Prospects of measurements of observables in $B^0 \to K^{*0}\mu^+\mu^-$ decays

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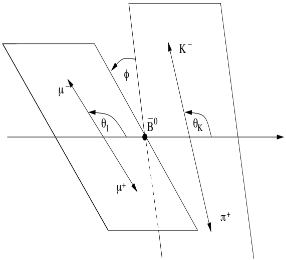
Disclaimer: This is part of a large ongoing effort within LHCb to define a path for the next round of this analysis.

Introduction

- As presented by Mitesh, LHCb measurements of $b \to s\ell\ell$ have revealed a host of interesting results.
- This talk focuses on the prospects for the full angular on the $3\,\text{fb}^{-1}$ update of $B^0 \to K^{*0}\mu^+\mu^-$.
- Thoughts on how to perform a "full" angular fit to $B^0 \to K^{*0}\mu^+\mu^-$ decays.
  - Results shown based on "realistic" $3\,\text{fb}^{-1}$-equivalent **toy data**.

$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$



- Differential decay rate of $\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$:

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = \frac{9}{32\pi}\Big[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l$$
$$+J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi$$
$$+(J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi$$
$$+J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big], \qquad (1)$$

- J_i terms depend on the spin amplitudes $A_0^{L,R}$,$A_{\|}^{L,R}$,$A_{\perp}^{L,R}$ (ignoring scalar contributions and in $m_\ell \sim 0$ limit)

Angular terms

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R^*} + A_\parallel^L A_\parallel^{R^*}\right),$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R^*})\right] + \beta_\ell^2 |A_S|^2,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right], \qquad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + |A_0^R|^2\right],$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + |A_\perp^R|^2 - |A_\parallel^R|^2\right], \qquad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L^*} + A_0^R A_\parallel^{R^*})\right],$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L^*} - A_0^R A_\perp^{R^*}) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^{R^*} A_S)\right],$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L^*} - A_\parallel^R A_\perp^{R^*})\right], \qquad J_{6c} = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_0^L A_S^* + A_0^{R^*} A_S),$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L^*} - A_0^R A_\parallel^{R^*}) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* - A_\perp^{R^*} A_S))\right],$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L^*} + A_0^R A_\perp^{R^*})\right], \qquad J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L^*} A_\perp^L + A_\parallel^{R^*} A_\perp^R)\right], \quad (3)$$

Amplitudes I

[JHEP 0901(2009)019] Altmannshofer et al.

$$A_{\perp}^{L(R)} = N\sqrt{2\lambda}\left\{\left[(\mathbf{C_9^{eff}} + \mathbf{C_9'^{eff}}) \mp (\mathbf{C_{10}^{eff}} + \mathbf{C_{10}'^{eff}})\right]\frac{\mathbf{V(q^2)}}{m_B + m_{K^*}} + \frac{2m_b}{q^2}(\mathbf{C_7^{eff}} + \mathbf{C_7'^{eff}})\mathbf{T_1(q^2)}\right\}$$

$$A_{\parallel}^{L(R)} = -N\sqrt{2}(m_B^2 - m_{K^*}^2)\left\{\left[(\mathbf{C_9^{eff}} - \mathbf{C_9'^{eff}}) \mp (\mathbf{C_{10}^{eff}} - \mathbf{C_{10}'^{eff}})\right]\frac{\mathbf{A_1(q^2)}}{m_B - m_{K^*}} + \frac{2m_b}{q^2}(\mathbf{C_7^{eff}} - \mathbf{C_7'^{eff}})\mathbf{T_2(q^2)}\right\}$$

$$A_0^{L(R)} = -\frac{N}{2m_{K^*}\sqrt{q^2}}\left\{\left[(\mathbf{C_9^{eff}} - \mathbf{C_9'^{eff}}) \mp (\mathbf{C_{10}^{eff}} - \mathbf{C_{10}'^{eff}})\right]\left[(m_B^2 - m_{K^*}^2 - q^2)(m_B + m_{K^*})\mathbf{A_1(q^2)} - \lambda\frac{\mathbf{A_2(q^2)}}{m_B + m_{K^*}}\right]\right.$$
$$\left. + 2m_b(\mathbf{C_7^{eff}} - \mathbf{C_7'^{eff}})\left[(m_B^2 + 3m_{K^*} - q^2)\mathbf{T_2(q^2)} - \frac{\lambda}{m_B^2 - m_{K^*}^2}\mathbf{T_3(q^2)}\right]\right\}$$

$$A_t = \frac{N}{\sqrt{q^2}}\sqrt{\lambda}\left\{2(\mathbf{C_{10}^{eff}} - \mathbf{C_{10}'^{eff}}) + \frac{q^2}{m_\mu}(\mathbf{C_P^{eff}} - \mathbf{C_P'^{eff}})\right\}\mathbf{A_0(q^2)}$$

$$A_S = -2N\sqrt{\lambda}(\mathbf{C_S} - \mathbf{C_S})\mathbf{A_0(q^2)}$$

- $\mathbf{C_i^{eff}}$ are the Wilson coefficients (including 4-quark operator contributions)
- $\mathbf{A_i}$, $\mathbf{T_i}$ and $\mathbf{V_i}$, are form factors typically treated as nuisance parameters

Amplitudes II

- At leading order in $1/m_b$, α_s and for large $E_{K^*} > \Lambda_{QCD}$ (large recoil), form factors reduce to $\xi_\perp$,$\xi_\parallel$:

$$A_\perp^{L,R} = \sqrt{2} N m_B (1-\hat{s}) \left[(\mathcal{C}_9^{\rm eff} + \mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10} + \mathcal{C}'_{10}) + \frac{2\hat{m}_b}{\hat{s}} (\mathcal{C}_7^{\rm eff} + \mathcal{C}_7^{\rm eff\prime}) \right] \xi_\perp(E_{K^*})$$

$$A_\parallel^{L,R} = -\sqrt{2} N m_B (1-\hat{s}) \left[(\mathcal{C}_9^{\rm eff} - \mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10} - \mathcal{C}'_{10}) + \frac{2\hat{m}_b}{\hat{s}} (\mathcal{C}_7^{\rm eff} - \mathcal{C}_7^{\rm eff\prime}) \right] \xi_\perp(E_{K^*})$$

$$A_0^{L,R} = -\frac{N m_B (1-\hat{s})^2}{2\hat{m}_{K^*}\sqrt{\hat{s}}} \left[(\mathcal{C}_9^{\rm eff} - \mathcal{C}_9^{\rm eff\prime}) \mp (\mathcal{C}_{10} - \mathcal{C}'_{10}) + 2\hat{m}_b (\mathcal{C}_7^{\rm eff} - \mathcal{C}_7^{\rm eff\prime}) \right] \xi_\parallel(E_{K^*})$$

- Can build form factor independent observables using ratios of bilinear amplitude combinations [JHEP 1301(2013)048] Descotes-Genon et al. e.g:

$$P'_5 \sim \frac{Re(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*})}{\sqrt{(|A_0^L|^2 + |A_0^R|^2)(|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2)}}$$

An experimentalist's view

- Experimentalists fit the angular distribution of $B^0 \to K^{*0}\mu^+\mu^-$ decays in a particular basis, to extract observables
- Theorists use these observables to extract Wilson coefficients
  - The observable basis is not set in stone
- **Goal: Build any observable basis and the correlations, from a single fit to the angular distribution**
- Fitting for the amplitudes will offer a complete description of the decay
- Fitting for amplitudes improves stability of fit relative to fitting for observables (complex relations between observables)
- Some important considerations
  1. Need to account for symmetries as they will exhibit themselves as degenerate regions in the fit
  2. Account for the q^2 dependence of the amplitudes

Symmetries of the angular distribution...

[JHEP10(2010)056] Egede et al.

- Symmetry: Transformations of the A_j's that leave the J_i's and hence the differential decay distribution invariant.
- Number of degrees of freedom of J_i's (“experimental”) and of A_j's (“theoretical”) must match.
- Account for dependencies between J_i's (n_d) and symmetries of A_j's (n_s) with $n_J - n_d = 2n_A - n_s$.
- $6 \times 2 = 12$ real A_j parameters, 8 independent J_i's (in the limit of $m_\ell = 0$ and no scalar operators).
- Thus 4 symmetry transformations.

A bit more on symmetries...

[JHEP10(2010)056] Egede et al.

- Define the basis:

$$n_{\parallel} = \begin{pmatrix} A_{\parallel}^{L} \\ A_{\parallel}^{R*} \end{pmatrix}, \quad n_{\perp} = \begin{pmatrix} A_{\perp}^{L} \\ -A_{\perp}^{R*} \end{pmatrix}, \quad n_{0} = \begin{pmatrix} A_{0}^{L} \\ A_{0}^{R*} \end{pmatrix}$$

- Continuous transformations:

$$n_i^{'} = U n_i = \begin{bmatrix} e^{i\phi_L} & 0 \\ 0 & e^{-i\phi_R} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cosh i\tilde{\theta} & -\sinh i\tilde{\theta} \\ -\sinh i\tilde{\theta} & \cosh i\tilde{\theta} \end{bmatrix} n_i$$

- $\phi_{L,R}$ are global phase changes of left and right handed amplitudes, θ and $\widetilde{\theta}$ are helicity+handedness transformations.

...and a bit more...

[JHEP10(2010)056] Egede et al.

- Implement symmetry as a constraint on a set of amplitudes in order to have a well defined minimum in the angular fit
- Can choose to fix any 4 components as long as:
  1. Solutions for $\phi_{L,R}$,θ,$\widetilde{\theta}$ exist,
  2. Transformed amplitudes can be parametrised by smooth function in q^2

NB: The form and symmetries of the amplitudes in the large recoil region, give rise to 8 observables, 6 of which are form factor independent and 2 which contain information on form factors.[JHEP 1204(2012)104] Descotes-Genon et al.

Parametrising amplitudes in q^2

- Focus on $1 < q^2 < 6\,\text{GeV}^2$ region mainly due to:

1. Potential resonant di-muon structures below 1 and above $6\,\text{GeV}^2$ complicate situation
2. Set of symmetries only apply only in $m_\mu \sim 0$ limit

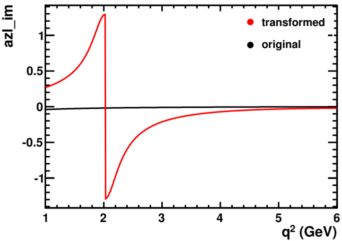
- Squinting at L.O. amplitude expression can see a general parametrisation:

$$A_i \sim \alpha_i + \beta_i q^2 + \gamma_i / q^2$$

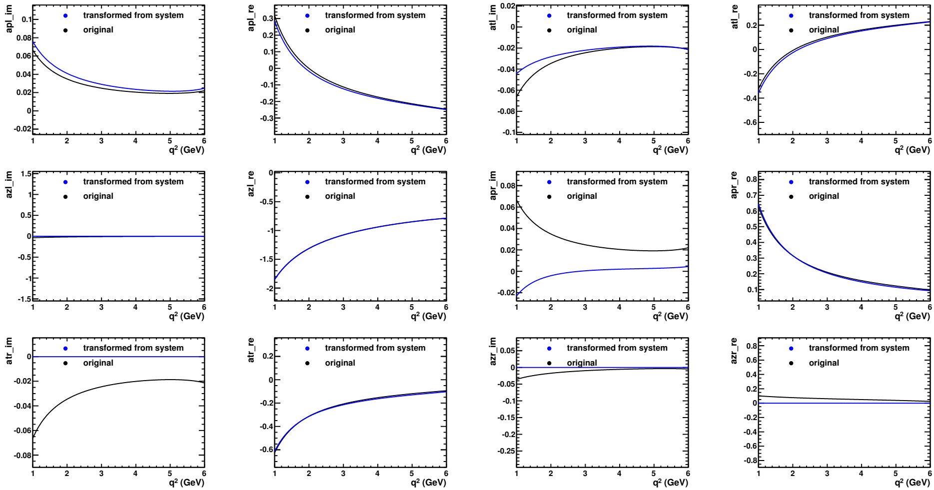
- 3 parameters per real amplitude component gives 24 amplitude parameters (accounting for constraints) per B^0 flavour

Fitting for the amplitudes

- Original attempt in [JHEP10(2010)056] Egede et al.
- Amplitudes parametrised as 2nd order polynomials in q^2
- Choice of $A_{\parallel}'^{L} = 0, Im(A_{\parallel}'^{R}) = 0, Im(A_{\perp}'^{L}) = 0$ leads to:



- Solution was to fit for $2.5 < q^2 < 6\,\text{GeV}$.
- Propose a (better/physics) choice: $A_0'^{R} = 0, Im(A_0'^{L}) = 0, Im(A_{\perp}'^{R}) = 0$

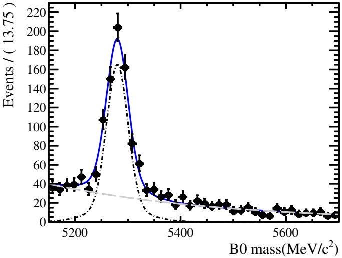


transformed, original

- Smooth behaviour of transformed amplitudes
- **Also holds for wide range of new physics scenarios**

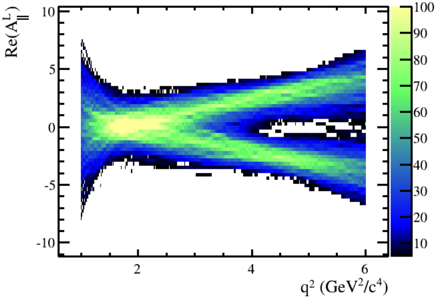
Fit procedure

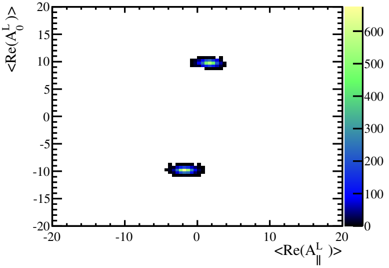
- Generate and fit toy datasets using rough estimates of signal and background yields in the $3\,\mathrm{fb}^{-1}$ LHCb dataset.
- Signal generated using `EOS` central value amplitude predictions in the SM. [JHEP1007(2010)098] Bobeth, van Dyk et al.
- Use "α, β, γ" parametrisation to fit it back.
- Effects not yet accounted for:
  1. S-wave in $K\pi$ system.
  2. Detector bias of angular distribution.
- Results should be treated as a proof of principle.



Results of amplitudes

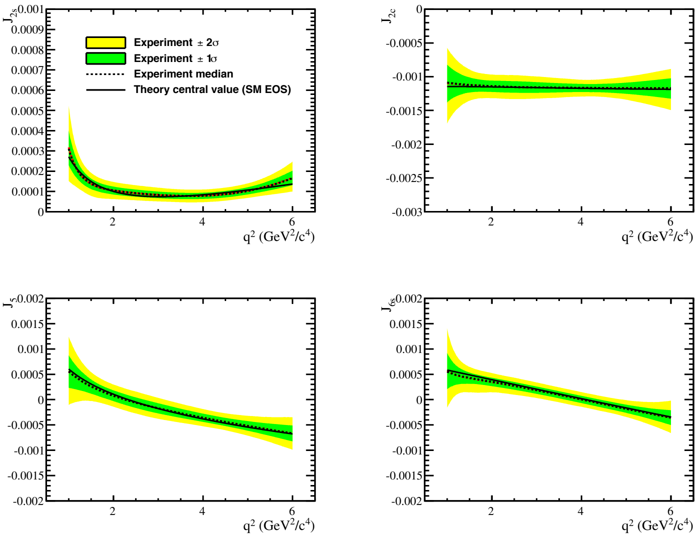
- Results from ensembles of toy-experiments
- Stable fit behaviour, well defined error matrix
- Discrete degeneracies of amplitudes in full angular fit due to form of angular coefficients
- Degeneracies not present in J_i's (bilinear combinations of amplitudes)





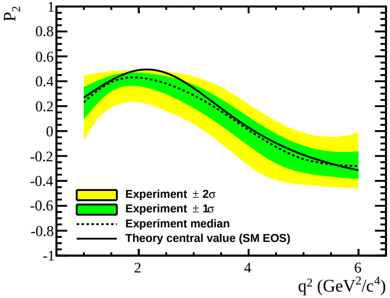
From amplitudes build the Js

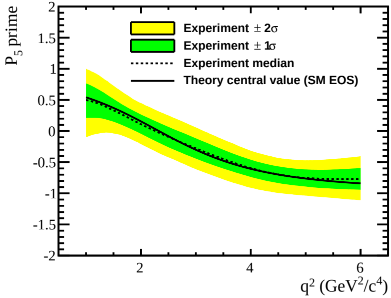
3 fb^{-1} toy data



From Js build observables

3 fb^{-1} toy data



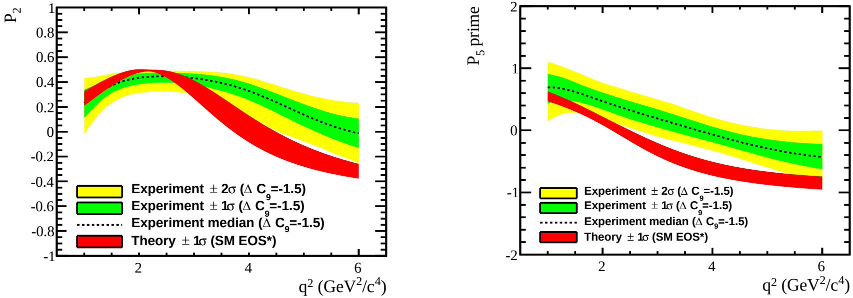


EOS [JHEP1007(2010)098] Bobeth, van Dyk et al.

- Can build any observable, e.g form factor independent P_2 and P_5' [JHEP 1301(2013)048] Descotes-Genon et al.
- Choice of Amplitude q^2 parametrisation faithfully reproduces q^2 dependence of observables

From Js build observables

3 fb^{-1} toy data



(*) EOS prediction with 15% uncertainty on sub-leading corrections [JHEP08(2012)030] Beaujean et al.

- Full q^2 shape information increases significance
- Combine observables to maximise sensitivity
  - p-value to SM can be calculated using various observable bases.
- Correlations need to be taken into account.

Other q^2 regions

- This study focused on $1 < q^2 < 6\,\mathrm{GeV}^2$ region
- A lot of interest for low recoil and $q^2 < 1\,\mathrm{GeV}$
- Separate treatment required. Ongoing effort in LHCb
  - Potential light and $c\bar{c}$ resonances
  - Different binning required, single bin preferable for theory predictions?
  - Could use resonances to extract additional information

Information transfer

- Accurate interpretation of measurements require correlations of observables, precise confidence intervals
- This information is available to experimentalists through the likelihood/dataset
- Ongoing discussion within LHCb how best to provide results to theory community
- Input required from interested theory groups
  - Is parametrisation of amplitudes satisfactory?
  - Are observables preferable
  - ...etc...
- Tools like `EOS` allow experimentalists to extract Wilson coefficients
  - Care needs to be taken on treatment of theory uncertainties
  - A general consensus on form of prior, size, correlations etc, would be useful

Conclusions

- Recent LHCb measurements have revealed interesting phenomena in the sector of electroweak penguin transitions
- A lot of work within LHCb to provide a comprehensive set of measurements in $B^0 \to K^{(*0)}\mu^+\mu^-$ and related decays
- Full angular analysis including a q^2 parametrisation seems possible with the current dataset $(1 < q^2 < 6\,\mathrm{GeV}^2)$
- A lot of work required. Ongoing effort on measurements in rest of q^2 region.
- Input from theory community is vital