Problematic CKM observables: inclusive $|V_{ub}|$

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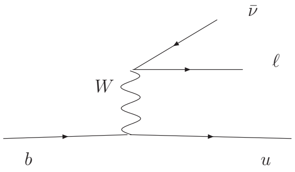
$|V_{ub}|$ and $b \to u\ell\bar{\nu}$ decays



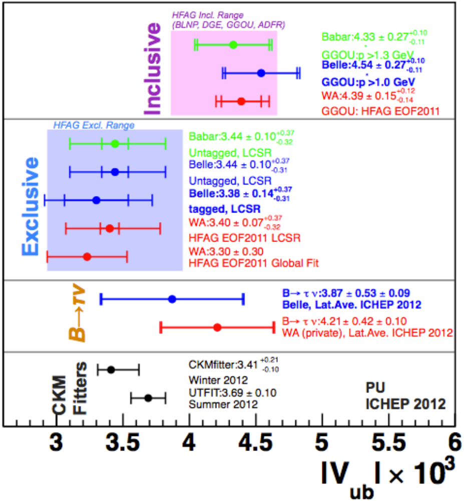
<u>Two observables</u>:

1) $\bar{B} \to \pi\ell\bar{\nu}$ = exclusive decay

2) $\bar{B} \to X_u\ell\bar{\nu}$ = inclusive decay

Simple, tree-level process in SM, should both yield same $|V_{ub}|$

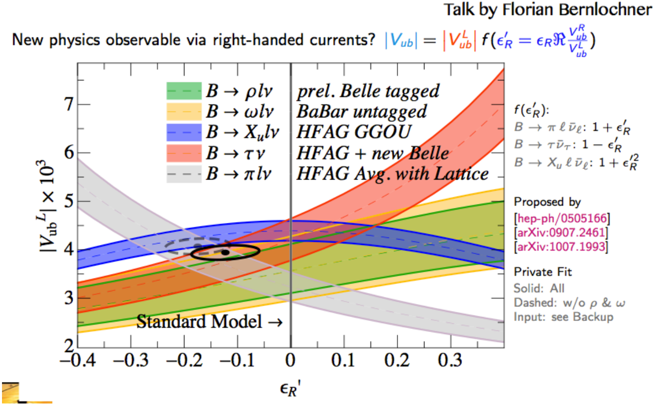
Inclusive vs. exclusive $|V_{ub}|$



$\sim 2\sigma$ discrepancy between inclusive and exclusive


[redacted] MESON
Don't Talk
B
French Extended Mix

New physics through right-handed currents?



A hard sell, to my eyes! Discrepancy more likely due to QCD.

The QCD challenge: Exclusive vs. Inclusive

Exclusive decays:

- clean experimentally
- need non-perturbative form factors from QCD sum rules and/or lattice QCD

Inclusive decays:

- require experimental cuts to suppress charm background
- cuts introduce sensitivity to non-perturbative shape functions (cannot use local OPE as in $\bar{B} \to X_c \ell \bar{\nu}$ or $\bar{B} \to X_s \gamma$ decays)
- exist an alphabet soup of "competing methods": BLNPGGOUSIMBADGEADFR, ...

QCD for inclusive $|V_{ub}|$

Model independent:

- "BLNP" approach based on SCET
  Bosch, Lange, Neubert, Paz (hep-ph/0402094, hep-ph/0504071)
- "GGOU" approach based on Wilsonian cutoff
  Gambino, Giordona, Ossola, Uraltsev (arXiv:0707.2493)
- "SIMBA" approach based on SCET plus global fits
  Bernlocher et. al. (e.g. arXiv:1101.3310)

Model dependent (calculate shape functions with model):

- "DGE" (Andersen, Gardi hep-ph/0609250)
- "ADFR" arXiv:0711.0860

Goal for talk: meaningfully compare GGOU and BLNP.
What is the same and what is different?

The roadmap

1) The theoretical minimum

- the shapefunction and OPE regions for differential decay rates

2) Contrasting BLNP and GGOU

- perturbative corrections
- subleading shape functions

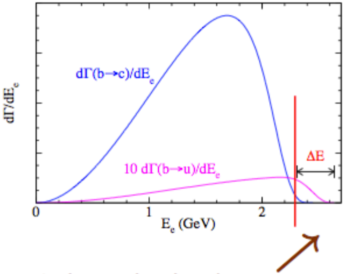
3) Connecting BLNP and GGOU

- shape function moments and the local OPE

Differential spectra and shape functions

- Total rate calculable with $\sim 4\%$ uncertainty, similar to $\mathcal{B}(B \to X_c \ell \bar{\nu})$
- To remove the huge charm background ($|V_{cb}/V_{ub}|^2 \sim 100$), need phase space cuts

  Can enhance pert. and nonpert. corrections



- Instead of being constants, the hadronic parameters are functions (like PDFs)

  Leading order: universal & related to $B \to X_s \gamma$;
  $\mathcal{O}(\Lambda_{QCD}/m_b)$: several new unknown functions

- Nonperturbative effects shift endpoint $\frac{1}{2} m_b \to \frac{1}{2} m_B$ & determine its shape
- Shape in the endpoint region is determined by b quark PDF in B ["shape function"]

  Related to $B \to X_s \gamma$ photon spectrum at lowest order [Bigi, Shifman, Uraltsev, Vainshtein; Neubert]

Decay rate with cut: $E_\ell > E_\ell^{\rm min}$: shapefunctions or not?

1) loose cut = "OPE region": $m_b - 2E_\ell^{\rm min} = \Delta \sim m_b \gg \Lambda_{\rm QCD}$

$$\Gamma(\Delta, m_b) \sim F\left(\frac{\Delta}{m_b}, \alpha_s\right) + \frac{\mu_\pi^2}{\Delta^2} K_\pi\left(\frac{\Delta}{m_b}, \alpha_s\right) + \frac{\mu_G^2}{\Delta^2} K_G\left(\frac{\Delta}{m_b}, \alpha_s\right) + \ldots$$

2) severe cut = "shapefunction region": $m_b - 2E_\ell^{\rm min} = \Delta \sim \Lambda_{\rm QCD}$

$$\Gamma(\Delta, m_b) \sim F_{\rm pert}\left(\frac{m_b^2}{\mu^2}, \frac{m_b\Delta}{\mu^2}, \frac{\Delta}{m_b}, \alpha_s\right) \otimes S(\Delta, \mu)$$
$$+ \frac{1}{m_b} \sum_i K_{i,\rm pert}\left(\frac{m_b^2}{\mu^2}, \frac{m_b\Delta}{\mu^2}, \frac{\Delta}{m_b}, \alpha_s\right) \otimes S_i^\Lambda(\Delta, \mu) + \ldots$$

But decay spectra don't include parametric labels on Δ!

GGOU and BNLP try to deal with both cuts in one code.

They differ conceptually for severe cuts.

Contrast I: perturbative corrections

$$\Gamma(\Delta, m_b) \sim F_{\text{pert}}\left(\frac{m_b^2}{\mu^2}, \frac{m_b\Delta}{\mu^2}, \frac{\Delta}{m_b}, \alpha_s\right) \otimes S(\Delta, \mu) + \frac{1}{m_b}\sum_i K_{i,\text{pert}}\left(\frac{m_b^2}{\mu^2}, \frac{m_b\Delta}{\mu^2}, \frac{\Delta}{m_b}, \alpha_s\right) \otimes S_i^{\Lambda}(\Delta, \mu) + \dots$$

Perturbative corrections: BLNP vs. GGOU

BNLP

$$F_{\rm pert}^{\rm BNLP} = H\left(\frac{m_b^2}{\mu^2}, \alpha_s\right) \times J\left(\frac{m_b\Delta}{\mu^2}, \alpha\right) + \frac{\Delta}{m_b} F_{\rm pert}^{(1)}\left(\frac{m_b^2}{\mu^2}, \frac{m_b\Delta}{\mu^2}, \alpha_s\right) + \ldots$$

- H, J are matching coefficients in EFT (known to NNLO)
- for any μ, logarithms of Δ/m_b appear: these are resummed

GGOU

$$F_{\rm pert}^{\rm GGOU} \sim \int_{\mu}^{E_g^{\rm max}} dE_g \frac{d\Gamma}{dE_g} \qquad \text{(schematic!)}$$

- perturbative contributions defined by phase-space slicing (Wilsonian cutoff $\mu \sim 1$ GeV on minimum gluon energy)
- no expansion in the ratio Δ/m_b, nor summation of logarithms
- known to NLO, plus partial NNLO (BLM terms $\mathcal{O}(\alpha_s^2\beta_0)$)

Perturbation corrections: open questions

1) Analytic agreement at leading power in Δ/m_b?

$$F_{\text{pert}}^{\text{GGOU}} \stackrel{?}{=} F_{\text{pert}}^{\text{BNLP}} + \mathcal{O}\left(\frac{\Delta}{m_b}\right)$$

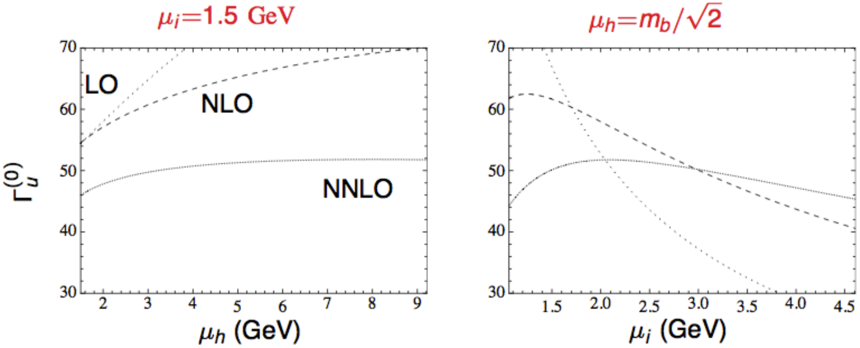
- this should be true since leading shapefunction is universal, but no comparison in literature

2) Are NNLO corrections smaller in one treatment or another?

$$F_{\text{pert}}^{\text{BLNP,NNLO}} = (H \times J)^{\text{NNLO}} + \mathcal{O}\left(\frac{\Delta}{m_b}\right)$$

- $(H \times J)^{\text{NNLO}}$ can give large negative corrections compared to BNLP default analysis (Greub, Neubert, BP '2009)
- Are there large cancellations between $(H \times J)^{\text{NNLO}}$ and subleading terms in Δ/m_b in GGOU, BNLP, or both?

NNLO corrections in BLNP (results for $P_+ < 0.66$ GeV)



- reduced dependence on μ_h, μ_i at NNLO
- large negative shift between NLO and NNLO
- largest uncertainty associated with μ_i (usually fixed at $\mu_i = 1.5$ GeV)

NNLO corrections in OPE

But NNLO corrections to $\Gamma_{\rm tot}$ are smallish, because of a cancellation between leading power and Δ/m_b terms. Question of how corrections depend on cut can be studied more using NNLO $b \to X_u \ell \bar{\nu}$ from Gao et. al. '12, Bruchseifer et. al. '13.

Example: Lepton energy cut ($\mu = m_b^{\rm pole} = 4.8$ GeV.)

$$\frac{\Gamma^{\rm NNLO}(E_l > 2.1\ {\rm GeV})}{\Gamma^{\rm LO}(E_l > 0\ {\rm GeV})} = 0.079 \times (1 - 0.32_{\,[\text{NLO corr.}]} - 0.33_{[\text{NNLO corr.}]}\,)$$

$$\frac{\Gamma^{\rm NNLO}(E_l > 0\ {\rm GeV})}{\Gamma^{\rm LO}(E_l > 0\ {\rm GeV})} = 1.000 \times (1 - 0.20_{\,[\text{NLO corr.}]} - 0.16_{[\text{NNLO corr.}]}\,)$$

- the NNLO corrections for restrictive cut are large and negative
- the NNLO corrections for total rate moderate (even in pole scheme)
- thanks to Jun Gao for proving NNLO numbers!

Contrast II: Subleading shape functions

BLNP

$$\Gamma(\Delta, m_b) \sim F_{\text{pert}} \otimes S^{\text{leading}} + \frac{1}{m_b} \sum_i K_{i,\text{pert}} \left(\frac{m_b^2}{\mu^2}, \frac{m_b \Delta}{\mu^2}, \frac{\Delta}{m_b}, \alpha_s \right) \otimes S_i^{\Lambda}(\Delta, \mu) + \ldots$$

- factorization of subleading terms based on effective field theory (Lee et. al, Bosch et. al., Beneke et. al '05)
- only 3 new S_i^{Λ} at tree level, but $\sim$ 20 in general (brick wall!)

GGOU

$$\Gamma(\Delta, m_b) \sim \sum_{i=1}^{3} \left(W_i = F_{\text{pert}} \otimes S_i^{\text{leading+subleading}} \right)$$

[very schematic!]

- W_i are the three form factors from hadronic tensor (backup slides), GGOU uses different shapefunction for each

BLNP and GGOU treatments not equivalent for $\Delta \sim \Lambda_{\text{QCD}}$

Subleading shape functions in BLNP

The contributions from subleading shape functions can be quite large!
But uncertainties small(ish)!!

(Examples below use default BLNP inputs/units, see hep-ph/0504071)

$$\Gamma\left(E_\ell > \frac{M_B^2 - M_D^2}{2M_B} \sim 2.3\ \mathrm{GeV}\right) = 7.3\,[\text{leading SF}] - 4.0\,[\text{subleading SF}]$$

$$\Gamma\left(P_+ > \frac{M_D^2}{M_B} \sim 0.66\ \mathrm{GeV}\right) = 58\,[\text{leading SF}] - 12.0\,[\text{subleading SF}]$$

E_0 [GeV]	Mean	Subl. SF	Pert.	Total
1.9	24.82	±0.54	$^{+1.91}_{-1.66}$	$^{+2.35}_{-2.15}$
2.0	19.00	±0.61	$^{+1.37}_{-1.21}$	$^{+1.96}_{-1.85}$
2.1	13.25	±0.71	$^{+0.85}_{-0.76}$	$^{+1.68}_{-1.63}$
2.2	7.99	±0.78	$^{+0.42}_{-0.37}$	$^{+1.54}_{-1.53}$
2.3	3.83	±0.86	$^{+0.18}_{-0.13}$	$^{+1.54}_{-1.53}$
2.4	1.31	±0.99	$^{+0.10}_{-0.14}$	$^{+1.61}_{-1.61}$

Δ_P [GeV]	E_0 [GeV]	Mean	Subl. SF	Pert.	Total
0.70	0.0	48.90	±1.15	$^{+2.83}_{-2.65}$	$^{+3.30}_{-3.15}$
0.65	0.0	45.34	±1.46	$^{+2.55}_{-2.41}$	$^{+3.20}_{-3.09}$
0.60	0.0	41.34	±1.76	$^{+2.26}_{-2.15}$	$^{+3.13}_{-3.05}$
0.55	0.0	36.91	±2.01	$^{+1.95}_{-1.87}$	$^{+3.08}_{-3.02}$
0.50	0.0	32.09	±2.34	$^{+1.64}_{-1.58}$	$^{+3.12}_{-3.09}$

Common ground

- The very important common ground is both match local OPE for wide cuts $\Delta \sim m_b$

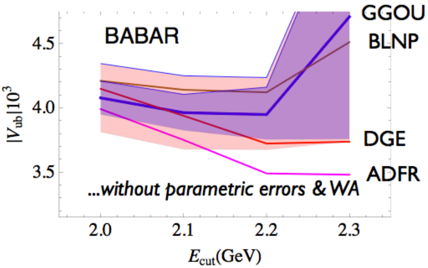
$$\int^{\Delta} d\Gamma^{\text{BLNP,GGOU}} = F\left(\frac{\Delta}{m_b}, \alpha_s\right) + \frac{\mu_\pi^2}{\Delta^2} K_\pi \left(\frac{\Delta}{m_b}, \alpha_s\right) + \frac{\mu_G^2}{\Delta^2} K_G \left(\frac{\Delta}{m_b}, \alpha_s\right) + \dots$$

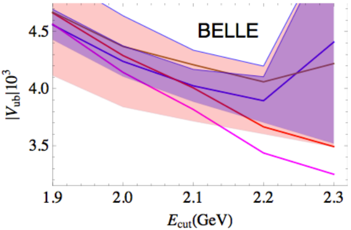
- This is accomplished through moment constraints on SFs

$$\int d\omega\, \omega\, S(\omega) \leftrightarrow m_b\,; \qquad \int d\omega\, \omega^2\, S(\omega) \leftrightarrow \mu_\pi^2; \quad \text{etc.}\dots$$

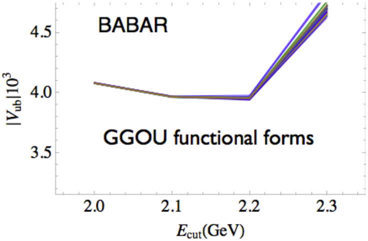
- If experimental cut is parametrically near $\Delta \sim m_b$, should have good agreement
- obvious point: cuts don't come with labels, so check numerically

Comparison with lepton energy cut





The spectrum does provide information: OPE based methods close to each other up to 2.2GeV, resummed methods show larger slope, *seem* to behave in same way



SIMBA

Theory framework for SIMBA very close to BLNP. Differs in that

- slicker shape-function treatment $\leftrightarrow$ matching to OPE
- seeks to use global fit to extract $\mathcal{B}(B \to X_s\gamma)$, $|V_{ub}|$, m_b, shapefunction(s)

Some thoughts:

- GGOU and BLNP vary shapefunction form a great deal and estimate small uncertainty. Seems moment constraints more important than actual shape.
- subleading shapefunctions make large numerical contributions and must also be extracted
- good idea and good luck, look forward to results!

The issue of m_b

NNLO results for two values of m_b in BNLP:

(uses shape-function scheme for m_b)

$$\frac{\Gamma_u(P_+ < \Delta)}{|V_{ub}|^2 \mathrm{ps}^{-1}} = 43.1^{+1.4}_{-1.8}\,, \quad m_b = 4.61\,\mathrm{GeV},\ \frac{m_b}{2} < \mu_h < m_b$$
$$= 51.3^{+1.5}_{-1.9}\,, \quad m_b = 4.71\,\mathrm{GeV},\ \frac{m_b}{2} < \mu_h < m_b$$

- the extracted value of $|V_{ub}|$ depends strongly on m_b
- the global fit method of SIMBA is potentially very useful here

Summary

- GGOU and BLNP (and other methods) differ quite a bit conceptually for severe cuts, but not so much for $|V_{ub}|$ extractions.

Some open issues:

- NNLO corrections should be settled and included
- Good agreement between methods indicative the OPE doesn't break down so badly for some cuts? After all, it's what connects GGOU and BLNP most clearly. Would be nice to see how far it can be pushed.

And look forward to SIMBA results