High Energy Jets (and NLO)

HEJ: All-Order Perturbative Corrections to Hard Multi-Jet Processes

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Overview of Talk

Multi-Jet Predictions

A new approach to resummation for multiple, hard, wide-angle emissions in the **hard scattering**:

High Energy Jets

Predictions for dijets, W+jets, H+jets,...

Theory vs. Data

Data vs. description of hard, higher order effects

Rôle of matching; a new beginning

High Energy Jets for merging (N)LO samples

Multiple (≥ 2) hard jets...

Special radiation pattern from **current-current** scattering
Look into **higher order corrections beyond** "inclusive K-factor"
Concentrate on the **hard, perturbative corrections** relevant for a description of the final state **in terms of jets**.

Goal

Build framework for **all-order summation** (virtual+real emissions). Exact in another limit than the usual soft&collinear. Better suited for describing **radiation relevant for multi-jet** production.

Reformulation for Merging of NLO Calculations

Use the approximate matrix elements (=basis of resummation) for merging **fixed order calculations**

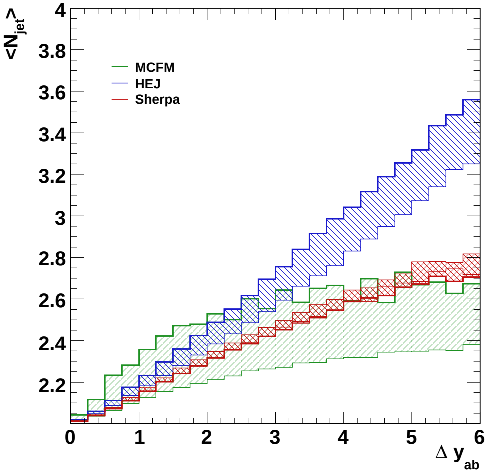
Insight

Can use the insight gained from studying the relevant limit to **guide and improve** analyses: *CP*-properties of the Higgs-boson couplings

Drivers of Emission

1. **Collinear** (jet profile)
2. **Soft** (p_t-hierarchies)
3. **Opening of phase space** (semi-hard emissions - not related to a divergence of $|M|^2$).
   Think (e.g.) multiple jets of fixed p_t, with increasing rapidity span (span=max difference in rapidity of two hard jets=Δy).
   All calculations will agree that number of additional jets increases - but the amount of radiation will differ (wildly) - e.g. due to **limitations** on the **number** (NLO) or **hardness** (shower) of additional radiation **allowed by theoretical assumptions**.

Increasing Rapidity Span → Increasing Number of Jets



h+dijets (at least 40GeV).
Δy_{ab}: Rapidity difference between most forward and backward hard jet

Compare NLO (green), CKKW matched shower (red), and High Energy Jets (blue).

All models show a clear increase in the number of hard jets as the rapidity span Δy_{ab} increases.

J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Collinear limit $\rightarrow$ enters many resummation formalisms, parton showers....

Like all good limits, the collinear approximation is applied **outside its strict region of validity.**

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider...

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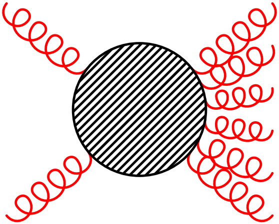
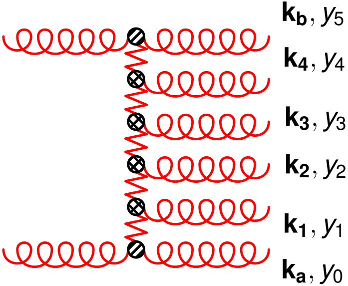
The Possibility for Predictions of *n*-jet Rates

The Power of Reggeisation



High Energy Limit $\longrightarrow$

$|\hat{t}|$ fixed, $\hat{s} \to \infty$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2}\left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)}\,\frac{V^{J_i}(q_i,q_{i+1})}{q_i^2 q_{i+1}^2}\right) e^{\omega(q_{n+1})(y_n-y_{n+1})}\,\frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|}\right)$$

to all orders in α_s.

At LL only gluon production; at NLL also quark–anti-quark pairs produced. Approximation of any-jet rate possible.

Comparison of 3-jet scattering amplitudes

Universal behaviour of scattering amplitudes in the **High Energy Limit**:

$$\forall i \in \{2, \ldots, n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i, j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}^{MRK}_{gg\to g\cdots g}\right|^2 = \frac{4\ s^2}{N_C^2 - 1}\ \frac{g^2\ C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\ C_A}{|p_{n\perp}|^2}.$$

$$\left|\overline{\mathcal{M}}^{MRK}_{qg\to qg\cdots g}\right|^2 = \frac{4\ s^2}{N_C^2 - 1}\ \frac{g^2\ C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\ C_A}{|p_{n\perp}|^2},$$

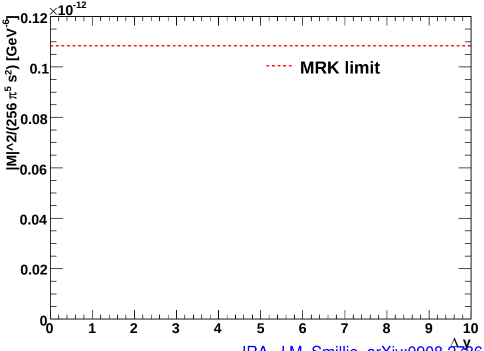
$$\left|\overline{\mathcal{M}}^{MRK}_{qQ\to qg\cdots Q}\right|^2 = \frac{4\ s^2}{N_C^2 - 1}\ \frac{g^2\ C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\ C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (**BFKL equation**).
However, **how well** does this actually **approximate the amplitude?**

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.



JRA, J.M. Smillie, arXiv:0908.2786

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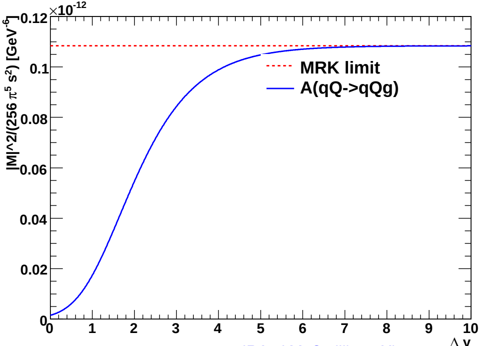

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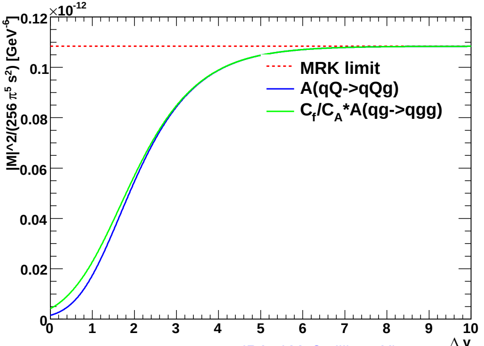

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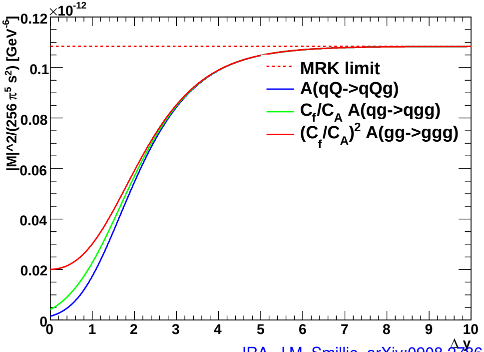

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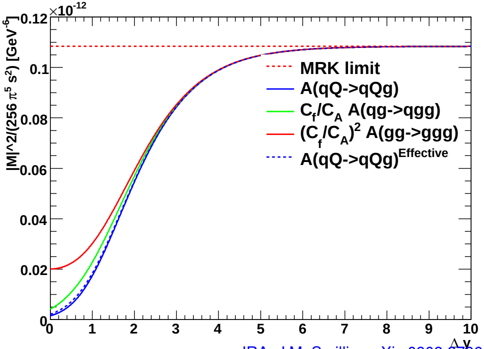

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High Energy Jets (HEJ):

1) Inspiration from Fadin&Lipatov: dominance by *t*-channel
2) No kinematic approximations in invariants (denominator)
3) Accurate definition of currents (coupling through *t*-channel exchange)
4) Gauge invariance. Not just asymptotically.

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$:

$$M_{q^- Q^- \to q^- Q^-} = \langle 1|\mu|a\rangle \frac{g^{\mu\nu}}{t} \langle 2|\nu|b\rangle$$

***t*-channel factorised**: Contraction of (local) currents across *t*-channel pole

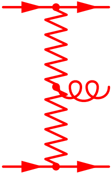
$$\left|\overline{\mathcal{M}}^t_{qQ\to qQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\ C_F\ \frac{1}{t_1}\right) \cdot \left(g^2\ C_F\ \frac{1}{t_2}\right).$$

Extend to $2 \to n \ldots$

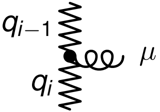
J.M.Smillie and JRA: arXiv:0908.2786

Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

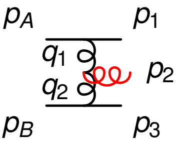


$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

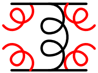


$$V^\mu(q_{i-1}, q_i)$$

$$j^\nu = \overline{\psi}\gamma^\nu\psi$$



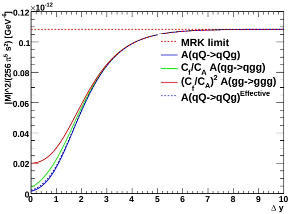
$$\begin{aligned} V^\rho(q_1, q_2) = &-(q_1+q_2)^\rho \\ &+ \frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_A} + \frac{p_2\cdot p_B}{p_A\cdot p_B} + \frac{p_2\cdot p_n}{p_A\cdot p_n}\right) + p_A \leftrightarrow p_1 \\ &- \frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_B} + \frac{p_2\cdot p_A}{p_B\cdot p_A} + \frac{p_2\cdot p_1}{p_A\cdot p_1}\right) - p_B \leftrightarrow p_3. \end{aligned}$$



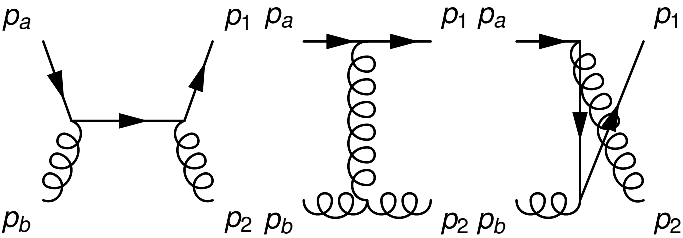
Building Blocks for an Amplitude

$p_g \cdot V = 0$ can easily be checked (**exact** gauge **invariance**).
The lowest order approximation for $qQ \to qgQ$ is given by

$$\left|\overline{\mathcal{M}}^t_{qQ\to qgQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2$$
$$\cdot\left(g^2\ C_F\ \frac{1}{t_1}\right)\cdot\left(g^2\ C_F\ \frac{1}{t_2}\right)$$
$$\cdot\left(\frac{-g^2 C_A}{t_1 t_2}\ V^\mu(q_1,q_2)V_\mu(q_1,q_2)\right)$$



"What happens in $2 \to 2$-processes with gluons? Surely the t-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \to q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p_b^-}{p_2^-}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p_2^-}{p_b^-}} \right) \langle b|\sigma|2\rangle \ \times \langle 1|\sigma|a\rangle.$$

Complete *t*-channel factorisation!

J.M.Smillie and JRA

Quark-Gluon Scattering

The t-channel current generated by a gluon in qg scattering is that generated by a quark, but with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

instead of C_F. Tends to C_A in MRK limit.

Similar results for e.g. $g^+g^- \to g^+g^-$. **Exact, complete** t-channel **factorisation**.

By using the formalism of **current-current scattering**, we get a better description of the ***t*-channel poles of the amplitude** than by using just the BFKL kinematic limit.

Performing the Explicit Resummation

Soft divergence from real radiation:

$$\left|\mathcal{M}_t^{p_a p_b \to p_0 p_1 p_2 p_3}\right|^2 \xrightarrow{\mathbf{p}_1^2 \to 0} \left(\frac{4 g_s^2 C_A}{\mathbf{p}_1^2}\right) \left|\mathcal{M}_t^{p_a p_b \to p_0 p_2 p_3}\right|^2$$

Integrate over the soft part $\mathbf{p}_1^2 < \lambda^2$ of phase space in $D = 4 + 2\varepsilon$ dimensions

$$\int_0^\lambda \frac{\mathrm{d}^{2+2\varepsilon}\mathbf{p}\ \mathrm{d}y_1}{(2\pi)^{2+2\varepsilon}\ 4\pi} \left(\frac{4 g_s^2 C_A}{\mathbf{p}^2}\right) \mu^{-2\varepsilon}$$
$$= \frac{4 g_s^2 C_A}{(2\pi)^{2+2\varepsilon} 4\pi}\ \Delta y_{02}\ \frac{\pi^{1+\varepsilon}}{\Gamma(1+\varepsilon)} \frac{1}{\varepsilon} (\lambda^2/\mu^2)^\varepsilon$$

Pole in ε cancels with that from the virtual corrections

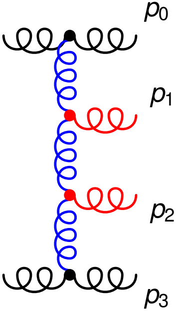
$$\frac{1}{t_1} \to \frac{1}{t_1} \exp\left(\hat{\alpha}(t)\Delta y_{02}\right) \qquad \hat{\alpha}(t) = -\frac{g_s^2 C_A \Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} \left(\mathbf{q}^2/\mu^2\right)^\varepsilon.$$

Expression for the Regularised Amplitude

$$\overline{\left|\mathcal{M}^{\rm reg}_{\rm HEJ}(\{p_i\})\right|}^2 = \frac{1}{4\,(N_C^2-1)}\,\left\|S_{f_1 f_2\to f_1 f_2}\right\|^2 \cdot \left(g^2\,K_{f_1}\,\frac{1}{t_1}\right)\cdot\left(g^2\,K_{f_2}\,\frac{1}{t_{n-1}}\right)$$

$$\cdot \prod_{i=1}^{n-2}\left(g^2 C_A\,\left(\frac{-1}{t_i t_{i+1}} V^\mu(q_i,q_{i+1})V_\mu(q_i,q_{i+1}) - \frac{4}{\mathbf{p}_i^2}\,\theta\left(\mathbf{p}_i^2<\lambda^2\right)\right)\right)$$

$$\cdot \prod_{j=1}^{n-1} \exp\left[\omega^0(q_j,\lambda)(y_{j-1}-y_j)\right], \qquad \omega^0(q_j,\lambda) = -\frac{\alpha_s N_C}{\pi}\log\frac{\mathbf{q}_j^2}{\lambda^2}.$$



Resummed (and Matched) Cross Section

The cross section is calculated as phase space integrals over explicit n-body phase space

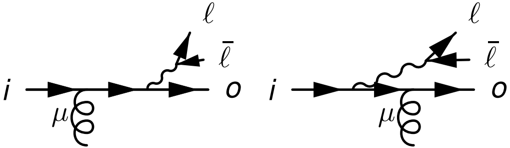
$$\sigma_{2j}^{\mathrm{resum,match}} = \sum_{f_1,f_2} \sum_{n=2}^{\infty} \prod_{i=1}^{n} \left(\int \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} \int \frac{\mathrm{d}y_i}{2} \right) \frac{\overline{\left|\mathcal{M}_{\mathrm{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})\right|}^2}{\hat{s}^2}$$
$$\times \quad \mathcal{O}_{2j}(\{p_i\}) \times \sum_m \mathcal{O}_{mj}^e(\{p_i\})\ w_{m-\mathrm{jet}}$$
$$\times \quad x_a f_{A,f_1}(x_a, Q_a)\ x_2 f_{B,f_2}(x_b, Q_b)\ (2\pi)^4\ \delta^2\!\left(\sum_{i=1}^{n} \mathbf{p}_{i\perp}\right).$$

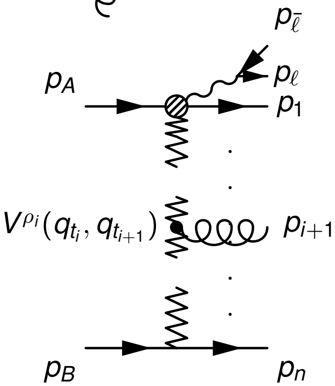
Matching to fixed order (tree-level so far) is obtained by clustering the n-parton phase space point into m-jet momenta and multiply by ratio of full to approximate matrix element:

$$w_{m-\mathrm{jet}} \equiv \frac{\overline{\left|\mathcal{M}^{f_1 f_2 \to f_1 g \cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}{\overline{\left|\mathcal{M}^{t, f_1 f_2 \to f_1 g \cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}.$$

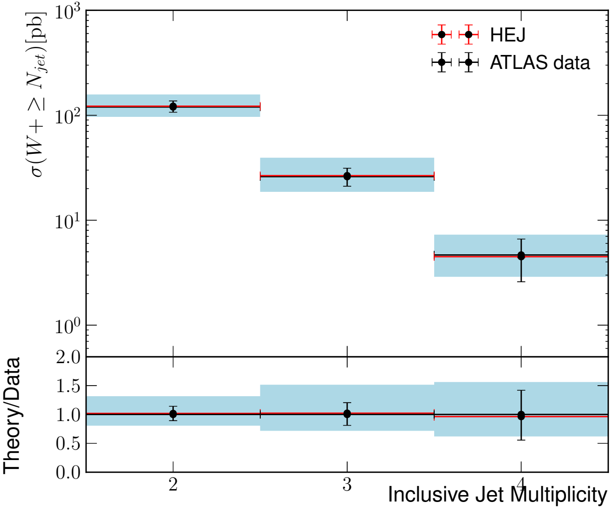
W+Jets

Two currents to calculate for $W + jets$:





Recent Comparisons to ATLAS Data



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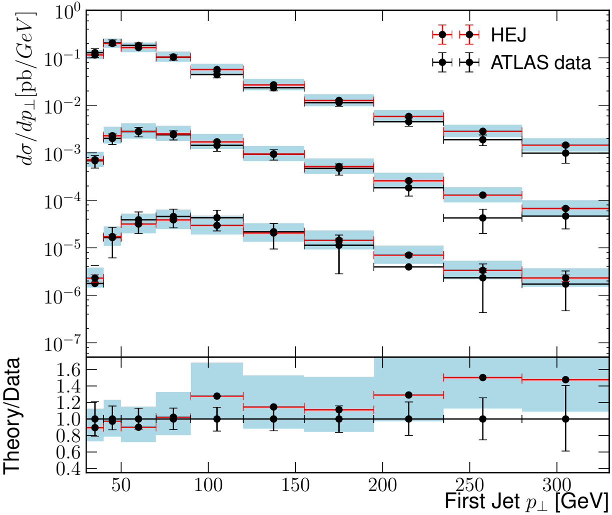

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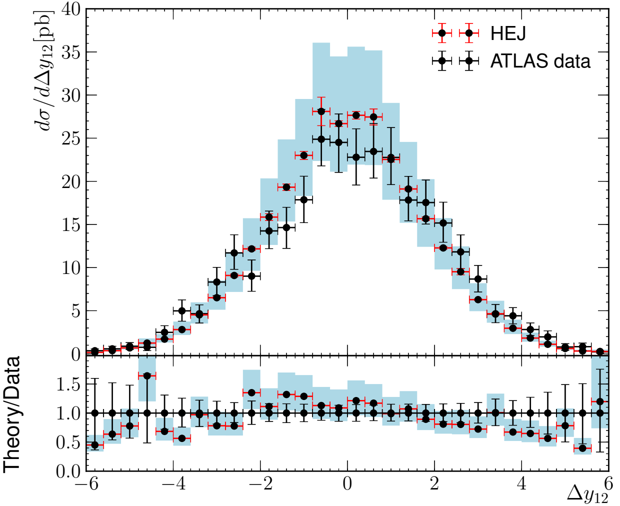
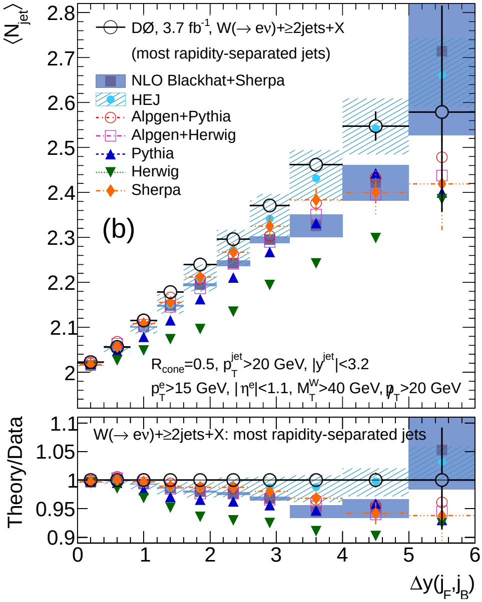


Recent Comparisons to D0 Data



PS: NLO predictions of $\langle\text{jets}\rangle$ formed as

$$\langle\text{jets}\rangle = 2 + \frac{\sigma_{3j}^{\text{nlo,incl}} + \sigma_{4j}^{\text{lo}}}{\sigma_{2j}^{\text{nlo,incl}}}$$

Including Perturbative Input of High Multiplicity NLO Caluclations

$$\sigma_{2j}^{\text{resum,match}} = \sum_{f_1,f_2} \sum_{n=2}^{\infty} \prod_{i=1}^{n} \left(\int \frac{\mathrm{d}^2 \mathbf{p}_{i\perp}}{(2\pi)^3} \int \frac{\mathrm{d}y_i}{2} \right) \frac{\overline{|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})|}^2}{\hat{s}^2}$$

$$\times \quad \mathcal{O}_{2j}(\{p_i\}) \times \sum_m \mathcal{O}_{mj}^e(\{p_i\}) \, w_{m-\text{jet}}$$

$$\times \quad x_a f_{A,f_1}(x_a, Q_a) \, x_2 f_{B,f_2}(x_b, Q_b) \, (2\pi)^4 \, \delta^2\left(\sum_{i=1}^{n} \mathbf{p}_{i\perp} \right).$$

Matching to NLO

$$w_{m-\text{jet}}^{NLO} \equiv \left(\frac{r_m^m(\{j_i\})}{r_m^{m,LL}(\{j_i\})} + \alpha_s \frac{r_m^{m+1}(\{j_i\}) - r_m^{m+1,LL}(\{j_i\}) \frac{r_m^m(\{j_i\})}{r_m^{m,LL}(\{j_i\})}}{r_m^{m,LL}(\{j_i\})} \right)$$

Rewriting Merging

$$\sigma_{2j}^{\text{resum,match}} = \sum_{f_1,f_2} \sum_{m} \prod_{i=1}^{m} \left(\int_{p_{i\perp}=0}^{p_{i\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{j,i\perp}}{(2\pi)^3} \int \frac{\mathrm{d}y_{j,i}}{2} \right) \delta^{(2)}\left(\sum_{i=1}^{n} \mathbf{p}_{j,i\perp} \right)$$

$$\times\ x_A^B\ f_A(x_A^B, Q_A^B)\ x_B^B\ f_B(x_B^B, Q_B^B)\ \frac{\overline{|\mathcal{M}^B|}^2}{\hat{s}_B^2}$$

$$\times\ \frac{w_{m-\text{jet}}}{\overline{|\mathcal{M}^B|}^2}$$

$$\times\ \sum_{n=2}^{\infty} \prod_{i=1}^{n} \left(\int_{p_{i\perp}=0}^{p_{i\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} \int \frac{\mathrm{d}y_i}{2} \right) \mathcal{O}_{mj}^{e} \prod_{l=1}^{m} \left(\delta^{(3)}(j_B^l - j^l) \right)$$

$$\times\ \frac{\overline{|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})|}^2}{\hat{s}^2} \times \hat{s}_B^2$$

$$\times\ \frac{x_a f_{A,f_1}(x_a, Q_a)\ x_2 f_{B,f_2}(x_b, Q_b)}{x_A^B\ f_A(x_A^B, Q_A^B)\ x_B^B\ f_B(x_B^B, Q_B^B)}\ (2\pi)^4\ \delta^2\left(\sum_{i=1}^{n} \mathbf{p}_{i\perp} \right)\ \mathcal{O}_{2j}(\{p_i\}).$$

Conclusions

- Colliders probe hard (=jets) perturbative corrections beyond pure NLO
  ... already at 1.98TeV!
- ***High Energy Jets****[*] provides a new approach to the perturbative description of LHC physics
  ... and compares favourably to data in several analyses
  ... already in its present, first iteration (several ongoing improvements in the theoretical description)

[*] http://cern.ch/hej