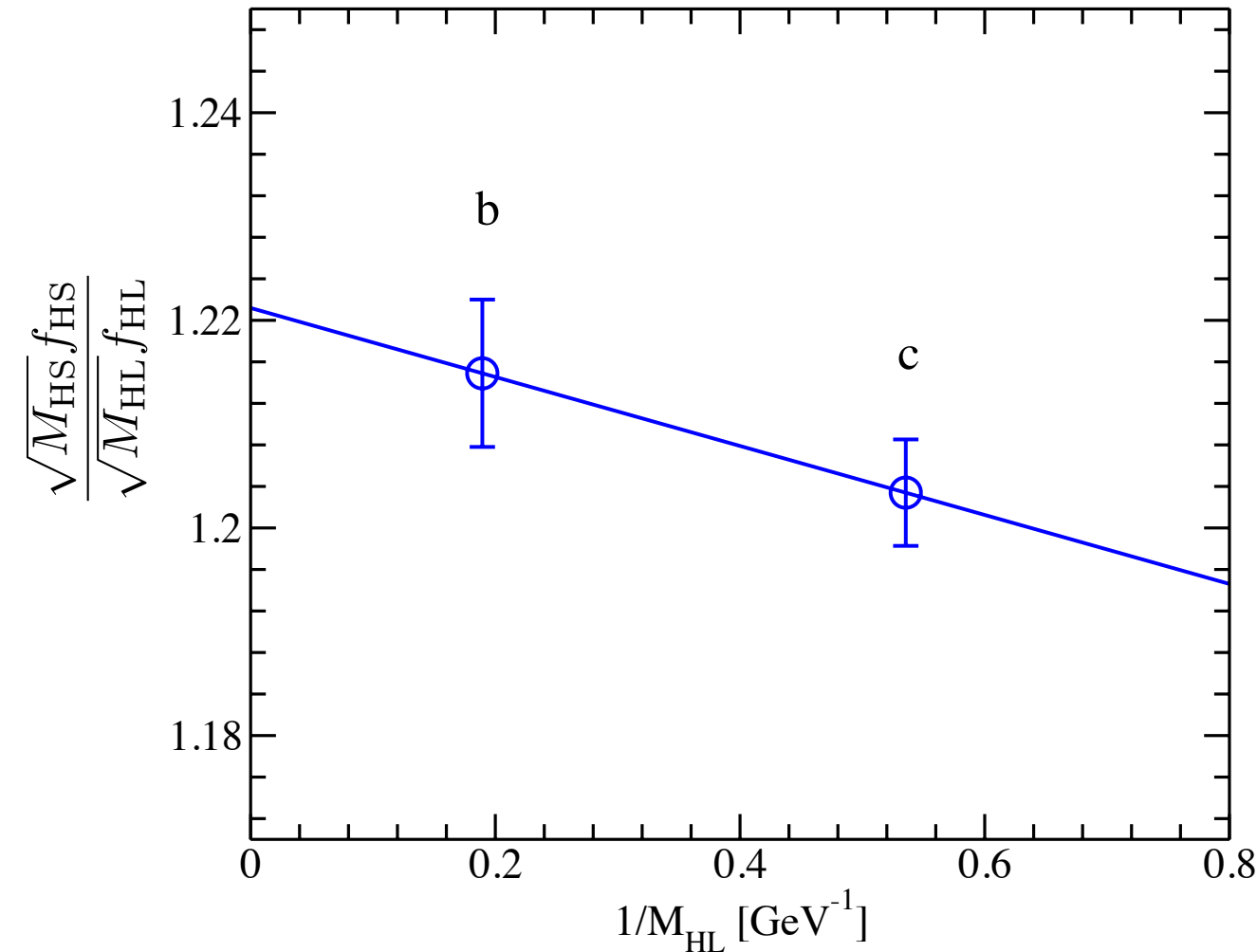
Fig. by C. Davies, HPQCD

Current status: f_{D_s} / f_D

$$f_{D_s} / f_D = 1.1717(20)_{\text{stat}} \left({}^{+52}_{-25} \right)_{\text{syst}}$$



Ratios of heavy-strange and heavy-light meson decay constants



- New determinations of ratios of decay constants (by FNAL/MILC [Lattice 2013] and HPQCD [arXiv:1302.2644], respectively)

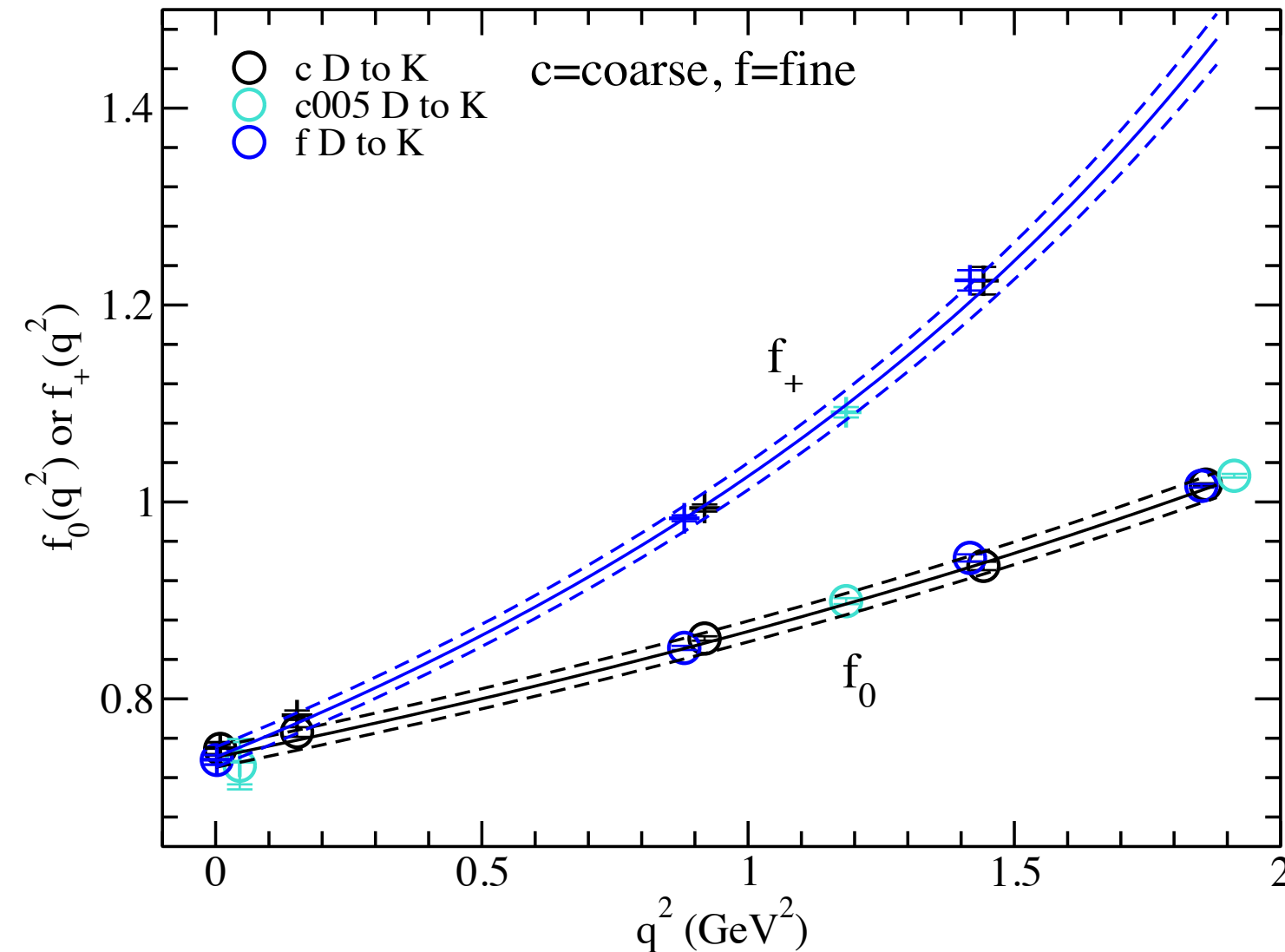
$$\frac{f_{D_s}}{f_D} = 1.172(5)$$

$$\frac{f_{B_s}}{f_B} = 1.205(7)$$

Fig. by C. Davies, HPQCD

Semileptonic decays, form factors

Shape of the form factors: $D \rightarrow K \ell \nu$



- Both scalar and vector form factor, f_0 and f_+ , as a function of q^2
- Kinematic constraint $f_+(0) = f_0(0)$

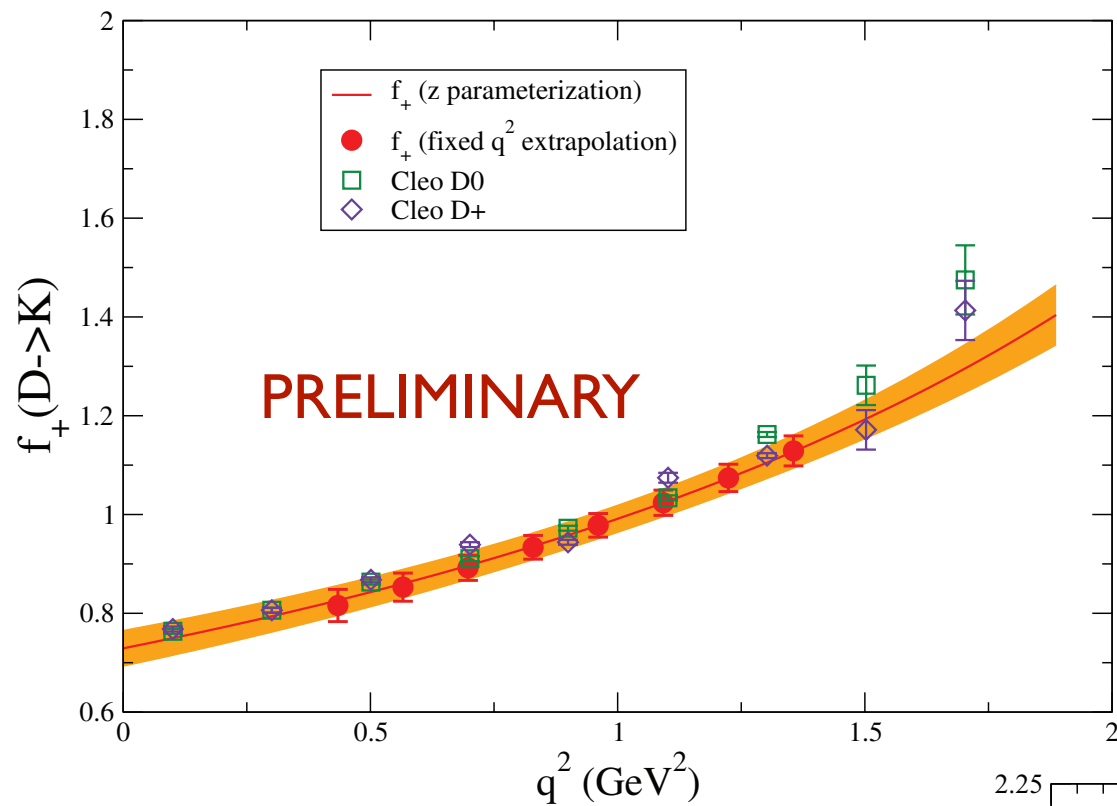
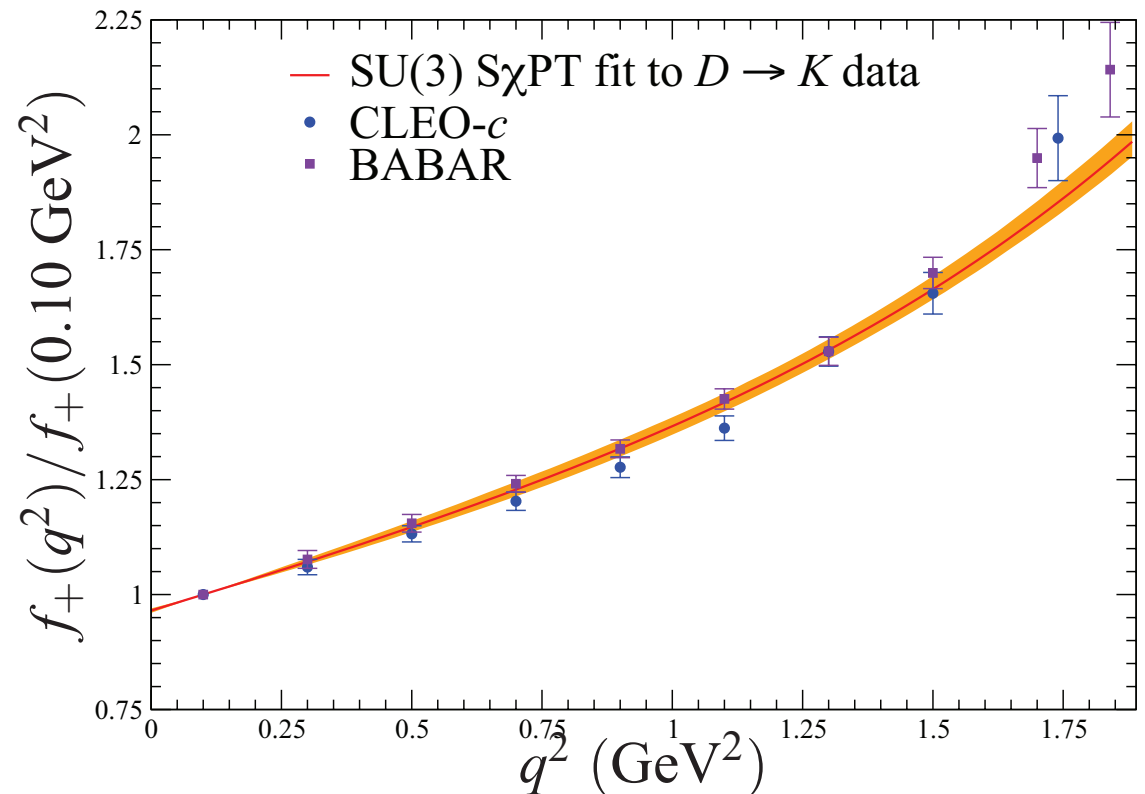


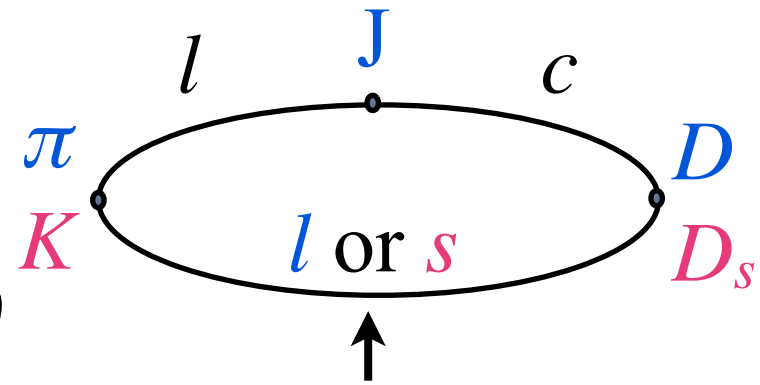
Fig. by F. Sanfilippo,
talk at Lattice 2013,
ETMC

Fig. by FNAL/MILC,
PoS(Lattice2012)272

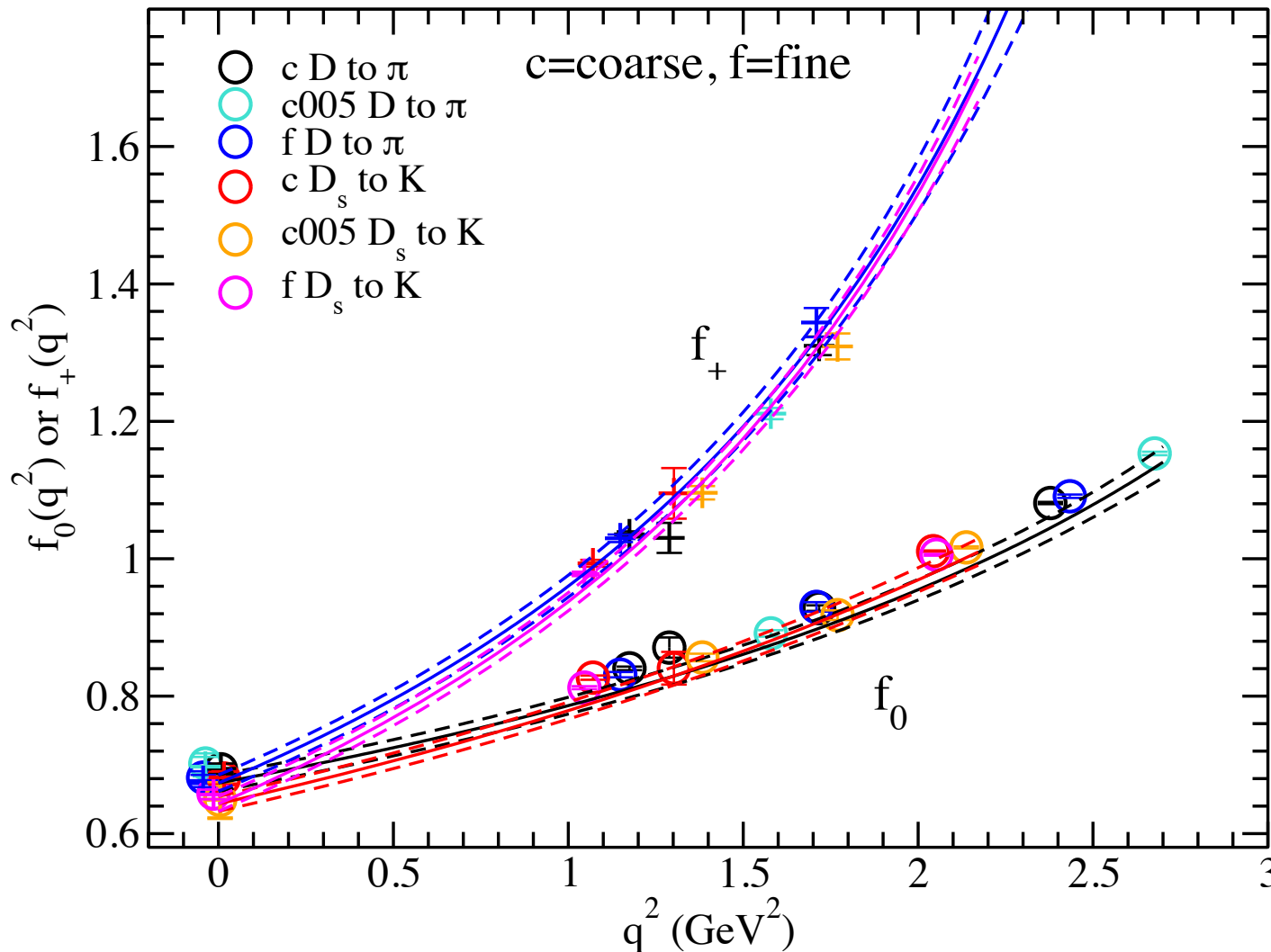


Shape of the FFs:

$D \rightarrow \pi \ell \nu$ and $D_s \rightarrow K \ell \nu$



↑
spectator quark



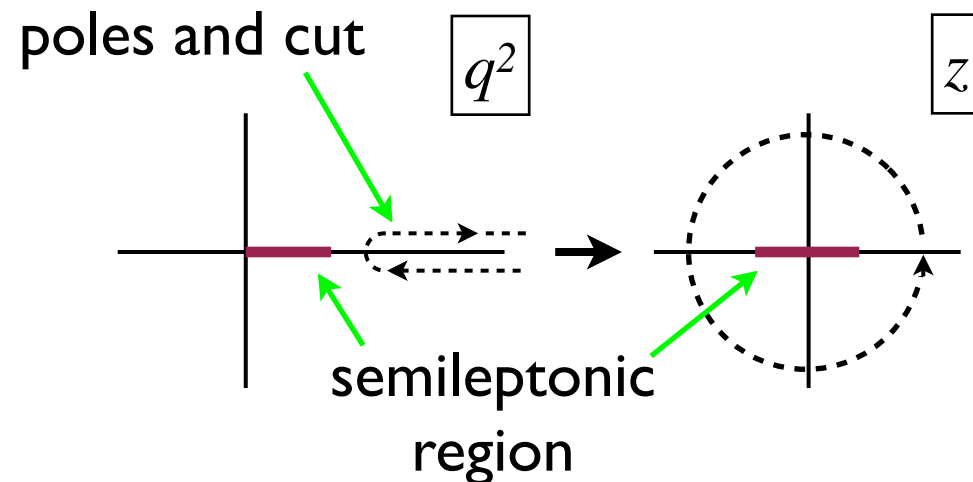
- Different spectator quark - light vs. strange
- Both decays experimentally accessible
- The shapes of the FFs are same within few %

The z-expansion

- Remove the poles

$$\tilde{f}_0^{D \rightarrow K}(q^2) = \left(1 - \frac{q^2}{M_{D_{s0}^*}^2}\right) f_0^{D \rightarrow K}(q^2),$$

$$\tilde{f}_+^{D \rightarrow K}(q^2) = \left(1 - \frac{q^2}{M_{D_s^*}^2}\right) f_+^{D \rightarrow K}(q^2)$$

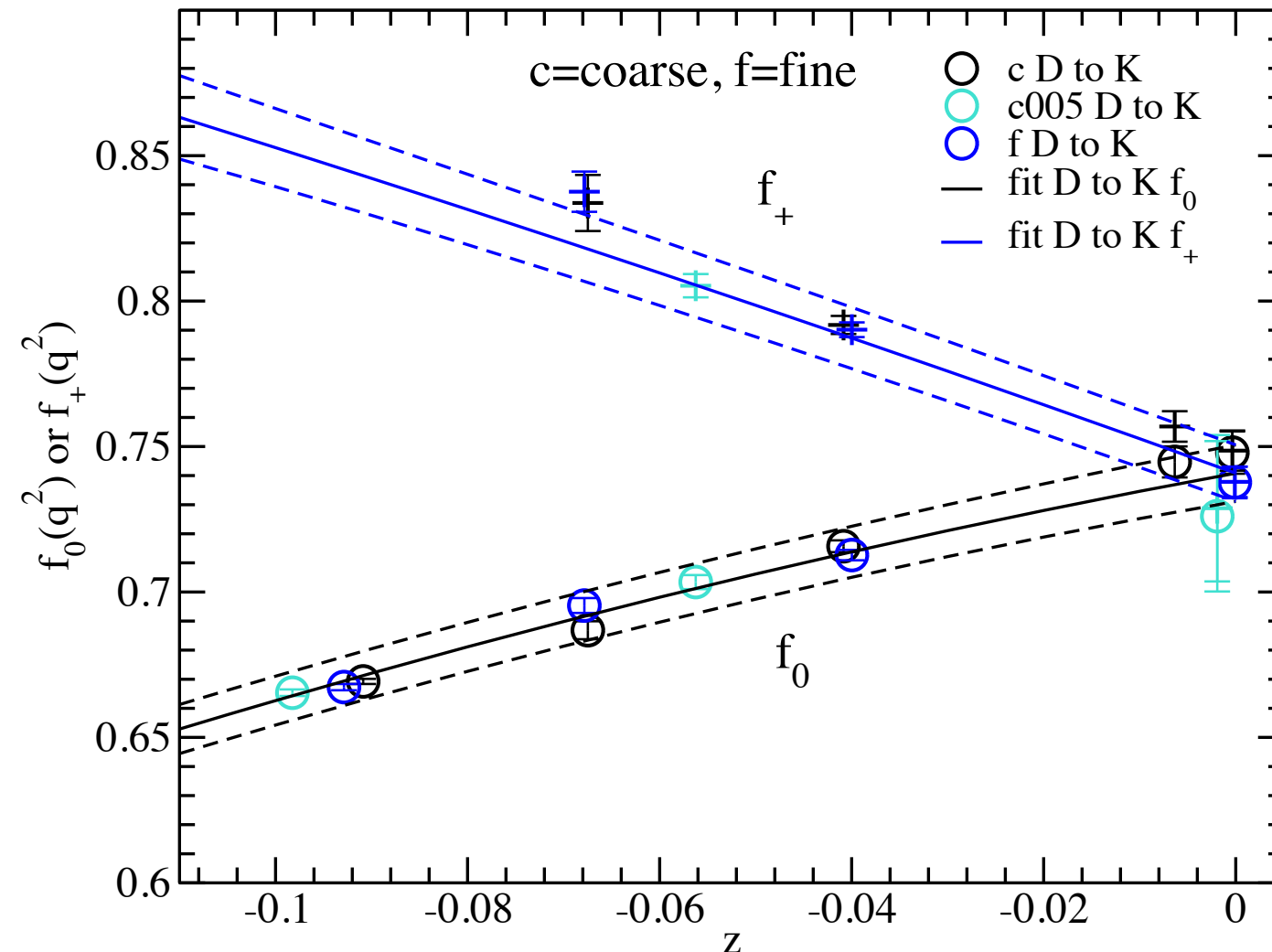


- Convert to z variable and fit $\tilde{f}$ as power series in z

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_D + m_K)^2,$$

$$\tilde{f}_0^{D \rightarrow K}(z) = \sum_{n \geq 0} c_n(a) z^n, \quad \tilde{f}_+^{D \rightarrow K}(z) = \sum_{n \geq 0} b_n(a) z^n, \quad c_0 = b_0$$

Continuum and chiral extrapolation

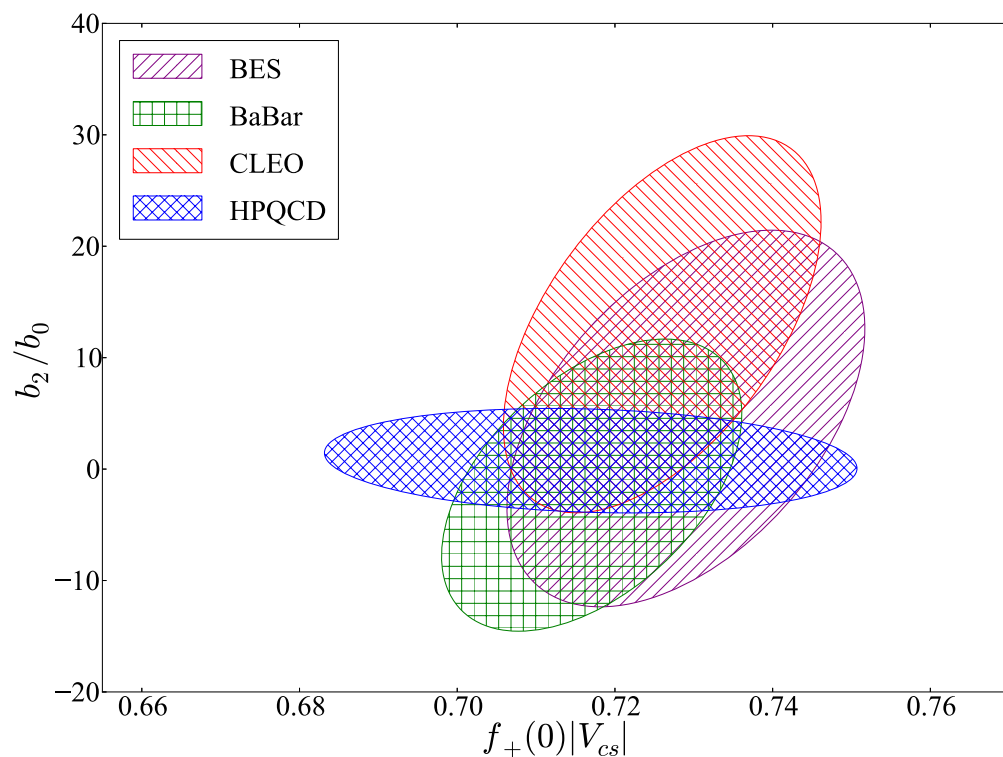
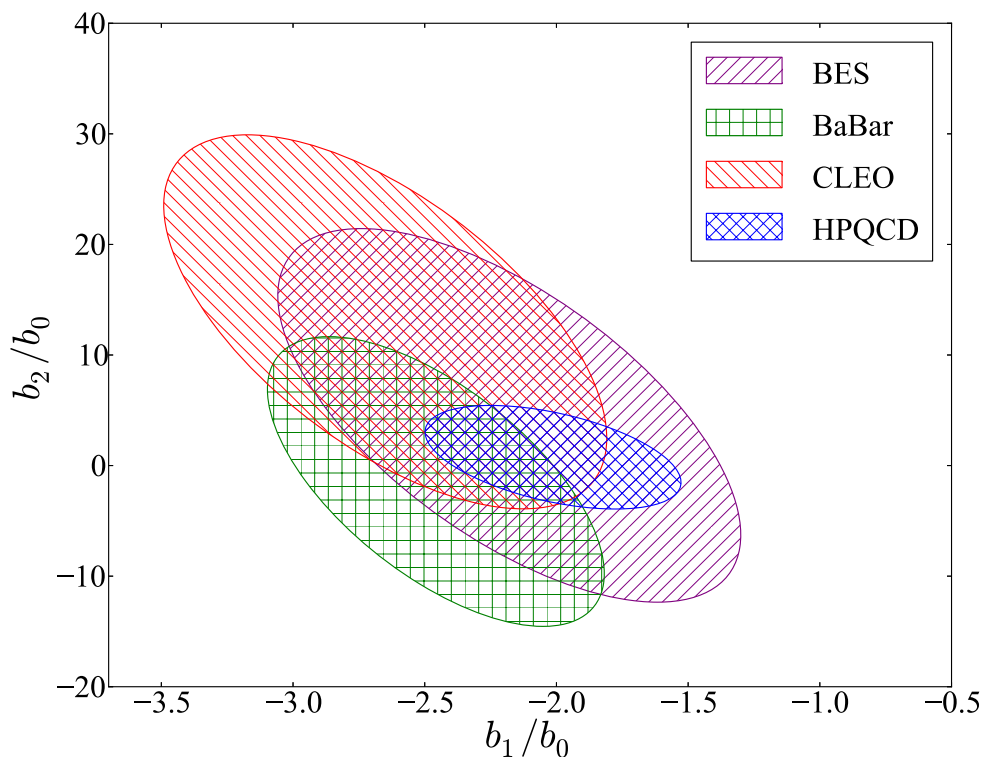
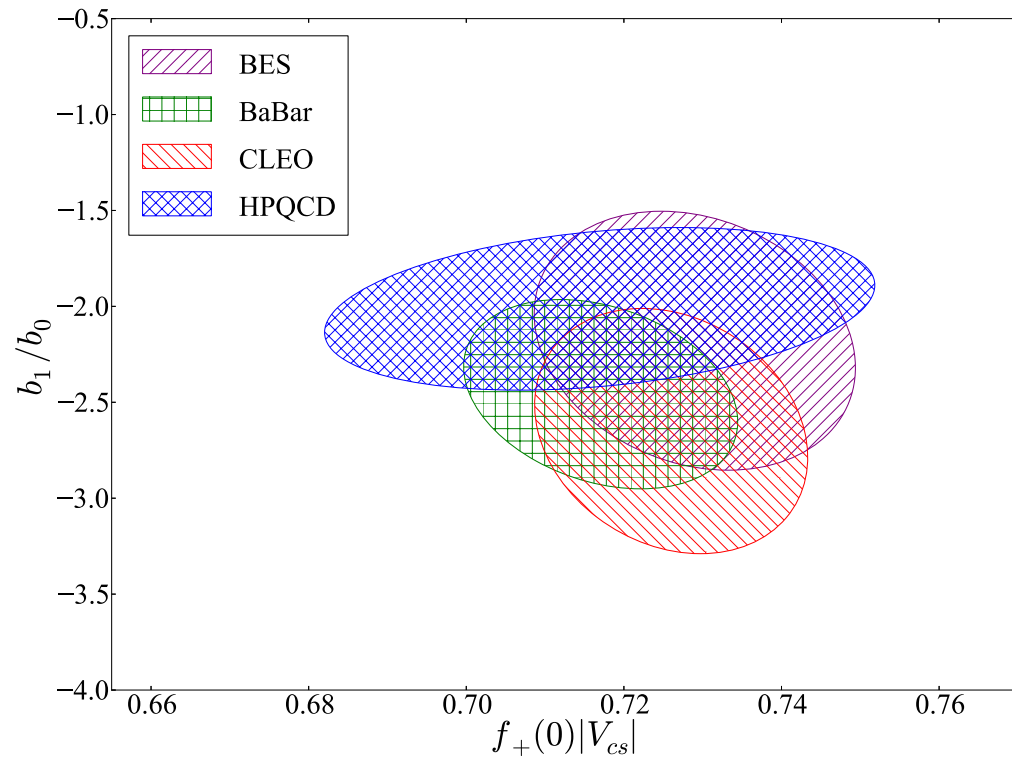


- Fit in z -space, including terms that depend on lattice spacing and quark masses
- Take $a=0$ and $m_q=m_q^{\text{phys}}$

f_+ shape parameters:

$$D \rightarrow K \ell \nu$$

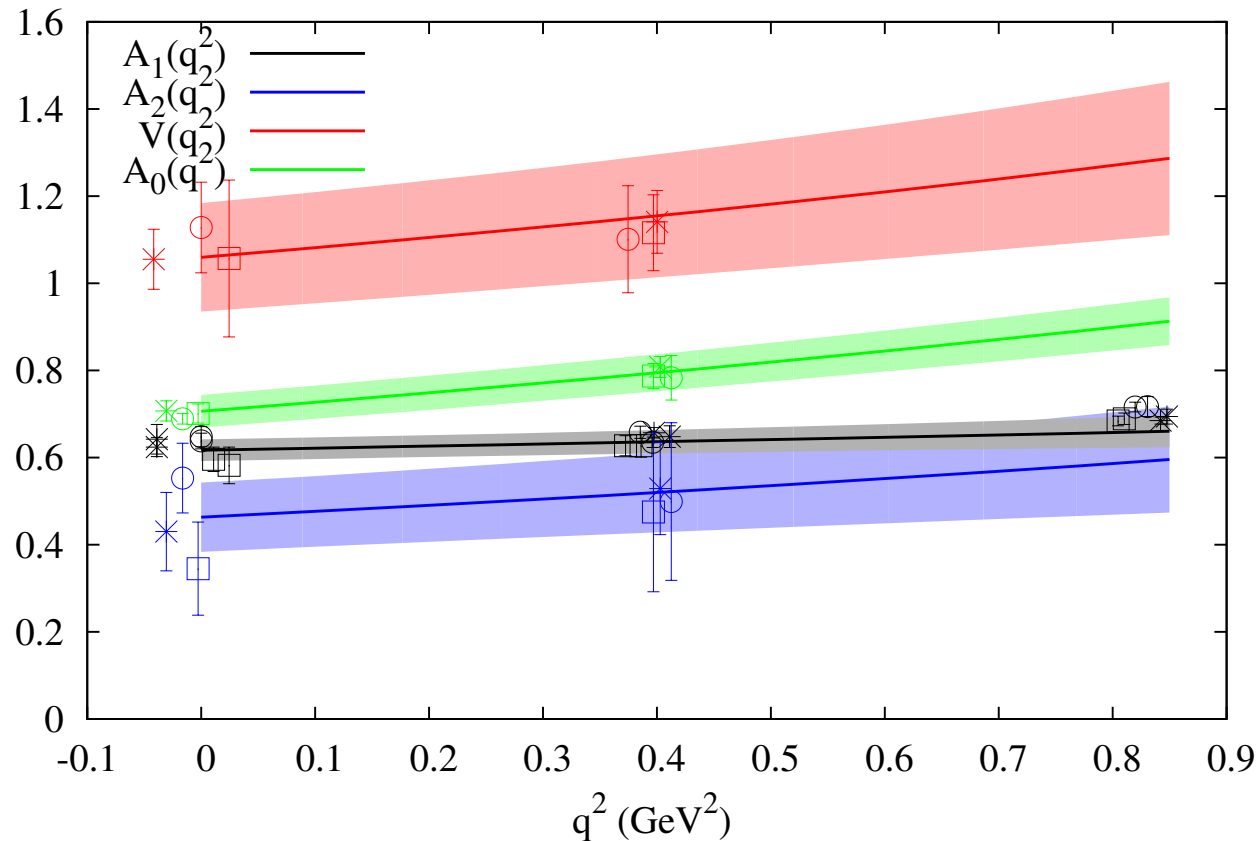
HPQCD, arXiv:1305.1462



Decays involving vector mesons

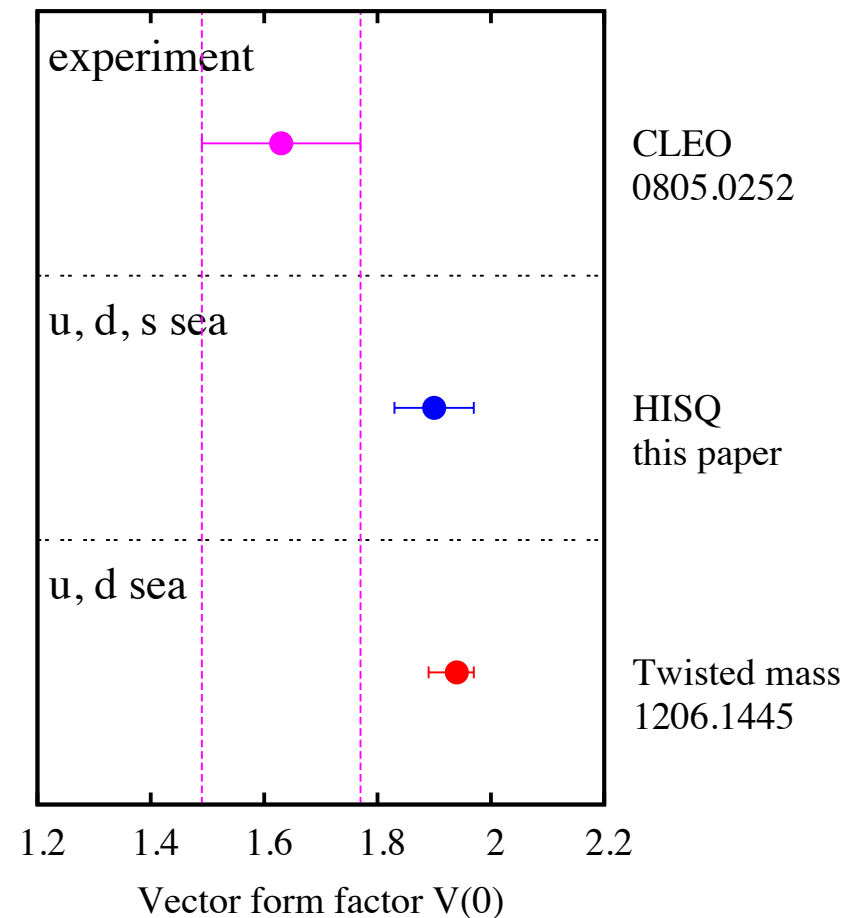
- Similar methods can be used to study decays that involve vector mesons

weak decay $D_s \rightarrow \phi \ell \nu$



Figs. by G. Donald, HPQCD, Lattice 2013
and arXiv:1208.2855

charmonium radiative
decay $J/\psi \rightarrow \eta_c \gamma$



$|V_{cs}|$ and $|V_{cd}|$

CKM elements from leptonic and semileptonic decays

- The branching fraction of D_s meson leptonic decay is

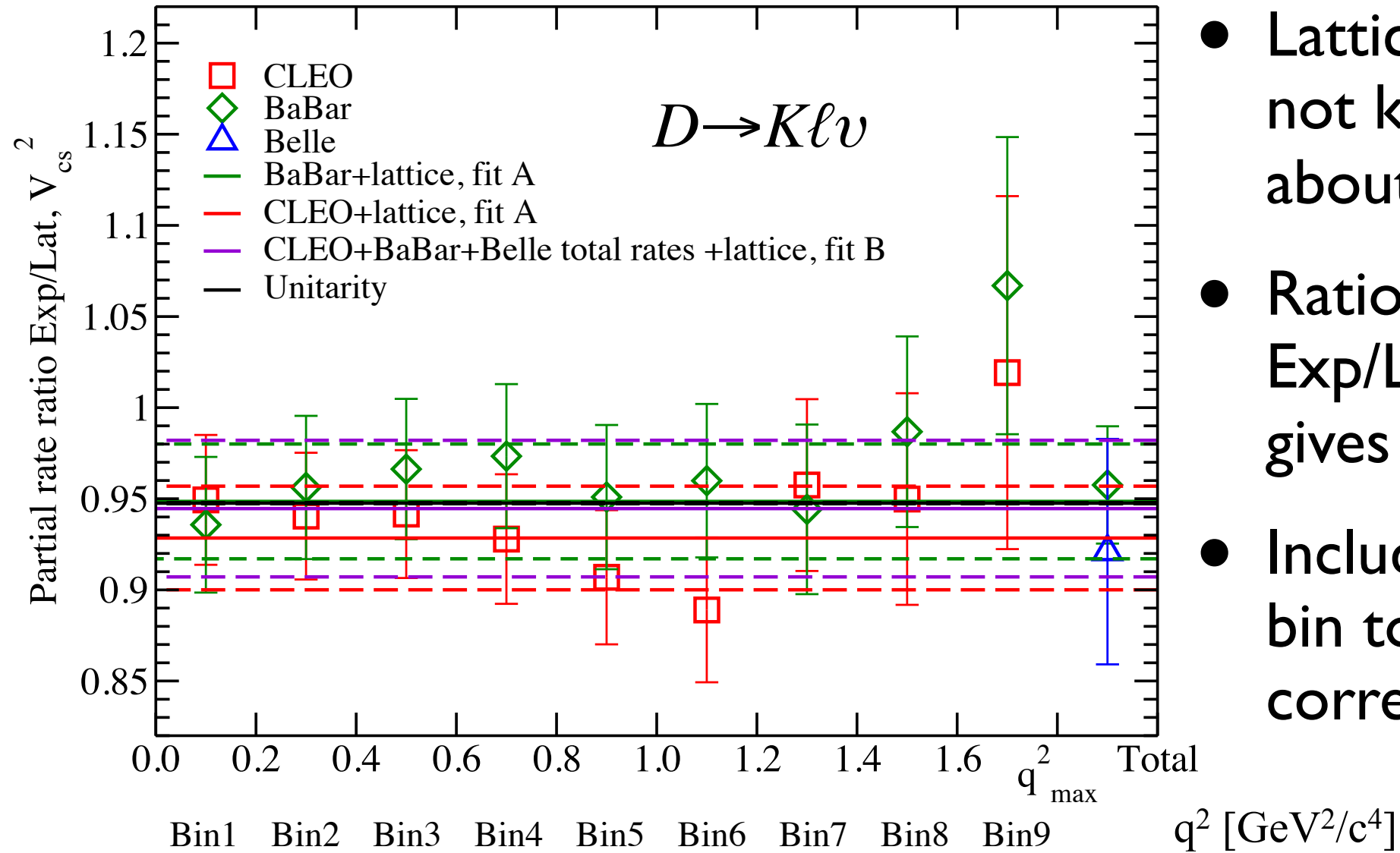
$$\mathcal{B}(D_s \rightarrow \ell \nu) = \frac{G_F^2}{8\pi} m_\ell^2 M_{D_s} \left(1 - \frac{m_\ell^2}{M_{D_s}^2}\right)^2 f_{D_s}^2 |V_{cs}|^2$$

- The partial decay rate of $D \rightarrow K$ semileptonic decay is

$$\frac{d\Gamma^{D \rightarrow K}}{dq^2} = \frac{G_F^2 p^3}{24\pi^3} |V_{cs}|^2 |f_+^{D \rightarrow K}(q^2)|^2$$

- Now we have the needed input from lattice QCD to determine CKM elements

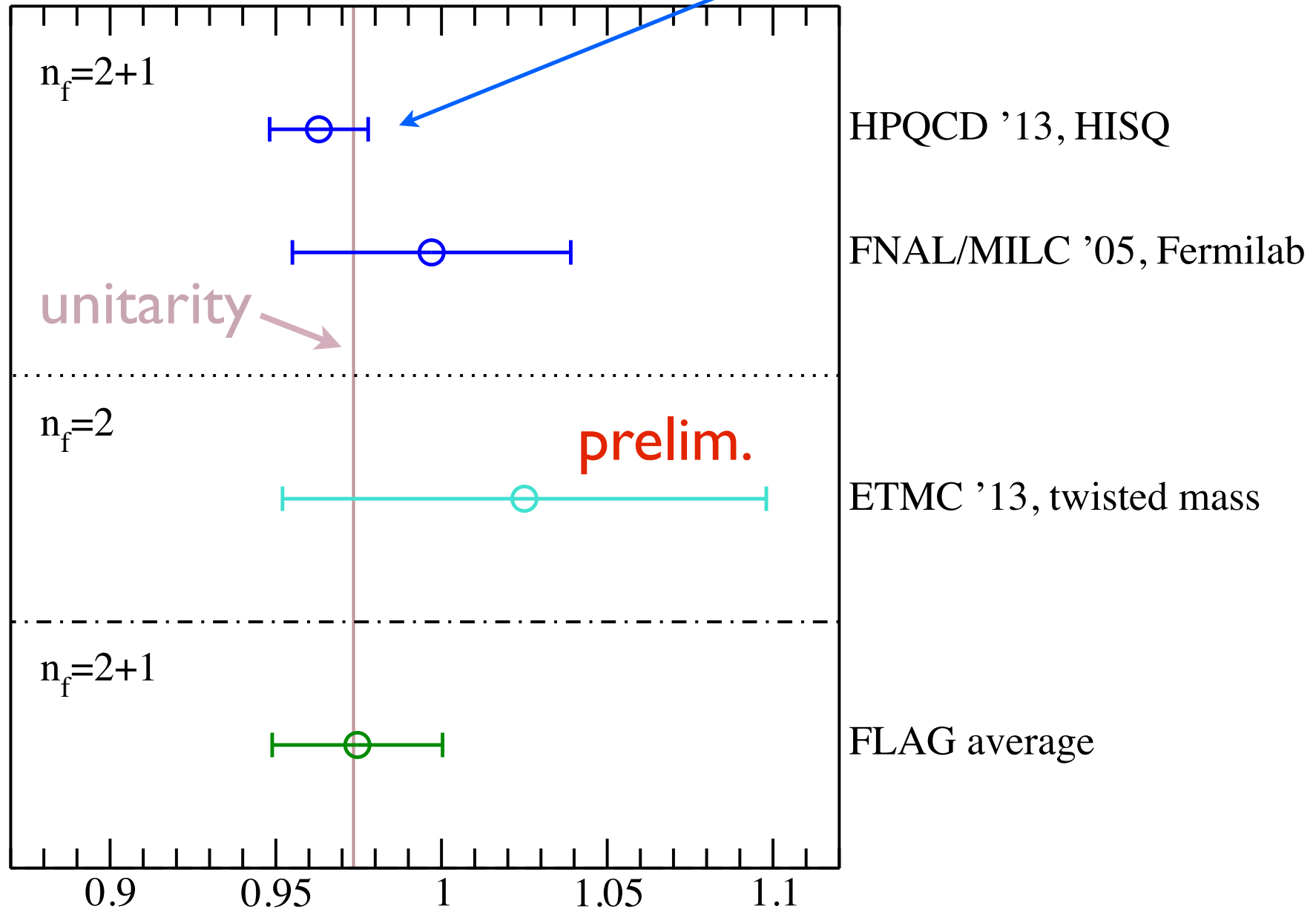
Extracting V_{cs}



- Lattice does not know about V_{cs}
- Ratio Exp/Lat gives V_{cs}^2
- Include bin to bin correlations

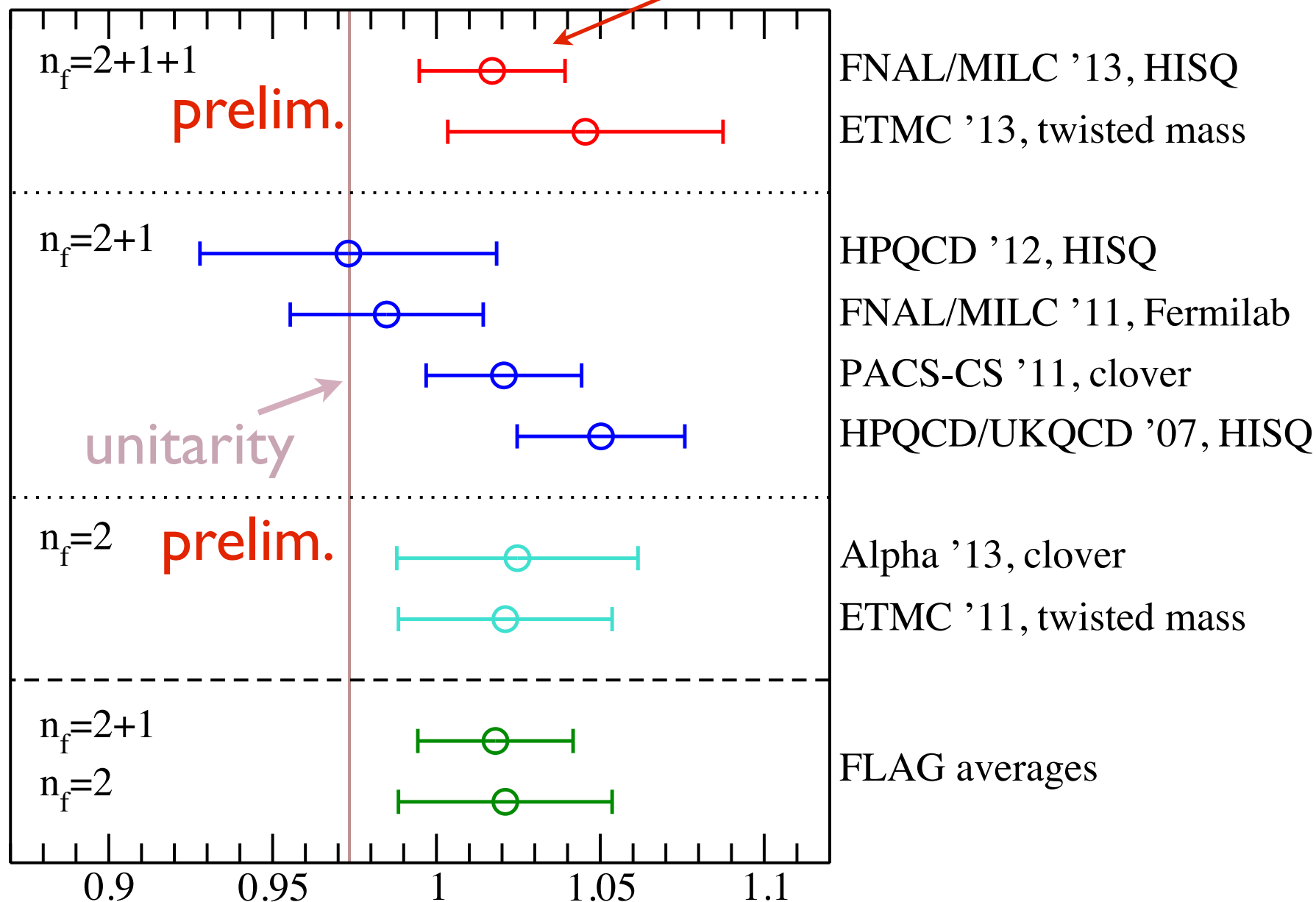
Current status: V_{cs} , semileptonic

$$V_{cs} = 0.963(5)_{\text{expt}}(14)_{\text{lattice}}$$



Current status: V_{cs} , leptonic

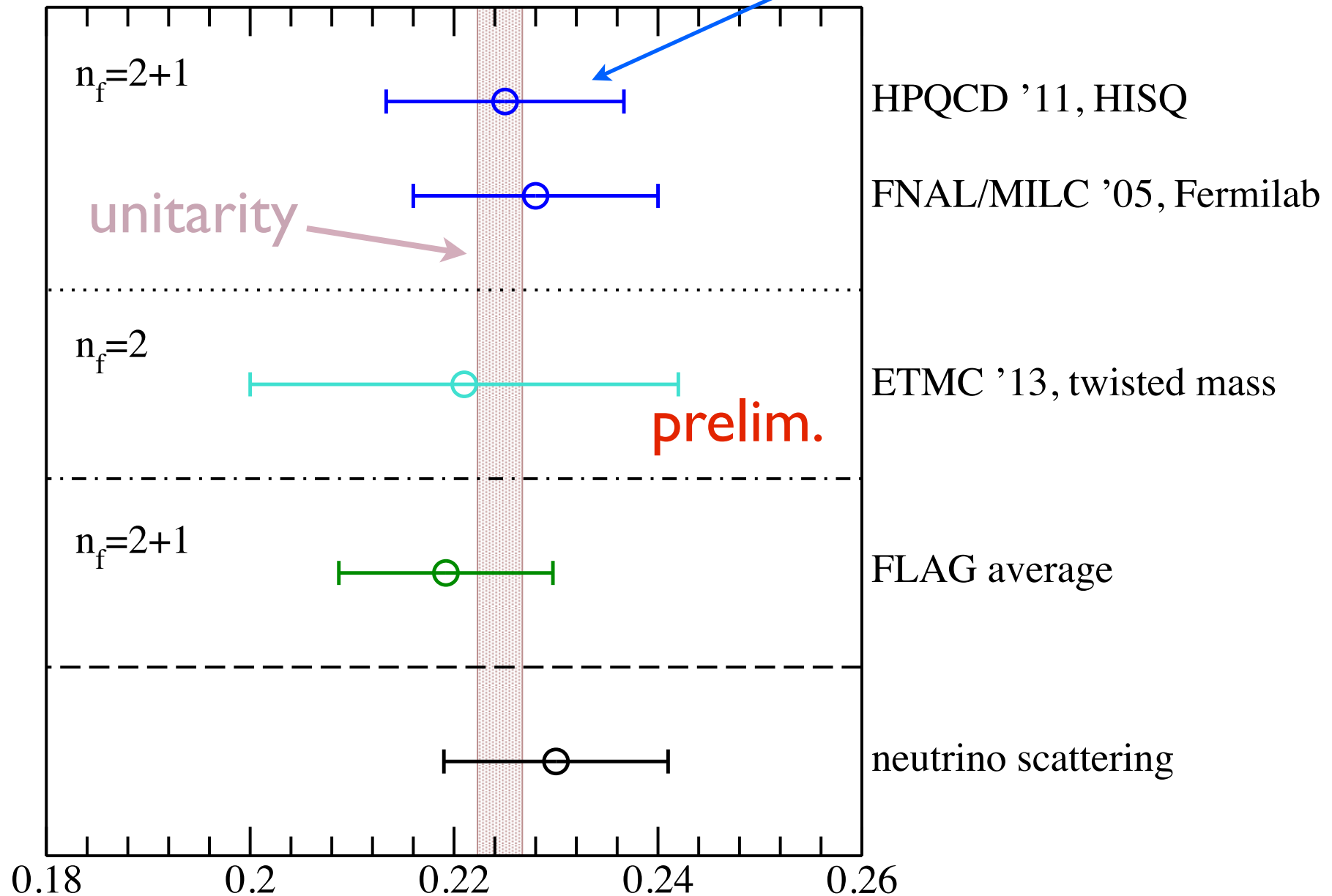
$$V_{cs} = 1.0169 \pm 0.02_{\text{expt}} {}^{+0.002}_{-0.007}|_{\text{lattice}}$$



N.B. Experimental input from PDG 2012. Results from Belle 2012 not included.

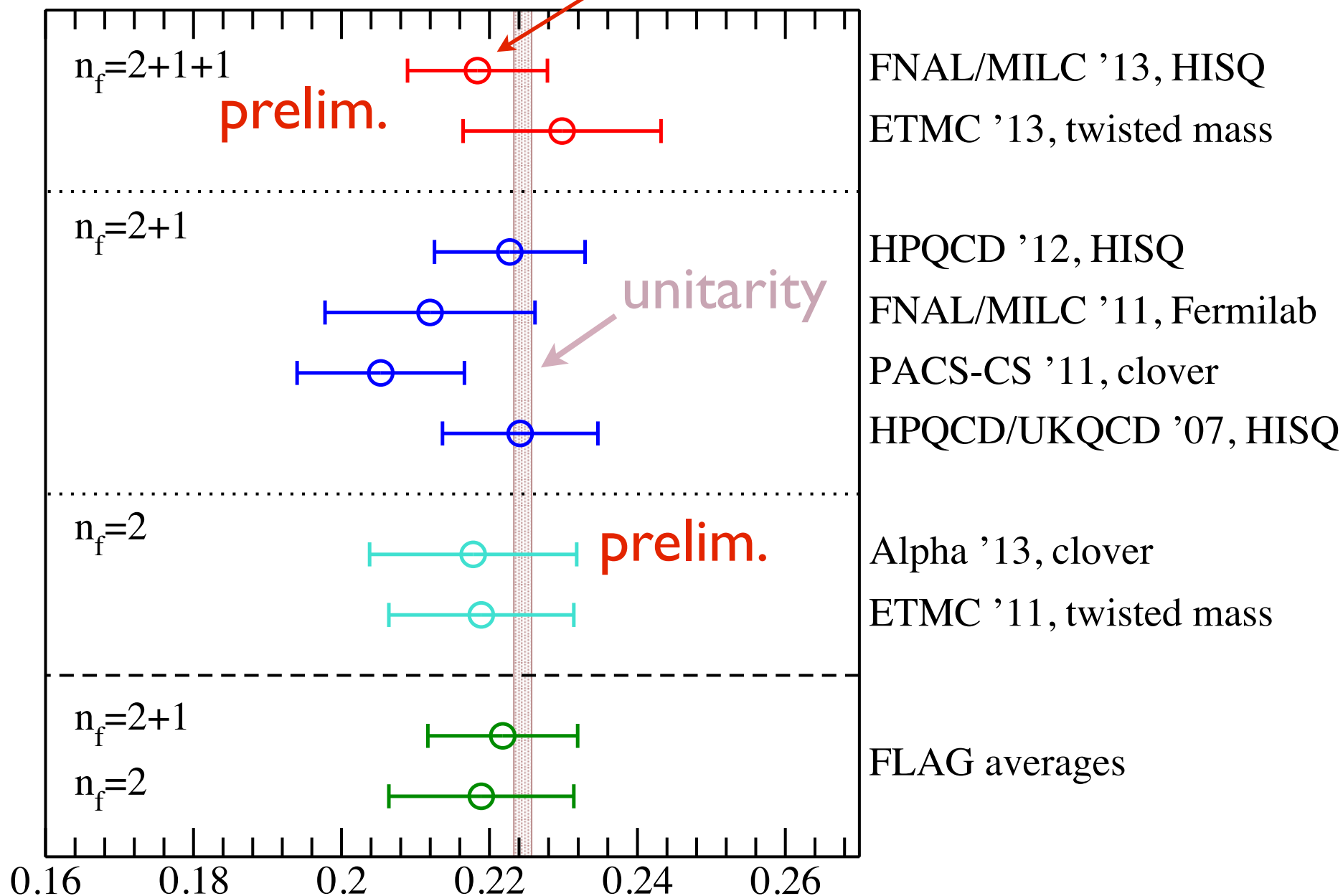
Current status: V_{cd} , semileptonic

$$V_{cd} = 0.225(6)_{\text{expt}}(10)_{\text{lattice}}$$



Current status: V_{cd} , leptonic

$$V_{cd} = 0.2184 \pm 0.009_{\text{expt}}^{+0.0008}_{-0.0016}|_{\text{lattice}}$$



N.B. Experimental input from PDG 2012. Results from BESIII 2012 not included.

Summary

- Can do precision tests of SM and search for new physics by extracting CKM elements from D and D_s meson leptonic and semileptonic decays
- Need input from theory (Lattice QCD):
decay constants f_D, f_{D_s}
form factors $D \rightarrow K\ell\nu, D \rightarrow \pi\ell\nu, D_s \rightarrow K\ell\nu$
- Also compare the shape of the form factors from Lattice QCD with experimental results
→ determine $|V_{cs}|$ using data from all q^2 bins

Summary of latest/best Lattice results

$$f_D = 212.5 \pm 0.5_{\text{stat}} \begin{smallmatrix} +0.6 \\ -1.5 \end{smallmatrix} |_{\text{syst}} \text{ MeV}$$

$$f_{D_s} = 248.9 \pm 0.2_{\text{stat}} \begin{smallmatrix} +0.5 \\ -1.6 \end{smallmatrix} |_{\text{syst}} \text{ MeV} \quad \text{Preliminary!}$$

$$f_{D_s} / f_D = 1.1717(20)_{\text{stat}} \begin{smallmatrix} +52 \\ -25 \end{smallmatrix} |_{\text{syst}}$$

$$V_{cd} = 0.2184 \pm 0.009_{\text{expt}} \begin{smallmatrix} +0.0008 \\ -0.0016 \end{smallmatrix} |_{\text{lattice}} \text{ (leptonic)}$$

$$V_{cs} = 1.0169 \pm 0.02_{\text{expt}} \begin{smallmatrix} +0.002 \\ -0.007 \end{smallmatrix} |_{\text{lattice}} \text{ (leptonic)}$$

$$V_{cd} = 0.225(6)_{\text{expt}}(10)_{\text{lattice}} \text{ (semileptonic)}$$

$$V_{cs} = 0.963(5)_{\text{expt}}(14)_{\text{lattice}} \text{ (semileptonic)}$$

Decay constants from C. Bernard's talk, Lattice 2013. CKM elements from semileptonic decays taken from PRD84 (2011) 114505 (arXiv:1109.1501) and arXiv:1305.1462.

References

ETMC '11: arXiv 1107.1441

FNAL/MILC '05: arXiv hep-lat/0506030

HPQCD/UKQCD '07: arXiv 0706.1726

HPQCD '10: arXiv 1008.4018

PACS-CS '11: arXiv 1104.4600

FNAL/MILC '11: arXiv 1112.3051

HPQCD '12: arXiv 1206.4936

FNAL/MILC '12: arXiv 1210.8431

HPQCD '13: arXiv 1305.1462

ETMC '13: L. Riggio's talk, Lattice 2013

Alpha '13: J. Heitger's poster, Lattice 2013

FNAL/MILC '13: C. Bernard's talk, Lattice 2013

FLAG averages: <http://itpwiki/unibe.ch/flag>

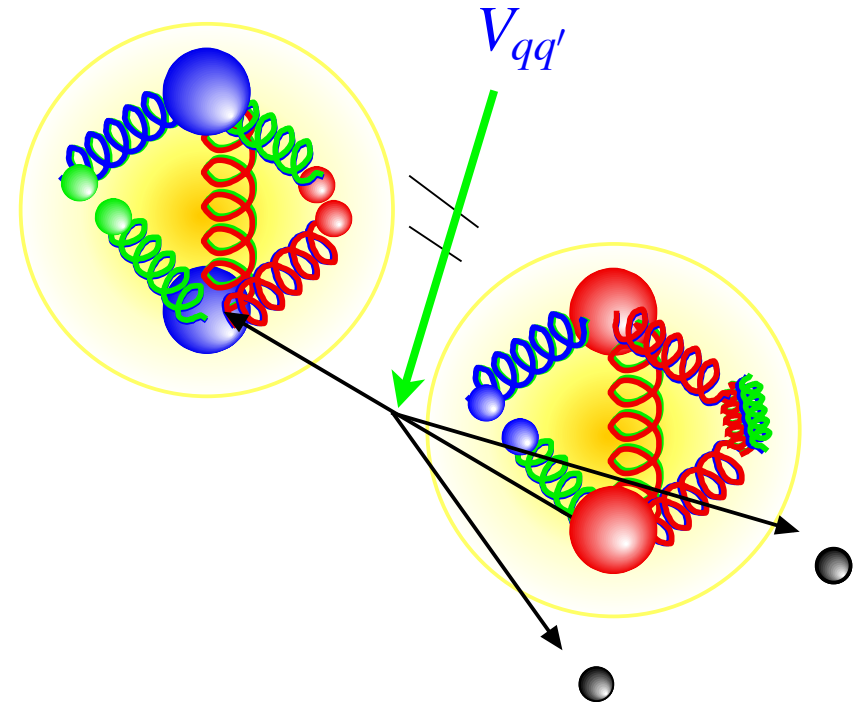
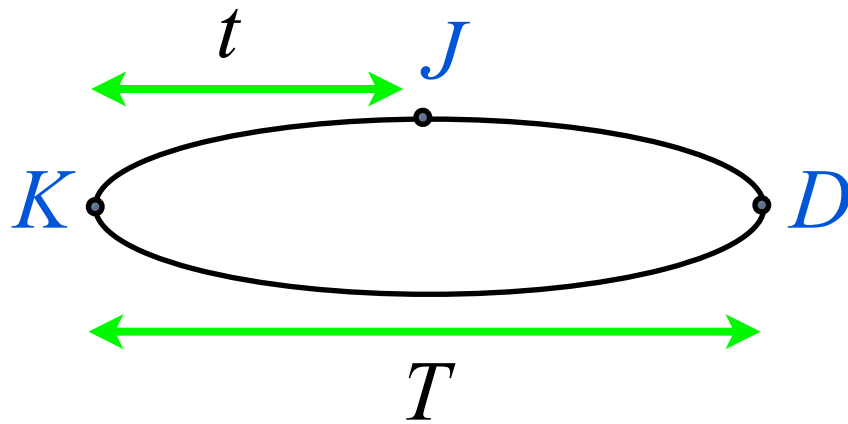
neutrino scattering: PDG, PRD86 (2012) 010001

Thank you!

Backup slides

Semileptonic form factors

= 3pt amplitudes



$$\langle K|S|D\rangle = f_0^{D\rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}}$$

$$\langle K|V^\mu|D\rangle = f_+^{D\rightarrow K}(q^2) \left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu \right] \quad q^\mu = p_D^\mu - p_K^\mu$$

$$+f_0^{D\rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{q^2} q^\mu \rightarrow f_0(0) = f_+(0)$$

Form factors: $D_s \rightarrow \phi \ell \nu$

$$\begin{aligned}\langle \phi(p', \epsilon) | V^\mu - A^\mu | D(p) \rangle = & \frac{2i\epsilon^{\mu\nu\alpha\beta}}{M + m_\phi} \epsilon_\nu^* p'_\alpha p_\beta V(q^2) + (M + m_\phi) \epsilon^{*\mu} A_1(q^2) \\ & + \frac{\epsilon^* \cdot q}{M + m_\phi} (p + p')^\mu A_2(q^2) + 2m_\phi \frac{\epsilon^* \cdot q}{q^2} q^\mu A_3(q^2) \\ & - 2m_\phi \frac{\epsilon^* \cdot q}{q^2} q^\mu A_0(q^2),\end{aligned}$$

where $V^\mu = \bar{q}' \gamma^\mu Q$, $A^\mu = \bar{q}' \gamma^\mu \gamma_5 Q$,

and $A_3(q^2) = \frac{M + m_\phi}{2m_\phi} A_1(q^2) - \frac{M - m_\phi}{2m_\phi} A_2(q^2)$ with $A_0(0) = A_3(0)$

$D_s \rightarrow \phi \ell \nu$ differential decay rate

$$\begin{aligned} \frac{d\Gamma(P \rightarrow V \ell \nu, V \rightarrow P_1 P_2)}{dq^2 d\cos \theta_V d\cos \theta_\ell d\chi} &= \frac{3}{8(4\pi)^4} G_F^2 |V_{q'Q}|^2 \frac{p_V q^2}{M^2} \mathcal{B}(V \rightarrow P_1 P_2) \\ &\times \{ (1 - \eta \cos \theta_\ell)^2 \sin^2 \theta_V |H_+(q^2)|^2 \\ &+ (1 + \eta \cos \theta_\ell)^2 \sin^2 \theta_V |H_-(q^2)|^2 \\ &+ 4 \sin^2 \theta_\ell \cos^2 \theta_V |H_0(q^2)|^2 \\ &- 4\eta \sin \theta_\ell (1 - \eta \cos \theta_\ell) \sin \theta_V \cos \theta_V \cos \theta_\chi H_+(q^2) H_0(q^2) \\ &+ 4\eta \sin \theta_\ell (1 + \eta \cos \theta_\ell) \sin \theta_V \cos \theta_V \cos \theta_\chi H_+(q^2) H_0(q^2) \\ &- 2 \sin^2 \theta_\ell \sin^2 \theta_V \cos 2\chi H_+(q^2) H_-(q^2) \}, \end{aligned}$$

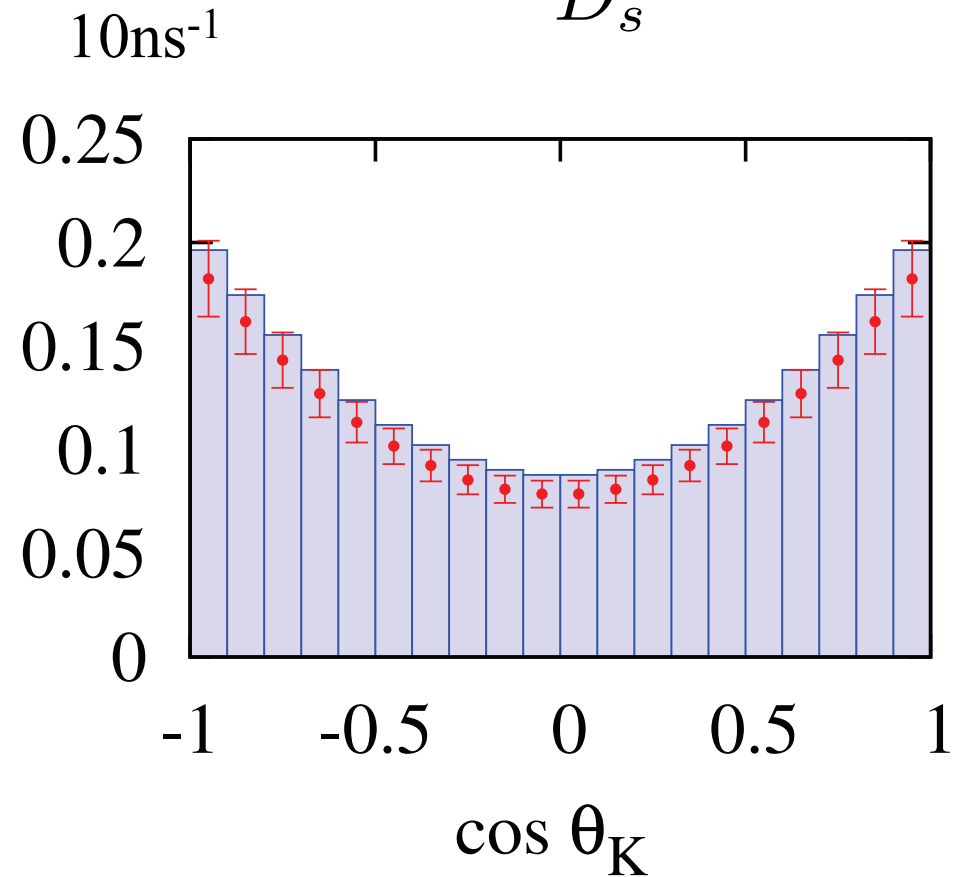
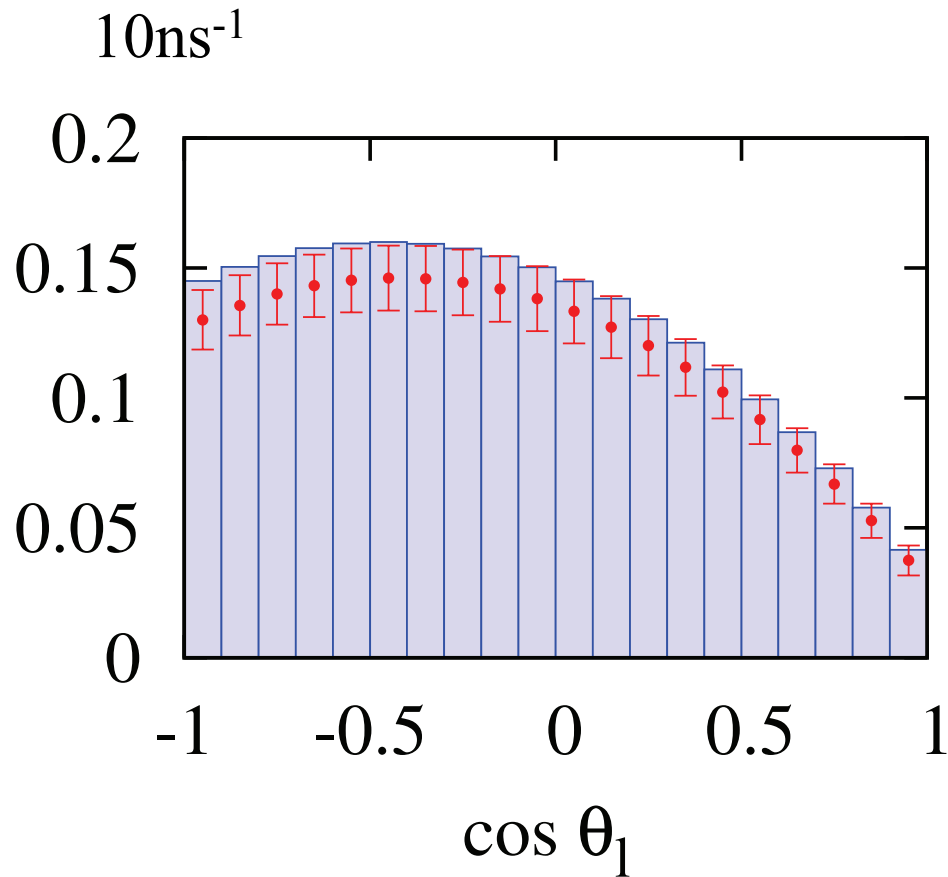
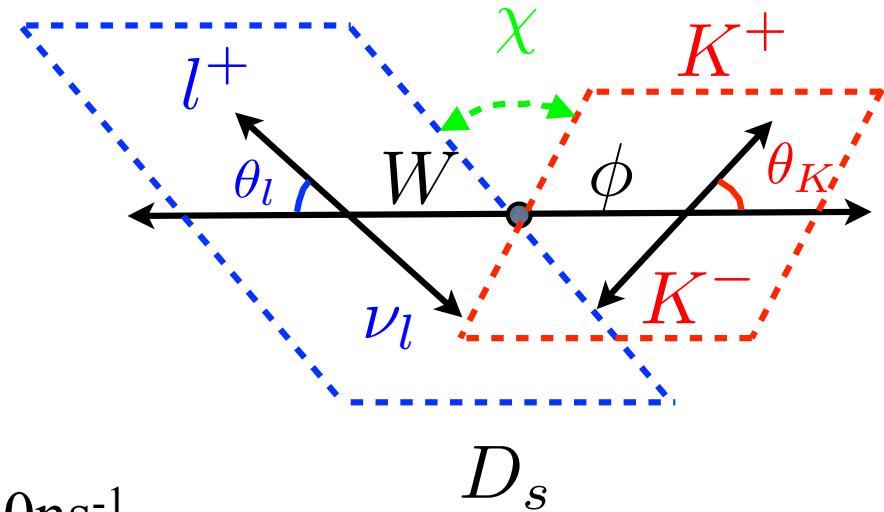
where the helicity amplitudes are

$$H_0(q^2) = \frac{1}{2m_\phi \sqrt{q^2}} \left[(M^2 - m_\phi^2 - q^2)(M + m_\phi) A_1(q^2) - 4 \frac{M^2 p_\phi^2}{M + m_\phi} A_2(q^2) \right]$$

$$H_\pm(q^2) = (M + m_\phi) A_1(q^2) \mp \frac{2M p_\phi}{M + m_\phi} V(q^2)$$

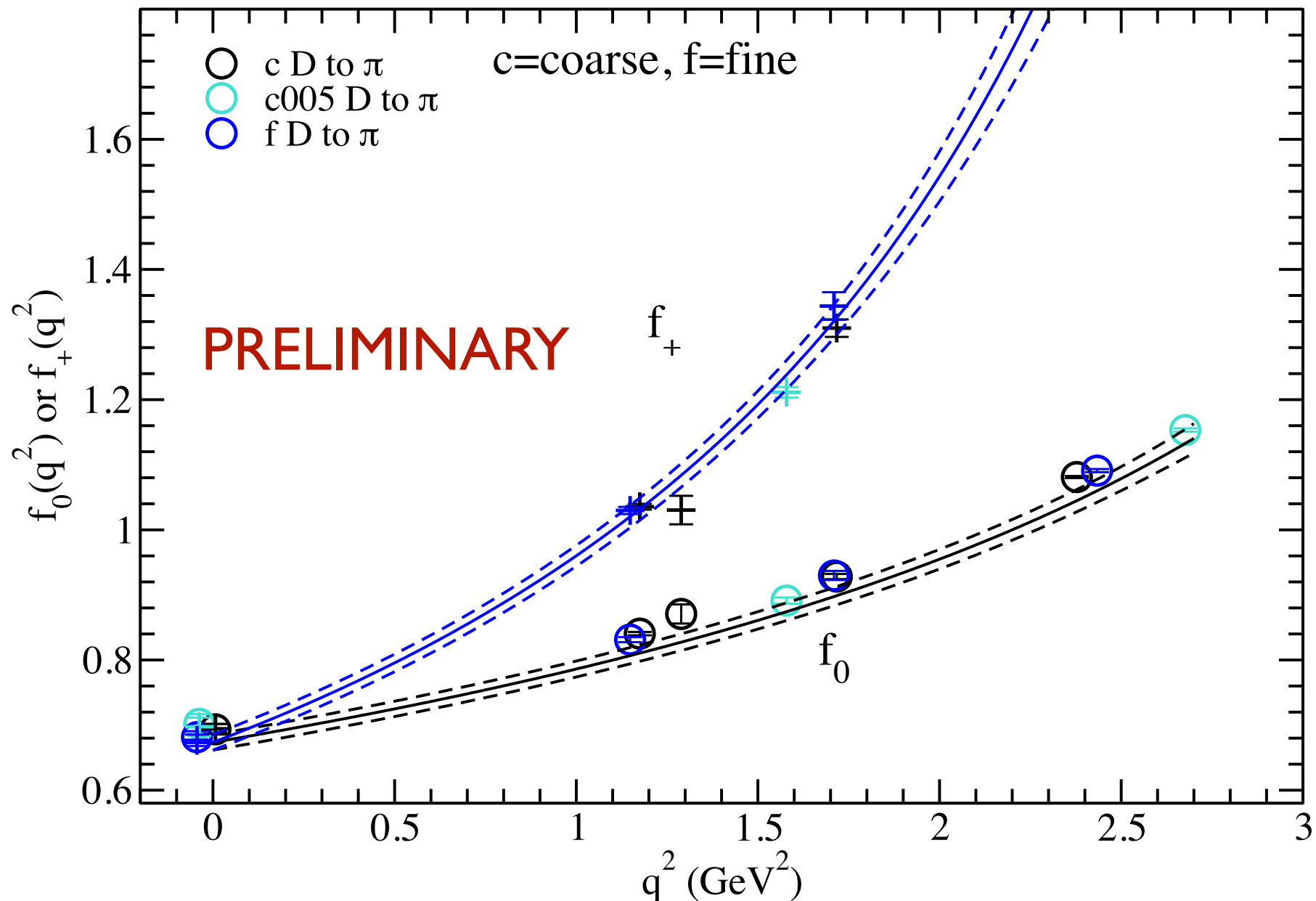
$D_s \rightarrow \phi \ell \nu$ angular distributions

G. Donald, HPQCD, Lattice 2013



Experimental data from BaBar, PRD 78, 051101(R) (2008)

Shape of the form factors: $D \rightarrow \pi \ell \nu$



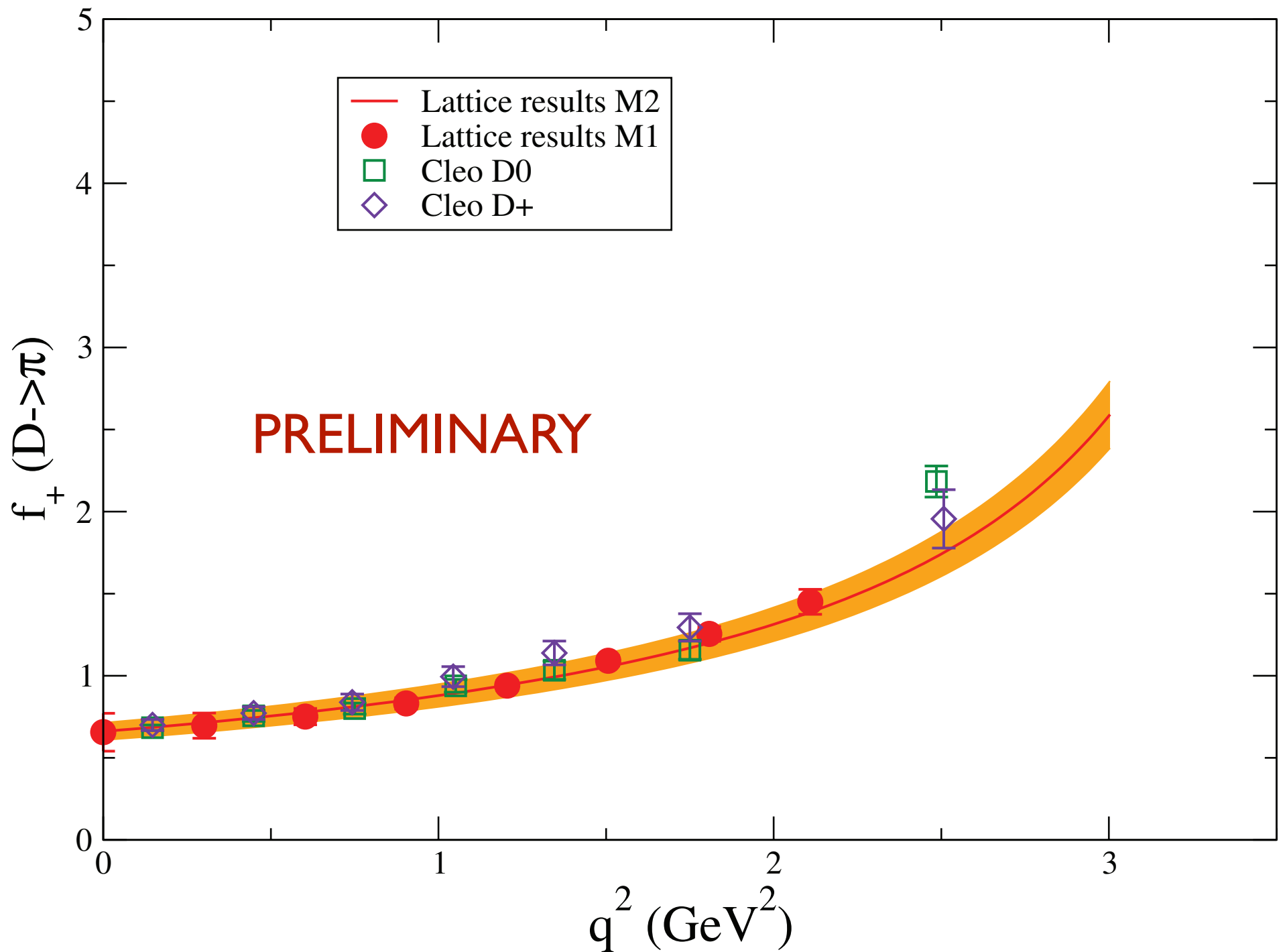
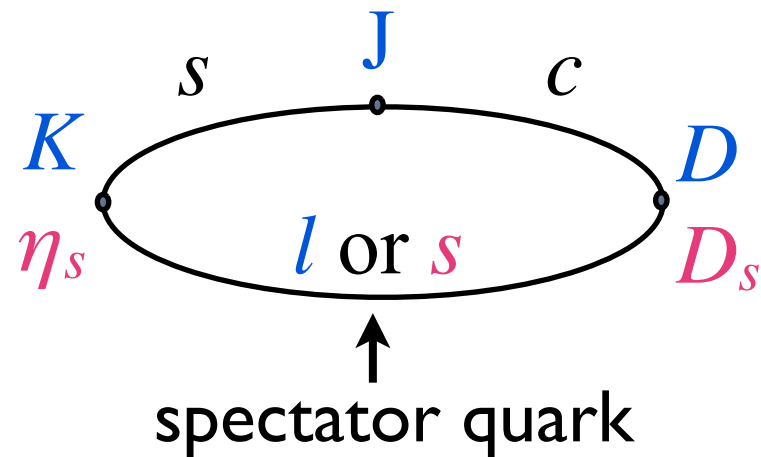
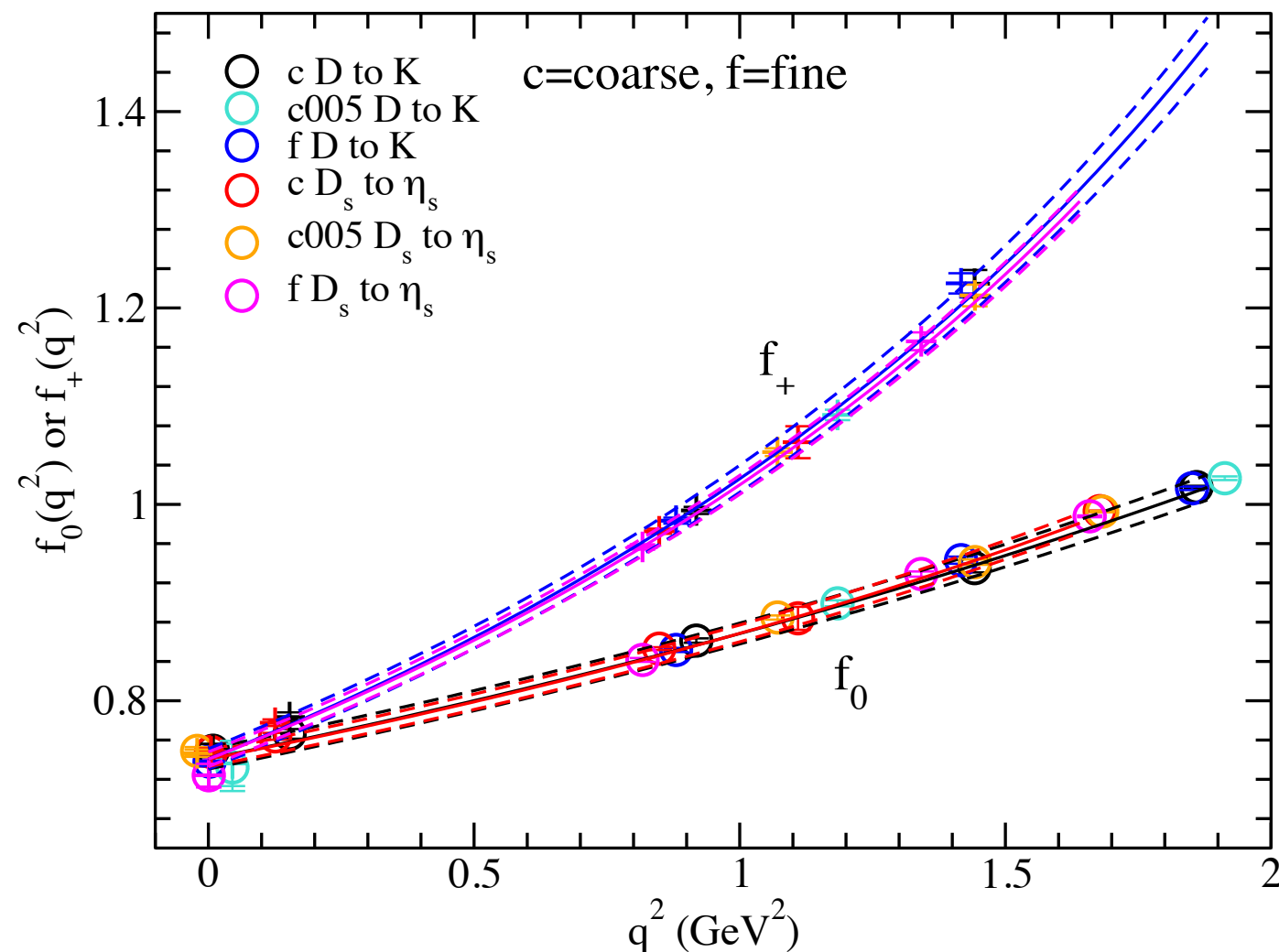


Fig. by F. Sanfilippo, talk at Lattice 2013, ETMC

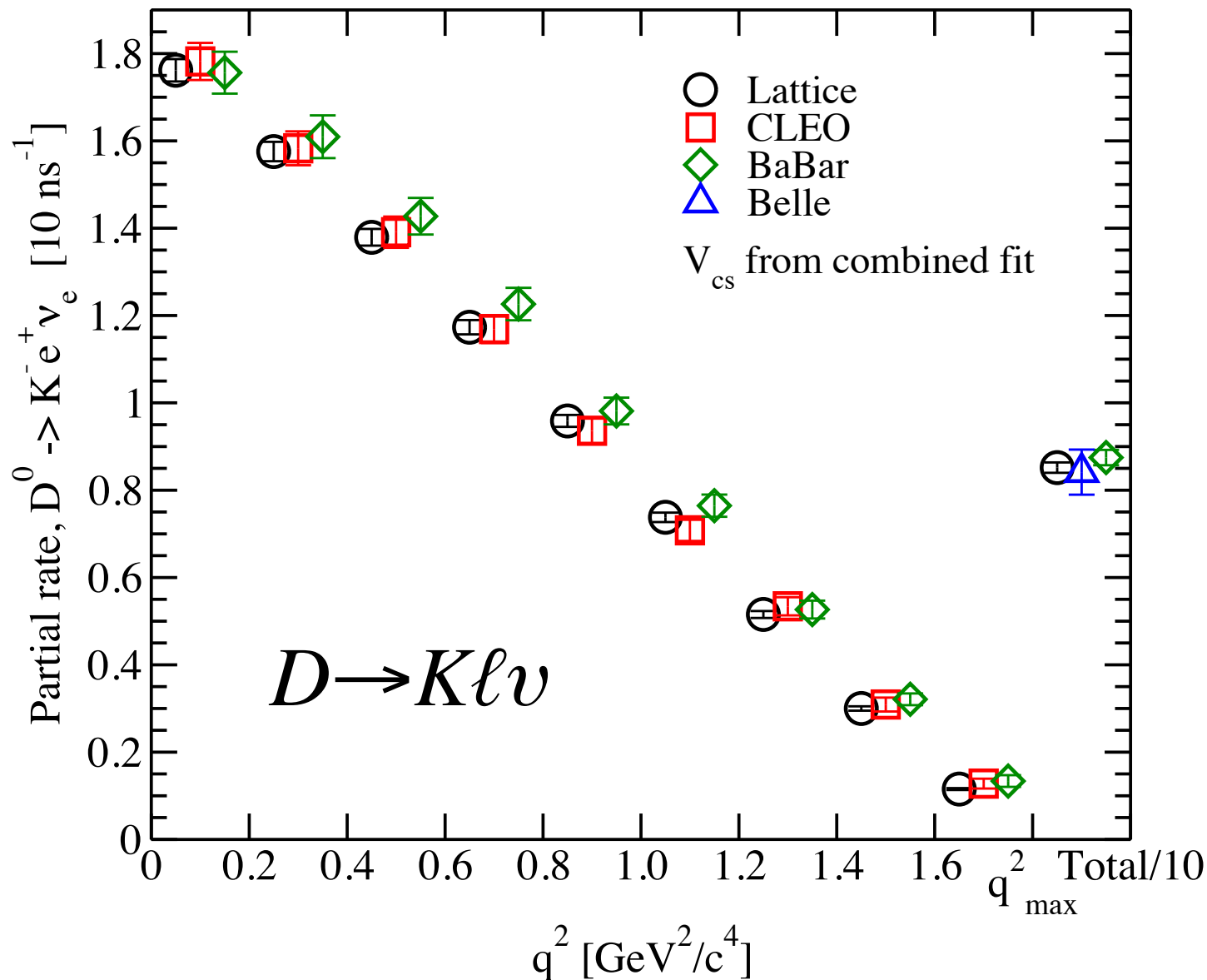
Shape of the FFs: spectator quark

HPQCD, arXiv:1305.1462



- The difference between $D \rightarrow K \ell \nu$ and $D_s \rightarrow \eta_s \ell \nu$ is the spectator quark - light vs. strange
- The shapes of the form factors are same within 3%

Decay rates in q^2 bins



HPQCD,
arXiv:1305.1462

Experimental data: CLEO, PRD 80, 032005 (2009); Belle, PRL 97, 061804 (2006); BaBar, PRD 76, 052005 (2007) and PRD 78, 051101(R) (2008)

Lattice averages (FLAG)

Latest results from Lattice 2013 NOT included here!
Only results from 2012 and earlier.

$$f_D = 209.2(3.3) \text{ MeV}$$

$$f_{D_s} = 248.6(2.7) \text{ MeV}$$

$$f_{D_s}/f_D = 1.187(12)$$

$$V_{cd} = 0.2218(35)(95) \text{ (leptonic)}$$

$$V_{cs} = 1.018(11)(21) \text{ (leptonic)}$$

$$V_{cd} = 0.2192(95)(45) \text{ (semileptonic)}$$

$$V_{cs} = 0.9746(248)(67) \text{ (semileptonic)}$$

<http://itpwiki/unibe.ch/flag>