QCD RESUMMATION FOR JET SUBSTRUCTURE OBSERVABLES

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with
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The boosted regime

- The LHC is exploring phenomena at energies above the EW scale
- $Z/W/H/top$ can no longer be considered heavy particles
- These particles are abundantly produced with a large boost
- Their hadronic decays are collimated and can be reconstructed within a single jet

\[ p_t \gg m \]

\[ R \]
Grooming and tagging

• The last few years have seen a rapid development in substructure techniques

• These tools identify subjets within the fat jet and try and remove the ones that only carries a small fraction of the jet’s energy

• I will analyse three of them in some detail
Jet mass calculations

- Calculations for p-p collisions both in pQCD and SCET
  - collinear branching only (process independent), no ISR
  

- Z+jet and dijets to NLL
  
  Dasgupta, Khelifa-Kerfa, S.M. and Spannowsky (2012)

- Photon + jet to (N)NLL
  
  Chien, Kelley, Schwartz and Zhu (2012)

- Higgs + jet and dijets to NNLL (different jet definition)
  
  Jouttenus, Stewart, Tackmann and Waalewijn (2013)

- Let's consider an isolated jet (small-R limit)
  - The NLL integrated jet mass distribution is

\[
\Sigma(\rho) \equiv \frac{1}{\sigma} \int_{\rho}^{\rho'} d\rho' \frac{d\sigma}{d\rho} = e^{-D(\rho)} \cdot \frac{e^{-\gamma E D'(\rho)}}{\Gamma(1 + D'(\rho))} \cdot \mathcal{N}(\rho)
\]

\[
\rho = \frac{m_j^2}{p_t^2 R^2}
\]
Non-global logarithms

- Independent emission is not the whole story
- The jet-mass is a non-global observable: single-log corrections from correlated emission
- This is a $C_F C_A$ term and it’s missed by single gluon exponentiation
- In principle we need to consider any number of gluons outside the jet
- Colour structure becomes intractable, so the resummation is performed in the large $N_c$ limit

Dasgupta and Salam (2001)
Banfi, Marchesini and Smye (2002)
The role of the jet algorithm

• With C/A and k_t algorithms two soft gluons can be the closest pair
  • In this case they are recombined, along the harder one
  • The jet is deformed

• This does not happen if we use anti-k_t algorithm: soft gluons are always far apart
  • The anti-k_t algorithm in the soft limit works as a perfect cone

• Beyond LL the jet algorithm does matter


  Cacciari, Salam, Soyez (2008)
Trimming

1. Take all particles in a jet and re-cluster them with a smaller jet radius $R_{\text{sub}} < R$
2. Keep all subjets for which $p_{t}^{\text{subj}} > z_{\text{cut}} p_{t}$
3. Recombine the subjets to form the trimmed jet

Krohn, Thaler and Wang (2010)
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Pruning
1. From an initial jet define pruning radius $R_{\text{prune}} \sim m / p_t$
2. Re-cluster the jet, vetoing recombination for which

$$z = \frac{\min(p_{ti}, p_{tj})}{|\vec{p}_{ti} + \vec{p}_{tj}|} < z_{\text{cut}}$$

$$d_{ij} > R_{\text{prune}}$$
i.e. soft and wide angle
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**Mass Drop Tagger**

1. Undo the last stage of the C/A clustering. Label the two subjets $j_1$ and $j_2$ ($m_1 > m_2$)
2. If $m_1 < \mu m$ (mass drop) and the splitting was not too asymmetric ($y_{ij} > y_{\text{cut}}$), tag the jet.
3. Otherwise redefine $j = j_1$ and iterate.

Krohn, Thaler and Wang (2010)

Butterworth, Davison, Rubin and Salam (2008)

Ellis, Vermillion and Walsh (2009)
Boost 2010 proceedings:

The [Monte Carlo] findings discussed above indicate that while [pruning, trimming and filtering] have qualitatively similar effects, there are important differences. For our choice of parameters, pruning acts most aggressively on the signal and background followed by trimming and filtering.

- To what extent are the taggers above similar?
- How does the statement of aggressive behaviour depend on the taggers’ parameters and on the jet’s kinematics?

- *Time to go back to basics*, i.e. to understand the perturbative behaviour of QCD jets with tagging algorithms
Comparison of taggers

The “right” MC study on QCD jets can be instructive
Comparison of taggers

Different taggers appear to behave quite similarly
Comparison of taggers

But only for a limited range of masses!
Comparison of taggers

Let’s translate from QCD variables to "search" variables:

\[ \rho \rightarrow m, \text{ for } p_t = 3 \text{ TeV} \]
Questions that arise

• Can we understand the different shapes (flatness vs peaks) ?
• What’s the origin of the transition points ?
• How do they depend on the taggers’ parameters ?

• What’s the perturbative structure of tagged mass distributions ?
• The plain jet mass contains (soft & collinear) double logs

\[ \Sigma(\rho) \equiv \frac{1}{\sigma} \int^{\rho} d\sigma \frac{d\sigma}{d\rho} d\rho' \sim \sum_{n} \alpha_{s}^{n} \ln^{2n} \frac{1}{\rho} + \ldots \]

• Do the taggers ameliorate this behaviour ?
• If so, what’s the applicability of FO calculations ?
Trimming

1. Take all particles in a jet and re-cluster them with a smaller jet radius $R_{\text{sub}} < R$

2. Keep all subjets for which $p_t^{\text{subjet}} > z_{\text{cut}} \, p_t$

3. Recombine the subjets to form the trimmed jet
LO calculation

• LO eikonal calculation is already useful
• Consider the emission of a gluon in soft/collinear limit (small $z_{\text{cut}}$ for convenience)

$$\frac{1}{\sigma} \frac{d\sigma}{dv} = \frac{\alpha_s C_F}{\pi} \int \frac{d\theta^2}{\theta^2} \int \frac{dx}{x} \Theta (R^2 - \theta^2) \left[ \Theta (R^2_{\text{sub}} - \theta^2) + \Theta (\theta^2 - R^2_{\text{sub}}) \Theta (x - z_c) \right] \delta (v - x\theta^2)$$

$$v = \frac{m_j^2}{p_i^2}$$
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\]

- Three regions:

\[
v = \frac{m_j^2}{P_i^2}
\]

![Graph showing coefficient of $C_F \alpha_s / \pi$ for trimming $R=0.8$, $R_{\text{sub}}=0.2$, $z_{\text{cut}}=0.03$]
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- Three regions: plain jet mass, single logs, jet mass with $R_{sub}$

![Graph showing coefficient of $C_F\alpha_s/\pi$ for trimming $R=0.8$, $R_{sub}=0.2$, $z_{cut}=0.03$.]

Subtraction with hard collinear and finite $z_c$
Trimming: all orders

- Emissions within $R_{\text{sub}}$ are never tested for $z_{\text{cut}}$: double logs
- Intermediate region in which $z_{\text{cut}}$ is effective: single logs
- Essentially one gets exponentiation of LO (+ running coupling)
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All-order calculation done in the small-$z_{\text{cut}}$ limit
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- Our calculation captures $\alpha_s^n L^{2n}$ and $\alpha_s^n L^{2n-1}$ in the expansion
- To go beyond that one faces the usual troubles: non-global logs, clustering effects, etc.
- The transition points are correctly identified by the calculations
- The shapes are understood
1. From an initial jet define pruning radius \( R_{\text{prune}} \sim m / p_t \)
2. Re-cluster the jet, vetoing recombination for which

\[
z = \frac{\min(p_{t_i}, p_{t_j})}{|\vec{p}_{t_i} + \vec{p}_{t_j}|} < z_{\text{cut}}
\]

\[d_{ij} > R_{\text{prune}}\]

i.e. soft and wide angle
LO calculation

- LO calculation similar to trimming
- Now the pruning radius is set dynamically \( R_{\text{prune}}^2 \sim x\theta^2 \)

\[
\frac{1}{\sigma} \frac{d\sigma}{dv} = \frac{\alpha_s C_F}{\pi} \int \frac{d\theta^2}{\theta^2} \frac{dx}{x} \Theta (R^2 - \theta^2) \left[ \Theta (R_{\text{prune}}^2 - \theta^2) + \Theta (\theta^2 - R_{\text{prune}}^2) \Theta(x - z_{\text{cut}}) \right] \delta (v - x\theta^2)
\]

\[
v = \frac{m_j^2}{p_t^2}
\]
LO calculation

- LO calculation similar to trimming
- Now the pruning radius is set dynamically $R_{\text{prune}}^2 \sim x\theta^2$

$$\frac{1}{\sigma} \frac{d\sigma}{d\ln v} = \frac{\alpha_s C_F}{\pi} \int \frac{d\theta^2}{\theta^2} \frac{dx}{x} \Theta (R^2 - \theta^2) \left[ \Theta (R_{\text{prune}}^2 - \theta^2) + \Theta (\theta^2 - R_{\text{prune}}^2) \Theta(x - z_{\text{cut}}) \right] \delta (v - x\theta^2)$$

- Two regions:

![Diagram](attachment:image.png)

- Transition point

$$v = \frac{m_j^2}{p_t^2}$$
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\]

- Two regions: plain jet mass and single-log region

\[
v = \frac{m_j^2}{p_t^2}
\]

**Subtraction with hard collinear and finite \( z_c \)**

**Coefficient of \( C_F \alpha_s / \pi \) for pruning \( R=0.8, z_{\text{cut}}=0.1 \)**

- Transition point
- Leading behaviour in each region
Beyond LO

**What pruning is meant to do**

Choose an $R_{\text{prune}}$ such that different hard prongs ($p_1, p_2$) end up in different hard subjets.

Discard any softer radiation.
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Discard any softer radiation.

**What pruning sometimes does**
Chooses $R_{\text{prune}}$ based on a soft $p_3$ (dominates total jet mass), and leads to a single narrow subjet whose mass is also dominated by a soft emission ($p_2$, within $R_{\text{prune}}$ of $p_1$, so not pruned away).
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Structure beyond LO

- Because of its I-component the logarithmic structure at NLO worsens: $\sim \alpha_s^2 L^4$ (as plain jet mass)
- Explicit calculation shows that the one-prong component is active for $\rho < z_{cut}^2$
- A simple fix: require at least one successful merging with $\Delta R > R_{prune}$ and $z > z_{cut}$ (Y-pruning)
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• A simple fix: require at least one successful merging with $\Delta R > R_{\text{prune}}$ and $z > z_{\text{cut}}$ ($\gamma$-pruning)

• It is convenient to resum the two components separately
• $\gamma$-pruning: essentially Sudakov suppression of LO $\sim \alpha_s^n L^{2n-1}$
• I-pruning: convolution between the pruned and the original mass $\sim \alpha_s^n L^{2n}$
All-order results

- Full Pruning: single-log region for $z_{\text{cut}}^2 < \rho < z_{\text{cut}}$
- We control $\alpha_s^n L^{2n}$ and $\alpha_s^n L^{2n-1}$ in the expansion
- NG logs present but parametrically reduced
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Pythia 6 MC: quark jets
$m$ [GeV], for $p_t = 3$ TeV, $R = 1$

Analytic Calculation: quark jets
$m$ [GeV], for $p_t = 3$ TeV, $R = 1$

All-order calculation done in the small-$z_{\text{cut}}$ limit
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All-order calculation done in the small-$z_{\text{cut}}$ limit

[Graphs showing comparisons between Pythia 6 MC and Analytic Calculation for gluon jets at $p_t = 3$ TeV, $R = 1$]
Mass Drop Tagger at LO

1. Undo the last stage of the C/A clustering. Label the two subjets $j_1$ and $j_2$ ($m_1 > m_2$)

2. If $m_1 < \mu m$ (mass drop) and the splitting was not too asymmetric ($y_{ij} > y_{cut}$), tag the jet.

3. Otherwise redefine $j = j_1$ and iterate.
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3. Otherwise redefine $j = j_1$ and iterate.

In the small-$y_{cut}$ limit the result is identical to LO pruning: single-log distribution
What MDT does wrong:
If the $\gamma_{ij}$ condition fails, MDT iterates on the more massive subjet. It can follow a soft branch ($p_2 + p_3 < y_{cut} \ p_{t\text{jet}}$), when the “right” answer was that the (massless) hard branch had no substructure.

- This can be considered a flaw of the tagger
- It worsens the logarithmic structure $\sim \alpha_s^2 L^3$
- It makes all-order treatment difficult
- It calls for a modification
Modified Mass Drop Tagger

1. Undo the last stage of the C/A clustering. Label the two subjets $j_1$ and $j_2$ ($m_1 > m_2$)
2. If $m_1 < \mu m$ (mass drop) and the splitting was not too asymmetric ($y_{ij} > y_{cut}$), tag the jet.
3. Otherwise redefine $j$ to be the subjet with highest transverse mass and iterate.

- In practice the soft-branch contribution is very small
- However, this modification makes the all-order structure particularly interesting

\[ \frac{1}{4} y_{cut}^2 \]
All-order structure of mMDT

• The mMDT has single logs to all orders (i.e. $\sim \alpha_s^n L^n$)
• In the small $\gamma_{\text{cut}}$ limit it is just the exponentiation of LO
• Beyond that flavour mixing can happen (under control)
All-order structure of mMDT

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Pythia 6 MC: gluon jets
$m$ [GeV], for $p_t = 3$ TeV, $R = 1$
$y_{cut}$ = 0.03
$y_{cut}$ = 0.13
$y_{cut}$ = 0.35

Analytic Calculation: gluon jets
$m$ [GeV], for $p_t = 3$ TeV, $R = 1$
$y_{cut}$ = 0.03
$y_{cut}$ = 0.13
$y_{cut}$ = 0.35 (some finite $y_{cut}$)
Properties of mMDT

- **Flatness** of the background is a desirable property (data-driven analysis, side bands)
- $\gamma_{\text{cut}}$ can be adjusted to obtain it (analytic relation)
- FO calculation might be applicable
- **Role of $\mu$, not mentioned so far**
- It contributes to subleading logs and has small impact if not too small ($\mu > 0.4$)
- **Filtering** only affects subleading terms
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- **Filtering** only affects subleading terms
- It has only **single logs**, which are of collinear origin
- Important consequence:
  
  **mMDT is FREE of non-global logs!**

- Very small sensitivity to hadronisation and UE
Non-perturbative effects

hadronisation summary (quark jets)

m [GeV], for $p_t = 3$ TeV, $R = 1$

UE summary (quark jets)

m [GeV], for $p_t = 3$ TeV, $R = 1$
Performances for finding signals (Ws)

Figure 17. Efficacies for tagging hadronically-decaying W's, for a range of taggers/groomers, shown as a function of the W transverse momentum generation cut in the Monte Carlo samples (Pythia 6, DW tune). Further details are given in the text.

- It receives O(\alpha_s) corrections from gluon radiation from the W → q\bar{q}' system. Monte Carlo simulation suggests these effects are responsible, roughly, for a 10% reduction in the tagging efficiencies.
- Secondly, Eq. (8.9) was for unpolarized decays. By studying lepton decay of the W in the pp → WZ process, one finds that the degree of polarization is p_t-dependent, and the expected tree-level tagging-efficiency ranges from about 76% at low p_t to 84% at high p_t.

These two effects explain the bulk of the modest differences between Fig. 17 and the result of Eq. (8.9). However, the main conclusion that one draws from Fig. 17 is that the ultimate performance of the different taggers will be driven by their effect on the background rather than by the fine details of their interplay with signal events. This provides an a posteriori justification of our choice to concentrate our study on background jets.

Y-pruning gives a visible improvements (but it is less calculable because sensitive to UE)
In summary ...

- Analytic studies of the taggers reveal their properties
- Particularly useful if MCs don’t agree
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BACKUP SLIDES
A large class of modern jet definitions is given by sequential recombination algorithms.

Starting with a list of particles, compute all distances $d_{ij}$ and $d_{iB}$.

Find the minimum of all $d_{ij}$ and $d_{iB}$.

If the minimum is a $d_{ij}$, recombine $i$ and $j$ and iterate.

Otherwise, call $i$ a final-state jet, remove it from the list and iterate.

Sequential recombination can be an external parameter (e.g. Jade algorithm), a distance from the beam ...

for a complete review see Salam, Towards jetography (2009)
Most common jet algorithms

Common choices for the distance are

\[ d_{ij} = \min \left( \frac{\Delta R_{ij}^2}{R^2} \right) \]

\( d_{iB} = p_{ti}^{2p} \)

with \( \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 \)

- Different algorithms serve different purposes
- Anti-\( k_t \) clusters around hard particles giving round jets (default choice for ATLAS and CMS)
- Anti-\( k_t \) is less useful for substructure studies, while C/A reflects QCD coherence