Higher-order soft and Coulomb corrections to squark and gluino production at the LHC

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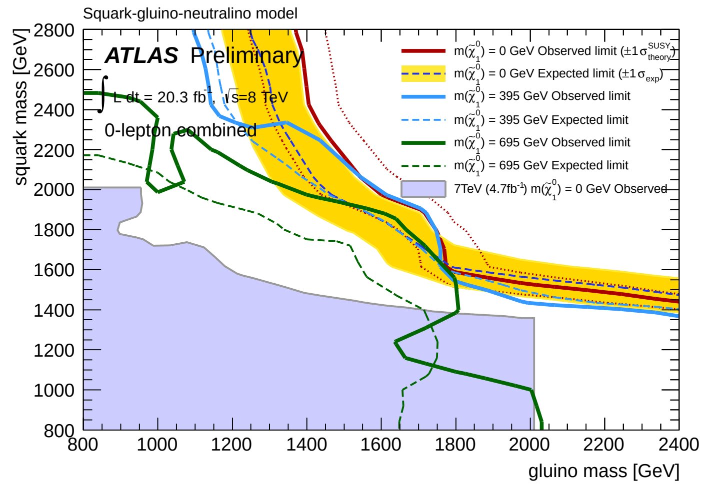
P.Falgari, CS, C.Wever, arXiv:1202.2260 [hep-ph], arXiv:1211.3408 [hep-ph]
+ M.Beneke, J.Piclum, in progress

No sign of SUSY after LHC at 8 TeV

...search will continue at $13-14$ TeV



Coloured sparticle production

main search channels at LHC

$$pp \to \{\tilde{q}\bar{\tilde{q}}\,, \quad \tilde{q}\tilde{q}\,, \quad \tilde{q}\tilde{g}\,, \quad \tilde{g}\tilde{g}\,, \quad \tilde{t}_i\bar{\tilde{t}}_i\}$$

Precise knowledge of cross sections:

- can help to distinguish models

  (if new particles observed)

- improve exclusion bounds

  (if no new particles observed)

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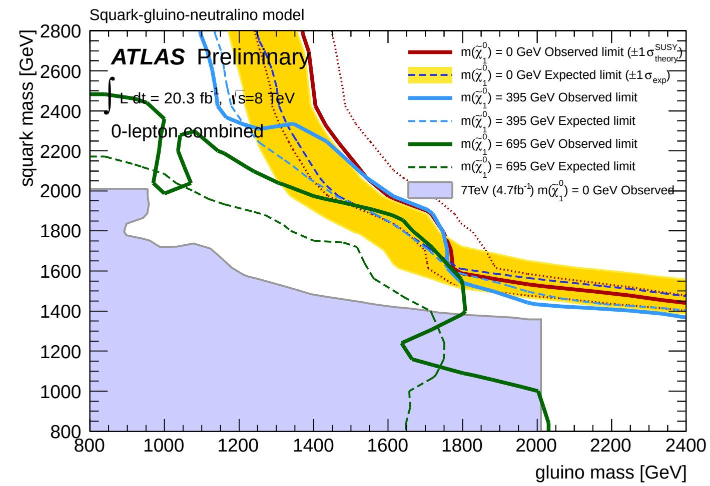

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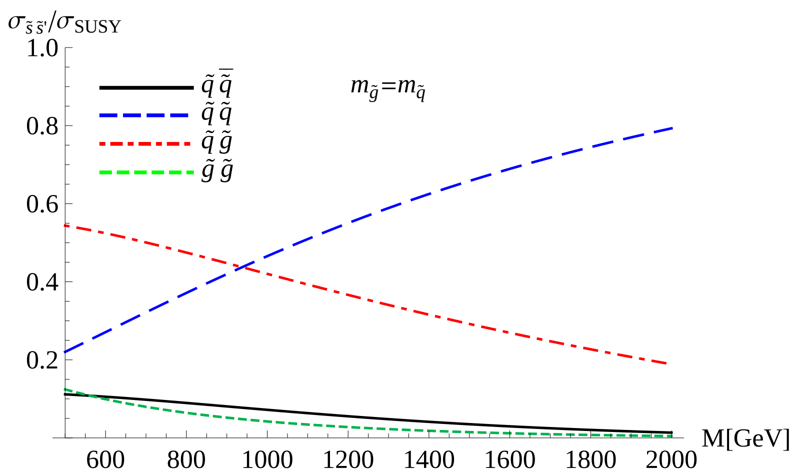
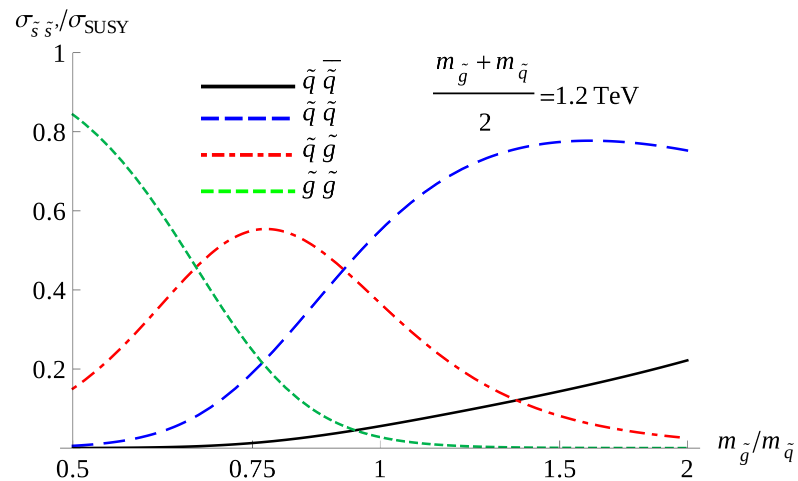
Theory status:

- NLO SUSY-QCD (Beenakker et al. 97, PROSPINO; Goncalves-Netto et al. 12, MADGOLEM, Parton-Shower matching: Gavin et al. 13)
- (N)NLL resummation (Beenakker et al. 09-13)
  NNLO$_{\text{approx}}$ for $\tilde{q}\bar{\tilde{q}}$, $\tilde{g}\tilde{g}$, $\tilde{t}\bar{\tilde{t}}$ (Langenfeld et al. 09–12, Broggio et al. 13)
- Bound state effects (Hagiwara, Yokoya 09, Kauth et al. 11)
- NLO corrections to $\tilde{q}\tilde{q}$ production and decay (Hollik et al. 12)
- EW corrections (Bornhauser et al. 07; Germer/Hollik/Mirabella/Trenkel 08-11)

Production processes $pp' \to \tilde{s}\tilde{s}'$, $\quad p, p' \in \{q, \bar{q}, g\}$, $\quad \tilde{s}, \tilde{s}' \in \{\tilde{q}, \bar{\tilde{q}}, \tilde{g}\}$





Colour structure of processes: $r \otimes r' \to R \otimes R' = \sum_\alpha R_\alpha$

$gg,\, q\bar{q} \to \tilde{q}\bar{\tilde{q}}$: $\quad 3 \otimes \bar{3} = 1 \oplus 8\,,$

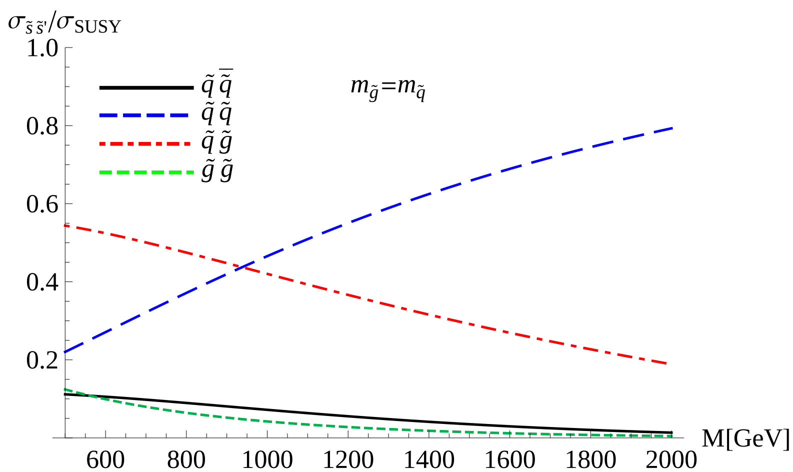
$qq \to \tilde{q}\tilde{q}$: $\quad 3 \otimes 3 = \bar{3} \oplus 6\,,$

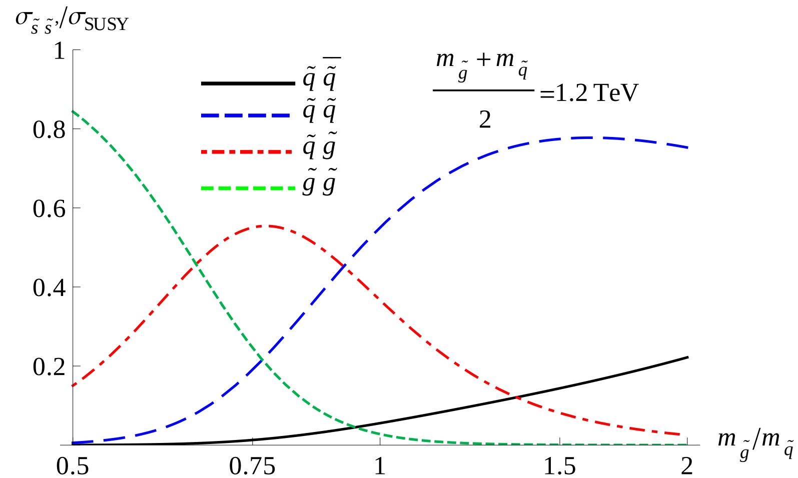
$gq \to \tilde{q}\tilde{g}$: $\quad 3 \otimes 8 = 3 \oplus \bar{6} \oplus 15\,,$

$gg, q\bar{q} \to \tilde{g}\tilde{g}$: $\quad 8 \otimes 8 = 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \overline{10} \oplus 27\,.$

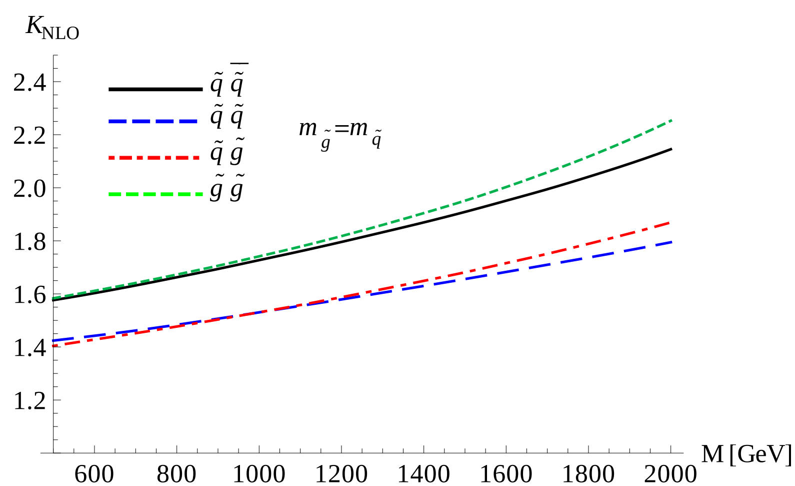
Squarks and gluinos

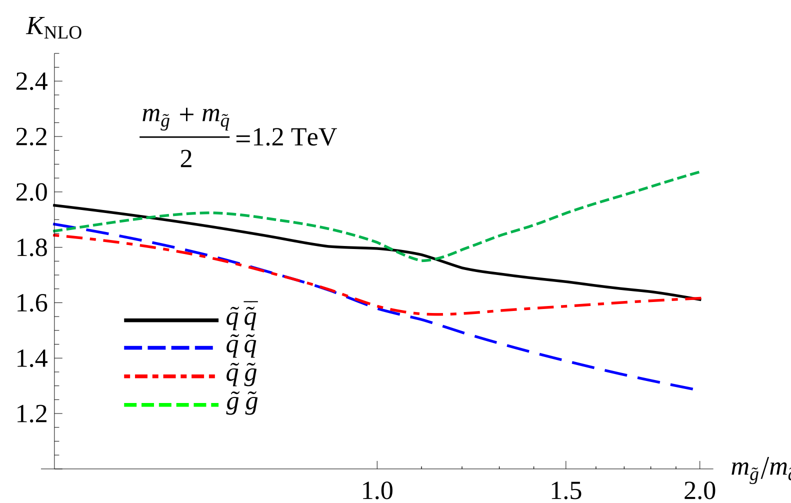
Production processes $pp' \to \tilde{s}\tilde{s}'$, $\quad p, p' \in \{q, \bar{q}, g\}$, $\quad \tilde{s}, \tilde{s}' \in \{\tilde{q}, \bar{\tilde{q}}, \tilde{g}\}$





NLO corrections up to $> 100\%$, scale uncertainty ± 20–30%





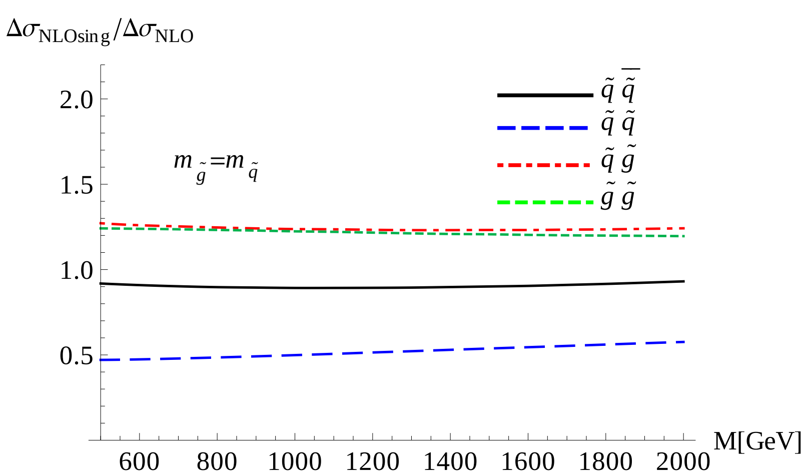
Universal limit $\beta = \sqrt{1 - \frac{4M^2}{\hat{s}}} \to 0$ (Beenakker et al. 97 , Beneke, Falgari, CS 09)

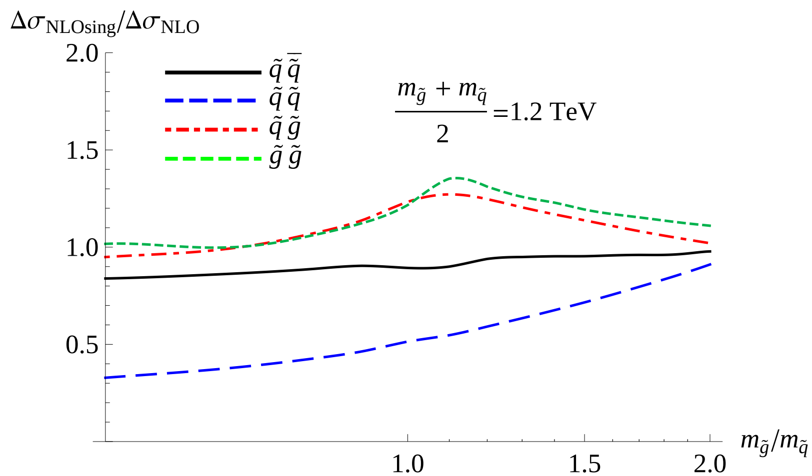
$$\sigma_{\text{NLO,app}} = \sigma^{(0)} \frac{\alpha_s}{(4\pi)} \left\{ -\frac{2\pi^2 D_{R_\alpha}}{\beta} \sqrt{\frac{2m_r}{M}} + 4(C_r + C_{r'}) \ln^2\left(\frac{8M\beta^2}{\mu_f}\right) \right.$$
$$\left. -4(C_{R_\alpha} + 4(C_r + C_{r'})) \ln\left(\frac{8M\beta^2}{\mu_f}\right) \right\}$$

(Average and reduced mass: $M = (m_s + m_{s'})/2$, $m_r = m_s m_{s'}/(m_s + m_{s'})$)

Coulomb correction: $D_{R_\alpha} = \mathbf{T}^{(R)} \cdot \mathbf{T}^{(R')} = \frac{1}{2}(C_{R_\alpha} - C_R - C_{R'})$

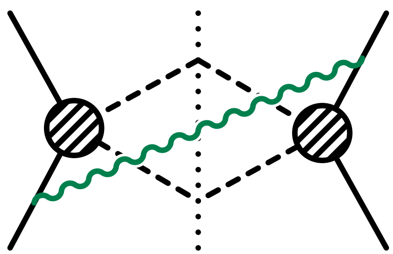
Accuracy of threshold approximation: (NLO:PROSPINO, Plehn et al.)



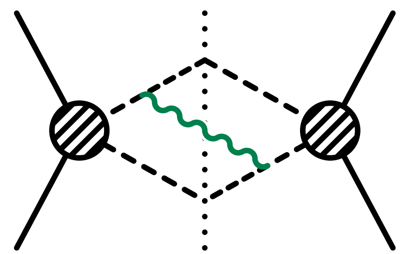


Threshold resummation

Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, ...)

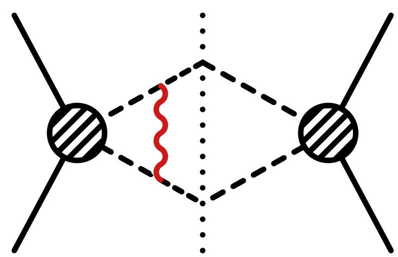


$\Rightarrow \alpha_s \; \log^2(8\beta^2)$



$\Rightarrow \alpha_s \; \log(8\beta^2)$

Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)



$\Rightarrow \alpha_s \, \frac{1}{\beta}$

Counting of threshold corrections:

$$\hat{\sigma}_{pp'} \quad \propto \quad \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$
$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \left\{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \dots\right\}:$$

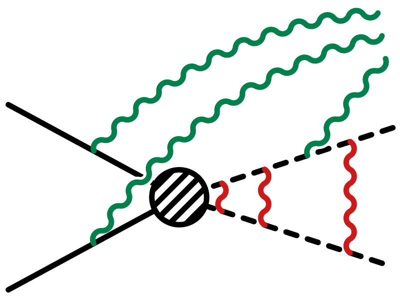
Combination of Coulomb- and soft effects?

Heavy particles nonrelativistic near threshold:

$$E \sim m\beta^2 \,, \qquad |\vec{p}| \sim m\beta$$

soft gluon momenta of same order: $q_s \sim m\beta^2 \sim E$

$\Rightarrow$ heavy particles "feel" soft radiation



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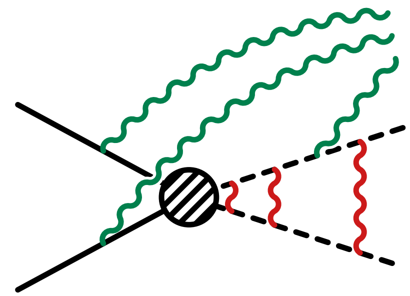
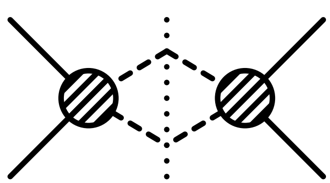
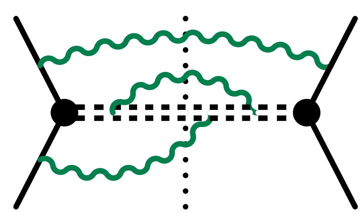
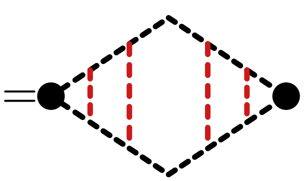


Factorization of cross section (Beneke, Falgari, CS 09/10)

$$\Rightarrow \hat{\sigma}_{pp'}(\hat{s},\mu) = \sum_{i,i'} H_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(\sqrt{\hat{s}} - 2M - \frac{\omega}{2}) W_{ii'}^{R_\alpha}(\omega,\mu)$$

Hard, soft and Coulomb functions:

$H_i =$  , $W_i{}^R =$  , $J^R =$ 

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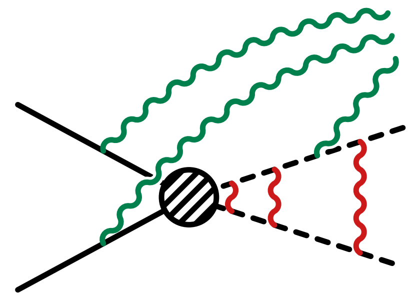

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Soft function (soft Wilson lines: $S_n(x) = \mathrm{P}\exp\left[ig_s \int_{-\infty}^0 dt\, n\cdot A_s^a(x+nt)T^a\right]$)

$$W^{R_\alpha}_{ii'}(\omega) = \int \frac{dz_0}{4\pi}\, e^{i\omega z_0/2} \langle 0|\overline{\mathrm{T}}[S_v S_v c^{(i')*} S^\dagger_{\bar{n}} S^\dagger_n](0) P^{R_\alpha} \mathrm{T}[S_n S_{\bar{n}} c^{(i)} S^\dagger_v S^\dagger_v](x_0)|0\rangle$$

Potential function: $J_{R_\alpha}(E) = 2\,\mathrm{Im}\,G_C^{R_\alpha}(0,0,E)$

(Same as for $e^-e^+ \to t\bar{t}$: Fadin, Khoze 87; Beneke, Signer, Smirnov; Hoang, Teubner 99,...)

Construction of colour basis tenors $c^{(i)}$

for all squark/gluino production processes $\tilde{q}\bar{\tilde{q}}$, $\tilde{q}\tilde{q}$, $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{g}$

- soft radiation off total colour charge of $(HH')_{R_\alpha}$ (Bonciani et.al. 98)
- one-loop basis (Kidonakis/Sterman 97; Kulesza/Moytka 08, Beenakker et.al. 09)
- all-order diagonalization of soft function (Beneke, Falgari, CS 09)
  - pairs of equivalent initial- and final state representations:

    e.g. $8 \otimes 8 \to 3 \otimes \bar{3}$: $\quad P_i \in \{(1,1),\ (8_S, 8),\ (8_A, 8)\}$
  - Clebsch-Gordan coefficients e.g. $8 \otimes 8 \to 8_A$, $3 \otimes \bar{3} \to 8$:

$$C^{(8_A)}_{\alpha a_1 a_2} = \frac{i}{\sqrt{3}} f^{a_2 \alpha a_1}\,, \qquad C^{(8)}_{\alpha a_1 a_2} = \sqrt{2} T^{\alpha}_{a_2 a_1}$$

  - construct basis tensors:

$$c^{(i)}_{\{a\}} = \frac{1}{\sqrt{\mathrm{dim}(r_\alpha)}} C^{r_\alpha}_{\alpha a_1 a_2} C^{R_\beta *}_{\alpha a_3 a_4} \qquad \text{e.g. } c^{(3)}_{\{a\}} = \frac{i}{\sqrt{12}} f^{a_2 \alpha a_1} T^{\alpha}_{a_3 a_4}$$

- Combine final-state Wilson lines: $\quad C^{R_\beta} S_v^{(R)} S_v^{(R')} = S_v^{R_\beta} C^{R_\beta}$

$\Rightarrow$ soft function for single final-state particle in R_α

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu} \Rightarrow$ IR singularities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

Resummation of threshold logarithms

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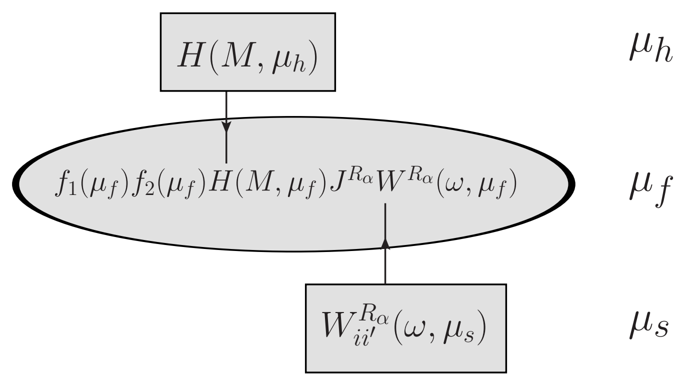
$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

Resummation:

- Momentum-space solution to RGE (Becher, Neubert, Pecjak 07)
- evolve hard function from $\mu_h \sim 2m_t$ to μ_f
- evolve soft function from $\mu_s \sim m_t\beta^2$ to μ_f



Potential function related to

Coulomb Green function: (Fadin, Khoze 87; Peskin, Strassler 90,...)

$$J_R(E) = 2\mathrm{Im}\, G_C^R(0,0;E) = \begin{cases} \frac{m_t^2 \pi D_R \alpha_s}{2\pi}\left(e^{\pi D_R \alpha_s \sqrt{\frac{m_t}{E}}} - 1\right)^{-1} & E > 0 \\ \sum_{n=1}^{\infty} \delta(E - E_n) 2R_n & E < 0 \end{cases}$$

Bound-state poles at

$$E_n = -\frac{\alpha_s^2 D_R^2 m_t}{4n^2}$$

Smeared out by finite decay with

$$E \to E + \mathrm{i}\Gamma$$

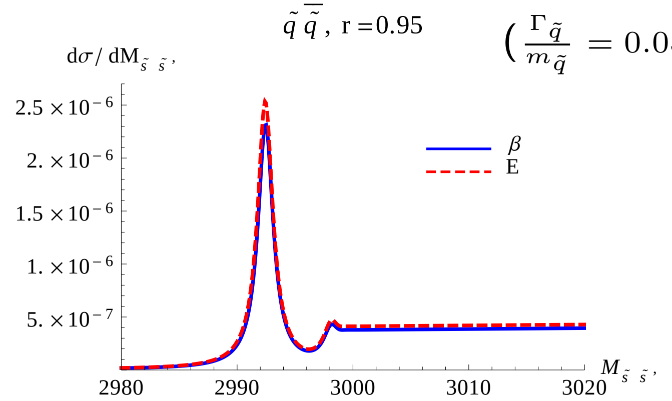
Default:

include bound-states with $\Gamma = 0$

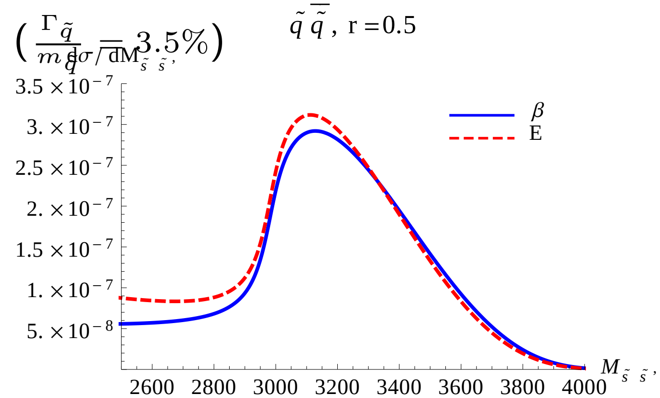
Finite-width effects negligible

for $\Gamma/M \lesssim 5\%$ (Falgari, CS, Wever 12)

$\left(\frac{\Gamma_{\tilde q}}{m_{\tilde q}} = 0.05\%\right)$



$\left(\frac{\Gamma_{\tilde q}}{m_{\tilde q}} = 3.5\%\right)$



One-loop hard functions $h_i^{(1)}(\mu_f)$ e.g. from threshold expansion
(Beenakker et al. 11/13; Kauth et al. 11; Langenfeld et al. 12)

$$\sigma_{\text{NLO,app}} = \sigma^{(0)} \frac{\alpha_s}{(4\pi)} \left\{ -\frac{2\pi^2 D_{R_\alpha}}{\beta} \sqrt{\frac{2m_r}{M}} + 4(C_r + C_{r'}) \left[\ln^2\left(\frac{8M\beta^2}{\mu_f}\right) + 8 - \frac{11\pi^2}{24} \right] - 4\left(C_{R_\alpha} + 4\left(C_r + C_{r'}\right)\right) \ln\left(\frac{8M\beta^2}{\mu_f}\right) + 12 C_{R_\alpha} + h_i^{(1)}(\mu_f) \right\}$$

One-loop soft function ($s = 1/(e^{\gamma_E} \mu e^{\rho/2})$) (Beneke/Falgari/CS 09)

$$\tilde{s}_i^R(\rho,\mu) = \int_{0_-}^{\infty} d\omega e^{-s\omega}\, \overline{W}_i^{R_\alpha}(\omega,\mu) = 1 + \frac{\alpha_s}{4\pi} \left[(C_r + C_{r'}) \left(\rho^2 + \frac{\pi^2}{6}\right) - 2C_R\,(\rho - 2) \right] + \mathcal{O}(\alpha_s^2)$$

Coulomb Green function (Beneke/Signer/Smirnov 98)

with insertion of NLO Coulomb/Non-Coulomb potential

$$\delta\tilde{V}(\boldsymbol{p},\boldsymbol{q}) = \frac{4\pi D_R \alpha_s(\mu_C)}{\boldsymbol{q}^2} \left[\frac{\alpha_s(\mu_C)}{4\pi} \left(a_1 - \beta_0 \ln\frac{\boldsymbol{q}^2}{\mu_C^2} + \frac{\pi^2 |\boldsymbol{q}|}{m_r} \left(\frac{D_R}{2} \frac{2m_r}{M} + C_A \right) \right) + \frac{\boldsymbol{p}^2}{m_1 m_2} + \frac{\boldsymbol{q}^2}{4m_r^2} v_{\text{spin}} \right],$$

$$v_{\text{spin}} = -\frac{2m_r}{4M}\,\text{(s-s)}, \qquad \frac{2m_r - M}{2M}\,\text{(f-f-triplet)}, \qquad -\frac{2m_r + 3M}{6M}\,\text{(f-f-singlet)}, \qquad -\frac{2m_r}{4M} - \frac{4m_r^2}{8m_f^2}\,\text{(s-f)}$$

Resummed cross section in momentum space

$$\hat{\sigma}^{\text{res}}_{pp'}(\hat{s},\mu_f) = \sum_i H_i(\mu_h)\, U_i \left(\frac{2M}{\mu_s}\right)^{-2\eta} \tilde{s}_i^{R_\alpha}(\partial_\eta,\mu_s)\, \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)} \int_0^\infty d\omega\, \frac{J^S_{R_\alpha}(E-\frac{\omega}{2})}{\omega} \left(\frac{\omega}{\mu_s}\right)^{2\eta}$$

- Resummation functions depend on scales μ_s, μ_f, μ_h:

$$U_i = e^{-\frac{\alpha_s \Gamma_{\text{cusp}}}{2\pi}\left(\log^2(\frac{\mu_s}{\mu_f}) - \log^2(\frac{\mu_h}{\mu_f}) - \log(\frac{4M^2}{\mu_f^2})\log(\frac{\mu_s}{\mu_h})\right)+\dots}, \eta = \frac{\alpha_s\Gamma_{\text{cusp}}}{2\pi}\log(\frac{\mu_s}{\mu_f}) + \dots$$

- fixed-order soft function generates $\log(E/\mu_s)$-terms

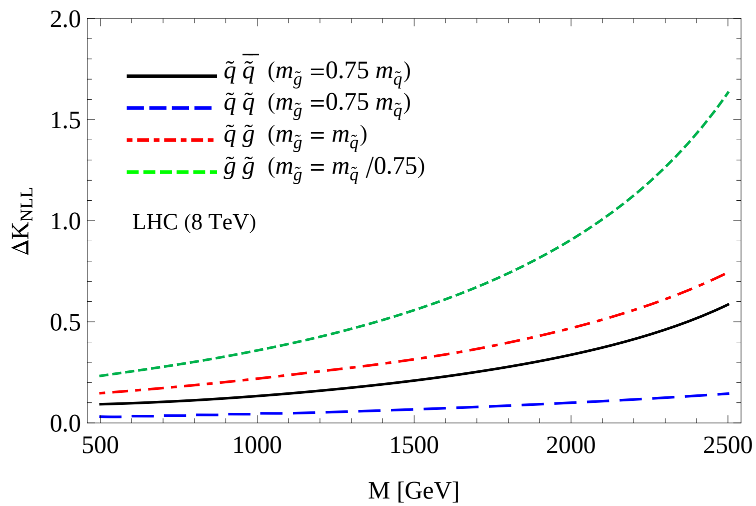
$$\tilde{s}_i^{R_\alpha}(\partial_\eta,\mu_s)\left(\frac{\omega}{\mu_s}\right)^{2\eta} = \left(\frac{\omega}{\mu_s}\right)^{2\eta} \tilde{s}_i^{R_\alpha}\left(2\ln\left(\frac{\omega}{\mu_s}\right) + \partial_\eta, \mu_s\right)$$

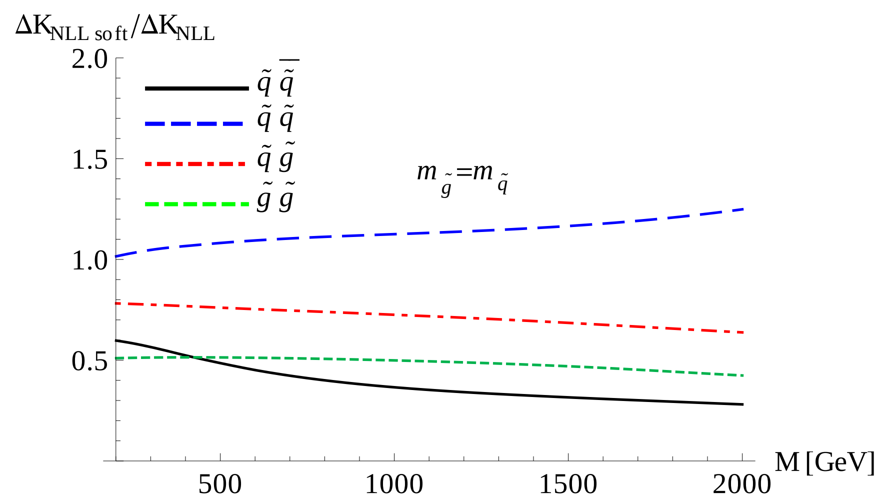
- Expansion in α_s generates all logs in $\hat{\sigma}$ for $\mu_s \sim M\beta^2$
- **RGE approach:** fixed μ_s that minimizes soft corrections to hadronic σ (Becher, Neubert, Xu 07)
- **Running scale** frozen at β_{cut} (Beneke/Falgari/Klein/CS 11)

$$\mu_s = 2M \max\{\beta^2, \beta^2_{\text{cut}}\}$$

NLL soft/Coulomb resummation (Falgari/CS/Wever 12)

- Large corrections depending on process: $\Delta K_{\text{NLL}} = \frac{\Delta\sigma_{\text{NLL}}}{\sigma_{\text{NLO}}} \sim 10\text{–}120\%$
- Coulomb effects can be large





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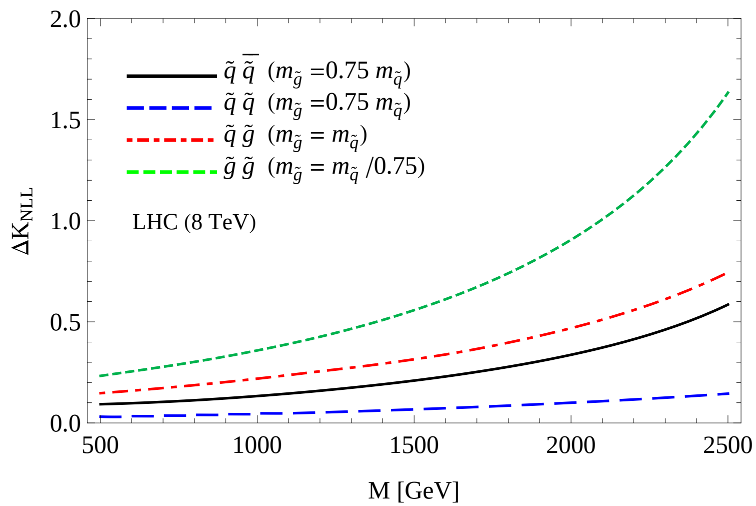
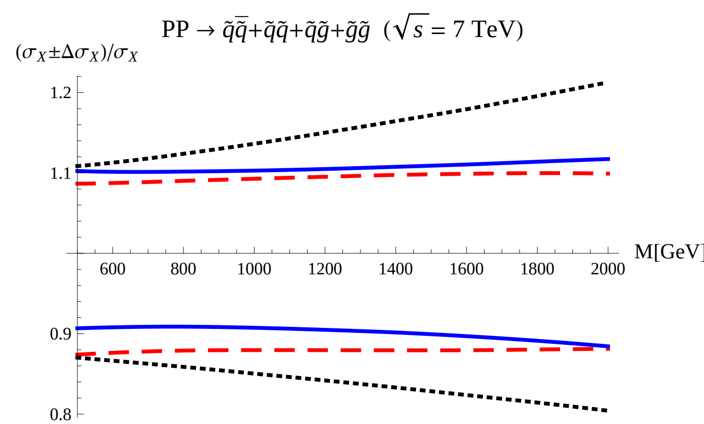
- Reduced scale dependence:

  $\pm 20\text{–}30\%$ (NLO)

  $\Rightarrow \pm 10\text{–}15\%$ (NLL)

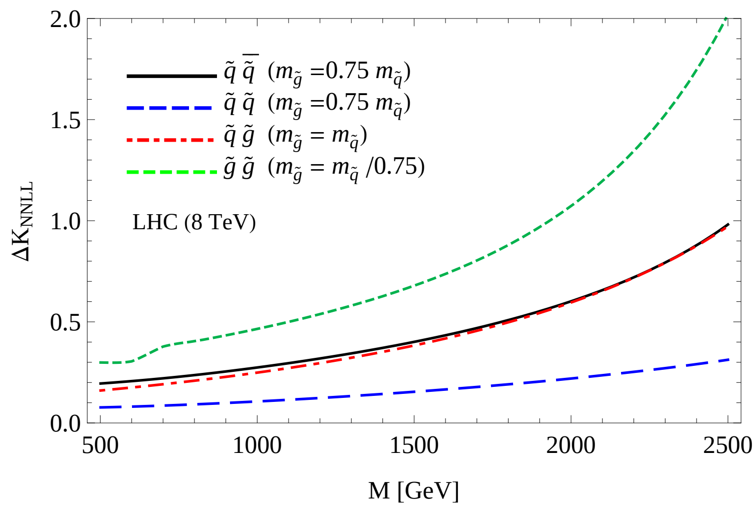
- interpolations provided



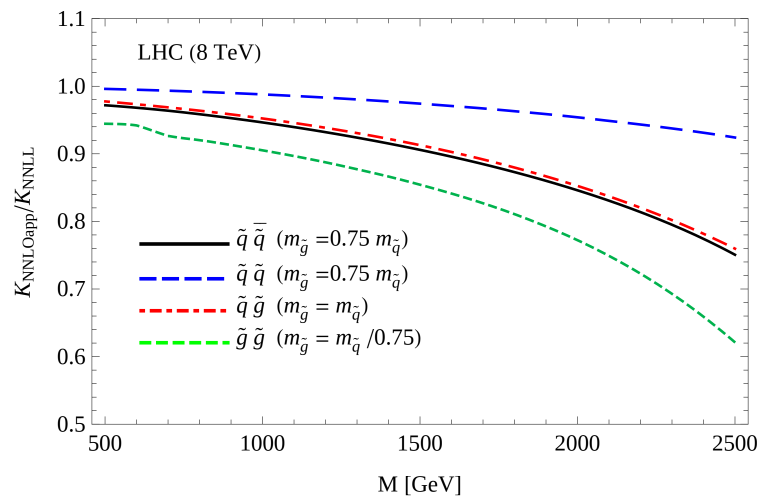


NNLL soft/Coulomb resummation (Beneke/Falgari/Piclum/CS/Wever)

- corrections $10 - 30\%$ relative to NLL
- Reduced scale dependence:
  ± 20–30% (NLO)
  $\Rightarrow \pm 10$–20% (NLL)
  $\Rightarrow \pm 5$–14% (NNLL)
- non-negligible corrections beyond NNLO for larger masses
- public program in preparation (based on `topixs`)



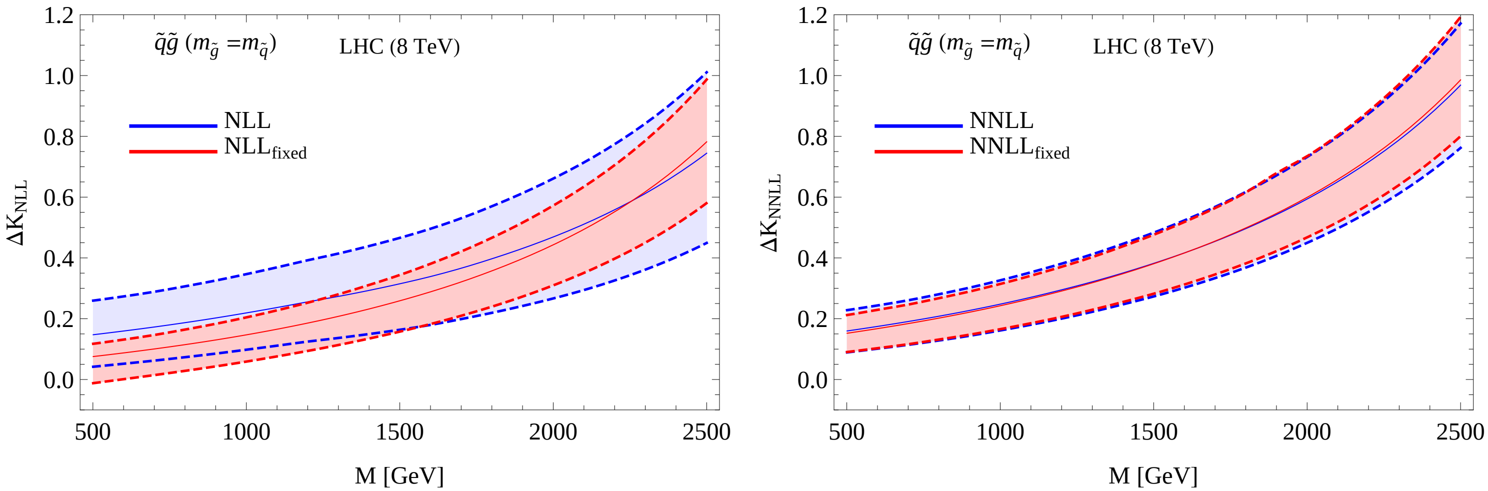
(preliminary, MSTW NLO PDFs)



Ambiguities in resummation

- variation of μ_f, μ_h, μ_C by $0.5, \ldots 2$
  ambiguity $E = M\beta^2 \Leftrightarrow \sqrt{\hat{s}} - 2M$
  estimate NNLO constant.
- Running soft scale: variation of β_{cut}, envelope of several approximations (matching to NLO/NNLO, approximate NNLO/N^{3}LO)
- Fixed soft scale: variation of μ_s

Resummation ambiguities reduced at NNLL



Threshold corrections $\sim \log^n \beta$, $\frac{1}{\beta^n}$

- Factorization of soft and Coulomb corrections
- $\log \beta$ resummation from momentum space solution to RGEs
- combined Soft and Coulomb resummation possible

NNLL resummation for squark and gluino production

- Corrections from $10 - 30\%$ ($\tilde{q}\tilde{q}$) to $30 - 200\%$ ($\tilde{g}\tilde{g}$)
- Coulomb corrections can be sizable
- Uncertainties reduced to ± 5–14%
- public program in preparation

Finite width effects negligible for $\Gamma/M \lesssim 5\%$,
total SUSY production rate.

Outlook: stop production (done at NLL, needs NLO P-wave Coulomb function),
non-degenerate squark masses

Bonus slides

Matching of scattering amplitude (for S-wave production)

$$\mathcal{A}_{pp'\to HH'X} = \sum_i C^{(i)}_{\{\alpha\}}(M,\mu)\, c^{(i)}_{\{a\}}\, \langle HH'X|\phi_{c;a_1\alpha_1}\phi_{\bar{c};a_2\alpha_2}\psi^{\dagger}_{a_3\alpha_3}\psi'^{\dagger}_{a_4\alpha_4}|pp'\rangle_{\mathrm{EFT}}$$

- $\psi^\dagger$, $\psi'^\dagger$: non-relativistic fields that create H and H'
  $\Rightarrow$ (P)NRQCD
- ϕ_c ($\phi_{\bar{c}}$): collinear (anti-collinear) fields that destroy p and p'
  $\Rightarrow$ SCET
- α_i: spin, a_i: colour indices, $c^{(i)}_{\{a\}}$: colour basis
- only soft hadronic final states X for threshold kinematics

Matching of scattering amplitude (for S-wave production)

$$\mathcal{A}_{pp'\to HH'X} = \sum_i C^{(i)}_{\{\alpha\}}(M,\mu)\, c^{(i)}_{\{a\}}\, \langle HH'X|\phi_{c;a_1\alpha_1}\phi_{\bar{c};a_2\alpha_2}\psi^\dagger_{a_3\alpha_3}\psi'^\dagger_{a_4\alpha_4}|pp'\rangle_{\rm EFT}$$

Collinear and nonrelativistic fields only connected by (u)soft gluons

$\Rightarrow$ Soft-gluon decoupling field redefinition in SCET and NRQCD

(Bauer, Pirjol, Stewart 01; Beneke, Falgari, CS 09/10)

$$\phi_c(x) = S_n(x_-)\phi_c^{(0)}(x), \qquad S_n(x) = \mathsf{P}\exp\left[ig_s\int_{-\infty}^0 dt\, n\cdot A_s^a(x+nt)T^a\right]$$

$$\psi(x) = S_v^{(R)}(x_0)\psi^{(0)}(x), \quad S_v^{(R)}(x) = \overline{\mathsf{P}}\exp\left[-ig_s\int_0^\infty ds\, v\cdot A^a(x+vs)\mathbf{T}^{(R)a}\right]$$

$$\mathcal{A}_{pp'\to HH'X} \Rightarrow \sum_i C^{(i)}\langle HH'|\psi^{(0)\dagger}\psi'^{(0)\dagger}|0\rangle\ \langle 0|\phi_c^{(0)}|p\rangle\langle 0|\phi_{\bar{c}}^{(0)}|p'\rangle\ \langle X|S_nS_{\bar{n}}c^{(i)}S_v^\dagger S_v^\dagger|0\rangle$$

$$\Rightarrow\cdots\Rightarrow \hat{\sigma}_{pp'}(\hat{s},\mu) = \sum_{i,i'} H_{ii'}(M,\mu)\int d\omega \sum_{R_\alpha} J_{R_\alpha}(\sqrt{\hat{s}}-2M-\frac{\omega}{2})W_{ii'}^{R_\alpha}(\omega,\mu)$$

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu} \Rightarrow$ IR singlarities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

$$\frac{d}{d\log\mu} W_i^{R_\alpha}(z^0,\mu) = \left(2\gamma_{\text{cusp}}(C_r + C_{r'})\log\left(\frac{iz_0\bar{\mu}}{2}\right) - 2(\gamma_{H,s}^{R_\alpha} + \underbrace{\gamma_s^r + \gamma_s^{r'}}_{\text{as for Drell-Yan/Higgs}})\right) W_i^{R_\alpha}(z^0,\mu)$$

Solution in Mellin space (Korchemsky/Marchesini 92);

momentum space (Becher/Neubert 06)

Soft anomalous dimension (Beneke, Falgari, CS 09; Czakon, Mitov, Sterman 09)

$$\gamma_{H,s}^{R_\alpha} = \frac{\alpha_s}{4\pi}(-2C_{R_\alpha}) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{R_\alpha}\left[-C_A\left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{40}{18}n_f\right] + \mathcal{O}(\alpha_s^3).$$

(extracted from Becher/Neubert 09, Korchemsky/Radyushkin 92, Kidonakis 09)

Resummation formula

$$\hat{\sigma}^{\text{res}}_{pp'}(\hat{s},\mu_f) = \sum_{S=|s-s'|}^{s+s'} \sum_i H_i^S(\mu_h)\, U_i(M,\mu_h,\mu_s,\mu_f) \left(\frac{2M}{\mu_s}\right)^{-2\eta}$$

$$\times\, \tilde{s}_i^{R_\alpha}(\partial_\eta,\mu_s) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \int_0^\infty d\omega\, \frac{J^S_{R_\alpha}(E-\frac{\omega}{2})}{\omega} \left(\frac{\omega}{\mu_s}\right)^{2\eta}$$

- Resummation functions:

$$\eta = 2a_\Gamma(\mu_s,\mu_f)$$

$$U_i^{R_\alpha} = \exp[4S(\mu_h,\mu_s) - 2a_i^V(\mu_h,\mu_s) + 2a^{\phi,r}(\mu_s,\mu_f) + 2a^{\phi,r'}(\mu_s,\mu_f)] \left(\frac{4M^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_s)}$$

$$S(\mu_h,\mu_s) = -\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu_s)} d\alpha_s \frac{\Gamma^r_{\text{cusp}}(\alpha_s) + \Gamma^{r'}_{\text{cusp}}(\alpha_s)}{2\beta(\alpha_s)} \int_{\alpha_s(\mu_h)}^{\alpha_s} \frac{d\alpha_s'}{\beta(\alpha_s')}\,,$$

$$a_\Gamma(\mu_a,\mu_b) = -\int_{\alpha_s(\mu_a)}^{\alpha_s(\mu_b)} d\alpha_s \frac{\Gamma^r_{\text{cusp}}(\alpha_s)+\Gamma^{r'}_{\text{cusp}}(\alpha_s)}{2\beta(\alpha_s)}\,, \qquad a_i^V(\mu_a,\mu_b) = -\int_{\alpha_s(\mu_a)}^{\alpha_s(\mu_b)} d\alpha_s \frac{\gamma_i^V(\alpha_s)}{\beta(\alpha_s)}.$$

Resummation in Mellin space

(Sterman 87; Catani, Trentadue 89; Korchemsky, Marchesini 92)

Mellin transform: ($\rho = 4m_t^2/\hat{s}$)

$$\sigma^N = \int_0^1 d\rho\rho^{N-1}\hat{\sigma}(4m_t^2/\rho)\ , \quad \int_0^1 d\rho\rho^N \beta \log^n \beta \propto \ln^n N + \ldots$$

Resummed cross section:

$$\hat{\sigma}^N(m_t^2,\mu)/\hat{\sigma}^{(0)N}(m_t^2,\mu) = g^0(M^2,\mu)\ \exp\left(G^{N+1}(m_t^2,\mu^2)\right)$$

Soft gluon effects resummed in exponent:

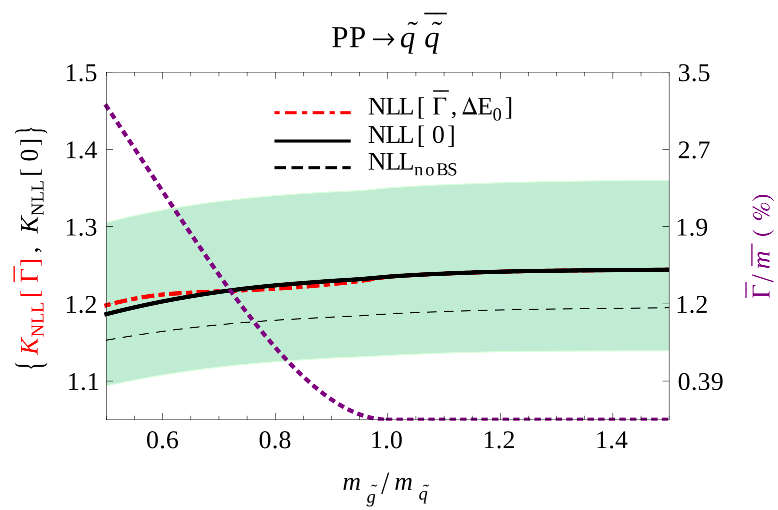
$$G^N(m_t^2,\mu^2)\ =\ \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\left[\int_\mu^{4m_t^2(1-z)^2}\frac{dq^2}{q^2}\left(2A\left(\alpha_s(q^2)\right)\right)\ +\ D\left(\alpha_s(4m_t^2(1-z)^2)\right)\right]$$

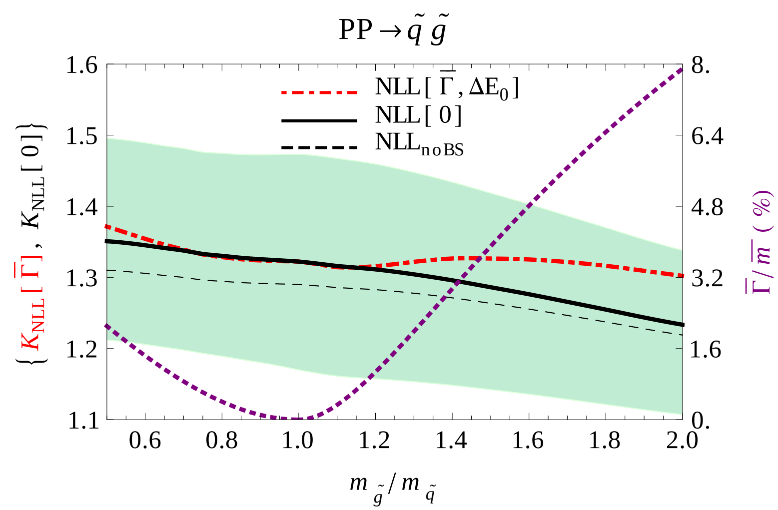
Inverse numerical transformation (Catani, Mangano, Nason, Trentadue 96)

(avoid Landau pole for $z \to 1$, power suppressed ambiguities,...)

Total cross sections, LO-SQCD decays as example:

$$\Gamma_{\tilde{q}\to q\tilde{g}} = \frac{\alpha_s C_F m_{\tilde{q}}}{2}(1-(\frac{m_{\tilde{g}}}{m_{\tilde{q}}})^2)^2\,, \qquad m_{\tilde{q}}>m_{\tilde{g}}, \qquad \Gamma_{\tilde{g}\to q\bar{\tilde{q}}} = \frac{\alpha_s n_f m_{\tilde{g}}}{2}(1-(\frac{m_{\tilde{g}}}{m_{\tilde{q}}})^{-2})^2\,, \qquad m_{\tilde{q}}<m_{\tilde{g}}.$$

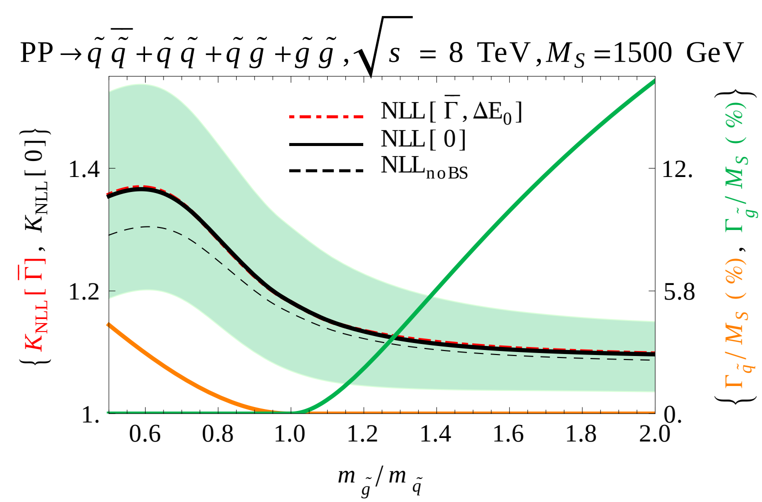




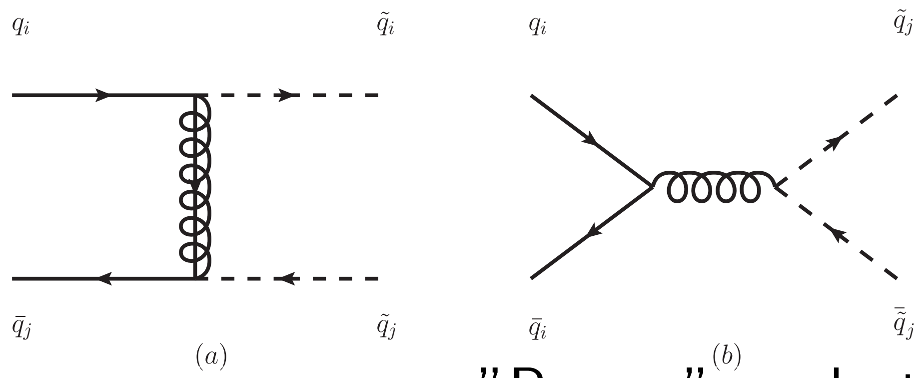
Negligible effect

on total SUSY production rate

but relevant bound-state corrections

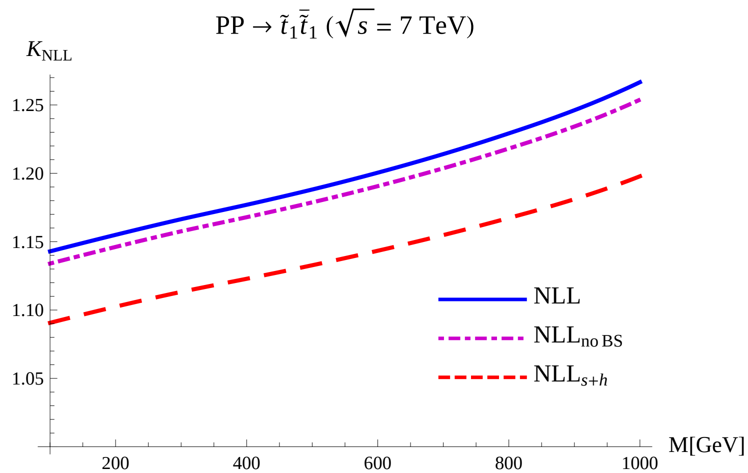


Difference to light-flavour squarks for $q\bar{q}$ initial state:



absent for stops $\Rightarrow$ "P-wave" production, $\sim \beta^3$

Resummation formalism works also for $q\bar{q}$ channel (Falgari, CS, Wever 12)
use Coulomb Green function for P-waves (Bigi/Fadin/Khoze 92)



Threshold expansion at $\mathcal{O}(\alpha_s^2)$

All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms (Beneke, Czakon, Falgari, Mitov, CS 09)

Pure soft corrections: (also Moch/Uwer+Langenfeld (08/09))

$$\Delta\sigma_{\mathsf{s}}^{(2)} \sim \alpha_s^2 (c_{\mathsf{LL}}^{(2)} \ln^4\beta + c_{\mathsf{NLL}}^{(2)} \ln^3\beta + c_{\mathsf{NNLL},2}^{(2)} \ln^2\beta + \underbrace{c_{\mathsf{NNLL},1}^{(2)} \ln\beta}_{\text{2-loop } \gamma_{H,s}})$$

Potential corrections: 2nd Coulomb, NLO potentials

$$\Delta\sigma_{\mathsf{p}}^{(2)} \sim \alpha_s^2 \left(\frac{c_{C^2}}{\beta^2} + \frac{1}{\beta}(c_{\mathsf{C},0}^{(2)} + c_{\mathsf{C},1}^{(2)} \log\beta) + \underbrace{c_{\mathsf{n\text{-}C}}^{(2)} \ln\beta}_{\text{spin-dependent}}\right)$$

(using Beneke, Signer, Smirnov 99, Czarnecki/Melnikov 97/01)

mixed Coulomb/soft/hard corrections:

$$\Delta\sigma_{\mathsf{p}\otimes\,\mathsf{sh}}^{(2)} \sim \frac{\alpha_s}{\beta}\alpha_s (c_{\mathsf{LL}}^{(1)} \ln\beta^2 + c_{\mathsf{NLL}}^{(1)} \ln\beta + c + H^{(1)})$$

$$\Delta\sigma_{\mathsf{s}\otimes\mathsf{h}}^{(2)} \sim \alpha_s^2 H^{(1)} (c_{\mathsf{LL}}^{(1)} \ln\beta^2 + c_{\mathsf{NLL}}^{(1)} \ln\beta)$$

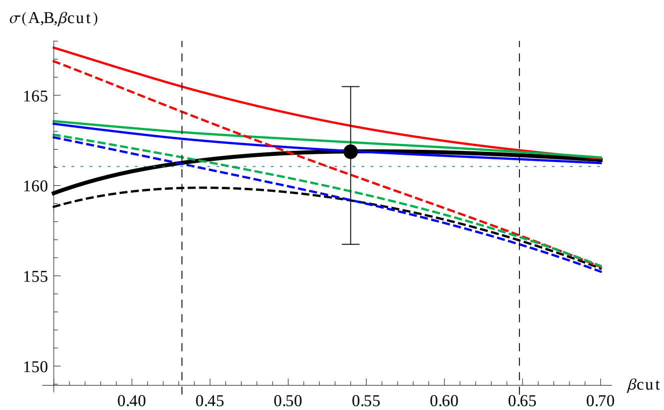
(H_1: process and colour-channel dependent, $t\bar{t}$: Czakon/Mitov 09)

Determination of β_{cut}

- allow for different implementations

  $\beta < \beta_{\text{cut}}$: NNLL ($\mu_s = k_s m_t \beta^2_{\text{cut}}$) with/without constant at $\mathcal{O}(\alpha_s^2)$

  $\beta > \beta_{\text{cut}}$: NNLL ($\mu_s = k_s m_t \beta^2$); NNLO$_{\text{approx}}$; NNNL$_3$(A/B)



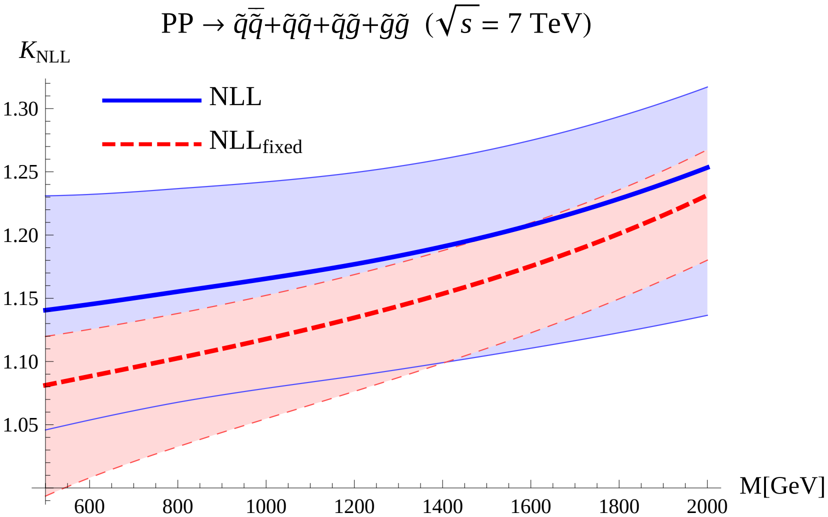
- Choose β_{cut} so that not too sensitive to
  - ambiguities for $\beta \to 1$
  - breakdown of perturbation theory for $\beta \to 0$

  (E.g. LHC7: $\mu_s = 2m_t\beta^2$, $\beta_{\text{cut}} = 0.54 \Rightarrow \mu_s > 100$ GeV)

Ambiguities in resummation

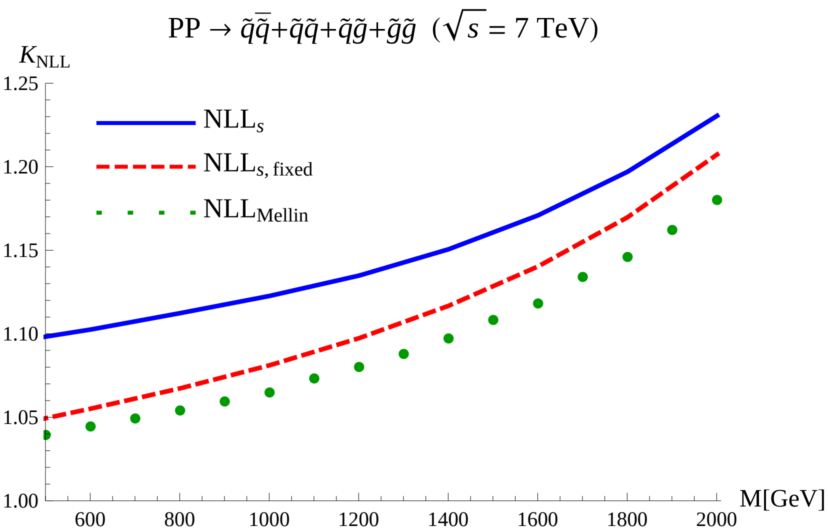
Running-scale and fixed-scale implementations agree within resummation uncertainties

(β_{cut}, μ_s variation...)



For soft resummation reasonable consistency with Mellin-space resummation

(Beenakker et al. 09)



(LHC7, $m_{\tilde{q}} = m_{\tilde{g}}$)

Comparison to other NNLO$_{\mathrm{app}}$/NNLL predictions

NNLL soft resummation (Cacciari/Czakon/Mangano/Mitov/Nason 11)

(β-expansion, Mellin space, no Coulomb resummation, different α_s^2 constant)

Resummed differential cross sections

- Pair-invariant mass : $\frac{d\hat{\sigma}(t\bar{t})}{dM_{t\bar{t}}}$, (Ahrens/Ferroglia/Neubert/Pecjak/Yang 10)
- One-particle inclusive: $\frac{d\hat{\sigma}(t+X)}{dp_T}$ (Kidonakis 11; Ahrens et al. 11)

Exact NNLO (Bärnreuther,Czakon,Fiedler,Mitov 12/13)