QCD threshold resummation for gluino pair production at NNLL

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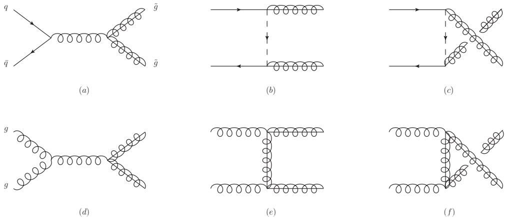
Outline

- Hadron colliders as the LHC are especially appropriated to produce color-charged particles, such as the gluino as Superpartner of the gluon in supersymmetric models.
- In models with R-parity conservation, SUSY particles can only be pair-produced.
- The gluino is special in that sense, that its inclusive hadronic production cross section basically depends on the cms energy $\sqrt{s}$ and the gluino mass $m_{\tilde{g}}$, but barely on the masses of the other SUSY particles.
- Production channels: gluon fusion, quark-antiquark annihilation (At NLO also gq annihilation, but suppressed near the threshold)
- The hadr. production cross section is mainly driven by gluon fusion (about 99%) !



State of the art

- Production cross section at NLO implemented in `Prospino(2)`
  [Beenakker, Hopker, Spira, Zerwas, 1997]
- Threshold limit with resummed threshold logarithms to NLL accuracy implemented in combination with the fixed NLO in the code `NLL-Fast`
  [Kulesza, Montyka, 2008,2009; Beenakker, Brensing, Krämer, Kulesza, Laenen, Motyka, Niessen, 2011]
- Bound-state effects at LO and NLO
  [Hagiwara, Yokaja, 2009; Kauth, Kühn, Marquard, Steinhauser, 2011]
- Combined soft and Coulomb resummation at NLL,
  extended discussion of finite-width effects [Falgari, Schwinn, Wever, 2012]
- Resummation of threshold logarithms to NNLL accuracy, approximated NNLO fixed order cross section [Langenfeld, Moch, TP, 2012; TP 2013]

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ATLAS and CMS analysis (see for instance arXiv:1208.0949 and 1212.6961)
[see also Krämer et al., 2012]

If gluinos are heavy, they are produced close to the threshold.

$\Rightarrow$ **Soft gluon exchange** is important. Do we have to go beyond NLL?

Threshold limit

Recall the (partonic) production cross section in the channel ij near threshold for a common renormalization and factorization scale $\mu_r = \mu_f = \mu$:

$$\sigma_{ij}^{\mathrm{NLO}}(\mu = m_{\tilde{g}}) = \sigma_{ij}^{\mathrm{Born}} \left(1 + \frac{\alpha_s}{\pi} \left[A_{ij} \ln^2(\beta) + B_{ij} \ln(\beta) + \frac{C_{ij}}{\beta} + C_1^{ij}(r) + \mathcal{O}(\beta)\right]\right)$$

- $\beta = \sqrt{1-\rho}\,, \quad \rho = 4m_{\tilde{g}}^2/\hat{s}\,, \quad r = m_{\tilde{q}}^2/m_{\tilde{g}}^2$ (squark masses assumed to be degenerated)
- threshold limit $\beta \longrightarrow 0$
- Born cross section factors out
- $C_1^{ij}(r)$: One-loop matching constant extracted from fixed NLO
- Soft and collinear gluon radiation $\Rightarrow$ $\ln^k(\beta)$
  Soft-gluon exchange between the final-state particles $\Rightarrow$ $1/\beta^l$
  (Coulomb corrections)
  - NLO: $k \leq 2\,, \quad l = 1$
  - NNLO: $k \leq 4\,, \quad l \leq 2\,,$ interference terms

1. Gluino pair production in a nutshell
2. Threshold resummation
3. Hadronic production cross section

Mellin-space approach

Threshold logarithms can be resummed to all orders in perturbation theory. This can be done in **Mellin space** for instance [Sterman 1987; Catani, Trentadue, 1989]

$$\hat{\sigma}_{ij}^{N}(m_{\tilde{g}}^2) = \int_0^1 d\rho\, \rho^{N-1}\, \hat{\sigma}_{ij}(\hat{s}, m_{\tilde{g}}^2)$$

- The threshold limit $\beta \to 0$ corresponds to $N \to \infty$.
- The resummation requires a decomposition of the amplitude w.r.t. color configurations $\mathbf{I}$ of the final-state gluino pair: $\hat{\sigma}_{ij\to\tilde{g}\,\tilde{g}} = \sum_{\mathbf{I}} \hat{\sigma}_{ij,\mathbf{I}}$

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Neglecting Coulomb terms, the general resummation formula reads:

$$\boxed{\hat{\sigma}_{ij,\mathbf{I}}^{N}(m_{\tilde{g}}^2) = \hat{\sigma}_{ij,\mathbf{I}}^{N,B}(m_{\tilde{g}}^2)\, g_{ij,\mathbf{I}}^{0}(m_{\tilde{g}}^2)\, \exp\Big[G_{ij,\mathbf{I}}(N+1)\Big] + \mathcal{O}(N^{-1}\ln^n N)}$$

$$G_{ij,\mathbf{I}}(N) = \underbrace{\ln(N)\cdot g_{ij}^{1}(\lambda)}_{\text{LL}} + \underbrace{g_{ij,\mathbf{I}}^{2}(\lambda)}_{\text{NLL}} + \underbrace{\frac{\alpha_s}{4\pi}\, g_{ij,\mathbf{I}}^{3}(\lambda)}_{\text{NNLL}} + \dots\,, \qquad \lambda = \frac{\alpha_s}{4\pi}\,\beta_0 \ln N$$

Exponent of the resummation formula

$$G_{ij,\mathbf{I}}(N) = \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \left\{ \int_{\mu_f^2}^{4m_{\tilde{g}}^2(1-z)^2} \frac{dq^2}{q^2} \Big(A_i\big(\alpha_s(q^2)\big) + A_j\big(\alpha_s(q^2)\big)\Big) + D_{ij,\mathbf{I}}\big(\alpha_s(4m_{\tilde{g}}^2(1-z)^2)\big) \right\}.$$

where $D_{ij,\mathbf{I}}(\alpha_s) = \frac{1}{2}\big(D_i(\alpha_s) + D_j(\alpha_s)\big) + D_{\tilde{g}\tilde{g},\mathbf{I}}(\alpha_s)$

Anomalous dimension functions:

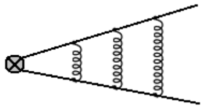
- $A_{i,j}$: initial-state collinear gluon radiation (process independent)
- $D_{i,j}$: initial-state soft radiation (process independent)
- $D_{\tilde{g}\tilde{g},\mathbf{I}}$: final-state soft radiation

The process independent functions are known up to $\mathcal{O}(\alpha_s^3)$ from top-quark pair production [Kodaira, Trentadue, 1982; S. Moch, J. .A. M. Vermaseren, A. Vogt, 2004]

The functions $D_{\tilde{g}\tilde{g},\mathbf{I}}$ are known to $\mathcal{O}(\alpha_s^2)$ [Beneke, Falgari, Schwinn, 2010]

Coulomb corrections

- Coulomb corrections can be summed in momentum (β)-space to all orders by means of NRQCD (pure Coulomb terms $\longrightarrow$ Sommerfeld factor).
  [Fadin, Khoze,Sjostrand, 1990; Beneke, Kiyo, Schuller, 2005]



- Interference of soft and Coulomb corrections requires combined soft and Coulomb resummation in momentum space.
  [Beneke, Falgari, Klein, Schwinn, 2010; 2011]
- A resummation in Mellin space would require the Mellin transforms of the NRQCD expressions at least in a semi-analytical form. Not available.

  $\Rightarrow$ include Coulomb terms at fixed order
  $\Rightarrow$ The matching constant depends on N: $g^{0}_{ij,\,\mathrm{I}}(m^2_{\tilde{g}}) \rightarrow g^{0}_{ij,\,\mathrm{I}}(N, m^2_{\tilde{g}})$

Color-decomposed NLO cross section

- For NNLL resummation one needs the color decomposition of the NLO cross section.
- We have taken results from gluino-bound state production.

  [Kauth, Kühn, Marquard, Steinhauser, 2011]

  Idea:

  In the threshold approx., only the Born term distinguishes between the $2 \to 1$ and the $2 \to 2$ process, the virtual and real quantum corrections agree. $\Rightarrow$ Replace Born term

$$\boxed{\hat{\sigma}_{ij\to T,\mathrm{I}} = \hat{\sigma}^{\mathrm{Born}}_{ij\to T,\mathrm{I}} \left(1 + \frac{\alpha_s(\mu_r)}{\pi}\,\overline{\mathcal{V}}_{ij,\mathrm{I}}\right)\left[\delta(1-z) + \frac{\alpha_s(\mu_r)}{\pi}\,\overline{\mathcal{R}}_{ij,\mathrm{I}}(z)\right]}$$

- Decomposition:
  - $ij = gg$: $\quad \mathbf{8} \times \mathbf{8} = \mathbf{1}_s + \mathbf{8}_s + \mathbf{8}_a + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}_s$

    only symmetric contributions near t.h.
  - $ij = q\bar{q}$: $\quad \mathbf{3} \times \bar{\mathbf{3}} = \mathbf{1}_s + \mathbf{8}_s + \mathbf{8}_a$

    only anti-symmetric contributions near t.h.

NLO results and one-loop matching constants

$$C_{1,\mathbf{I}}^{gg\,\overline{\mathrm{MS}}}(r) = C_{\mathbf{I}}\left(4+\ln(2)-\frac{\pi^2}{8}\right)+36-6\ln^2(2)-\pi^2+\frac{n_f}{18}A_{\mathbf{I}}^{gg}(r)$$

$$C_{1,\mathbf{8}_a}^{q\bar{q}\,\overline{\mathrm{MS}}}(r) = n_f\left(\ln(2)-\frac{5}{9}\right)+\frac{181}{6}-\frac{8}{3}\ln^2(2)-\frac{43}{36}\pi^2-\frac{1}{3}\ln\left(\frac{m_t^2}{m_{\tilde{g}}^2}\right)-\frac{n_f}{6}\ln(r)+A_{\mathbf{8}_a}^{q\bar{q}}(r)$$

- Numerical cross check with Prospino near the threshold agrees for gluon fusion at the per-mill level, but differs in $q\bar{q}$-annihilation by several %.
- The deviation can be explained in part by a different treatment of m_t, which has only been kept within logarithms in the bound-state calculation.
- Recently, the color-decomposed NLO cross section has been recalculated. This revealed an error in $A_{\mathbf{8}_a}^{q\bar{q}}(r)$. [Beenakker et al. 2013]

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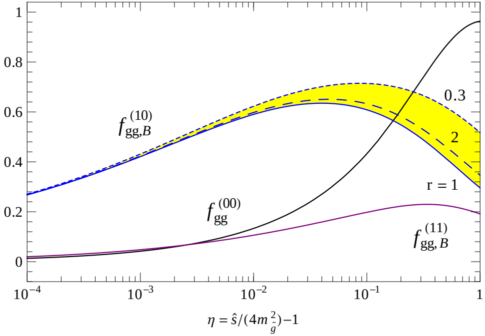
However, remember that the hadronic cross section is based on gluon fusion by about 99%!
$\Rightarrow$ Error negligible for practical purposes!

Dependence on $r = m_{\tilde{q}}^2 / m_{\tilde{g}}^2$

$$\hat{\sigma}_{ij,\mathbf{I}} = \frac{\alpha_s^2}{m_{\tilde{g}}^2}\left[f_{ij,\mathbf{I}}^{(00)} + 4\pi\alpha_s\left(f_{ij,\mathbf{I}}^{(10)} + f_{ij,\mathbf{I}}^{(11)} \ln\left(\frac{\mu^2}{m_{\tilde{g}}^2}\right)\right) + \mathcal{O}\left(\alpha_s^2\right)\right]$$

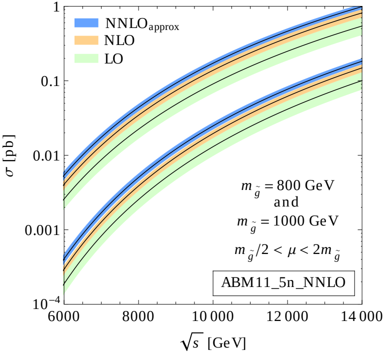
$f_{ij,B}^{(10)} = \sum_{\mathbf{I}} f_{ij,\mathbf{I}}^{(00)} f_{ij,\mathbf{I}}^{(10)}$, etc. $\qquad f_{ij,\mathbf{I}}^{(10)}$ and $f_{ij,\mathbf{I}}^{(11)}$ in the threshold limit



First phenomenological application: NNLO approximation

The expansion of the NNLL resummation formula up to $\mathcal{O}(\alpha_s^2)$ produces all threshold logarithms at NNLO. After the expansion, the Mellin inversion can easily be done analytically in the leading $1/N$ approximation. [Moch, Uwer, 2008]

Adding the $\mathcal{O}(\alpha_s^2)$ part to the full NLO result gives a first hint of the threshold effects. [formulas given in Langenfeld, Moch, TP, 2012]



Momentum-space approach

Originally introduced for DIS. [Becher, Neubert 2006; Becher, Neubert, Pecjak 2007]

- Uses framework of soft-collinear effective theory and, for heavy-particle pair production, is based on factorization of the cross section into a hard, a soft, and a potential function: [Beneke, Falgari, Schwinn, 2011]

$$\hat{\sigma}_{ij\to\tilde{g}\tilde{g}}(\hat{s},\mu) = \sum_S \sum_{\mathbf{I}} H^S_{ij,\mathbf{I}}(\mu) \int_0^\infty d\omega \, J^S_{\mathbf{I}}(E - \frac{\omega}{2}) \, W_{\mathbf{I}}(\omega,\mu)$$

  - $E = \sqrt{\hat{s}} - 2m_{\tilde{g}}$ = energy relative to the threshold
  - S = spin of the produced heavy-particle pair (enters at NNLO)
  - A convolution of the soft function with the potential function sums Coulomb terms.

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- Solving RGEs for $W_{\mathbf{I}}(\omega,\mu_s) \to W_{\mathbf{I}}(\omega,\mu_f)$ sums soft logs.
- Also evolve $H_{\mathbf{I}}(\mu_h) \to H_{\mathbf{I}}(\mu_f)$ for a separate treatment of the scale of the hard interaction and the factorization scale. $\Rightarrow$ Enhanced error estimation

Matching of the hard function

Master formula: [Beneke, Falgari, Schwinn, 2011]

$$\begin{aligned}\hat{\sigma}^{\mathrm{res}}_{ij\to\tilde{g}\tilde{g}}(\hat{s},\mu_f) \quad &= \quad \sum_S \sum_{\mathbf{I}} H^S_{ij,\,\mathbf{I}}(\mu_h)\, U_{ij,\,\mathbf{I}}(m_{\tilde{g}},\mu_h,\mu_s,\mu_f) \\ &\times \int_0^\infty d\omega\, \frac{J^S_{\mathbf{I}}\left(E-\frac{\omega}{2}\right)}{\omega} \left(\frac{\omega}{2m_{\tilde{g}}}\right)^{2\eta} \tilde{s}_{ij,\,\mathbf{I}}\left(2\ln\left(\frac{\omega}{\mu_s}\right)+\partial_\eta,\mu_s\right) \frac{e^{-2\gamma_E\,\eta}}{\Gamma(2\eta)}\end{aligned}$$

$U_{ij,\,\mathbf{I}}$ evolution function, $\tilde{s}_{ij,\,\mathbf{I}}$ Laplace transform of the soft function, η auxiliary variable

The soft and potential functions depend on the color configuration, but not on the specific process under consideration. Ingredients for NNLL resummation are given in the literature.

[see Becher, Neubert, Xu, 2008 (soft fuction), and Beneke, Falgari, Klein, Schwinn, 2011 (potential function)]

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One missing piece for gluino pair production: Hard function at NLO

Expand master formula in α_s and match to the fixed NLO result.

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$\overline{\mathrm{MS}}$ scheme, $\mu=\mu_r=\mu_f$, $L_\mu=\ln(\mu^2/m^2_{\tilde{g}})$: [TP, 2013]

$$H^{(1)}_{gg,\,\mathbf{I}}(\mu) = 4\,C^{gg}_{1,\,\mathbf{I}} - 12\,C_{\mathbf{I}} - C_A\left(64-\frac{11\pi^2}{3}\right) - \Big(2\,C_{\mathbf{I}} - 8\,C_A\ln(2)\Big)L_\mu - 2\,C_A L_\mu^2$$

$$H^{(1)}_{q\bar{q},\,\mathbf{8}_a}(\mu) = 4\,C^{q\bar{q}}_{1,\,\mathbf{8}_a} - 12\,C_A - C_F\left(64-\frac{11\pi^2}{3}\right) - \Big(2\,C_A - 8\,C_F\ln(2) - 2\beta_0 + 6\,C_F\Big)L_\mu - 2\,C_F L_\mu^2$$

Implementation of the resummation formulas

Both approaches are kind of tricky when it comes to the numerical implementation:

- Within the momentum-space approach, the integral over the energy requires analytic continuation to negative values of the variable η.
  [Becher, Neubert, Pecjak, 2007]
  In general, the analytic continuation has to be generalized such that the hadronic convolution is well defined, i.e. the partonic cross section has to be understood as a distribution. [Beneke, Falgari, Klein, Schwinn, 2012]
- Within the Mellin-space approach, a numerical Mellin inversion is required. Here, it is common to exclude the Landau pole singularity from the integration contour. [Catani, Mangano, Nason, Trentadue, 1996]
- In both approaches, one wants to keep the information of the full fixed order result. Therefore, a matching procedure is required in order to avoid double-counting.
- The matching procedure is not unique, different approaches differ by higher order terms.

Implementation of the Mellin-space formalism

- Match resummed cross section $\hat{\sigma}^{\mathrm{NNLL}}_{ij\to\tilde{g}\tilde{g}}$ to the NNLO threshold approx. by subtracting all terms up to $\mathcal{O}(\alpha_s^2)$ of the expanded resummation formula $\hat{\sigma}^{\mathrm{NNLL}(2)}_{ij\to\tilde{g}\tilde{g}}$:

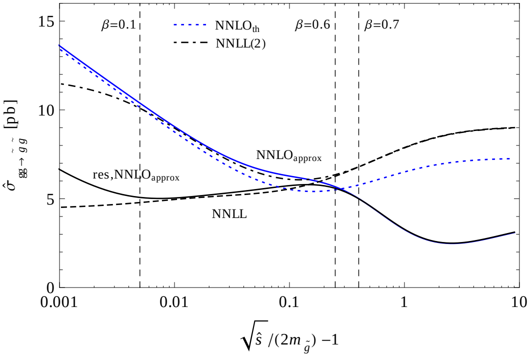
$$\hat{\sigma}^{\mathrm{res,NNLO_{approx}}}_{ij\to\tilde{g}\tilde{g}} = \mathbf{M}^{-1}\left[\hat{\sigma}^{\mathrm{NNLL,\,N}}_{ij\to\tilde{g}\tilde{g}} - \hat{\sigma}^{\mathrm{NNLL(2),\,N}}_{ij\to\tilde{g}\tilde{g}}\right] + \hat{\sigma}^{\mathrm{NNLO_{approx}}}_{ij\to\tilde{g}\tilde{g}}$$

- The Mellin inversion $\mathbf{M}^{-1}$ can be implemented according to

$$f(\rho) = \frac{1}{\pi}\int_0^\infty dz\,\mathrm{Im}\left[e^{i\Phi}\rho^{-c(z)}\mathbf{M}[f](c(z))\right]$$

where Mellin-N is identified with $c(z) = c_0 + ze^{i\Phi}$. [Blümlein, 2000]
The parameter integral over z is divided logarithmically into 20 pieces, where each segment is performed by the 32-point Gauss formula.

Cross check: Near the threshold, the inverted expansion $\hat{\sigma}^{\mathrm{NNLL}(2)}_{ij\to\tilde{g}\tilde{g}}$ should be in the vicinity of the threshold limit $\mathrm{NNLO_{th}} = \mathrm{NLO_{th}} + \mathrm{NNLO}$ logs. It can not fit exactly as $1/N$ terms have been dropped in the construction of the resummation formula.



(For very small energies ($\beta < 0.1$), the quality of the numerical inversion seems to decrease. However, this hardly affects the outcome of the hadronic convolution, which mainly follows from the region $\beta > 0.1$.)

Hadronic convolution

$$\sigma_{pp\to\tilde{g}\tilde{g}X}(s, m_{\tilde{g}}^2, m_{\tilde{q}}^2, \mu_f^2, \mu_r^2) = \sum_{i,j=q,\bar{q},g} \int_{4m_{\tilde{g}}^2/s}^{1} d\tau\ L_{ij}(\tau, \mu_f^2)\ \hat{\sigma}_{ij\to\tilde{g}\tilde{g}}(\hat{s}=\tau s, m_{\tilde{g}}^2, m_{\tilde{q}}^2, \mu_f^2, \mu_r^2)$$

$$L_{ij}(\tau, \mu_f^2) = \int_0^1 dx_1 \int_0^1 dx_2\ \delta(x_1 x_2 - \tau) f_{i/p}\left(\mu_f^2, x_1\right) f_{j/p}\left(\mu_f^2, x_2\right)$$

$\sqrt{s} = 7\ \mathrm{TeV}$, $m_{\tilde{q}}/m_{\tilde{g}} = 4/5$, $\mu = m_{\tilde{g}}$
numbers in brackets: $\mu = m_{\tilde{g}}/2$ (upper number) and $\mu = 2m_{\tilde{g}}$

$m_{\tilde{g}}$ [GeV]	σ^{LO} [pb]	σ^{NLO} [pb]	$\sigma^{\mathrm{NNLO_{approx}}}$ [pb]	$\sigma^{\mathrm{res,NNLO_{approx}}}$ [pb]
MSTW 2008 NNLO, $\alpha_s(M_Z) = 0.1171 \pm 0.0014$, PDF errors not included				
800	$0.0198\ \binom{0.0293}{0.0139}$	$0.0329\ \binom{0.0385}{0.0270}$	$0.0425\ \binom{0.0477}{0.0373}$	$0.0351\ \binom{0.0407}{0.0306}$
1000	$0.0020\ \binom{0.0030}{0.0014}$	$0.0034\ \binom{0.0041}{0.0028}$	$0.0046\ \binom{0.0052}{0.0039}$	$0.0037\ \binom{0.0044}{0.0032}$

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1000	$0.0020 \binom{0.0030}{0.0014}$	$0.0034 \binom{0.0041}{0.0028}$	$0.0046 \binom{0.0052}{0.0039}$	$0.0037 \binom{0.0044}{0.0032}$

$\Rightarrow K_{\mathrm{NNLL}} = \sigma_{\mathrm{res,NNLO}_{\mathrm{approx}}}/\sigma_{\mathrm{NLO}} \approx 1.07\,(m_{\tilde{g}} = 800\ \mathrm{GeV}), 1.09\,(m_{\tilde{g}} = 1\ \mathrm{TeV})$

Compare to $K_{\mathrm{NLL}} = \sigma_{\mathrm{res,NLO}}/\sigma_{\mathrm{NLO}} \approx 1.16\,(m_{\tilde{g}} = 800\ \mathrm{GeV}), 1.21\,(m_{\tilde{g}} = 1\ \mathrm{TeV})$ [Beenakker et al. 2011]

Going beyond NLL stabilizes the cross section in the vicinity of the fixed NLO!

The error due to scale variation slightly decreases compared to the fixed order.

Hadronic convolution

- Result crucially depends on the high-x region of the chosen PDF set and the related value of $\alpha_s(M_Z)$!
- The central value can differ by a factor of about 2!
- Deviations are not covered by the individual PDF errors!

$\sqrt{s} = 7\ \mathrm{TeV}$, $m_{\tilde{q}}/m_{\tilde{g}} = 4/5$, $\mu = m_{\tilde{g}}$
numbers in brackets: $\mu = m_{\tilde{g}}/2$ (upper number) and $\mu = 2m_{\tilde{g}}$

$m_{\tilde{g}}$ [GeV]	σ^{LO} [pb]	σ^{NLO} [pb]	$\sigma^{\mathrm{NNLO_{approx}}}$ [pb]	$\sigma^{\mathrm{res,NNLO_{approx}}}$ [pb]
MSTW 2008 NNLO, $\alpha_s(M_Z) = 0.1171 \pm 0.0014$, PDF errors not included				
800	$0.0198 \binom{0.0293}{0.0139}$	$0.0329 \binom{0.0385}{0.0270}$	$0.0425 \binom{0.0477}{0.0373}$	$0.0351 \binom{0.0407}{0.0306}$
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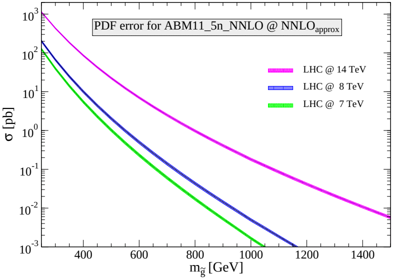
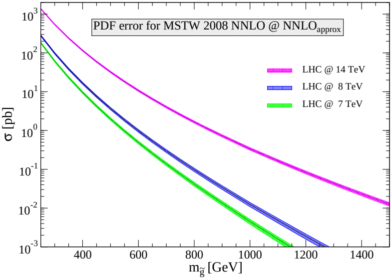
ABM11 NNLO, $\alpha_s(M_Z) = 0.1134 \pm 0.0011$, PDF errors not included				
800	$0.0087 \binom{0.0119}{0.0065}$	$0.0135 \binom{0.0151}{0.0117}$	$0.0175 \binom{0.0195}{0.0156}$	$0.0142 \binom{0.0166}{0.0125}$
1000	$0.0008 \binom{0.0011}{0.0006}$	$0.0013 \binom{0.0014}{0.0011}$	$0.0017 \binom{0.0019}{0.0015}$	$0.0014 \binom{0.0016}{0.0012}$

PDF errors



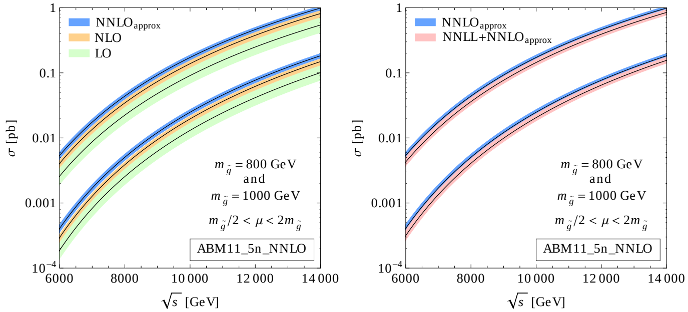


The LHC analysis use an 'envelope' of MSTW2008 and CTEQ6.6 PDF sets.
[see explanations in Krämer et al., 2012]

These sets are rather similar, especially in the high-x region.
They go along with a rather high value of $\alpha_s(M_Z)$.

$\Rightarrow$ Gluino-pair production cross section overestimated?

Conclusions



- The implementation of the NNLL resummation formula stabilizes the prediction of the production cross section between the fixed NLO and NNLO approximation.
- The central values of the different approximations agree within their errors due to scale variation.
- The NNLL-resummed cross section is close to the fixed NLO: $K_{\mathrm{NNLL}} \approx 1.04 - 1.09$ depending on $m_{\tilde{g}}$, $\sqrt{s}$, $\alpha_s(M_Z)$, and the PDF set

Conclusions

- The main uncertainty resides in the non-perturbative input.
- The current LHC analysis features MSTW2008 and CTEQ6.6 PDF sets, which give rise to rather large predictions for the hadronic cross section. In addition, the applied NLL results also produce (slightly) bigger numbers than the NNLL formulas.
- It seems that the current exclusion bounds may be too pessimistic.
- Nevertheless, the use of the public code `NLL-Fast` should already give a good estimate, provided that the discrepancies in the non-perturbative input are understood.

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$\longrightarrow$ Likely to be answered by the experts soon.

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Thank you for your attention!