The UNLOPS procedure for merging multi-jet NLO calculations

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* Most work done by Stefan Prestel. Similar work done by Simon Plätzer (arXiv:1211.5467)

Introduction

- Introduction
- Improving unitarity for CKKW(-L) $\rightarrow$ UMEPS
- Multi-jet merging to NLO $\rightarrow$ UNLOPS

General Philosophy

Keep the Parton Shower description intact as far as possible, but improve description for partonic configuration with hard, well separated partons using fixed-order matrix elements.

ME region typically defined by a *merging scale* cutoff, regularizing soft and collinear divergencies. Everything should be stable wrt. the merging scale.

Fixed-Order Matrix Elements

Assume that we have a ME generator that can give us samples (eg. in LHE files) of some Born-level configurations, and also samples with $+n$ extra partons ($n \leq N$).

For $n \leq M < N$ these may be calculated to NLO.

We want to combine these together and add parton showers.

To avoid double counting, we need to have exclusive cross sections, or take inclusive ones and make them exclusive.

Parton Showers

- All-order resummation to (N)LL accuracy
- Process-independent (more or less)
- Exclusive final states with arbitrary multiplicities
- Prerequisite for any hadronization model
- Any Parton Shower will do
  (as long as it has on-shell intermediate states)
  (PYTHIA8)
- Parton Showers are unitary

The Unitary nature of Parton Showers

Start with a state from a Born-level ME

$$\frac{d\sigma_0^{inc}}{d\phi_0} \equiv F_0|\mathcal{M}_0|^2,$$

A parton shower will turn this into a +1-parton event with a according to the cross section

$$\frac{d\sigma_1^{first}}{d\phi_0} = F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho dz\,\Gamma_0(\rho_0,\rho).$$

Using a splitting function and a no-emission probability (the *first* or *hardest* splitting).

The PS does not only add a state with an extra parton, it also subtracts the total cross section for this to happen:

$$-\int F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho dz \Gamma_0(\rho_0,\rho).$$

The *exclusive* zero-parton cross section that is left is

$$\begin{aligned}\frac{d\sigma_0^{excl}}{d\phi_0} &= F_0|\mathcal{M}_0|^2\left(1-\int_{\rho_c}\alpha_s\mathcal{P}_1 d\rho dz \Gamma_0(\rho_0,\rho)\right)\\ &= F_0|\mathcal{M}_0|^2\exp\left(-\int_{\rho_c}^{\rho_0}\alpha_s\mathcal{P}_1 d\rho dz\right)\\ &= F_0|\mathcal{M}_0|^2\Gamma_0(\rho_0,\rho_c)\end{aligned}$$

The PS then continues to turn the 1-parton state into a 2-parton state with cross section

$$\frac{d\sigma_2^{first}}{d\phi_0} = F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1\Gamma_0(\rho_0,\rho_1)\alpha_s\mathcal{P}_2 d\rho_2 dz_2\Gamma_1(\rho_1,\rho_2).$$

Again it adds the emission and subtracts the corresponding 1-parton state (integrated over the second emission) leaving the exclusive 1-jet cross-section

$$\frac{d\sigma_1^{excl}}{d\phi_0} = F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1\Gamma_0(\rho_0,\rho_1)\Gamma_1(\rho_1,\rho_c).$$

And so on with a third parton, etc.

CKKW(-L)

We can now use full tree-level matrix elements instead, by multiplying them with appropriate no-emission probabilities, thus making them exclusive:

- $F_0|\mathcal{M}_0|^2\Gamma_0(\rho_0, \rho_{\text{MS}}) \to F_0|\mathcal{M}_0|^2\Gamma_0(\rho_0, \rho_{\text{MS}})$
- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1\Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\text{MS}})$

  $\to F_1|\mathcal{M}_1|^2 d\rho_1 dz_1\Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\text{MS}})$
- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1\Gamma_0(\rho_0, \rho_1)\alpha_s\mathcal{P}_2 d\rho_2 dz_2\Gamma_1(\rho_1, \rho_2)$

  $\to F_2|\mathcal{M}_2|^2 d\rho_1 dz_1\Gamma_0(\rho_0, \rho_1) d\rho_2 dz_2\Gamma_1(\rho_1, \rho_2)$

Where ρ_{MS} is some merging scale (defined in the PS evolution variable). ρ_i and z_i are (PS) reconstructed splittings.

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  $\to F_1|\mathcal{M}_1|^2 d\rho_1 dz_1\Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\rm MS})$
- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1\Gamma_0(\rho_0, \rho_1)\alpha_s\mathcal{P}_2 d\rho_2 dz_2\Gamma_1(\rho_1, \rho_2)$

  $\to F_2|\mathcal{M}_2|^2 d\rho_1 dz_1\Gamma_0(\rho_0, \rho_1) d\rho_2 dz_2\Gamma_1(\rho_1, \rho_2)$

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  $\rightarrow F_1|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\text{MS}})$
- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\alpha_s\mathcal{P}_2 d\rho_2 dz_2 \Gamma_1(\rho_1, \rho_2)$

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  $\rightarrow F_2|\mathcal{M}_2|^2 d\rho_1 dz_1\Gamma_0(\rho_0, \rho_1) d\rho_2 dz_2\Gamma_1(\rho_1, \rho_2)$

Where $\rho_{\rm MS}$ is some merging scale (defined in the PS evolution variable). ρ_i and z_i are (PS) reconstructed splittings.

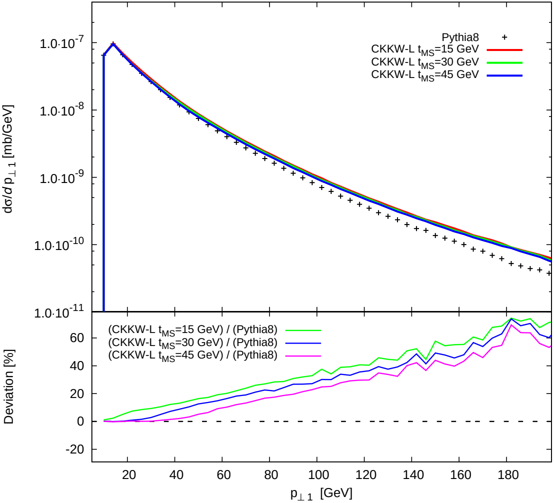
We let eg. MadEvent generate 0-, 1-, and 2-jet samples. We make the 0- and 1-jet samples exclusive and the 2-jet sample *hardest* inclusive by reweighting with no-emission probabilities. We can now add a normal PS below $\rho_{\rm MS}$ (or below ρ_2 in the 2-jet case), and add all samples together avoiding all double-counting.

However, what we add is no longer what we subtract.

- We add the full tree-level ME
- We subtract the PS-approximation

This will give us a dependence of the inclusive cross section on the merging scale.

W+jets


Pythia8
CKKW-L t_{MS}=15 GeV
CKKW-L t_{MS}=30 GeV
CKKW-L t_{MS}=45 GeV
dσ/d $p_{\perp 1}$ [mb/GeV]
1.0·10^-7
1.0·10^-8
1.0·10^-9
1.0·10^-10
1.0·10^-11
(CKKW-L t_{MS}=15 GeV) / (Pythia8)
(CKKW-L t_{MS}=30 GeV) / (Pythia8)
(CKKW-L t_{MS}=45 GeV) / (Pythia8)
Deviation [%]
60
40
20
0
-20
20
40
60
80
100
120
140
160
180
$p_{\perp 1}$ [GeV]

Even far above the merging scales we have a 5-10% merging scale dependence.

No problem for a tree-level calculation, as the scale uncertainties are larger.

But if we want to use this procedure as a starting point for an NLO matching we need to worry.

UMEPS

Instead of making the tree-level ME-samples exclusive, make all of them *hardest* inclusive:

- $F_0|\mathcal{M}_0|^2$
  $-\int F_1|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1)$
- $F_0|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1)$
  $-d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1)\int F_2|\mathcal{M}_2|^2 d\rho_2 dz_2 \Gamma_1(\rho_1,\rho_2)$
- $F_0|\mathcal{M}_2|^2 d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1) d\rho_2 dz_2 \Gamma_1(\rho_1,\rho_2)$

For each extra parton we add the reweighted ME sample but we also subtract the integrated version from the parton multiplicity below making them exclusive.

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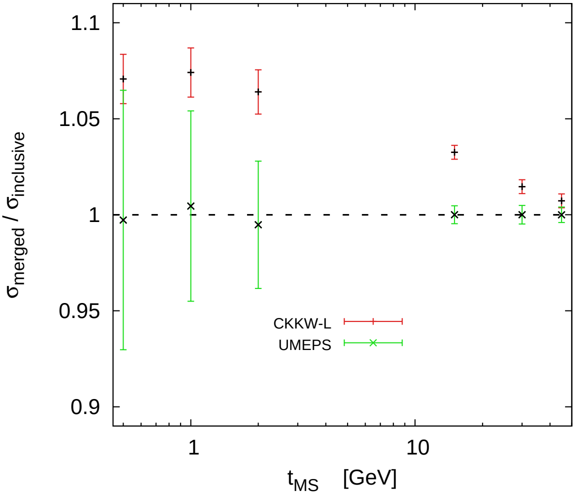
  $-d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1)\int F_2|\mathcal{M}_2|^2 d\rho_2 dz_2 \Gamma_1(\rho_1,\rho_2)$
- $F_0|\mathcal{M}_2|^2 d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1) d\rho_2 dz_2 \Gamma_1(\rho_1,\rho_2)$

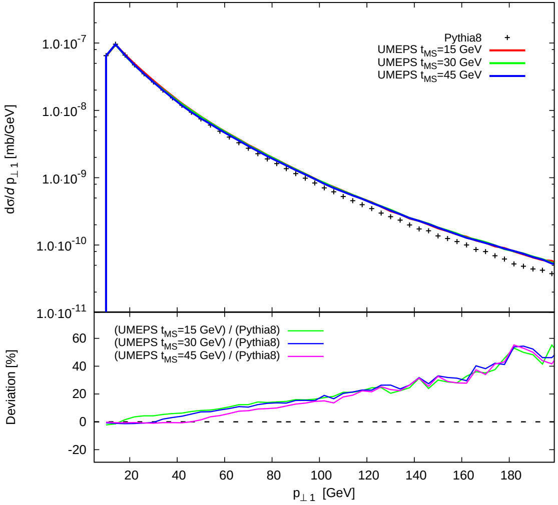
For each extra parton we add the reweighted ME sample but we also subtract the integrated version from the parton multiplicity below making them exclusive.

We can still add a normal PS below ρ_{MS} (or below ρ_2 in the 2-jet case), to avoid all double-counting.

But the procedure is now (almost) completely unitary.

Lönnblad & Prestel arxiv:1211.4827 [hep-ph]


1.1
1.05
1
0.95
0.9
1
10
$\sigma_{merged} / \sigma_{inclusive}$
t_{MS} [GeV]
CKKW-L
UMEPS


Pythia8
UMEPS tMS=15 GeV
UMEPS tMS=30 GeV
UMEPS tMS=45 GeV
1.0·10-7
1.0·10-8
1.0·10-9
1.0·10-10
1.0·10-11
dσ/d p⊥1 [mb/GeV]
(UMEPS tMS=15 GeV) / (Pythia8)
(UMEPS tMS=30 GeV) / (Pythia8)
(UMEPS tMS=45 GeV) / (Pythia8)
60
40
20
0
-20
Deviation [%]
20
40
60
80
100
120
140
160
180
p⊥1 [GeV]

Caveats

We can use any merging scale definition - no need for truncated showers. We still need vetoed showers, but only the first shower emission need to be vetoed.

Only states where the n hardest partons according to the PS are above the merging scale, will be ME-correct.

When reclustered, an n-parton state above the merging scale may result in a $n-1$-parton state below the merging scale. Rather than subtracting this from the exclusive $n-1$ parton sample, it is instead reclustered again and subtracted from the $n-2$ sample.

Negative weights

For small merging scales, the 0-jet exclusive cross section is very small, and the the 0-jet inclusive sample is almost completely canceled by reclustered 1-jet events (with negative weights).

Not a problem in principle, but statistics is an issue.

It would be nice if we could bias our ME-generator to generate LHE-files with suitable weights.

UNLOPS†

We can now go on to also add multi-jet NLO calculations.

- From the NL^3 NLO-merging we know how to expand out the no-emission probabilities in orders of α_s, and subtract any given order.
- We also know how to expand out PDF-ratios with running factorization scales used in the PS to any given order.
- Likewise, the running of α_s in the PS can be trivially expanded.
- If we want we can multiply the UMEPS samples with a K-factor - again, trivially expanded.

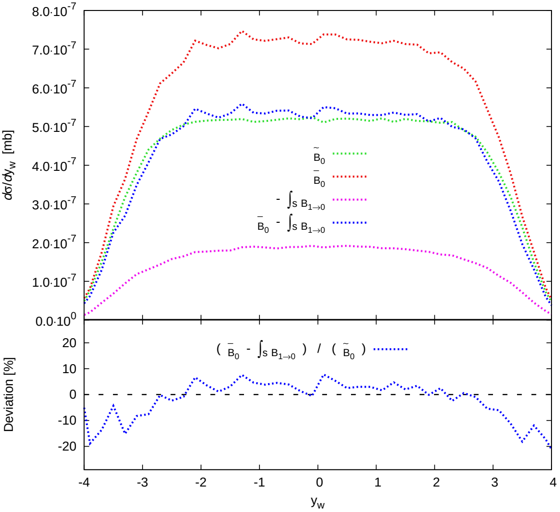
† Lönnblad & Prestel arxiv:1211.7278 [hep-ph]

For each exclusive UMEPS multiplicity sample we can subtract the α_s^n and α_s^{n+1} terms by reweighing, and instead add a sample generated according to the *exclusive* NLO cross section.

There are no generators for exclusive NLO states available, but it is possible to feed NLO ($n+1$) states from POWHEG into PYTHIA8 which are then combined with tree-level states by carefully remapping the radiative phase space of POWHEG into the one used by PYTHIA8.

(a bit complicated, but hidden from the user)

Phase space mappings in PYTHIA8 and POWHEG



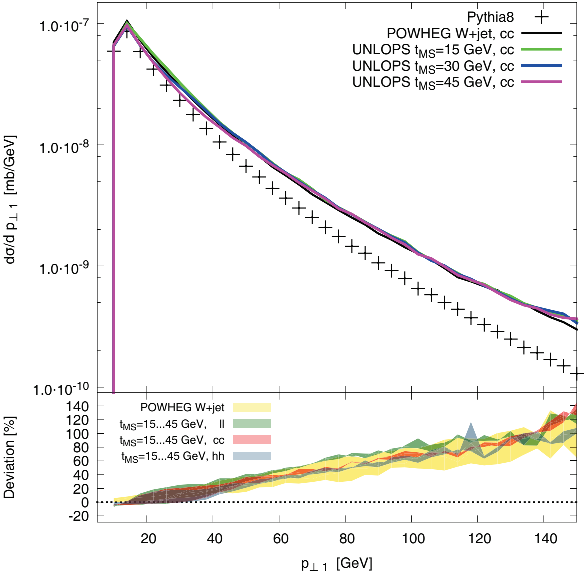
But we also need to subtract what we add.

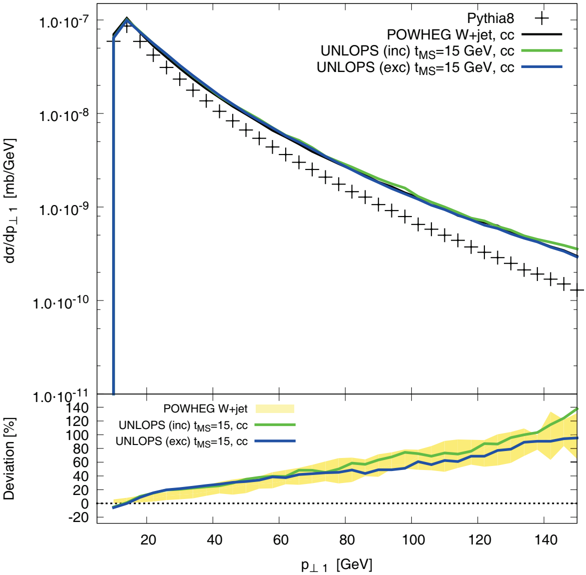
We take the exclusive NLO sample minus the α_s-terms we subtracted from UMEPS reweighted tree-level ME, integrate them over the last emission and subtract them from the multiplicity below.

We are still unitary:

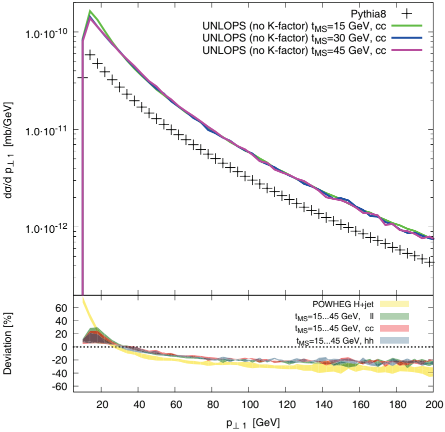
- The inclusive total cross section will be given by the NLO calculation.
- The inclusive 1-parton cross section will be given by the corresponding NLO calculation
- ...

NNLO is also possible in this framework.


Pythia8
POWHEG W+jet, cc
UNLOPS t_{MS}=15 GeV, cc
UNLOPS t_{MS}=30 GeV, cc
UNLOPS t_{MS}=45 GeV, cc
$d\sigma/d\,p_{\perp 1}$ [mb/GeV]
1.0·10^-7
1.0·10^-8
1.0·10^-9
1.0·10^-10
POWHEG W+jet
t_{MS}=15...45 GeV, ll
t_{MS}=15...45 GeV, cc
t_{MS}=15...45 GeV, hh
Deviation [%]
140
120
100
80
60
40
20
0
-20
20
40
60
80
100
120
140
$p_{\perp 1}$ [GeV]


Pythia8
POWHEG W+jet, cc
UNLOPS (inc) t_{MS}=15 GeV, cc
UNLOPS (exc) t_{MS}=15 GeV, cc
dσ/dp⊥1 [mb/GeV]
1.0·10^-7
1.0·10^-8
1.0·10^-9
1.0·10^-10
1.0·10^-11
POWHEG W+jet
UNLOPS (inc) t_{MS}=15, cc
UNLOPS (exc) t_{MS}=15, cc
Deviation [%]
140
120
100
80
60
40
20
0
-20
20
40
60
80
100
120
140
p⊥1 [GeV]

Higgs production



Summary

Multi-jet NLO merging with parton showers is a solved problem. Several algorithms exists.

UNLOPS (and UMEPS) has a couple of attractive features:

- Low jet-multiplicity cross section explicitly preserved without merging scale dependence.
- Merging scale can be taken arbitrarily low (in principle down to the shower cutoff).
- Works for arbitrary multiplicities.
- Extension to NNLO is “straight forward” (“trivial” for the lowest multiplicity).

Still, there are downsides:

- Need full exclusive n-parton states calculated to (N)NLO (can be provided by POWHEG and aMC@NLO)
- Resolution scale must be defined similar to the PS evolution scale.
- Need biased ME event samples to get reasonable statistics for low merging scales.
- For exclusive observables, resummation of higher orders is never better than what the PS gives.

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