

RESUMMATION

FOR LHC PHENOMENOLOGY

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RESUMMATION & PARTON SHOWERS

IPPP, DURHAM, JULY 15, 2013

RESUMMATION ACCORDING TO SPIRES (2010-2013)

The screenshot shows the INSPIRE website interface. The search bar contains the text "find t resummation and date after 2009". The search results are displayed as a "Citations summary". The summary includes the following data:

	Citeable papers	Published only
Total number of papers analyzed:	180	103
Total number of citations:	2,348	2,153
Average citations per paper:	13.0	20.9
Breakdown of papers by citations:		
Renowned papers (500+)	0	0
Famous papers (250-499)	1	1
Very well-known papers (100-249)	4	4
Well-known papers (50-99)	6	6
Known papers (10-49)	44	39
Less known papers (1-9)	84	45
Unknown papers (0)	41	8
h_{HEP} index [?]	23	23

Below the table, there is a section titled "See additional metrics" with a link "Exclude self-citations or RPP". A warning message at the bottom states: "Warning: The citation search should be used and interpreted with great care. Read the fine print".

DISCLAIMER & APOLOGY

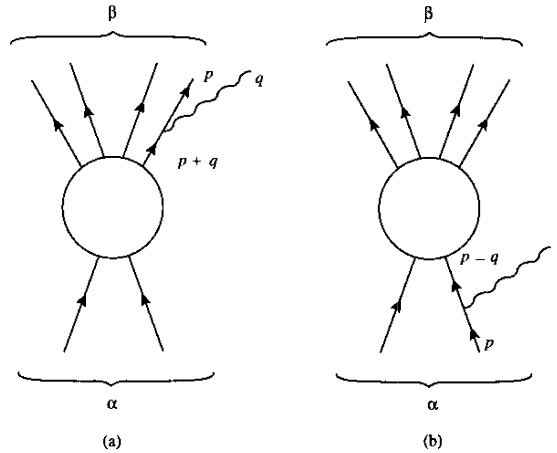
- FOCUS ON LHC PHENOMENOLOGY
- A PERSONAL SELECTION OF TOPICS
- NO ATTEMPT OF PROVIDING A COMPREHENSIVE REVIEW
- EVEN LESS A COMPLETE SET REFERENCES

SUMMARY

- INTRODUCTION
 - EIKONAL EMISSION AND RESUMMATION: THRESHOLD, p_T , AND HIGH-ENERGY
 - THE STRUCTURE OF RESUMMED RESULTS
 - THE STATE OF THE ART
- DRELL-YAN: WHEN IS RESUMMATION RELEVANT?
 - PARTONIC VS HADRONIC KINEMATICS
 - RESUMMED PDFs
 - PERTURBATIVE DIVERGENCE AND RESUMMATION PRESCRIPTIONS
- HIGGS: RESUMMATION AMBIGUITIES
 - THE ROLE OF SUB-EIKONAL TERMS
 - APPROXIMATION FROM RESUMMATION
 - CHOICES OF SCALE: QCD vs. SCET
 - RESUMMING CONSTANTS
- HEAVY QUARKS AND TOP: MULTISCALE PROBLEMS
 - MATCHING FIXED ORDER AND RESUMMATION
 - RESUMMATION VS MONTE CARLO
- JETS: LESS INCLUSIVE OBSERVABLES
 - VETOS
 - SHAPES AND SUBSTRUCTURE
- OUTLOOK
 - THE FRONTIER
 - GOALS

WHAT IT IS ALL ABOUT

WHERE IT ALL COMES FROM: EIKONAL EMISSION...



EMISSION OF A SOFT ($q^\mu \rightarrow 0$) GAUGE PARTICLE FROM **EXTERNAL LINE**

$$\sigma(\alpha \rightarrow \beta) \rightarrow \sigma(\alpha \rightarrow \beta) \frac{ep^\mu}{p \cdot q - i\epsilon}$$

(Bloch, Nordseck, 1937; Yenni, Frautschi, Suura, 1955; Weinberg, 1964)

- **SOFT EMISSION** $\Rightarrow$ **EIKONAL FACTOR**
- **CROSS SECTION** FOR SINGLE (DOUBLE. . .) EMISSION **INFRARED DIVERGENT**; **DIVERGENCE CANCELLED** BY VIRTUAL CORRECTIONS
- **1,2,...,N EMISSIONS EXPONENTIATE** $\Rightarrow \Gamma \sim \exp - \left[\alpha \ln^2 \left(1 - \frac{M_\beta^2}{s} \right) \right]$ (Sudakov, 1956)
- AFTER CANCELLATION, **LEFTOVER SOFT LOGS**:

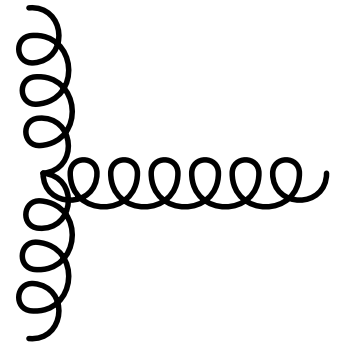
$$\sigma(\alpha \rightarrow \beta) \rightarrow \sigma(\alpha \rightarrow \beta) \ln^2 \left(1 - \frac{M_\beta^2}{s} \right)$$

...AND RESUMMATION

- EXPONENTIATION OF LEFTOVER LOGS $\Rightarrow$
THRESHOLD RESUMMATION OF $\alpha_s \ln^2(1-x)$, $x = \frac{M^2}{s}$
- LOGS COME IN PAIRS: **SOFT+COLLINEAR** $\rightarrow \ln p_t$ WHEN INTEGRAL OVER p_t NOT PERFORMED $\Rightarrow$
TRANSVERSE MOMENTUM RESUMMATION OF $\alpha_s \ln^2 \frac{q_T^2}{M^2}$
- IN GLUON CHANNEL **SYMMETRY OF THE TRIPLE GLUON VERTEX** $\rightarrow$ **LARGE LOGS** ALSO WHEN THE **EXCHANGED GLUON** IS SOFT: NO COLLINEAR CONTRIBUTION, **SINGLE LOGS** $\rightarrow \ln \frac{s}{Q^2} \Rightarrow$
HIGH ENERGY RESUMMATION OF $\alpha_s \ln \frac{1}{x}$

GLUON RADIATION

$$\sigma(\tau, M^2) = \int_y^1 \frac{dy}{y} P\left(\frac{x}{y}\right) \int_{\mu^2}^{(s-M^2)^2/s} \frac{dk_t^2}{k_t^2} \hat{\sigma}(y, M^2)$$



THE GLUON SPLITTING FUNCTION:

$$P_{gg}(x) = 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \beta_0 \delta(1-x)$$

LOGARITHMICALLY ENHANCED TERMS

- **INFRARED LOGS:** $\int_{\tau}^1 dy \frac{1}{1-y} \sim \ln(1-\tau)$
- **UV LOGS:** $\int_{\tau}^1 dy \frac{1}{y} \sim \ln(\tau)$
- **COLLINEAR LOGS:** $\int_{\mu^2}^{(s-M^2)^2/s} \frac{dk_t^2}{k_t^2} \sim \ln \left[\frac{Q^2}{\mu^2} \frac{(1-\tau)^2}{\tau} \right] = \ln \frac{Q^2}{\mu^2} + \ln(1-\tau)^2 + \ln \tau$

SOME DEFINITIONS

THE FACTORIZED CROSS SECTION

$$\sigma(\tau, M^2) = \tau \sum_{ij} \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ij} \left(\frac{\tau}{z}, \mu_F^2 \right) \frac{1}{z} \hat{\sigma}_{ij} \left(z, M^2, \alpha_s(\mu_R^2), \frac{M^2}{\mu_F^2}, \frac{M^2}{\mu_R^2} \right) \quad \tau = \frac{M^2}{s}$$

PARTON LUMINOSITIES

$$\mathcal{L}_{ij}(z, \mu^2) = \int_z^1 \frac{dx}{x} f_i \left(\frac{z}{x}, \mu^2 \right) f_j(x, \mu^2)$$

COEFFICIENT FUNCTIONS

$$\hat{\sigma}_{ij} \left(z, M^2, \alpha_s(\mu_R^2), \frac{M^2}{\mu_F^2}, \frac{M^2}{\mu_R^2} \right) = z \sigma_0 \left(M^2, \alpha_s(\mu_R^2) \right) C_{ij} \left(z, \alpha_s(\mu_R^2), \frac{M^2}{\mu_F^2}, \frac{M^2}{\mu_R^2} \right)$$
$$C_{ij}(z, \alpha_s) = \delta(1-z) \delta_{ig} \delta_{jg} + \alpha_s C_{ij}^{(1)}(z) + \alpha_s^2 C_{ij}^{(2)}(z) + \alpha_s^3 C_{ij}^{(3)}(z) + \mathcal{O}(\alpha_s^4)$$

MELLIN-SPACE FACTORIZATION

$$\sigma(N, M^2) = \sigma_0 \left(M^2, \alpha_s \right) \mathcal{L}(N) C(N, \alpha_s),$$
$$\sigma(N, M^2) \equiv \int_0^1 d\tau \tau^{N-2} \sigma(\tau, M^2); \mathcal{L}(N) \equiv \int_0^1 dz z^{N-1} \mathcal{L}(z) C(N, \alpha_s) \equiv \int_0^1 dz z^{N-1} C(z, \alpha_s)$$

EXPONENTIATION FROM RG INVARIANCE

(Contopanagos, Laenen, Sterman, 1997; S.F., Ridolfi, 2003)

EXAMPLE: THRESHOLD RESUMMATION

- CLOSE TO PARTONIC THRESHOLD, $C(z) \sim \ln^k(1-z)/(1-z)$
- $\int_0^1 dz z^{N-1} \left[\frac{\ln^p(1-z)}{1-z} \right]_+ = \frac{1}{p} \ln^{p-1} N \Rightarrow C(N) \sim \ln^k N$ AT LARGE N

THE RESUMMED CROSS SECTION

$$C(N, M^2) = g_0(\alpha_s) \exp S(\alpha_s L, \alpha_s); \quad S(\alpha_s L, \alpha_s) = \frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \dots;$$
$$\alpha_s = \alpha_s(M^2); \quad L \equiv \ln \frac{1}{N^2}; \quad \text{PHYSICAL AN. DIM.: } \gamma(\alpha_s(M^2), N) = \frac{\partial \ln \sigma(M^2, N)}{\partial \ln M^2}$$

RG INVARIANCE

- REGULARIZED FCTN OF α_s AND SINGLE SCALE M^2 CAN ONLY DEPEND ON $\alpha_s(M^2)$:

$$\gamma(\alpha(M^2)) = \gamma^{(c)}\left(\frac{M^2}{\mu^2}, \alpha(\mu^2)\right) + \gamma^{(l)}\left(\frac{M^2}{\mu^2 N^2}, \alpha(\mu^2)\right);$$

- RGI $\mu^2 \frac{\partial}{\partial \mu^2} \gamma = 0 \Rightarrow \mu^2 \frac{\partial}{\partial \mu^2} \gamma^c = -g(\alpha(\mu^2)) = -\mu^2 \frac{\partial}{\partial \mu^2} \gamma^l$
- $\Rightarrow \gamma^l = \int_{M^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} g(\alpha(\mu^2))$

RESUMMATION FOLLOWS FROM INTEGRATING UP TO “SOFT” SCALE $\frac{Q^2}{N}$

- EXPONENTIATION FOLLOWS FROM FACTORIZATION OF THE DEP OF. THE PARTONIC CROSS SECTION ON THE SCALE WHICH IS BEING RESUMMED (Contopanagos, Laenen, Sterman, 1997)
- IN $\overline{\text{MS}}$ SCHEME, RESUMMATION ENTIRELY CONTAINED IN COEFFICIENT FUNCTION ($\ln N$ HIGHEST POWER IN ANOMALOUS DIM.)

THE STRUCTURE OF RESUMMED EXPRESSIONS

THRESHOLD RESUMMATION

LOG COUNTING

$$C_{\text{res}}(N, \alpha_s) = g_0(\alpha_s) \exp \left[\frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \dots \right];$$

$$g_0(\alpha_s) = 1 + \alpha_s g_{0,1} + \alpha_s^2 g_{0,2} + \mathcal{O}(\alpha_s^3); \quad g_1(\lambda) = \sum_{k=2}^{\infty} g_{1,k} \lambda^k, \quad g_i(\lambda) = \sum_{k=1}^{\infty} g_{i,k} \lambda^k \quad \text{FOR } i \geq 2$$

LOG APPROX.	XSECT ACCURACY	EXP. ACCURACY: $\alpha_s^n L^k$	g_0 ACCURACY: α_s^i
LL	$k = 2n$	$k = n + 1$	0
NLL	$2n - 2 \leq k \leq 2n$	$k = n$	1
NNLL	$2n - 4 \leq k \leq 2n$	$k = n - 1$	2

THE RESUMMED EXPONENT

$$S \left(M^2, \frac{M^2}{N^2} \right) = \int_{M^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} \bar{\gamma} \left(\alpha_s(\mu^2), \frac{M^2}{N^2 \mu^2} \right)$$

$$= \int_{M^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} \left[-A(\alpha_s(\mu^2)) \ln \left(\frac{M^2/N^2}{\mu^2} \right) + B[\alpha_s(\mu^2)] \right].$$

- A, B ARE POWER SERIES IN α_s
- A IS UNIVERSAL, COEFFICIENT OF $\ln N$ IN (DIAGONAL QUARK OR GLUON) ANOMALOUS DIMENSION AT EACH ORDER
- B CONTAINS PROCESS-DEPENDENT TERMS, STARTS AT NLL

THE STATE OF THE ART

THRESHOLD RESUMMATION

- KNOWLEDGE OF **FINITE-ORDER** RESULT ALLOWS FOR RESUMMATION AT THE **CORRESPONDING LOGARITHMIC ORDER**: $N^k \text{LO} \Rightarrow N^k \text{LL}$
- IN MANY CASES, **HIGHER ORDER CALCULATION IN SOFT LIMIT AVAILABLE** THAN AT FIXED ORDER
- **NNLL AVAILABLE** FOR PROCESSES WITH **TWO PARTONS AT BORN LEVEL**: DRELL-YAN, HIGGS (Catani, Grazzini, de Florian, Moch, Vogt, Laenen, Magnea... 2000s)
- **NNLL AVAILABLE** FOR HEAVY QUARKS (Czakon, Mitov, Sterman, 2009)
- **GRADUALLY EXTENDED TO VARIOUS PROCESSES** WITH THREE OR MORE PARTONS AT BORN LEVEL: PROMPT PHOTONS, W , Z AND HIGGS AT HIGH p_t , JETS, ONE-PARTICLE INCLUSIVE (Sterman, de Florian, Vogelsang, Grazzini, Catani... late 2000s)
- CAN BE PERFORMED BOTH FOR TOTAL CROSS-SECTIONS AND **RAPIDITY DISTRIBUTIONS**

THE STRUCTURE OF RESUMMED EXPRESSIONS

P_t RESUMMATION

THE CROSS SECTION

- PARTONIC DIFFERENTIAL p_T DISTRIBUTION: $\Sigma = \sum_n \alpha_s^n(M^2) \Sigma^{(n)}(p_T, M^2)$;
$$\Sigma^{(n)}(p_T, M^2) = \left[\frac{P_n(\ln(p_T^2/M^2))}{p_T^2/M^2} \right]_+ + Q_n(p_T^2/M^2) + D_n \delta(p_T^2/M^2)$$
- **DIVERGES** AS $p_T \rightarrow 0$ TO ANY FINITE ORDER

RESUMMATION

- PHASE-SPACE FACTORIZATION: LONGITUDINAL $\leftrightarrow$ MELLIN; **TRANSVERSE** $\leftrightarrow$ **FOURIER**
- $\Sigma(\alpha_s, p_t^2/M^2) = \frac{M^2}{2\pi} \int d^2b e^{-i\vec{p}_T \cdot \vec{b}} \Sigma(\alpha_s, \alpha_s L) = \int_0^{+\infty} db b J_0(bq_T) \Sigma(\alpha_s, \alpha_s L)$;
 $\alpha_s = \alpha_s(M^2)$; $L \equiv -\ln b^2 M^2$
- EXPONENTIATION: $\Sigma(\alpha_s, \alpha_s L) = \exp S(\alpha_s, \alpha_s L)$

THE RESUMMED EXPONENT

$$S(\alpha_s, \alpha_s L) \equiv - \int_{\frac{1}{b^2}}^{Q^2} \frac{d\mu^2}{\mu^2} \left[\ln \frac{Q^2}{\mu^2} A(\alpha_s(\mu^2)) + B(\alpha_s(\mu^2)) \right],$$
$$\Rightarrow \Sigma(\alpha_s, \alpha_s L) = H(\alpha_s) \exp \left[L g^{(1)}(\alpha_s L) + g^{(2)}(\alpha_s L) + \dots \right]$$

- A, B ARE POWER SERIES IN α_s
- A CONTAINS **SOFT** CONTRIBUTIONS, B **PURELY COLLINEAR** (FLAVOR-CONSERVING) CONTRIBUTIONS
- RESUMMATION SET UP SO THAT A AND B ARE **PROCESS-INDEPENDENT** (ONLY DEPEND ON QUARK VS GLUON) (Catani, de Florian, Grazzini, 2000)

THE STATE OF THE ART

P_t RESUMMATION

- CAN BE PERFORMED FOR TOTAL CROSS-SECTIONS BUT ALSO LESS INCLUSIVE QUANTITIES (E.G. RAPIDITY DISTRIBUTIONS): FOURIER-MELLIN
- CAN BE COMBINED WITH THRESHOLD RESUMMATION (Laenen, Sterman, Vogelsang, 2002)
- KNOWLEDGE OF FINITE-ORDER RESULT ALLOWS FOR RESUMMATION AT THE CORRESPONDING LOGARITHMIC ORDER: $N^k \text{LO} \Rightarrow N^k \text{LL}$
- IN MANY CASES, HIGHER ORDER CALCULATION IN SOFT LIMIT AVAILABLE THAN AT FIXED ORDER
- CLASSIC APPLICATION TO MATCHED NLL+NLO (FONLL) HEAVY QUARK PRODUCTION (Cacciari, Mangano, Nason, Ridolfi, 2000s)
- NNLL AVAILABLE, MATCHED TO FIXED ORDER FOR DRELL-YAN (QUARK), HIGGS (GLUON) (Catani, Grazzini, de Florian... 2000s)
- NLL GRADUALLY EXTENDED TO VARIOUS PROCESSES WITH MORE EXCLUSIVE DESCRIPTION OF FINAL STATES SUCH AS HIGGS PRODUCTION & DECAY CHANNELS (de Florian, Ferrera, Grazzini, 2012)
- ALSO EXTENDED TO NEW RESUMMATION VARIABLES (Banfi, Dasgupta, Marzani, Tomlinson, 2010s)
- IMPORTANT RECENT APPLICATION TO HIGGS OR W WITH JET VETOS (Tackmann, Monni, Banfi, Salam, Zanderighi, Petriello Ferrera, Grazzini, 2012)

THE STRUCTURE OF RESUMMED EXPRESSIONS

HIGH ENERGY RESUMMATION

- HIGH-ENERGY (SIMPLE) LOGS ARE NOT KINEMATICALLY DETERMINED:
AFFECT BOTH COEFFICIENT FUNCTION & ANOMALOUS DIMENSION
- ENTER AT LEADING LOG IN GLUON-DOMINATED CHANNELS
- RELEVANT FACTORIZATION

$$\sigma(\tau, Q^2) = \int_{\tau}^1 \frac{dz}{z} \int_0^{s/4} \frac{dk^2}{k^2} C\left(\frac{z}{\tau}, \frac{Q^2}{k^2}; \alpha_s(\mu^2)\right) \mathcal{L}(z, k^2; \mu^2)$$

ONLY PROVEN AT LEADING LOG LEVEL

- FACTORIZES UPON DOUBLE MELLIN: (note unusual def. of Mellin, $N - 1 \rightarrow N$)

$$\sigma_{NM}(\mu^2) \equiv \int_0^\infty dx x^N \int_0^\infty \frac{dQ^2}{Q^2} \left(\frac{\Lambda^2}{Q^2}\right)^M \sigma\left(x, \frac{Q^2}{\mu^2}; \mu^2\right); \quad \sigma_{NM} = C_{NM}(\alpha_s(\mu^2)) f_{NM}(\mu^2).$$

$$\frac{\ln^{k-1} 1/x}{x} \Rightarrow k! \frac{1}{(N-1)^k}$$

- RESUMMED ANOMALOUS DIMENSION:

$$\gamma(\alpha_s, N) = \gamma_s\left(\frac{\alpha_s}{N}\right) + \alpha_s \gamma_{ss}\left(\frac{\alpha_s}{N}\right) + \dots$$

- ONCE $C(N, M)$ KNOWN, RESUMMATION OF THE COEFFICIENT FUNCTION DETERMINED FROM RESUMMED ANOMALOUS DIMENSION:

$$C(N, \alpha_s) = C(N, M) \Big|_{M=\gamma_s(\alpha/N)}$$

THE STRUCTURE OF RESUMMED EXPRESSIONS

HIGH ENERGY RESUMMATION: DUALITY OF THE ANOMALOUS DIMENSIONS

(T. Jaroszewicz, 1982; R. Ball & S.F., 1995)

THE ALTARELLI-PARISI EQN IS AN INTEGRO-DIFFERENTIAL EQUATION $\Rightarrow$ IT CAN BE EQUIVALENTLY VIEWED AS Q^2 -EVOLUTION EQUATION FOR x -MOMENTS (usual RG eqn.), OR x -EVOLUTION EQUATION FOR Q^2 -MOMENTS (BFKL eqn.)

EVOLUTION IN $t = \ln Q^2$

$$\frac{d}{dt} G(N, t) = \gamma(N, \alpha_s) G(N, t)$$

MELLIN x -MOMENTS

$$G(N, t) = \int_0^\infty d\xi e^{-N\xi} G(\xi, t)$$

EVOLUTION IN $\xi = \ln 1/x$

$$\frac{d}{d\xi} G(\xi, M) = \chi(M, \alpha_s) G(\xi, M)$$

MELLIN Q^2 -MOMENTS

$$G(\xi, M) = \int_{-\infty}^\infty dt e^{-Mt} G(\xi, t)$$

THE TWO EQUATIONS HAVE THE SAME SOLUTIONS
PROVIDED THE EVOLUTION KERNELS ARE RELATED BY

$$\begin{aligned}\chi(\gamma(N, \alpha_s), \alpha_s) &= N \\ \gamma(\chi(M, \alpha_s), \alpha_s) &= M\end{aligned}$$

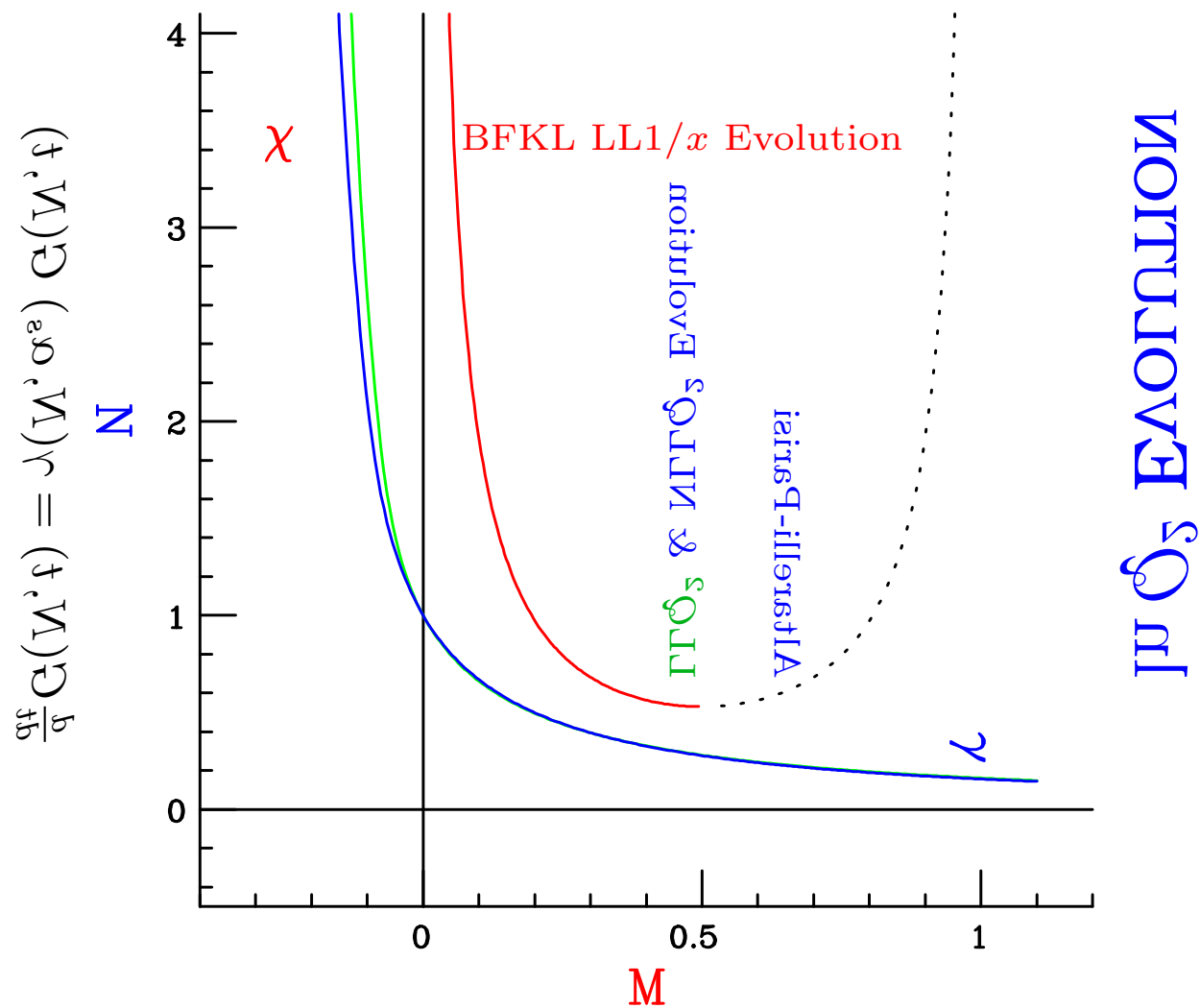
& BOUNDARY CONDITIONS RELATED BY

$$H_0[M] \rightarrow G_0(N) = H_0[\gamma(N, \alpha_s)] / \chi'(\gamma(N, \alpha_s))$$

... CAN SWITCH FROM LLQ^2 TO $LL1/x$

CHOOSING THE EVOLUTION KERNEL

$\ln 1/x$ EVOLUTION

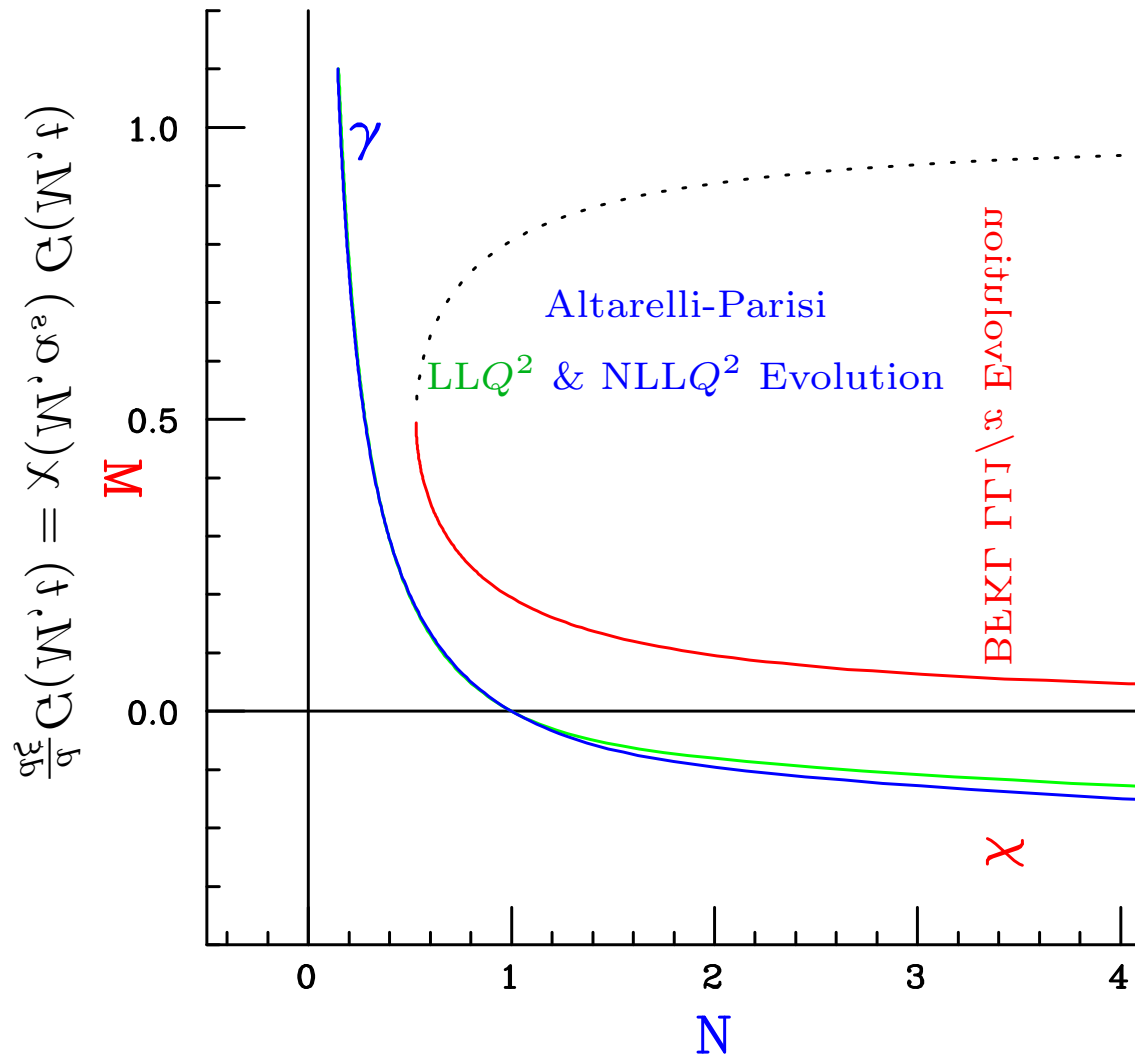


$$\frac{d}{d\xi} G(M, t) = \chi(M, \alpha_s) G(M, t)$$

$\ln Q_s^2$ EVOLUTION

... IN EITHER EQUATION!

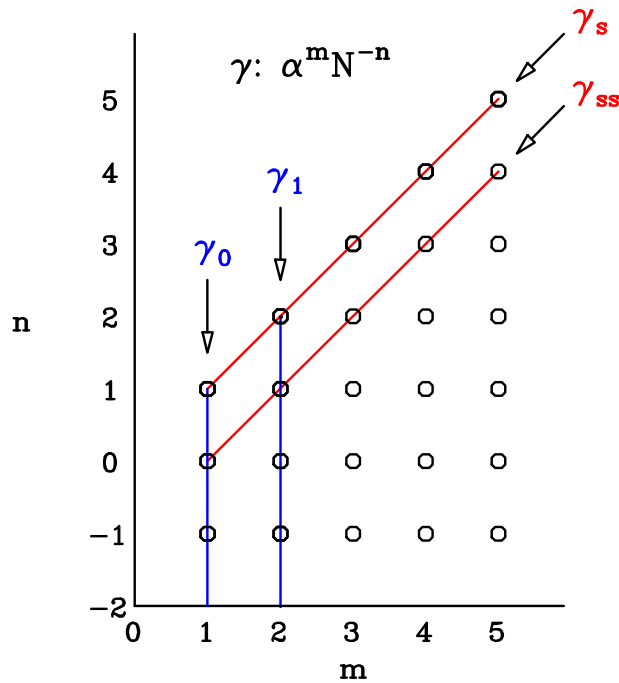
$\ln Q^2$ EVOLUTION



$\ln 1/\alpha_s$ EVOLUTION

$$\frac{d}{dt} G(N, t) = \gamma(N, \alpha_s) G(N, t)$$

DUAL PERTURBATIVE EXPANSIONS



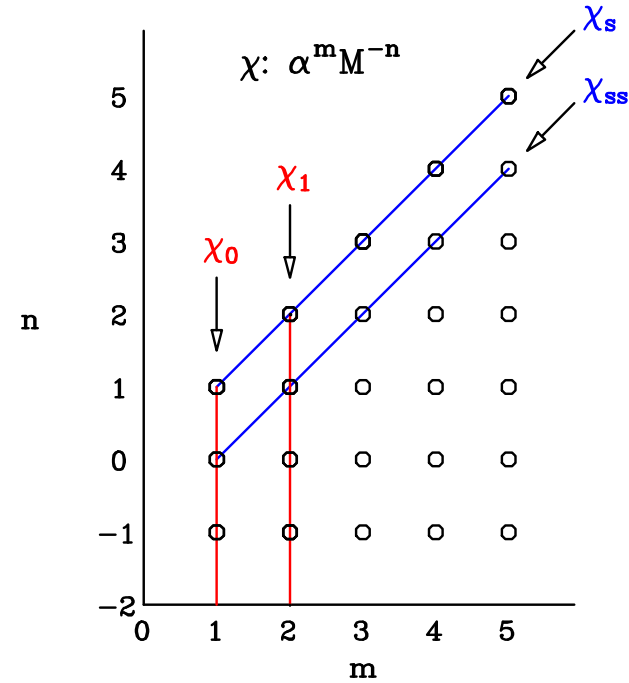
$\ln Q^2$ EVOLUTION

$$\gamma(N) = \alpha \left(\frac{c_{-1}^{(1)}}{N} + c_0^{(1)} + \dots \right) + \alpha^2 \left(\frac{c_{-2}^{(2)}}{N^2} + \frac{c_{-1}^{(2)}}{N} + \dots \right)$$

$$\gamma_s(N) = c_{-1}^{(1)} \frac{\alpha}{N} + c_{-2}^{(2)} \frac{\alpha^2}{N^2} + \dots$$

$1/N$ POLES $\Leftrightarrow \ln 1/x$

$$\begin{aligned} \gamma_0(N) &\Leftrightarrow \chi_s(\alpha_s/M) \\ \gamma_s(\alpha_s/N) &\Leftrightarrow \chi_0(M) \end{aligned}$$



$\ln 1/x$ EVOLUTION

$$\chi(M) = \alpha \left(\frac{\tilde{c}_{-1}^{(1)}}{M} + \tilde{c}_0^{(1)} + \dots \right) + \alpha^2 \left(\frac{\tilde{c}_{-2}^{(2)}}{Q^2} + \frac{\tilde{c}_{-1}^{(2)}}{M} + \dots \right)$$

$$\chi_s(M) = \tilde{c}_{-1}^{(1)} \frac{\alpha}{M} + \tilde{c}_{-2}^{(2)} \frac{\alpha^2}{Q^2} + \dots$$

$1/M$ POLES $\Leftrightarrow \ln Q^2$

THE STATE OF THE ART

HIGH ENERGY RESUMMATION

- SMALL x RESUMMATION OF ANOMALOUS DIMENSIONS KNOWN UP TO NLL (Fadin-Lipatov, 1997)
- LARGE SUBLEADING TERMS, IMPORTANT CORRECTIONS FROM PHYSICAL CONSTRAINTS (RUNNING COUPLING): FULL MATCHED RESUMMATION IN GLUON CHANNEL NEEDED (Altarelli, Ball, SF, Ciafaloni, Colferai, Salam, Stasto, mid 2000s), AT NLL MIXING WITH QUARKS IMPORTANT (Altarelli, Ball, SF, 2008)
- LL RESUMMATION OF COEFFICIENT FUNCTION KNOWN FOR DRELL-YAN, HIGGS, HEAVY QUARKS, PROMPT PHOTON (Ball, SF, K.Ellis, Diana, Marzani, del Duca, Vicini, late 2000)
- STILL NOT DONE FOR JETS, THOUGH MANY MC TOOLS AVAILABLE (J. Andersen et al, 2000s)
- RESUMMATION ALSO AVAILABLE FOR RAPIDITY DISTRIBUTIONS (Caola, SF, Marzani, 2009), ONLY DONE FOR HIGGS
- INTRIGUING RECENT THEORETICAL CONNECTION WITH IR SINGULARITIES: LL EXPONENTIATION (“REGGEIZATION”) DERIVED FROM RESUMMATION OF SOFT AND COLLINEAR SINGULARITIES (Del Duca, Duhr, Gardi, Magnea, White, 2012)

DRELL-YAN

WHEN IS RESUMMATION RELEVANT?

- IN FACTORIZED PERTURBATIVE COMPUTATIONS ONE DETERMINES A PARTONIC CROSS-SECTION
⇒ PARTONIC KINEMATICS DETERMINES WHETHER RESUMMATION IS NECESSARY
- PARTONIC KINEMATICS CAN BE QUITE DIFFERENT FROM HADRONIC (EG: GLUONS CARRY ON AVERAGE SMALL FRACTION OF PARENT HADRON'S MOMENTUM); IT IS INTEGRATED OVER
- HOW CAN ONE DETERMINE IT?

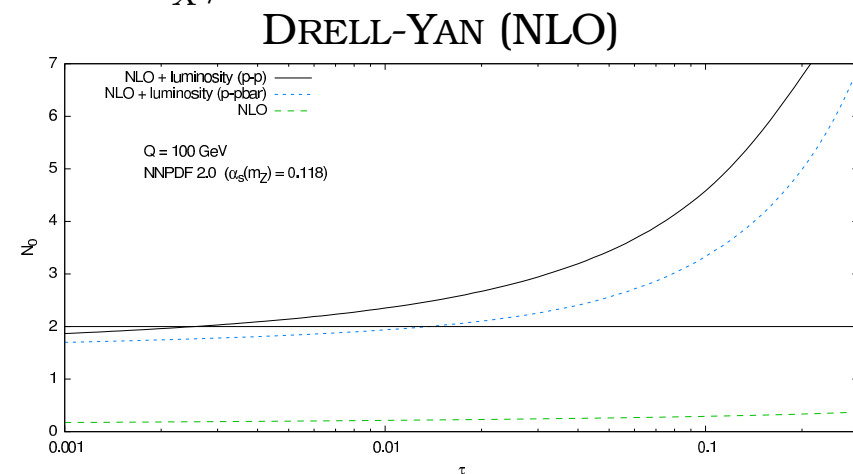
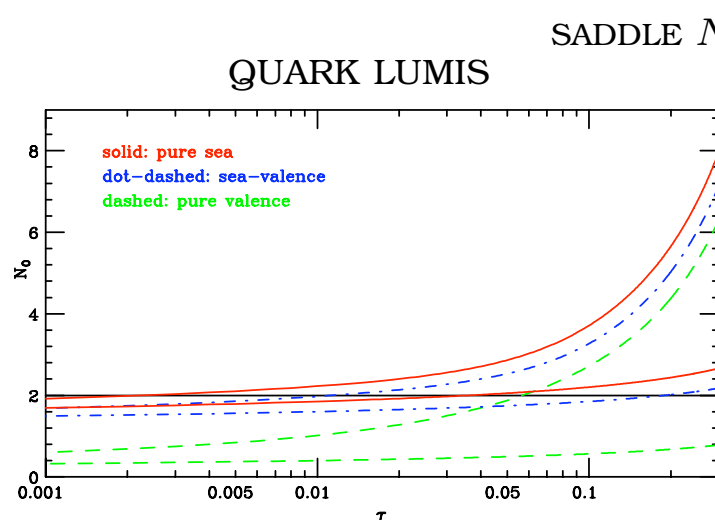
SADDLE POINT

- CROSS SECTION **DETERMINED** FROM N -SPACE RESULT **BY MELLIN INVERSION**:

$$\frac{\sigma(\tau, M^2)}{\tau} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \tau^{-N} \sigma(N, M^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN e^{E(N, \tau, M^2)}$$

$$E(N, \tau, M^2) \equiv N \ln \frac{1}{\tau} + \ln \sigma(N, M^2)$$

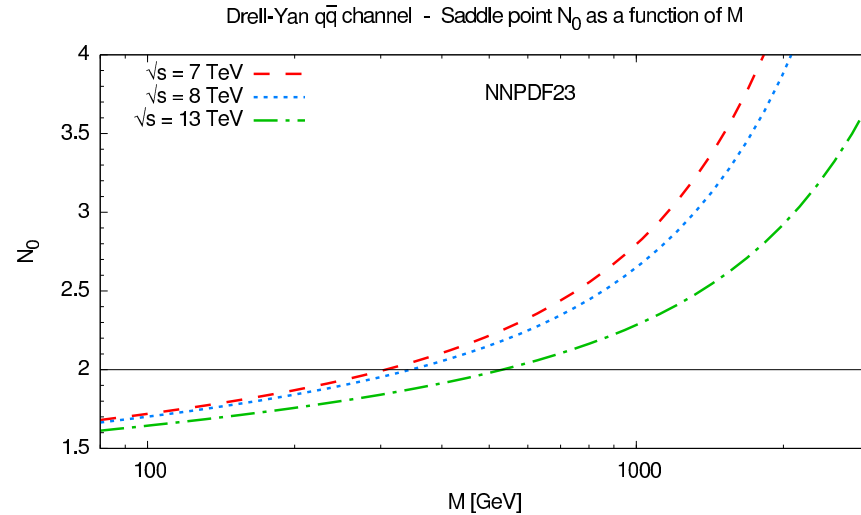
- DOMINATED BY SADDLE POINT** $\left. \frac{\partial E(N, \tau, M^2)}{\partial N} \right|_{N=N_0} = 0$
- SADDLE N_0 ON THE REAL AXIS: $\sigma(N)$ MUST BE A **DECREASING FUNCTION** OF N ; AS $\tau \rightarrow 0$, N_0 **MOVES LEFTWARDS TOWARDS THE RIGHTMOST SING.**
- MOST OF THE **CONTRIBUTION** COMES FROM A **SMALL NEIGHBORHOOD** OF N_0
- POSITION OF **SADDLE** MOSTLY **DETERMINED BY LUMINOSITY**
- FOR **FIXED HADRONIC** τ , **LARGER SADDLE FOR GLUON, SEA (CLOSER TO THRESHOLD)** THAN FOR VALENCE



(Bonvini, SF, Ridolfi, 2009,2012)

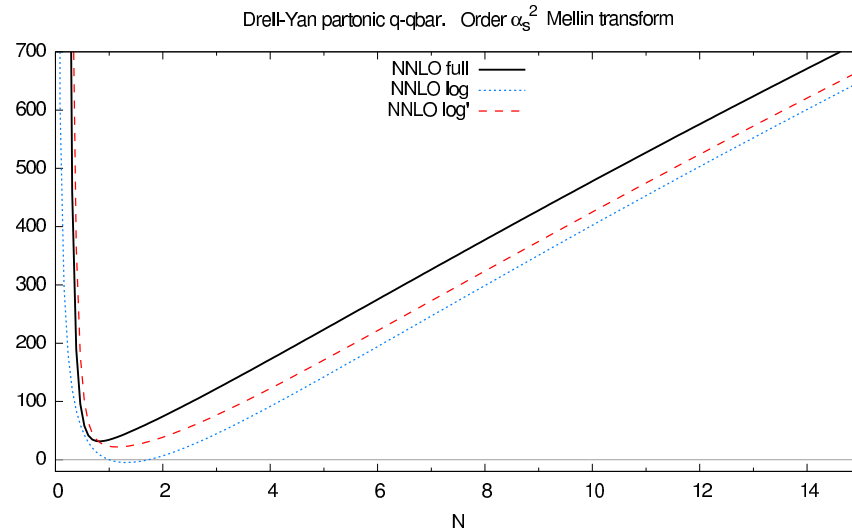
CASE STUDY I: DRELL-YAN

SADDLE

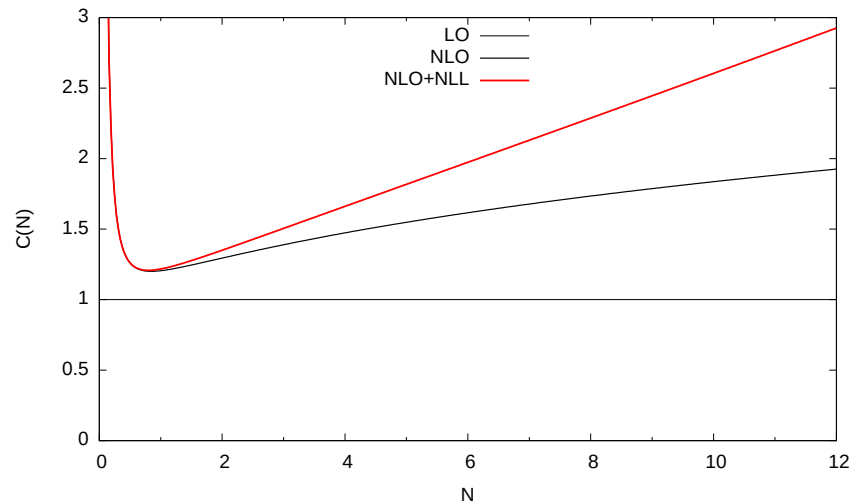


- ASSUME PRODUCTION OF A Z' WITH $M = 2 \text{ TeV}$ AT LHC 8 TeV
- SADDLE AT $N_0 \sim 4$
- CROSS SECTION COMPLETELY DOMINATED BY SOFT CONTRIBUTION
- $NLO/LO \sim 15 \frac{\alpha_s}{\pi} \Rightarrow$ RESUM!

NNLO COEFFICIENT FUNCTION



NLO VS RESUMMED



RESUMMED PDFS?

- IN PRINCIPLE, RESUMMED PARTON DISTRIBUTIONS MUST BE USED WITH RESUMMED PARTONIC CROSS SECTIONS
- IN PRACTICE DOES IT MAKE ANY DIFFERENCE?

RESUMMED PDFS?

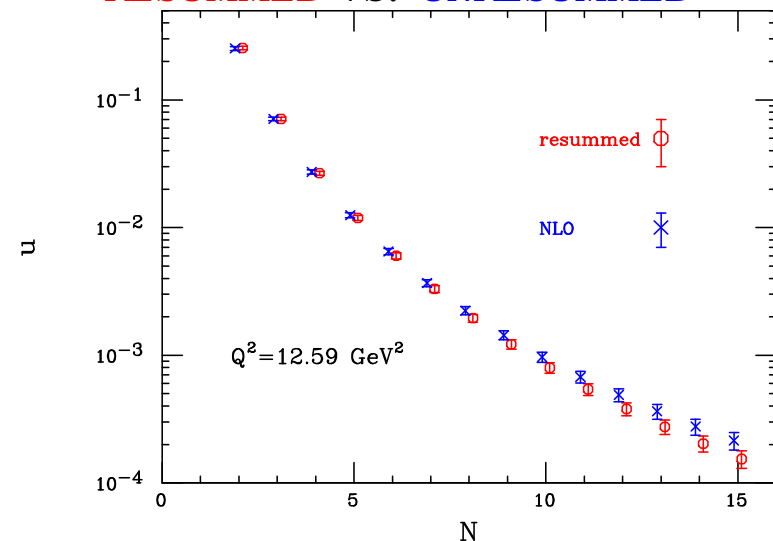
- IN PRINCIPLE, RESUMMED PARTON DISTRIBUTIONS MUST BE USED WITH RESUMMED PARTONIC CROSS SECTIONS
- IN PRACTICE DOES IT MAKE ANY DIFFERENCE?

NO

- PRESENT-DAY PDFs MOSTLY DETERMINED BY DEEP-INELASTIC SCATTERING, SUPPLEMENTED BY DRELL-YAN AND JET DATA
- LARGEST x DATA ARE MOSTLY DIS (QUARKS) AND JETS (GLUON)
- RESUMMATION CORRECTIONS TO DIS MODERATE:
ONLY ONE INCOMING PARTON, $\ln N$ (DIS) VS. $\ln N^2$ (DY)
- CORRECTION ON QUARK DUE TO RESUMMATION FOR $N \sim 4$ BELOW 1%

UP QUARK PDF IN MELLIN SPACE

RESUMMED VS. UNRESUMMED



(Corcella, Magnea, 2005)

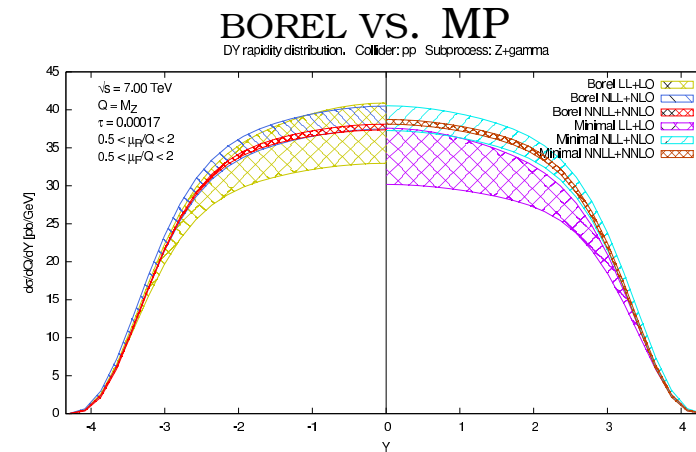
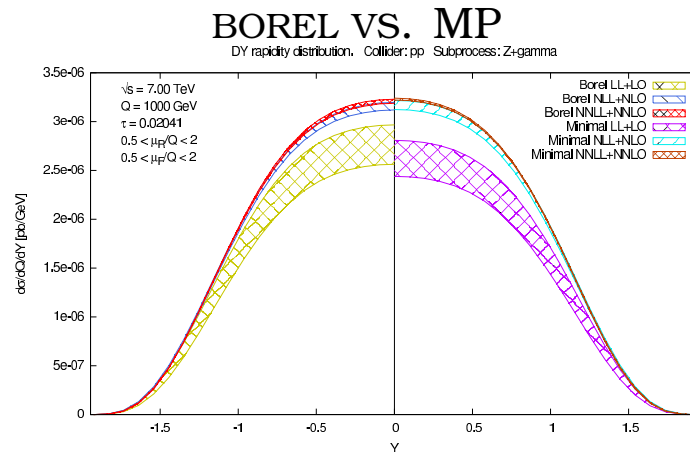
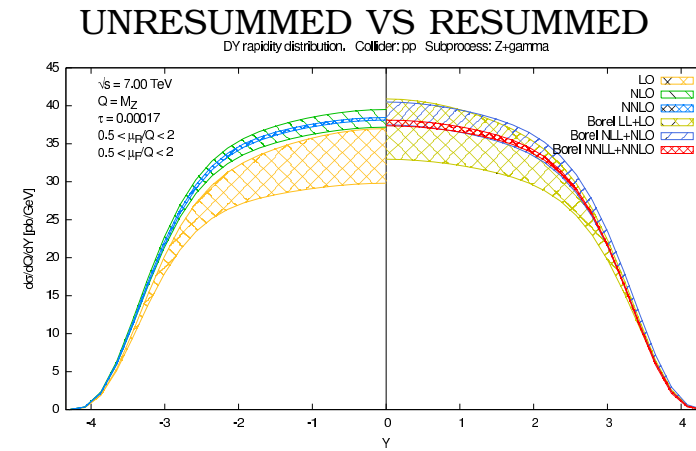
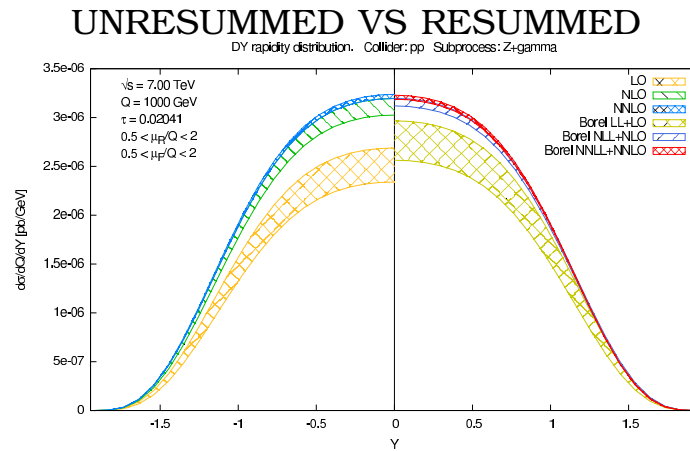
DRELL-YAN AT LHC

RESUMMATION AMBIGUITIES

- EFFECT OF RESUMMATION OF A FEW PERCENT FOR Z' BUT ALSO FOR Z
 - RESULT DEPENDS ON RESUMMATION PRESCRIPTION (MINIMAL VS. BOREL)
- ⇒ SIZABLE AMBIGUITIES!

DY PRODUCTION AT THE LHC 7TeV

1 TeV Z'
 Z



(Bonvini, SF, Ridolfi, 2012)

AMBIGUITIES $\Leftrightarrow$ PERTURBATIVE DIVERGENCE

γ DEPENDS ON $\alpha_s(M^2/N^2) \Rightarrow$ **LANDAU POLE** SINGULARITY AT LARGE N :

$$\gamma_{\text{LL}}(\alpha_s(M^2), N) = g_1 \int_1^N \frac{dn}{n} \alpha_s(M^2/n) = -\frac{g_1}{\beta_0} \ln \left(1 + \beta_0 \alpha_s(M^2) \ln \frac{1}{N} \right),$$

CUT ON REAL N AXIS FOR $N \geq N_L \equiv e^{\frac{1}{\beta_0 \alpha_s(M^2)}} \Rightarrow$ **CANNOT BE A MELLIN**

DIVERGENCE ORDER-BY-ORDER INVERSE MELLIN: $P_{\text{LL}}(\alpha_s(M^2), x) = \frac{g_1}{\beta_0} \left[\frac{R(\alpha_s(M^2), x)}{1-x} \right]_+$

$$R(\alpha_s(M^2), x) = \lim_{K \rightarrow \infty} \sum_{n=0}^K \Delta^{(n)}(1) [-\beta_0 \alpha_s(M^2(1-x))]^{n+1} \text{ WHERE}$$

$$\frac{1}{\Gamma(1-z)} = \sum_{n=0}^{\infty} (-1)^n \frac{\Delta^{(n)}(1)}{n!} z^n$$

- **DIVERGES FACTORIALY!**
- DIVERGENCE ONLY KICKS IN AT VERY HIGH ORDER, EG FOR $\alpha_s \sim 0.1$, $\tau \sim 0.1$ BEYOND THE 20^{th} ORDER
- **NEED A RESUMMATION PRESCRIPTION**

PRESCRIPTIONS

THE MINIMAL PRESCRIPTION

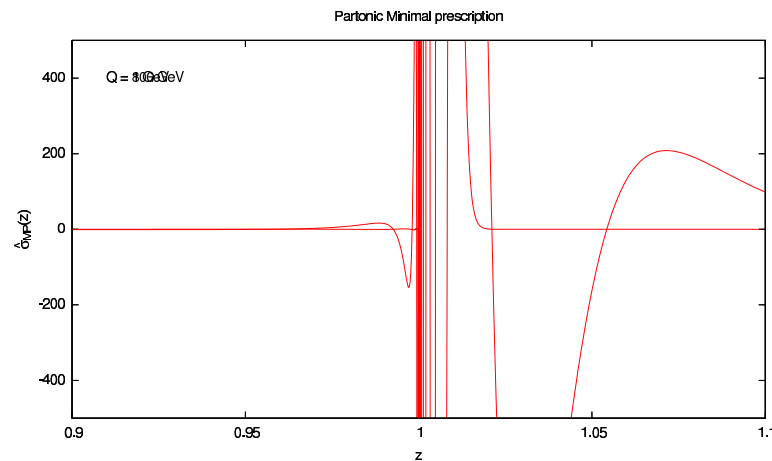
(Catani, Mangano, Nason, Trentadue, 1996)

JUST DEFINE CROSS-SECTION AS MELLIN INVERSION INTEGRAL:

$$\sigma(M^2, \tau) = \frac{1}{2\pi i} \int_{\bar{N}-i\infty}^{\bar{N}+i\infty} dN \tau^{-N} \sigma(M^2, N)$$

$\bar{N}$ TO THE LEFT OF LANDAU POLE, TO THE RIGHT OF ALL OTHER SINGULARITIES

- CROSS-SECTION **DOES NOT VANISH** WHEN $\tau > 1 \Leftrightarrow M^2 > s$
- HOWEVER CONTRIBUTION IN $\tau > 1$ REGION **EXPONENTIALLY SUPPRESSED**
 $\sim \exp -\frac{\Lambda^2}{Q^2}$
- **STRONGLY OSCILLATING** IN $\tau \sim 1$ REGION



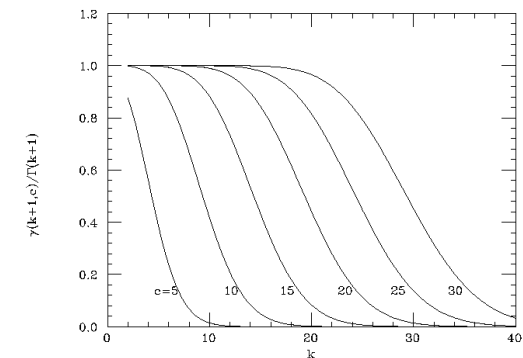
(Bonvini, s.f., Ridolfi, 2011)

PRESCRIPTIONS

THE BOREL PRESCRIPTION

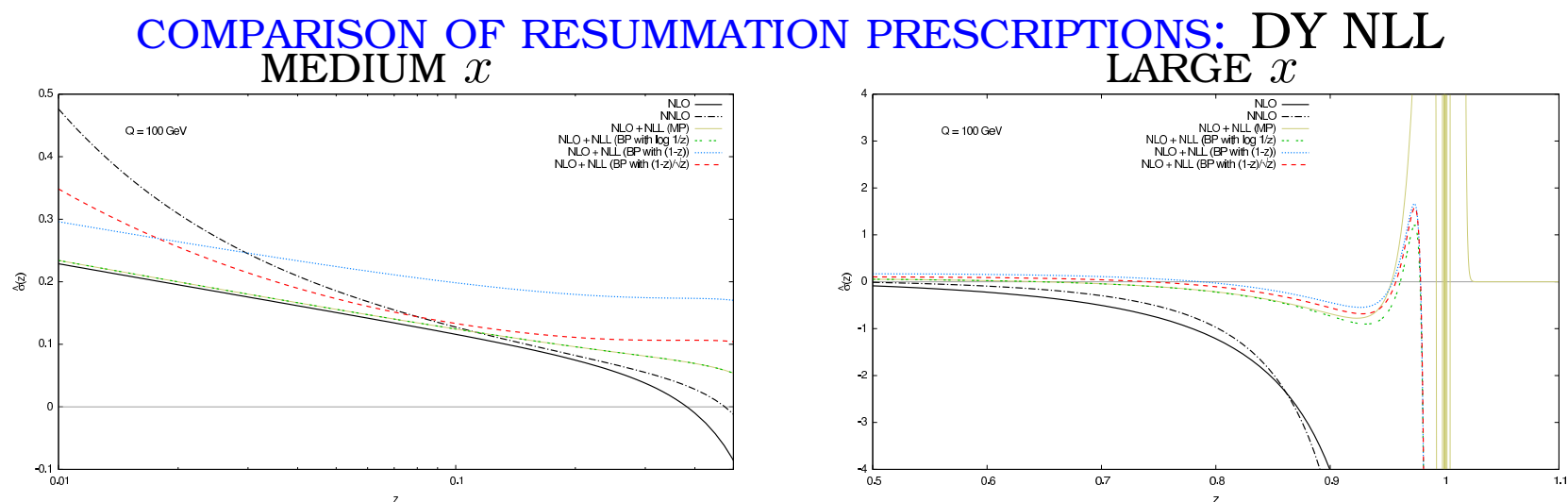
(s.f., Ridolfi, Rojo, Ubiali, 2006)

- SUM THE DIVERGENT SERIES BY THE BOREL METHOD
- BOREL INVERSION INTEGRAL DIVERGES $\Rightarrow$ CUT OFF
- THE CUT-OFF SERIES (“BOREL PRESCRIPTION”) IS AN ASYMPTOTIC SUM OF THE DIVERGENT SERIES
- CORRECTION TERM IS HIGHER TWIST $O\left[\left(\frac{\Lambda^2}{Q^2}\right)^C\right]$ C DETERMINED BY THE VALUE OF THE CUTOFF
- IN PRACTICE, THE HIGHER TWIST CORRECTION DAMPS HIGHER PERTURBATIVE ORDERS
- DAMPING DETERMINED BY $c = \frac{C}{2\beta_0\alpha_s}$: $\alpha = 0.118$
 $\Rightarrow c \approx 15 \Rightarrow$ DAMPING BETWEEN 10TH AND 20TH ORDER



AMBIGUITIES $\Leftrightarrow$ SUB-EIKONAL TERMS

- WHY ARE DIFFERENCES BETWEEN PRESCRIPTIONS LARGER FOR SMALLER M_X ?
- BOREL PRESCRIPTION ALLOWS FOR DIFFERENT TREATMENTS OF SUB-EIKONAL TERMS
E.G. INVERSE MELLIN OF POWERS $\ln N$ REALLY LEADS TO POWERS OF $\ln \ln(1/x) = \ln(1-x)[1 + O(1-x)]$
- COMPARE DIFFERENT VERSIONS OF BOREL PRESCRIPTION (DIFFERENT SUB-EIKONAL TERMS) & MINIMAL PRESCRIPTION
- DIFFERENCE BETWEEN MINIMAL & BOREL ONLY VISIBLE AT EXTREMELY LARGE x
- DIFFERENCES DUE TO SUB-EIKONAL TERMS SIZABLE
- RESUMMATION USED IN A PERTURBATIVE REGION

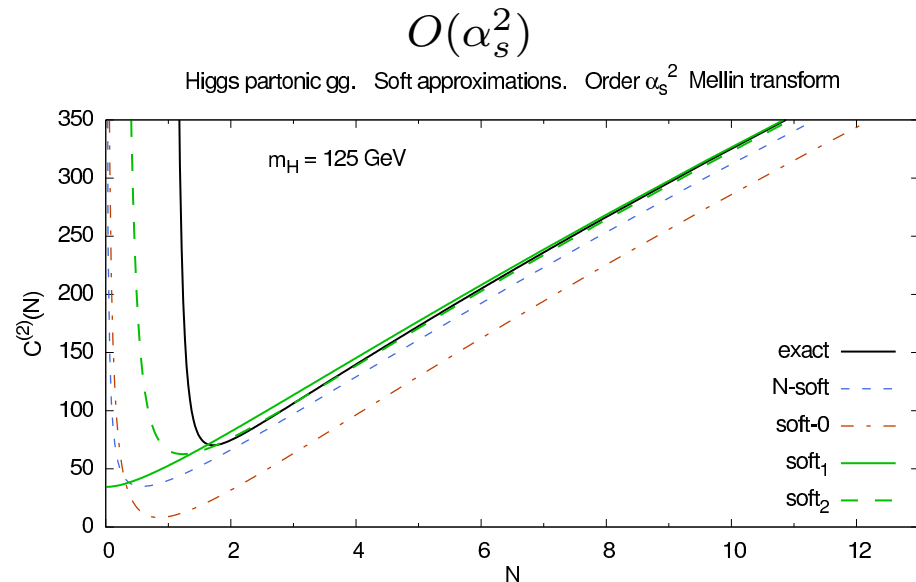
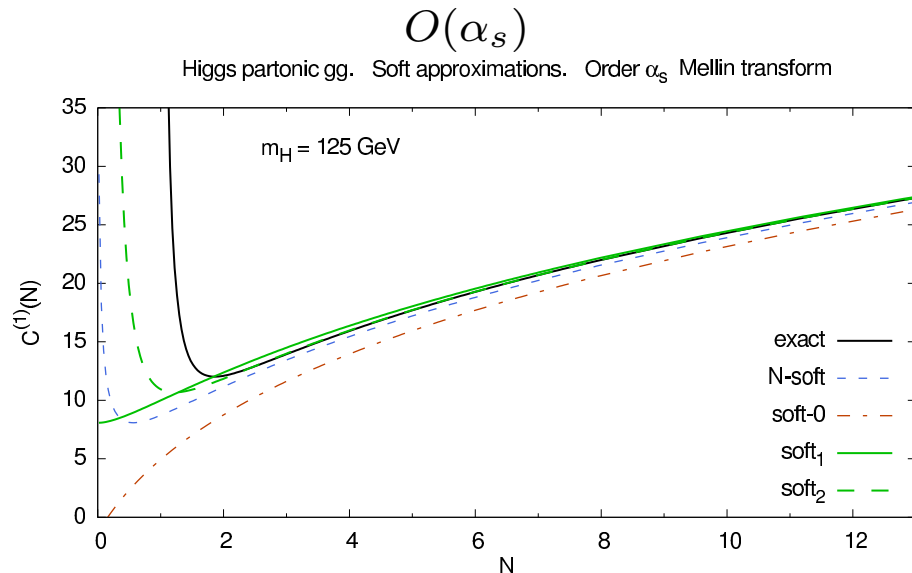
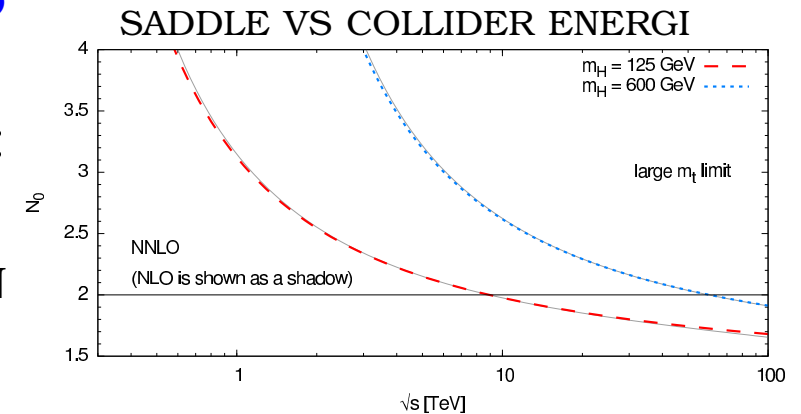


HIGGS

CASE STUDY II: HIGGS IN GLUON FUSION

“PERTURBATIVE” RESUMMATION

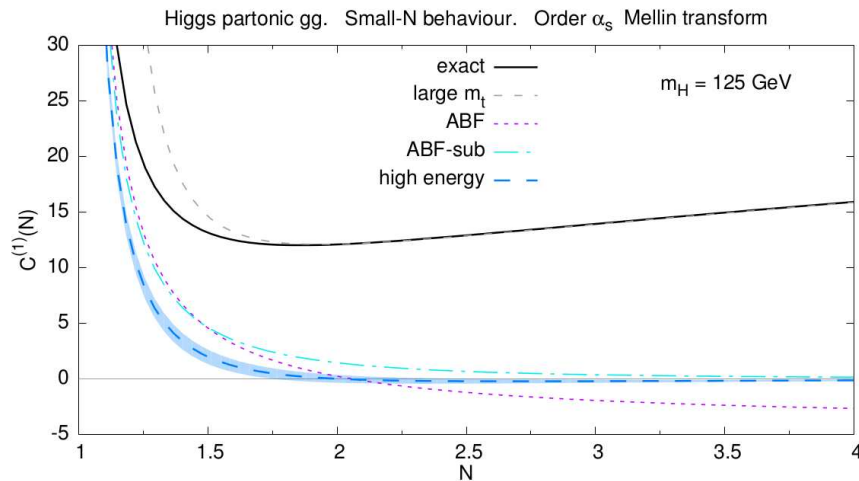
- STANDARD PREDICTIONS FOR THIS PROCESS OFTEN INCLUDE NNLL THRESHOLD RESUMMATION (de Florian, Grazzini 2012)
- HOWEVER, SADDLE $N_0 \sim 2$, NOT LARGE: $\alpha_s \ln N_0 \ll 1$
- SOFT APPROXIMATION TO CROSS-SECTION SURPRISINGLY GOOD
- DIFFERENT FORMS OF SOFT APPROX DIFFER SUBSTANTIALLY



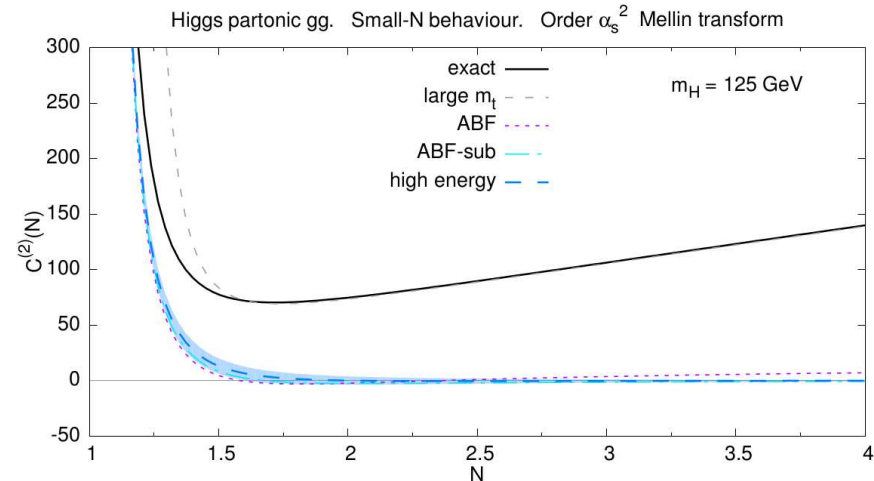
HIGGS AT SMALL x/N

- SMALL N SINGULARITIES ARE NOT THE SAME IN $m_t \rightarrow \infty$ LIMIT
(LARGE m_t APPROX FAILS WHEN $s \gg m_t$)
 $\Rightarrow$ **SPURIOUS DOUBLE LOGS** $\alpha_s \ln^2 x \leftrightarrow$ **DOUBLE POLES** $\alpha_s \frac{1}{(N-1)^2}$
COULD BE RESUMMED (Hautmann 2002)
- SOFT EXPANSION OF $C(z)$ ABOUT $z = 1$ BREAKS DOWN FOR $z < \frac{m_H^2}{m_t}$ (Harlander, Ozeren 2009), CAN **FIX MATCHING TO KNOWN CORRECT BEHAVIOUR** FROM SMALL x **RESUMMATION** (Harlander, Mantler, Marzani, Ozeren, 2010)

$O(\alpha_s)$



$O(\alpha_s^2)$



SINGULARITIES AND SUB-EIKONAL TERMS

Q: WHICH SOFT APPROX IS THE BEST?

A: THAT WHICH RESPECTS THE SMALL N SINGULARITY STRUCTURE

- EXAMPLE: THRESHOLD RESUMMATION + MP $\Rightarrow$ POWERS OF $\ln N \Rightarrow$ CUTS AT $N = 0$
- INVERSE MELLIN OF $\ln^2 N \Rightarrow \left. \frac{\ln \ln 1/x}{\ln 1/x} \right|_+$
- PARTONIC CROSS SECTION: CONTAINS $\psi_i(N) \Rightarrow$ POLES AT $N = 0$: MELLIN OF $\left. \frac{\ln(1-x)}{1-x} \right|_+$
- CAN USE ANALYTICITY TO CONSTRUCT OPTIMAL SOFT APPROX (Bonvini, s.f., Marzani, Ridolfi, 2013)
- CAN CLASSIFY NEXT-TO-EIKONAL TERMS BASED ON KINEMATIC AND DIAGRAMMATIC ARGUMENTS (Laenen, Magnea, Stavenga, White, 2008-2011)
- COULD ONE CONSTRUCT DOUBLE-RESUMMED RESULTS, WITH CORRECT ALL-ORDER SINGULARITIES?

RESUMMATION vs APPROXIMATIONS

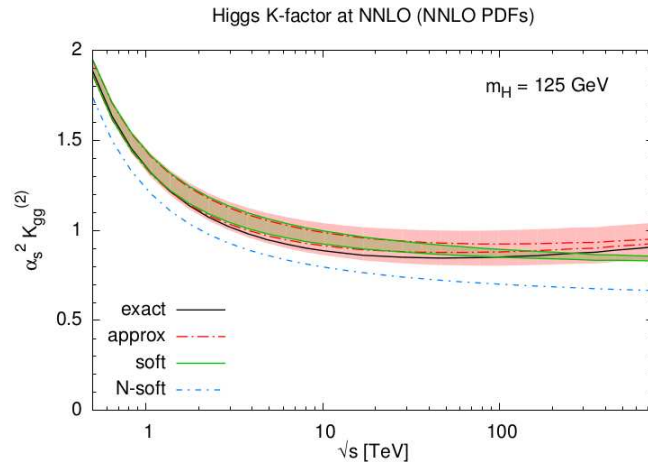
- APPROXIMATE HIGHER ORDERS FROM THRESHOLD RESUMMATION ARE ROUTINELY USED FOR JETS (Kidonakis, Owens, 2001); IMPLEMENTED IN STANDARD CODES (`fastNLO`, (Kluge, Rabbertz, Wobisch)) & USED E.G. FOR PDF FITS
- DOES NOT SEEM TO WORK VERY WELL FOR JETS AT LHC
- AVAILABLE FOR SEVERAL PROCESSES (PROMPT PHOTON, SINGLE TOP, W JET...) (Kidonakis et al, 200s)
- RECENTLY COMBINATION OF SMALL AND LARGE x ATTEMPTED FOR NNLO HEAVY QUARKS (Moch, Uwer, Vogt, 2012), DID NOT AGREE WELL WITH FINAL EXACT RESULT
- SUGGESTED FOR N^3 LO HIGGS (Bonvini et al, 2013) BASED ON “OPTIMAL” APPROX BASED ON ANALYTICITY

RESUMMATION vs APPROXIMATIONS:

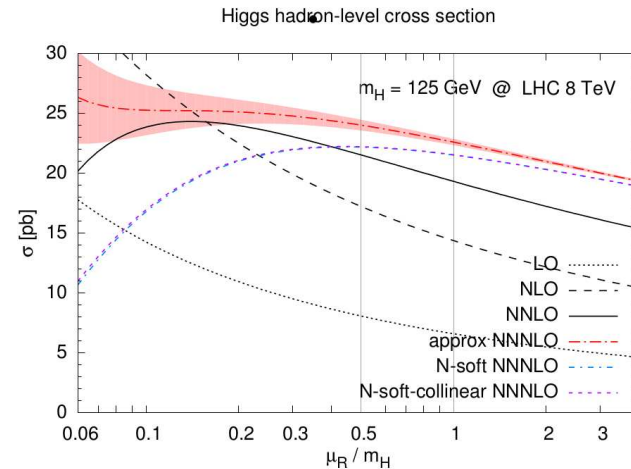
HIGGS AND PERTURBATIVE RESUMMATION

HIGGS IN GLUON FUSION

NNLO K -FACTOR VS CM ENERGY



N^3 LO @ LHC8 VS RENORM. SCALE



- NNLO: ‘OPTIMAL’ FOUND TO AGREE MUCH BETTER WITH EXACT THAN (α_s^2) SOFT FROM RESUMMATION+MP
- ‘OPTIMAL’ N^3 LO (FROM ANALYTICITY) FOUND TO CORRECT NNLO BY ABOUT 15% IF $\mu_r = m_H$
- IF N^3 LO TAKEN FROM RESUMMATION+MP (Moch, Vogt, 2005) CORRECTION IS ABOUT 6%
- RESUMMED RESULT (de Florian, Grazzini, 2012) CORRECTS BY 8%, REMAINING 2% FROM HIGHER ORDERS $\Rightarrow$ RESUMMATION IS PERTURBATIVE
SHOULD ONE CHOOSE DIFFERENT μ_r (m_h VS $m_h/2$) AT RESUMMED LEVEL?
- COLLINEAR IMPROVEMENT (SUB-EIKONAL) HAS LARGE EFFECT IF OPTIMAL APPROX USED, NEGLIGIBLE IF MP USED

RESUMMING CONSTANTS?

- RECALL $C_{\text{res}}(N, \alpha_s) = g_0(\alpha_s) \exp \left[\frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \dots \right];$
 $g_0(\alpha_s) = 1 + \alpha_s g_{0,1} + \alpha_s^2 g_{0,2} + \mathcal{O}(\alpha_s^3);$
- IN PRACTICE, g_0 **COEFFNS MAY BE LARGE**: FOR HIGGS $g_{0,1} \sim 10, g_{0,2} \sim 40$
- **SIZABLE PART OF THESE COEFFICIENTS COMES FROM MELLIN TRANSFORM WHICH RELATES $\frac{\ln^{k-1}(1-x)}{1-x}$ TO $\ln^k N$**
- AFTER SUBTRACTING THIS, FOR HIGGS, NON-MELLIN $\bar{g}_{0,1} \sim 5, \bar{g}_{0,2} \sim 10$

- SHOULD WE **EXPONENTIATE** IT?
- **CLASSIC OBSERVATION** (Parisi, 1980): FOR DRELL-YAN/HIGGS, RESUM $\ln^2 - M_X^2 \sim \pi^2$
- **CLASSIFICATION OF UNIVERSAL CONSTANTS** AVAILABLE (Eynck, Laenen, Magnea, 2003)

	DIS		$\overline{\text{MS}}$	
	ESTIMATE	EXACT	ESTIMATE	EXACT
COEFFICIENTS OF C_F^2				
$l = 0$	38.06	39.03	3.33	1.79
$l = 1$	- 13.09	- 14.41	0	0
$l = 2$	9.85	9.85	5.16	5.16
COEFFICIENTS OF $C_A C_F$				
$l = 0$	5.63	18.12	13.12	6.82
$l = 1$	9.83	- 0.25	1.51	- 0.47
$l = 2$	- 0.69	0.35	0	2.08
COEFFICIENTS OF $n_f C_F$				
$l = 0$	- 1.02	- 4.40	- 2.38	- 0.80
$l = 1$	- 1.79	0.35	- 0.27	- 0.52
$l = 2$	0.12	- 0.15	0	- 0.56

CONSTANTS AND SCALES

HIGGS IN GLUON FUSION

- IN SCET APPROACH, RESUMMATION CHARACTERIZED BY A SOFT SCALE μ_s :
EG THRESHOLD RESUMMATION $\Leftrightarrow \mu_s = \frac{Q^2}{N}$ (Manohar, Pecjak, Idilbi, Ji, 2003-2005)
- π^2 RESUMMATION NATURALLY OBTAINED BY CHOOSING $\mu_s^2 \sim -m_h^2$
CLAIMED TO LEAD TO LARGE CORRECTION TO RESUMMED HIGGS
CROSS-SECTION 5-6% (Ahrens, Becher, Neubert, Yang, 2008)
- USE OF A HADRONIC SCALE $\mu_s^2 = Q^2(1 - \tau)^2$ SUGGESTED
NO LANDAU POLE IN $\alpha_s \rightarrow$ PERTURBATIVE DIVERGENCE REMOVED (Becher, Neubert, 2006)
- IN SCET W. HADRONIC SCALE CHOICE, NNNLL RESUMMATION FOR HIGGS
HAS VERY LITTLE IMPACT: 1% VS 7% OF CONVENTIONAL APPROACH
(Ahrens, Becher, Neubert, Yang, 2008)
- WHAT IS THE RELATION OF SCET WITH HADRONIC SCALE CHOICE TO
STANDARD THRESHOLD RESUMMATION?

THRESHOLD RESUMMATION: SCET vs STANDARD

(Bonvini, Ghezzi, SF, Ridolfi, 2012)

SCET RESULT DIFFERS FROM STANDARD PERTURBATIVE RESULT THROUGH

- THE **REPLACEMENT OF $\frac{Q^2}{N^2}$ WITH THE SOFT SCALE μ_s^2** :

$$\exp \int_{M^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} \bar{\gamma} \rightarrow \exp \int_{M^2}^{\mu_s^2} \frac{d\mu^2}{\mu^2} \bar{\gamma}$$

- **THE PRESENCE OF A MATCHING FUNCTION**

$$C_{\text{QCD}}(N, M^2) = C_r(N, M^2, \mu_s^2) C_{\text{SCET}}(N, M^2, \mu_s^2)$$

$$C_r(N, M^2, \mu_s^2) = \frac{\exp \int_{\mu_s^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} \bar{\gamma} \left(\alpha_s(\mu^2), \frac{M^2}{N^2 \mu^2} \right)}{E(N, M^2, \mu_s^2)}$$
$$= e^{\int_{\mu_s^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} \left[\bar{\gamma} \left(\alpha_s(\mu^2), \frac{M^2}{N^2 \mu^2} \right) - \gamma_E \left(\alpha_s(\mu^2), \frac{M^2}{N^2 \mu^2} \right) \right]}$$

- RESUMMATION $\bar{\gamma} \left(\alpha_s(\mu^2), \frac{M^2}{N^2 \mu^2} \right) = \left[-A(\alpha_s(\mu^2)) \ln \left(\frac{M^2/N^2}{\mu^2} \right) + B[\alpha_s(\mu^2)] \right]$
- MATCHING $\gamma_E \left(\alpha_s(\mu^2), \frac{M^2}{N^2 \mu^2} \right) = \frac{A_1 \alpha_s(\mu^2)}{4} \ln \left(\frac{M^2}{N^2} / \mu^2 \right) - \frac{A_1}{8} \beta(\alpha_s(\mu^2)) \ln^2 \left(\frac{M^2}{N^2} / \mu^2 \right)$

MATCHING FUNCTION $\Rightarrow$ **REMOVES** SCALE MISMATCH UP TO GIVEN LOG ACCURACY

RESUMMATION SCALE CHOICES

$$C_{\text{QCD}}(N, M^2) = C_r(N, M^2, \mu_s^2) C_{\text{SCET}}(N, M^2, \mu_s^2)$$

$$C_r(N, M^2, \mu_s^2) = e^{\int_{\mu_s^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} \left[\bar{\gamma} \left(\alpha_s(\mu^2), \frac{M^2}{N^2 \mu^2} \right) - \gamma_E \left(\alpha_s(\mu^2), \frac{M^2}{N^2 \mu^2} \right) \right]}$$

- $\mu_s = M^2/N^2 \Rightarrow$ **MELLIN-SPACE QCD & SCET EXPRESSIONS COINCIDE**
- **DIVERGENCE OF** $C_{\text{QCD}}(N, M^2)$ **CONTAINED IN** $C_r(N, M^2, \mu_s^2)$
- WITH $\mu_s^2 = M^2(1 - z)^2$,
 - **SCET EXPRESSION COINCIDES WITH** z **SPACE PERTURBATIVE QCD** FOR $z < 1$
 - FOR $z = 1$ **FACTORIAL DIVERGENCE** (DUE TO MOMENTUM-NONCONSERVATION, UNRELATED TO LANDAU POLE) **ALREADY IN PERTURBATIVE QCD, WORSE IN SCET**
- IF $\mu_s^2 = M^2(1 - \tau)^2$
 - RESUMMATION EXPRESSED IN TERMS OF A PHYSICAL (HADRONIC) SCALE $\Rightarrow$ **NO LANDAU POLE**
 - **PERTURBATIVE FACTORIZATION SPOILED** (PARTON XSECT DEPS ON HADRON KINEMATICS)

$$\sigma_{\text{SCET}}(\tau, M^2) = \int_{\tau}^1 \frac{dz}{z} C_{\text{SCET}}\left(\frac{\tau}{z}, M^2, M^2(1 - \tau)^2\right) \mathcal{L}(z)$$

DOES NOT FACTORIZE UPON MELLIN TRANSFORM

- IF NO LOGS IN PARTON LUMI, SCET-QCD DIFFER BY SUBLEADING LOGS, BUT COULD BE SPOILED BY PDFs
- IF $\tau \ll 1$, $\mu_s^2 \sim M^2$ SO
 $C_r \sim \alpha_s^2(M^2) \ln^3 \frac{M^2}{N^2 \mu_s^2}$: **SCET RESUMMATION ALREADY FAILS AT NLL!**

HEAVY QUARKS

MULTISCALE PROBLEMS

CASE STUDY III: HEAVY QUARKS

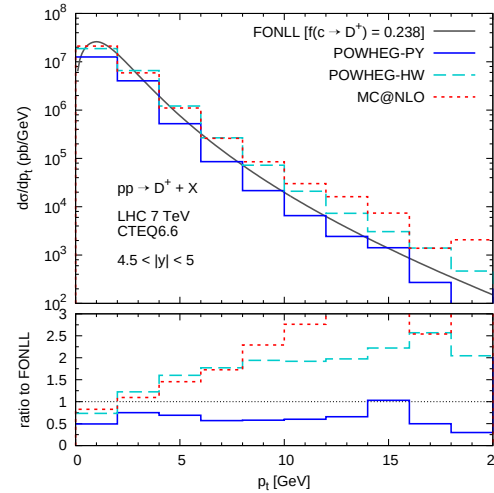
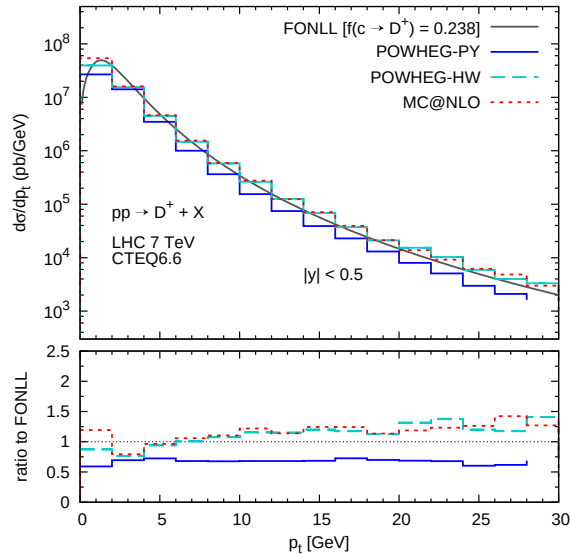
- COMBINATION OF RESUMMATION AND FIXED ORDER MANDATORY: FONLL
 - COMBINE FIXED ORDER CALCULATION (FO) WITH LEADING-LOG (E.G. p_T) RESUMMATION (NLL) (Cacciari, Greco, Nason, 1998)
 - ORIGINALLY PERFORMED AT NLO+NLL, EACH OF TWO INGREDIENTS CAN BE PUSHED TO HIGH ORDERS
 - AVAILABLE ALSO FOR PHOTOPRODUCTION (Cacciari, Frixione, Nason, 2001) & ELECTROPRODUCTION (S.F., Laenen, Nason, Rojo, 2010)
 - HADRONIZATION REQUIRES FRAGMENTATION FUNCTION
- ANALYTIC RESUMMATION MAY NOT BE ENOUGH: NLO+PS
 - COMBINE NLO CALCULATION WITH PARTON SHOWERING: POWHEG (Frixione, Nason, Ridolfi, 2007), MC@NLO (Frixione, Nason, Webber, 2003)
 - CAN MATCH TO HADRONIZATION, FULLY EXCLUSIVE DESCRIPTION OF FINAL STATE
 - ONLY A SUBSET OF LARGE LOGS $\ln P_T/m_q$ RESUMMED

CHARM PRODUCTION:

OPEN CHARM

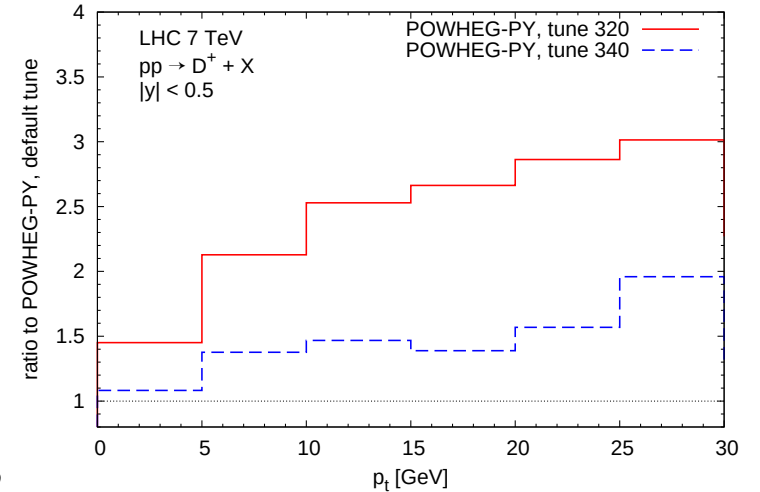
high rapidity

low rapidity



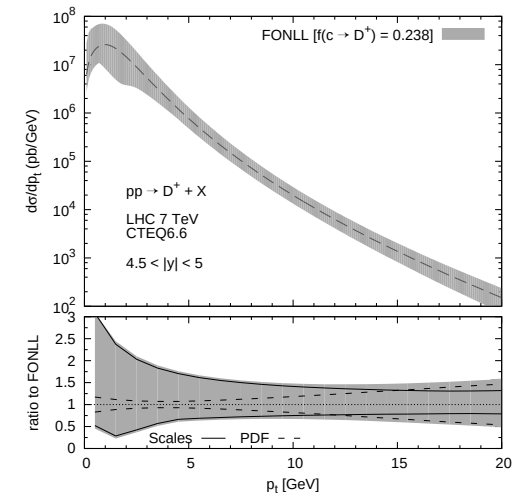
(Cacciari, Frixione, Houdeau, Mangano, Nason, Ridolfi 2012)

DEPENDENCE ON MC TUNES



high rapidity **TH. UNC.**

- **NONPERTURBATIVE FRAGMENTATION** WITH PARM FROM ALEPH
- **NLO+PS LOOSES ACCURACY** AT LARGE p_t
- **SIGNIFICANT DEPENDENCE ON MC TUNE**
- **PDF & SCALE UNCERTAINTY** COMPARABLE



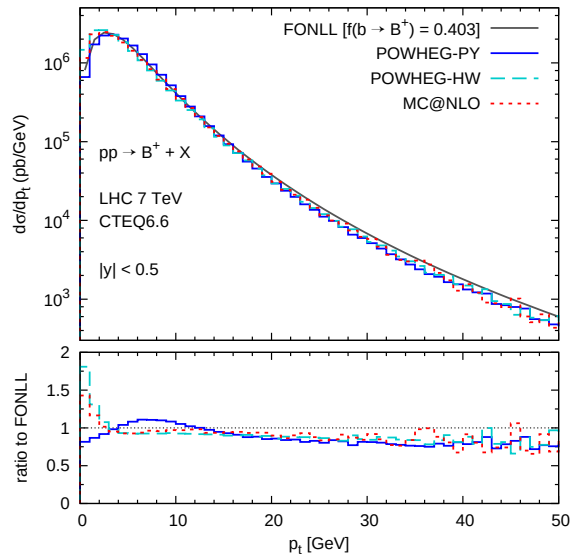
BOTTOM PRODUCTION:

PAST PROBLEMS & SOLUTIONS

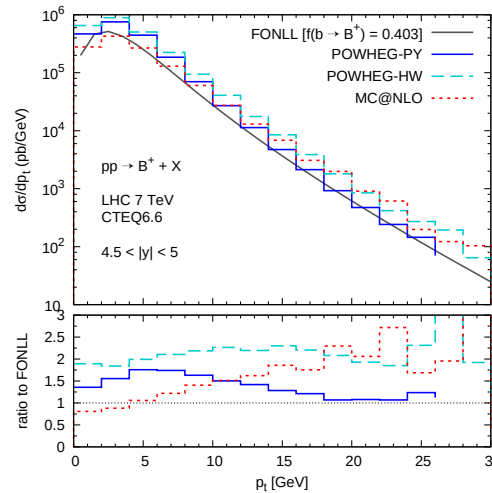
- FONLL $\Rightarrow$ CONSISTENT EXTRACTION OF FRAGMENTATION FUNCTIONS
- FONLL $\Rightarrow$ ENHANCEMENT AT LARGE p_T
- MC@NLO/POWHEG: ACCURATE TREATMENT AT LOW p_T

OPEN BOTTOM FROM $H_b \rightarrow J/\psi, \psi(2s)$

low rapidity

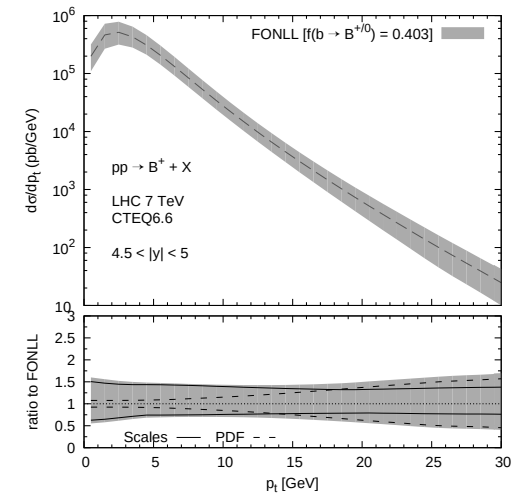


high rapidity



(Cacciari, Frixione, Houdeau,
Mangano, Nason, Ridolfi 2012)

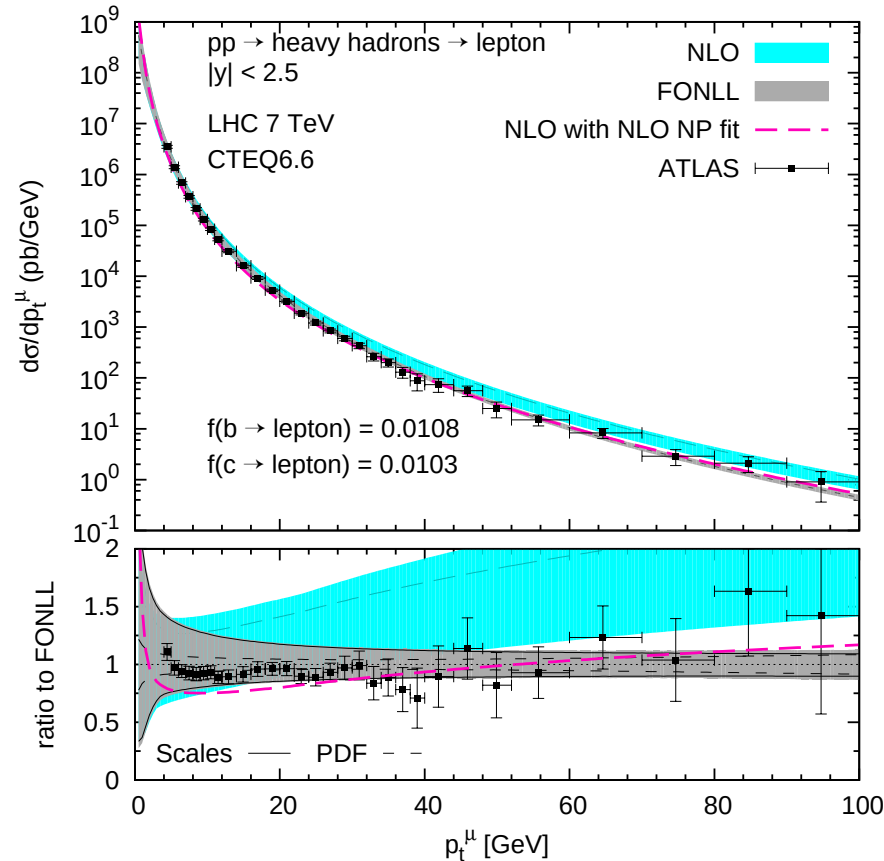
high rapidity: th. unc.



- DEPENDENCE ON MC TUNE MODERATE
- FRAGM. FUNCTION FITTED TO A VARIETY OF LEP DATA
- UNCERTAINTY PATTERN SIMILAR TO CHARM, BUT SMALLER

CHARM+BOTTOM PRODUCTION:

OPEN CHARM AND BOTTOM FROM SEMILEPTONIC DECAYS



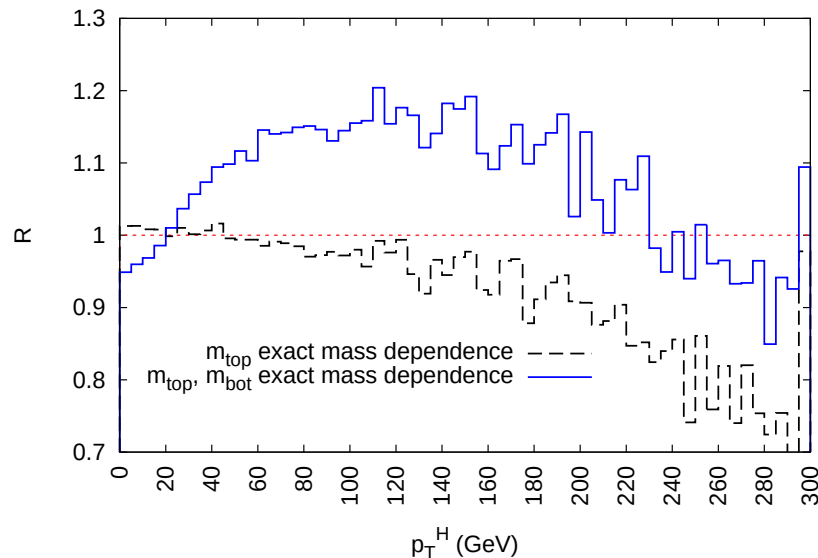
(Cacciari, Frixione, Houdeau, Mangano, Nason, Ridolfi 2012)

- SUM OF $H_c \rightarrow \ell$, $H_b \rightarrow \ell$, $H_b \rightarrow H_c \rightarrow \ell$
- **SIGNIFICANT DISCREPANCY** BETWEEN FONLL & MC@NLO @ LARGE p_T
- **REABSORBED** IF NLO ALSO USED FOR FRAGMENTATION FUNCTION FIT
BUT CAN ONLY USE WHEN $p_t \sim M_Z$ SCALE OF FRAGMENTATION FUNCTION

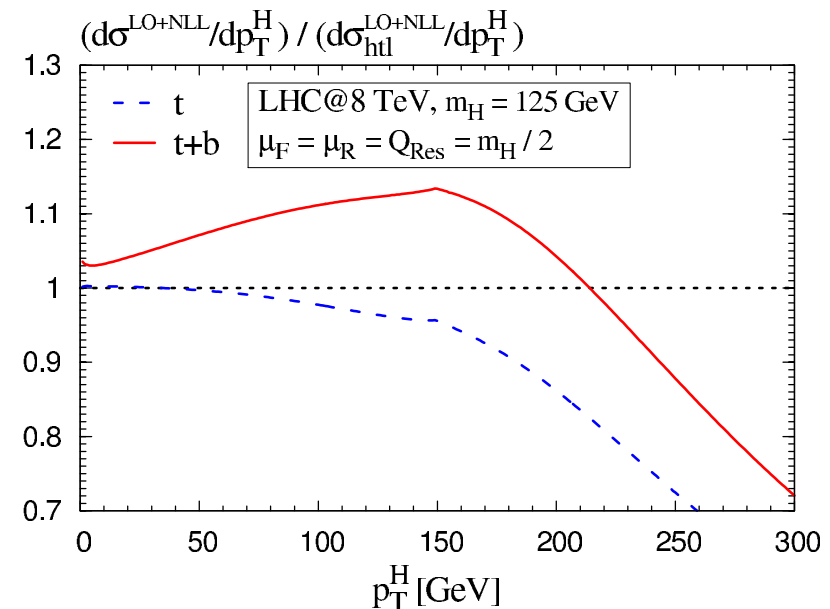
HQ MASSES IN HIGGS PRODUCTION

- AN **EXTREME MULTISCALE** PROBLEM: p_T SPECTRUM IN HIGGS PRODUCTION IN GLUON FUSION
- SCALES: m_t , m_b , m_H , p_T , s
- **EFFECT OF FINITE** m_B IS **VISIBLE** AT SMALL p_T
- **DISCREPANCY** BETWEEN POWHEG AND NLO+NLL COMPUTATION
- **GLUONS** WITH $m_b \lesssim p_T \lesssim m_H$ ARE **TREATED AS SOFT** IN **NLO+NLL** (NO EFFECT ON b LOOP), WHILE THEY **DO AFFECT THE b LOOP** IN **POWHEG** APPROACH $\rightarrow$ **NEITHER** APPROACH **FULLY CAPTURES** ALL THE RELEVANT PHYSICS

FINITE m_t , m_b , RATIO TO POINTLIKE
POWHEG NLO+NLL



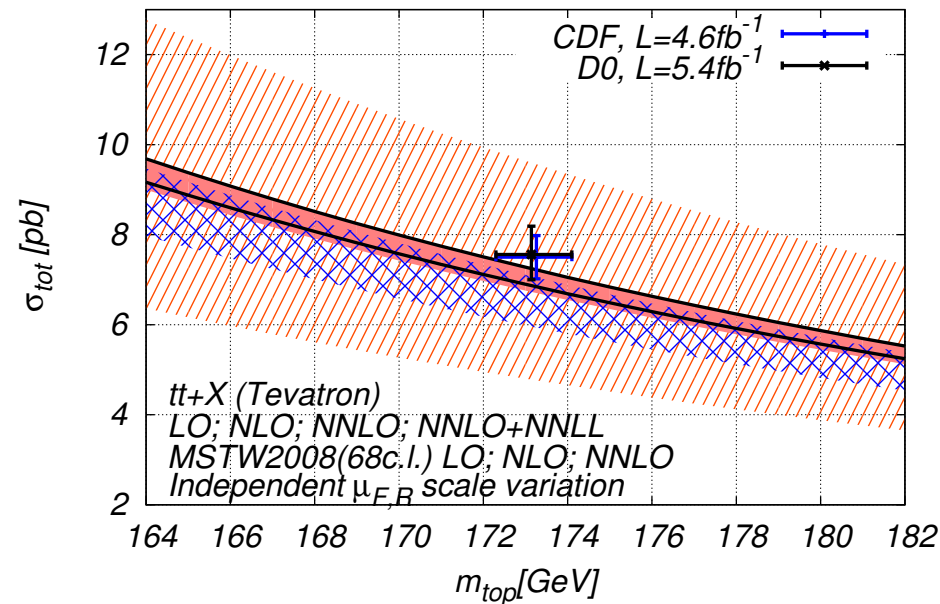
(Bagnaschi, Deggrasi, Slavich, Vicini, 2012)



(Mantler, Wiesemann, 2012)

TOP PRODUCTION

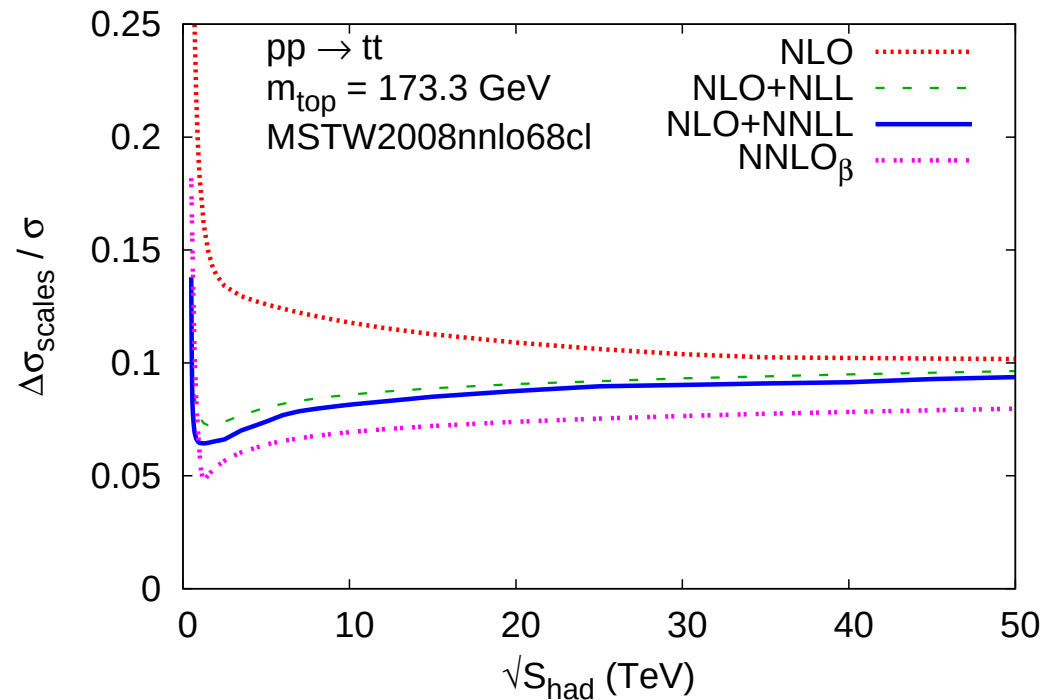
- RECENTLY FIXED-ORDER RESULT DETERMINED UP TO NNLO (Czakon, Mitov+ coll, 2012-2013)
- NNLL THRESHOLD RESUMMATION OF $\ln \beta$ ($\beta = \sqrt{1 - 4m_t^2/s}$) AVAILABLE (Czakon, Mitov, Sterman; Beneke, Falgari, Schwinn, 2009)
- RESUMMATION OF “COULOMB” $\frac{1}{\beta}$ KNOWN, NEGLIGIBLE
- PERTURBATIVE RESUMMATION AGAIN:
- RESUMMATION HAS MINOR IMPACT ON CENTRAL VALUE, BUT DOES REDUCE UNCERTAINTY ON TOTAL XSECT AT THE TEVATRON



(Bärnreuther, Czakon, Mitov, 2012)

TOP PRODUCTION

TH UNCERTAINTY (SCALE VARIATION)



(Cacciari, Czakon, Mangano, Mitov, Nason, 2012)

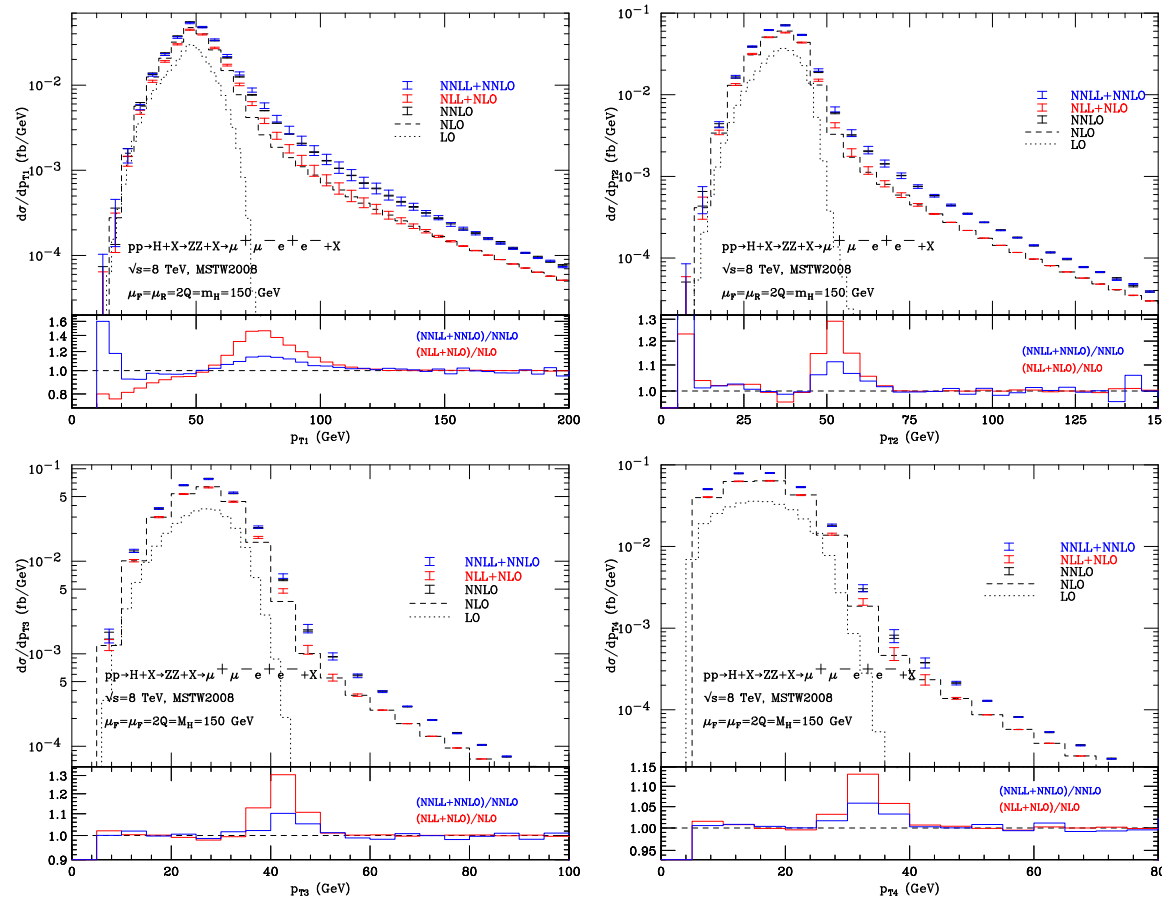
- ..AND **EVEN AT THE LHC** (THOUGH **IMPACT ON CENTRAL VALUE LESS** THAN 1%)
- **MODERATE BUT NON-NEGLIGIBLE IMPACT** ON CROSS SECTION
- IN SCET APPROACH (Beneke, Falgari, Schwinn, 2009), μ_s CHOSEN AS PARTONIC SCALE, FACTORIAL DIVERGENCE AS $\beta \rightarrow 1$ TREATED BY FREEZING

JETS

EXCLUSIVE FINAL STATES IN HIGGS PRODUCTION

- COMBINATION OF FIXED ORDER AND RESUMMED CALCULATIONS (THRESHOLD+ p_T)
- $\Rightarrow$ MORE ACCURATE DESCRIPTION OF FULLY EXCLUSIVE FINAL STATES
- FULL COMBINATION OF ALL PRODUCTION CHANNELS+DECAY, AVAILABLE FOR ALL DECAY CHANNELS

p_T SPECTRA OF FOUR FINAL-STATE LEPTONS IN $pp \rightarrow H + X \rightarrow \mu^+ \mu^- e^+ e^- + X$

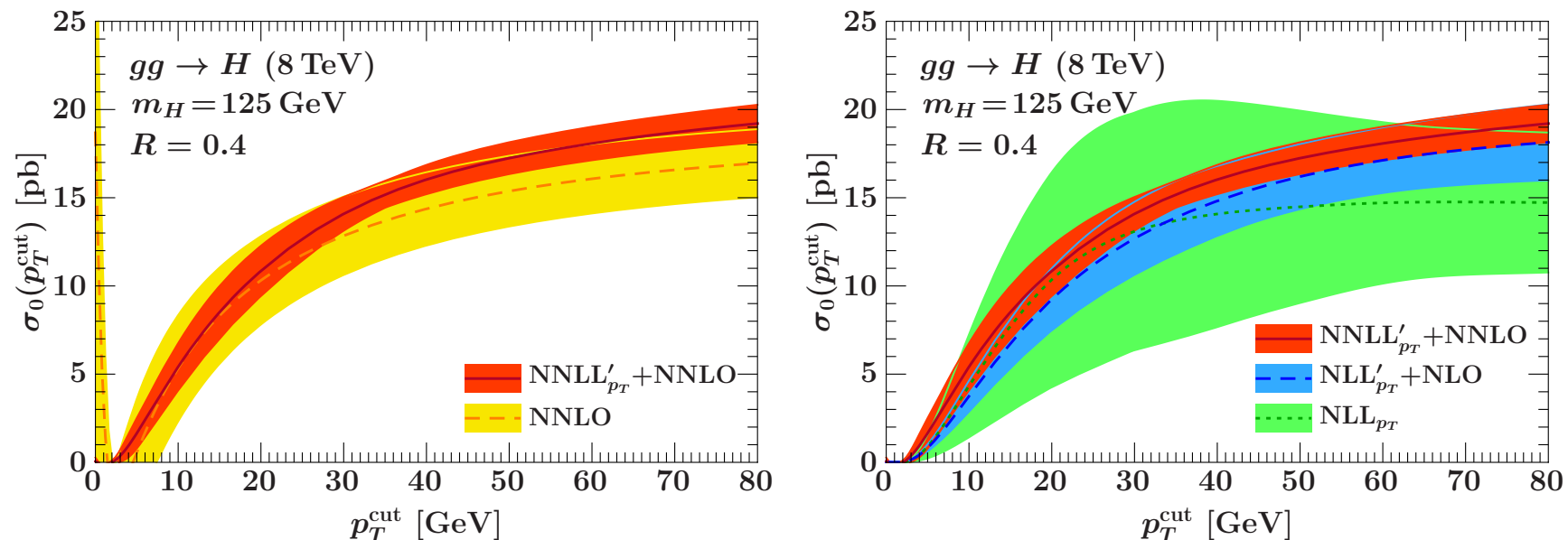


(de Florian, Ferrera, Grazzini, Tommasini, 2012),

HIGGS + JETS & JET VETOS

- CROSS-SECTION FOR HIGGS+ AT LEAST ONE JET CONTAINS DOUBLE LOGS OF MINIMAL p_T OF JET
- $\Rightarrow$ **SO DOES CROSS SECTION WITH JET VETO:**
 $\sigma_{\geq 1} \sim (\alpha L^2)^n$; $\sigma_{tot} \sim \alpha^n \rightarrow \sigma_o \equiv \sigma_{tot} - \sigma_{\geq 1} \sim (\alpha L^2)^n$;
- **RESUMMATION PERFORMED** UP TO NNLL+NNLO BOTH WITH PERTURBATIVE APPROACH (Banfi, Monni, Salam, Zanderighi, 2012) & **SCET** (Stewart, Tackmann, Walsh, Zuberi, 2012-2013; Becher, Neubert, Rothen, 2013)
- EXCLUSIVE ONE-JET CROSS-SECTION RESUMMED USING SCET (Liu, Petriello 2012)
- **GOOD PERTURBATIVE STABILITY; SIGNIFICANTLY IMPROVED UNCERTAINTY**

0-JET CROSS-SECTION FOR HIGGS IN GLUON FUSION



(Stewart, Tackmann, Walsh, Zuberi, 2013)

JET VETOS

EFFICIENCIES AND UNCERTAINTIES

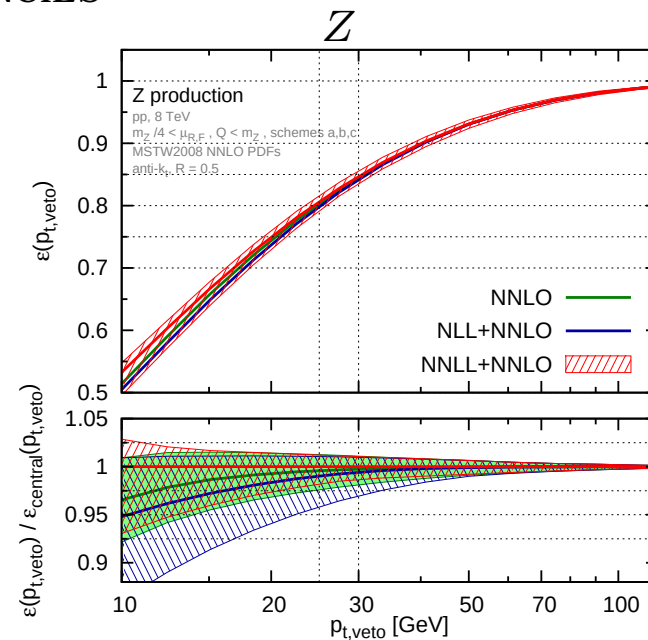
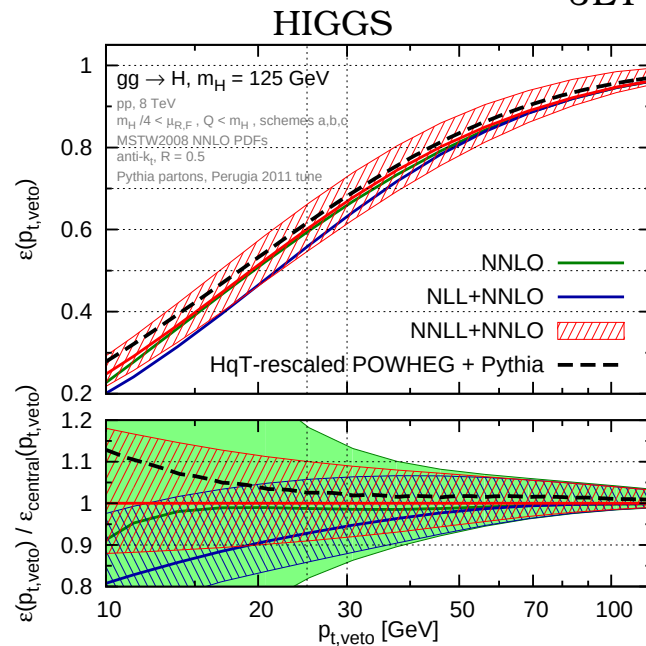
- EXPERIMENTALLY, ONE IS INTERESTED IN THE **EFFICIENCY**, DEFINED AS

$$\epsilon_0(p_T^{\text{cut}}) \equiv \frac{\sigma_0(p_T^{\text{cut}})}{\sigma_{\text{tot}}}$$

- SUBTLE **ISSUES IN UNCERTAINTY ESTIMATION**:

- WHAT IF ONE INSTEAD DEFINES $\epsilon_0(p_T^{\text{cut}}) \equiv 1 - \frac{\sigma_{\geq 1}(p_T^{\text{rmcut}})}{\sigma_{\text{tot}}}$?
EQUIVALENT UP TO HIGHER ORDERS
- HOW DOES ONE **KEEP INTO ACCOUNT CORRELATIONS** BETWEEN UNCERTAINTIES?

JET VETO EFFICIENCIES

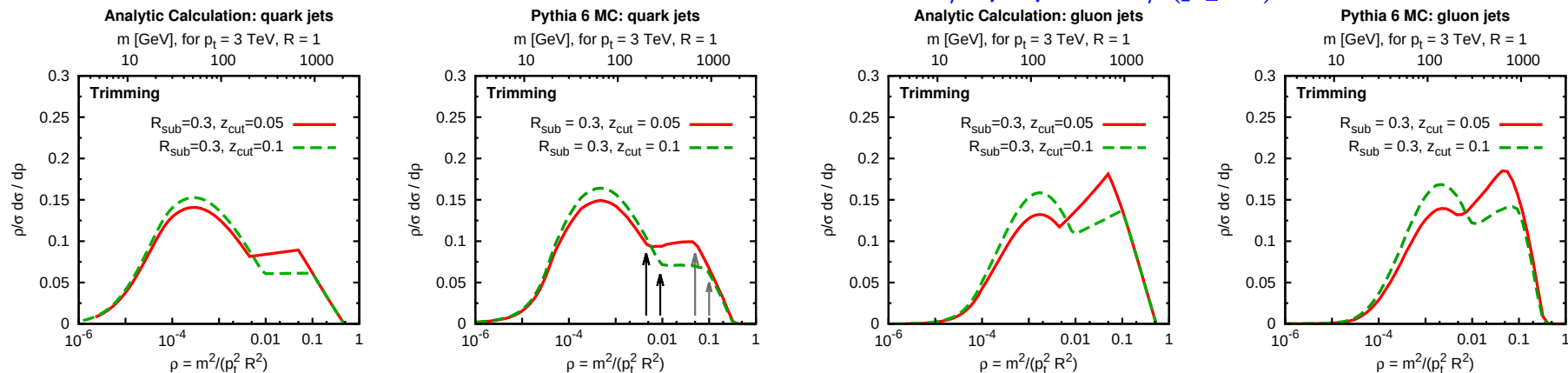


(Banfi, Monni, Salam, Zanderighi, 2012)

JET SHAPES AND SUBSTRUCTURE

- SEVERAL JET PROPERTIES STUDIED ANALYTICALLY AND RESUMMED
EXTENSION OF SEMINAL WORK ON NLO+NNLO (Banfi, Salam, Zanderighi 2003-2010)
 - BROADENING (MOMENTUM ORTHOGONAL TO THRUST AXIS, $b_t = \sum_i |p_i^T|$): NNLL RESUMMATION USING SCET (Becher, Bell, 2012)
 - INVARIANT MASS OF HARDEST JET RESUMMED AT NLL USING PERTURBATIVE (Dasgupta, Khelifa-Kerfa, Marzani, Spannowsky, 2012) OR SCET AT NLL (Chien, Kelley, Schwartz, Zhu, 2012) AND NNLL (Jouttenus, Stewart, Tackmann, Waalewijn, 2013)
 - GENERALLY “NON-GLOBAL” LOGS WHICH CHARACTERIZE A SINGLE JET RESUMMED TO NLL (Dasgupta, Salam, 2001; Banfi, Dasgupta, Khelifa-Kerfa, Marzani, 2012)
 - DIJET INVARIANT MASS SPECTRA RESUMMED USING MULTISCALE EXTENSION OF SCET HIERARCHY $m_l \lesssim M_{jj} \lesssim \hat{s}$ (Bauer, Tackmann, Walsh, Zuberi, 2012)
 - ...
- ANALYTIC RESULTS AVAILABLE FOR JET SUBSTRUCTURE TOOLS (TRIMMING, PRUNING...)
 - SUCCESSFUL COMPARISON WITH MONTE CARLO (Dasgupta, Fregoso, Marzani, Salam, 2012)

TRIMMED JET MASS DISTRIBUTION $d\sigma/d\rho$; $\rho = m/(p_T R)$

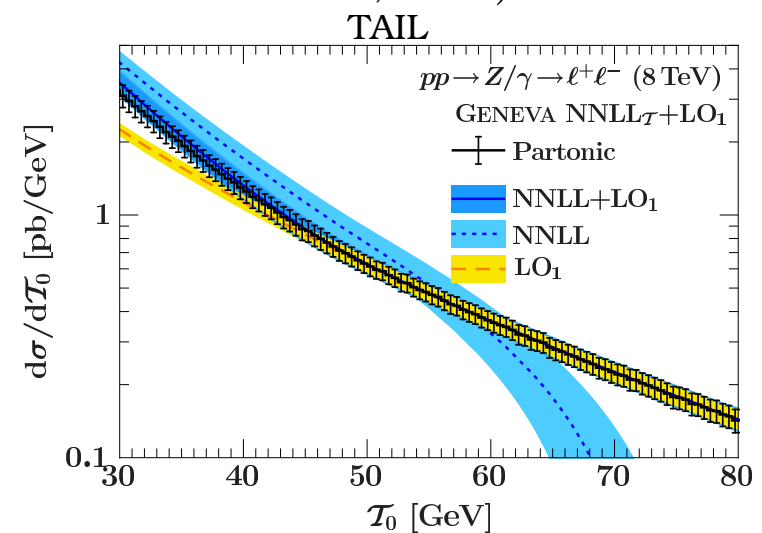
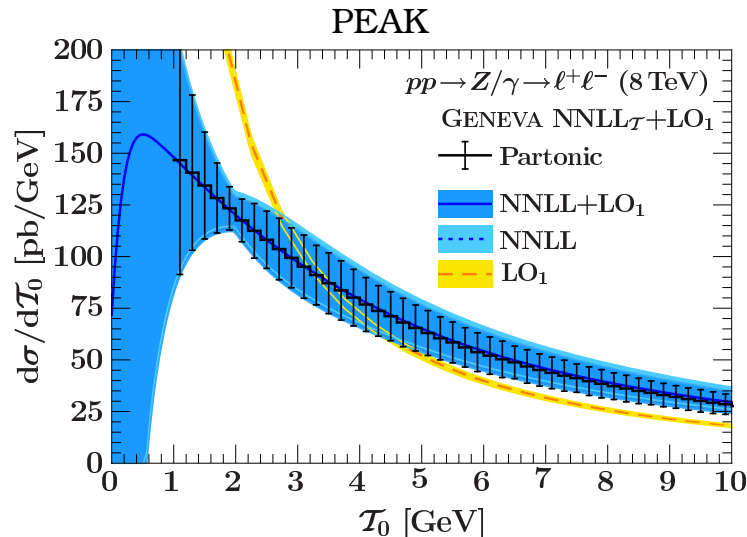


OUTLOOK

THE FRONTIER

- **STRETCHING FACTORIZATION: DOES FACTORIZATION HOLD NOT ONLY FOR INCLUSIVE PROCESSES, BUT ALSO FOR PARTONIC AMPLITUDES?**
 - **NO: VIOLATIONS AT THREE-LOOP LEVEL** FOR PROCESSES WITH TWO INCOMING COLORED PARTONS (Catani, de Florian, Rodrigo, 2012; Forshaw, Seymour, Siodmok, 2013)
 - VIOLATION **CANCELS IN INCLUSIVE OBSERVABLE** (Aybat, Sterman, 2012)
WILL SHOW UP IN OBSERVABLES ABOVE THE SCALE UP TO WHICH REAL RADIATION IS INCLUSIVE
 - **RELATED TO UNDERLYING EVENT:** FACTORIZATION VIOLATION MUST BE VIEWED AS PERTURBATIVELY CALCULABLE CORRECTION TO IT
IMPORTANT FOR MATCHING TO MONTE CARLO
- **MATCHING TO MONTE CARLO:** MANY EFFORTS TO
COMBINE ANALYTIC RESUMMATION TO PARTON SHOWERING

BEAM THRUST IN DRELL-YAN (GENEVA: Alioli et al, 2012)



THE FUTURE

- CAN WE STRECTCH FACTORIZATION IN ORDER TO EXTEND RESUMMATION TO NEW PROCESSES/CONTEXTS?

EXAMPLE: SMALL x RESUMMATION BEYOND LL

- CAN WE DO A BETTER JOB AT COMBINING INFORMATION FROM DIFFERENT KINDS OF RESUMMATION?

EXAMPLE: COMBINED SMALL AND LARGE x RESUMMATION

- CAN WE DO A BETTER JOB AT MATCHING RESUMMATION TO MONTE CARLOS?

EXAMPLE: MC-RESUMMED PDFs