Soft Gluon Effects in Four Parton Hard-Scattering Processes

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Outline

1. Introduction: Sudakov Logs & Threshold Resummation

2. One-particle inclusive cross section: a general NLO calculation of the large logarithmic terms

3. Soft-gluon resummation at fixed rapidity

4. Conclusions
Threshold Resummation

- **Perturbative** QCD predictions are at the heart of our understanding of hard-scattering processes at high-energy colliders.

- At the **partonic threshold**, the imbalance between real emission (strongly inhibited) and virtual corrections leads to *enhanced logarithmic terms*, already at NLO.

- The large logarithms spoil the perturbative expansion ($\alpha_s L \sim 1$). A reliable evaluation of any cross-section in the near-threshold region requires the **all-order resummation** of these logarithms.

  [Sterman '87]
  [Catani, Trentadue '89]
  [Kidonakis, Laenen, Oderda, Sterman '98]
  [Bonciani, Catani, Mangano, Nason '98]
Motivation

- Resummation nowadays available for several processes, involving only two partons (DY and H production) or more partons (photoproduction, high-$p_T$ vector and H bosons, heavy quarks, jet and dihadron, single-hadron inclusive production) in QCD and SCET.

- We consider the **single-hadron** inclusive production at **high hadron transverse momentum**. Easily measurable at hadron colliders, it offers both a relevant test of the QCD factorization and quantitative information on the parton fragmentation functions.

- Soft-gluon resummation up to NLL for this process was performed in [De Florian, Vogelsang ’05]. The quantitative effect is large, especially at the typical energies of fixed-target collisions.
Aim

We want to study soft-gluon resummation for the transverse-momentum cross section at fixed rapidity of the observed hadron.

- To this aim we compute the NLO QCD corrections close to the partonic threshold, directly factorized in color space.
- Using our general expression of the NLO cross section, we determine the one-loop hard-virtual amplitude that enters into the colour-space factorization structure of the resummation formula.

Related works (threshold resummation):
- Prompt-photon [Catani, Mangano, Nason '98] [Becher, Schwartz '10]
- One-particle integrated over rapidity [De Florian, Vogelsang '05]
- Double-particle at large invariant mass [Kelley, Schwartz '11]
One-hadron-inclusive cross-section

\[ h_1 h_2 \rightarrow h_3 X \quad \text{and} \quad a_1 a_2 \rightarrow a_3 X \]

\[ \frac{d\sigma_{h_1 h_2 h_3}}{d^3 P_3/E_3}(P_i) = \sum_{a_i} f_{h_1/a_1}^{(\mu_F)} \otimes f_{h_2/a_2}^{(\mu_F)} \otimes d_{h_3/a_3}^{(\mu_f)} \otimes \frac{d\hat{\sigma}_{a_1 a_2 a_3}}{d^3 p_3/p_3^0} (p_i, \mu_F, \mu_f) \]

\[ d\eta \, d^2 p_\perp \]

- At high \( p_T \): leading contributions at the partonic threshold.
One-hadron-inclusive cross-section

\[ h_1 h_2 \rightarrow h_3 X \quad a_1 a_2 \rightarrow a_3 X \]

\[
\frac{d\sigma_{h_1 h_2 h_3}}{d^3 P_3/E_3}(P_i) = \sum \frac{d\sigma_{a_1 a_2 a_3}}{d^3 p_3/p_3^0}(p_i, \mu_F, \mu_f) \\
\sum a_i f_{h_1/a_1}^{(\mu_F)} \otimes f_{h_2/a_2}^{(\mu_F)} \otimes d_{h_3/a_3}^{(\mu_f)}
\]

- At high \( p_T \): leading contributions at the partonic threshold.
- If \( x = (s + t + u)/s \rightarrow 0 \), these are:
  - Constant terms \( \delta(x) \rightarrow 1 \)
  - Large Logs \( \left( \frac{\ln^\ell x}{x} \right)_+ \rightarrow (\ln N)^{\ell+1} \)
- While regular terms are suppressed: \( \mathcal{O}(1/N) \)
NLO result near threshold

Already in the literature [Aversa, Chiappetta, Greco, Guillet '89]

In terms of the kinematical variables \( v = 1 + t/s \) and \( w = -u/(s + t) \), the partonic cross section can be written as

\[
\frac{d\hat{\sigma}^{a_1 a_2 a_3}_{a_1 a_2 a_3}(p_i; \mu_F, \mu_f)}{d^3 p_3/p_3^0} = \frac{\alpha_s^2(\mu_R^2)}{\pi s} \left[ \frac{1}{v} \frac{d\hat{\sigma}^{(0)}(s, v)}{dv} \delta(1 - w) \right. \\
+ \left. \alpha_s(\mu_R^2) \frac{1}{2 \pi} \frac{1}{v s} C^{(1)}(s, v, w; \mu_R, \mu_F, \mu_f) + O(\alpha_s^2) \right]
\]

where the NLO term \( C^{(1)} \) has the structure

\[
C^{(1)}(s, v, w; \mu_R, \mu_F, \mu_f) = C_3(v) \left( \frac{\ln(1 - w)}{1 - w} \right) + C_2(v; s, \mu_F, \mu_f) \left( \frac{1}{1 - w} \right) + \\
+ C_1(v; s, \mu_R, \mu_F, \mu_f) \delta(1 - w) + C_0(1 - w, v; s, \mu_R, \mu_F, \mu_f).
\]
We have performed an independent calculation, by using soft and collinear approximations.

\[ a_1 a_2 \rightarrow a_3 a_4 \text{ @ 1-loop: } [\text{Kunszt, Signer, Trocsanyi '94}] \]

\[
|\mathcal{M}^{(1)}\rangle = I^{(1)}_{\text{sing}} |\mathcal{M}^{(0)}\rangle + |\mathcal{M}^{(1)}_{\text{fin}}\rangle.
\]

The color operator \( I^{(1)}_{\text{sing}} \) embodies the one-loop IR divergence, while \( |\mathcal{M}^{(1)}_{\text{fin}}\rangle \) is finite as \( \epsilon \to 0 \). We use the expression

\[
I^{(1)}_{\text{sing}} = \frac{1}{2} \frac{1}{\Gamma(1-\epsilon)} \left[ \frac{1}{\epsilon^2} \sum_{i,j=1, i \neq j}^{4} T_i T_j \left( \frac{4\pi \mu_R^2 e^{-i\lambda_{ij}\pi}}{2p_i \cdot p_j} \right)^\epsilon - \frac{1}{\epsilon} \sum_{i=1}^{4} \gamma_{ai} \left( \frac{4\pi \mu_R^2 s}{u t} \right)^\epsilon \right]
\]

Full color structure: \( T_i \) color operators.
\[ a_1 a_2 \rightarrow a_3 X, \text{ with real emission of } X = \{2 \text{ partons}\}. \]

We use soft and collinear factorization formulae [Catani, Seymour '97]

- The soft configuration is treated via eikonal approximation.
- \( X = \{\text{hard-collinear pair}\} \)? Matching to the full AP behaviour.
- Collinear-divergent counterterms (NLO PDFs).
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We use soft and collinear factorization formulae [Catani,Seymour '97]
- The soft configuration is treated via eikonal approximation.
- \( X = \{ \text{hard-collinear pair} \} \)? Matching to the full AP behaviour.
- Collinear-divergent counterterms (NLO PDFs).

- Our result has a rather compact form: one simple formula valid for **all** the flavour and colour channels.
  - It is factorized on the colour space: the coefficients of \((1/(1 - x))_+\) and \(\delta(1 - x)\) depend on colour-correlation operators \(T_i T_j\).
  - It is consistent with (the dominant contribution of) known results:
    - Photoproduction from \(qg\) and \(q\bar{q}\) channels [Gordon and Vogelsang '93]
    - \(qq\) and \(qg\) scattering [Aversa,Chiappetta,Greco,Guillet '89]
$$16\pi N^{(in)} C^{(1)} = \langle M^{(0)} | C^{(1)} | M^{(0)} \rangle + (\langle M^{(0)} | M^{(1)\text{fin}} \rangle + \text{c.c.}) \delta(1 - w) + \mathcal{O} \left( (1 - w)^0 \right)$$

$$C^{(1)}_{a_1 a_2 a_3 a_4} (s, v, w; \mu_R, \mu_F, \mu_f) = 2 \left( \frac{\ln(1 - w)}{1 - w} \right) + \left[ 2 \sum_{i=1}^{3} T_i^2 - T_4^2 \right]$$

$$- \left( \frac{1}{1 - w} \right) \left[ 2 \sum_{i=1}^{3} T_i^2 \left( \ln \frac{1 - v}{v} + \ln \frac{\mu_{Fi}^2}{s} \right) - 2 T_4^2 \ln(1 - v) \right.$$  

$$+ \gamma_{a_4} + 8 \left( T_1 \cdot T_3 \ln(1 - v) + T_2 \cdot T_3 \ln v \right) \right]$$

$$+ \delta(1 - w) \left\{ \frac{\pi^2}{2} \left( T_1^2 + T_2^2 + 3 T_3^2 - \frac{4}{3} T_4^2 \right) - \sum_{i=1}^{3} \gamma_{a_i} \ln s \frac{\mu_{Fi}^2}{v(1 - v)} + \gamma_{a_4} \ln(1 - v) \right.$$ 

$$- 2 T_3^2 \ln v \ln \frac{\mu_f^2}{s} + 2 T_2^2 \ln \frac{1 - v}{v} \ln \frac{\mu_F^2}{s} + \ln v \ln(1 - v) \left( T_4^2 - T_1^2 - T_2^2 - T_3^2 \right)$$

$$+ T_2 \cdot T_3 \left( 2\pi^2 + 2 \ln v \left( 2 \ln(1 - v) - 3 \ln v \right) \right) + \ln^2 v \ln(1 - v) \left( T_1^2 + T_3^2 - T_4^2 \right)$$

$$+ T_1 \cdot T_3 \left( 2\pi^2 + 2 \ln(1 - v) \left( \ln(1 - v) - 2 \ln v \right) \right) + \ln^2 v \left( T_2^2 + T_3^2 \right) + K_{a_4} \right\}$$
We introduce \( x_\omega = -\frac{t + u}{s}, \quad r = \frac{u}{t}, \quad p_T^2 = \frac{t u}{s}, \)

The threshold region is \( x_\omega \to 1. \)

The threshold variable \( x_\omega \) is symmetric w.r.t. \( t \leftrightarrow u. \) Otherwise the truncation of the resummed series would produce unphysical asymmetries in the angular distribution.

In terms of these variables

\[
\frac{d\hat{\sigma}_{a_1 a_2 a_3}}{d^3 p_3 / p_3^0} = \frac{|\mathcal{M}_a^{(0)}(r, p_T^2)|^2}{(4\pi s)^2} \sum_{a_1 a_2 \to a_3} \left( x_\omega, r; p_T^2, \mu_F, \mu_f \right).
\]

We perform resummation in the Mellin space of \( N \) conjugated to \( x_\omega \), neglecting \( \mathcal{O}(1/N) \) contributions. The all-order expression of \( \sum_{a_1 a_2 \to a_3, N} \) is obtained by using the BCMN resummation formalism.
Resummed Multiparton Cross Section [Bonciani, Catani, Mangano, Nason ’03]

\[ \sum_{a_1 a_2 a_3, N}^{\text{res}} = \]

\begin{align*}
&\Delta a_i, N_i(Q^2_i, \mu^2_i) \\
&J a_4, N_4(Q^2_4) \\
\end{align*}

\( \langle M_h | \Delta (\text{int}) | M_h \rangle \bigg|_{M(0)} \bigg|^{\mu^2} \)

\( N_1 = N_r + r, \quad N_2 = N_1 + r, \quad N_3 = N, \quad N_4 = N_r(1 + r) \)

\( Q^2_i = p_{T}^2 \)
Resummed Multiparton Cross Section [Bonciani, Catani, Mangano, Nason ’03]

$$\sum_{a_1a_2a_3,N}^{\text{res}} = \prod_{i=1,2,3} \Delta_{a_i,N_i}(Q_i^2, \mu_i^2)$$

$$\Delta_{a_i,N_i} : \text{IS-like radiation (soft-collinear)}$$
Resummed Multiparton Cross Section [Bonciani, Catani, Mangano, Nason ’03]

\[ \sum_{a_1a_2a_3, N}^{\text{res}} = \prod_{i=1,2,3} \Delta_{a_i, N_i}(Q_i^2, \mu_i^2) J_{a_4, N_4}(Q_4^2) \]

- \( \Delta_{a_i, N_i} \): IS-like radiation (soft-collinear)
- \( J_{a_4, N_4} \): Jet function (collinear, soft and hard, radiation)
Resummed Multiparton Cross Section [Bonciani, Catani, Mangano, Nason ’03]

\[
\Sigma_{a_1 a_2 a_3, N}^{\text{res}} = \prod_{i=1,2,3} \Delta_{a_i, N_i}(Q_i^2, \mu_i^2) J_{a_4, N_4}(Q_4^2) \Delta^{(\text{int})}_N
\]

\(\Delta_{a_i, N_i}\): IS-like radiation (soft-collinear)

\(J_{a_4, N_4}\): Jet function (collinear, soft and hard, radiation)

\(\Delta^{(\text{int})}_N\): Color-correlated large-angle soft emission
Resummed Multiparton Cross Section  [Bonciani, Catani, Mangano, Nason ’03]

\[
\sum_{a_1a_2a_3,N}^{\text{res}} = \prod_{i=1,2,3} \Delta_{a_i,N_i}(Q_i^2, \mu_i^2) J_{a_4,N_4}(Q_4^2) \frac{\langle M_H | \Delta_N^{(\text{int})} | M_H \rangle}{|M(0)|^2}
\]

\(\Delta_{a_i,N_i}\): IS-like radiation (soft-collinear)

\(J_{a_4,N_4}\): Jet function (collinear, soft and hard, radiation)

\(\Delta_N^{(\text{int})}\): Color-correlated large-angle soft emission

\(M_H\): Process-dependent constant terms \(\sim\) hard virtual corrections
Resummed Multiparton Cross Section [Bonciani, Catani, Mangano, Nason ’03]

\[ \sum_{a_1 a_2 a_3, N}^{\text{res}} = \prod_{i=1,2,3} \Delta_{a_i, N_i}(Q_i^2, \mu_i^2) J_{a_4, N_4}(Q_4^2) \frac{\langle \mathcal{M}_H | \Delta_N^{(\text{int})} | \mathcal{M}_H \rangle}{|\mathcal{M}(0)|^2} \]

\( \Delta_{a_i, N_i} \): IS-like radiation (soft-collinear)

\( J_{a_4, N_4} \): Jet function (collinear, soft and hard, radiation)

\( \Delta_N^{(\text{int})} \): Color-correlated large-angle soft emission

\( \mathcal{M}_H \): Process-dependent constant terms \( \sim \) hard virtual corrections

\[ N_1 = \frac{N - r}{1 + r}, \quad N_2 = \frac{N}{1 + r}, \quad N_3 = N, \quad N_4 = \frac{N - r}{(1 + r)^2}, \quad Q_i^2 = p_T^2 \]
Collinear radiation

The collinear radiation is diagonal in colour-space:

\[
\ln \Delta_{a,N}(Q^2, \mu^2) = \int_0^1 \frac{z^{N-1} - 1}{1 - z} \int_{\mu^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} A_a(\alpha_S(q^2)) \\
\ln J_{a,N}(Q^2) = \int_0^1 \frac{z^{N-1} - 1}{1 - z} \left[ \int_{(1-z)^2 Q^2}^{(1-z) Q^2} \frac{dq^2}{q^2} A_a(\alpha_S(q^2)) \\
+ \frac{1}{2} B_a(\alpha_S((1-z)Q^2)) \right]
\]

The coefficients \(A_a\) and \(B_a\) have perturbative expansions:

\[
A_a(\alpha_S) = \sum_{k=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n A_a^{(n)} \quad B_a(\alpha_S) = \sum_{k=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n B_a^{(n)}
\]

\[
A_a^{(1)} = C_a \quad A_a^{(2)} = \frac{1}{2} C_a \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} n_F T_R \right] \\
B_a^{(1)} = -\gamma_a
\]
Soft large-angle radiation

The colour-space radiative factor embodies all the quantum-interference effects induced by soft-gluons radiated at large angles:

\[
\Delta^{(\text{int})}_N = V_N^\dagger V_N
\]

\[
V_N = P_z \exp \left\{ \sum_{i \neq j} \int_0^1 \frac{z^{N-1} - 1}{1 - z} \Gamma \left( \alpha_S \left( (1 - z)^2 p_T^2 \right), r \right) \right\}
\]

\( P_z \) denotes z-ordering in the expansion of the exponential matrix

The anomalous-dimension matrix \( \Gamma(\alpha_S, r) \) has the perturbative expansion

\[
\Gamma(\alpha_S, r) = \frac{\alpha_S}{\pi} \Gamma^{(1)}(r) + \mathcal{O}(\alpha_S^2),
\]

\[
\Gamma^{(1)}(r) = T_t^2 \ln(1 + r) + T_u^2 \ln \frac{1 + r}{r} + i\pi T_s^2,
\]

\[
T_s^2 = (T_1 + T_2)^2, \quad T_t^2 = (T_1 + T_3)^2, \quad T_u^2 = (T_2 + T_3)^2.
\]

A.Torre 24 September 2013
Hard components

- $|\mathcal{M}_H\rangle$ embodies the residual terms of $\Sigma_N$ that are constant and it is perturbatively computable as a power series in $\alpha_S$.

- $|\mathcal{M}_H^{(1)}\rangle$ can be obtained from the result of our NLO calculation:
  - Expand & truncate the resummed formula, compare with Mellin transformed fixed order result;
  - All the logs must match $\rightarrow$ the constant terms give $\mathcal{M}_H$.

- The colour-space factorization of our NLO is essential to obtain $|\mathcal{M}_H^{(1)}\rangle$ as an amplitude!
@NLO: \[ |\mathcal{M}^{(1)}_H\rangle = |\mathcal{M}^{(1)}\rangle - I^{(1)}_H |\mathcal{M}^{(0)}\rangle \]

\[
I^{(1)}_H = I^{(1)}_{\text{sing}} + \frac{\pi^2}{4} \left( T^2_1 + T^2_2 + T^2_3 + 4 T^2_4 \right) + \frac{1}{2} \sum_{i=1}^{3} \gamma_{a_i} \ln \frac{\mu_{F_i}^2}{p_T^2} \\
- \frac{1}{2} \ln(1 + r) \ln \frac{1 + r}{r} \left( T^2_1 + T^2_2 - 3 T^2_3 + T^2_4 \right) \\
- T^2_t \left( \frac{\pi^2}{2} + \frac{1}{2} \ln^2(1 + r) + \ln(1 + r) \ln \frac{1 + r}{r} \right) \\
- T^2_u \left( \frac{\pi^2}{2} + \frac{1}{2} \ln^2 \frac{1 + r}{r} + \ln(1 + r) \ln \frac{1 + r}{r} \right) - \frac{1}{2} K_{a_4} \\
\]

\[ K_q = K_{\bar{q}} = \left( \frac{7}{2} - \frac{\pi^2}{6} \right) C_F, \quad K_g = \left( \frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{10}{9} n_F T_R \]
The extension to NNLL resummation still requires:

- $A^{(3)}_a$ in $\Delta_{a,N}$ and $J_{a,N}$ [Moch, Vermaseren, Vogt '04]
- $B^{(2)}_a$ in $J_{a,N}$
- $|M^{(2)}_H\rangle$ in $|M_H\rangle$
- $\Gamma^{(2)} \sim \left( K/2 \right) \Gamma^{(1)}$ in $\Delta^{(\text{int})}_N$
The extension to NNLL resummation still requires:

- $A^{(3)}_a$ in $\Delta_{a,N}$ and $J_{a,N}$ [Moch, Vermaseren, Vogt '04]
- $B^{(2)}_a$ in $J_{a,N}$
- $|\mathcal{M}^{(2)}_H\rangle$ in $|\mathcal{M}_H\rangle$
- $\Gamma^{(2)} \sim (K/2 \, \Gamma^{(1)})$ in $\Delta^{(\text{int})}_N$

Note that $\langle \mathcal{M}_H | \Delta^{(\text{int})}_N | \mathcal{M}_H \rangle$ leads to the second-order contribution

$$\alpha_s^2 \ln N \left( \langle \mathcal{M}^{(0)}_H | \left( \Gamma^{(1)} + \Gamma^{(1)\dagger} \right) | \mathcal{M}^{(1)}_H \rangle + \text{c.c.} \right)$$

The colour interferences between $\Delta^{(\text{int})}_N$ and $|\mathcal{M}_H\rangle$ are relevant, starting from $\mathcal{O}(\alpha_s(\alpha_s \ln N)^n)$.  

A. Torre 24 September 2013
Conclusions

- We have considered the one-particle-inclusive cross section at high transverse energies in hadron collisions.

- We have presented the general structure of the logarithmically enhanced terms at NLO.
  - Agreement with previous specific results in the literature.

- We have presented the all-order resummation formula of the logarithmically enhanced terms at fixed rapidity and extracted the colour structure of the hard coefficient at $\mathcal{O}(\alpha_s)$.
  - These resummation results are valid for both spin-unpolarized and spin-polarized hard scattering.
  - The same technique can be applied to other multiparton hard scattering processes, such as jet and heavy-quark production.
Thank you!
Backup: Rapidity-integrated cross section

Scaling variable: \( x_T = 2p_T/\sqrt{s} \) s.t. \( x_\omega = x_T \cosh \eta \).

\[
\frac{d\hat{\sigma}_{a_1a_2\rightarrow a_3}}{d^2p_T} = \frac{1}{(4\pi s)^2} \sum_{a_1a_2\rightarrow a_3} (x_T; p_T^2, \mu_F, \mu_f)
\]

\[
\tilde{\Sigma}(x_T) = \int d\eta \ |M^{(0)}_{a_1a_2a_3a_4}(r = e^{2\eta})|^2 \Theta(1 - x_T \cosh \eta) \Sigma(x_T \cosh \eta, r = e^{2\eta})
\]

The threshold limit \( x_T \rightarrow 1 \) kinematically forces \( \eta \rightarrow 0, r \rightarrow 1 \). Since \( \Sigma(x_\omega, r) \) is smooth in this limit, we can use \( \Sigma(x_\omega, r = 1) \). The resulting convolution is diagonalized in Mellin space:

\[
\tilde{\Sigma}_{a_1a_2\rightarrow a_3,N}(p_T^2, \mu_{f_i}) = \sum_{a_1a_2\rightarrow a_3,N} \left[ \sum_{a_1a_2\rightarrow a_3a_4,N} \Sigma_{res}(r = 1; p_T^2, \mu_{f_i}) + O(1/N) \right]
\]

✓ Consistent with the NLL resummed result of [De Florian, Vogelsang '05]

NB: \( \Gamma^{(1)}(r = 1) = T_s^2(-\ln 2 + i\pi) + \sum_{i=1}^{4} C_{a_i} \ln 2 \).

A. Torre 24 September 2013