Review of NNLO and subtraction

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Resummation and Parton Showers, IPPP

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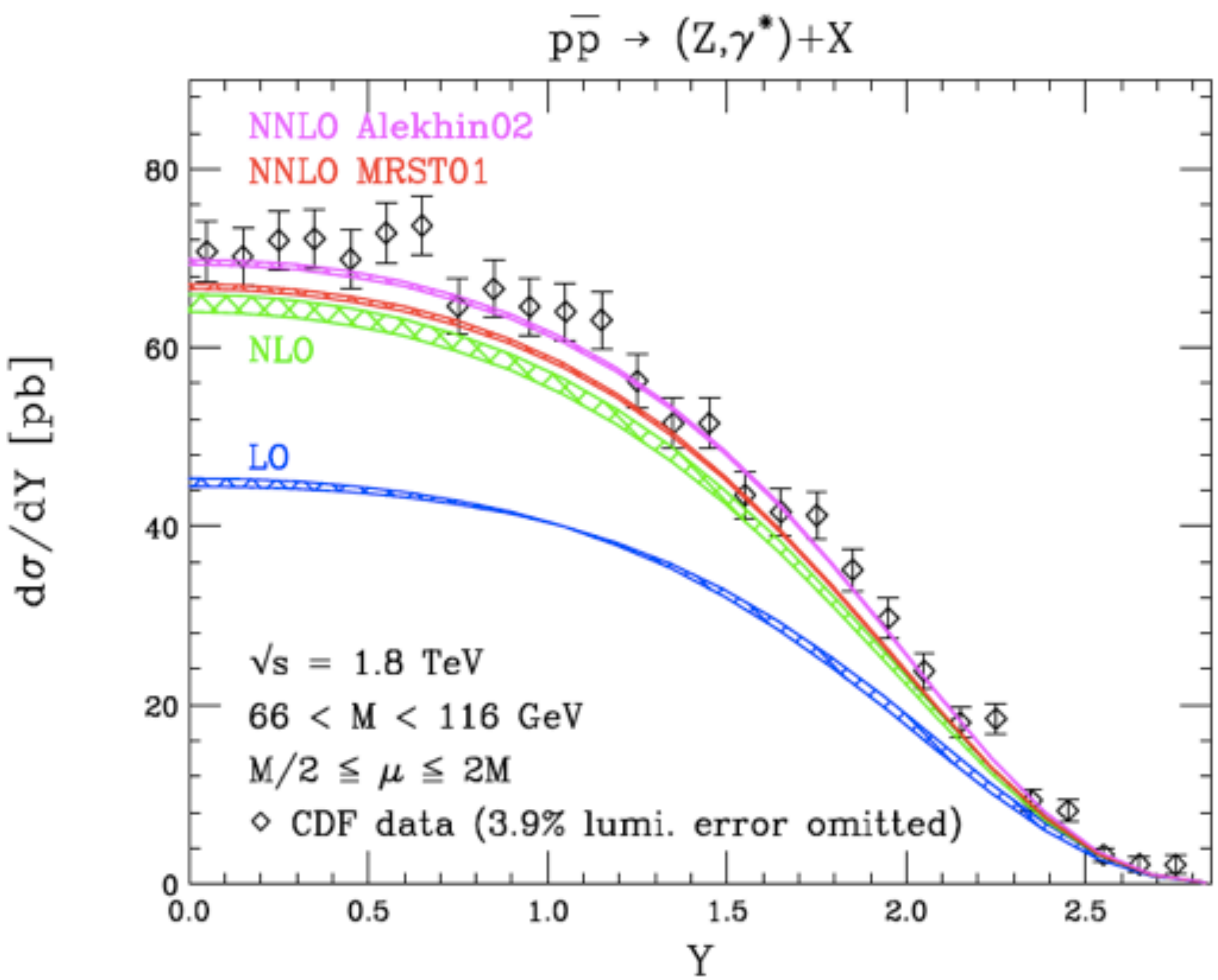
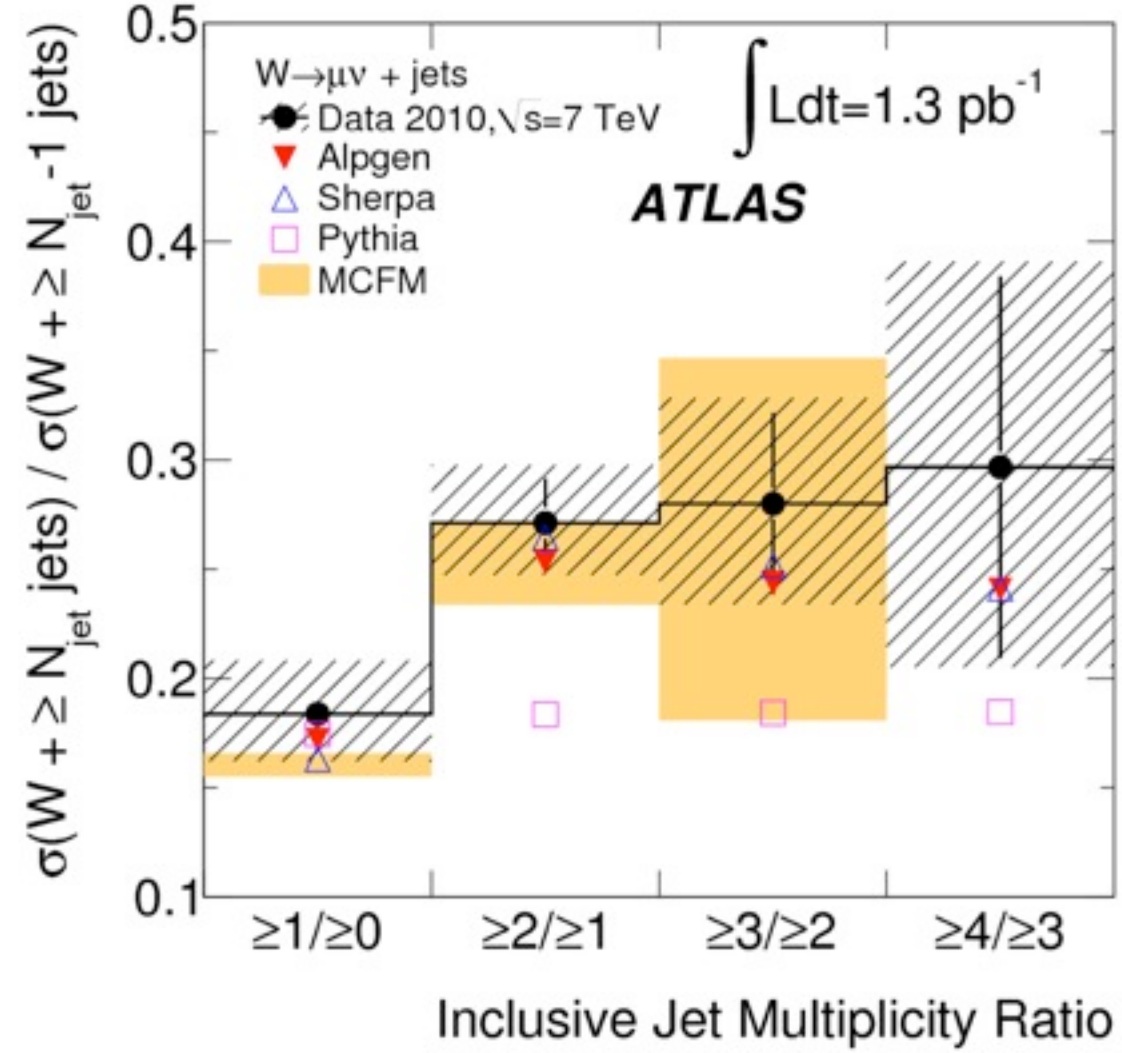
Outline

- Will attempt to motivate the necessity of NNLO in the presence of advanced FO+PS and resummation tools
- The major bottleneck is (was!) the construction of a subtraction scheme for double-real radiation at NNLO. I'll explain the generic issues that made this an unsolved problem for many years.
- I will attempt to show the details of the \`sector-improved' subtraction approach, which has been successfully applied to two non-trivial 2→2 calculations at NNLO. I will mention some differences between this and the \`antennae subtraction' scheme, also successfully used for 2→2 at NNLO.
- I'll use both Z→ee in QED and H+jet in QCD to illustrate the techniques.
- At the end I'll try to motivate some discussion on NNLO+PS

The need for higher-order QCD



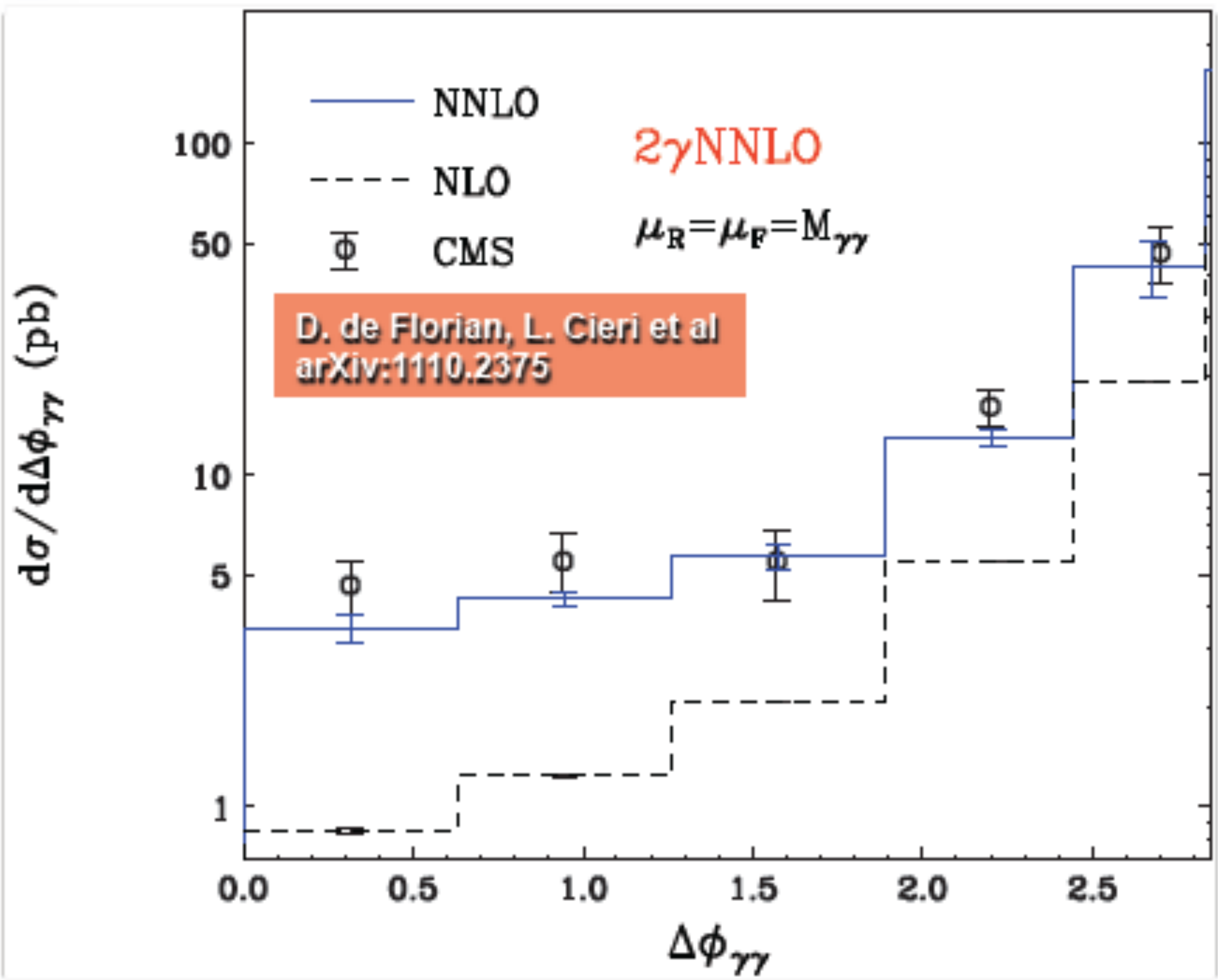
Anastasiou, Dixon, Melnikov, FP 2004



- The need to go beyond leading order QCD, or the parton-shower approximation, to understand hadron-collider data is by now unquestioned.
- NLO and matched parton-shower+NLO now standard tools used.

LHC examples of NLO versus data

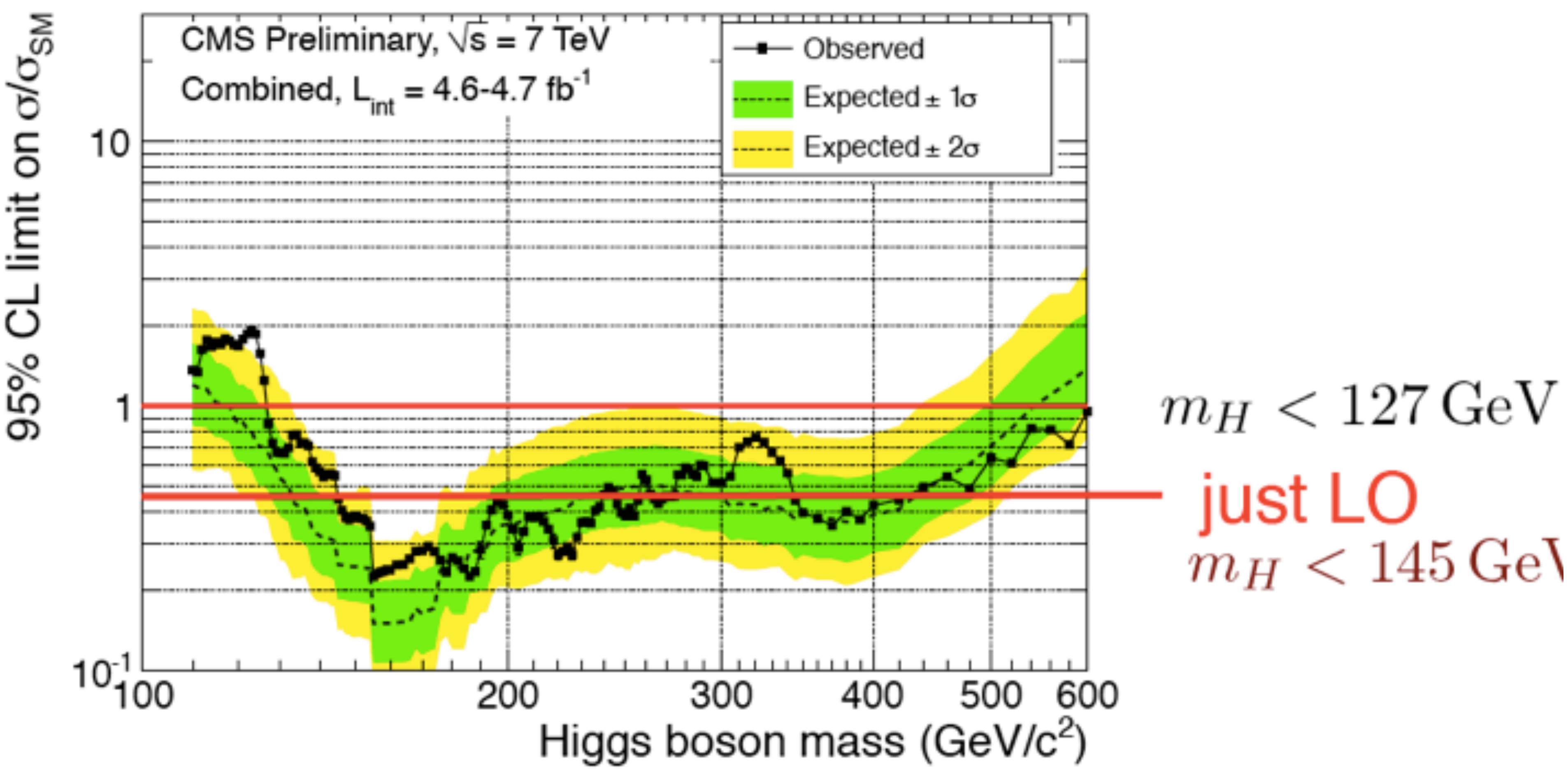
- Sometimes even NLO is not enough... now there's data to illustrate the point

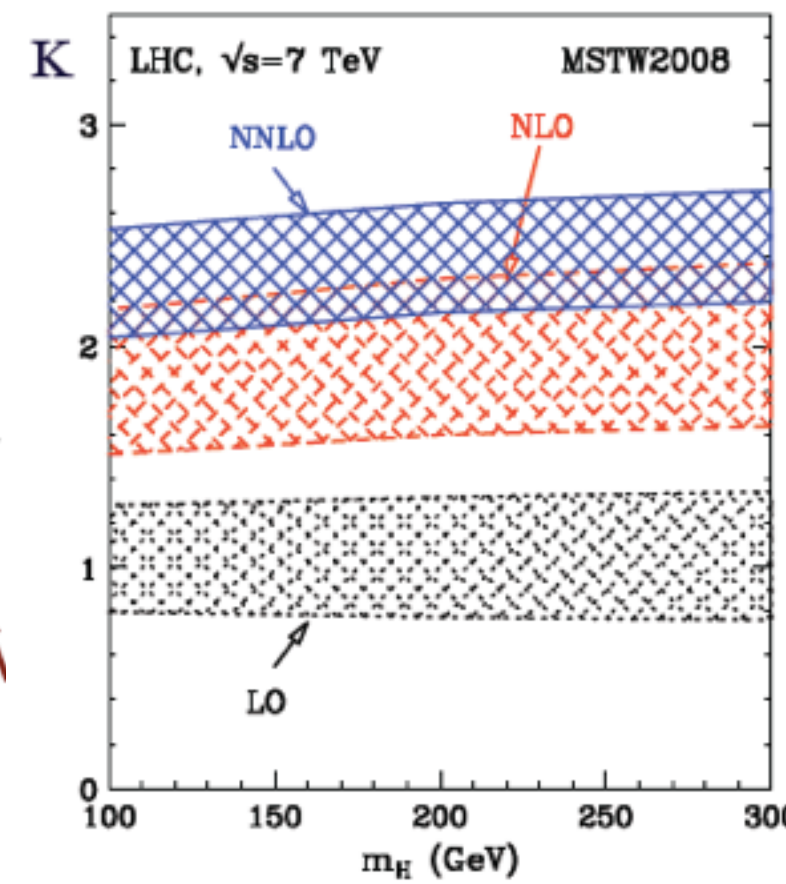


- At LO, opening angle in the transverse plane is π
- Distribution begins only at NLO
- NLO→NNLO shift large for two reasons: large first correction to the large qg channel which first opens at NLO, and new gg channel

The Higgs in gluon-fusion

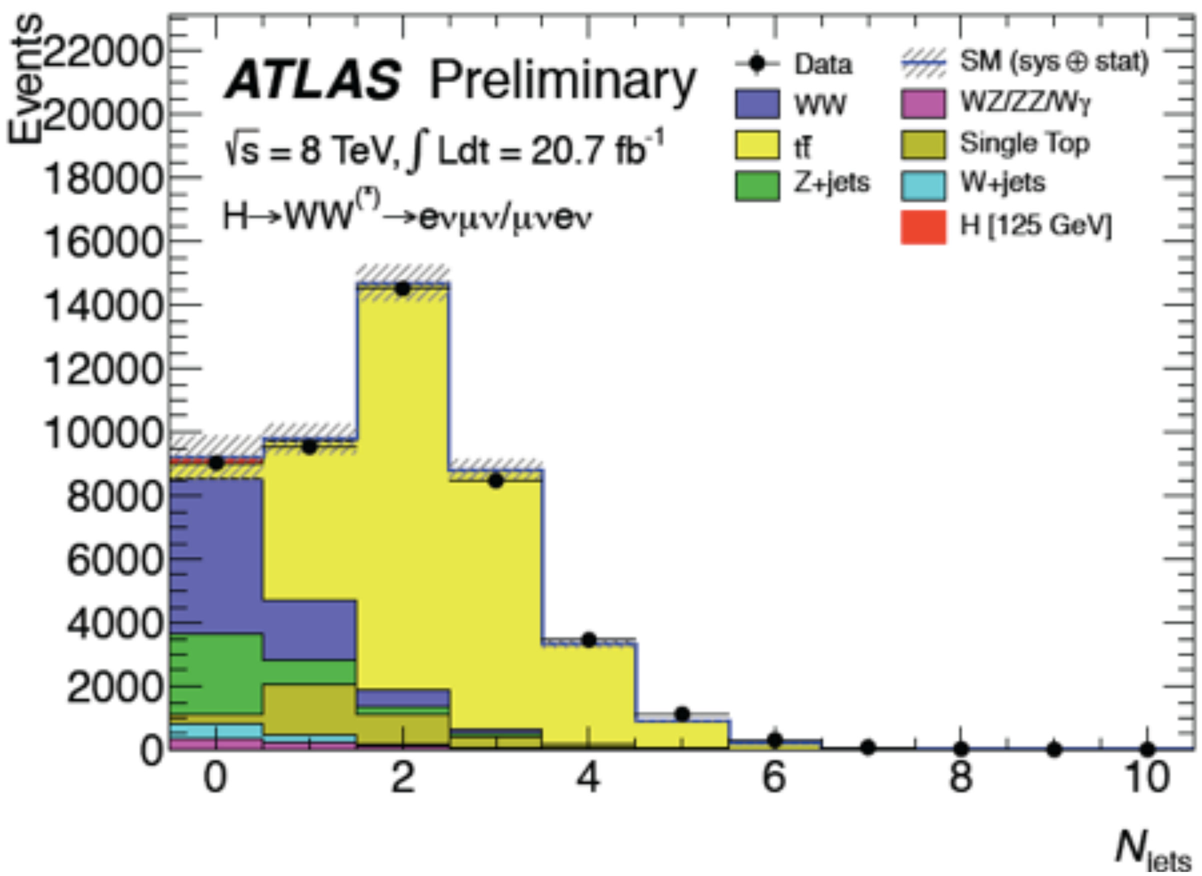
- Can't rely upon LO or even NLO for Higgs production in gluon-fusion





from de Florian, Higgs Magnificent Mile 2012

Jet vetoes and the Higgs



- Theory errors worsen when the requisite division into exclusive jet bins is performed
- 25-30 GeV jet cut used; restriction of radiation leads to large logs

Source	Signal processes (%) $N_{\text{jet}}=0$	Signal $N_{\text{jet}}=1$	Signal $N_{\text{jet}}\geq 2$	Background processes (%) $N_{\text{jet}}=0$	Background $N_{\text{jet}}=1$	Background $N_{\text{jet}}\geq 2$
Theoretical uncertainties						
QCD scale for ggF signal for $N_{\text{jet}}\geq 0$	13	-	-	-	-	-
QCD scale for ggF signal for $N_{\text{jet}}\geq 1$	10	27	-	-	-	-
QCD scale for ggF signal for $N_{\text{jet}}\geq 2$	-	15	4	-	-	-
QCD scale for ggF signal for $N_{\text{jet}}\geq 3$	-	-	4	-	-	-
Parton shower and UE model (signal only)	3	10	5	-	-	-
PDF model	8	7	3	1	1	1
$H\to WW$ branching ratio	4	4	4	-	-	-
QCD scale (acceptance)	4	4	3	-	-	-
WW normalisation	-	-	-	1	2	4
Experimental uncertainties						
Jet energy scale and resolution	5	2	6	2	3	7
b-tagging efficiency	-	-	-	-	7	2
f_{recoil} efficiency	1	1	-	4	2	-

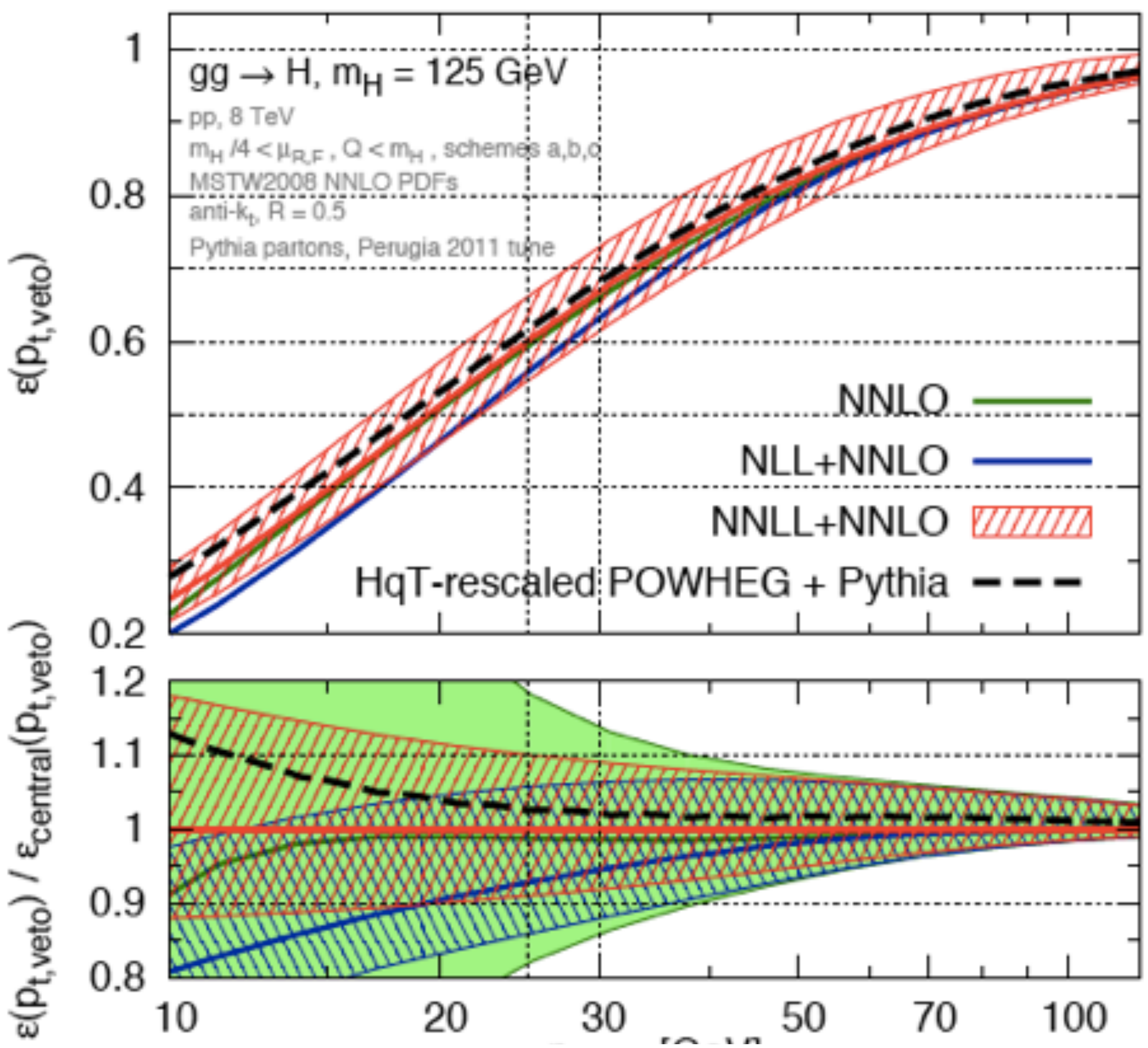
ATLAS

- Theory (NNLO for 0 jets, NLO for 1 jet) becoming a limiting systematic in the 0-jet and 1-jet bins

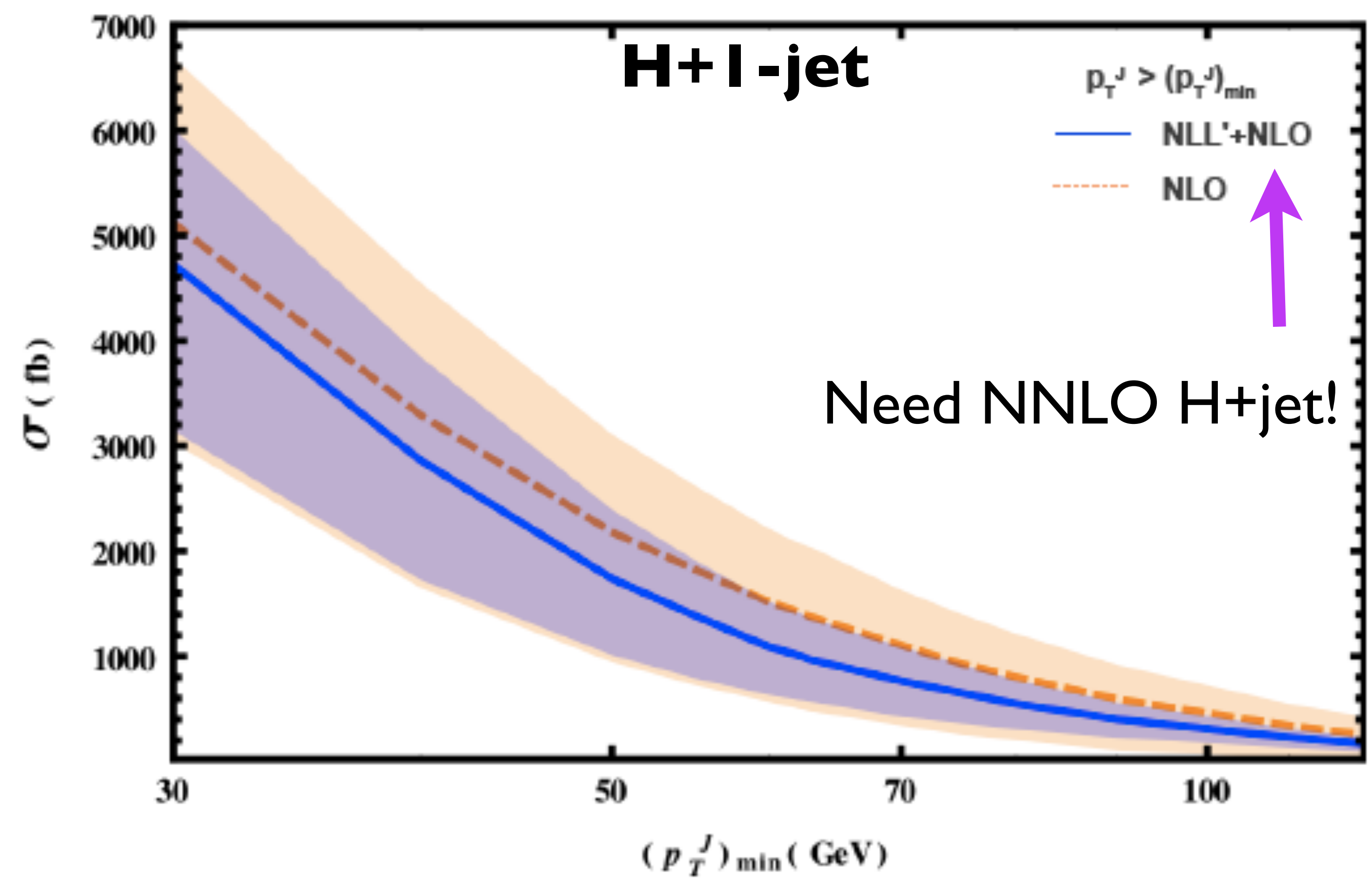
Jet vetoes and the Higgs

- Although resummation can help tame these large logs, need further fixed-order progress to improve the resummation, both to obtain the required anomalous dimensions and for the matching... relevant kinematics is in the transition region between resummation and fixed order

Banfi, Monni, Salam, Zanderighi 2012



X. Liu, FP 2013

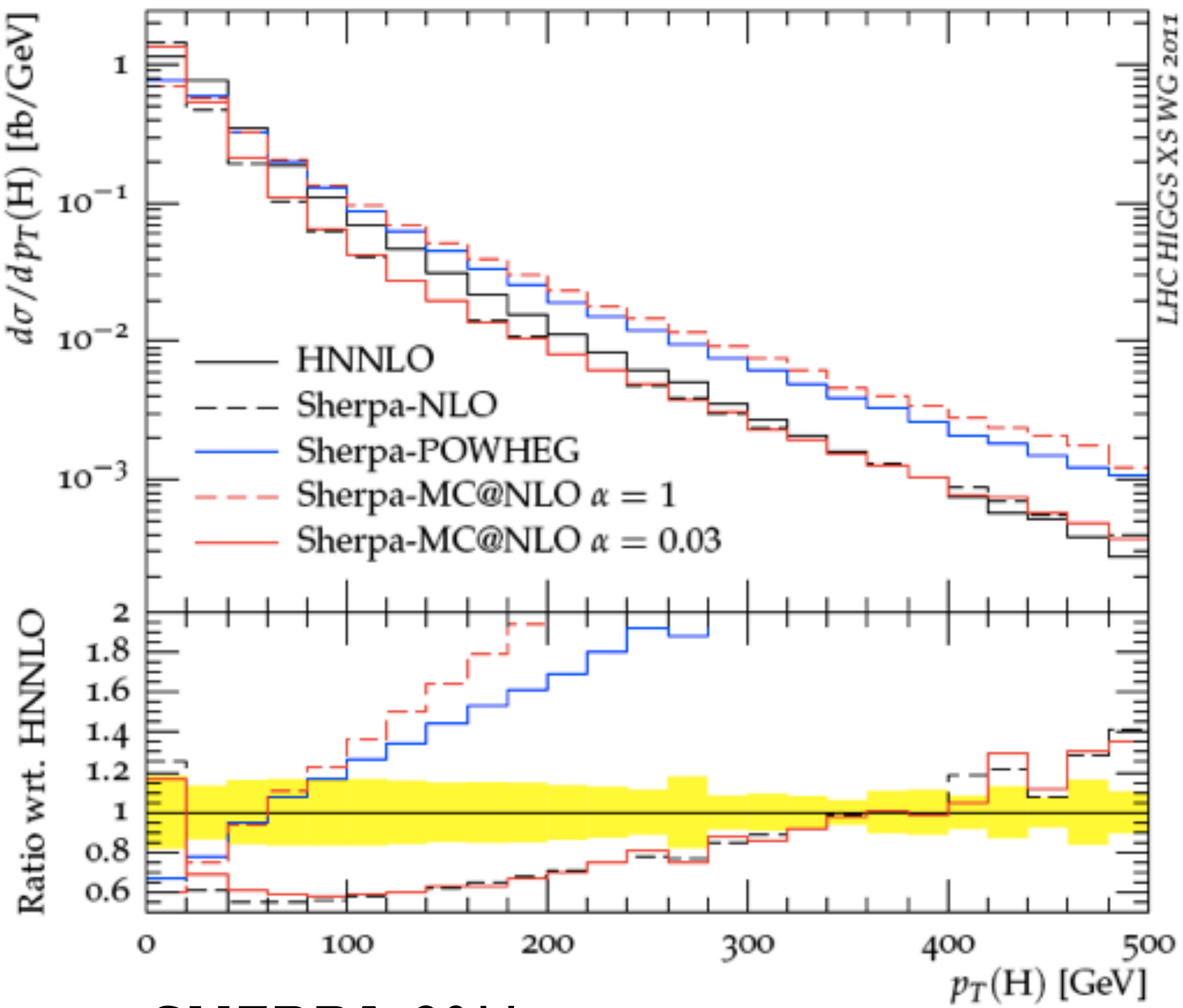


$$\epsilon(p_{t,\text{veto}}) = 1 - \frac{\sigma^{\text{NLO}}_{\text{1-jet}}(p_{t,\text{veto}})}{\sigma_0 + \sigma_1}$$

Need NNLO H+jet!

NNLO and NLO+parton showers

- NLO+parton-shower tools are indispensable, but can have very large uncertainties for exactly the interesting variables

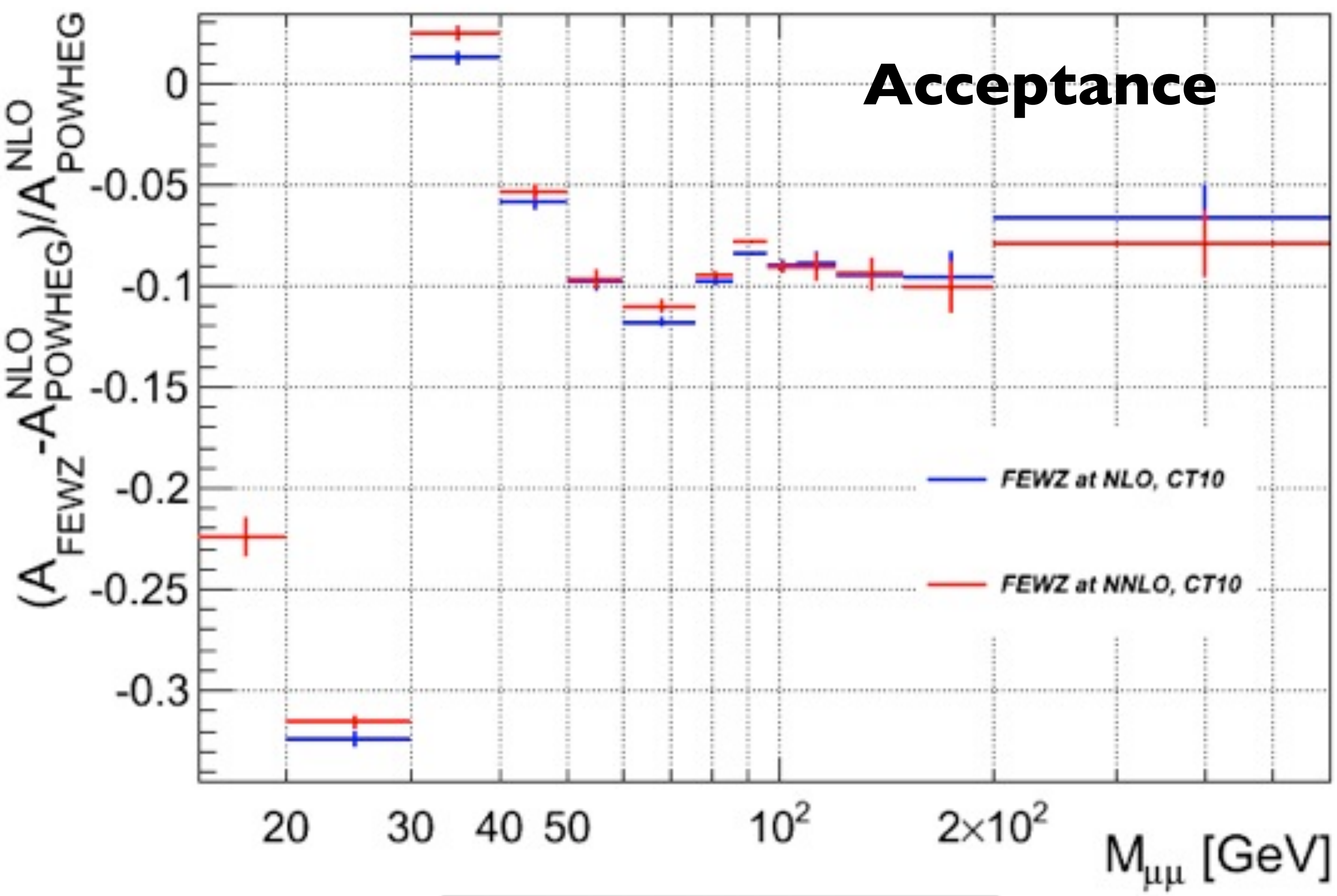


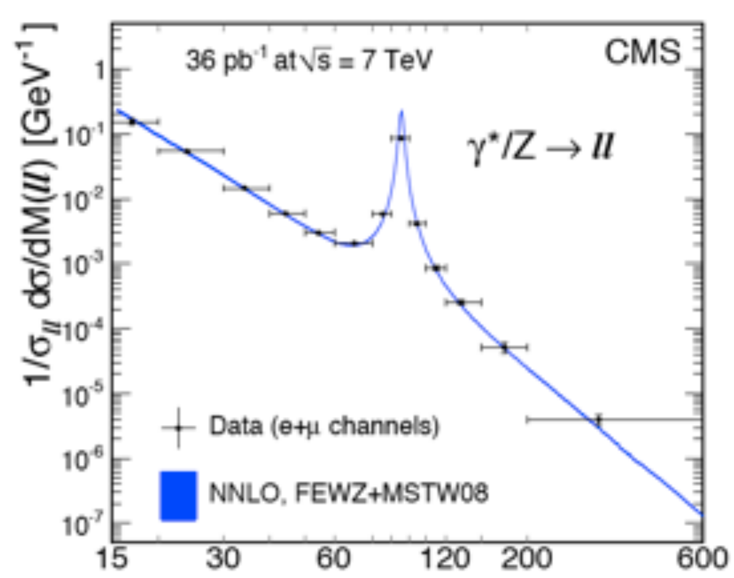
SHERPA, 2011

- What exactly is used in the exponent in the various curves modifies the pT spectrum
- Gives an indication of NNLO corrections to Higgs+jet

Low-mass Drell-Yan and NLO+PS

- An interesting example of NLO+PS versus data from $pp \to \mu^+\mu^-$





- Double muon trigger: p_{T1}>16 GeV, p_{T2}>7 GeV
- For M=[15,20], [20,30] GeV: NLO→LO, NNLO→NLO, need a hard jet to generate this configuration
- α_S(15 GeV)≈0.17, K-factor≈1.9 when going from 'N'LO→'N'NLO
- Corrections to POWHEG acceptance of ≈1.5-2
- Would a consistent combination of NLO+PS for DY+0 jets and DY+1 jets correctly describe this data?

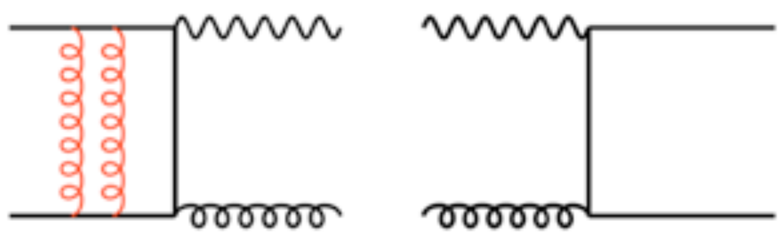
Recap

- Many other examples to give (ttbar, dijet cross sections for gluon PDF, $e^+e^- \to 3$ jets for α_s extraction)
- Moral: Need NNLO for most interesting processes at the LHC, too much potential interplay between QCD and analysis cuts for LO/NLO. NLO+PS is not always sufficient.
- Until very recently, only a special class of observables currently computed: at NNLO colorless final state (W, Z, Higgs, WH, $\gamma\gamma$) or initial state ($e^+e^- \to 3$ jets)
- Need at least the capability for $2 \to 2$ with colored final states; would like a method in principle extendable to higher multiplicities

Structure of NNLO cross section

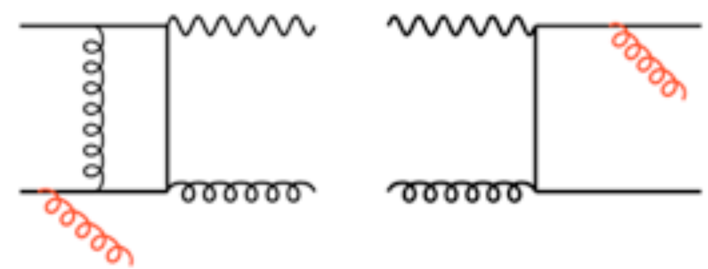
- Need the following ingredients for a NNLO cross section

2-loop matrix elements, m partons



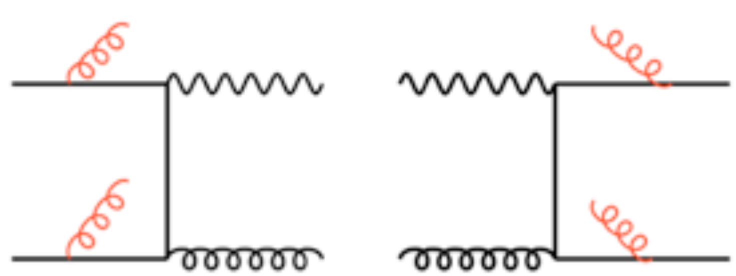
- Explicit IR poles from loop integrals

1-loop matrix elements, m+1 partons



- Explicit IR poles from loops
- Implicit IR poles from single unresolved radiation

Tree level matrix elements, m+2 partons



Implicit IR poles from double unresolved radiation

- IR singularities cancel in the sum of real and virtual corrections and mass factorization counterterms but only after phase space integration for real radiations
- Need a procedure to extract poles before phase-space integration to allow for differential observables

How to calculate at NLO

- Well-honed techniques for calculating and combining real+virtual at NLO
- Virtual corrections with Feynman diagrams or new unitarity techniques (Blackhat, Rocket, CutTools, GoSam, Openloops,...)
- To deal with IR singularities of real emission, have dipole subtraction (Catani, Seymour 1996), FKS subtraction (Frixione, Kunszt, Signer 1996)

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^S_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^V_{NLO}\right]$$

Approximates real-emission matrix elements in all singular limits so this difference is numerically integrable

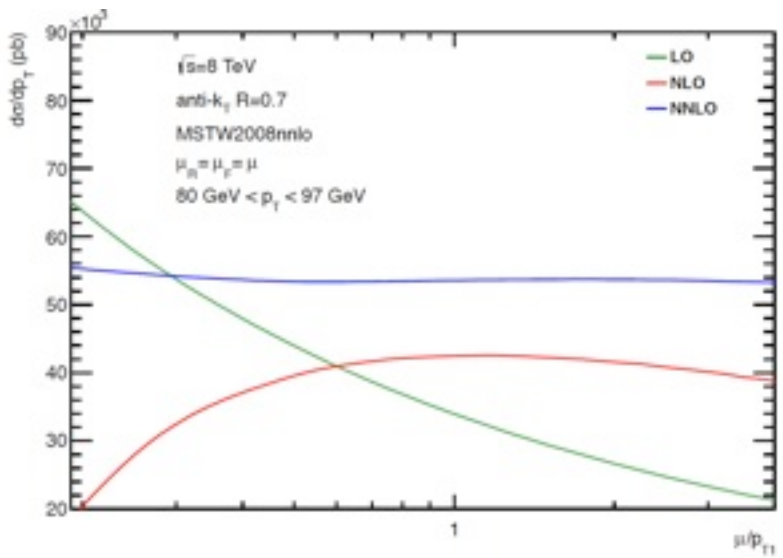
Simple enough to integrate analytically so that 1/ε poles can be cancelled against virtual corrections

What's known at NNLO

- Two-loop amplitudes for dijet, γ+jet, H+jet, V+jet, known, some for over 10 years (Anastasiou, Glover, Oleari, Tejeda-Yeomans 2000-2002; Gehrmann et al. 2010-2013)
- One-loop corrections to real emission (real-virtual) known
- Singular limits of double-real emission, real-virtual, known for over 10 years (Campbell, Glover 1997; Catani, Grazzini 1999; Kosower, Uwer 1999)
- The problem is how to use the singular limits of the double-real emission
- Until recently, only special processes with colorless initial states or colorless final states were known at the differential level to NNLO
    - $pp \to H$: Anastasiou, Melnikov, FP 2005; Catani, Grazzini 2007
    - $pp \to V$: Melnikov, FP 2006; Catani, Cieri, Ferrera, de Florian, Grazzini 2009
    - $e^+e^- \to 3$ jets: Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 2007; Weinzierl 2008
    - $pp \to \gamma\gamma$, VH: Catani et al. 2011; Ferrera, Grazzini, Tramontano 2011

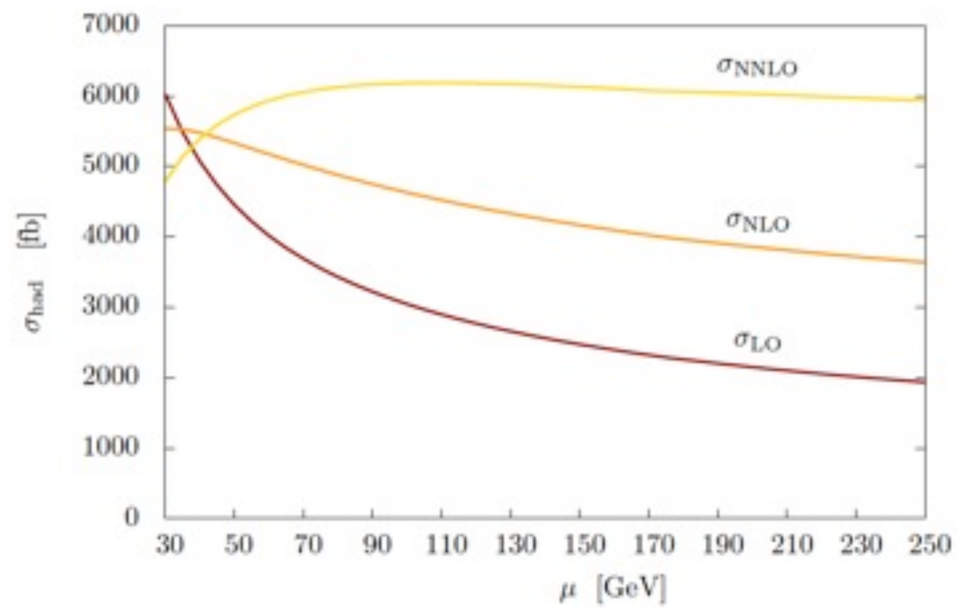
2013: the year of NNLO

- After more than a decade of research we finally know how to generically handle NNLO QCD corrections to processes with both colored initial and final states



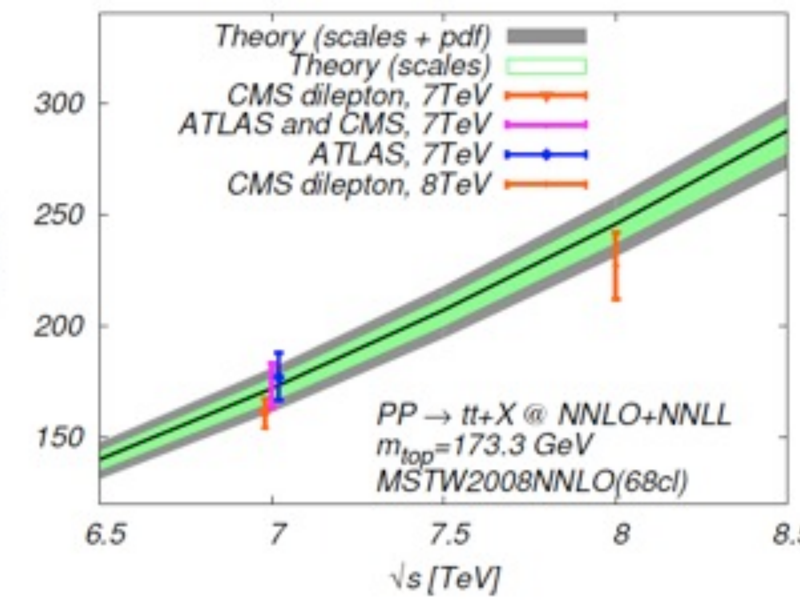
Gehrmann-de Ridder, Gehrmann, Glover, Pires (2013)

dijet: gg-channel



Boughezal, Caola, Melnikov, FP, Schulze (2013)

H+1j:gg-channel



Czakon, Fiedler, Mitov (2013)

ttbar: all-channels

Based on Antenna subtraction scheme

Based on sector-improved subtraction scheme

2013: the year of NNLO

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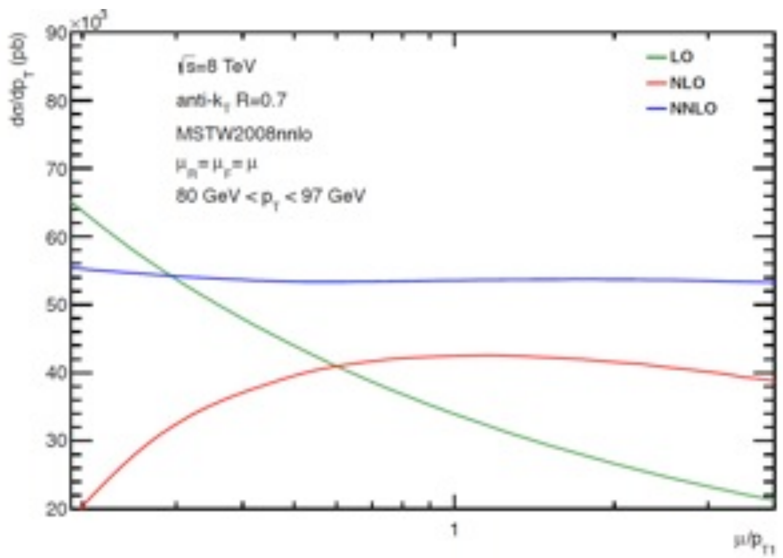

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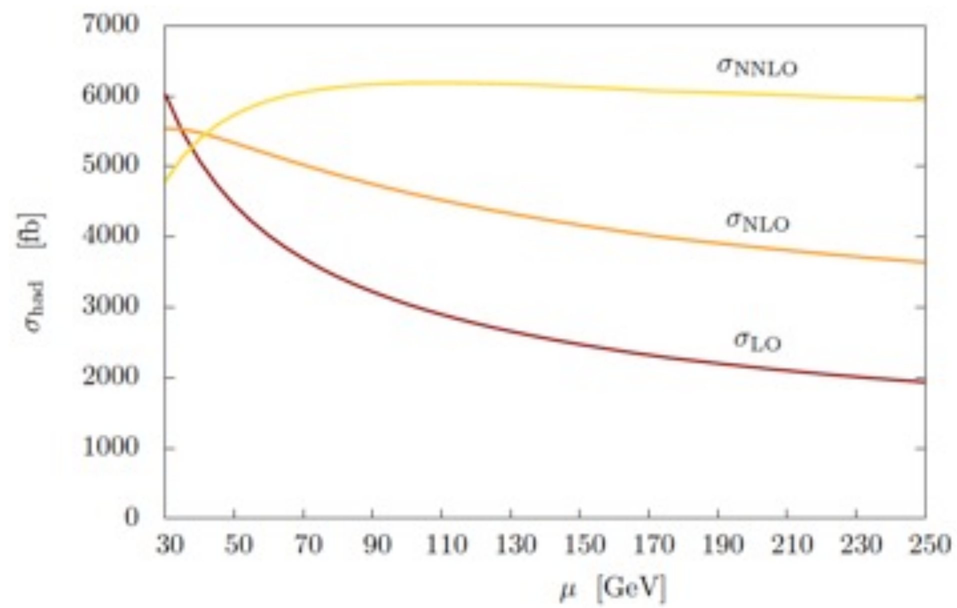

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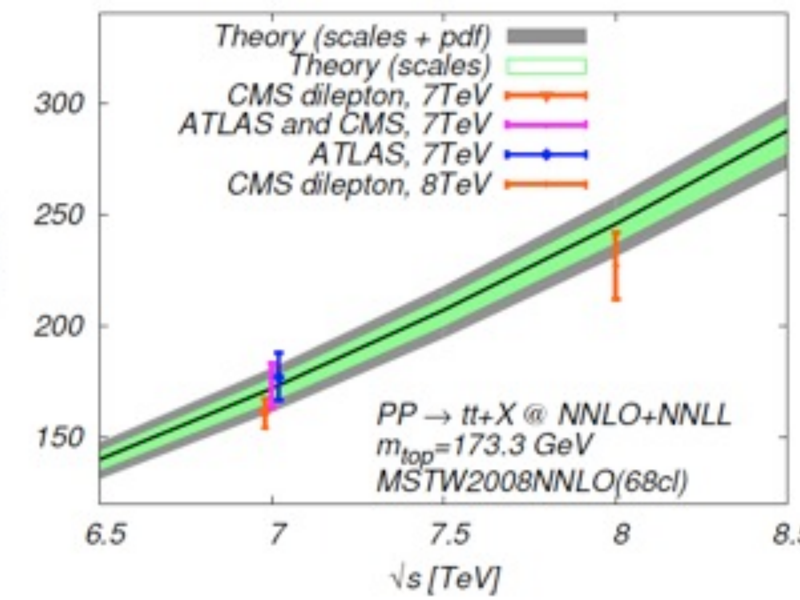


Czakon, Fiedler, Mitov (2013)

ttbar: all-channels

I will focus on describing this technique here

Based on sector-improved subtraction scheme

Subtraction at NNLO

- The generic form of an NNLO subtraction scheme is the following:

$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} = & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^R_{NNLO} - \mathrm{d}\sigma^S_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^S_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} \\
& + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^{V,2}_{NNLO} ,
\end{aligned}
$$

from T. Gehrmann

- Maximally singular configurations at NNLO can have two collinear, two soft singularities
- Subtraction terms must account for all of the many possible singular configurations: triple-collinear ($p_1||p_2||p_3$), double-collinear ($p_1||p_2, p_3||p_4$), double-soft, single-soft, soft +collinear, etc.

- The factorization of the matrix elements in all singular configurations is known in the literature

The triple-collinear example

- To illustrate the problems that occur when trying to use these formulae, consider the triple-gluon collinear limit. The factorization of the matrix element squared in this limit is the following.

$$|\mathcal{M}(\ldots,p_1,p_1,p_3)|^2 \approx \frac{4g_s^4}{s_{123}^2}\mathcal{M}^\mu(\ldots,p_1+p_2+p_3)\mathcal{M}^{\nu*}(\ldots,p_1+p_2+p_3)\,P^{\mu\nu}_{g_1g_2g_3}$$

$z_i = E_i/(\sum E_j)$

$$\begin{aligned}
\hat{P}^{\mu\nu}_{g_1g_2g_3} &= C_A^2\left\{\frac{(1-\epsilon)}{4s_{12}^2}\left[-g^{\mu\nu}t_{12,3}^2+16s_{123}\frac{z_1^2z_2^2}{z_3(1-z_3)}\left(\frac{\widetilde{k}_2}{z_2}-\frac{\widetilde{k}_1}{z_1}\right)^\mu\left(\frac{\widetilde{k}_2}{z_2}-\frac{\widetilde{k}_1}{z_1}\right)^\nu\right]\right.\\
&- \frac{3}{4}(1-\epsilon)g^{\mu\nu}+\frac{s_{123}}{s_{12}}g^{\mu\nu}\frac{1}{z_3}\left[\frac{2(1-z_3)+4z_3^2}{1-z_3}-\frac{1-2z_3(1-z_3)}{z_1(1-z_1)}\right]\\
&+ \frac{s_{123}(1-\epsilon)}{s_{12}s_{13}}\left[2z_1\left(\widetilde{k}_2^\mu\widetilde{k}_2^\nu\frac{1-2z_3}{z_3(1-z_3)}+\widetilde{k}_3^\mu\widetilde{k}_3^\nu\frac{1-2z_2}{z_2(1-z_2)}\right)\right.\\
&+ \frac{s_{123}}{2(1-\epsilon)}g^{\mu\nu}\left(\frac{4z_2z_3+2z_1(1-z_1)-1}{(1-z_2)(1-z_3)}-\frac{1-2z_1(1-z_1)}{z_2z_3}\right)\\
&+ \left.\left.\left(\widetilde{k}_2^\mu\widetilde{k}_3^\nu+\widetilde{k}_3^\mu\widetilde{k}_2^\nu\right)\left(\frac{2z_2(1-z_2)}{z_3(1-z_3)}-3\right)\right]\right\}+(5\text{ permutations})\ .
\end{aligned}$$

Catani, Grazzini 1999

Entangled singularities

•To illustrate the problems that occur when trying to use these formulae, consider the triple-gluon collinear limit. The factorization of the matrix element squared in this limit is the following.

$$|\mathcal{M}(\ldots,p_1,p_1,p_3)|^2 \approx \frac{4g_s^4}{s_{123}^2}\mathcal{M}^{\mu}(\ldots,p_1+p_2+p_3)\mathcal{M}^{\nu *}(\ldots,p_1+p_2+p_3)\,P^{\mu\nu}_{g_1 g_2 g_3}$$

•When one introduces an explicit parameterization:
$s_{123} \sim E_1E_2(1-c_{12})+E_1E_3(1-c_{13})+E_2E_3(1-c_{23})$

•What goes to zero quicker? E_1,E_2,E_3,$(1-c_{12})$,$(1-c_{13})$, or $(1-c_{23})$?

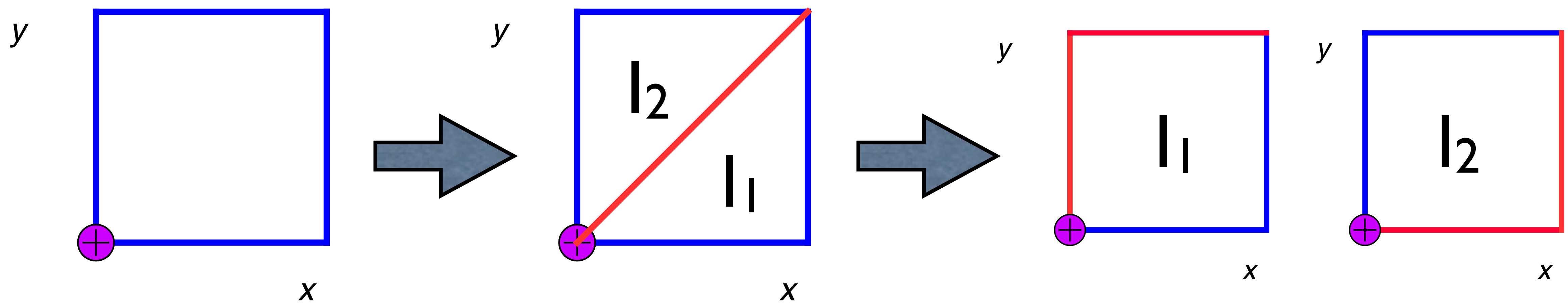
•Need to order the limits, since singularities must be extracted from integrals of the schematic form:

$$\int_0^1 dx dy \frac{x^{\epsilon}y^{\epsilon}}{(x+y)^2}F_J(x,y)$$

•Need a systematic technique for ordering limits, too many of such issues appear

Sector decomposition

- Can define a systematic procedure to order limits



$$I = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^\epsilon y^\epsilon}{(x+y)^2}$$

$$I_1 = \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \frac{x^\epsilon y^\epsilon}{(x+y)^2} \qquad I_1 = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^{-1+2\epsilon} y^\epsilon}{(1+y)^2}$$

$$I_2 = \int_0^1 \mathrm{d}y \int_0^y \mathrm{d}x \frac{x^\epsilon y^\epsilon}{(x+y)^2} \qquad I_2 = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{y^{-1+2\epsilon} x^\epsilon}{(1+x)^2}$$

$$y^{-1-\epsilon} = -\frac{\delta(y)}{\epsilon} + \left[\frac{1}{y}\right]_+ - \epsilon\left[\frac{\ln y}{y}\right]_+ + \mathcal{O}(\epsilon^2)$$

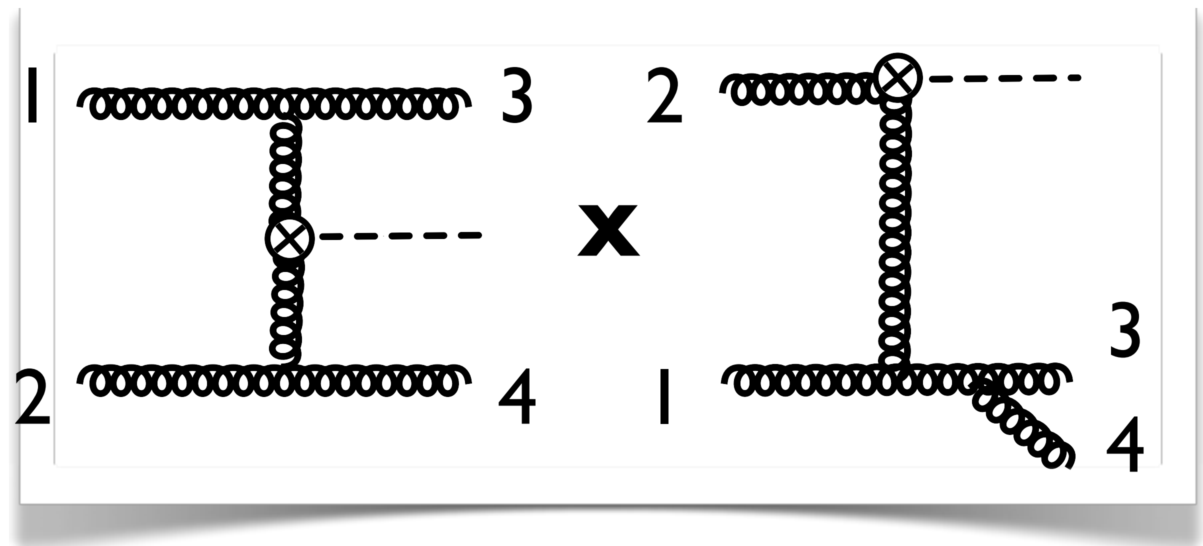
Binoth, Heinrich; Anastasiou, Melnikov, FP 2003-2005

Sector decomposition

- Give up on the idea of analytic cancellation of poles; calculate the coefficients of $1/\varepsilon^n$ Laurent expansion numerically
- In its original incarnation, was applied directly to each interference of diagrams which appears.
- Used for the first differential NNLO calculations at hadron colliders: Higgs, W/Z Anastasiou, Melnikov, FP; Melnikov, FP 2005-2006
- The one-loop single-emission corrections (the real-virtual contribution) was simple enough for these processes to calculate completely analytically
- The (major) drawback: originally used a *global* phase-space parameterization for a given interference

Higgs production

- To illustrate the drawbacks, use Higgs production as an example. Consider one of the diagrammatic contributions to the double-real radiation correction.



- Invariants that occur in this topology : s_{13}, s_{24}, s_{134}, s_{34}. These contain collinear singularities $p_1||p_3$, $p_2||p_4$, $p_3||p_4$, $p_1||p_3||p_4$
- The structure of these singularities makes it difficult to find a suitable global parameterization amenable to sector decomposition.
- Would need to start over with entirely new parameterization for Higgs+jet
- However, can only have $p_1||p_3$ & $p_2||p_4$ or $p_1||p_3||p_4$ in a given phase space region. Not all invariants above can have collinear singularities simultaneously.

FKS@NNLO

- A suggestion recently that removes drawback of previous slide: pre-partitioning of the phase space leads to a phase-space parameterization applicable to NNLO real-radiation corrections for any process, regardless of multiplicity (Czakon, 2010).
- Partition the phase space such that in each partition only a subset of particles leads to singularities, and only one triple collinear or one double collinear singularity can occur. This is effectively an extension of the FKS subtraction technique to NNLO.
- Allows use of known soft/collinear limits, and is extendable to higher multiplicity. Let's see these points explicitly in a simple test case.

Z decay at NNLO in QED

- We will illustrate the details with $Z \to e^+e^-$ to NNLO in QED (Boughezal, Melnikov, FP 2011). Retains the features of the QCD computation, but makes the formulae a bit simpler to show.
- Study the double-real radiation correction: $Z \to e^+(p_+)e(p_-)\gamma(p_1)\gamma(p_2)$
- The starting point is the partitioning of phase space:

$$1 = \delta_{12}^{--} + \delta_{12}^{++} + \delta_{12}^{-+} + \delta_{12}^{+-}$$

$$\delta_{12}^{--} = \frac{1-\hat{n}_1\cdot\hat{n}_+}{2-\hat{n}_1\cdot\hat{n}_+-\hat{n}_1\cdot\hat{n}_-}\,\frac{1-\hat{n}_2\cdot\hat{n}_+}{2-\hat{n}_2\cdot\hat{n}_+-\hat{n}_2\cdot\hat{n}_-}$$

- Focus on this triple-collinear partition as an example. Has only p_1,p_2 soft and $p_1||p_2||p_-$. We don't care how ugly the invariants s_{1+},s_{2+} are. They contain no collinear singularities, only (simple) energy singularities.

The triple-collinear decomposition

- The most complicated invariant appearing in this partition is s_{-12}

$$s_{-12} \sim A\xi_1\eta_1 + B\xi_2\eta_2 + C\xi_1\xi_2(\eta_1-\eta_2)^2$$

$$\cos\theta_i = 1-2\eta_i$$

$$E_i = \xi_i M_Z/2$$

- Perform the following sector decompositions to disentangle singularities

  - Order energies, focus on $\xi_1 > \xi_2$, $\xi_2 \to \xi_1\xi_2$

    $$s_{-12} \sim \xi_1(A\eta_1 + B\xi_2\eta_2 + C\xi_1\xi_2(\eta_1-\eta_2)^2)$$

  - Order angles, focus on $\eta_2 > \eta_1$, $\eta_1 \to \eta_1\eta_2$

    $$s_{-12} \sim \xi_1\eta_2(A\eta_1 + B\xi_2 + C\xi_1\xi_2\eta_2(1-\eta_1)^2)$$

  - Order η_1, ξ_2, focus on $\eta_1 > \xi_2$, $\xi_2 \to \xi_2\eta_1$

    $$s_{-12} \sim \xi_1\eta_2\eta_1(A + B\xi_2 + C\xi_1\xi_2\eta_2(1-\eta_1)^2)$$

All singularities extracted as overall multiplicative factors

Bracket is finite in all limits

The triple-collinear decomposition

- We're left with the following variable changes to factorize singularities

1. S_1^{--}, where $\xi_1 = x_1$, $\xi_2 = x_{\max} x_2 x_1$, $\eta_1 = x_3$, $\eta_2 = x_4 x_3$, $\kappa = x_5$;
2. S_2^{--}, where $\xi_1 = x_1$, $\xi_2 = x_{\max} x_2 x_4 x_1$, $\eta_1 = x_3 x_4$, $\eta_2 = x_3$, $\kappa = x_5$;
3. S_3^{--}, where $\xi_1 = x_1$, $\xi_2 = x_{\max} x_2 x_1$, $\eta_1 = x_2 x_3 x_4$, $\eta_2 = x_3$, $\kappa = x_5$.

Crucial point: sectors are identical for *any* NNLO QED correction. Just as we didn't care about the form of s_{1+}, s_{2+}, we don't care about s_{1j}, s_{2j} in this partition, where j indicates any other particle we add to the process. We are working with a *local* parameterization suitable for any triple-collinear partition.

For sector S_1^{--}:

$$\boxed{E_1 = \frac{m_Z}{2} x_1, \quad E_2 = \frac{m_Z}{2} x_1 x_2 x_{\max},}$$

$$\boxed{\cos\theta_1 = 1 - 2x_3, \quad \cos\theta_2 = 1 - 2x_3 x_4.}$$

The triple-collinear decomposition

- We have reduced our calculation to the following objects:

$$\int \underline{\mathrm{dLips}}^{--}_{S_1} F_1(x_1, x_2, x_3, x_4, x_5)$$

with

regular functions of x_i

$$\underline{\mathrm{dLips}}^{--}_{S_1} = \mathrm{dNorm}\ \mathrm{PS}_w\ (\mathrm{PS})^{-\epsilon}$$
$$\times \frac{\mathrm{d}x_1}{x_1^{1+4\epsilon}} \frac{\mathrm{d}x_2}{x_2^{1+2\epsilon}} \frac{\mathrm{d}x_3}{x_3^{1+2\epsilon}} \frac{\mathrm{d}x_4}{x_4^{1+\epsilon}} \frac{\mathrm{d}\kappa}{\pi(\kappa(1-\kappa))^{1/2}}$$

expandable in plus distributions

and

$$F_1(\{x_{i=1..5}\}) = \left[x_1^4 x_2^2 x_3^2 x_4 m_Z^2 \delta_{12}^{--}\right] |\mathcal{M}_{Z\to e^+e^-\gamma\gamma}|^2$$

Let's look at some of the singularities that can occur

The double-soft limit

$$E_1 = \frac{m_Z}{2} x_1, \quad E_2 = \frac{m_Z}{2} x_1 x_2 x_{\max},$$

$$\cos\theta_1 = 1 - 2x_3, \quad \cos\theta_2 = 1 - 2x_3 x_4.$$

- What happens if $x_1 = 0$? → $E_1 = E_2 = 0$ → double soft limit

the QED matrix element factorizes completely, use known singular limits

$$|\mathcal{M}_{Z\to e^+e^-\gamma\gamma}|^2 \to e^4 J_1 J_2 |\mathcal{M}_{Z\to e^-e^+}|^2$$

with
$$J_i = \frac{2p_- \cdot p_+}{(p_- \cdot p_i)(p_+ \cdot p_i)}$$

derive the following formula

$$F_1|_{x_1=0} = \frac{16e^4}{m_Z^2} |\mathcal{M}_{Z\to e^-e^+}|^2$$

easy to calculate numerically

The soft+collinear limit

$$\boxed{E_1 = \frac{m_Z}{2}x_1, \quad E_2 = \frac{m_Z}{2}x_1 x_2 x_{\max},} \qquad \boxed{\cos\theta_1 = 1 - 2x_3, \quad \cos\theta_2 = 1 - 2x_3 x_4.}$$

- What happens if $x_2 = 0$ & $x_3 = 0$? $\Rightarrow$ $E_2 = 0$ & $p_1 \parallel p_-$ $\Rightarrow$ soft-collinear limit

The matrix element factorizes in two steps:

soft factorization of γ_2 $\quad |\mathcal{M}_{Z\to e^+e^-\gamma_1\gamma_2}|^2 \to e^2 J_2 |\mathcal{M}_{Z\to e^+e^-\gamma_1}|^2$

collinear factorization of γ_1 $\quad |\mathcal{M}_{Z\to e^+e^-\gamma_1}|^2 \approx \frac{2e^2}{s_{1e}} P_{e\gamma}(\epsilon, z) |\mathcal{M}_{Z\to e^+\tilde{e}^-}|^2$

derive the following formula

$$F_1|_{x_2=0,x_3=0} = \frac{16e^4 x_1}{m_Z E_- x_{\max}^2 \Delta_{12}} P_{e\gamma}(\epsilon, z) \times |\mathcal{M}_{Z\to e^+\tilde{e}^-}|^2.$$

easy to calculate numerically

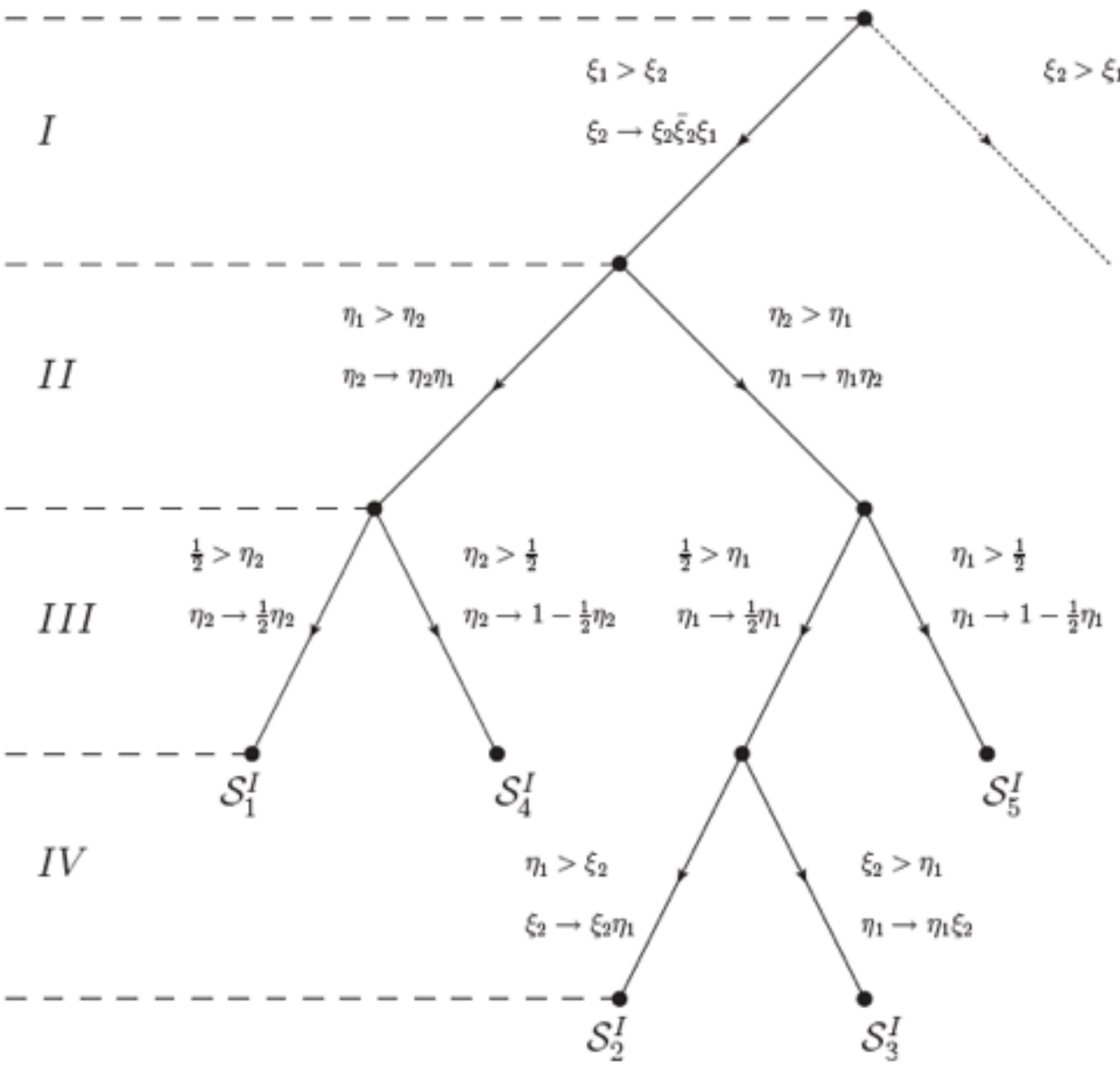
Moving onto Higgs+jet

- What differences occur when considering a more complex process such as Higgs+jet? Let's look at the double-real radiation.
- First introduce a transverse-momentum partitioning to ensure that at least one hard parton is in the final state:

$$\Delta = \frac{p_{T3}}{p_{T3} + p_{T4} + p_{T5}}$$

- Perform an angular partitioning similar to that for $Z \to e^+e^-$
- Left with the following partitions: $p_5||p_4||p_1$, $p_5||p_4||p_2$, $p_5||p_4||p_3$, $p_5||p_1\&p_4||p_2$, $p_5||p_2\&p_4||p_1$, $p_5||p_1\&p_4||p_3$, $p_5||p_3\&p_4||p_1$, $p_5||p_2\&p_4||p_3$, $p_5||p_3\&p_4||p_2$

Sector structure



from Czakon, 2010

- Follow same procedure as for the QED example
- Five sectors for the triple-collinear partition, not three as in QED, from $g \to gg$ splitting
- This same sector tree applies to all three triple-collinear partitions
- Very helpful to use rotational invariance to use different reference frames in each partition. For $p_5||p_4||p_1$ set $p_1=E_1(1,0,0,1)$. For $p_5||p_4||p_3$, rotate and set $p_3=E_3(1,0,0,1)$.

Real-virtual

- Treatment of the real-virtual corrections possible with same technique
- Phase-space is that of an NLO real-emission correction, so FKS@NLO is suitable.
- However, the amplitudes now have branch cuts, which change the overall fractional powers appearing in the integral we must perform.

$$\mathrm{RV}_i = \int \{dy\} \frac{dx_1}{x_1^{1+2\epsilon}} \frac{dx_2}{x_2^{1+\epsilon}} \left(F_{i,1} + (x_1^2 x_2)^{-\epsilon} F_{i,2} + x_1^{-2\epsilon} F_{i,3} \right)$$

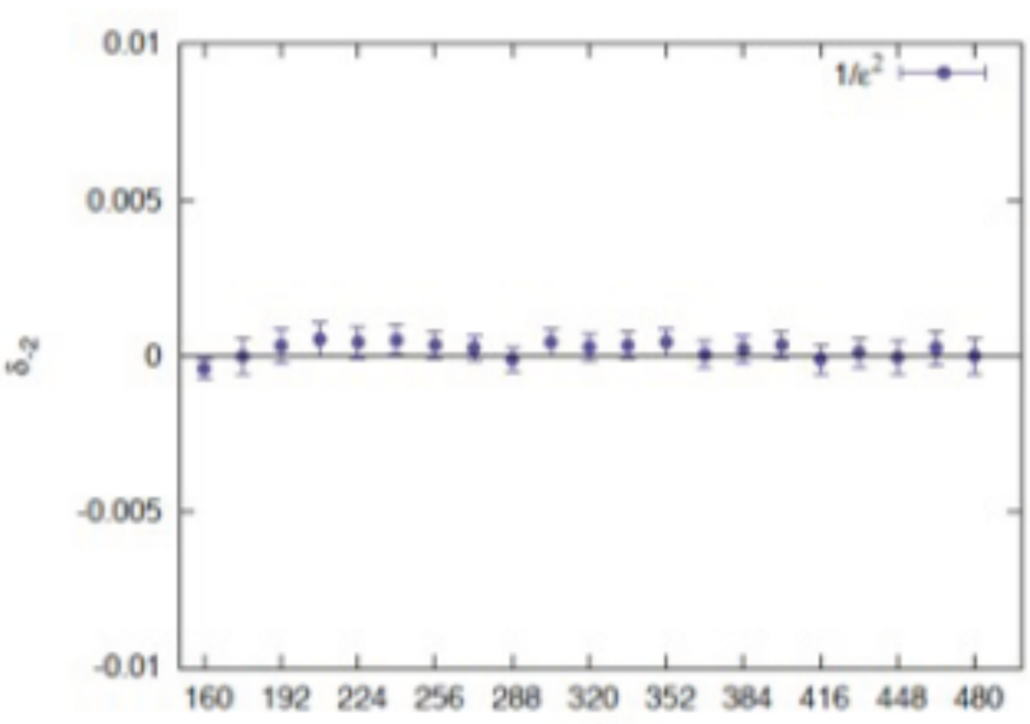
energy variable

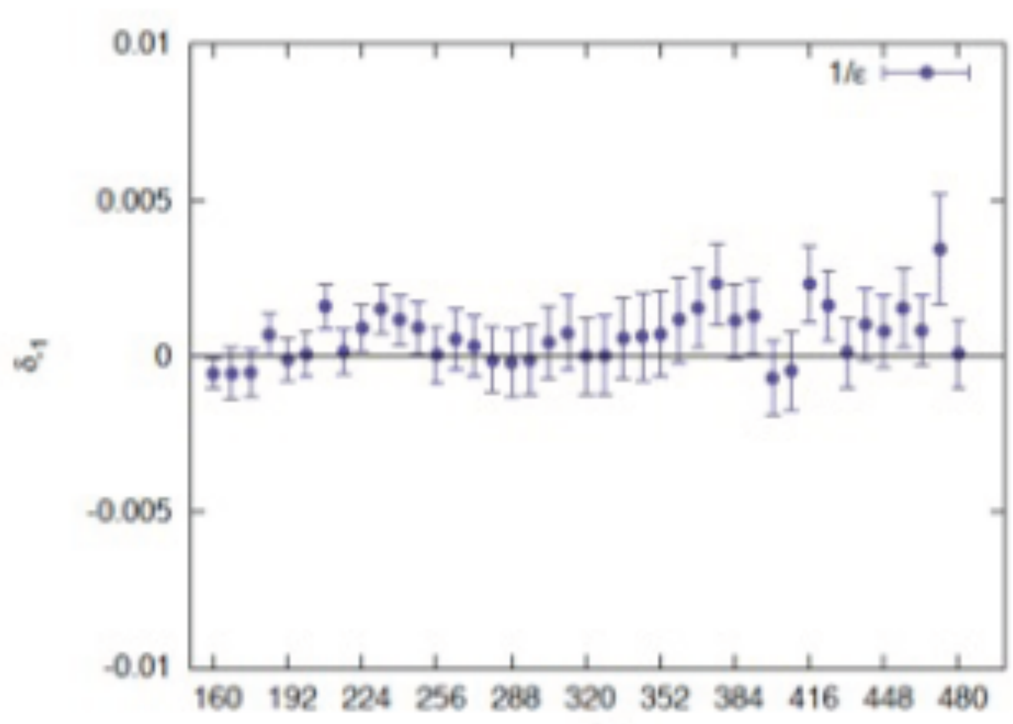
angular variable

must keep track of the different fractional powers which can appear, to properly expand in plus distributions

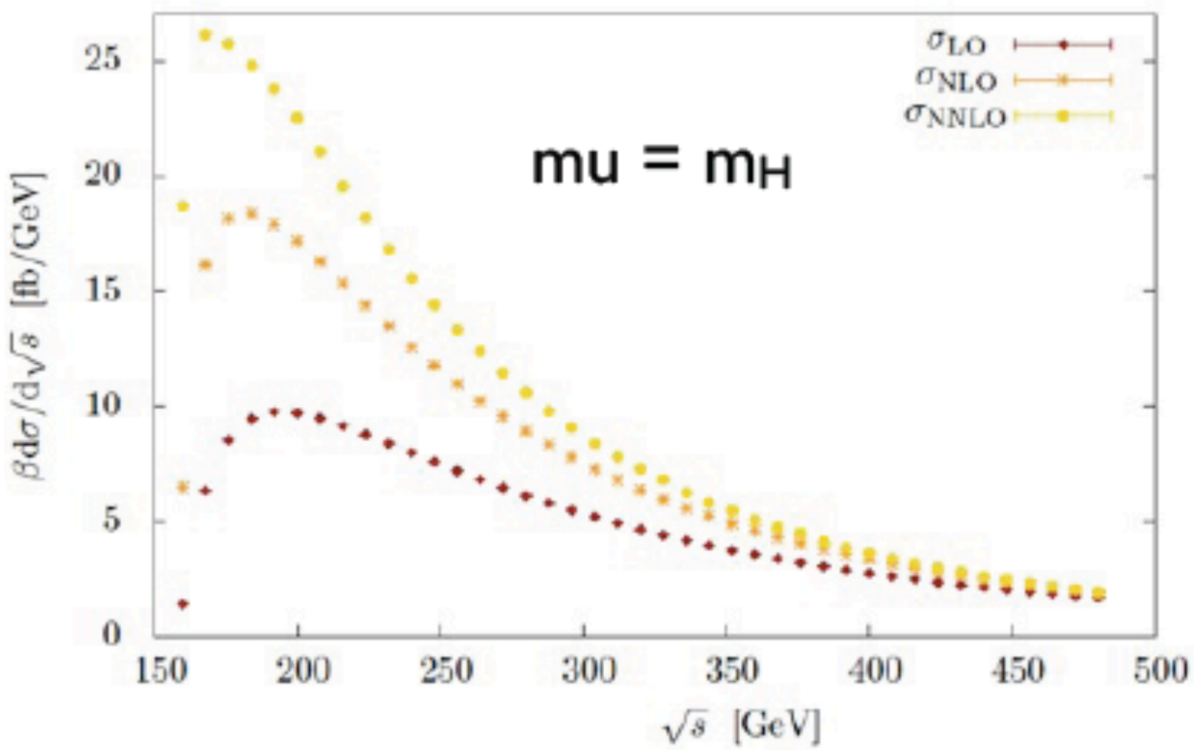
Checks for H+jet

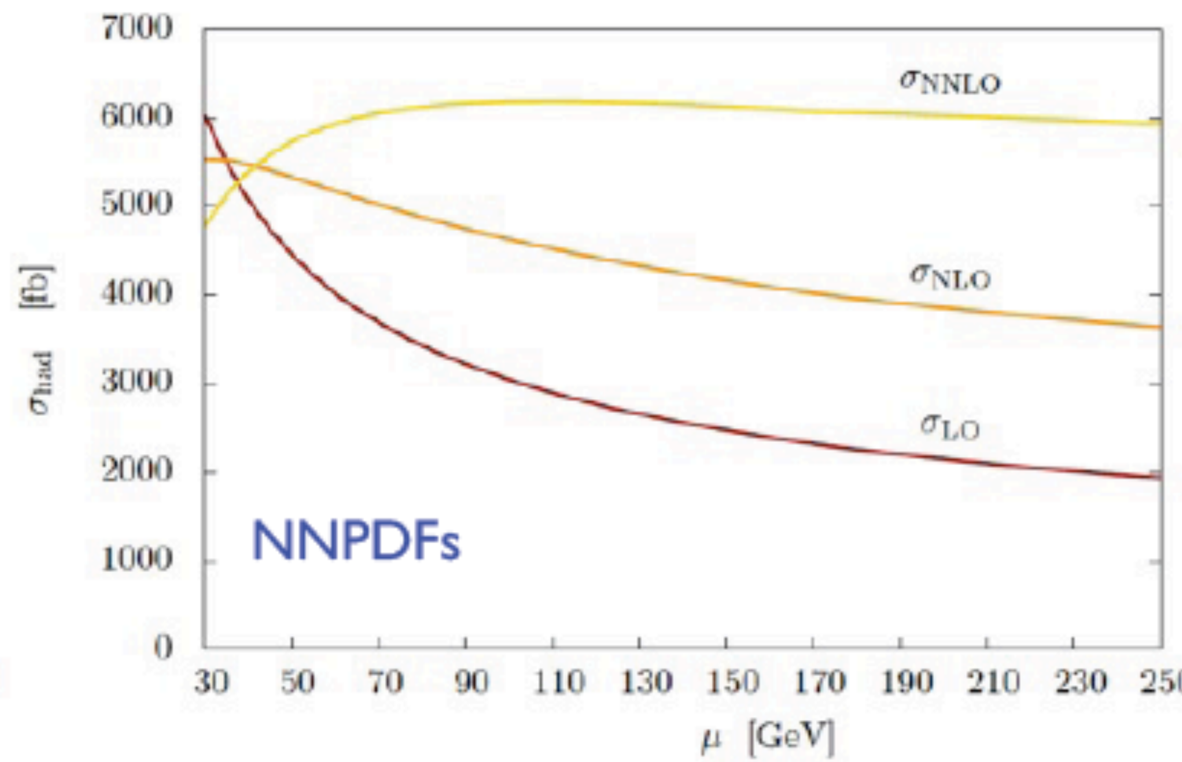
- Two independent calculations and codes
- Correct d-dimensional phase-space volume in each partition
- Tree/loop level amplitudes tested against the literature; internal calculations using multiple techniques agreed
- Checks that the full amplitudes match the subtraction terms in the singular limits
- Pole cancellation:





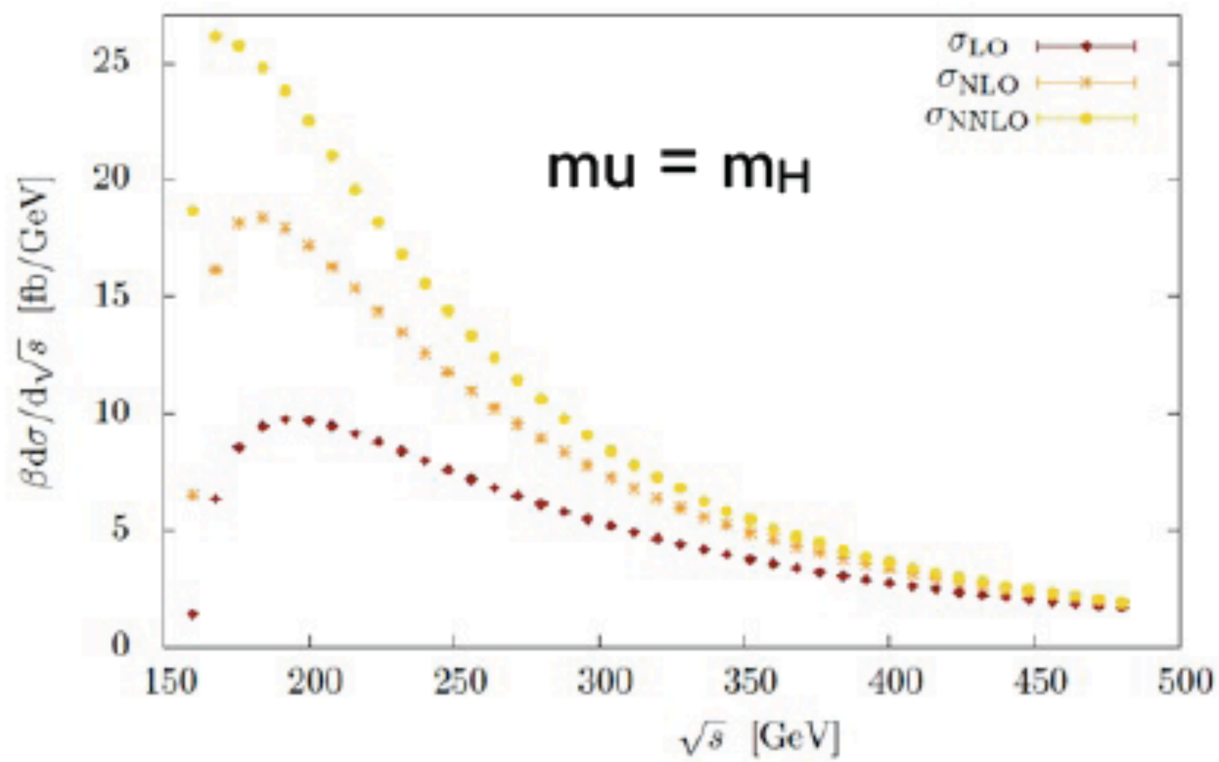
Initial results (gg only)

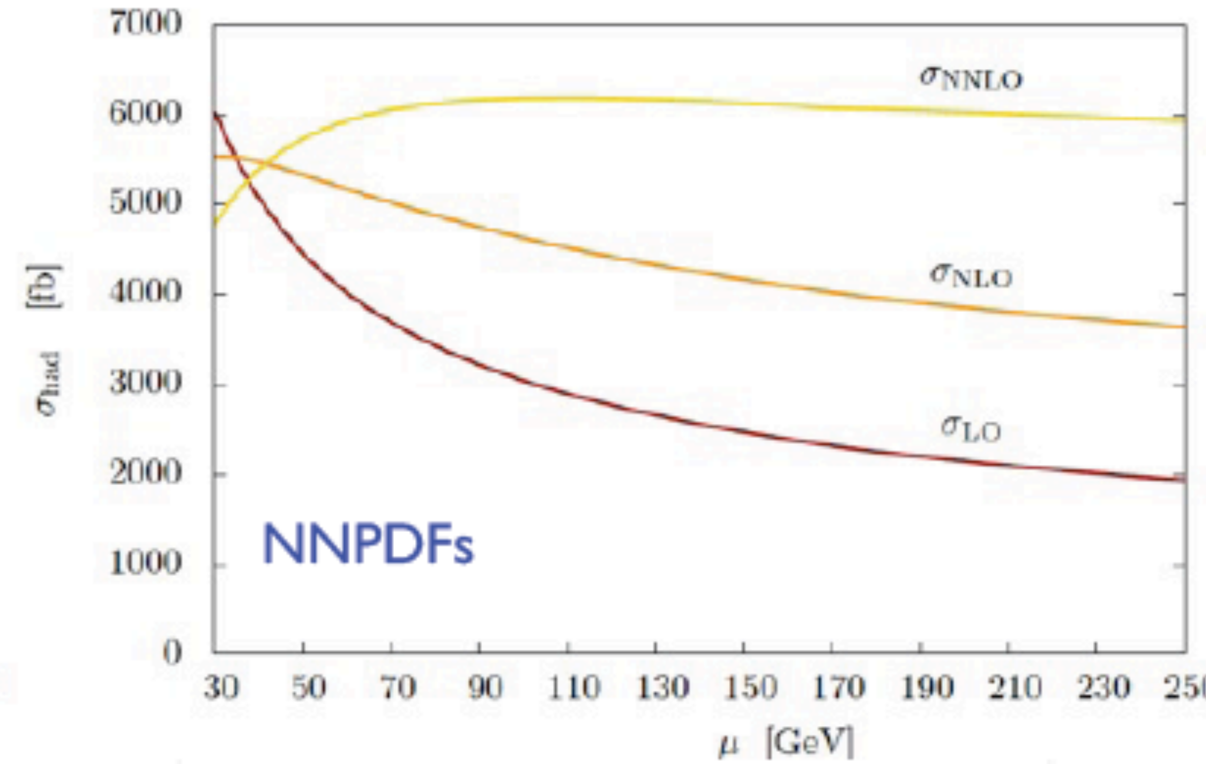




- Partonic cross section for gg → Hj @ LO, NLO, NNLO
- Realistic jet algorithm, k_T with R=0.5, p_T > 30 GeV
- Hadronic cross-section pp → Hj using latest NNPDF sets
- Scale variation in the range $m_H/2 < \mu < 2\, m_H$, m_H = 125 GeV

Initial result (gg only)





$$\sigma_{\rm LO}(pp \to Hj) = 2713^{+1216}_{-776} \text{ fb},$$
$$\sigma_{\rm NLO}(pp \to Hj) = 4377^{+760}_{-738} \text{ fb},$$
$$\sigma_{\rm NNLO}(pp \to Hj) = 6177^{-204}_{+242} \text{ fb}.$$

Large K-factors

$\sigma_{NLO}/\sigma_{LO} = 1.6$

$\sigma/_{NNLO}/\sigma_{NLO} = 1.3$

Significantly reduced O(4%) scale dependence

Questions on NNLO+PS

- We might be interested in such a tool, to generate events with respect to the correct NNLO distributions in the fixed-order region while getting the correction Sudakov suppression in the resummation region.
- What do we want from NNLO+PS? Are the circled terms enough, if only for a first attempt? Would have

$$
\begin{aligned}
\sigma(\tau_{cut}) = {} & 1 \\
& + \alpha_s L^2 + \alpha_s L + \alpha_s \\
& + \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 \\
& + \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \dots \\
& + \vdots
\end{aligned}
$$

Questions on NNLO+PS

- For what processes do we need this level of description? If only W/Z/H, probably special techniques can be used to accomplish this (for example, the q_T subtraction scheme of Catani et al. can be used to get NNLO for colorless final states, because the p_T recoil of the colorless system against the radiation completely controls the singularity structure, and the resummation of p_T is known through NNLL). If not, we need to understand the combination of a general subtraction scheme with parton shower.
- Any questions the audience wants to raise?