Threshold corrections to inclusive jet production at hadron colliders

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Outline

1. Inclusive jet production at hadron colliders

2. Soft-gluon resummation formalism

3. Cross sections to NNLO at NLL accuracy

4. numerical results

5. Summary

Inclusive jet production

1. The process is given by

$$P_a + P_b \longrightarrow J + X$$

2. Dominant process at hadron colliders

3. Useful in the extraction of parton distribution functions

   - MSTW 2008 PDFs

     EPJC 63, 189 (2009)

   - CT10 PDFs

     PRD 82, 074024 (2010)

   - NN PDFs

     NPB 838, 136 (2010)

4. Useful in the measurement of strong coupling constant α_s.
   D0 collaboration **PRD 80, 111107 (2009)**

State of the art for jet cross sections

1. Three jet production in electron-positron annihilation to NNLO
   Garland, Gehrmann, Glover, Koukoutsakis, Remiddi, NPB 642, 227 (2002)
   Ridder, Gehrmann, Glover, Heinrich, PRL 100, 172001 (2008)
2. Inclusive jet production to NLO at hadron colliders
   Ellis, Kunszt and Soper, PRL 64, 2121 (1990)
3. Di-jet production to NLO at hadron colliders
   Ellis, Kunszt and Soper, PRL 69, 1496 (1992)
   Giele, Glover and Kosower, PRL 73, 2019 (1994)
4. Threshold corrections to inclusive jet production at hadron colliders
   Kidonakis and Owens, PRD 63, 054019 (2001)
5. Di-jet production to NNLO at hadron colliders (only gluonic channel)
   Ridder, Gehrmann, Glover, Pires, PRL 110 (2013)
   see also the talk by J. Pires on NNLO results.

Soft gluon resummation formalism

1. Color basis and color decomposition of QCD amplitudes
2. Soft function
3. Hard function
4. Soft anomalous dimension Γ_S
5. Jet functions
6. parton level cross section
7. Hadronic cross section

N. Kidonakis, G. Oderda and G. Sterman, Nucl. Phys. B531 (1998) 365
N. Kidonakis, G. Oderda and G. Sterman, Nucl. Phys. B525 (1998) 299
N. Kidonakis and G. Sterman, Nucl. Phys. B505 (1997) 321

Color basis, Soft function, Hard function ....

1. The t-channel color basis for a process $i\ j \to k\ l$ is given by

$$c_1 = \delta_{ik}\,\delta_{jl}, \quad c_2 = T^c_{ki}\,T^c_{jl}$$

2. The soft function for this basis is

$$S_{IJ} = |C_I >< C_J|, \quad \text{with} \quad |C_I >= \{c_1, c_2\}$$

3. If $\mathcal{M}$ denotes the QCD amplitude for a given subprocess, then the color decomposed matrix elements H_I are obtained from

$$|H_I >= \mathcal{M}|P_I > \quad \text{and} \quad < H_I| =< P_I|\mathcal{M}^*$$

where $|P_I >= \{p_1, p_2\}$ such that $p_i = c_i/S_{ii}$

4. The hard function for that given subprocess is then given as

$$H_{IJ} = |H_I >< H_J|$$

5. The squared matrix elements at Born level are then simply obtained from

$$|M|^2 = S_{IJ}H_{JI} = Tr[S.H]$$

and the trace is taken in color space.

Color basis for different parton channels

1. The t-channel color basis for a $qq \to qq$ process $i\, j \to k\, l$ is given by

   $$c_1 = \delta_{ik}\, \delta_{jl}, \quad c_2 = T^c_{ki}\, T^c_{jl}$$

   The soft function for this basis is given by

   $$\begin{bmatrix} N_c^2 & 0 \\ 0 & (N_c^2-1)/4 \end{bmatrix}$$

2. The t-channel color basis for a $qg \to qg$ process $i\, j \to k\, l$ is given by

   $$c_1 = \delta_{ik}\, \delta_{jl}, \quad c_2 = d^{jlc} T^c_{ki} \quad c_3 = i\, f^{jlc}\, T^c_{ki}$$

   The soft function for this basis is given by

   $$\begin{bmatrix} N_c(N_c^2-1) & 0 & 0 \\ 0 & (N_c^2-4)(N_c^2-1)/(2N_c) & 0 \\ 0 & 0 & N_c(N_c^2-1)/2 \end{bmatrix}$$

3. For the processes that are related to the above by charge conjugation, the basis needs to be defined accordingly.
4. $i, j, k, l = 1, 3$ for quarks and $i, j, k, l = 1, 8$ for gluons.

Color basis for different parton channels contd...

The t-channel color basis for a $gg \to gg$ process $i\ j \to k\ l$ is given by

$$
\begin{aligned}
c_1 &= \frac{i}{4}[f^{ijm} d^{klm} \; - \; d^{ijm} f^{klm}]\, \delta_{ik}\, \delta_{jl}, \\
c_2 &= \frac{i}{4}[f^{ijm} d^{klm} \; + \; d^{ijm} f^{klm}], \\
c_3 &= \frac{i}{4}[f^{ikm} d^{jlm} \; + \; d^{ikm} f^{jlm}], \\
c_4 &= \frac{1}{8}\delta_{ik}\delta_{jl}, \\
c_5 &= \frac{3}{5} d^{ikn}\; d^{jln}, \\
c_6 &= \frac{1}{3} f^{ikn}\; f^{jln}, \\
c_7 &= \frac{1}{2}\left(\delta_{ij}\delta_{kl} - \delta il \delta_{jk}\right) - \frac{1}{3} f^{ikn}\; f^{jln}, \\
c_8 &= \frac{1}{2}\left(\delta_{ij}\delta_{kl} + \delta il \delta_{jk}\right) - \frac{1}{8}\delta_{ik}\delta_{jl} - \frac{3}{5} d^{ikn}\; d^{jln}
\end{aligned}
$$

Soft function for $gg \to gg$ subprocess

The soft function for this basis is given by

$$S_{8\times 8} = \begin{bmatrix} G_{3\times 3} & 0_{3\times 5} \\ 0_{5\times 3} & G_{5\times 5} \end{bmatrix}$$

where $G_{3\times 3} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and $G_{5\times 5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 27 \end{bmatrix}$

N. Kidonakis, G. Oderda and G. Sterman

Nucl. Phys. B. 531 (1998)

Γ_S for different parton subprocesses

1. Soft anomalous function for $qq \to qq$ subprocess is given by

$$\frac{\alpha_s}{\pi}\begin{bmatrix} \frac{-1}{N_c}(T+U)+2C_F U & 2U \\ \frac{C_F}{N_c}U & 2C_F\,T \end{bmatrix}$$

2. Soft anomalous function for $qg \to qg$ subprocess is given by

$$\frac{\alpha_s}{\pi}\begin{bmatrix} (C_F+C_A)T & 0 & U \\ 0 & C_F T+\frac{C_A}{2}U & \frac{C_A}{2}U \\ 2U & \frac{N_c^2-4}{2N_c}U & C_F T+\frac{C_A}{2}U \end{bmatrix}$$

where

$$T = \ln\left(\frac{-t}{s}\right)+i\pi$$

$$U = \ln\left(\frac{-u}{s}\right)+i\pi$$

Γ_S for $gg \to gg$ subprocesses

1. The soft anomalous dimension can be written as

$$\Gamma_S = \begin{bmatrix} G_{3\times 3} & 0_{3\times 5} \\ 0_{5\times 3} & G_{5\times 5} \end{bmatrix} \quad \text{where} \quad G_{3\times 3} = \frac{\alpha_s}{\pi} \begin{bmatrix} 3T & 0 & 0 \\ 0 & 3U & 0 \\ 0 & 0 & 3(T+U) \end{bmatrix}$$

$$G_{5\times 5} = \frac{\alpha_s}{\pi} \begin{bmatrix} 6T & 0 & -6U & 0 & 0 \\ 0 & 3T + \frac{3U}{2} & \frac{-3U}{2} & -3U & 0 \\ -\frac{3U}{4} & -\frac{3U}{2} & 3T + \frac{3U}{2} & 0 & \frac{-9U}{4} \\ 0 & -\frac{6U}{5} & 0 & 3U & -\frac{9U}{5} \\ 0 & 0 & -\frac{2U}{3} & -\frac{4U}{3} & -2T + 4U \end{bmatrix}$$

Jet functions

1. Jet functions have the information about collinear configurations.

2. The initial state functions $\mathcal{J}_a^I$ are given by

$$\begin{aligned} \mathcal{J}_a^I &= -2\int_{\mu_F}^{2p_a.\zeta} \frac{d\mu}{\mu} C_a \frac{\alpha_s(\mu^2)}{\pi} \ln N_a \\ &- \int_0^1 dz \frac{z^{N_a-1}}{1-z} \left[\int_{(1-z)^2}^1 \frac{d\lambda}{\lambda} A^{(f_a)}[\alpha_s(\lambda(2p_a.\zeta)^2)] + \frac{1}{2}\nu^{f_i}[\alpha_s((1-z)^2(2p_a.\zeta)^2)] \right] \end{aligned}$$

3. The final state functions $\mathcal{J}_a^F$ are given by

$$\begin{aligned} \mathcal{J}_a^F &= \int_0^1 dz \frac{z^{N-1}}{1-z} \left[\int_{(1-z)^2}^{(1-z)} \frac{d\lambda}{\lambda} A^{(f_a)}[\alpha_s(\lambda(p_T^2))] \right. \\ &+ \left. B_a^{(1)}[\alpha_s((1-z)p_T^2)] + B_a^{(2)}[\alpha_s((1-z)^2 p_T^2)] \right] \end{aligned}$$

Parton level cross section

1. The parton level resummed cross section for a generic subprocess is given by

$$
\begin{aligned}
d\hat{\sigma}_{12\,\to\,34} \quad = \quad & \text{exp}\left\{\sum_{a=1,2} \mathcal{J}_a^I\right\} \times \text{exp}\left\{\sum_{b=3,4} \mathcal{J}_b^I\right\} \\
& \times \text{exp}\left[2\sum_{a=1,2}\int_{\mu_F}^{p_T} \frac{d\mu}{\mu}\gamma_a[\alpha_s(\mu^2)]\right] \times \text{exp}\left[4\int_{\mu_R}^{p_T} \frac{d\mu}{\mu}\beta(\alpha_s(\mu^2))\right] \\
& \times \text{Trace}\left\{H(\alpha_s(\mu_R^2))\ \bar{P}\text{exp}\left[\int_{p_T}^{p_T/N} \frac{d\mu}{\mu}\Gamma_S^\dagger(\alpha_s(\mu^2))\right]\right. \\
& \left. \times\ S(\alpha_s(p_T^2/N^2))\ P\text{exp}\left[\int_{p_T}^{p_T/N} \frac{d\mu}{\mu}\Gamma_S(\alpha_s(\mu^2))\right]\right\}
\end{aligned}
$$

with

$$
\begin{aligned}
H(x) &= H^{(0)}(x) + \frac{\alpha_s}{\pi} H^{(1)}(x) \\
S(x) &= S^{(0)}(x) + \frac{\alpha_s}{\pi} S^{(1)}(x)
\end{aligned}
$$

Hadron level cross section

1. The hadron level cross section is given by

$$S^2 \frac{d\sigma}{dT\, dU} = \int_{x_1-}^{1} \frac{dx_1}{x_1} \int_{x_2-}^{1} \frac{dx_2}{x_2}\, f_1(x_1, \mu_F)\, f_2(x_2, \mu_F)\, s^2 \frac{d\hat{\sigma}}{dtdu}$$

   where

$$s = x_1 x_2 S, \quad t = x_1 T, \quad u = x_2 U, \quad x_1- = \frac{-U}{S+T} \quad \text{and} \quad x_2- = \frac{-x_1 T}{x_1 S + U}$$

2. Born kinematics : $s + t + u = 0$
3. Threshold kinematics : $s + t + u = s_4$, with $0 < s_4 < x_1(S+T) + U$
4. The hadron level cross section in terms of s_4 variable is given

$$S^2 \frac{d\sigma}{dT\, dU} = \int_{x_1-}^{1} \frac{dx_1}{x_1} \int_{s_4-}^{s_4+} \frac{1}{s_4 - x_1 T} ds_4\; f_1(x_1, \mu_F)\; f_2\left[\frac{s_4 - x_1 T}{x_1 S + U}, \mu_F\right]\; s^2 \frac{d\hat{\sigma}}{dtdu}$$

5. The total cross section is given as

$$\sigma = \int_0^S d(-T) \int_0^{S+T} d(-U)\, S^2 \frac{d\sigma}{dT\, dU}$$

Expansion of the resummed result to NLL

1. At parton level, the resummed result can be expanded to NLO at NLL accuracy as

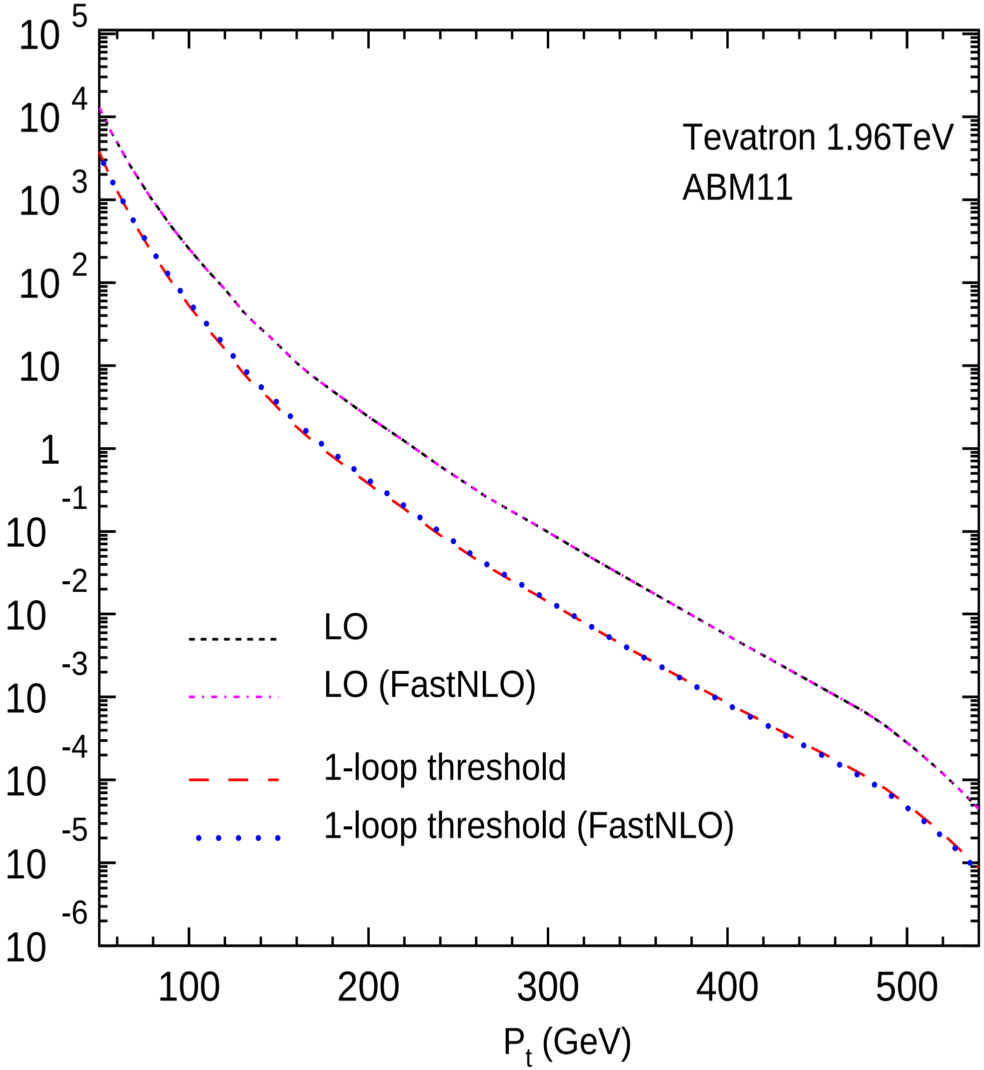
$$s^2 \frac{d^2\hat{\sigma}}{dtdu} = \frac{\alpha_s}{\pi}\sigma^{(0)} \left\{ c_3 \left[\frac{\ln(s_4/p_T^2)}{s4} \right]_+ + c_2 \left[\frac{1}{s_4} \right]_+ + c_1 \delta(s_4) \right\}$$

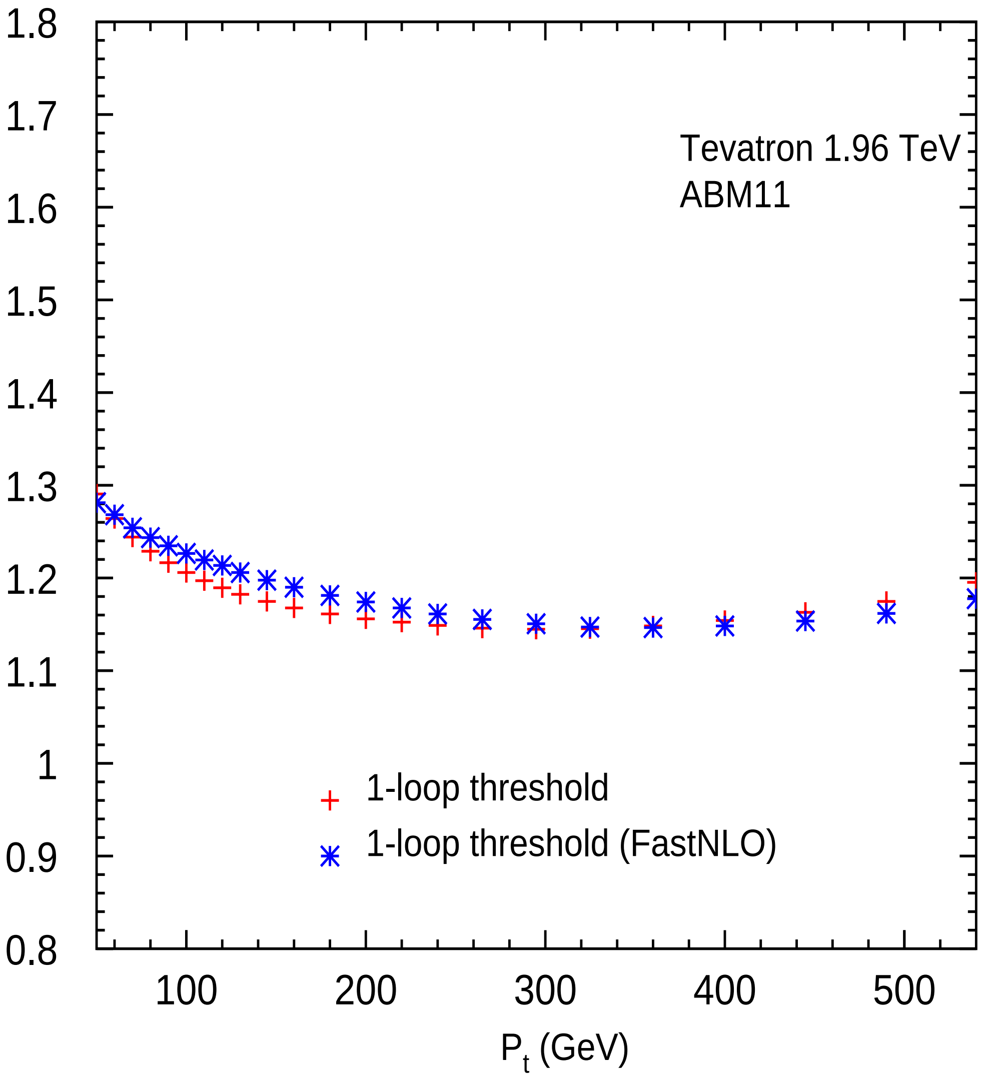
2. The resummed result expanded to two-loop level at NLL accuracy is given by

$$s^2 \frac{d^2\hat{\sigma}}{dtdu} = \left(\frac{\alpha_s}{\pi}\right)^2 \sigma^{(0)} \left\{ b_3 \left[\frac{\ln^3(s_4/p_T^2)}{s4} \right]_+ + b_2 \left[\frac{\ln^2(s_4/p_T^2)}{s_4} \right]_+ + b_1 \left[\frac{\ln(s_4/p_T^2)}{s_4} \right]_+ \right\}$$

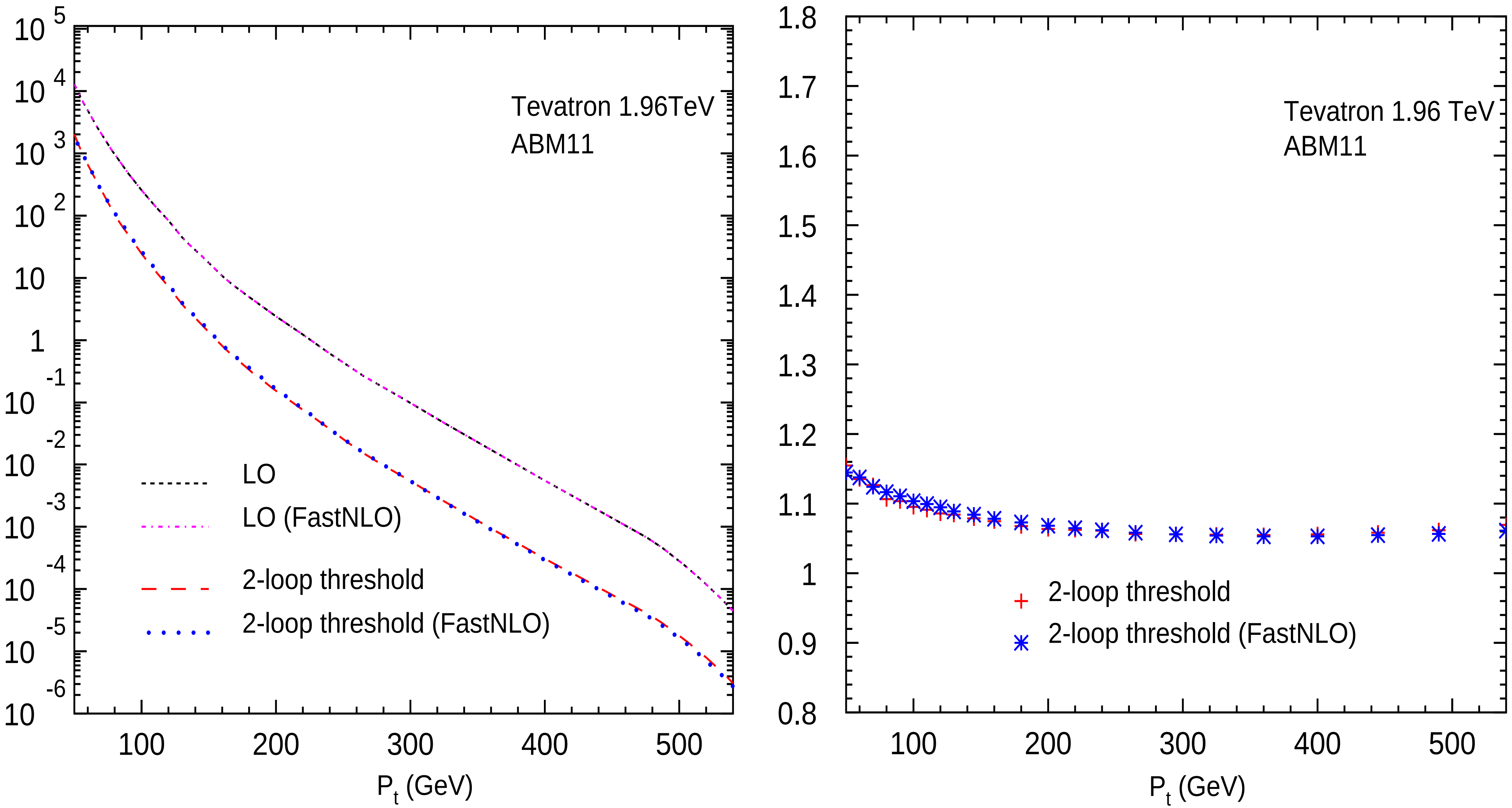
3. The coefficients of c_3 and b_3 are leading logarithms (LL)
4. The coefficients of c_2 and b_2 are next-to-leading logarithms (NLL)
5. To determin the coefficients c_1 and b_1, we need the hard matching functions $H^{(1)}$.

One-loop threshold corrections (Tevatron)

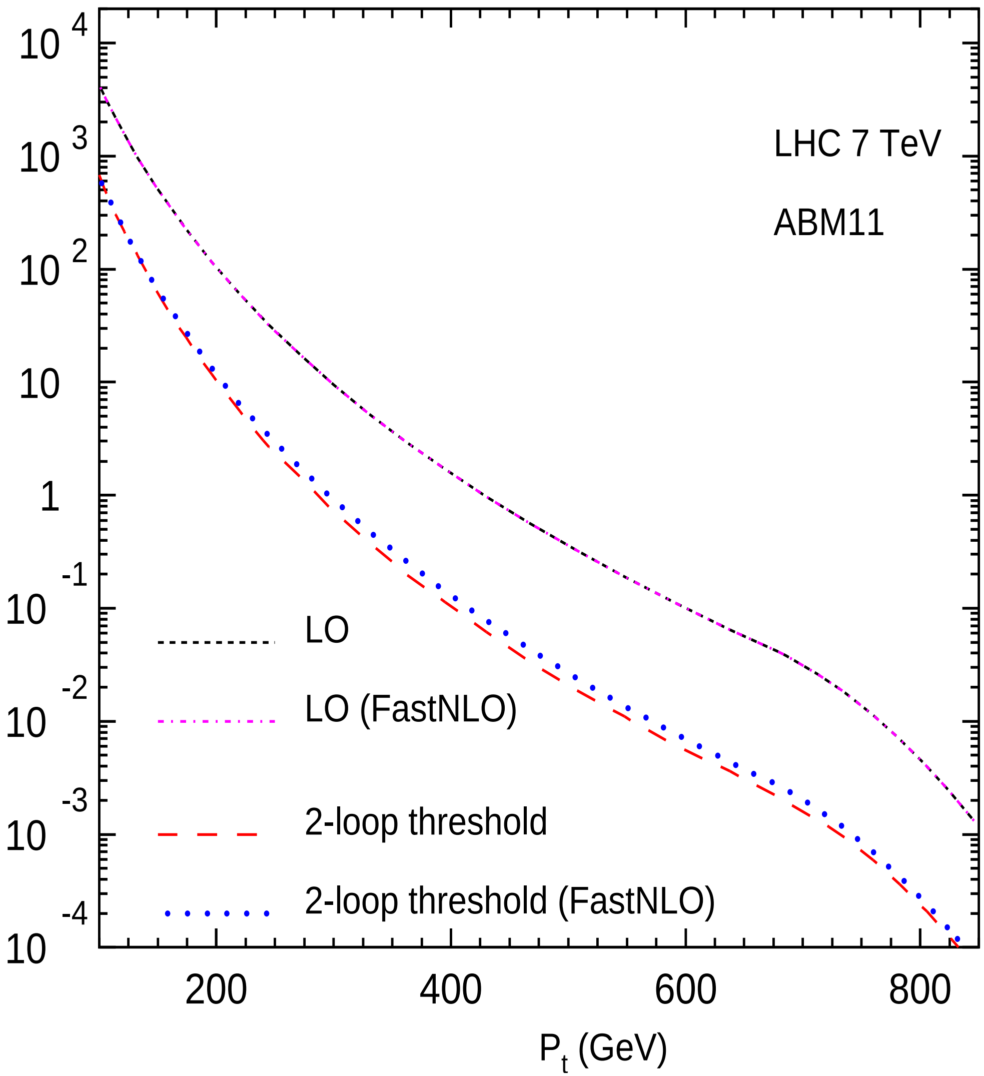


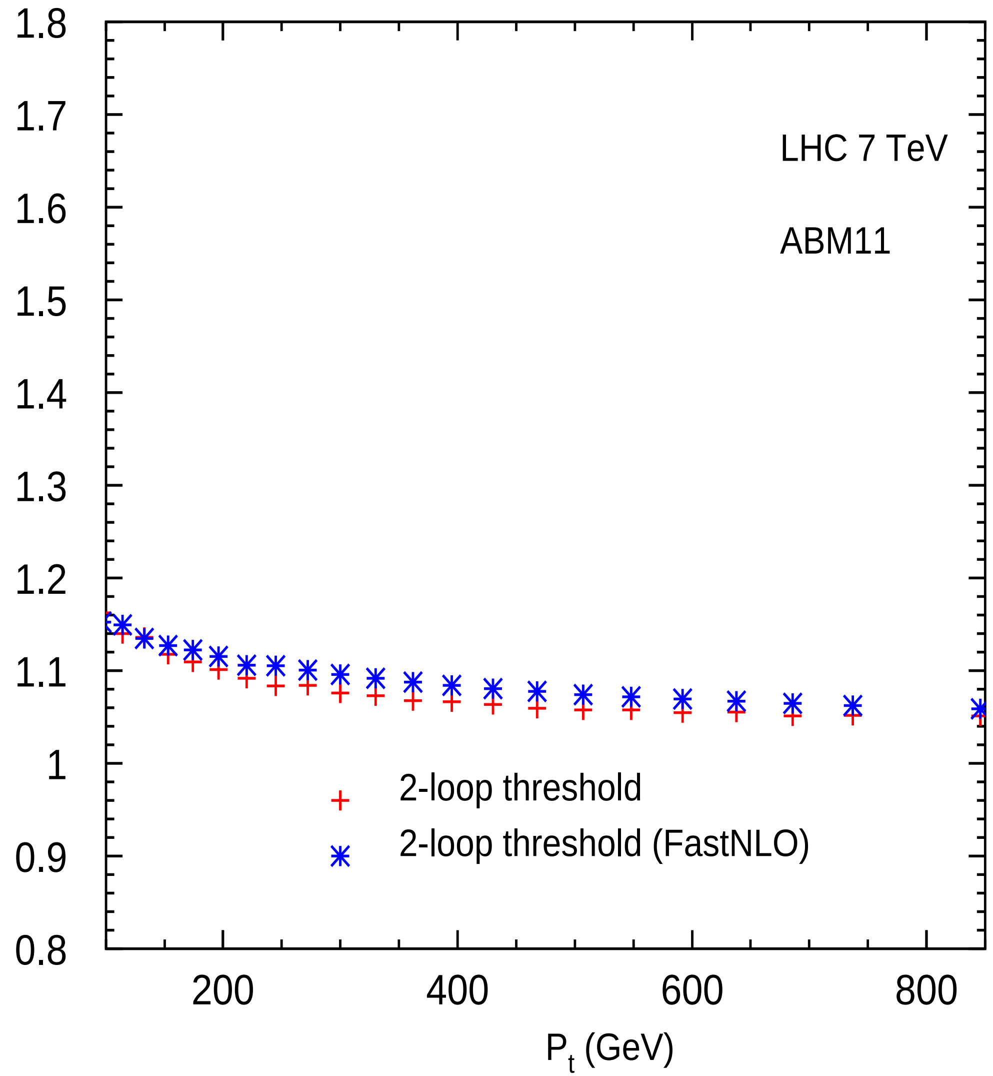


Two-loop threshold corrections (Tevatron)

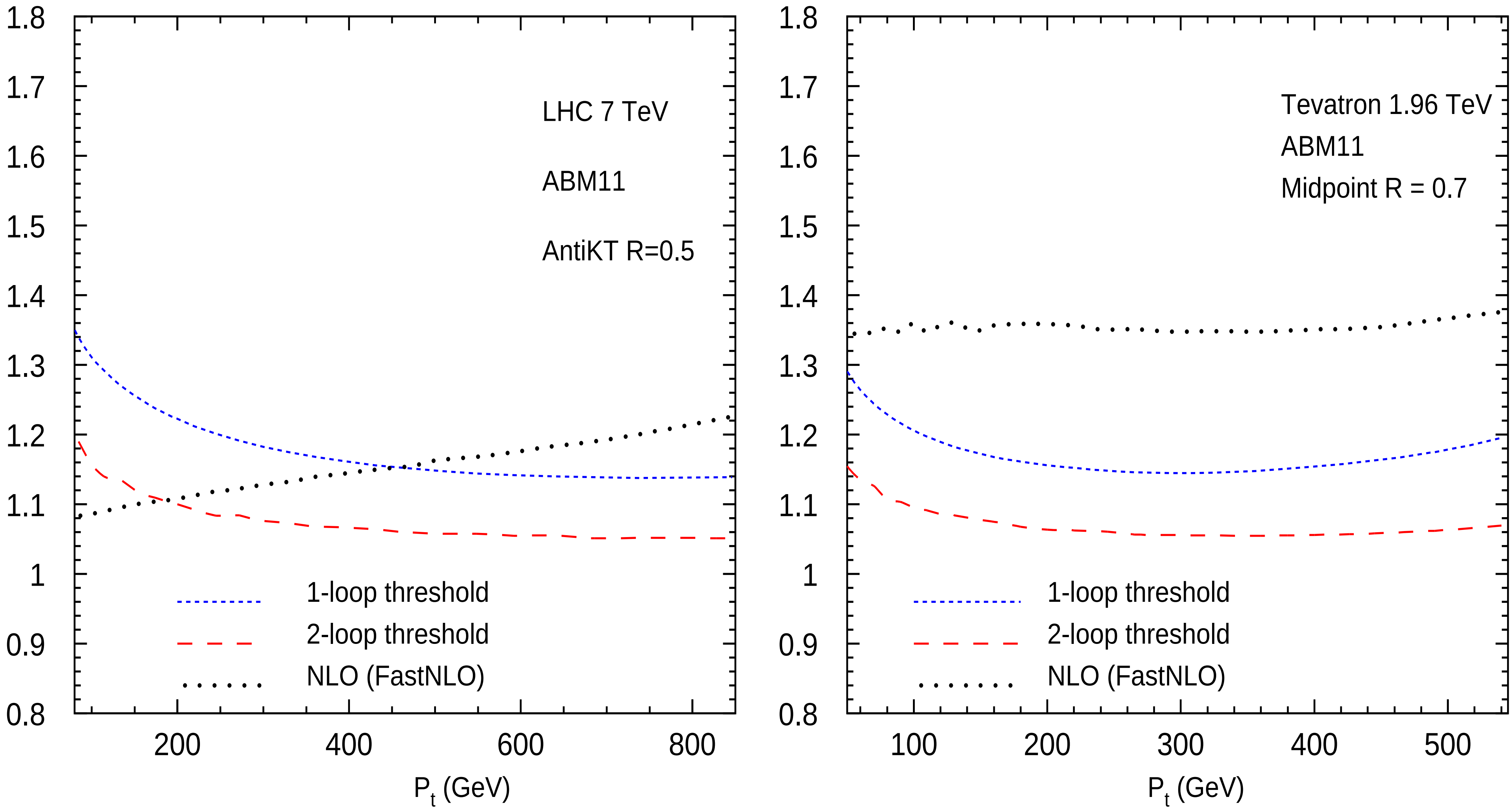


Two-loop threshold corrections (LHC 7TeV)

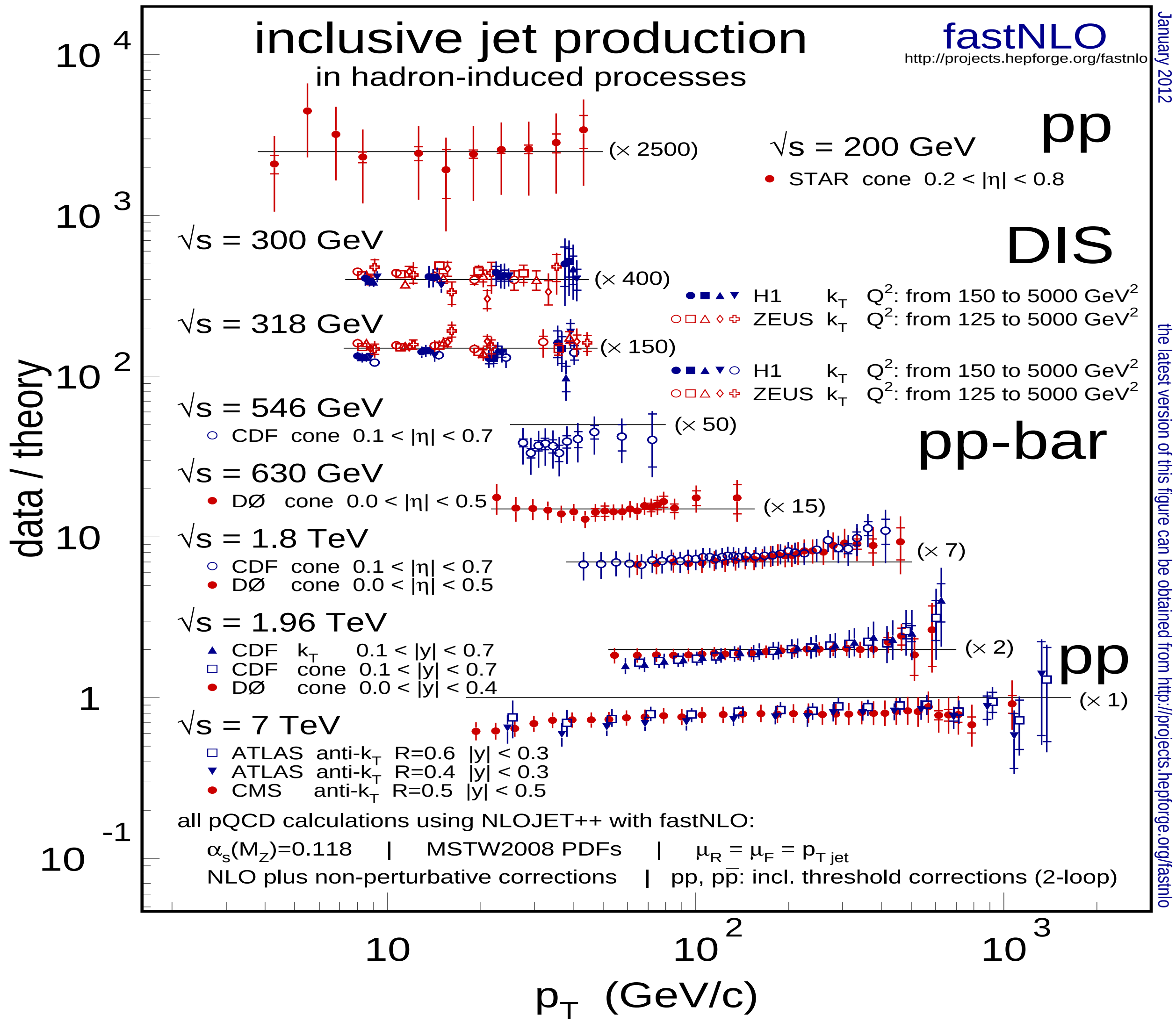




K-factors for LHC and Tevatron



Comparison to the data



D. Britzger, K. Rabbertz, F. Stober and M. Wobisch; **arXiv:1208.3641**.

Fixed order results vs threshold corrections

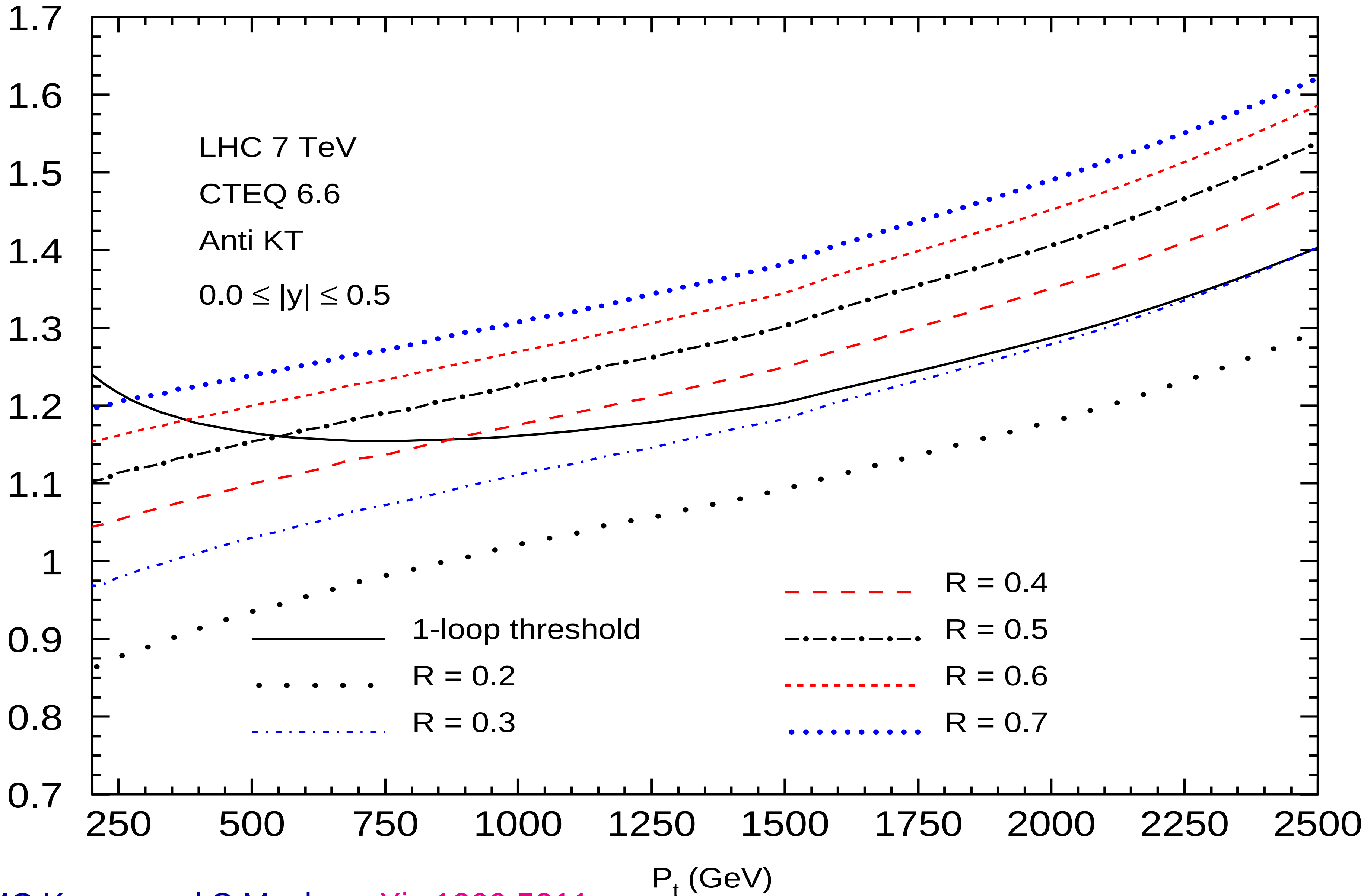
- Fixed order results

1. Complete real and virtual contributions
2. Jet definition
3. Jet algorithm, recombination scheme, dependence on R
4. `NLOJET++` : Z. Nagy; PRD68, 094002 (2003)
5. `MEKS`: For inclusive jet cross sections to NLO (interface with `LHAPDF`)
   Gao, Liang, Soper, Lai, Nadolsky, Yuan; Comp. Phys. Comm. 184, 1626 (2013)

- Threshold corrections

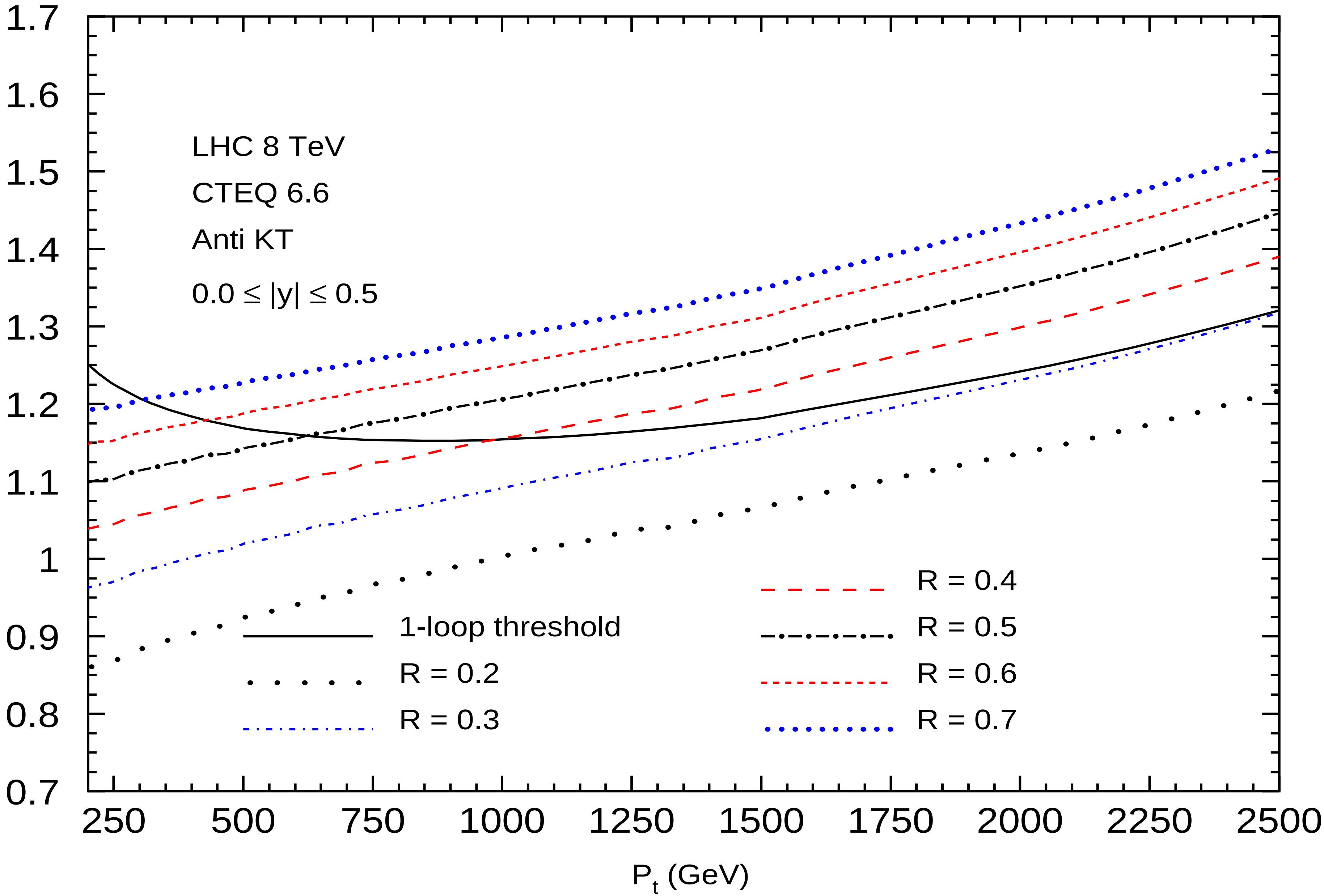
1. Soft and collinear contributions only, no matching functions yet.
2. No jet definition
3. No dependence on R

Cone size dependence of jet distributions for LHC 7 TeV

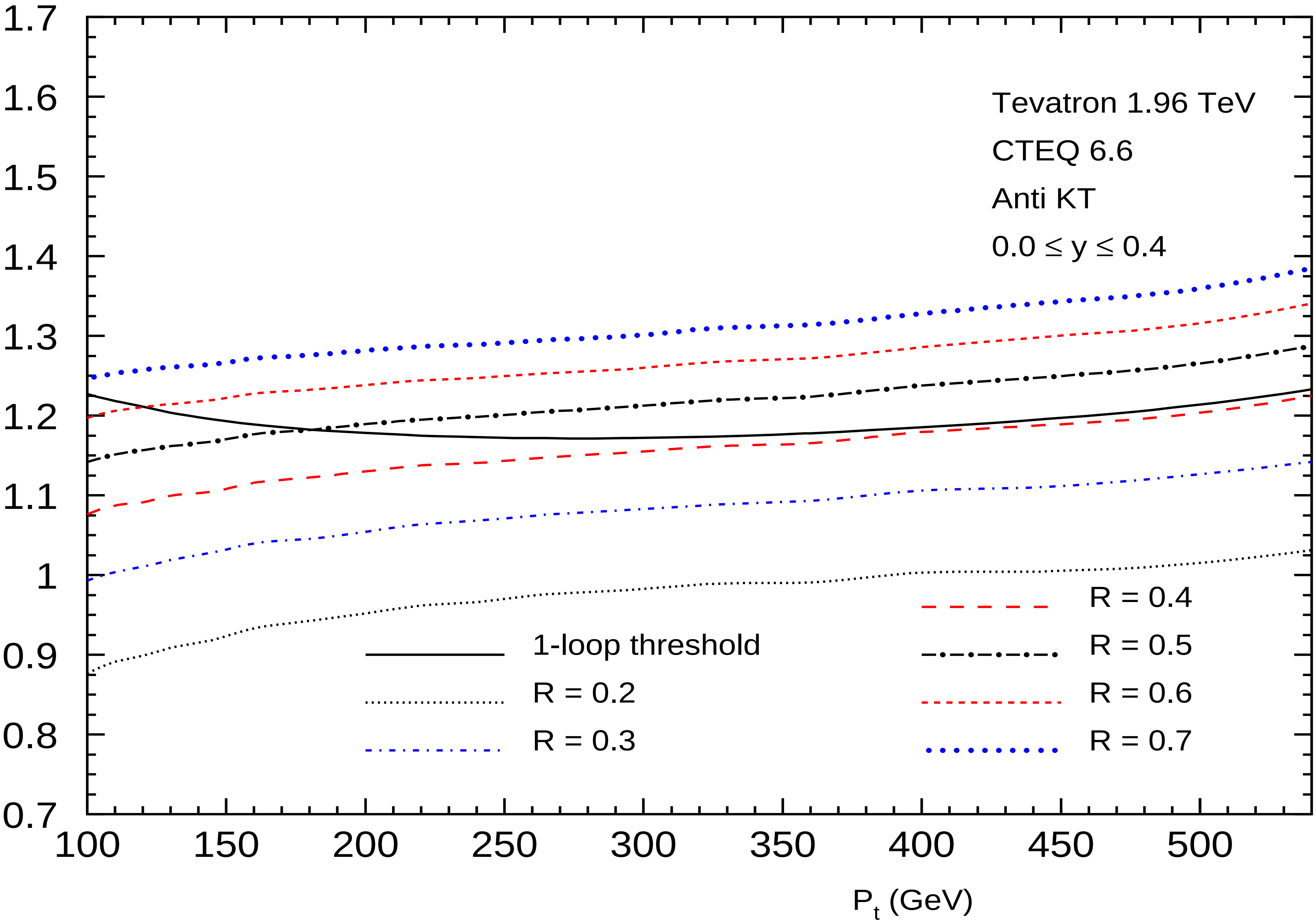


MC Kumar and S Moch; arXiv:1309.5311

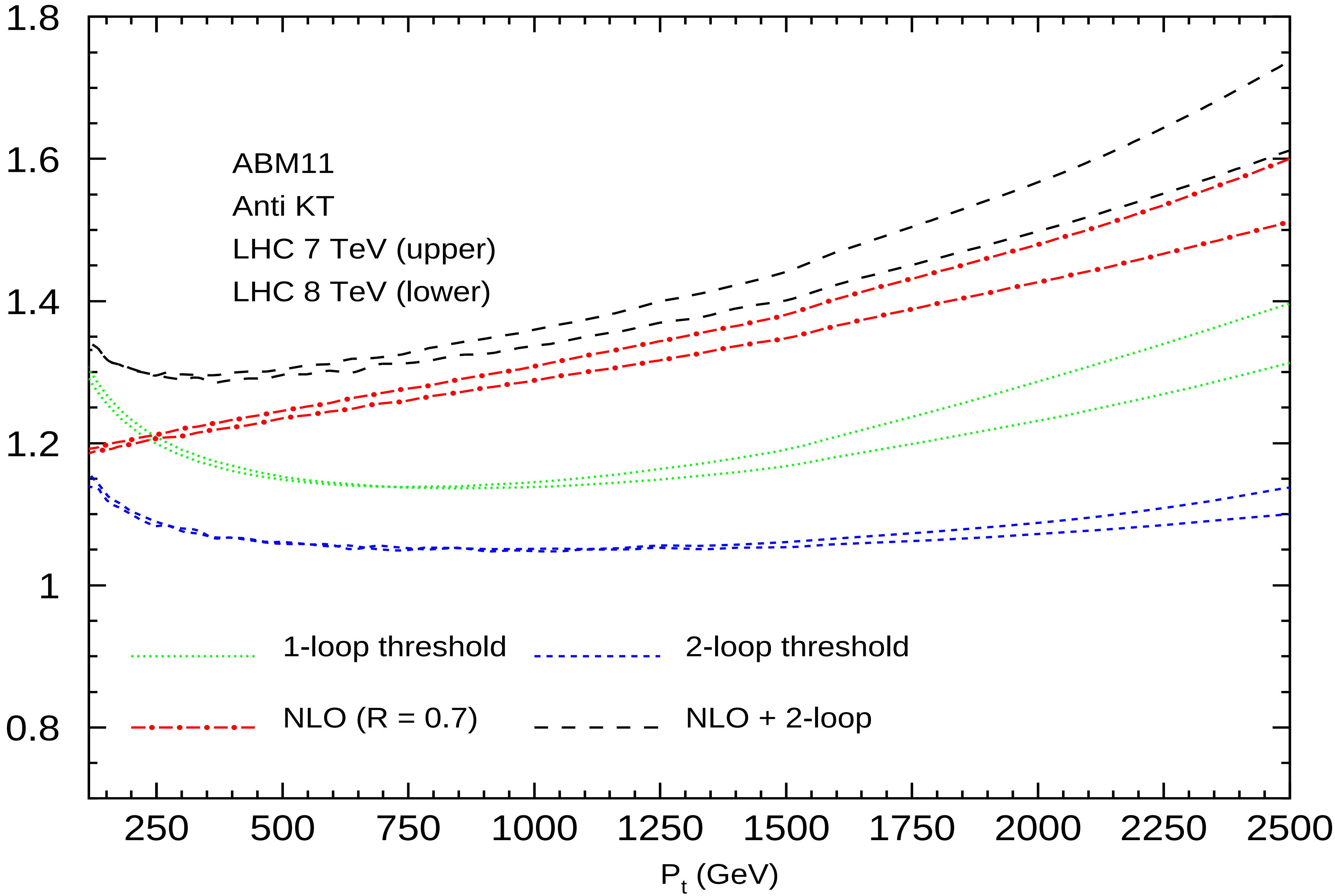
Cone size dependence of jet distributions for LHC 8 TeV



Cone size dependence of jet distributions for Tevatron



K-factors for LHC 7 TeV and 8 TeV



Summary

1. Threshold corrections for inclusive jet process have been re-calculated to NNLO at NLL accuarcy.

2. Transverse momentum distributions of the jet are presented for LHC and Tevatron, and the kinematical range of the validity of the threshold corrections has been investigated.

3. For large p_T values of the jet, threshold corrections have similar behaviour to that of fixed order NLO results but underestimate them for the current values of R used in the jet analysis.

4. Large theory uncertainties for smaller p_T values of the jet, at LHC.

5. Plenty of room for improvement, e.g. including the hard matching functions, dependence on the cone size R.