Resummation properties of MEPS@NLO

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Resummation properties of parton showers

$$\langle O\rangle^{\mathsf{PS}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\left[\Delta^{(\mathcal{K})}(t_c,\mu_Q^2)\, O(\Phi_B) + \int\limits_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \mathcal{K}(\Phi_1)\, \Delta^{(\mathcal{K})}(t,\mu_Q^2)\, O(\Phi_R)\right]$$

- splitting kernel $\mathcal{K}(\Phi_1) \propto \frac{\alpha_s}{t} P(z)$, $\Phi_1 = \{t,z,\phi\}$
- Sudakov form factor

$$\Delta^{(\mathcal{K})}(t,t') = \exp\left[-\int_t^{t'} \mathrm{d}\Phi_1\, \mathcal{K}(\Phi_1)\right] = \exp\left[c_1\alpha_s L^2 + c_2\alpha_s L + \ldots\right]$$

- parton shower starting scale μ_Q plays role of resummation scale, at LO commonly identified with μ_F to recover PDF evolution
- resummation in evolution variable t, c_1 correctly described, c_2 at most in $N_c \to \infty$ approximation
- $\alpha_s \to \alpha_s(k_\perp)$ catches dominant terms of higher log. order

$\Rightarrow$ **crucial in defining "parton shower accuracy"**

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Resummation properties of MEPS

$$\langle O \rangle^{\text{MEPS}}$$

$$= \int \mathrm{d}\Phi_n \, \mathrm{B}_n \left[\Delta_n^{(\mathcal{K})}(t_c, \mu_Q^2) \, O_n + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \, \mathcal{K}_n \, \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \, \Theta(Q_{\text{cut}} - Q) \, O_{n+1} \right]$$

$$+ \int \mathrm{d}\Phi_{n+1} \, \mathrm{B}_{n+1} \, \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \, \Theta(Q - Q_{\text{cut}})$$

$$\times \left[\Delta_{n+1}^{(\mathcal{K})}(t_c, t_{n+1}) \, O_{n+1} + \int_{t_c}^{t_{n+1}} \mathrm{d}\Phi_1 \, \mathcal{K}_{n+1} \, \Delta_{n+1}^{(\mathcal{K})}(t_{n+2}, t_{n+1}) \, O_{n+2} \right]$$

- LOPS for n-jet process restricted to region $Q < Q_{\text{cut}}$
- LOPS for $n+1$-jet process with additional Sudakov wrt. n-jet process
  $\rightarrow$ implements correct resummation behaviour wrt. incl. sample
- truncated showering to account for mismatch of t and Q Nason JHEP11(2004)040

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Resummation properties of MEPS

Two instructive ways of rewriting $\langle O \rangle^{\text{MEPS}}$:

1) composite emission kernel for ME and PS region

$$
\begin{aligned}
\langle O \rangle^{\text{MEPS}}
&= \int \mathrm{d}\Phi_n \, \mathrm{B}_n \Bigg[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2)\, O_n \\
&\qquad\qquad + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1 \left(\mathcal{K}_n \, \Theta(Q_{\text{cut}} - Q) + \frac{\mathrm{B}_{n+1}}{\mathrm{B}_n} \, \Theta(Q - Q_{\text{cut}}) \right) \\
&\qquad\qquad\qquad\qquad \times \; \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2)\, O_{n+1} \Bigg] \\
&\quad + \int \mathrm{d}\Phi_{n+1} \, \mathrm{B}_{n+1} \, \Theta(t_{n+1} - \mu_Q^2)\, O_{n+1}
\end{aligned}
$$

- α_s scales in $\mathrm{B}_n \cdot \mathcal{K}_n$ and B_{n+1} must be the same to retain resummation properties of the parton shower
- necessity to interprete B_{n+1} as PS splitting on top of B_n

Resummation properties of MEPS

Two instructive ways of rewriting $\langle O\rangle^{\text{MEPS}}$:

2) standard parton shower and correction terms

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$$+ \int \mathrm{d}\Phi_n \int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1\left[\mathrm{B}_{n+1} - \mathrm{B}_n\cdot\mathcal{K}_n\right]\Theta(Q-Q_{\text{cut}})\,\Delta_n^{(\mathcal{K})}(t_{n+1},\mu_Q^2)\,O_{n+1}$$

$$+ \int \mathrm{d}\Phi_{n+1}\,\mathrm{B}_{n+1}\,\Theta(t_{n+1}-\mu_Q^2)\,O_{n+1}$$

- α_s scales in $\mathrm{B}_n\cdot\mathcal{K}_n$ and B_{n+1} must be the same for second line not to spoil resummation properties of the parton shower (first line)
- necessity to interprete B_{n+1} as PS splitting on top of B
  $\rightarrow$ need to use inverse parton shower

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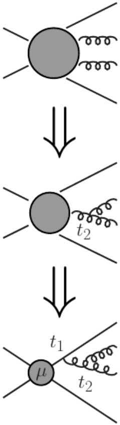
Scales:



$$\alpha_s^{k+n}(\mu_{\text{eff}}) = \alpha_s^k(\mu)\, \alpha_s(t_1) \cdots \alpha_s(t_n)$$

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mismatch of $\mathcal{O}(\frac{1}{N_c}\,\alpha_s\,L)$

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Resummation properties of SHERPA's MC@NLO

$$\langle O\rangle^{\text{MC@NLO}} = \int \mathrm{d}\Phi_B\, \bar{\mathrm{B}}^{(\mathsf{A})}(\Phi_B)\left[\Delta^{(\mathsf{A})}(t_0,\mu_Q^2)\,O(\Phi_B) + \int_{t_0}^{\mu_Q^2}\mathrm{d}\Phi_1\,\frac{\mathrm{D}^{(\mathsf{A})}(\Phi_B,\Phi_1)}{\mathrm{B}(\Phi_B)}\,\Delta^{(\mathsf{A})}(t,\mu_Q^2)\,O(\Phi_R)\right] + \int \mathrm{d}\Phi_R\left[\mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^{(\mathsf{A})}(\Phi_R)\right]O(\Phi_R)$$

- SHERPA: $\mathrm{D}_i^{(\mathsf{A})} = \mathrm{D}_i^{(\mathsf{S})}\,\Theta(\mu_Q^2 - t)$ ($N_c = 3$ CS kernels) as kernels
  → incorporation of subleading colour dipoles, spin-dependence
- Sudakov form factor

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- c_1 and c_2 correct in $N_c = 3$
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⇒ **improves accuracy of resummation for first emission**

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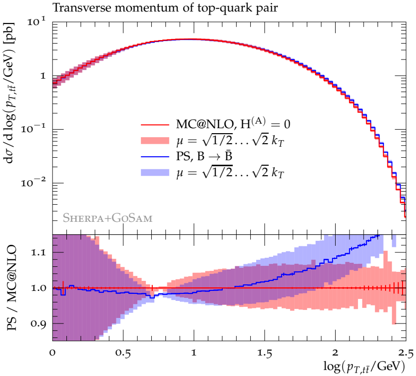

Importance of
$N_c = 3$ colour coherence vs.
$N_c \to \infty$ colour coherence

- small effect on standard (rapidity blind) observables, e.g. $p_{\perp,t\bar{t}}$ $\to$ some destructive interference at large $p_{\perp,t\bar{t}}$
- large effect on $A_{FB}(p_{\perp,t\bar{t}})$ $\to$ subleading colour terms lead to asymmetric radiation pattern

$$A_{\mathrm{FB}}(\mathcal{O}) = \frac{\frac{d\sigma_{t\bar{t}}}{d\mathcal{O}}\Big|_{\Delta y>0} - \frac{d\sigma_{t\bar{t}}}{d\mathcal{O}}\Big|_{\Delta y<0}}{\frac{d\sigma_{t\bar{t}}}{d\mathcal{O}}\Big|_{\Delta y>0} + \frac{d\sigma_{t\bar{t}}}{d\mathcal{O}}\Big|_{\Delta y<0}}$$

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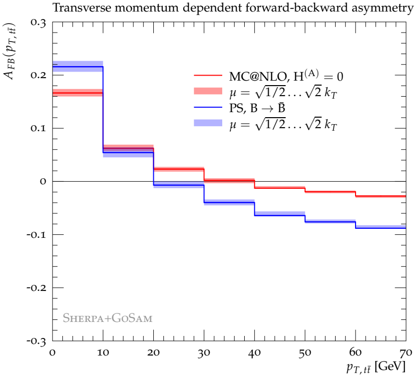

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$N_c \to \infty$ colour coherence

- small effect on standard (rapidity blind) observables, e.g. $p_{\perp,t\bar{t}}$
  $\to$ some destructive interference at large $p_{\perp,t\bar{t}}$
- large effect on $A_{FB}(p_{\perp,t\bar{t}})$
  $\to$ subleading colour terms lead to asymmetric radiation pattern

$$A_{\text{FB}}(\mathcal{O}) = \frac{\frac{d\sigma_{t\bar{t}}}{d\mathcal{O}}\big|_{\Delta y>0} - \frac{d\sigma_{t\bar{t}}}{d\mathcal{O}}\big|_{\Delta y<0}}{\frac{d\sigma_{t\bar{t}}}{d\mathcal{O}}\big|_{\Delta y>0} + \frac{d\sigma_{t\bar{t}}}{d\mathcal{O}}\big|_{\Delta y<0}}$$

Resummation properties of MePs@Nlo

Höche, Krauss, MS, Siegert JHEP04(2013)027

Consider a $2 \to n$ process and an observable senstive to (at least) $j = n + k$ jets according to measure some jet measure $Q_{n+k+i} = Q(\Phi_{n+k+i})$

$$\langle O \rangle_j = \sum_{i=0}^{\infty} \langle O_{j+i} \rangle = \sum_{i=0}^{\infty} \langle O_{n+k+i} \rangle \,,$$

such that

$$\langle O \rangle_j^{\text{excl}} = \sum_{i=0}^{\infty} \langle O_{j+i} \, \Theta(Q_j - Q_{\text{cut}}) \, \Theta(Q_{\text{cut}} - Q_{j+1}) \rangle \,,$$

$$\langle O \rangle_j^{\text{incl}} = \sum_{i=0}^{\infty} \langle O_{j+i} \, \Theta(Q_j - Q_{\text{cut}}) \rangle \,.$$

Aim: a) the $(n + k + i)$ jet state is NLO correct
b) log. structure of the parton shower wrt. $2 \to n$ process is guaranteed
$\to$ include truncated showering to accommodate arbitrary Q and t

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$\to$ include truncated showering to accommodate arbitrary Q and t

Resummation properties of MEPS@NLO

Define

$$\langle O\rangle_{n+k}^{\text{excl}} = \int \mathrm{d}\Phi_{n+k}\,\Theta(Q_{n+k}-Q_{\text{cut}})\,\tilde{\mathrm{B}}_{n+k}^{(\mathsf{A})} \times \Bigg[\tilde{\Delta}_{n+k}^{(\mathsf{A})}(t_c,\mu_Q^2)\,O_{n+k} + \int_{t_c}^{t_{n+k}} \mathrm{d}\Phi_1\,\frac{\tilde{\mathrm{D}}_{n+k}^{(\mathsf{A})}}{\mathrm{B}_{n+k}}\,\tilde{\Delta}_{n+k}^{(\mathsf{A})}(t_{n+k+1},\mu_Q^2) \times \Theta(Q_{\text{cut}}-Q_{n+k+1})\;O_{n+k+1}\Bigg]$$
$$+\int \mathrm{d}\Phi_{n+k+1}\,\Theta(Q_{n+k}-Q_{\text{cut}})\,\tilde{\mathrm{H}}_{n+k}^{(\mathsf{A})}\,\tilde{\Delta}_{n+k}^{(\mathrm{PS})}(t_{n+k+1},\mu_Q^2;>Q_{\text{cut}}) \times \Theta(Q_{\text{cut}}-Q_{n+k+1})\;O_{n+k+1}$$

$\Rightarrow$ **structurally similar to MC@NLO**

emission terms restricted from above by Q_{cut}, as in MEPS

twiddled quantities incorporate truncated emissions

Resummation properties of MEPS@NLO

Truncated shower emission manifest in compound emission kernel

$$\tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}(\bar{\Phi}_1) \;=\; \mathrm{D}^{(\mathrm{A})}_{n+k}\,\Theta(t_{n+k}-\bar{t}) \;+\; \mathrm{B}_{n+k}\sum_{i=n}^{n+k-1}\mathrm{K}_i\,\Theta(t_i-\bar{t})\,\Theta(\bar{t}-t_{i+1})$$

⇒ kernels for PS emissions in between reconstructed scales t_i starting at core process $2\to n$ up to $2\to n+k$

⇒ hierarchy $\mu_Q^2 = t_n > t_{n+1} > \ldots t_{n+k-1} > t_{n+k}$ implicit

⇒ $n+k+1$-th emission by $\tilde{\mathrm{D}}^{(\mathrm{A})}$, intermediate emissions by PS

Compound Sudakov form factor

$$\tilde{\Delta}^{(\mathrm{A})}_{n+k}(t,t') = \exp\left\{-\int_t^{t'}\mathrm{d}\bar{\Phi}\,\frac{\tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}}{\mathrm{B}_{n+k}}\right\} = \Delta^{(\mathrm{A})}_{n+k}(t,t')\prod_{i=n}^{n+k-1}\Delta^{(\mathrm{PS})}_i(t,t')$$

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$$\Delta^{(\mathsf{PS})}(t,t') \;=\; \Delta^{(\mathsf{PS})}(t,t';{<}Q_{\mathrm{cut}})\,\Delta^{(\mathsf{PS})}(t,t';{>}Q_{\mathrm{cut}})$$

Resummation properties of MEPS@NLO

Compound emission kernels require modified $\bar{\mathrm{B}}$-functions

$$\begin{aligned}\tilde{\mathrm{B}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k}) \;=&\; \bar{\mathrm{B}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k}) \\ &+ \mathrm{B}_{n+k}(\Phi_{n+k}) \sum_{i=n}^{n+k-1} \int \mathrm{d}\Phi_1\, \mathrm{K}_i(\Phi_1)\, \Theta(t_i - t_{n+k+1})\, \Theta(t_{n+k+1} - t_{i+1})\end{aligned}$$

and, similarly, H-functions

$$\begin{aligned}\tilde{\mathrm{H}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1}) \;=&\; \mathrm{H}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1}) \\ &- \mathrm{B}_{n+k}(\Phi_{n+k}) \sum_{i=n}^{n+k-1} \mathrm{K}_i(\Phi_1)\, \Theta(t_i - t_{n+k+1})\, \Theta(t_{n+k+1} - t_{i+1})\end{aligned}$$

Necessary, because truncated shower emissions enter at $\mathcal{O}(\alpha_s)$.

Resummation properties of MePs@Nlo

Compound emission kernels require modified $\bar{\mathrm{B}}$-functions

$$\tilde{\mathrm{B}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k}) = \bar{\mathrm{B}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k}) + \mathrm{B}_{n+k}(\Phi_{n+k}) \sum_{i=n}^{n+k-1} \int \mathrm{d}\Phi_1 \, \mathrm{K}_i(\Phi_1)\, \Theta(t_i - t_{n+k+1})\, \Theta(t_{n+k+1} - t_{i+1})$$

and, similarly, H-functions

$$\tilde{\mathrm{H}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1}) = \mathrm{H}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1}) - \mathrm{B}_{n+k}(\Phi_{n+k}) \sum_{i=n}^{n+k-1} \mathrm{K}_i(\Phi_1)\, \Theta(t_i - t_{n+k+1})\, \Theta(t_{n+k+1} - t_{i+1})$$

Necessary, because truncated shower emissions enter at $\mathcal{O}(\alpha_s)$.

Resummation properties of MEPS@NLO

This leads to

$$
\begin{aligned}
\langle O\rangle^{\text{excl}}_{n+k} = \int \mathrm{d}\Phi_{n+k}\,\Theta(Q_{n+k}-Q_{\text{cut}})\,\tilde{\mathrm{B}}^{(\mathsf{A})}_{n+k}\,\tilde{\Delta}^{(\mathsf{A})}_{n+k}(t_c,\mu_Q^2;{>}Q_{\text{cut}})\\
\times\Bigg[\tilde{\Delta}^{(\mathsf{A})}_{n+k}(t_c,\mu_Q^2;{<}Q_{\text{cut}})\,O_{n+k}\\
+\int_{t_c}^{t_{n+k}}\mathrm{d}\Phi_1\,\frac{\tilde{\mathrm{D}}^{(\mathsf{A})}_{n+k}}{\mathrm{B}_{n+k}}\,\tilde{\Delta}^{(\mathsf{A})}_{n+k}(t_{n+k+1},\mu_Q^2;{<}Q_{\text{cut}})\\
\times\;\Theta(Q_{\text{cut}}-Q_{n+k+1})\;O_{n+k+1}\Bigg]\\
+\int \mathrm{d}\Phi_{n+k+1}\,\Theta(Q_{n+k}-Q_{\text{cut}})\,\tilde{\mathrm{H}}^{(\mathsf{A})}_{n+k}\,\tilde{\Delta}^{(\mathrm{PS})}_{n+k}(t_{n+k+1},\mu_Q^2;{>}Q_{\text{cut}})\\
\times\;\Theta(Q_{\text{cut}}-Q_{n+k+1})\;O_{n+k+1}
\end{aligned}
$$

$\Rightarrow$ Sudakovs for veto on $Q > Q_{\text{cut}}$ explicit
$\tilde{\mathrm{B}}^{(\mathsf{A})}$, $\tilde{\mathrm{H}}^{(\mathsf{A})}$ contain correct subtraction terms for correct $\mathcal{O}(\alpha_s)$ terms

Resummation properties of MePs@Nlo

- proof on NLO accuracy proceeds as for Mc@Nlo, $\mathcal{O}(\alpha_s)$ terms of compound Sudakovs cancel with additional terms in $\tilde{\mathrm{B}}$ and $\tilde{\mathrm{H}}$

- proof of logarithmic accuracy

  1. emissions below Q_{cut} on $(n+k)$ jet configuration generated by $\Delta^{(\mathrm{A})}_{n+k}(t_{n+k+1}, t_{n+k})$
  2. emissions above Q_{cut} are in the $(n+k+1)$ jet configuration, they may not generate unwanted logarithmic corrections wrt. $(n+k)$ jet configuration
  3. iterate 1 and 2 for $(n+k+1)$ jet configuration

- for 1 and 2 rewrite

$$\begin{aligned}\langle O\rangle^{\text{incl}}_{n+k} &= \langle O\rangle^{\text{excl}}_{n+k} + \langle O\rangle^{\text{excl}}_{n+k+1} + \ldots \\ &= \langle O\rangle^{\text{MC@NLO}}_{n+k} + \langle O\rangle^{\text{corr}}_{n+k}\end{aligned}$$

Resummation properties of MEPS@NLO

- proof on NLO accuracy proceeds as for MC@NLO, $\mathcal{O}(\alpha_s)$ terms of compound Sudakovs cancel with additional terms in $\tilde{\mathrm{B}}$ and $\tilde{\mathrm{H}}$

- proof of logarithmic accuracy
  1. emissions below Q_{cut} on $(n+k)$ jet configuration generated by $\Delta^{(\mathrm{A})}_{n+k}(t_{n+k+1}, t_{n+k})$
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Resummation properties of MEPS@NLO

$$\begin{aligned}\langle O\rangle_{n+k}^{\mathrm{MC@NLO}} &= \int \mathrm{d}\Phi_{n+k}\,\Theta(Q_{n+k}-Q_{\mathrm{cut}})\,\tilde{\mathrm{B}}_{n+k}^{(\mathsf{A})}\\ &\quad\times\left[\tilde{\Delta}_{n+k}^{(\mathsf{A})}(t_c,\mu_Q^2)\,O_{n+k}+\int\limits_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1\,\frac{\tilde{\mathrm{D}}_{n+k}^{(\mathsf{A})}}{\mathrm{B}_{n+k}}\,\tilde{\Delta}_{n+k}^{(\mathsf{A})}(t,\mu_Q^2)\,O_{n+k+1}\right]\\ &\quad+\int \mathrm{d}\Phi_{n+k+1}\,\Theta(Q_{n+k}-Q_{\mathrm{cut}})\,\tilde{\mathrm{H}}_{n+k}^{(\mathsf{A})}\,\tilde{\Delta}_{n+k}^{(\mathrm{PS})}(t_{n+k+1},\mu_Q^2;>Q_{\mathrm{cut}})\,O_{n+k+1}\end{aligned}$$

MC@NLO expression with

- additional Sudakov form factors to correctly resum all scale hierarchies $\mu_Q^2 = t_n > t_{n+1} > \ldots t_{n+k-1} > t_{n+k}$
- truncated shower terms allowing for emissions off intermediate legs

⇒ **has exactly the right resummation behaviour**

Resummation properties of MEPS@NLO

$$
\begin{aligned}
&\langle O\rangle^{\text{corr}}_{n+k}\\
&\quad= \int \mathrm{d}\Phi_{n+k+1}\;\Theta(Q_{n+k+1}-Q_{\text{cut}})\;\tilde{\Delta}^{(\text{PS})}_{n+k+1}(t_c,\mu_Q^2)\;O_{n+k+1}\\
&\qquad\times\;\Bigg\{\tilde{\mathrm{D}}^{(\text{A})}_{n+k}\left[1-\frac{\tilde{\mathrm{B}}^{(\mathsf{A})}_{n+k}}{\mathrm{B}_{n+k}}\,\frac{\Delta^{(\mathsf{A})}_{n+k}(t_{n+k+1},t_{n+k})}{\Delta^{(\mathsf{PS})}_{n+k}(t_{n+k+1},t_{n+k})}\right]\\
&\qquad\qquad\qquad-\mathrm{B}_{n+k+1}\left[1-\frac{\tilde{\mathrm{B}}^{(\mathsf{A})}_{n+k+1}}{\mathrm{B}_{n+k+1}}\,\frac{\Delta^{(\mathsf{A})}_{n+k+1}(t_c,t_{n+k+1})}{\Delta^{(\mathsf{PS})}_{n+k+1}(t_c,t_{n+k+1})}\right]+\mathcal{O}(\alpha_s^2)\Bigg\}
\end{aligned}
$$

- Similarly to MEPS case, this is the difference between the $(n+k+1)$ parton state and the emission kernel off the $(n+k)$ parton state

$\Rightarrow$ **this term is of $\mathcal{O}(\frac{1}{N_C}\,\alpha_s^2 L^3)$ relative to the $(n+k)$ parton process**

Resummation properties of MEPS@NLO

$$
\begin{aligned}
\langle O\rangle^{\text{corr}}_{n+k}
&= \int \mathrm{d}\Phi_{n+k+1}\,\Theta(Q_{n+k+1}-Q_{\text{cut}})\,\tilde{\Delta}^{(\text{PS})}_{n+k+1}(t_c,\mu_Q^2)\,O_{n+k+1}\\
&\quad\times\left\{\tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}\left[1-\frac{\tilde{\mathrm{B}}^{(\mathsf{A})}_{n+k}}{\mathrm{B}_{n+k}}\,\frac{\Delta^{(\mathsf{A})}_{n+k}(t_{n+k+1},t_{n+k})}{\Delta^{(\mathsf{PS})}_{n+k}(t_{n+k+1},t_{n+k})}\right]\right.\\
&\qquad\left.-\,\mathrm{B}_{n+k+1}\left[1-\frac{\tilde{\mathrm{B}}^{(\mathsf{A})}_{n+k+1}}{\mathrm{B}_{n+k+1}}\,\frac{\Delta^{(\mathsf{A})}_{n+k+1}(t_c,t_{n+k+1})}{\Delta^{(\mathsf{PS})}_{n+k+1}(t_c,t_{n+k+1})}\right]+\mathcal{O}(\alpha_s^2)\right\}
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$$

identical at $\mathcal{O}(\alpha_s L^2)$ and $\mathcal{O}(\alpha_s L)$
no $\mathcal{O}(\alpha_s^2)$ terms

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Resummation properties of MEPS@NLO

$$
\begin{aligned}
\langle O\rangle^{\rm corr}_{n+k}
&= \int \mathrm{d}\Phi_{n+k+1}\,\Theta(Q_{n+k+1}-Q_{\rm cut})\,\tilde\Delta^{\rm (PS)}_{n+k+1}(t_c,\mu_Q^2)\,O_{n+k+1}\\
&\times\left\{\tilde{\rm D}^{\rm (A)}_{n+k}\left[1-\frac{\tilde{\rm B}^{\rm (A)}_{n+k}}{{\rm B}_{n+k}}\,\frac{\Delta^{\rm (A)}_{n+k}(t_{n+k+1},t_{n+k})}{\Delta^{\rm (PS)}_{n+k}(t_{n+k+1},t_{n+k})}\right]\right.\\
&\left.\qquad-{\rm B}_{n+k+1}\left[1-\frac{\tilde{\rm B}^{\rm (A)}_{n+k+1}}{{\rm B}_{n+k+1}}\,\frac{\Delta^{\rm (A)}_{n+k+1}(t_c,t_{n+k+1})}{\Delta^{\rm (PS)}_{n+k+1}(t_c,t_{n+k+1})}\right]+\mathcal{O}(\alpha_s^2)\right\}
\end{aligned}
$$

mismatch at $\mathcal{O}(\frac{1}{N_C}\,\alpha_s L)$

identical at $\mathcal{O}(\alpha_s L^2)$ and $\mathcal{O}(\alpha_s L)$
no $\mathcal{O}(\alpha_s^2)$ terms

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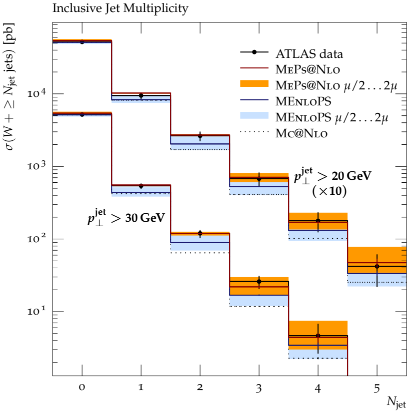
Results – pp → W+jets

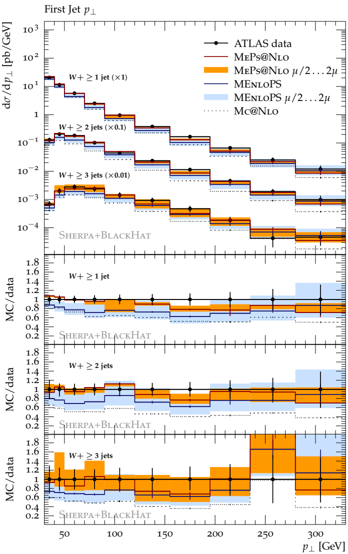


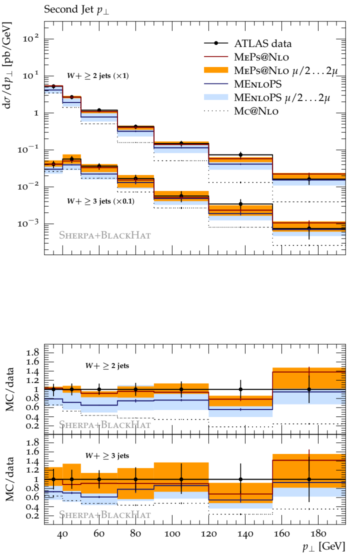
$pp \to W$+jets (0,1,2 @ NLO; 3,4 @ LO)

- $\mu_{R/F} \in [\frac{1}{2}, 2]\,\mu_{\text{def}}$ scale uncertainty much reduced
- NLO dependence for $pp \to W$+0,1,2 jets LO dependence for $pp \to W$+3,4 jets
- $Q_{\text{cut}} = 30$ GeV
- good data description

ATLAS data Phys.Rev.D85(2012)092002

Results – pp → W+jets





Results – pp → W+jets

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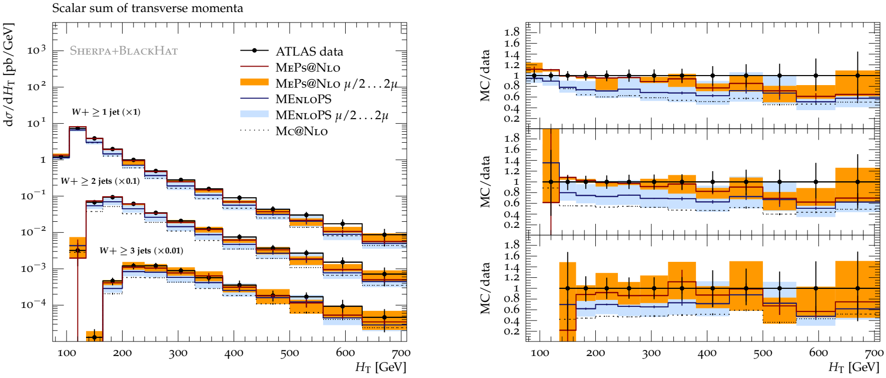
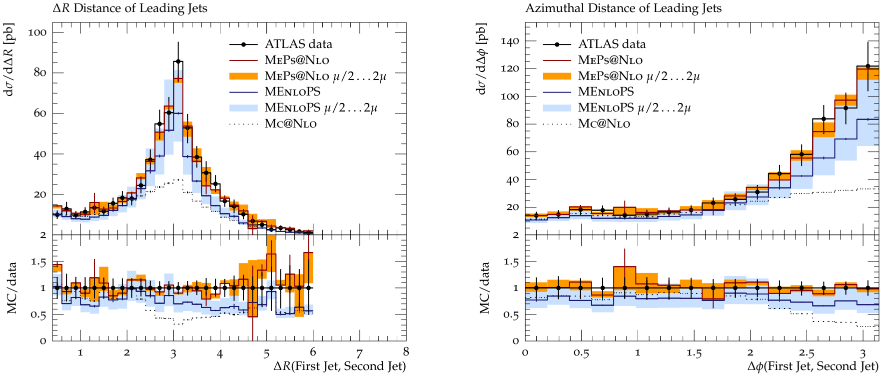


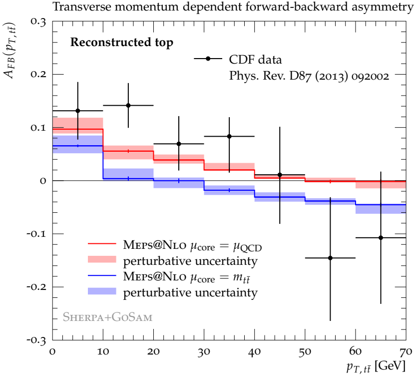
Results – pp → W+jets

ATLAS data Phys.Rev.D85(2012)092002



Results – $p\bar{p} \to t\bar{t} + \text{jets}$

CDF data Phys.Rev.D87(2013)092002



$p\bar{p} \to t\bar{t}$+jets (0,1 @ NLO)

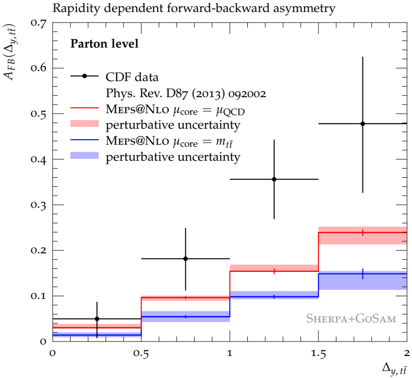
S. Höche, J. Huang, G. Luisoni, MS, J. Winter

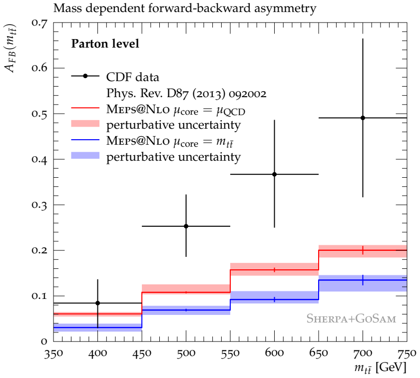
arXiv:1306.2703

- $A_{FB}(p_{\perp,t\bar{t}})$ NLO accurate in all but the first bin
- reconstructed tops
- no EW corrections

Results – $p\bar{p} \to t\bar{t} + \text{jets}$

CDF data Phys.Rev.D87(2013)092002





- parton level
- no EW corrections ($\approx 20\%$) effected
- right qualitative bahviour, but consistently below data

Conclusions

- parton showers have a well defined "parton shower accuracy"
  $\rightarrow$ more than just leading log
- merging methods must recover that accuracy to improve upon pure PS

- multijet merging at NLO proceeds schematically as at LO
  $\rightarrow$ truncated vetoed showers integrate smoothly into formalism
- scale setting essential for recovering PS resummation

- NLOPS matching is LO+(N)LL matching
- can be improved by adding higher order calculations
  - (N)NLL resummation
  - NNLO corrections

current release SHERPA-2.0.β_2, when fully tuned SHERPA-2.0.0

http://sherpa.hepforge.org

Thank you for your attention!

MEPS@NLO – Multijet merging at NLO

Höche, Krauss, MS, Siegert JHEP04(2013)027
Gehrmann, Höche, Krauss, MS, Siegert JHEP01(2013)144

$$\langle O\rangle^{\text{MEPS@NLO}}$$

$$
\begin{aligned}
&= \int \mathrm{d}\Phi_n\, \bar{\mathrm{B}}_n^{(\mathsf{A})}\left[\Delta_n^{(\mathsf{A})}(t_0,\mu_Q^2)\,O_n \right.\\
&\qquad\left. + \int_{t_0}^{\mu_Q^2}\mathrm{d}\Phi_1\,\frac{\mathrm{D}_n^{(\mathsf{A})}}{\mathrm{B}_n}\,\Delta_n^{(\mathsf{A})}(t_{n+1},\mu_Q^2)\,\Theta(Q_{\text{cut}}-Q)\,O_{n+1}\right]\\
&\quad + \int \mathrm{d}\Phi_{n+1}\left[\mathrm{R}_n-\mathrm{D}_n^{(\mathsf{A})}\right]\Theta(Q_{\text{cut}}-Q)\,O_{n+1}\\
&\quad + \int \mathrm{d}\Phi_{n+1}\,\bar{\mathrm{B}}_{n+1}^{(\mathsf{A})}\left[1+\frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}}\int_{t_{n+1}}^{\mu_Q^2}\mathrm{d}\Phi_1\,\mathrm{K}_n\right]\Delta_n^{(\mathsf{A})}(t_{n+1},\mu_Q^2)\,\Theta(Q-Q_{\text{cut}})\\
&\qquad\times\left[\Delta_{n+1}^{(\mathsf{A})}(t_0,t_{n+1})\,O_{n+1}+\int_{t_0}^{t_{n+1}}\mathrm{d}\Phi_1\,\frac{\mathrm{D}_{n+1}^{(\mathsf{A})}}{\mathrm{B}_{n+1}}\,\Delta_{n+1}^{(\mathsf{A})}(t_{n+2},t_{n+1})\,O_{n+2}\right]\\
&\quad + \int \mathrm{d}\Phi_{n+2}\left[\mathrm{R}_{n+1}-\mathrm{D}_{n+1}^{(\mathsf{A})}\right]\Delta_n^{(\mathsf{A})}(t_{n+1},\mu_Q^2)\,\Theta(Q-Q_{\text{cut}})\,O_{n+2}
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&\quad + \int \mathrm{d}\Phi_{n+1} \left[\mathrm{R}_n - \mathrm{D}_n^{(\mathsf{A})}\right] \Theta(Q_{\text{cut}}-Q)\, O_{n+1}
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&\quad + \int \mathrm{d}\Phi_{n+1}\; \bar{\mathrm{B}}_{n+1}^{(\mathsf{A})} \left[1 + \frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathcal{K}_n \right] \Delta_n^{(\mathcal{K})}(t_{n+1},\mu_Q^2)\, \Theta(Q-Q_{\text{cut}}) \\
&\qquad\qquad \times \left[\Delta_{n+1}^{(\mathsf{A})}(t_0,t_{n+1})\, O_{n+1} + \int_{t_0}^{t_{n+1}} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_{n+1}^{(\mathsf{A})}}{\mathrm{B}_{n+1}}\, \Delta_{n+1}^{(\mathsf{A})}(t_{n+2},t_{n+1})\, O_{n+2} \right] \\
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$$\langle O\rangle^{\text{MEPS@NLO}}$$

$$\begin{aligned}
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&\qquad + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_n^{(\mathsf{A})}}{\mathrm{B}_n}\, \Delta_n^{(\mathsf{A})}(t_{n+1},\mu_Q^2)\, \Theta(Q_{\text{cut}}-Q)\, O_{n+1}\Bigg] \\
&\quad + \int \mathrm{d}\Phi_{n+1}\left[\mathrm{R}_n - \mathrm{D}_n^{(\mathsf{A})}\right] \Theta(Q_{\text{cut}}-Q)\, O_{n+1} \\
&\quad + \int \mathrm{d}\Phi_{n+1}\, \bar{\mathrm{B}}_{n+1}^{(\mathsf{A})} \left[1 + \frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathcal{K}_n\right] \Delta_n^{(\mathcal{K})}(t_{n+1},\mu_Q^2)\, \Theta(Q-Q_{\text{cut}}) \\
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&\quad + \int \mathrm{d}\Phi_{n+2}\left[\mathrm{R}_{n+1} - \mathrm{D}_{n+1}^{(\mathsf{A})}\right] \Delta_n^{(\mathcal{K})}(t_{n+1},\mu_Q^2)\, \Theta(Q-Q_{\text{cut}})\, O_{n+2}
\end{aligned}$$

NLO merging – Generation of MC counterterm

$$\left[1+\frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}}\int_{t_{n+1}}^{\mu_Q^2}\mathrm{d}\Phi_1\,\mathcal{K}_n\right]$$

- same form as exponent of Sudakov form factor $\Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)$
- truncated parton shower on n-parton configuration underlying $n+1$-parton event
  1. no emission $\to$ retain $n+1$-parton event as is
  2. first emission at t' with $Q>Q_{\text{cut}}$, multiply event weight with $\mathrm{B}_{n+1}/\bar{\mathrm{B}}_{n+1}^{(\mathsf{A})}$, restart evolution at t', do not apply emission kinematics
  3. treat every subsequent emission as in standard truncated vetoed shower
- generates

$$\left[1+\frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}}\int_{t_{n+1}}^{\mu_Q^2}\mathrm{d}\Phi_1\,\mathcal{K}_n\right]\Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)$$

$\Rightarrow$ **identify $\mathcal{O}(\alpha_s)$ counterterm with the omitted emission**

NLO merging

Renormalisation scales:

- determined by clustering using PS probabilities and taking the respective nodal values t_i

$$\alpha_s(\mu_R^2)^{n+k} = \alpha_s(\mu_{\text{core}})^n \cdot \prod_{i=1}^{k} \alpha_s(t_i)$$

- change of scales $\mu_R \to \tilde{\mu}_R$ in MEs necessitates one-loop counter term

$$\alpha_s(\tilde{\mu}_R^2)^k \left(1 - \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi}\, \beta_0 \sum_{i=1}^{k} \ln \frac{t_i}{\tilde{\mu}_R^2}\right)$$

Factorisation scale:

- μ_F determined from core n-jet process
- change of scales $\mu_F \to \tilde{\mu}_F$ in MEs necessitates one-loop counter term

$$\mathrm{B}_n(\Phi_n)\, \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi}\, \log \frac{\mu_F^2}{\tilde{\mu}_F^2} \left(\sum_{c=q,g}^{n} \int_{x_a}^{1} \frac{\mathrm{d}z}{z}\, P_{ac}(z)\, f_c(x_a/z, \tilde{\mu}_F^2) + \ldots \right)$$