QCD RESUMMATION FOR JET SUBSTRUCTURE OBSERVABLES

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Resummation and Parton Showers
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with
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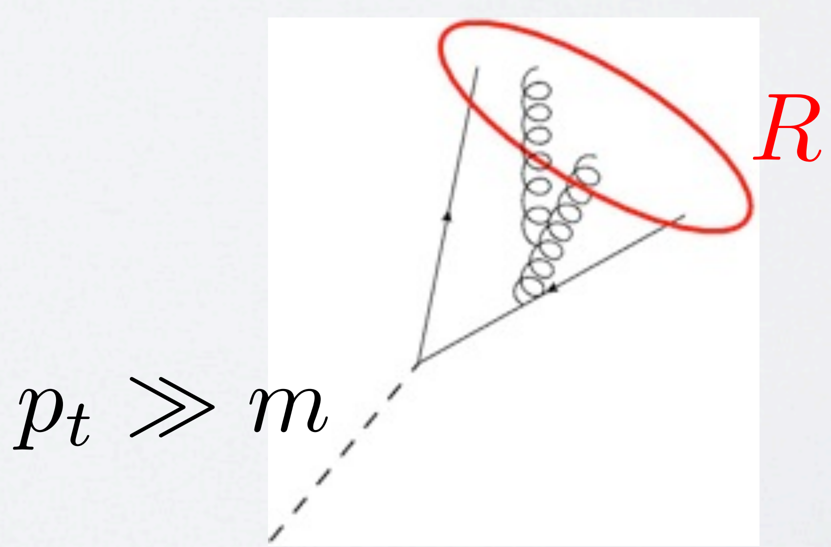
The boosted regime

- The LHC is exploring phenomena at energies above the EW scale

- Z/W/H/top can no longer be considered heavy particles

- These particles are abundantly produced with a large boost

- Their hadronic decays are collimated and can be reconstructed within a single jet

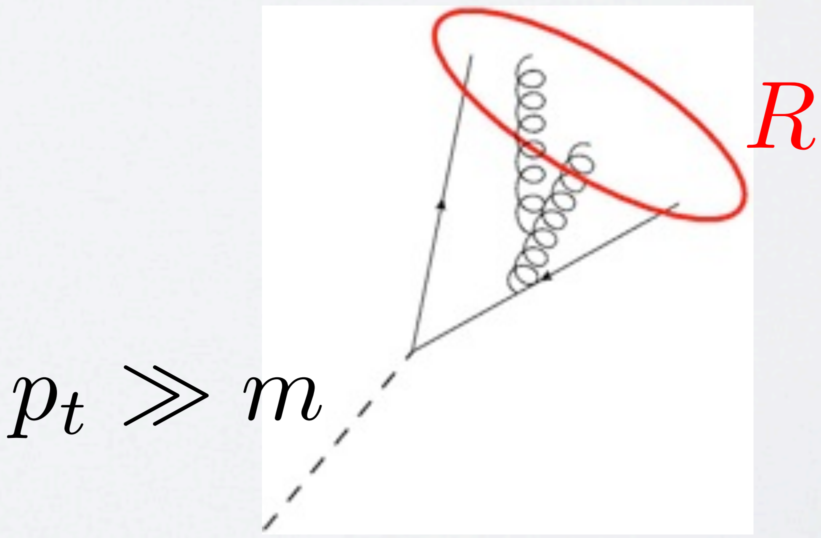


Grooming and tagging

- The last few years have seen a rapid development in substructure techniques

- These tools identify subjets within the fat jet and try and remove the ones that only carries a small fraction of the jet's energy

- I will analyse three of them in some detail



Jet mass calculations

- Calculations for p-p collisions both in pQCD and SCET
  - collinear branching only (process independent), no ISR

H. Li, Z. Li and C.P. Yuan (2011,2012)

  - Z+jet and dijets to NLL

Dasgupta, Khelifa-Kerfa, S.M. and Spannowsky (2012)

  - Photon + jet to (N)NLL

Chien, Kelley, Schwartz and Zhu (2012)

  - Higgs + jet and dijets to NNLL (different jet definition)

Jouttenus, Stewart, Tackmann and Waalewijn (2013)

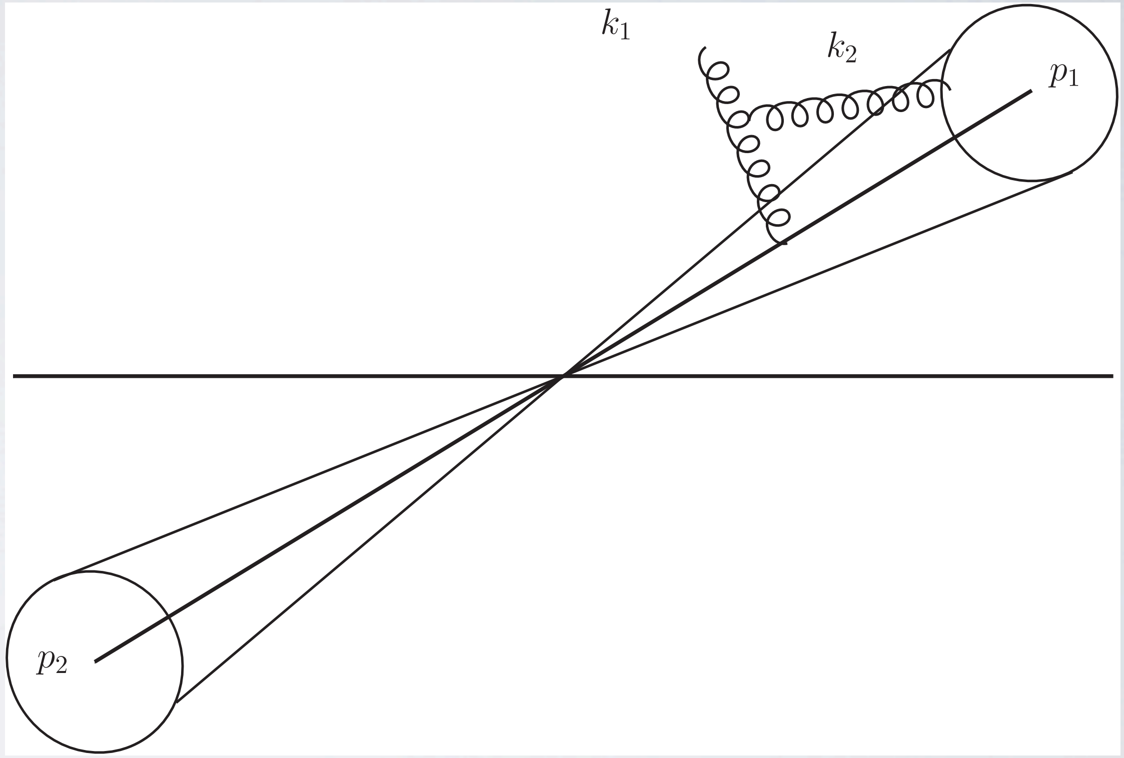
- Let's consider an isolated jet (small-R limit)
- The NLL integrated jet mass distribution is

$$\Sigma(\rho) \equiv \frac{1}{\sigma} \int^{\rho} d\,\rho' \frac{d\sigma}{d\rho'} = e^{-D(\rho)} \cdot \frac{e^{-\gamma_E D'(\rho)}}{\Gamma(1+D'(\rho))} \cdot \mathcal{N}(\rho)$$

$$\rho = \frac{m_j^2}{p_t^2 R^2}$$

Non-global logarithms

- Independent emission is not the whole story
- The jet-mass is a non-global observable: single-log corrections from correlated emission
- This is a $C_F C_A$ term and it's missed by single gluon exponentiation
- In principle we need to consider any number of gluons outside the jet
- Colour structure becomes intractable, so the resummation is performed in the large N_c limit



Recent progress by Hatta and Ueda: non-global logs at finite N_c

Dasgupta and Salam (2001)
Banfi, Marchesini and Smye (2002)

The role of the jet algorithm

- With C/A and k_t algorithms two soft gluons can be the closest pair
  - In this case they are recombined, along the harder one
  - The jet is deformed

- This does not happen if we use anti-k_t algorithm: soft gluons are always far apart
- The anti-k_t algorithm in the soft limit works as a perfect cone

Cacciari, Salam, Soyez (2008)

- Beyond LL the jet algorithm does matter

Delenda, Appleby, Banfi, Dasgupta (2006), Kelley, Walsh and Zuberi (2012), (Delenda) and K. Kerfa (2011,2012)

Trimming

1. Take all particles in a jet and re-cluster them with a smaller jet radius $R_{sub} < R$
2. Keep all subjets for which $p_t^{subjet} > z_{cut}\, p_t$
3. Recombine the subjets to form the trimmed jet

Krohn, Thaler and Wang (2010)

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Pruning

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$$z = \frac{\min(p_{ti}, p_{tj})}{|\vec{p}_{ti} + \vec{p}_{tj}|} < z_{\mathrm{cut}}$$

$$d_{ij} > R_{\mathrm{prune}}$$

i.e. soft and wide angle

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Mass Drop Tagger

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2. If $m_1 < \mu m$ (mass drop) and the splitting was not too asymmetric ($y_{ij} > y_{cut}$), tag the jet.
3. Otherwise redefine $j = j_1$ and iterate.

Butterworth, Davison, Rubin and Salam (2008)

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Our current understanding

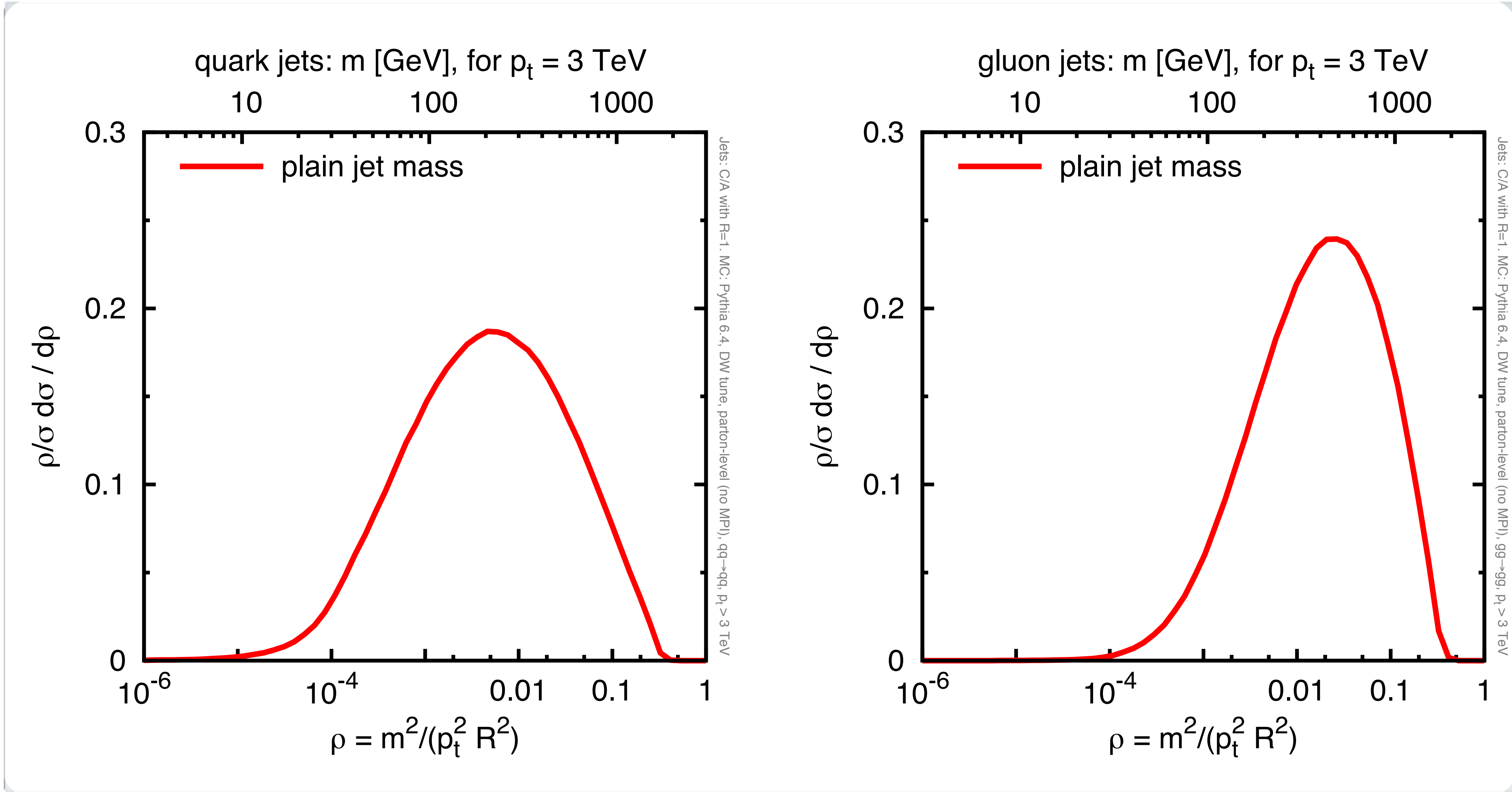
Boost 2010 proceedings:

The [Monte Carlo] findings discussed above indicate that while [pruning, trimming and filtering] have qualitatively similar effects, there are important differences. For our choice of parameters, pruning acts most aggressively on the signal and background followed by trimming and filtering.

- To what extent are the taggers above similar ?
- How does the statement of aggressive behaviour depend on the taggers' parameters and on the jet's kinematics ?

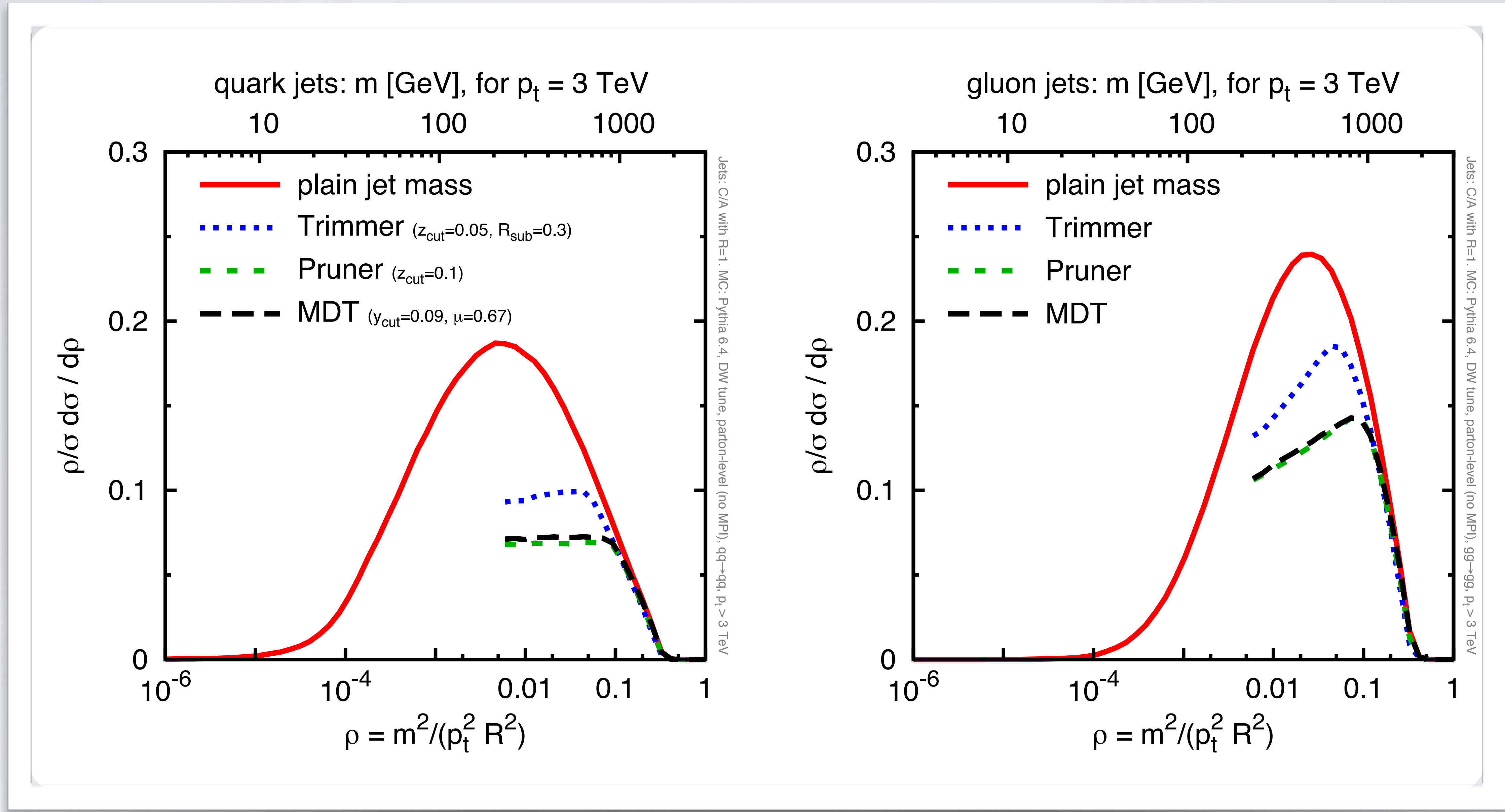
- *Time to go back to basics,* i.e. to understand the perturbative behaviour of QCD jets with tagging algorithms

Comparison of taggers



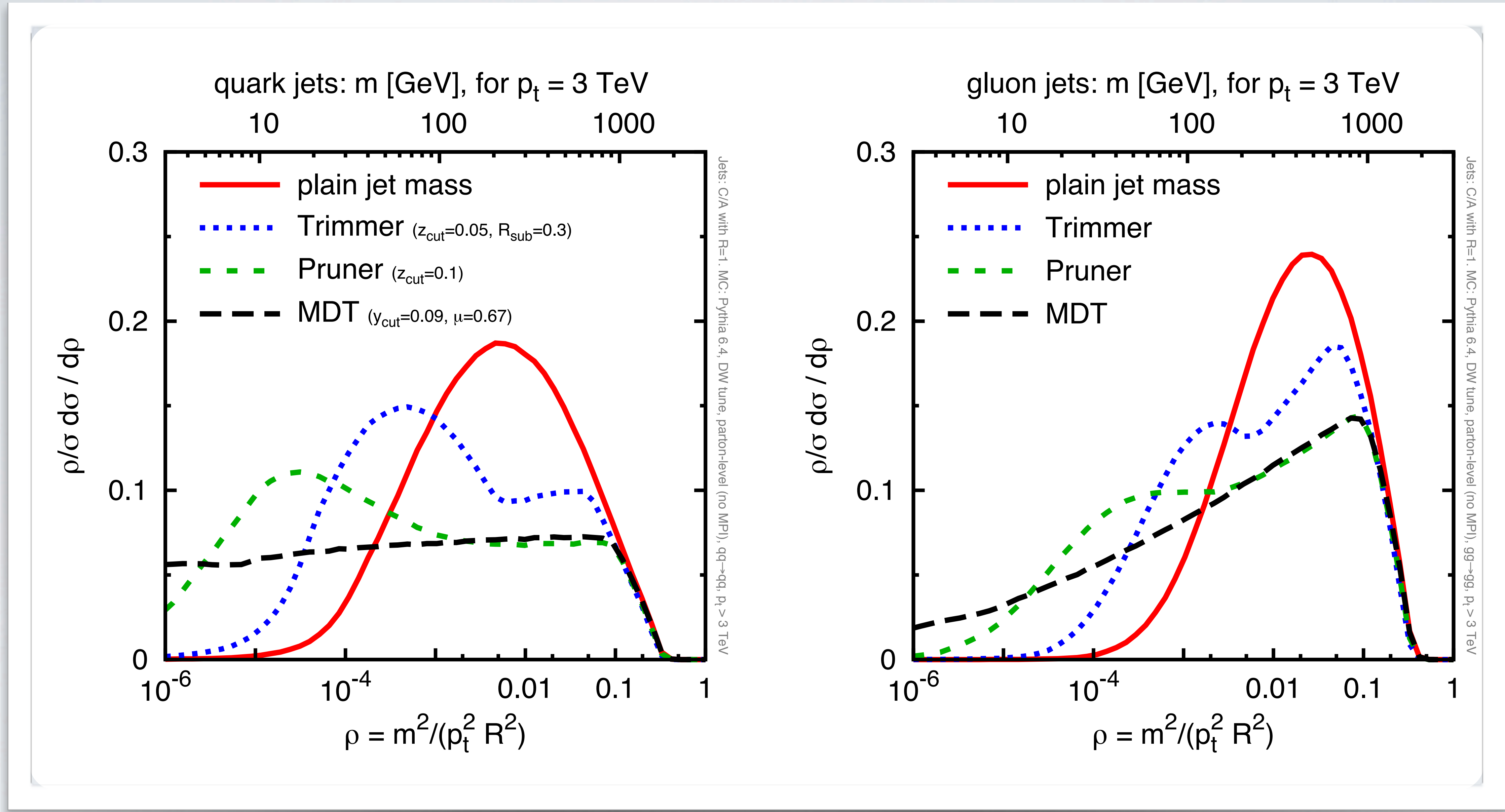
The "right" MC study on QCD jets can be instructive

Comparison of taggers



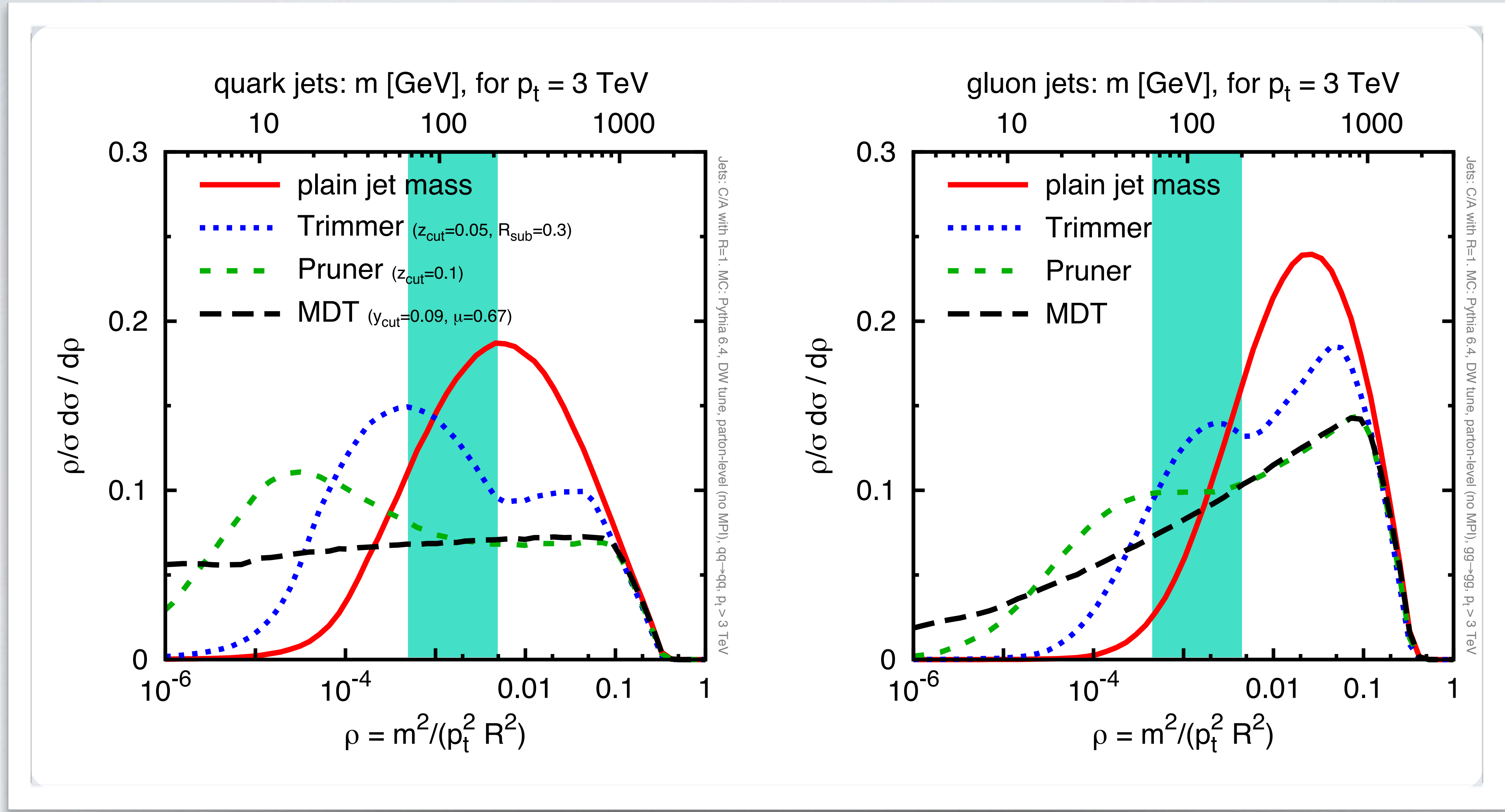
Different taggers appear to behave quite similarly

Comparison of taggers



But only for a limited range of masses !

Comparison of taggers



Let's translate from QCD variables to ``search'' variables:

$\boldsymbol{\rho} \rightarrow$ m, for p_t = 3 TeV

Questions that arise

- Can we understand the different shapes (flatness vs peaks) ?
- What's the origin of the transition points ?
- How do they depend on the taggers' parameters ?

- What's the perturbative structure of tagged mass distributions ?
- The plain jet mass contains (soft & collinear) double logs

$$\Sigma(\rho) \equiv \frac{1}{\sigma} \int^{\rho} \frac{d\sigma}{d\rho'} d\rho' \sim \sum_n \alpha_s^n \ln^{2n} \frac{1}{\rho} + \ldots$$

- Do the taggers ameliorate this behaviour ?
- If so, what's the applicability of FO calculations ?

Trimming

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2. Keep all subjets for which $p_t^{subjet} > z_{cut}\ p_t$

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LO calculation

- LO eikonal calculation is already useful
- Consider the emission of a gluon in soft/collinear limit

(small z_{cut} for convenience)

$$\frac{1}{\sigma}\frac{d\sigma}{dv} = \frac{\alpha_s C_F}{\pi}\int\frac{d\theta^2}{\theta^2}\int\frac{dx}{x}\Theta\left(R^2-\theta^2\right)\left[\Theta\left(R_{\text{sub}}^2-\theta^2\right)+\Theta\left(\theta^2-R_{\text{sub}}^2\right)\Theta(x-z_c)\right]\delta\left(v-x\theta^2\right)$$

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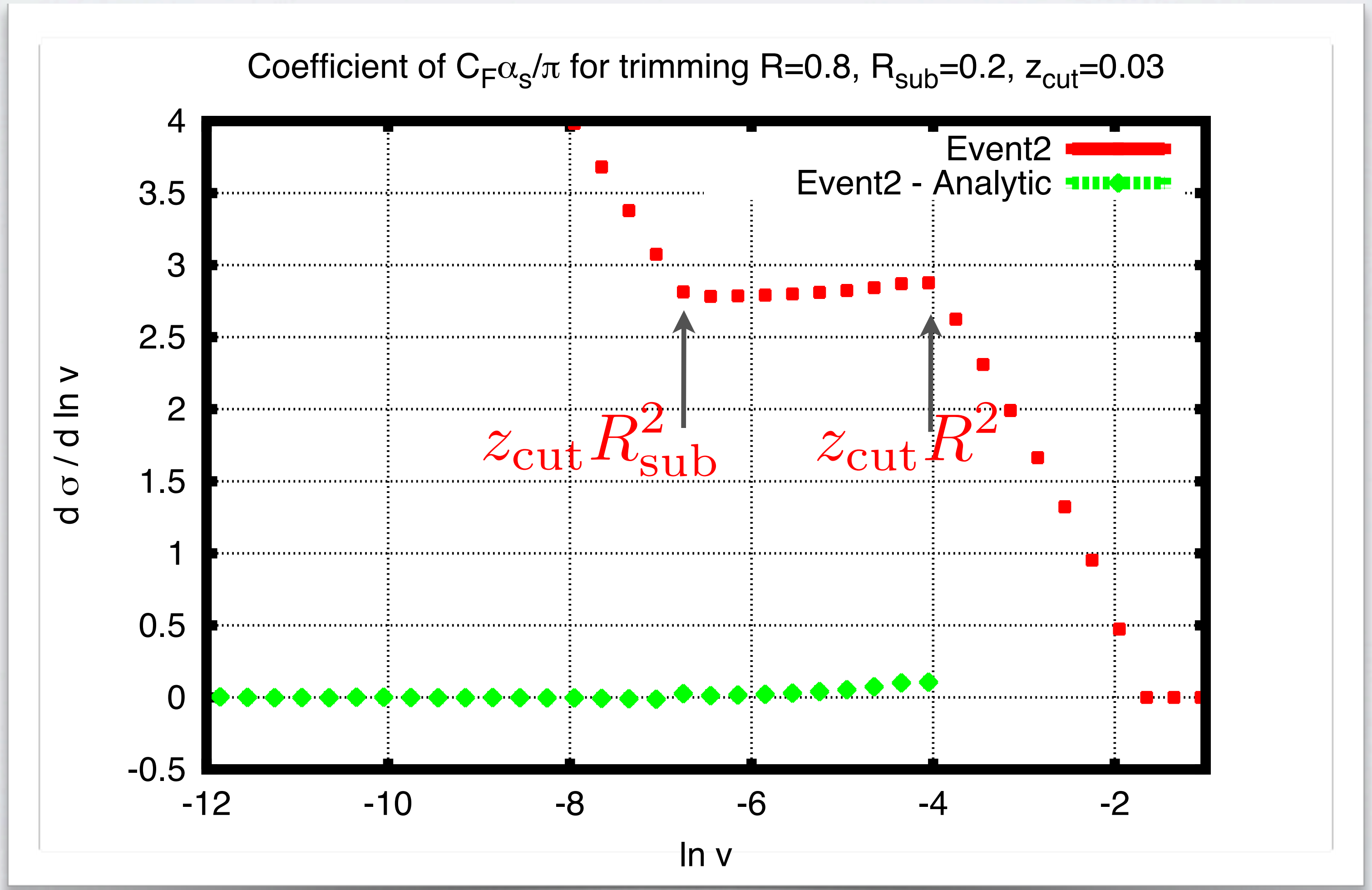
- *Three regions:*

transition points



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- *Three regions*: plain jet mass, single logs, jet mass with R_{sub}

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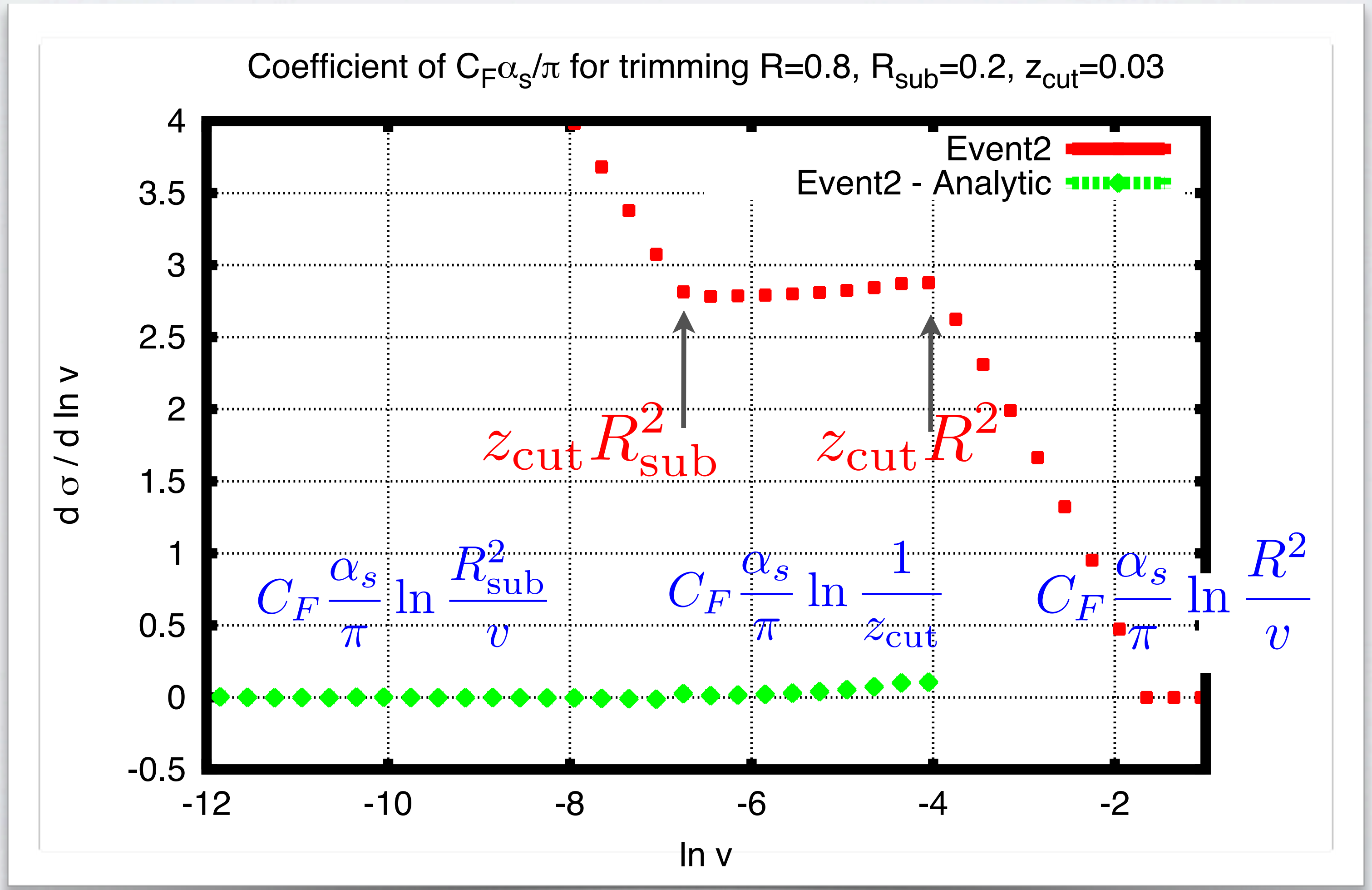
transition points

leading behaviour in each region



Subtraction with hard collinear and finite z_c

Trimming: all orders

- Emissions within R_{sub} are never tested for z_{cut}: double logs
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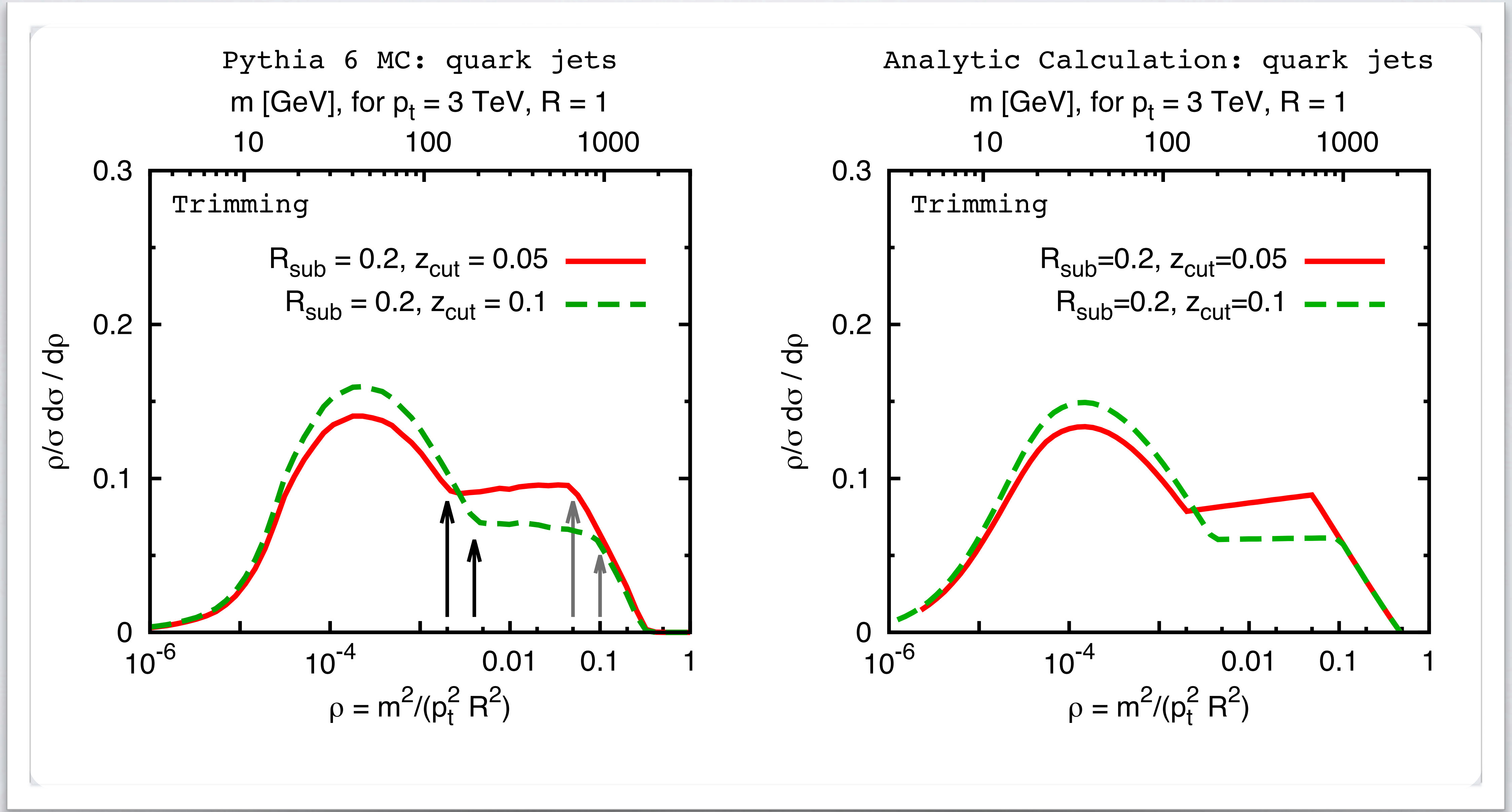

All-order calculation done in the small-z_{cut} limit

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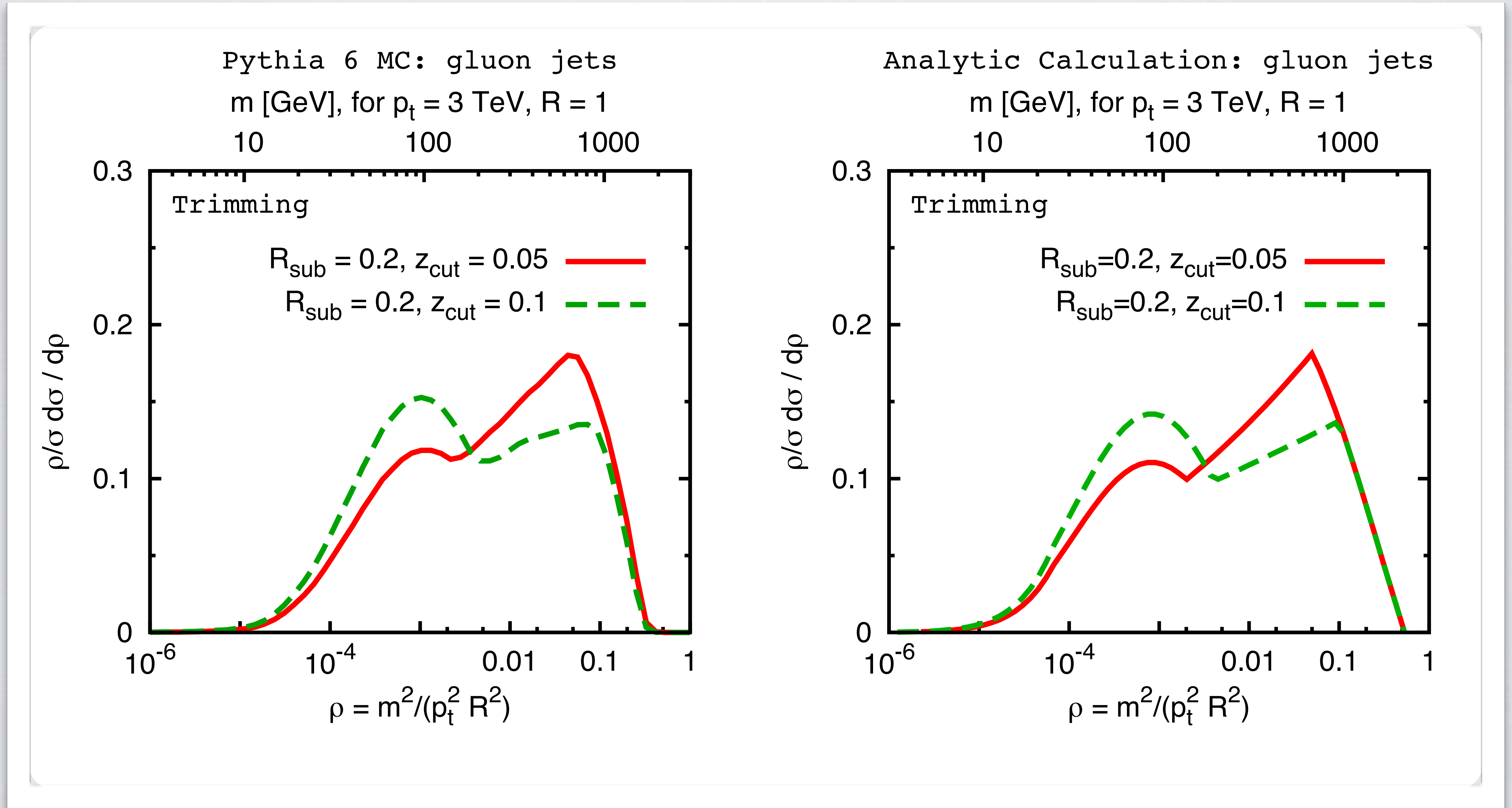

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- Our calculation captures $\alpha_s^n L^{2n}$ and $\alpha_s^n L^{2n-1}$ in the expansion
- To go beyond that one faces the usual troubles: non-global logs, clustering effects, etc.
- The transition points are correctly identified by the calculations
- The shapes are understood

Pruning

1. From an initial jet define pruning radius $R_{prune} \sim m / p_t$
2. Re-cluster the jet, vetoing recombination for which

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LO calculation

- LO calculation similar to trimming
- Now the pruning radius is set dynamically $R^2_{\text{prune}} \sim x\theta^2$

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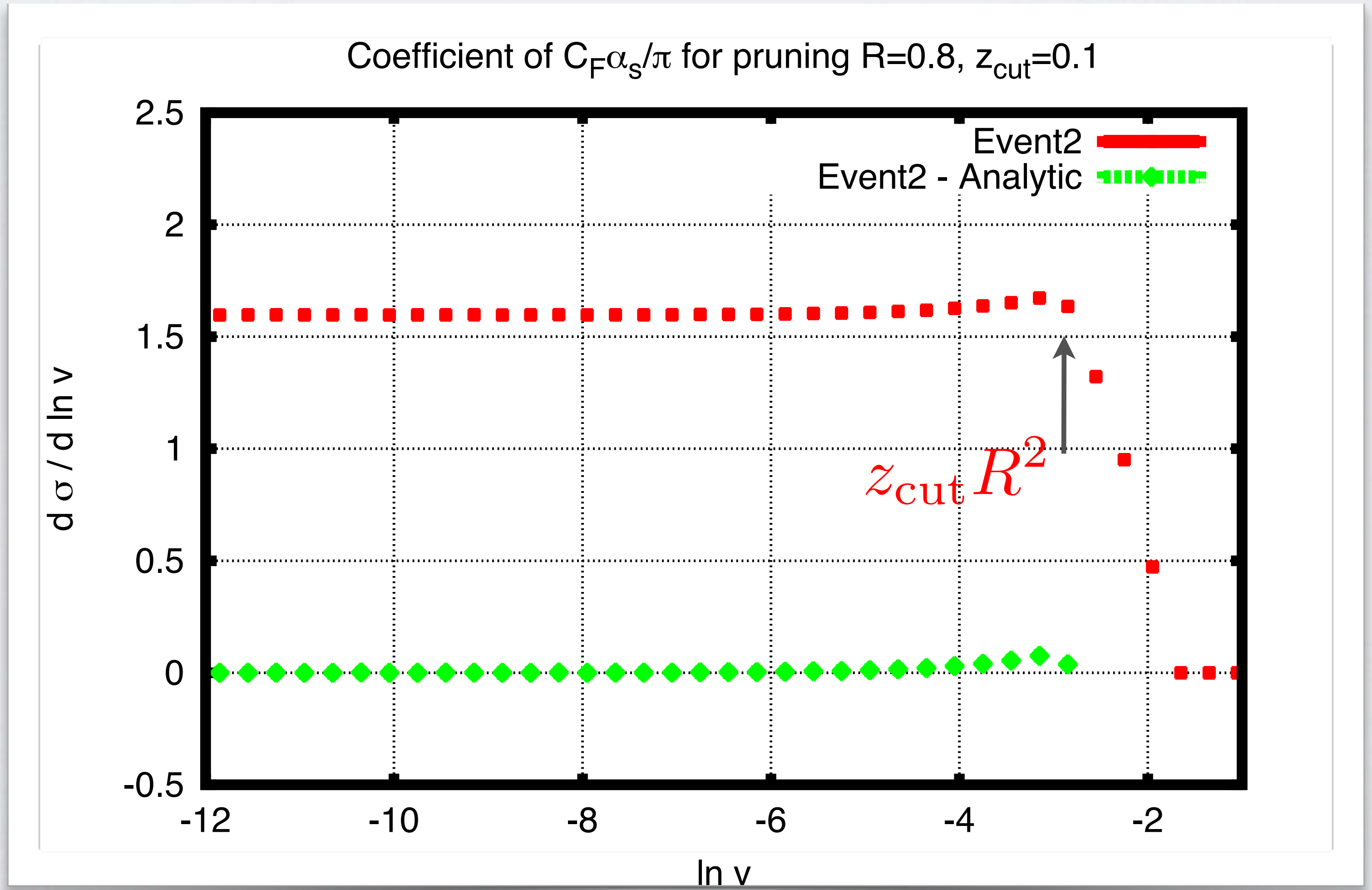
- Two regions:

transition point



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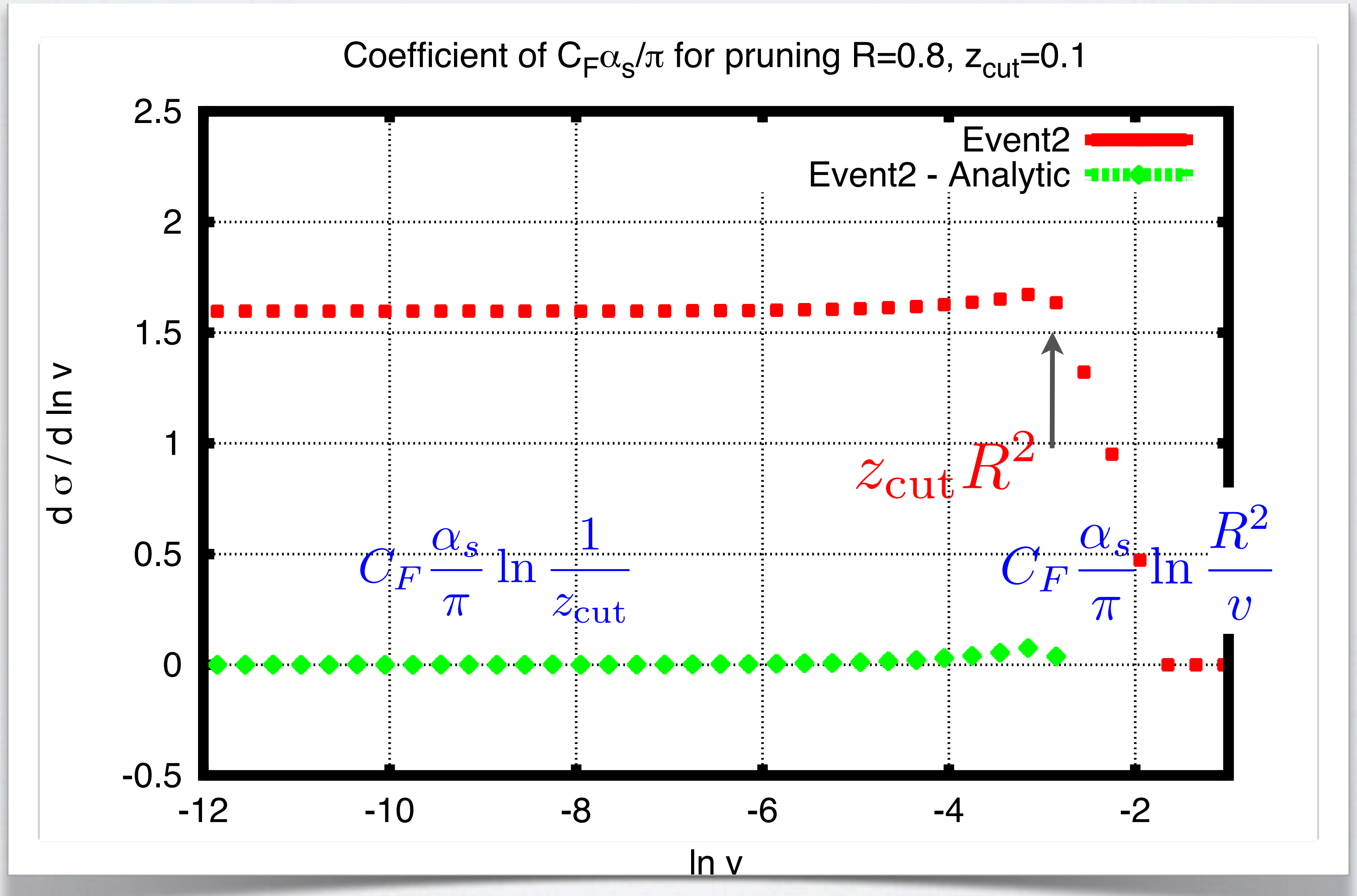
- Two regions: plain jet mass and single-log region !

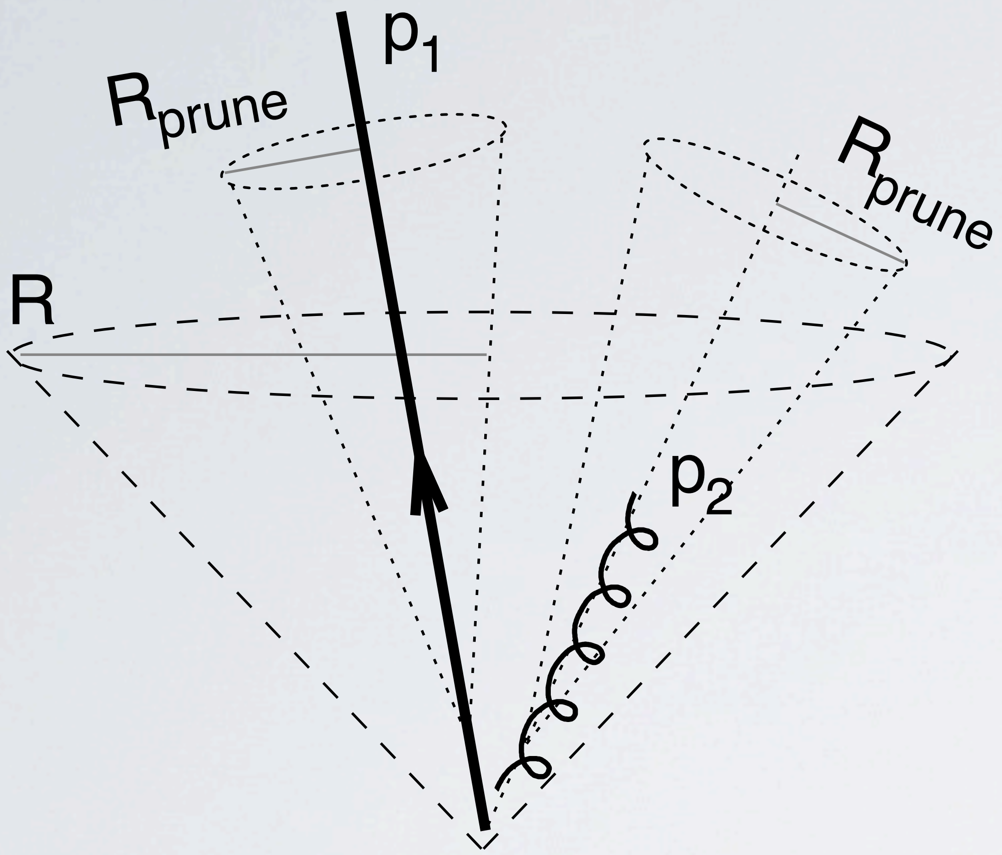
transition point

leading behaviour in each region



Subtraction with hard collinear and finite z_c

Beyond LO



What pruning is meant to do

Choose an R_{prune} such that different hard prongs (p_1, p_2) end up in different hard subjets.

Discard any softer radiation.

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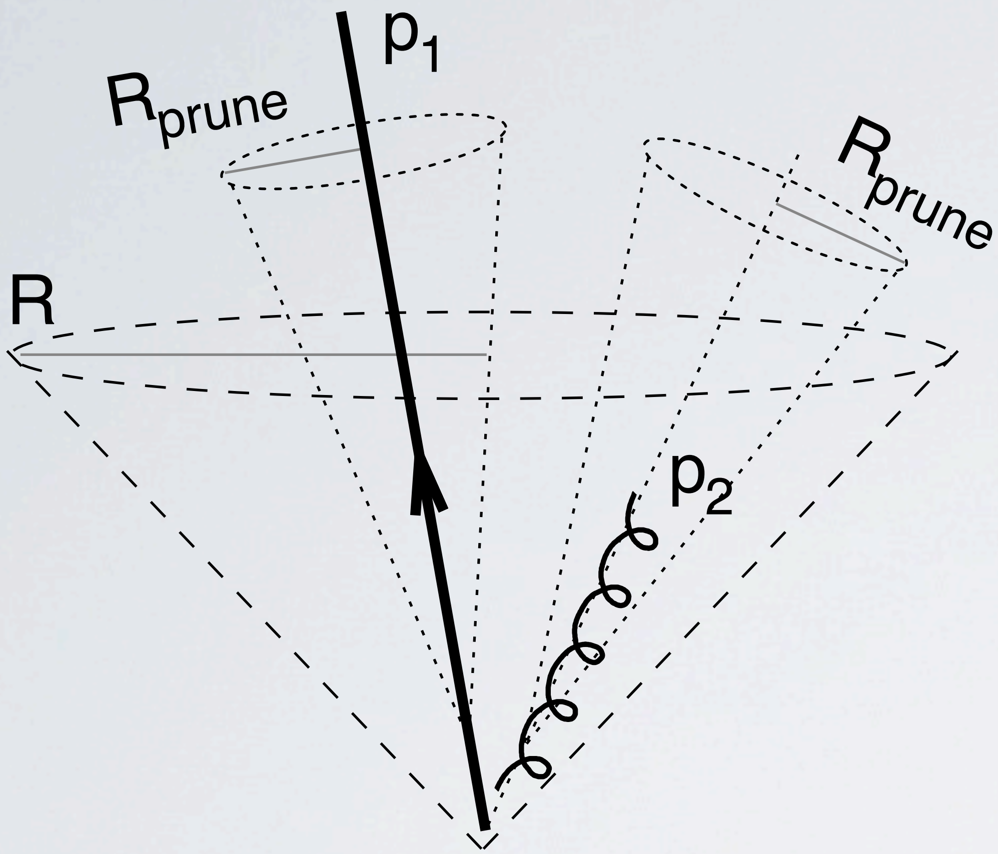

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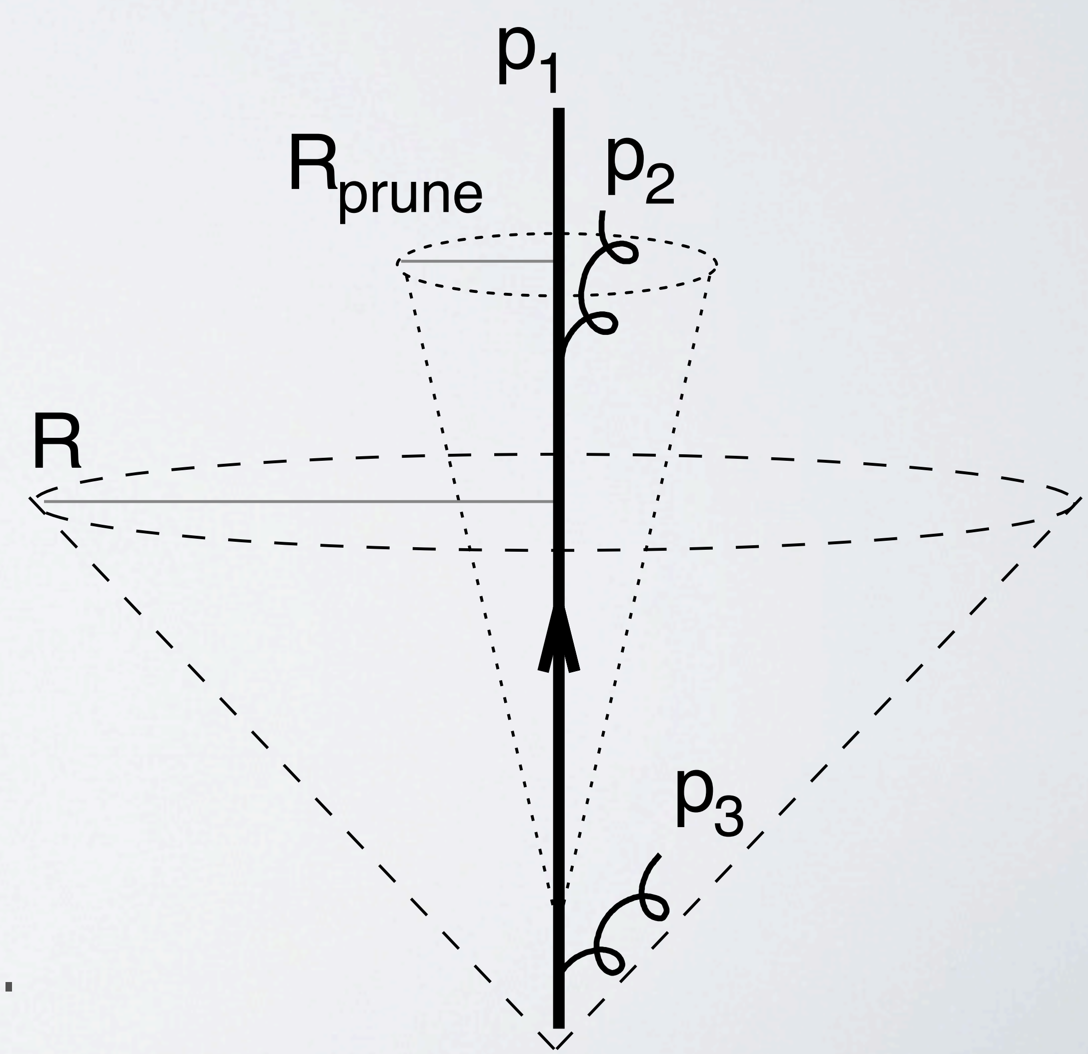
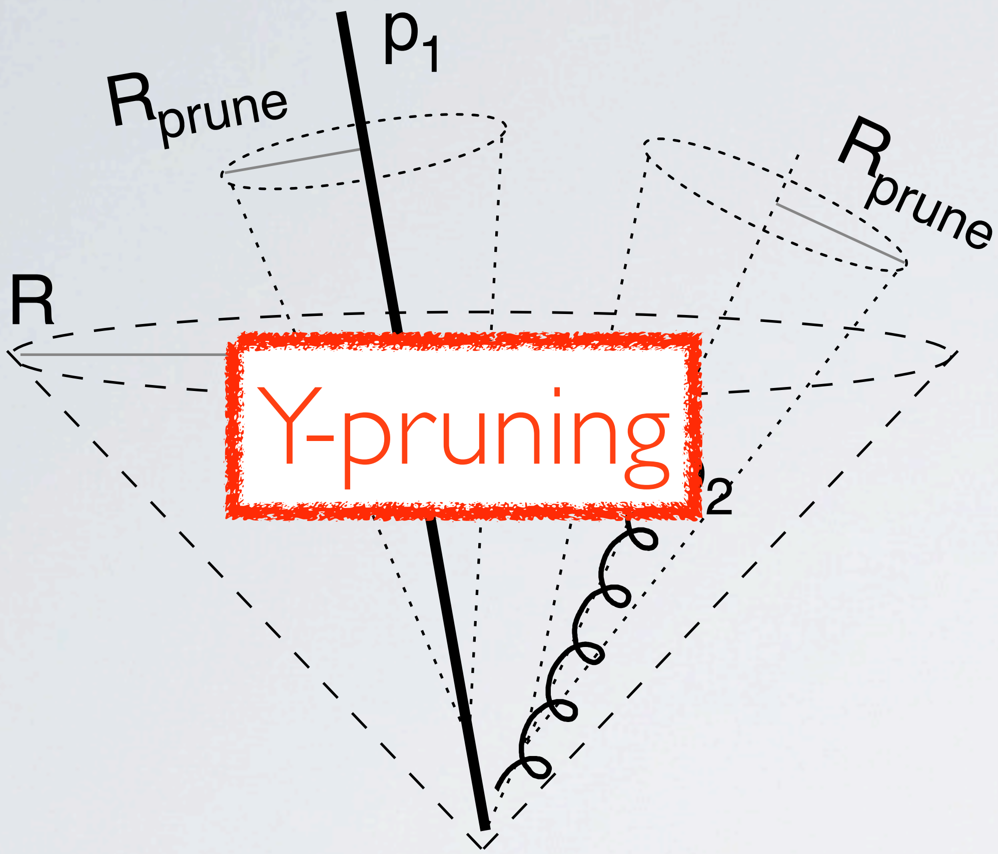


What pruning sometimes does

Chooses R_{prune} based on a soft p_3 (dominates total jet mass), and leads to a single narrow subjet whose mass is also dominated by a soft emission (p_2, within R_{prune} of p_1, so not pruned away).

Beyond LO



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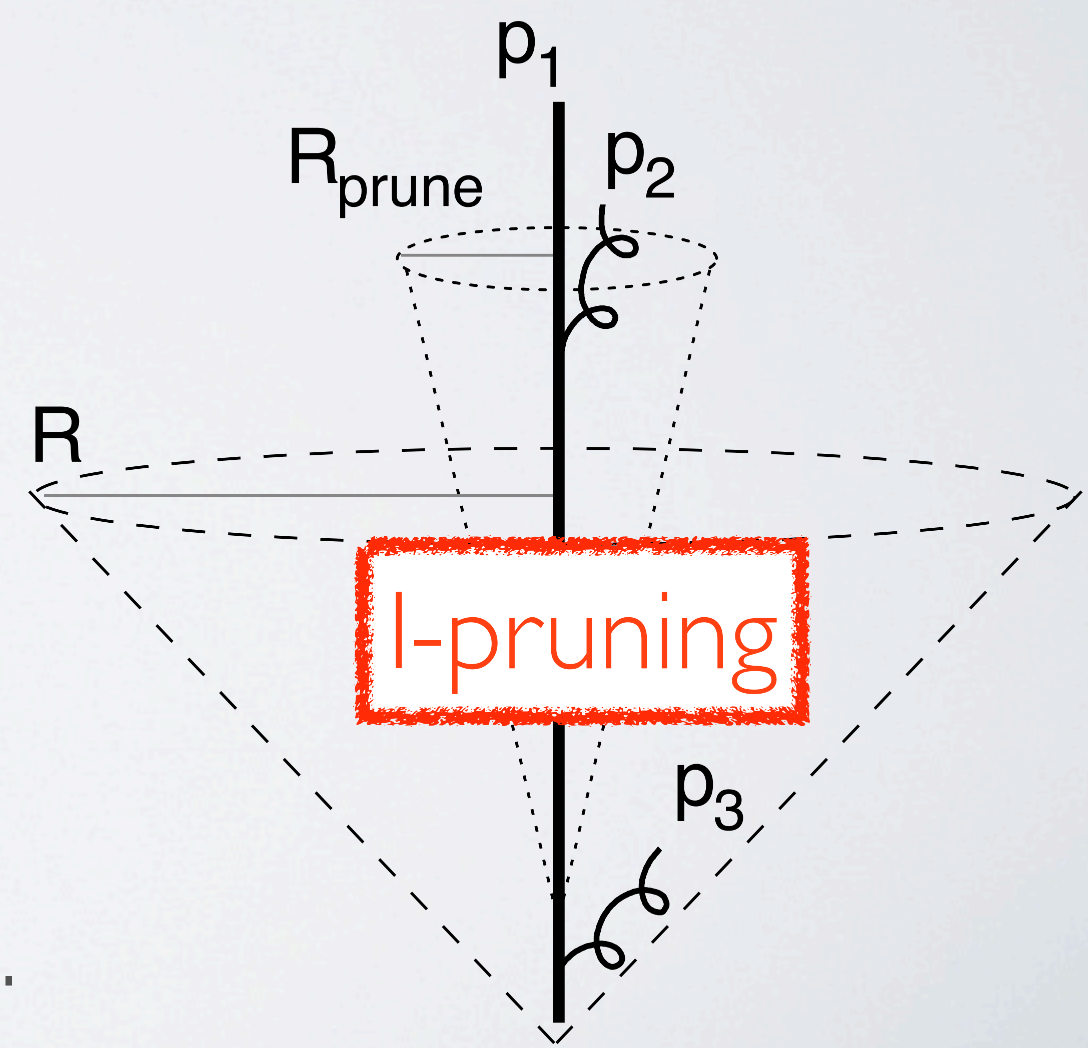

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Structure beyond LO

- Because of its I-component the logarithmic structure at NLO worsens: $\sim \alpha_s^2 L^4$ (as plain jet mass)
- Explicit calculation shows that the one-prong component is active for $\rho < z_{cut}^2$
- A simple fix: require at least one successful merging with $\Delta R > R_{prune}$ and $z > z_{cut}$ (Y-pruning)

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- It is convenient to resum the two components separately
- Y-pruning: essentially Sudakov suppression of LO $\sim \alpha_s^n L^{2n-1}$
- I-pruning: convolution between the pruned and the original mass $\sim \alpha_s^n L^{2n}$

All-order results

- Full Pruning: single-log region for $z_{cut}^2 < \rho < z_{cut}$
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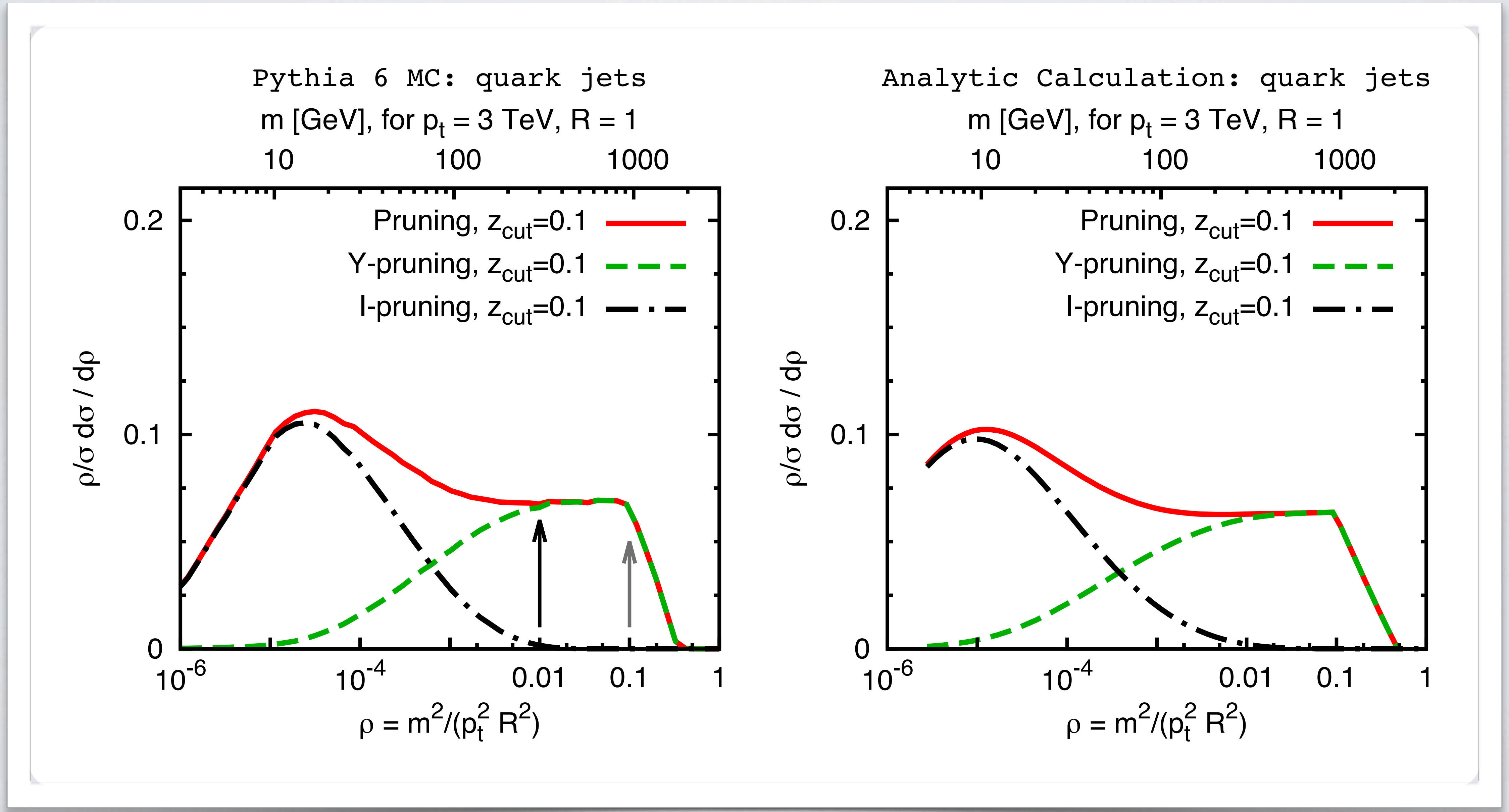

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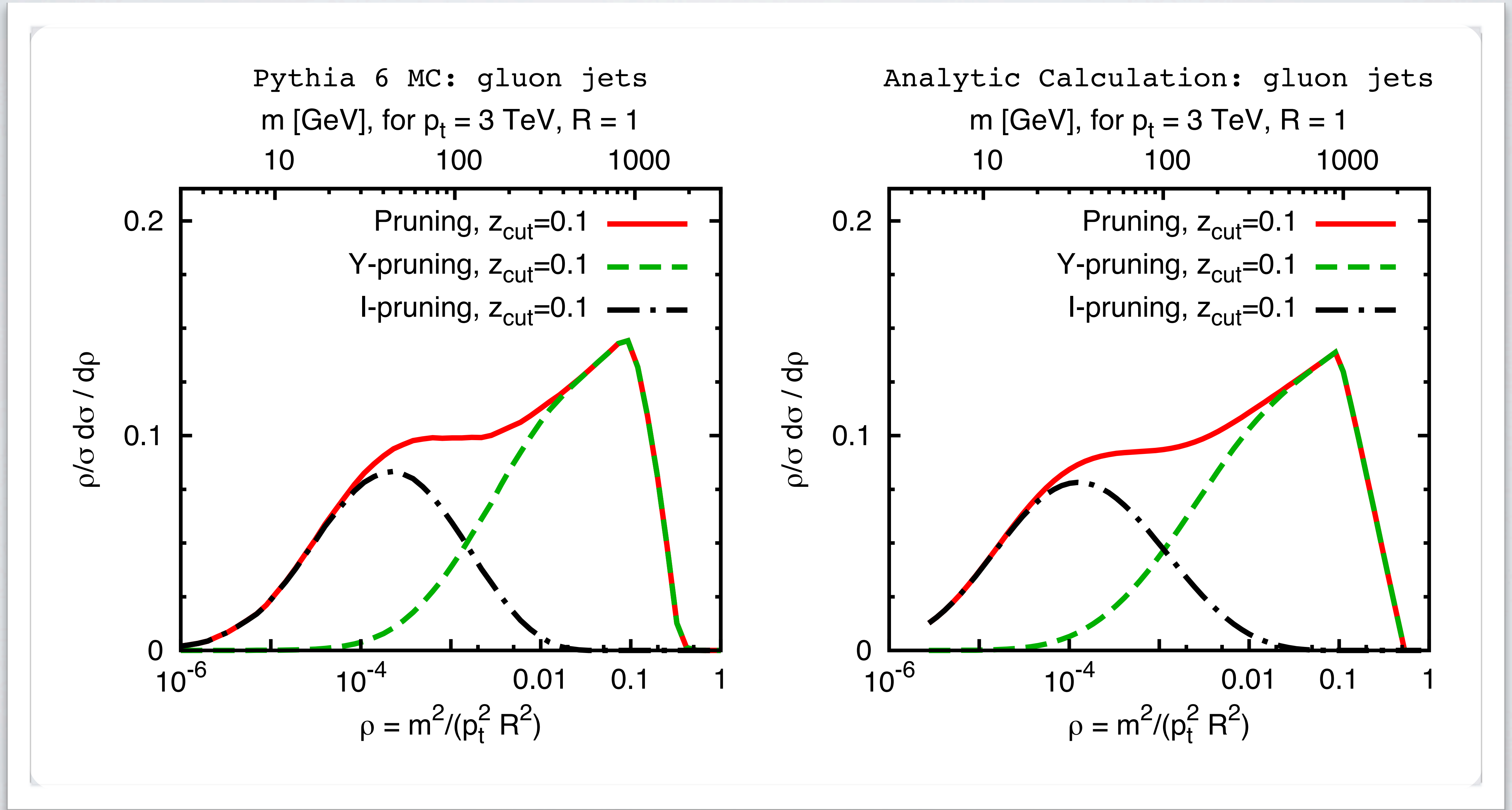

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Mass Drop Tagger at LO

1. Undo the last stage of the C/A clustering. Label the two subjets j_1 and j_2 ($m_1 > m_2$)
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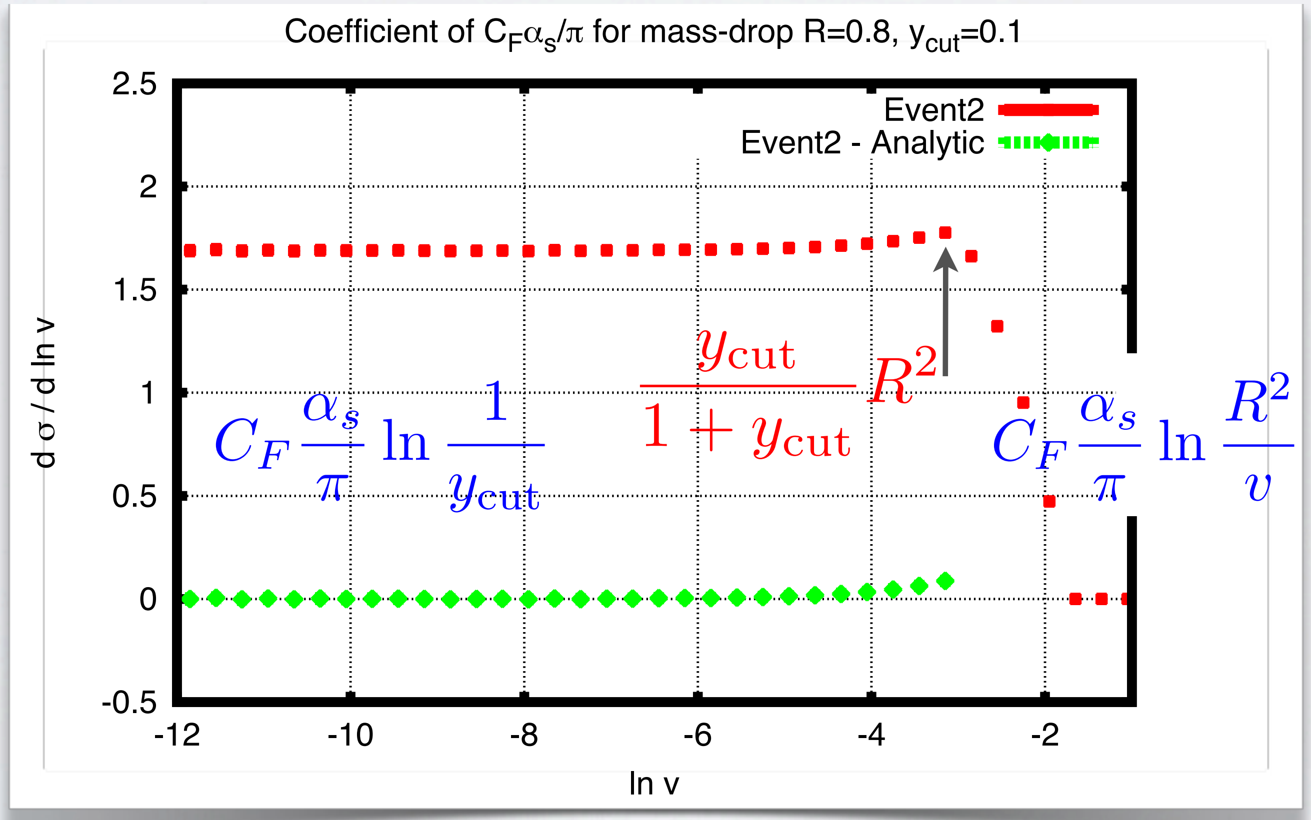
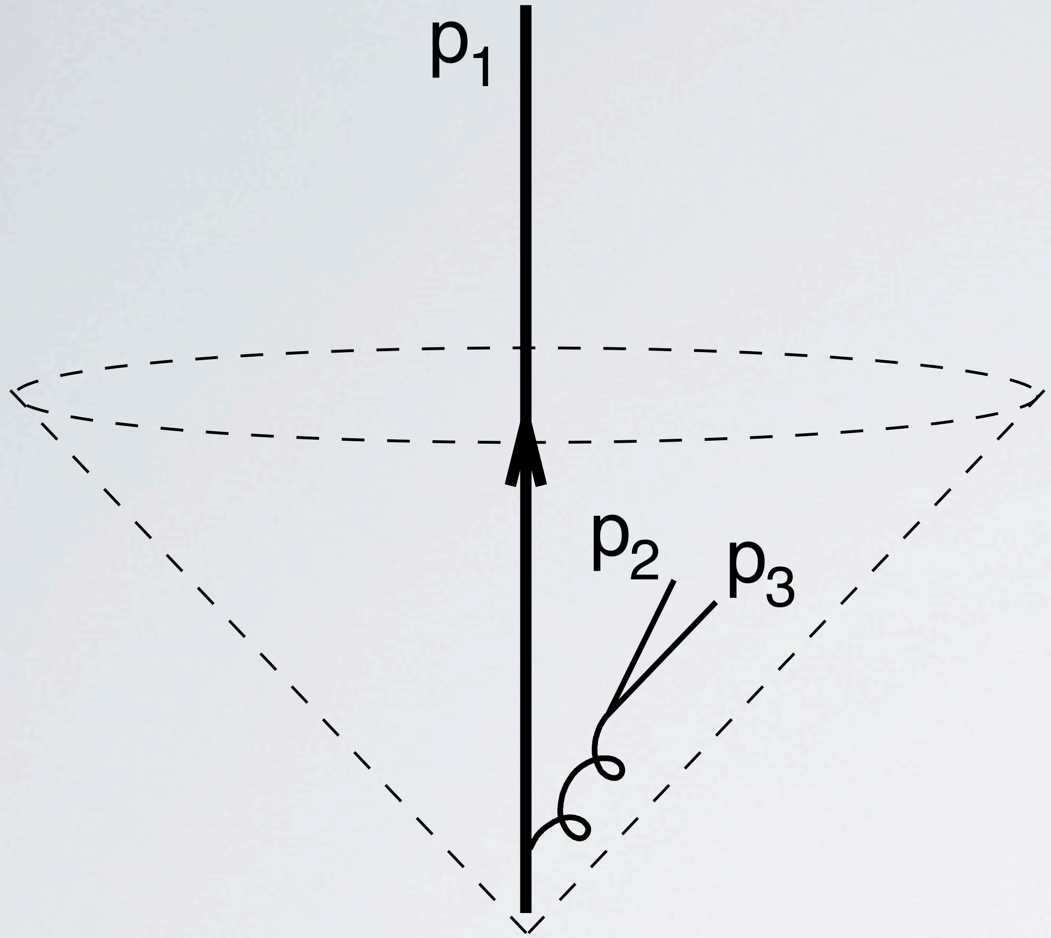
In the small-y_{cut} limit the result is identical to LO pruning: single-log distribution



Subtraction with hard collinear and finite y_c

Problems beyond LO



<u>What MDT does wrong:</u>

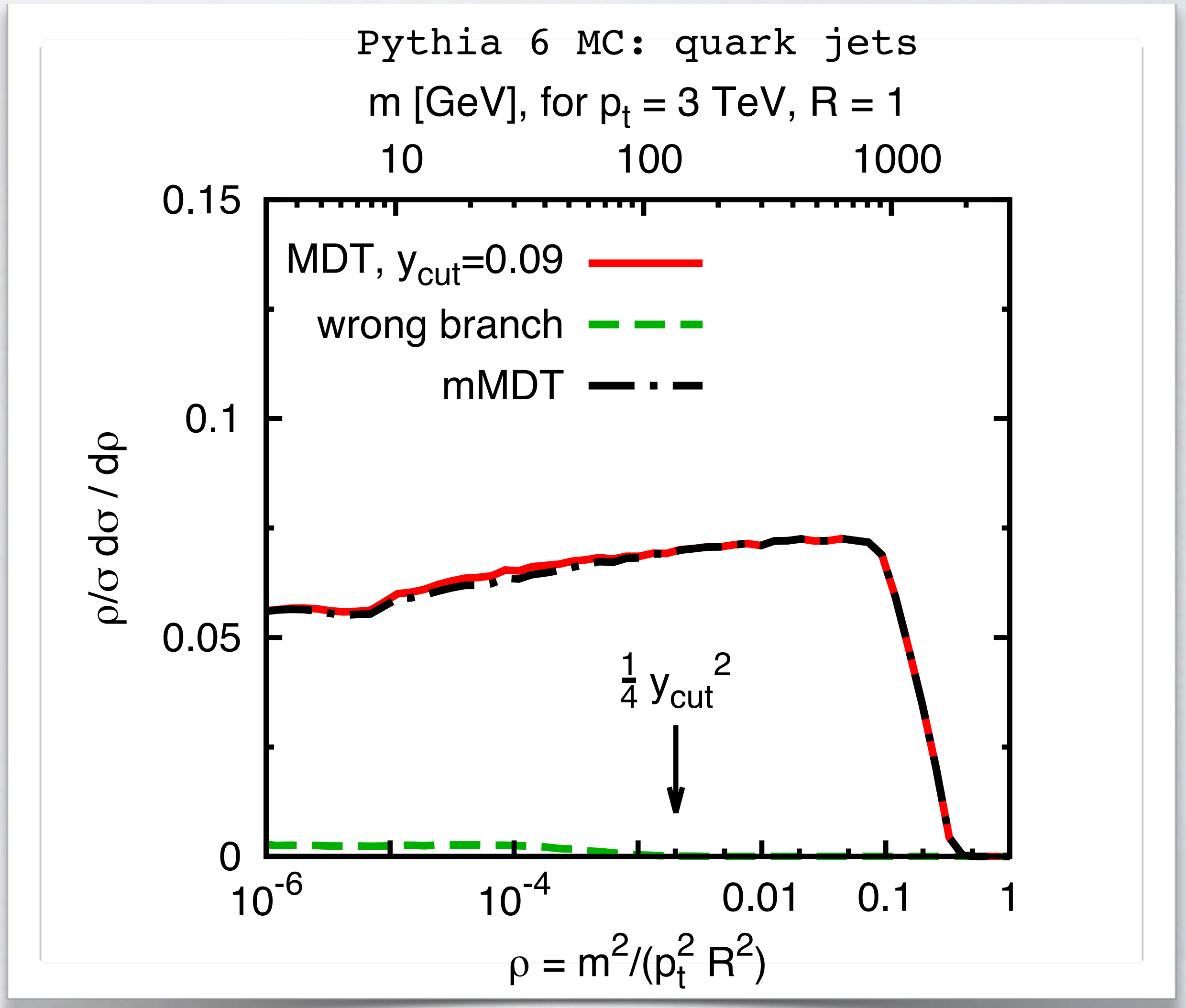
If the y_{ij} condition fails, MDT iterates on the more massive subjet. It can follow a soft branch ($p_2+p_3 < y_{cut}\ p_{tjet}$), when the "right" answer was that the (massless) hard branch had no substructure

- This can be considered a flaw of the tagger
- It worsens the logarithmic structure $\sim\alpha_s^2 L^3$
- It makes all-order treatment difficult
- It calls for a modification

Modified Mass Drop Tagger

1. Undo the last stage of the C/A clustering. Label the two subjets j_1 and j_2 ($m_1 > m_2$)
2. If $m_1 < \mu m$ (mass drop) and the splitting was not too asymmetric ($y_{ij} > y_{cut}$), tag the jet.
3. Otherwise redefine j to be the subjet with highest transverse mass and iterate.

- In practice the soft-branch contribution is very small
- However, this modification makes the all-order structure particularly interesting



All-order structure of mMDT

- The mMDT has single logs to all orders (i.e. $\sim\alpha_s^n L^n$)
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- Beyond that flavour mixing can happen (under control)

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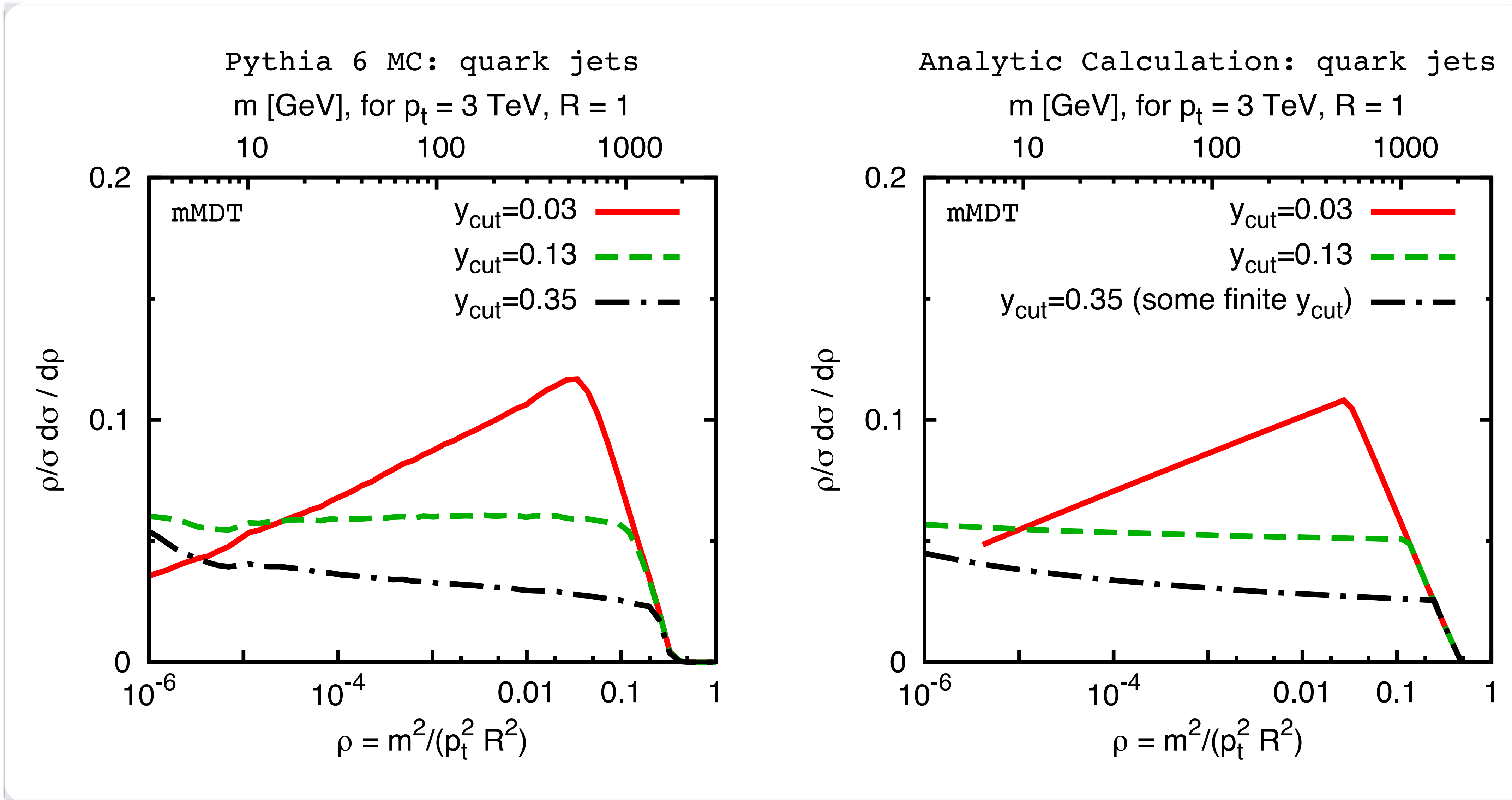

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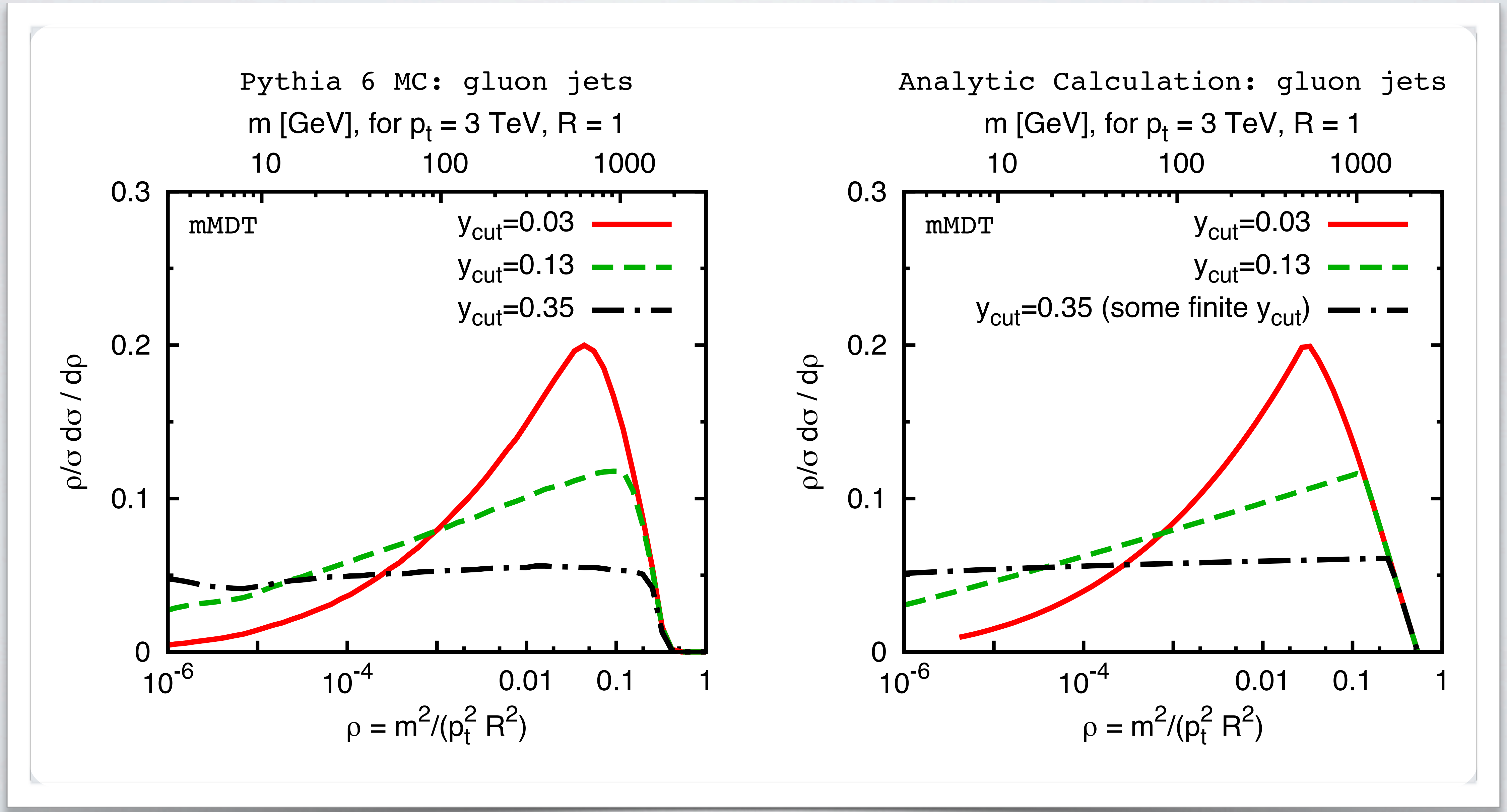
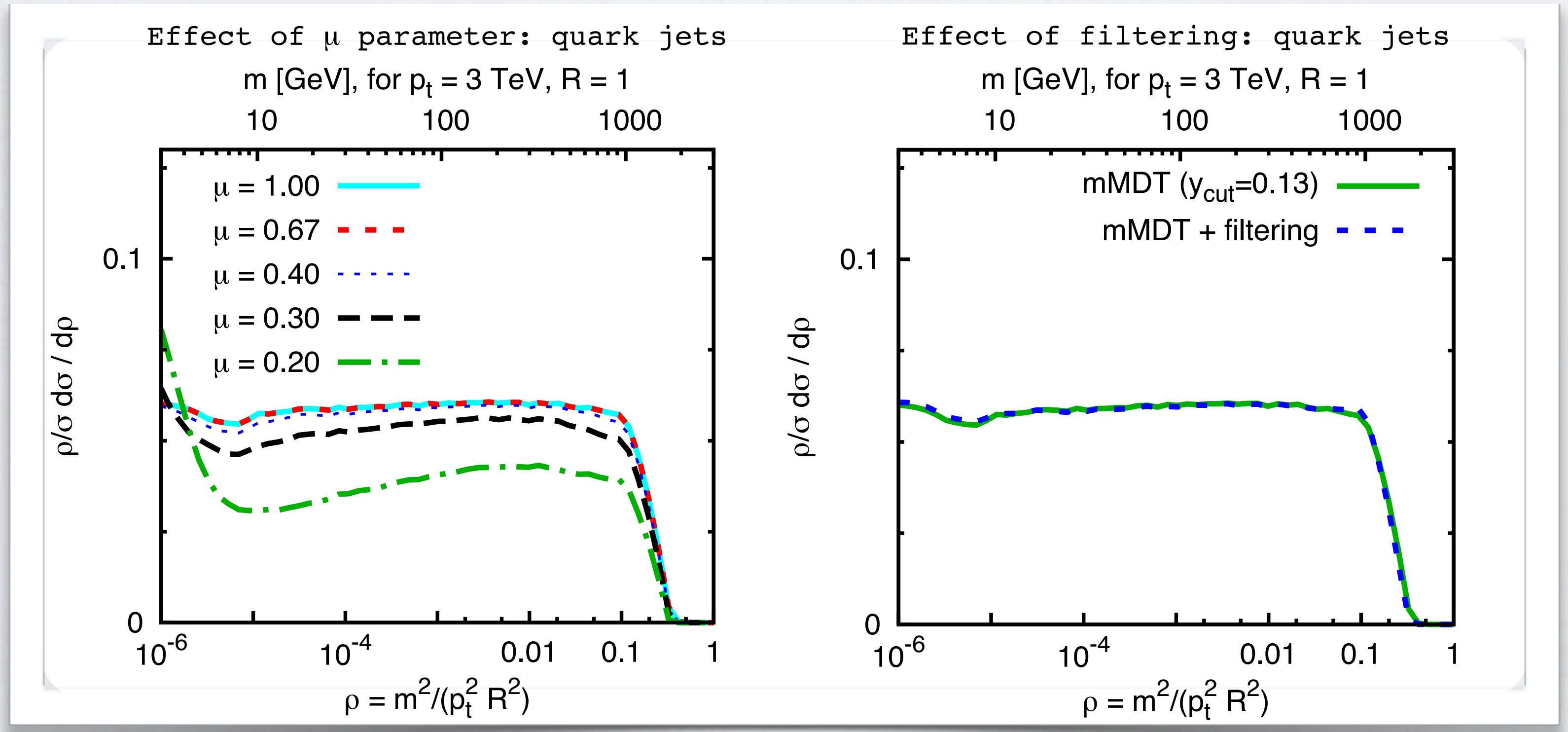


Properties of mMDT

- Flatness of the background is a desirable property (data-driven analysis, side bands)
- y_{cut} can be adjusted to obtain it (analytic relation)
- FO calculation might be applicable
- **Role of μ**, not mentioned so far
- It contributes to subleading logs and has small impact if not too small (μ>0.4)
- Filtering only affects subleading terms



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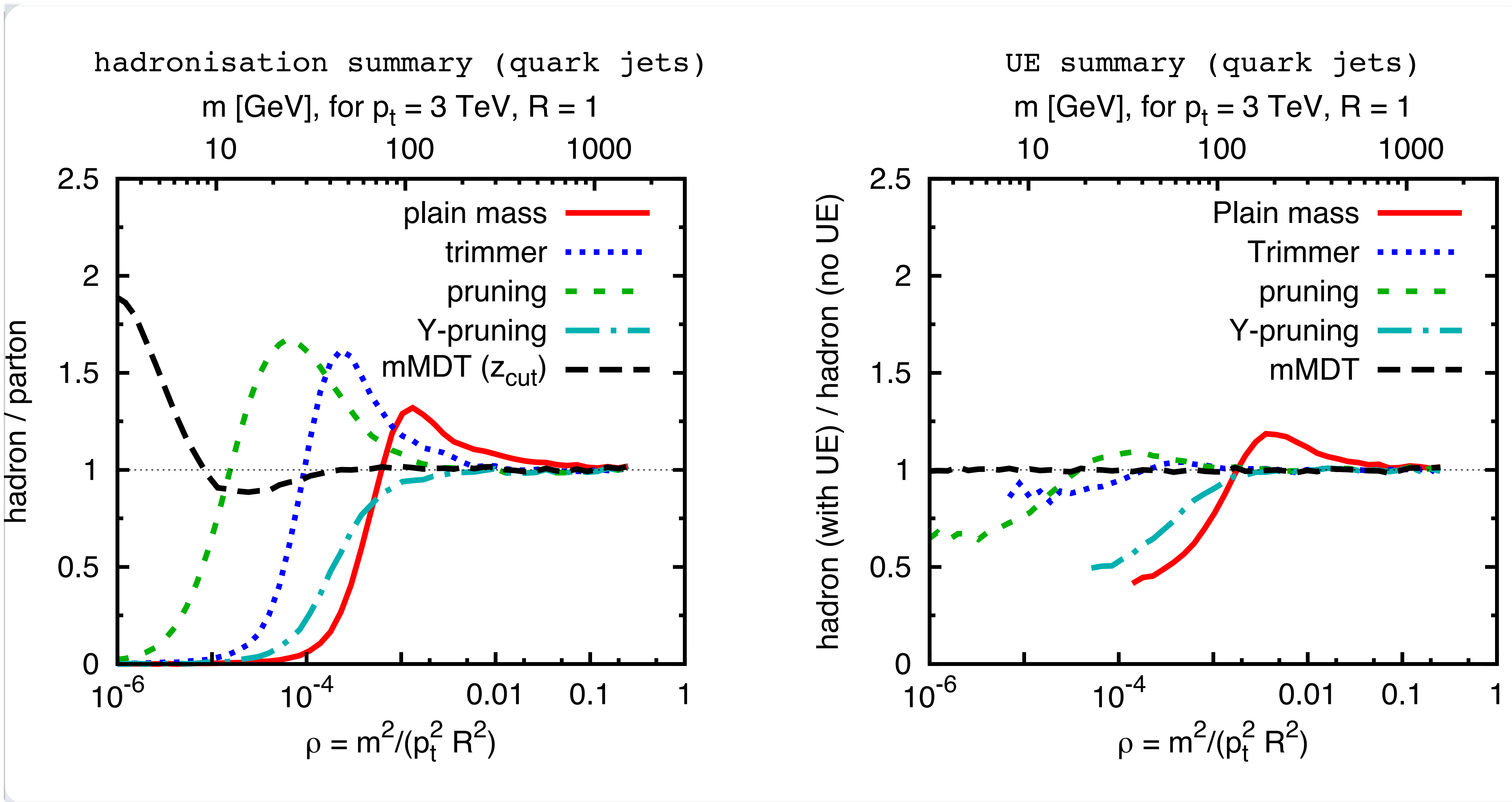
- It has only single logs, which are of collinear origin
- Important consequence:

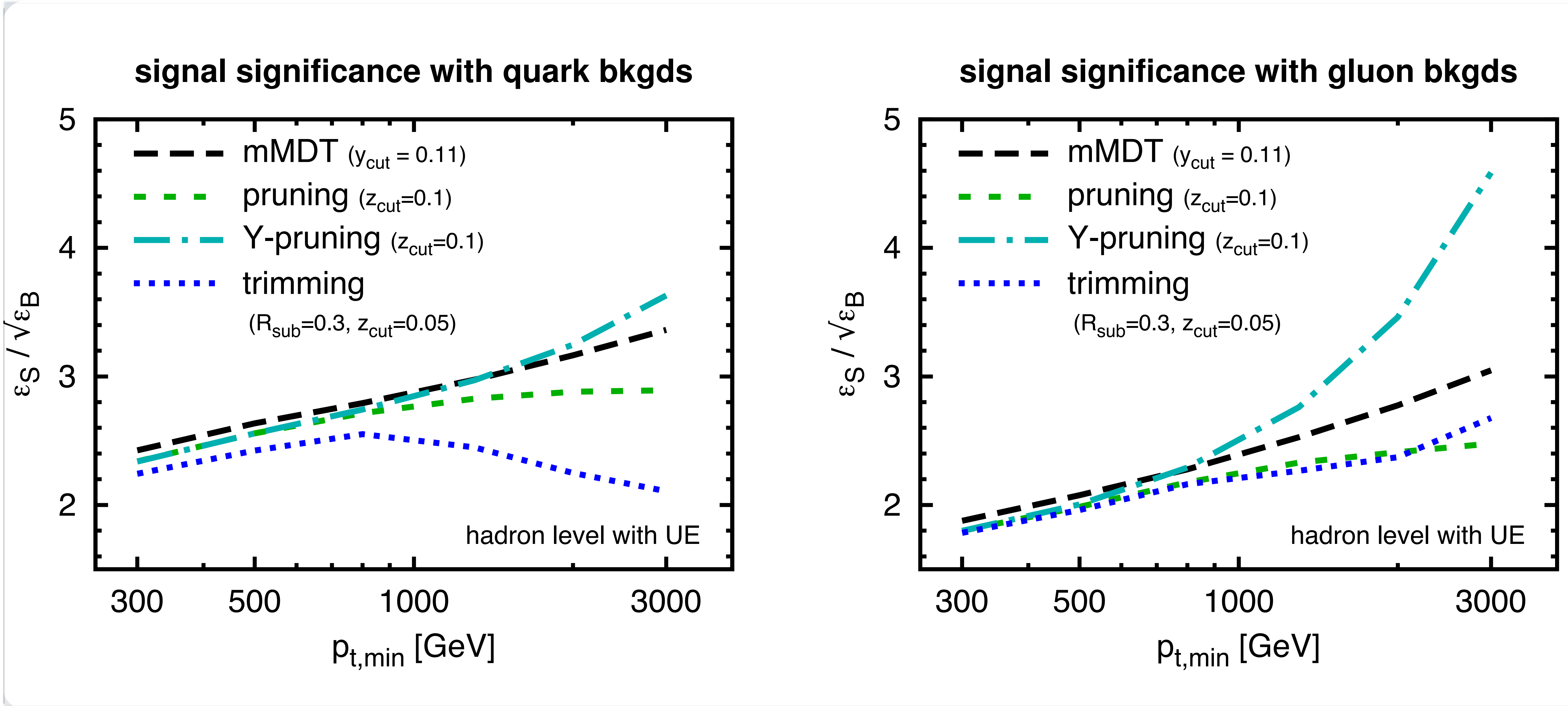
mMDT is FREE of non-global logs!

- Very small sensitivity to hadronisation and UE

Non-perturbative effects



Performances for finding signals (Ws)



Y-pruning gives a visible improvements
(but it is less calculable because sensitive to UE)

In summary ...

- Analytic studies of the taggers reveal their properties
- Particularly useful if MCs don't agree

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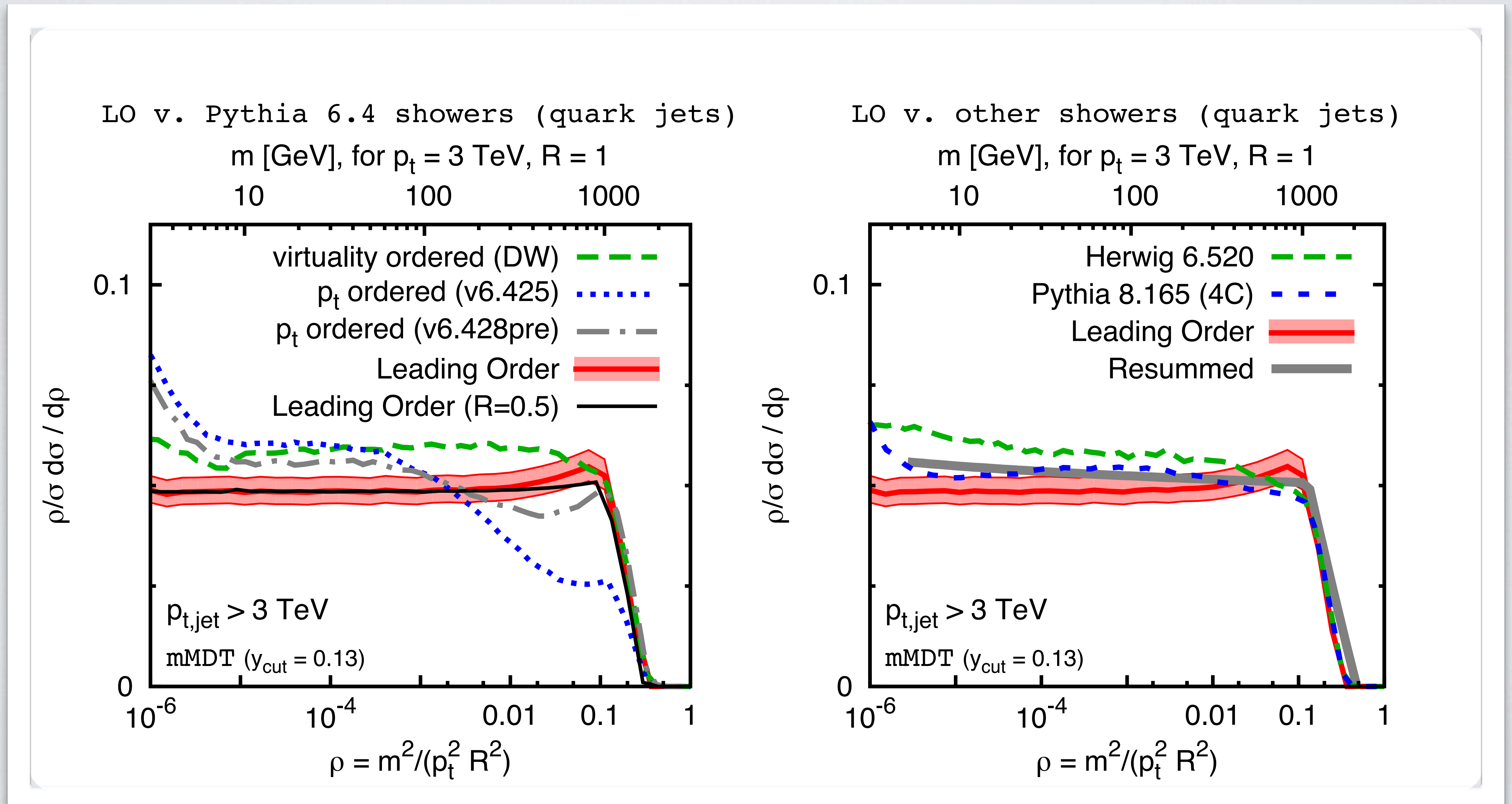

BACKUP SLIDES

Sequential recombination

- A large class of modern jet definitions is given by sequential recombination algorithms

- Starting with a list of particles, compute all distances d_{ij} and d_{iB}

- Find the minimum of all d_{ij} and d_{iB}

can be an external parameter (e.g. Jade algorithm), a distance from the beam ...

- If the minimum is a d_{ij}, recombine i and j and iterate

- Otherwise call i a final-state jet, remove it from the list and iterate

for a complete review see Salam, Towards jetography (2009)

Most common jet algorithms

Common choices for the distance are

$$d_{ij} = \min\left(p_{ti}^{2p}, p_{tj}^{2p}\right)\frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = p_{ti}^{2p}$$

$$\text{with} \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

p = 1 k_t algortihm
(Catani *et al.*, Ellis and Soper)
p = 0 Cambridge / Aachen
(Dokshitzer *et al.*, Wobish and Wengler)
p = -1 anti-k_t algorithm
(Cacciari, Salam, Soyez)

- Different algorithms serve different purposes
- Anti-k_t clusters around hard particles giving round jets (default choice for ATLAS and CMS)
- Anti-k_t is less useful for substructure studies, while C/A reflects QCD coherence