CKM triangle fit inputs

Denis Derkach

University of Oxford

UK Flavour Workshop 2013
Durham, UK
06 September 2013

Credits

http://utfit.org/

Adrian Bevan, Marcella Bona,
Marco Ciuchini, Denis Derkach,
Enrico Franco, Vittorio Lubicz,
Guido Martinelli, Fabrizio Parodi,
Maurizio Pierini, Carlo Schiavi,
Luca Silvestrini, Viola Sordini,
Achille Stocchi, Cecilia Tarantino,
Vincenzo Vagnoni

Use the Bayesian statistics to extract the observables. Extract the credibility interval from the fit.

We use likelihoods of measurements where possible. Gaussian PDFs are used to represent statistical and systematic uncertainties otherwise.

The results included into this talk are based on experimental studies that were public by August (Lattice and EPS conferences)

Bayesian Method is very straightforward to understand:

- Probability that the event A occurs given that B also occurs:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Bayes theorem says: if $P(A|B) = \frac{P(A \cap B)}{P(B)}$ and $P(B|A) = \frac{P(A \cap B)}{P(A)}$

than $$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Before the observation *B*, our degree of belief of *A* is *P(A) (prior* probability*)*
After observing *B*, our degree of belief changes into *P(A|B) (posterior* probability*)*

Thus, having several observables with their probabilities, we are able to understand the value and uncertainty of the parameter needed.

Technically, this means that we construct

$$\mathcal{F}(x_1, x_2, ..., x_N | c_1, ..., c_M) \propto \mathcal{F}(c_1, ..., c_M | x_1, ..., x_N) \mathcal{F}_0(x_1, ..., x_N)$$

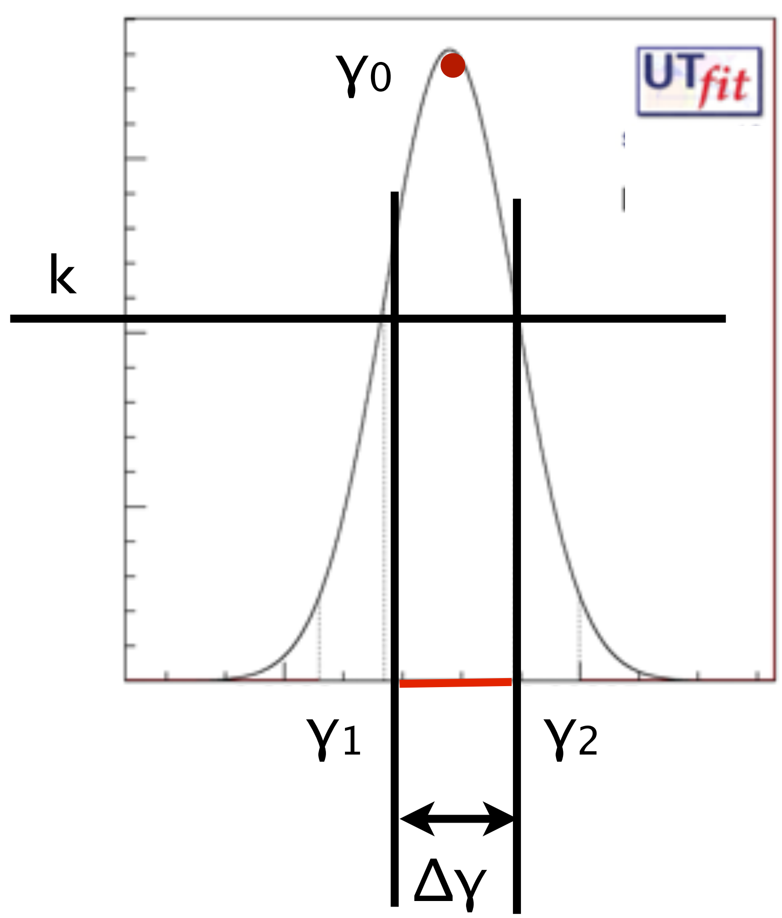
where F is the PDF for the constraints $\{c_i\}$ and F_0 is the prior probability for the parameters of interest $\{x_i\}$. In order to obtain the posterior PDF, in case of absence of mutual correlations for parameter $\gamma = x_1$ we have

$$\mathcal{G}(\gamma) \propto \int \prod_{j=1}^{M} \mathcal{F}_j(c_j | \gamma, x_2, ..., x_N) \prod_{i=2}^{N} \mathcal{F}_0(x_i) \mathcal{F}_0(\gamma)$$

This means, that we have to "just" integrate the PDFs used.

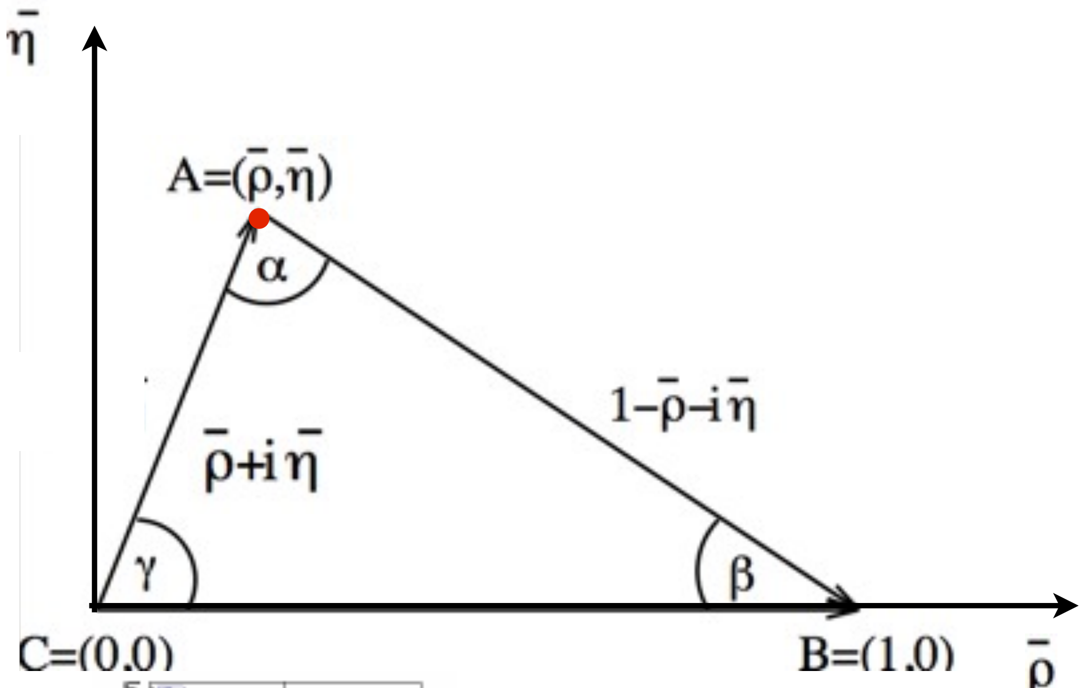
We than obtain 68% and 95% credibility intervals looking at the integral of posterior PDF.
We look at minimum k fulfilling:

$$\frac{\int_{\mathcal{G}(\gamma)>k} d\gamma \, \mathcal{G}(\gamma)}{\int d\gamma \, \mathcal{G}(\gamma)} > 0.683$$



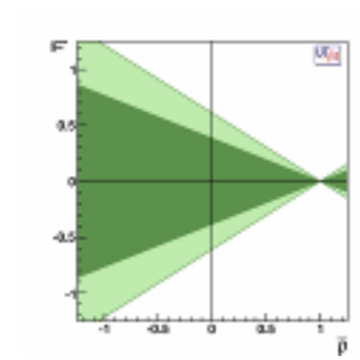
In the example, we take 68% credibility interval to be $[\gamma_1; \gamma_2]$.
We quote $\gamma_0 \pm \Delta\gamma/2$,
where $\Delta\gamma = \gamma_2 - \gamma_1$ (in case of symmetric distributions)

We can write down the unitarity conditions to the CKM matrix. In the Wolfenstein parameterisation defining the $\{\bar{\rho};\bar{\eta}\}$, we can represent these conditions in graphical way.



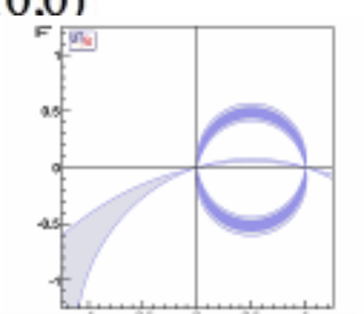
We now want to test this picture using different possible constraints.

$\cos(2\beta)$



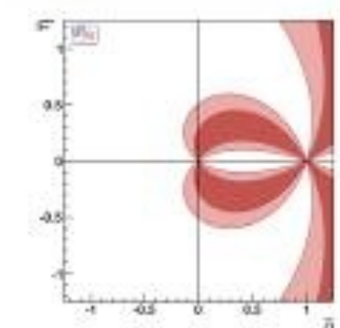
B→DK, B→Dπ

α



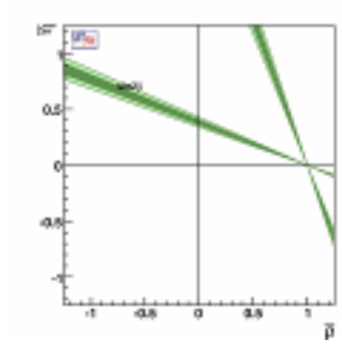
B→ππ, B→ρρ, B→ρπ

$2\beta+\gamma$



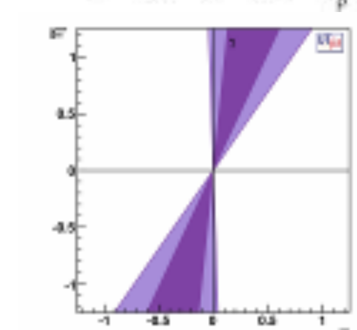
B→DK, B→Dπ

$\sin(2\beta)$

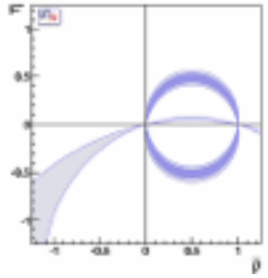


B→J/ΨK

γ



B→DK



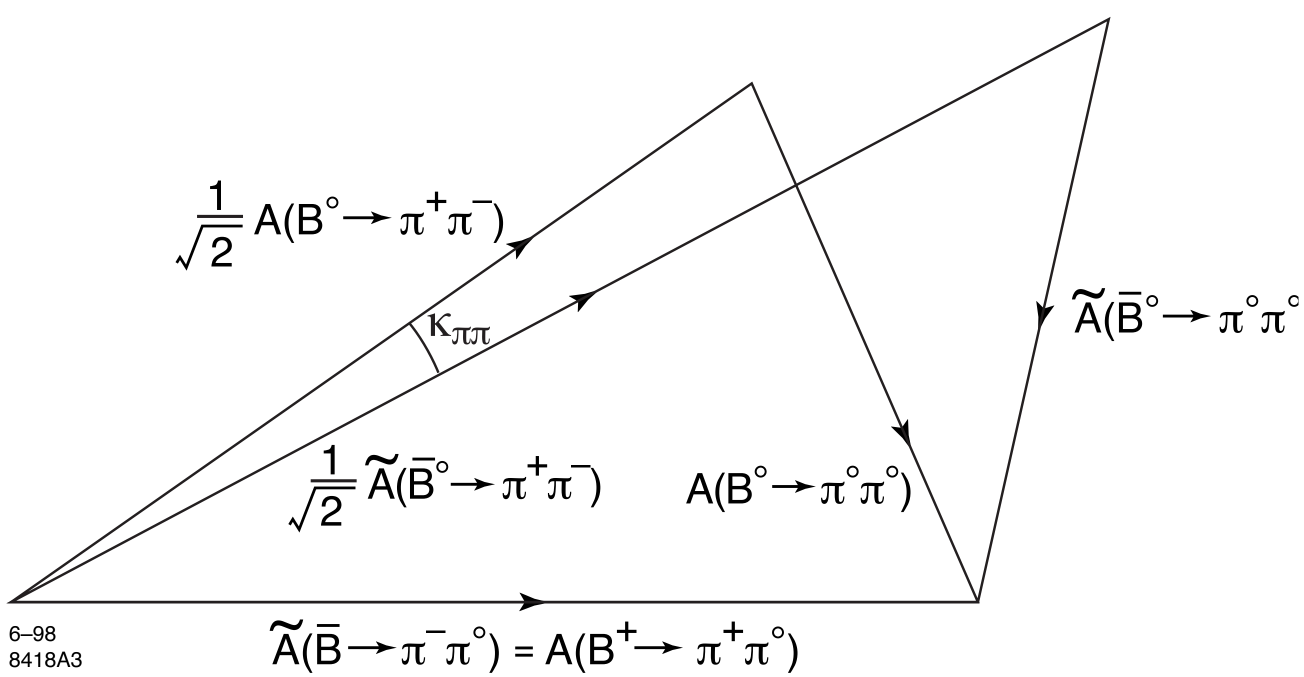
Alpha from B→ππ

B→$\pi^+\pi^-$, B→$\pi^0\pi^0$, B→$\pi^+\pi^0$ decays are connected from isospin relations. ππ states can have I = 2 or I = 0

the gluonic penguins contribute only to the I = 0 state (ΔI=1/2)

$\pi^+\pi^0$ is a pure I = 2 state (ΔI = 3/2) and it gets contribution only from the tree diagram

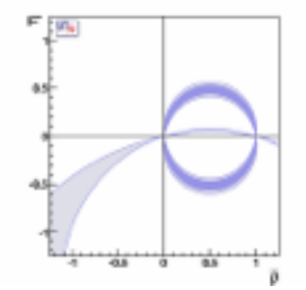
triangular relations allow for the determination of the phase difference induced on α



$$
\begin{aligned}
A^{+-} &= -Te^{-i\alpha} + Pe^{i\delta_P} \\
A^{+0} &= -\frac{1}{\sqrt{2}}\left[e^{-i\alpha}(T + T_c\, e^{i\delta_{T_c}})\right] \\
A^{00} &= -\frac{1}{\sqrt{2}}\left[e^{-i\alpha}T_c\, e^{i\delta_{T_c}} + P\, e^{i\delta_P}\right] ,
\end{aligned}
$$

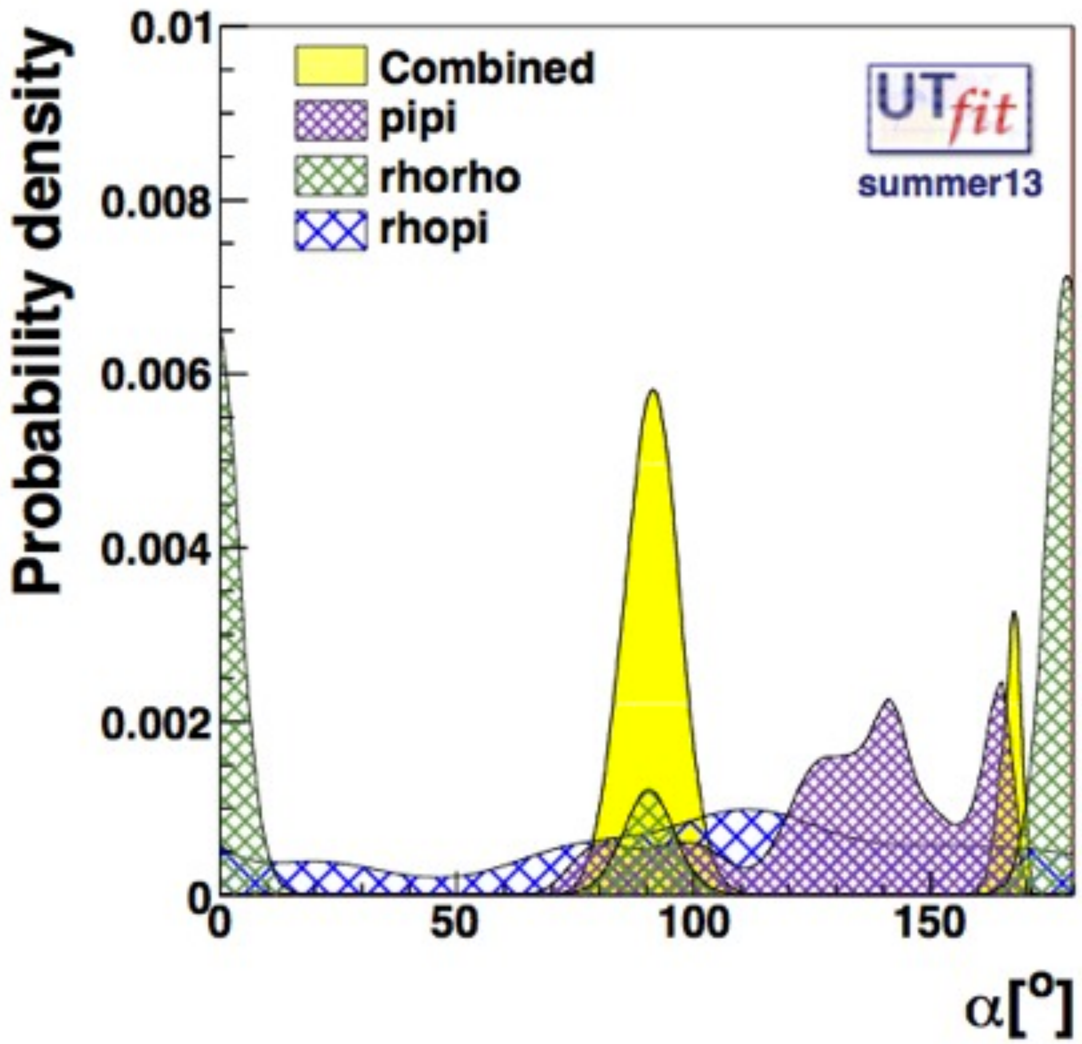
We can construct the observables, like *CP* asymmetries and branching fractions from amplitudes and solve the equation on α. The same method also can be used for the B→ρρ system

PRL 65 (1990) 3381



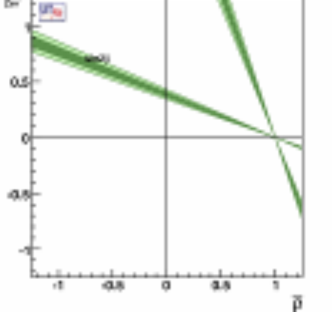
Another point is adding the B→ρπ analysis

This is a completely different analysis: The time-dependent Dalitz plot analysis of the decays of the neutral B allows one to infer the value of α without any dependence on the hadronic parameter.



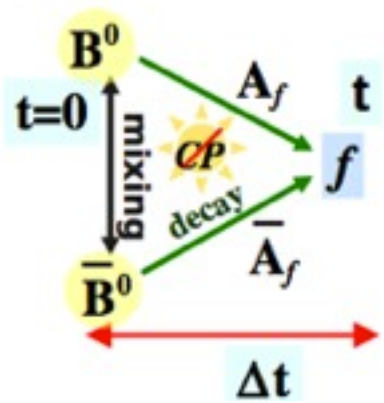
$$\alpha = (90.7 \pm 7.4)^\circ$$

Phys. Rev. D76 (2007) 014015



$B^0 \rightarrow J/\Psi K^0$

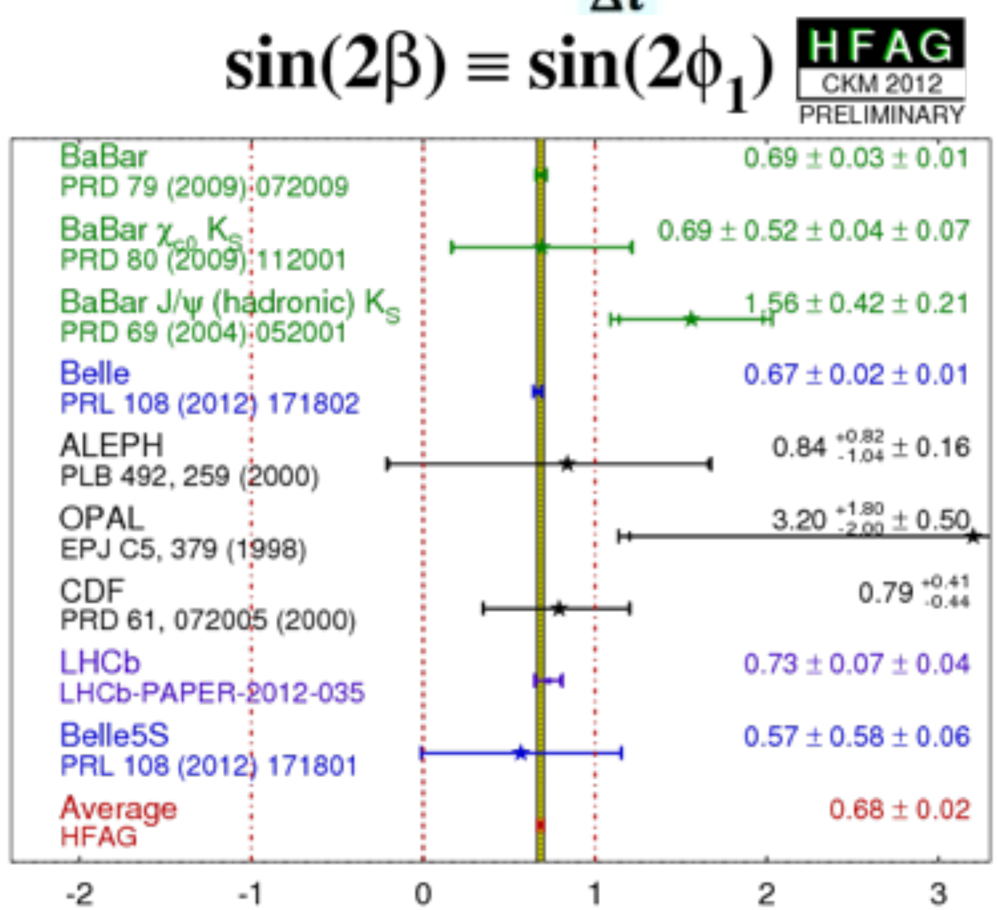
sin2β from time-dependent A_{CP} in $B \rightarrow J/\psi K$



$$a_{f_{CP}}(t) = \frac{\mathrm{Prob}(B^0(t) \rightarrow f_{CP}) - \mathrm{Prob}(\overline{B}^0(t) \rightarrow f_{CP})}{\mathrm{Prob}(\overline{B}^0(t) \rightarrow f_{CP}) + \mathrm{Prob}(B^0(t) \rightarrow f_{CP})} = C_f \cos \Delta m_d t \; + \; S_f \sin \Delta m_d t$$

The decay is dominated by a single (tree level) amplitude, thus a can be simplified:

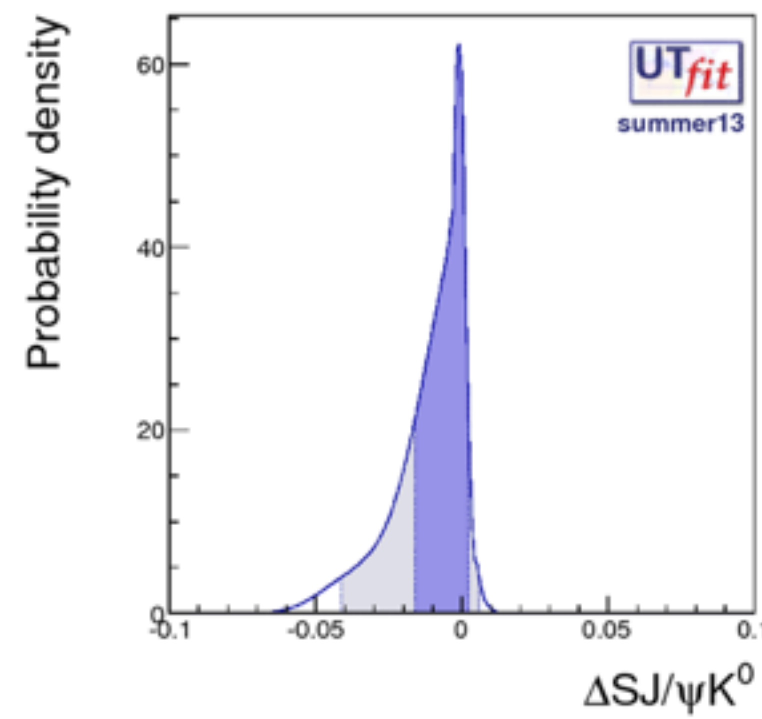
$$a_{f_{CP}}(t) = -\eta_{CP} \sin \Delta m_d \Delta t \sin 2\beta$$



We also analise $\bar{B}^0 \rightarrow J/\psi \pi^0$ to obtain the theoretical uncertainty in data-driven way. This gives us an additional correction:

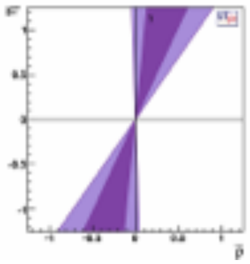
data-driven theoretical uncertainty

$\Delta S \in [-0.02, 0.00]$ at 68% prob.



$$\sin(2\beta) = (0.680 \pm 0.023)$$

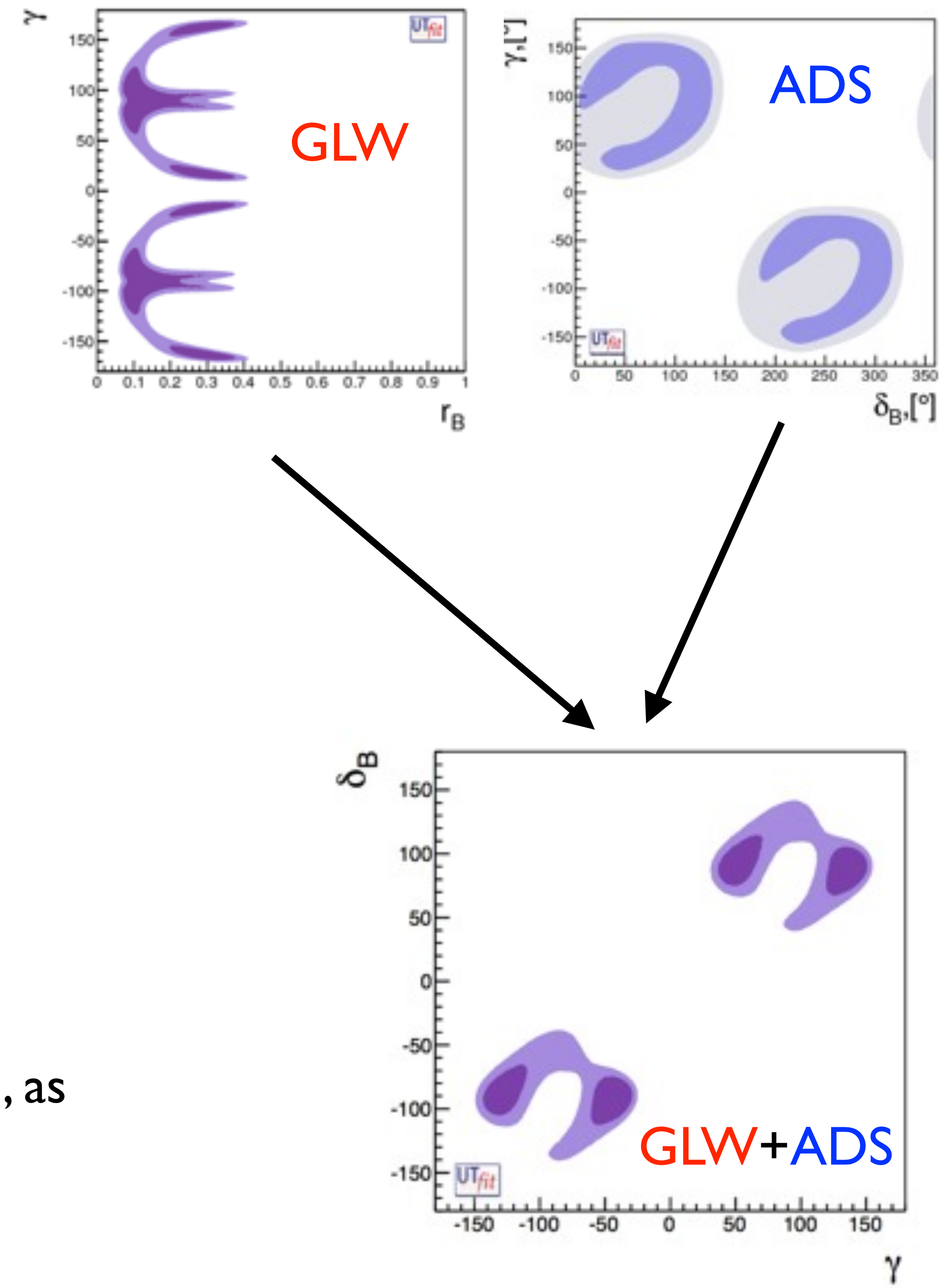
Phys. Rev. Lett. 95, 221804 (2005)



We use the available information coming from the three methods:

- GLW (M. Gronau, D. London, D. Wyler, PLB253,483 (1991); PLB 265, 172 (1991))
- ADS (D. Atwood, I. Dunietz and A. Soni, PRL 78, 3357 (1997))
- GGSZ (A. Giri, Yu. Grossman, A. Soffer, J. Zupan, PRD 68, 054018(2003))

For the decays: $B^+ \to D^{(*)}K^{(*)+}$ and $B^0 \to D^{(*)}K^{(*)0}$

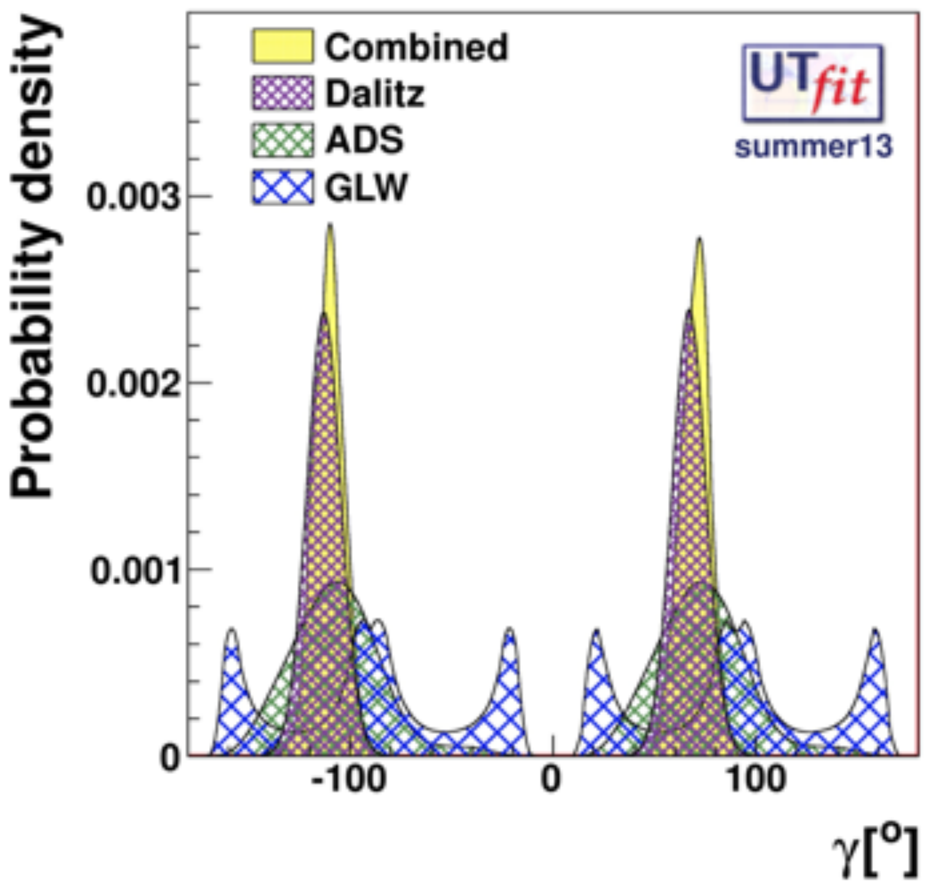


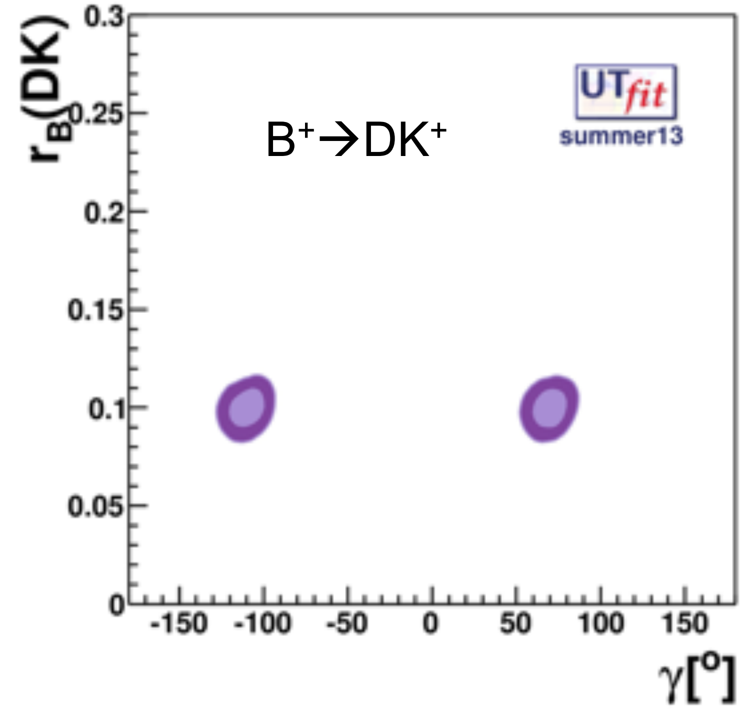
The combination is performed starting from the HFAG averages. The main problem is treatment of the nontrivial likelihoods for $\{\gamma, \delta_B, r_B\}$ observables.

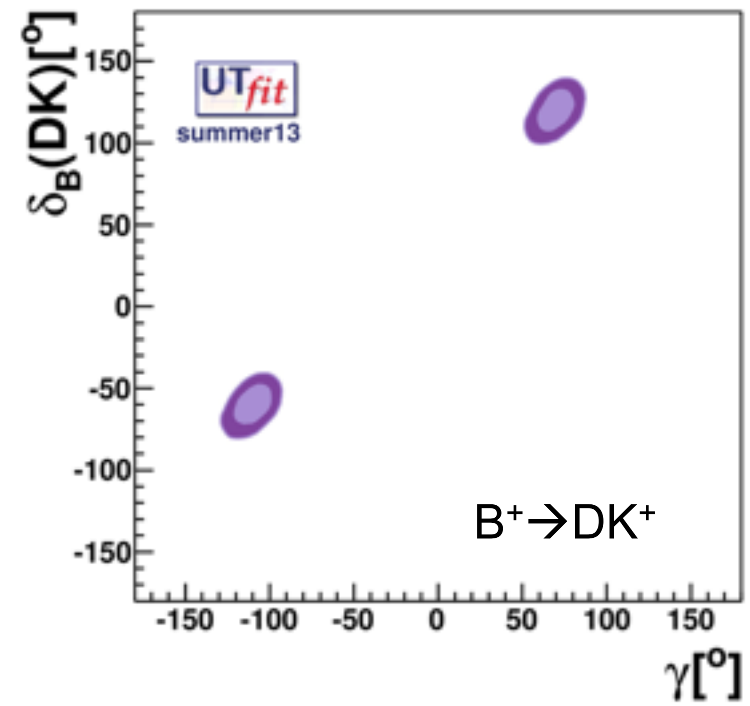
We also use CLEOc results in the ADS reconstruction.

Currently, we do not include D^0 mixing in the combination, as the effect is small in $B \to DK$ system

Results of Combination







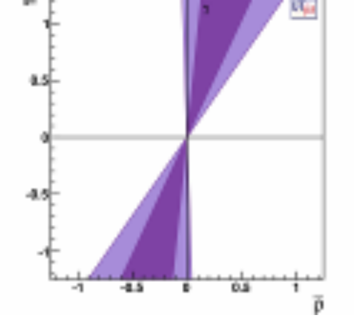
$\gamma_{all} = (70.1 \pm 7.1)^\circ$

The results show gaussian behaviour in the most sensitive channels $B^+ \to DK^+$

With new results in B^0 system, we are able to have the combined value more than 4 sigmas away from 0.

	DK^+	D^*K^+	DK^{*+}	DK^{*0}
δ_B	$(120.2\pm8.2)^\circ$	$(-51\pm13)^\circ$	$(124\pm34)^\circ$	$(-55 \pm44)^\circ$
r_B	(0.100 ± 0.006)	(0.118 ± 0.018)	(0.13 ± 0.06)	(0.26 ± 0.06)

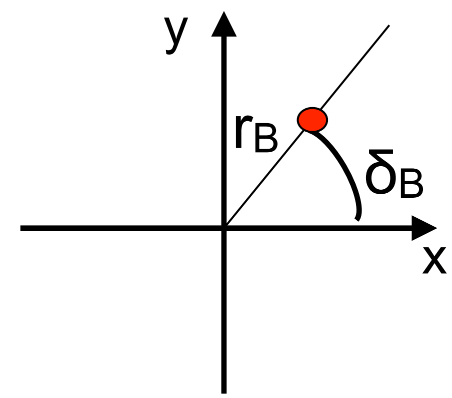
It is very important to understand that constructing predictions observables out of values of $\{\gamma, \delta_B, r_B\}$ will still require a similar likelihood analysis (for example, asymmetries will not be gaussian).



Gamma combination: prior studies and strong phases

We have tested the behavior of the gamma average for different priors including:

- Flat cartesian coordinates {x;y}:
- Jeffreys prior on r_B (weight ~ $1/\sqrt{r_B}$)



The results are stable against all the reasonable priors and do not give more than 1 degree difference in central values

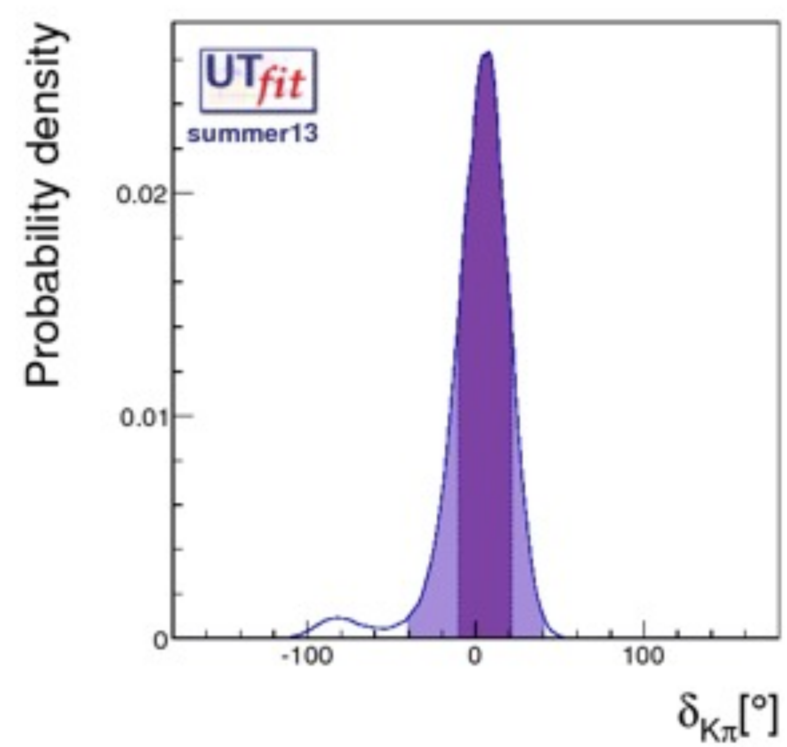
Another important result is that we are able to measure the strong phase $\delta_{D\to K\pi}$.

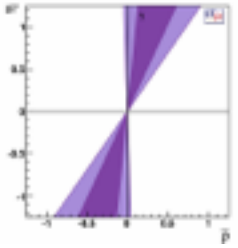
at 68.27% prob [-10,21]
at 95.45% prob [-40,40]

The results are consistent with our mixing studies and with most recent BES III results:

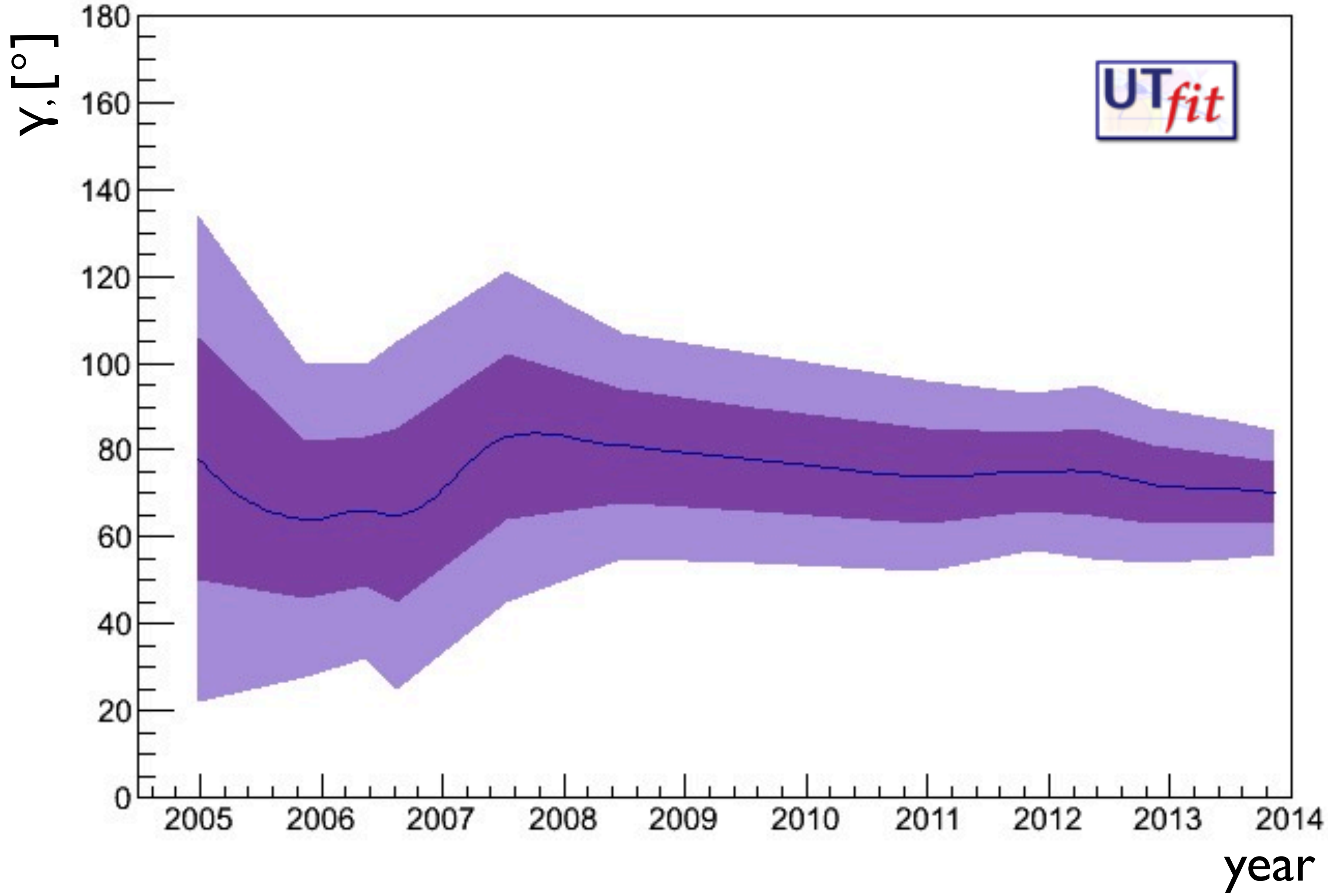
$$\delta_D = (18^{+11}_{-17})^\circ$$

Removing CLEOc information inflates the errors by 0.5 degrees





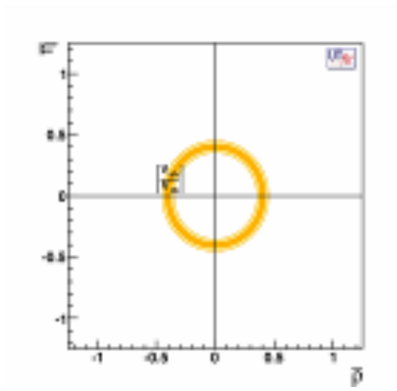
History of Combination



The world average error has decreased by a factor 3 in 10 years

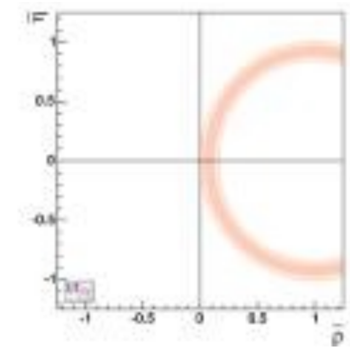
https://www.utfit.org/foswiki/bin/view/UTfit/GammaFromTrees

$|V_{ub}/V_{cb}|$



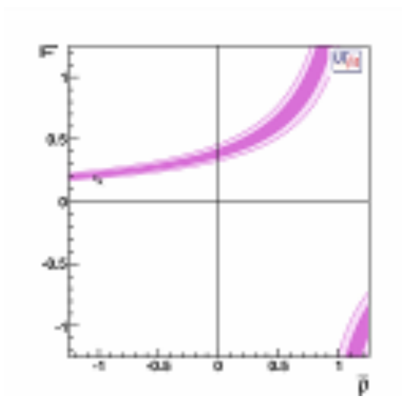
exclusive B→Dlν (B→π(ρ)lν) determination
inclusive b→c (b→u) determination

$\Delta m_d/\Delta m_s$



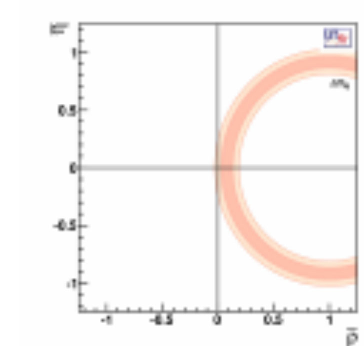
$B_{d,s}$ mixing

ε_K



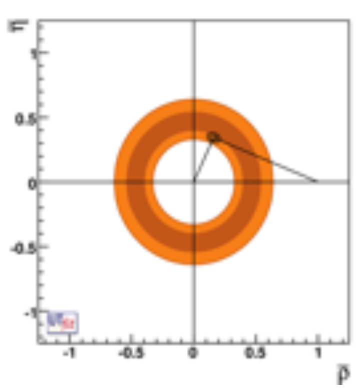
indirect *CP* violation in K_L decays

Δm_d



B_d mixing

Rare decays



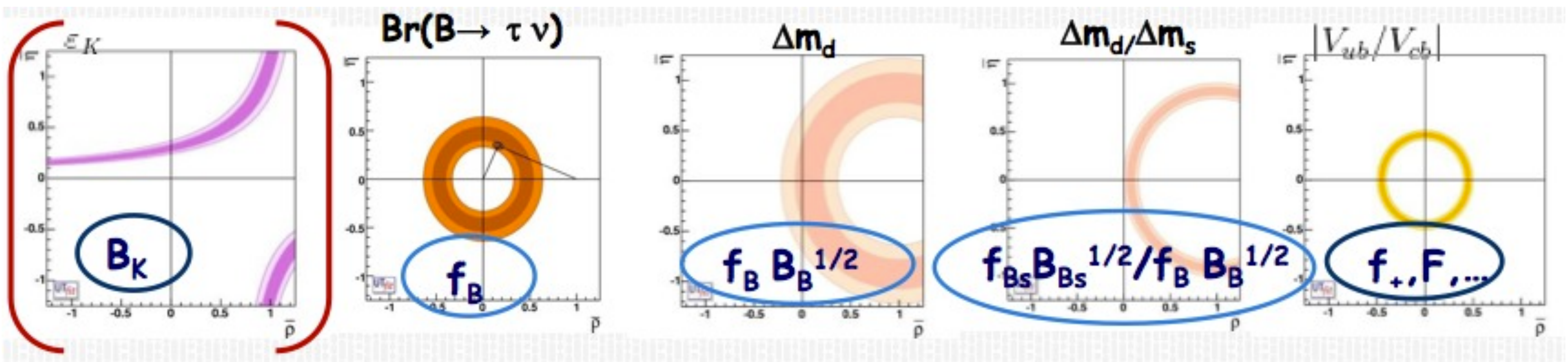
B→τν, B→μμ

For most of other CKM fit inputs we need several parameters calculated on lattice. We use:

- B-parameter in the Kaon sector
- K, D, and B mesons decay constants f_{Bs}, f_B, f_D, f_K
- Matrix elements for K, B, and D mixing
- s quark mass, V_{us}, V_{ud} (FlaviaNet values)

We use the most updated values from FLAG working group and our averages for the full full basis of B-parameters for K-Kbar, D-Dbar and B-Bbar mixing

B_K	0.766 ± 0.010
f_{B_s}	0.2277 ± 0.0045
f_{B_s}/f_{B_d}	1.202 ± 0.022
$\hat{B}_{B_s}$	1.33 ± 0.06
$\hat{B}_{B_s}/\hat{B}_{B_d}$	1.06 ± 0.11



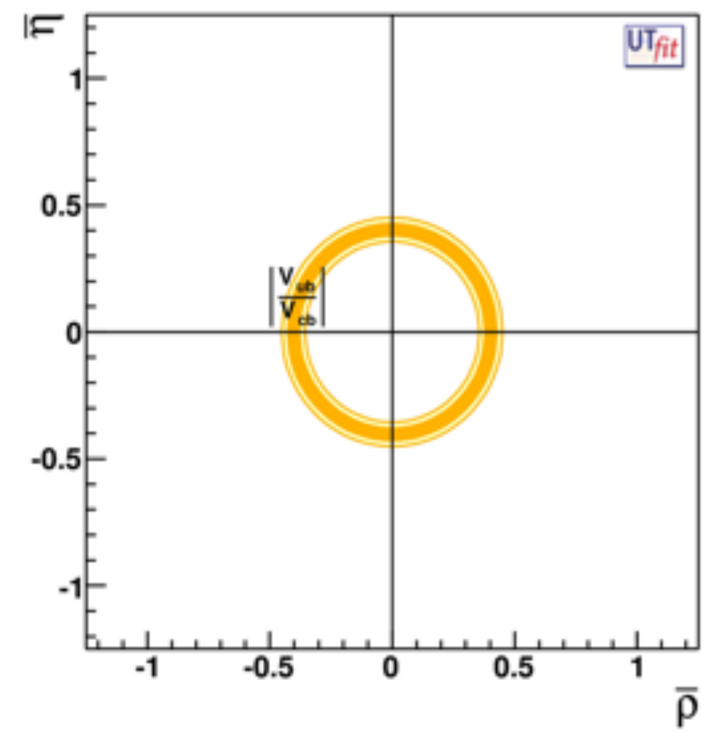
Nuovo Cim.B123:674-688,2008

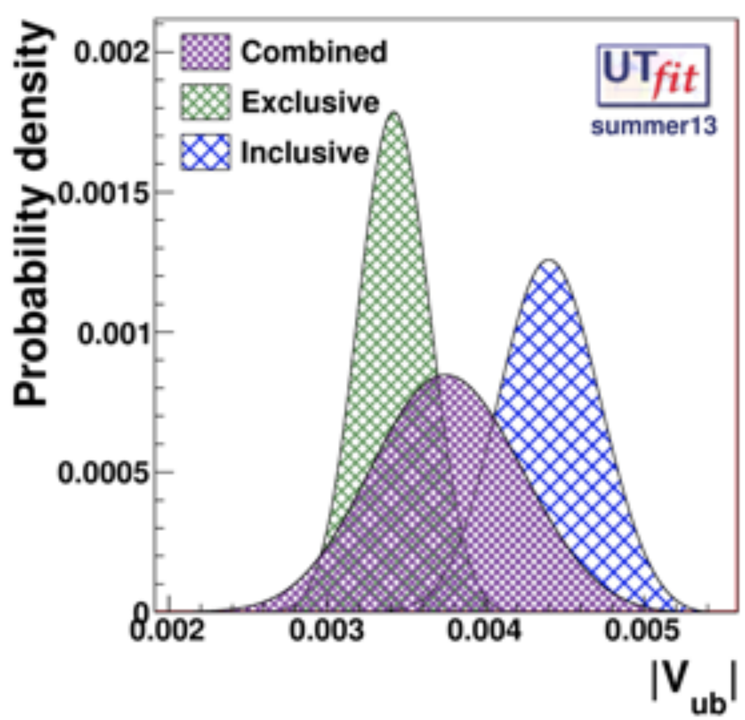
The relative ratio of CKM elements is easily calculable:

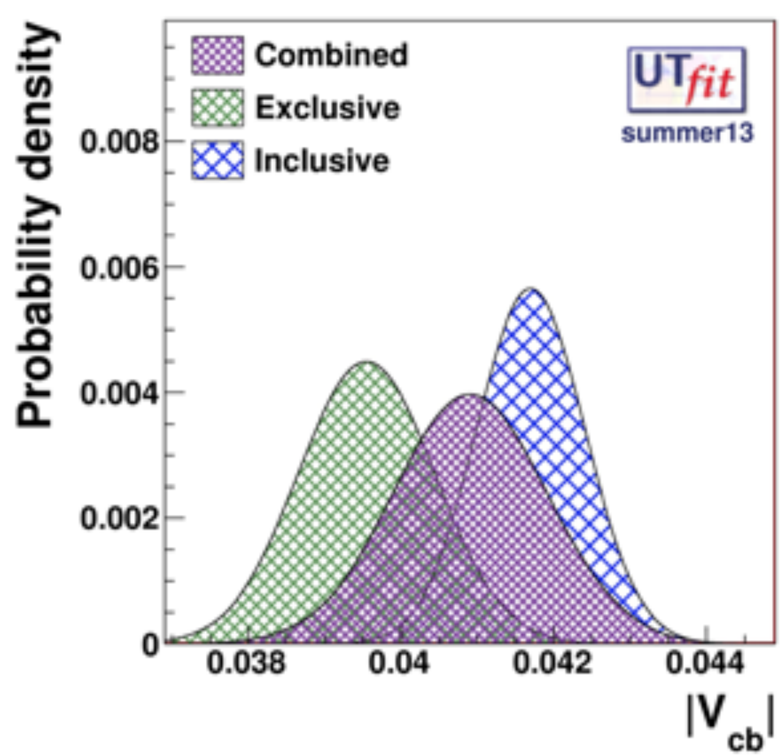
$$\left|\frac{V_{ub}}{V_{cb}}\right| = \frac{\lambda}{1 - \frac{\lambda^2}{2}}\sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

QCD corrections to be considered

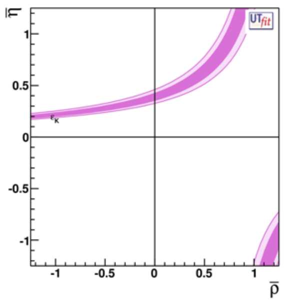
- inclusive measurements: OPE
- exclusive measurements: form-factors from lattice QCD







There is still an inconsistency between inclusive and exclusive measurements. We take this into account inflating the combined uncertainty.



ε_K

Indirect CP violation in the Kaon system is usually expressed in terms of $|\varepsilon_K|$ parameter which is the fraction of CP violating component in the mass eigenstates.

$$|\varepsilon_K| = C_\varepsilon B_K A^2 \lambda^6 \bar{\eta} \{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^4 (1-\bar{\rho})\}$$

S_0 - Inami-Lim functions for c-c, c-t, e t-t contributions (from perturbative calculations)

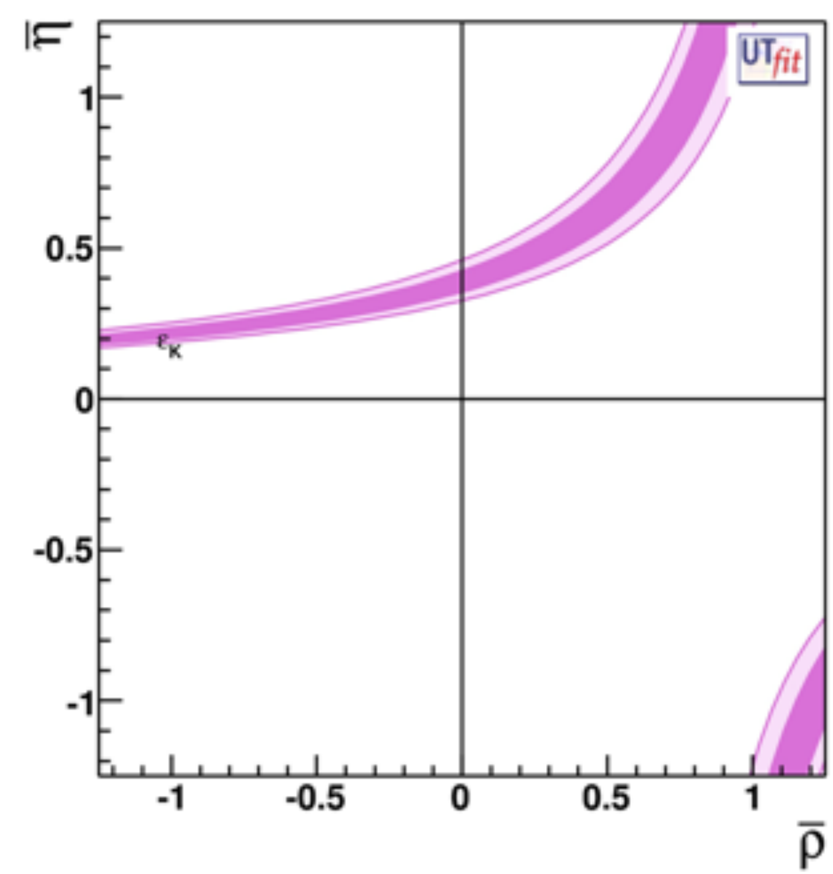
We also have a corrections for long-distance effects (Phys.Rev.D78:033005, PLB688 (2010) 309).

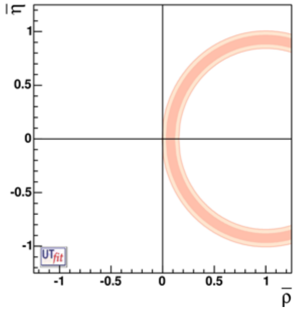
$$\epsilon_K = \sin\phi_\epsilon e^{i\phi_\epsilon} \left[\frac{\mathrm{Im} M_{12}^{(6)}}{\Delta m_K} + \rho\xi \right]$$

We use:

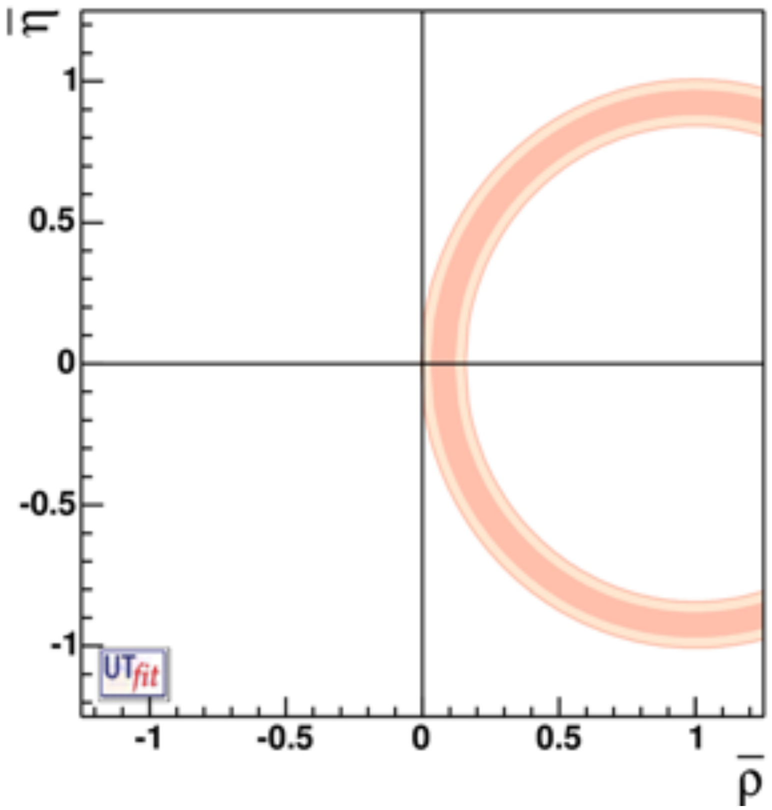
$$|\epsilon_K| = (2.23 \pm 0.11) \cdot 10^{-4}$$

Introducing the NNLO charm-top-quark contribution (from PRL108 (2012) 121801) increases the uncertainty by 0.01.





Δm_s or Δm_d



We include the oscillation of B_d and B_s as inputs of the fit using two observables Δm_d and $\Delta m_d/\Delta m_s$

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_W^2 \eta_b S(x_t) m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{tb}|^2 |V_{td}|^2 = $$

$$= \frac{G_F^2}{6\pi^2} m_W^2 \eta_b S(x_t) m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{cb}|^2 \lambda^2 ((1-\bar{\rho})^2 + \bar{\eta}^2)$$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d} f_{B_d}^2 \hat{B}_{B_d}}{m_{B_s} f_{B_s}^2 \hat{B}_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2} =$$

$$= \frac{m_{B_d} f_{B_d}^2 \hat{B}_{B_d}}{m_{B_s} f_{B_s}^2 \hat{B}_{B_s}} \left(\frac{\lambda}{1-\lambda^2/2}\right)^2 \frac{((1-\bar{\rho})^2 + \bar{\eta}^2)}{\left(1 + \frac{\lambda^2}{1-\lambda^2/2}\bar{\rho}\right)^2 + \lambda^4 \bar{\eta}^2}$$

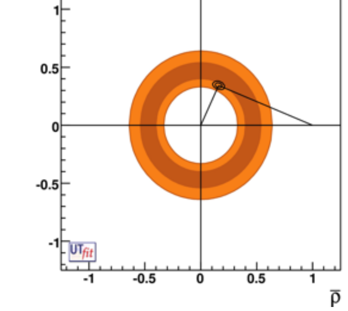
We use the following approximation

$$\Delta m_d \approx [(1-\rho)^2 + \eta^2] \frac{f_{B_s}^2 B_{B_s}}{\xi^2}$$

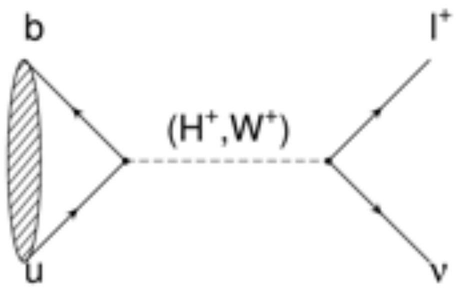
$$\Delta m_s \approx f_{B_s}^2 B_{B_s}$$

$$\Delta m_s = 17.768 \pm 0.024 \text{ ps}^{-1}$$

$$\Delta m_d = 0.510 \pm 0.004 \text{ ps}^{-1}$$



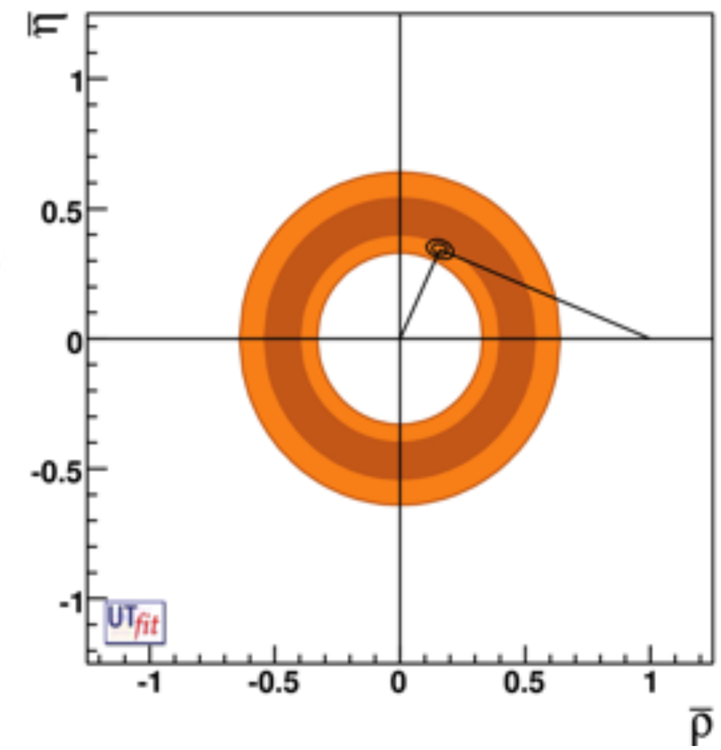
We use the combination of B→τν measurements by BaBar and Belle



$$\mathcal{B}(B \to \ell\nu) = \frac{G_F^2 m_B m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

We use:

$$B(B \to \tau\nu) = (1.14 \pm 0.22) \cdot 10^{-4}$$

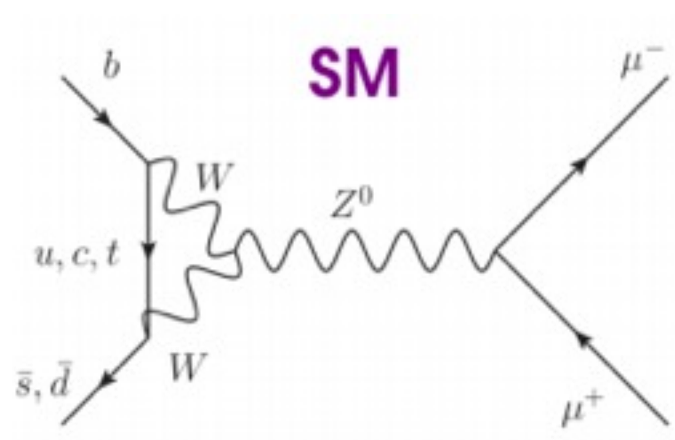


Brand new constraint from $B_{(s)}$→μμ measurements by LHCb and CMS

Experimental value needs to be corrected for the Bs oscillation to be compared to the theoretical predictions (see PRL 109, 041801 (2012))

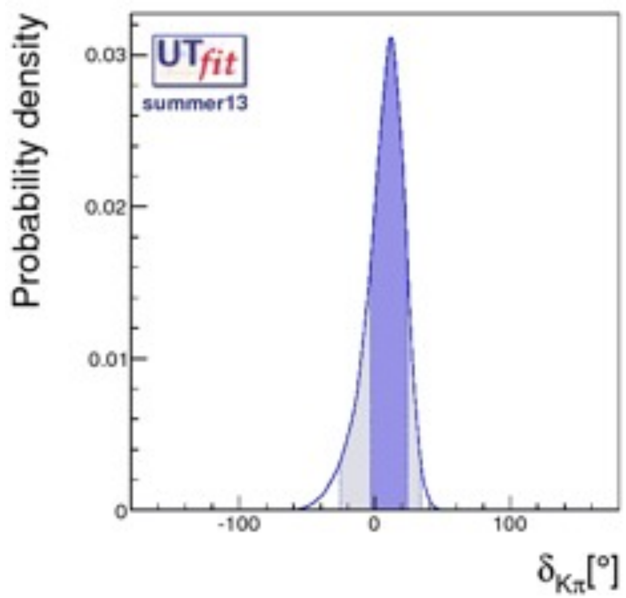
We use LHCb+CMS combination:

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}$$
$$\mathcal{B}(B^0 \to \mu^+\mu^-) = \left(3.6\,^{+1.6}_{-1.4}\right) \times 10^{-10}$$



Charm mixing for generic new physics fit

We perform a fit to the charm sector results allowing for CP violation in the singly-Cabibbo suppressed decays and receive the following results.



$\delta_D = (10.5 \pm 13.5)°$

$|q/p|-1=-0.015 \pm 0.077$

$\phi=(0.3 \pm 2.6)°$

$x = (4.2 \pm 1.8) \cdot 10^{-3}$

$y = (6.4 \pm 0.8) \cdot 10^{-3}$

This does not include the most recent LHCb results announced this week

For the purpose of constraining NP, it is useful to express the fit results in terms of the $\Delta C = 2$ effective Hamiltonian matrix elements.

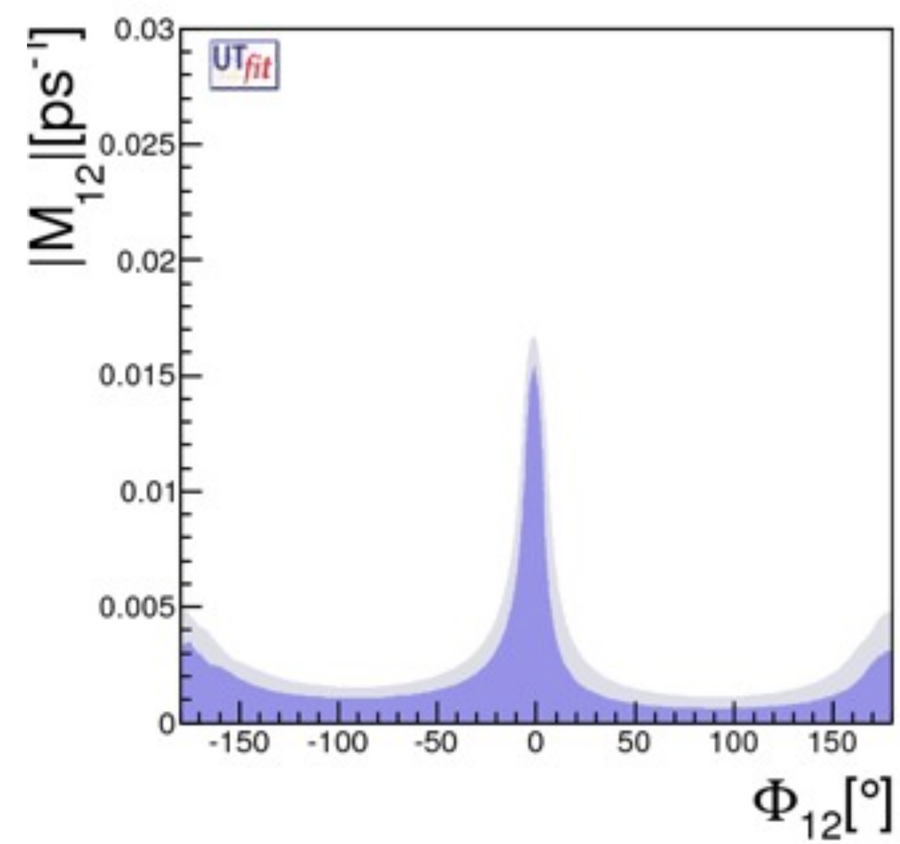
$$|M_{12}| = \frac{1}{\tau_D}\sqrt{\frac{x^2+\delta^2 y^2}{4(1-\delta^2)}}, \quad |\Gamma_{12}| = \frac{1}{\tau_D}\sqrt{\frac{y^2+\delta^2 x^2}{1-\delta^2}},$$

$$\sin\Phi_{12} = \frac{|\Gamma_{12}|^2 + 4|M_{12}|^2 - (x^2+y^2)|q/p|^2/\tau_D^2}{4|M_{12}\Gamma_{12}|}$$

$M_{12}=(0.005 \pm 0.002)\ ps^{-1}$

$\Gamma_{12}=(0.016 \pm 0.002)\ ps^{-1}$

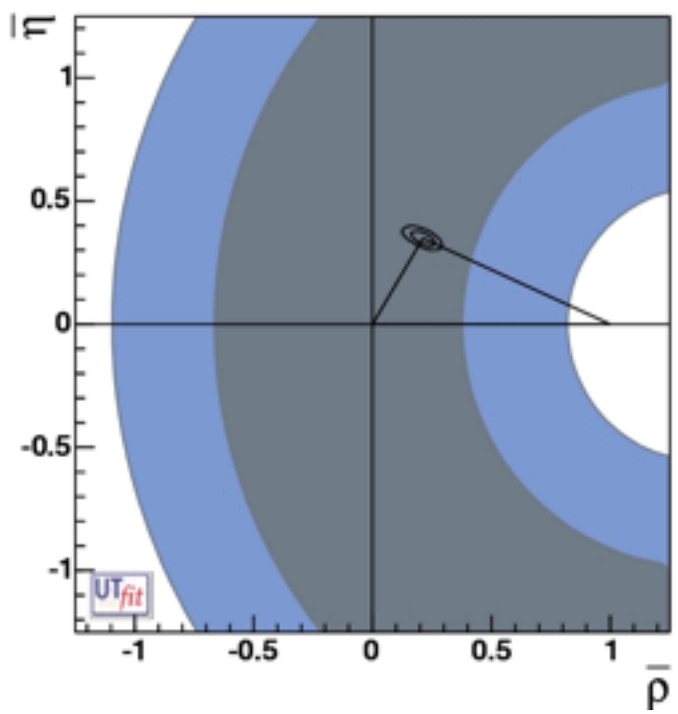
$\phi_{12}=(2 \pm 11)°$

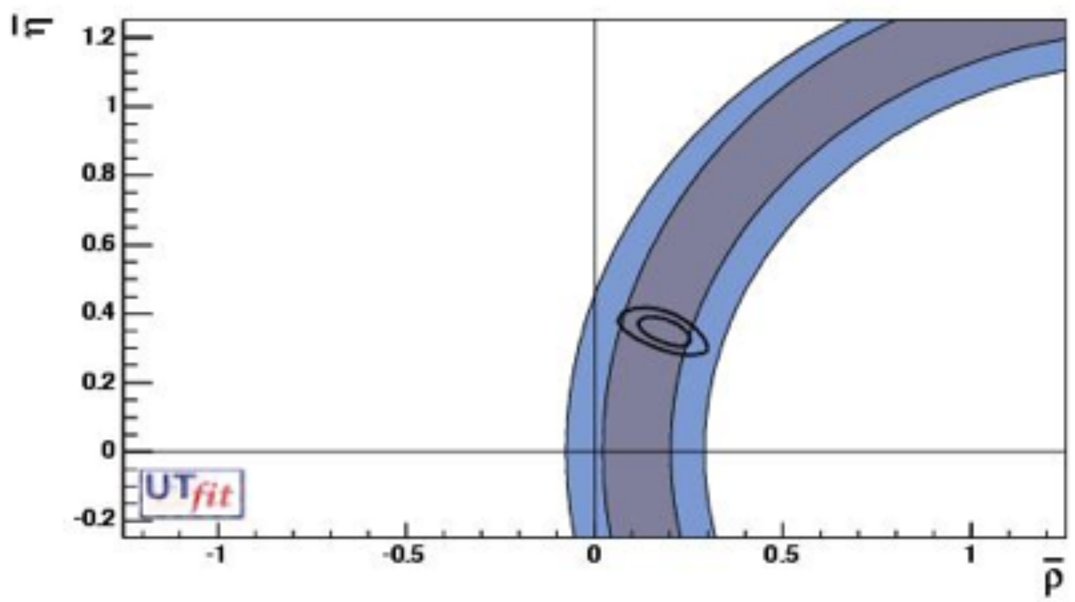


JHEP 1210 (2012) 068

$K^+ \to \pi^+ \nu \bar{\nu}$ decay is under study by NA62 (CERN) and ORKA (Fermilab) projects.

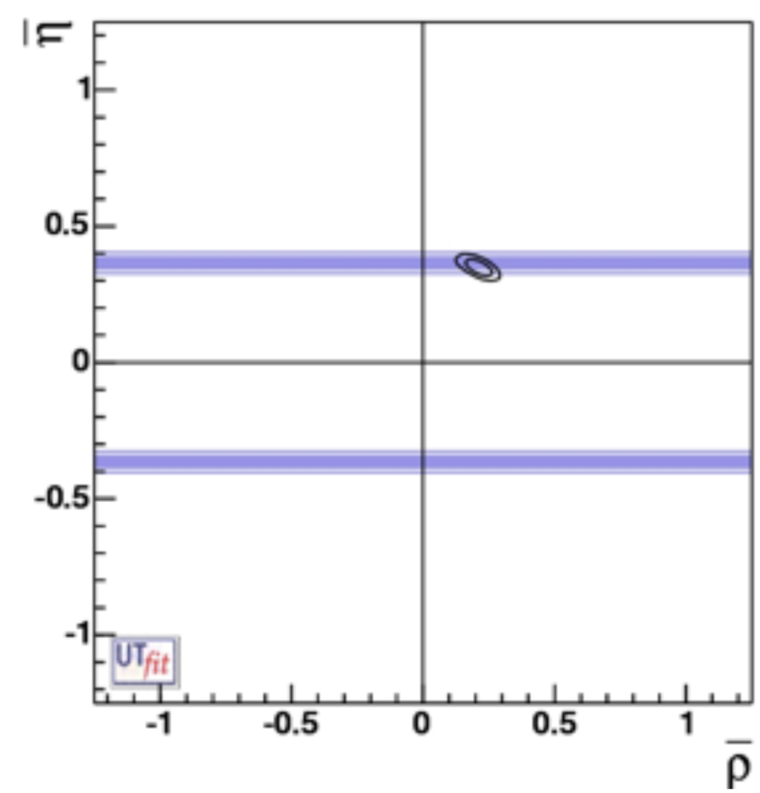
NA62 expects to collect ~100 events by 2016





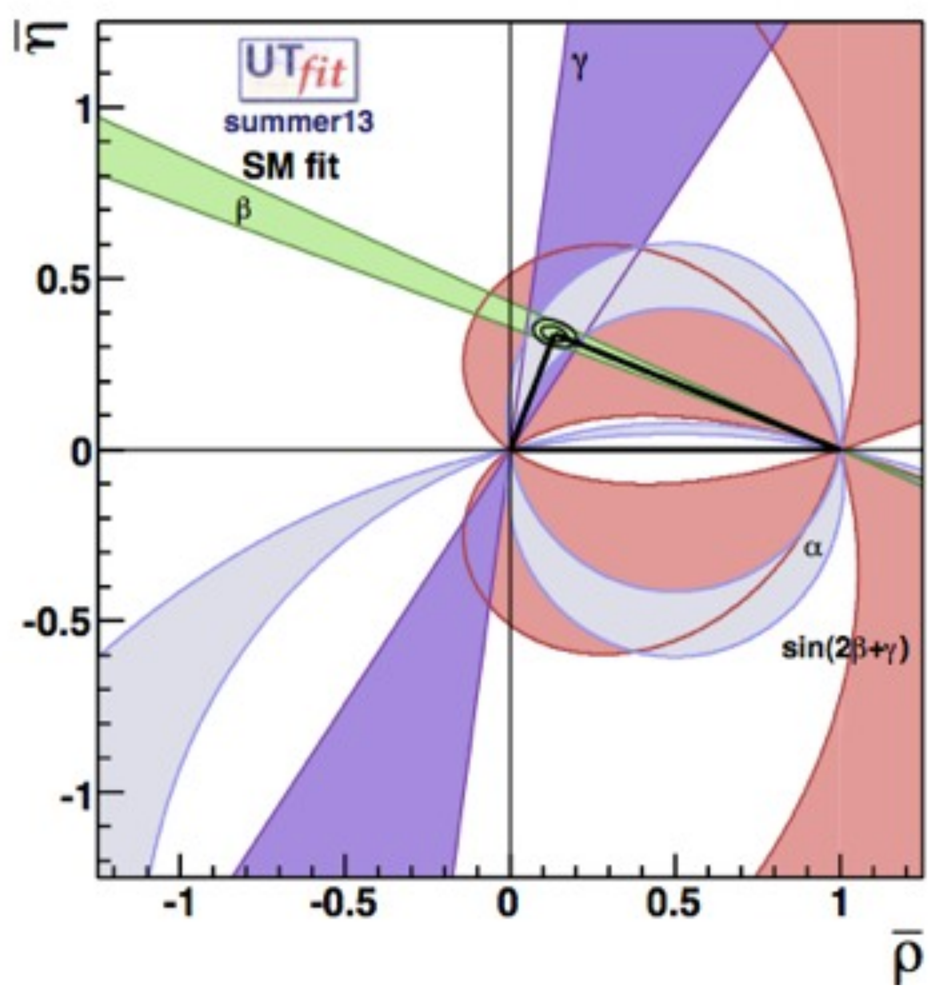
KOTO (JPARC) experiment is due to be operational soon. It will be searching for $K^0 \to \pi^0 \nu \bar{\nu}$

It aims to discover this decay at Step I and continue its measurements. For 10% error on BF

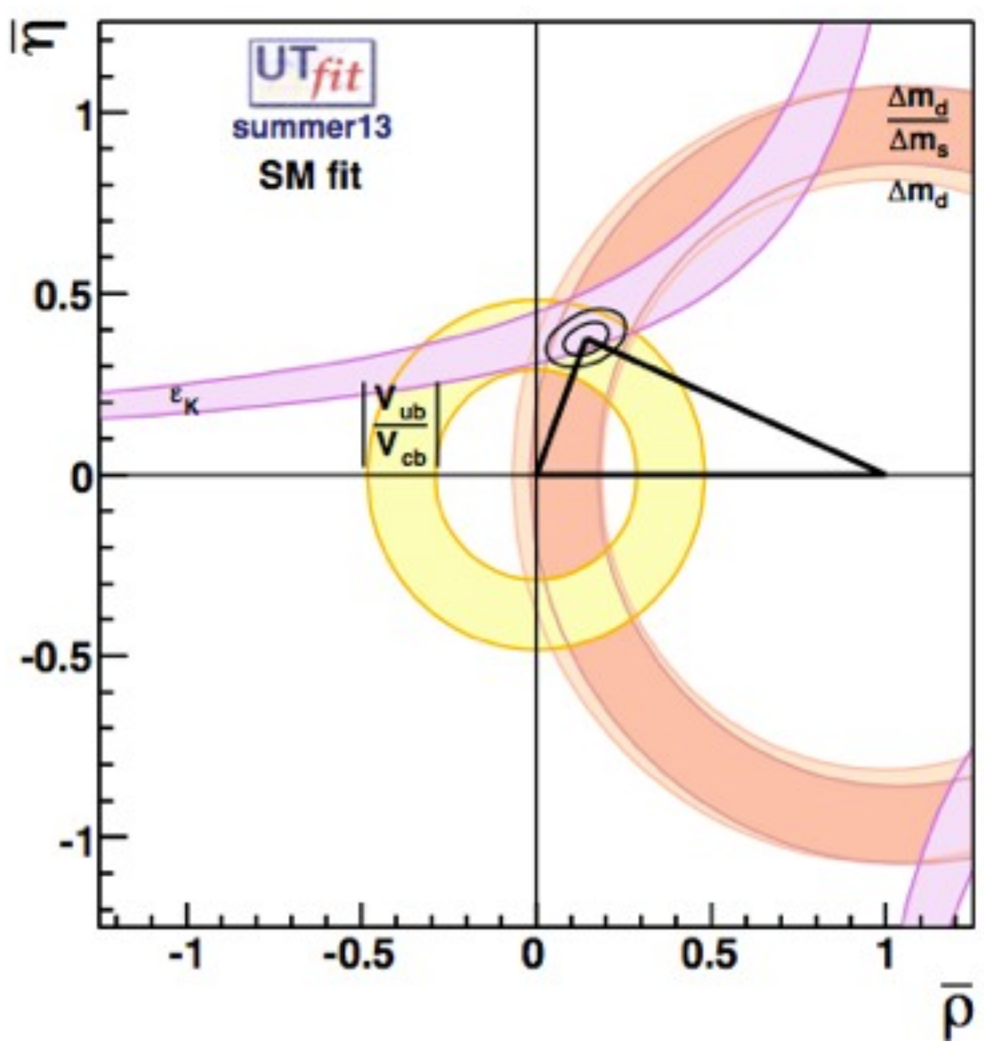


Outlook

Some more results did not make it inside the talk: lifetimes, their differences and quark masses.



vs.



See next talk by Marcella for the outcome of different fits.