NLO EW corrections to $pp \rightarrow WW \rightarrow 4f$ in double-pole approximation at the LHC

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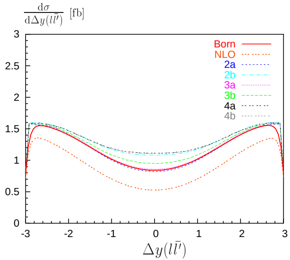
Motivation

W^+W^- production at the LHC

- probe non-abelian structure of the Standard Model
- measurement of anomalous gauge couplings
- irreducible background to $H \to WW^\star$

For precise theoretical predictions we need

- QCD corrections
- EW corrections (enhancement by **Sudakov logarithms** at high scales)



EW corrections could fake New Physics!

[Accomando,Kaiser '05]
(logarithmic approximation)

Recent results

QCD corrections

- NLO QCD corrections (+ parton shower matching @NLO)
  [Cascioli,Hoeche,Krauss,Maierhofer,Pozzorini,Siegert '13]
  $\rightarrow$ talk by P. Maierhöfer
- approximate NNLO QCD results [Dawson,Lewis,Zeng '13]
- step towards NNLO QCD corrections $\rightarrow$ talk by L. Tancredi
  numerical results only valid in high energy limit
  [Chachamis,Czakon,Eiras '08]

Fixed order calculations including EW corrections

- one-loop corrections to on-shell $\mathrm{W}^+\mathrm{W}^-$ production
  [Bierweiler,Kasprzik,Kühn,Uccirati '12/13] $\rightarrow$ talk by J. Kühn
- one-loop corrections to on-shell $\mathrm{W}^+\mathrm{W}^-$ production including γq-induced contributions [Baglio,Ninh,Weber '13]
  - EW corrections for integrated cross sections: $\mathcal{O}(1\%)$
  - EW corrections for distribution at high scales: up to -40%

Our calculation: RacoonWW approach

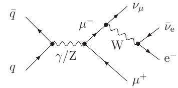
RacoonWW: $\mathrm{e}^+\mathrm{e}^- \to$ 4 fermions in double-pole approximation

[Denner,Dittmaier,Roth,Wackeroth '00]

Leading order: full off-shell calculation

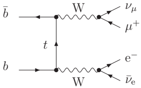
- $\bar{q}q$-induced contribution ($q = \mathrm{u}, \mathrm{d}, \mathrm{c}, \mathrm{s}$)

$$\bar{q}q \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e}$$



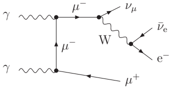
- $\bar{\mathrm{b}}\mathrm{b}$-induced contribution ($<$ 2% @LO)

$$\bar{\mathrm{b}}\mathrm{b} \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e}$$



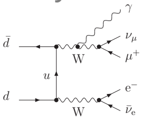
- $\gamma\gamma$-induced contribution ($<$ 1% @LO)

$$\gamma\gamma \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e}$$



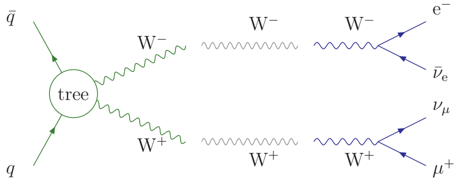
EW corrections to $\bar{q}q$-induced contribution only

- **Virtual** corrections:
  Double-pole approximation (DPA)
- **Real** corrections: full off-shell calculation



Double Pole Approximation: Basic Idea

LO



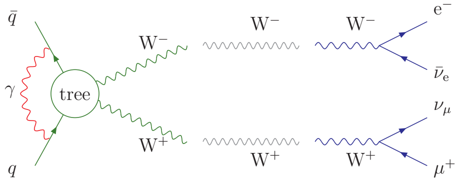
on-shell production   Breit-Wigner   on-shell decay

$$\mathcal{M}_{\text{Born,DPA}} = \sum_{\lambda_+,\lambda_-} \frac{1}{\left[k_+^2 - M_W^2 + iM_W\Gamma_W\right]\left[k_-^2 - M_W^2 + iM_W\Gamma_W\right]} \times \left\{ \mathcal{M}_{\text{Born}}^{\bar{q}q\to W^+W^-} \mathcal{M}_{\text{Born}}^{W^+\to\nu_\mu\mu^+} \mathcal{M}_{\text{Born}}^{W^-\to e^-\bar{\nu}_e} \right\}$$

- consider on-shell prodution of W pair and on-shell W decay
- add off-shell propagators including Breit-Wigner distribution
- sum over the polarization states
- for gauge invariance: on-shell projection of momenta needed!
- error estimate: Γ_W/M_W

Double Pole Approximation: Basic Idea

NLO

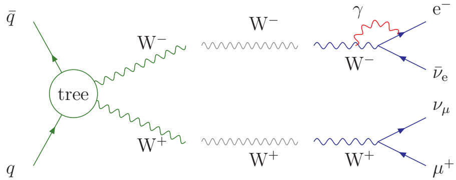


on-shell production   Breit-Wigner   on-shell decay

One distinguishes between factorizable (corrections to the decay OR to the production) and non-factorizable corrections (only soft photon exchange).

Double Pole Approximation: Basic Idea

NLO



on-shell production Breit-Wigner on-shell decay

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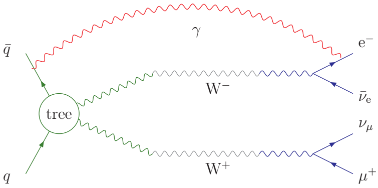
Double Pole Approximation: Basic Idea

NLO



on-shell production   Breit-Wigner   on-shell decay

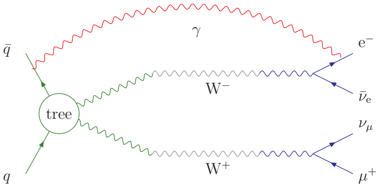
One distinguishes between factorizable (corrections to the decay OR to the production) and non-factorizable corrections (only soft photon exchange).

- selection of all doubly resonant one-loop diagrams to $\mathrm{pp} \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e}$
- expansion around the poles
- error estimate: $\alpha/\pi \times \Gamma_\mathrm{W}/M_\mathrm{W}$

Advantages

- # diagrams reduced
- diagrams much easier to calculate (without DPA EW corrections to $2 \to 4$ process!) $\to$ faster code

Double Pole Approximation: Basic Idea
NLO



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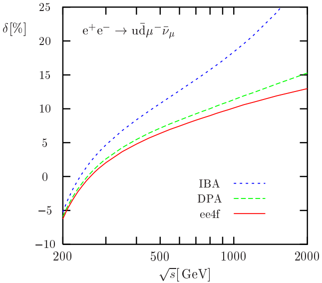


[Denner,Dittmaier,Roth,Wieders '06]

full off-shell calculation of $e^+e^- \to 4f$

Virtual corrections in DPA

$$\int_4 \mathrm{d}\hat{\sigma}_{\bar{q}q}^{\mathrm{virt,DPA}} \sim \int_4 \mathrm{d}\Phi_4 \left\{ 2\,\mathrm{Re}\left[\left(\mathcal{M}_{\mathrm{Born,DPA}}^{\bar{q}q\to\,\mathrm{WW}\to\,\nu_\mu\mu^+\mathrm{e}^-\bar{\nu}_\mathrm{e}}\right)^\star \delta\mathcal{M}_{\mathrm{virt,fact,DPA}}^{\bar{q}q\to\,\mathrm{WW}\to\,\nu_\mu\mu^+\mathrm{e}^-\bar{\nu}_\mathrm{e}}\right] + \left|\mathcal{M}_{\mathrm{Born,DPA}}^{\bar{q}q\to\,\mathrm{WW}\to\,\nu_\mu\mu^+\mathrm{e}^-\bar{\nu}_\mathrm{e}}\right|^2 \delta_{\mathrm{nfact}}^{\mathrm{virt}} \right\}$$

Factorizable Corrections

$$\delta\mathcal{M}_{\mathrm{virt,fact,DPA}}^{\bar{q}q\to\mathrm{W^+W^-}\to\nu_\mu\mu^+\mathrm{e}^-\bar{\nu}_\mathrm{e}} = \sum_{\lambda_+,\lambda_-} \frac{1}{\left[k_+^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}\right]\left[k_-^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}\right]} \times \left\{ \delta\mathcal{M}^{\bar{q}q\to\mathrm{W^+W^-}} \mathcal{M}_{\mathrm{Born}}^{W^+\to\nu_\mu\mu^+} \mathcal{M}_{\mathrm{Born}}^{W^-\to\mathrm{e}^-\bar{\nu}_\mathrm{e}} + \mathcal{M}_{\mathrm{Born}}^{\bar{q}q\to\mathrm{W^+W^-}} \delta\mathcal{M}^{W^+\to\nu_\mu\mu^+} \mathcal{M}_{\mathrm{Born}}^{W^-\to\mathrm{e}^-\bar{\nu}_\mathrm{e}} + \mathcal{M}_{\mathrm{Born}}^{\bar{q}q\to\mathrm{W^+W^-}} \mathcal{M}_{\mathrm{Born}}^{W^+\to\nu_\mu\mu^+} \delta\mathcal{M}^{W^-\to\mathrm{e}^-\bar{\nu}_\mathrm{e}} \right\}$$

Non-factorizable corrections

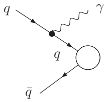
- finite part: very small (for $\mathrm{e}^+\mathrm{e}^- \to 4f$) [Denner,Dittmaier,Roth '97]
- singular part: important for cancellation of IR singularities

Real corrections: $\bar{q}q \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e} \gamma$

Subprocesses with final-state photon

- Dipole subtraction formalism [Dittmaier '99]

$$\int_5 \mathrm{d}\hat{\sigma}^{\mathrm{real}}_{\bar{q}q} + \int_0^1 \mathrm{d}x \int_4 \mathrm{d}\hat{\sigma}^{\mathrm{fact}}_{\bar{q}q} = \underbrace{\int_5 \left[\mathrm{d}\hat{\sigma}^{\mathrm{real}}_{\bar{q}q} - \mathrm{d}\hat{\sigma}^{\mathrm{real,sing}}_{\bar{q}q}\right]}_{\mathrm{IR\ finite}} + \int_4 \left[\int_1 \mathrm{d}\hat{\sigma}^{\mathrm{real,sing}}_{\bar{q}q}\right] + \int_0^1 \mathrm{d}x \int_4 \mathrm{d}\hat{\sigma}^{\mathrm{fact}}_{\bar{q}q}$$



- Convolution part with collinear IS singularities absorbed into PDFs (Factorization)
- Endpoint contribution for cancellation of IR singularities in virtual corrections

Cancellation of IR singularities

Soft and collinear singularities are regularized by introducing an infinitesimal photon mass λ and small fermion masses m_i.
(checked against pure dim. regularization)
$\rightarrow$ cancellation between virtual and real corrections

Endpoint contribution

$$\int_5 \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{real,sing}} = \int_0^1 \mathrm{d}x \int_4 \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{real,conv}} + \int_4 \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{real,endp}}$$

$$\int_4 \mathrm{d}\hat{\sigma}_{\bar{q}q}^{\mathrm{virt,fin}} = \int_4 \mathrm{d}\hat{\sigma}_{\bar{q}q}^{\mathrm{virt}} + \int_4 \mathrm{d}\hat{\sigma}_{\bar{q}q}^{\mathrm{real,endp}}$$

$$\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{real,endp,DPA}} = - \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{Born,DPA}} \frac{\alpha}{2\pi} \sum_{i=1}^{6} \sum_{j=i+1}^{6} (-1)^{i+j} Q_i Q_j \left[\mathcal{L}(\hat{s}_{ij}, m_i^2) + \mathcal{L}(\hat{s}_{ij}, m_j^2) + const. \right]$$

$$\mathcal{L}(s, m^2) = \ln\left(\frac{m^2}{s}\right) \ln\left(\frac{\lambda^2}{s}\right) + \ln\left(\frac{\lambda^2}{s}\right) - \frac{1}{2}\ln^2\left(\frac{m^2}{s}\right) + \frac{1}{2}\ln\left(\frac{m^2}{s}\right)$$

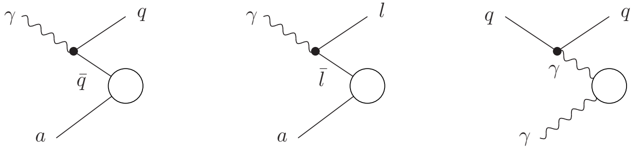
$\rightarrow$ IR-finite!

Real corrections: $\gamma q \to \nu_\mu \mu^+ e^- \bar{\nu}_e q$

Photon-quark induced subprocesses

no soft, only collinear singularities occur

- $\gamma \to \bar{q}q$: dipole subtraction
- $\gamma \to \bar{l}l$: cut on transverse momentum of charged leptons
- $q \to \gamma^\star q$: effective collinear factor
  $\to$ restores regulator mass dependence in singular limit
  $\to$ mass dependence drops out by redefinition of pdfs



Further details on our calculation

- for $\sqrt{\hat{s}} < 2M_{\mathrm{W}} + x$: Improved Born Approximation
  [Denner,Dittmaier,Roth,Wackeroth '01]

- Matrix elements based on RacoonWW
- all LO MEs checked against `Madgraph`
- multi-channel integrator based on `Coffer`$\gamma\gamma$ for $\gamma\gamma \to 4f$
  [Bredenstein,Dittmaier,Roth '05]
- Tools for virtuals: In general we use
  - `FeynArts`
  - inhouse `Mathematica` routines
  - `Collier` [Denner,Dittmaier,Hofer]

  $\to$ Fortran code
- Setup for independent check only:
  - `FeynArts, FormCalc, LoopTools` [Hahn et al.]

Our calculation: Summary

$$\sigma_{\rm pp}^{\rm NLO} = \int_0^1 {\rm d}x_1 \int_0^1 {\rm d}x_2 \Bigg\{ \Bigg(\sum_{q={\rm u,d,c,s}} f_{\bar{q}}(x_1,\mu_{\rm F}) f_q(x_2,\mu_{\rm F}) \\ \times \left[\int_4 {\rm d}\hat{\sigma}_{\bar{q}q}^{\rm LO} + \int_4 {\rm d}\hat{\sigma}_{\bar{q}q}^{\rm virt} + \int_5 {\rm d}\hat{\sigma}_{\bar{q}q}^{\rm real} + \int_0^1 {\rm d}x \int_4 {\rm d}\hat{\sigma}_{\bar{q}q}^{\rm fact} \right] + (q \leftrightarrow \bar{q}) \Bigg) \\ \to \delta_{\bar{q}q}$$

Our calculation: Summary

$$
\begin{aligned}
\sigma_{\mathrm{pp}}^{\mathrm{NLO}} \quad = \quad & \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \left\{ \left(\sum_{q=\mathrm{u,d,c,s}} f_{\bar{q}}(x_1,\mu_{\mathrm{F}}) f_q(x_2,\mu_{\mathrm{F}}) \right.\right. \\
& \times \left[\int_4 \mathrm{d}\hat{\sigma}_{\bar{q}q}^{\mathrm{LO}} + \int_4 \mathrm{d}\hat{\sigma}_{\bar{q}q}^{\mathrm{virt}} + \int_5 \mathrm{d}\hat{\sigma}_{\bar{q}q}^{\mathrm{real}} + \int_0^1 \mathrm{d}x \int_4 \mathrm{d}\hat{\sigma}_{\bar{q}q}^{\mathrm{fact}} \right] + (q \leftrightarrow \bar{q}) \Bigg) \\
& \hspace{20em} \to \delta_{\bar{q}q} \\
& + f_\gamma(x_1,\mu_{\mathrm{F}}) \sum_{q=\mathrm{u,d,c,s}} \left(f_q(x_2,\mu_{\mathrm{F}}) \left[\int_5 \mathrm{d}\hat{\sigma}_{\gamma q}^{\mathrm{real}} + \int_0^1 \mathrm{d}x \int_4 \mathrm{d}\hat{\sigma}_{\gamma q}^{\mathrm{fact}} \right] + (q \leftrightarrow \bar{q}) \right) \\
& + f_\gamma(x_2,\mu_{\mathrm{F}}) \sum_{q=\mathrm{u,d,c,s}} \left(f_{\bar{q}}(x_1,\mu_{\mathrm{F}}) \left[\int_5 \mathrm{d}\hat{\sigma}_{\bar{q}\gamma}^{\mathrm{real}} + \int_0^1 \mathrm{d}x \int_4 \mathrm{d}\hat{\sigma}_{\bar{q}\gamma}^{\mathrm{fact}} \right] + (\bar{q} \leftrightarrow q) \right) \\
& \hspace{20em} \to \delta_{\gamma q}
\end{aligned}
$$

Our calculation: Summary

$$\begin{aligned}
\sigma^{\rm NLO}_{\rm pp} \;=\; & \int_0^1 {\rm d}x_1 \int_0^1 {\rm d}x_2 \Bigg\{ \Bigg(\sum_{q={\rm u,d,c,s}} f_{\bar q}(x_1,\mu_{\rm F}) f_q(x_2,\mu_{\rm F}) \\
& \times \left[\int_4 {\rm d}\hat\sigma^{\rm LO}_{\bar q q} + \int_4 {\rm d}\hat\sigma^{\rm virt}_{\bar q q} + \int_5 {\rm d}\hat\sigma^{\rm real}_{\bar q q} + \int_0^1 {\rm d}x \int_4 {\rm d}\hat\sigma^{\rm fact}_{\bar q q} \right] + (q \leftrightarrow \bar q) \Bigg) \\
& \hspace{10cm} \rightarrow \delta_{\bar q q} \\
& + f_\gamma(x_1,\mu_{\rm F}) \sum_{q={\rm u,d,c,s}} \Bigg(f_q(x_2,\mu_{\rm F}) \left[\int_5 {\rm d}\hat\sigma^{\rm real}_{\gamma q} + \int_0^1 {\rm d}x \int_4 {\rm d}\hat\sigma^{\rm fact}_{\gamma q} \right] + (q \leftrightarrow \bar q) \Bigg) \\
& + f_\gamma(x_2,\mu_{\rm F}) \sum_{q={\rm u,d,c,s}} \Bigg(f_{\bar q}(x_1,\mu_{\rm F}) \left[\int_5 {\rm d}\hat\sigma^{\rm real}_{\bar q \gamma} + \int_0^1 {\rm d}x \int_4 {\rm d}\hat\sigma^{\rm fact}_{\bar q \gamma} \right] + (\bar q \leftrightarrow q) \Bigg) \\
& \hspace{10cm} \rightarrow \delta_{\gamma q} \\
& + \Bigg(f_{\bar{\rm b}}(x_1,\mu_{\rm F}) f_{\rm b}(x_2,\mu_{\rm F}) \int_4 {\rm d}\hat\sigma^{\rm LO}_{\bar{\rm b}{\rm b}} + ({\rm b} \leftrightarrow \bar{\rm b}) \Bigg) \hspace{3cm} \rightarrow \delta_{\bar{\rm b}{\rm b}} \\
& + f_\gamma(x_1,\mu_{\rm F}) f_\gamma(x_2,\mu_{\rm F}) \int_4 {\rm d}\hat\sigma^{\rm LO}_{\gamma\gamma} \Bigg\} \hspace{3cm} \rightarrow \delta_{\gamma\gamma}
\end{aligned}$$

Numerical Setup

Default Setup

- NNPDF2.3qed
- factorization scale $\mu_{\mathrm{F}} = M_{\mathrm{W}}$
- photon recombination

Minimal Cuts

- $p_{\mathrm{T},\ell} > 20\,\mathrm{GeV}\,,\ |y_\ell| < 2.5$
- jet veto: $p_{\mathrm{T,jet}} > 100\,\mathrm{GeV}$

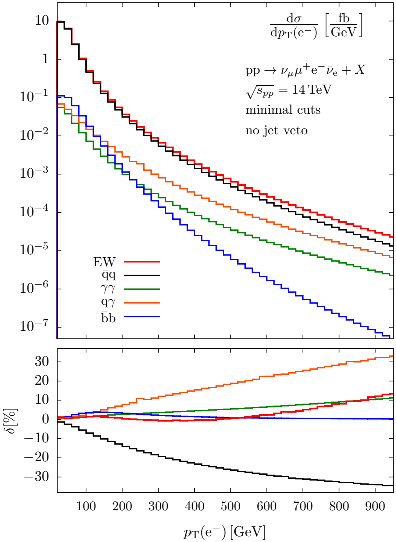
ATLAS Cuts

- $p_{\mathrm{T},\ell} > 20\,\mathrm{GeV}\,,\ |y_\ell| < 2.5$
- $p_{\mathrm{T},\ell}^{\mathrm{leading}} > 25\,\mathrm{GeV},\ E_{\mathrm{T}}^{\mathrm{miss}} > 25\,\mathrm{GeV},\ R_{\mathrm{e}\mu} > 0.1,\ M_{\mathrm{e}\mu} > 10\,\mathrm{GeV}$
- jet veto: $p_{\mathrm{T,jet}} > 25\,\mathrm{GeV}$

	$\sigma_{\bar{q}q}^{\mathrm{LO}}$ [fb]	$\delta_{\bar{q}q}$ [%]	$\delta_{\gamma q}$ [%]	$\delta_{\gamma\gamma}$ [%]	$\delta_{\bar{\mathrm{b}}\mathrm{b}}$ [%]
LHC14	412.5	-2.7	0.6	0.7	1.7
LHC8	236.8	-2.8	0.5	0.8	0.9
ATLAS cuts	163.8	-3.0	-0.3	1.0	1.0

δ_{x}: relative correction by contribution x compared to $\sigma_{\bar{q}q}^{\mathrm{LO}}$

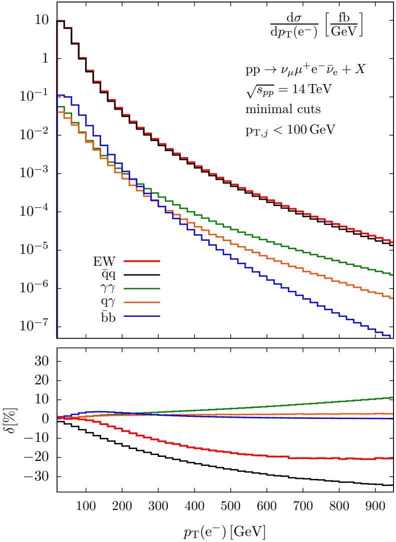
LHC14: transverse momentum distribution



without jet veto

- $\delta_{\bar{q}q}$: -30% at $p_T = 900\,\text{GeV}$ (Sudakov logs)
- $\delta_{\gamma\gamma}$: up to $+10\%$
- $\delta_{\gamma q}$: large due to **soft** W **emission**
  $\to$ same effect in QCD corrections leads to huge K-factors
  $\to$ apply jet veto

LHC14: transverse momentum distribution



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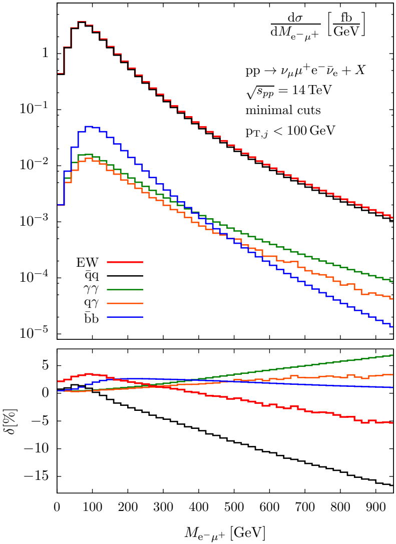
- $\delta_{\gamma\gamma}$: up to $+10\%$
- $\delta_{\gamma q}$: large due to **soft** W **emission**
  $\rightarrow$ same effect in QCD corrections leads to huge K-factors
  $\rightarrow$ apply jet veto

with jet veto $p_{\mathrm{T,jet}} > 100\,\mathrm{GeV}$

- $\delta_{\gamma q} < 5\%$ even in the region of high transverse momentum
- $\delta_{EW} (= \delta_{\bar{q}q} + \delta_{\gamma q} + \delta_{\gamma\gamma} + \delta_{\bar{b}b})$
  $= -20\%$ at $p_T = 900\,\mathrm{GeV}$

LHC14: invariant mass distribution of charged leptons



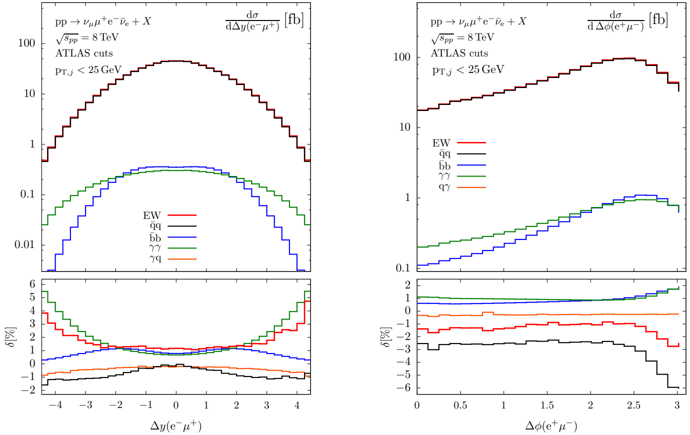
with jet veto

- relatively large negative EW corrections partially compensated by positive contributions, especially $\delta_{\gamma\gamma}$

  $\to$ moderate corrections even at high invariant mass ($< 10\%$)

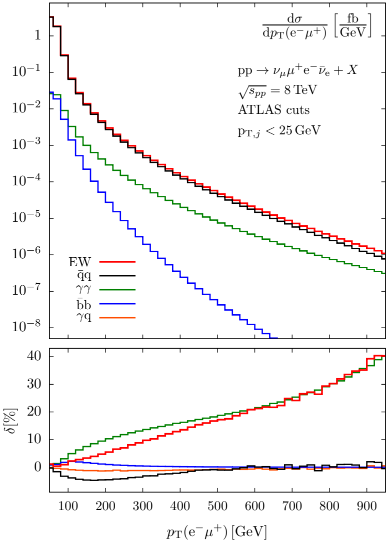
- same behaviour found for transverse-mass distribution $M_{T,WW}$

ATLAS cuts: angular distributions



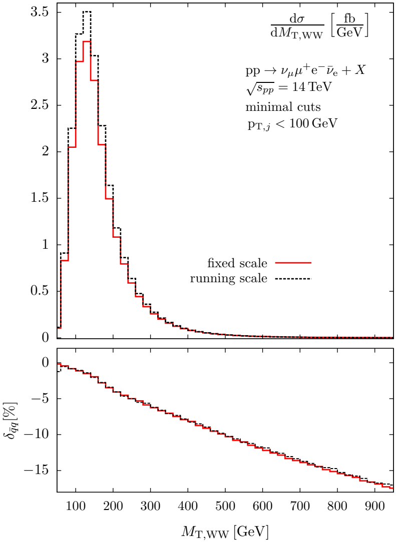
- angular distributions in general less affected by EW corrections
- $\gamma\gamma$ contribution dominates the forward/backward emission of the charged leptons (reported by Bierweiler et al.)

ATLAS cuts: transverse momentum $p_{\mathrm{T}}(\mathrm{e}^-\mu^+)$



- real radiation of **hard** photon: large impact by $\mathrm{pp} \to \mathrm{W}^+\mathrm{W}^-\gamma$ with **hard** photon
- large photon recoil

LHC14: Scale dependence



- variation of μ_F by a factor 2 changes the inclusive cross section by around $\pm 8\%$
- but $\delta_{\bar{q}q}$ shows no scale dependence

fixed scale $\mu_F = M_W$,
running scale $\mu_F = M_{WW}$

$\rightarrow$ **factorization** of EW corrections:

$$d\sigma = d\sigma^{QCD}_{qq} \times (1 + \delta_{\bar{q}q}) + d\sigma_{gg} + d\sigma_{\gamma\gamma} + d\sigma_{\gamma q}$$

$d\sigma^{QCD}_{qq}$: state-of the-art QCD prediction

Conclusion + Outlook

Conclusion

We have calculated the EW corrections to $\nu_\mu \mu^+ e^- \bar{\nu}_e$ production at the LHC in double-pole approximation:

- first evaluation of EW corrections to W-pair production with decays (realistic event selection)
- previous (idealized) on-shell result confirmed within some %
  - EW corrections to integrated cross sections small
  - but sizable effects due to Sudakov logarithms in distributions at high scale
  - $\gamma\gamma$ contribution has to be taken into account
  - γq contribution is suppressed by jet veto
  - scale dependence low
- first step towards calculation of full EW corrections to $4f$ production

Outlook

- full EW corrections to $\nu_\mu \mu^+ e^- \bar{\nu}_e$ production at the LHC
- implementation of subtraction terms for non-collinear-save observables [Dittmaier,Kabelschacht,Kasprzik '08]
- implementation of semi-hadronic decay

Backup slides

Improved Born Approximation

$$\mathrm{d}\hat{\sigma}^{\mathrm{IBA}}_{\bar{q}q} = F(x_1 x_2)\,\mathrm{d}\Phi_4 \overline{\sum} \left|\mathcal{M}^{\bar{q}q\to\nu_\mu\mu^+\mathrm{e}^-\bar{\nu}_\mathrm{e}}_{\mathrm{IBA}}\right|^2 \left[1+\delta_{\mathrm{Coul}}(\hat{s},k_+^2,k_-^2)\right] g(\bar{\beta})$$

Coulomb singularity [Fadin et al., Beenakker et al.]:

$$\delta_{\mathrm{Coul}}(\hat{s},k_+^2,k_-^2) = \frac{\alpha(0)}{\bar{\beta}}\mathrm{Im}\left\{\ln\left(\frac{\beta-\bar{\beta}+\Delta_M}{\beta+\bar{\beta}+\Delta_M}\right)\right\},$$

$$\bar{\beta} = \frac{\sqrt{\hat{s}^2+k_+^4+k_-^4-2\hat{s}k_+^2-2\hat{s}k_-^2-2k_+^2k_-^2}}{\hat{s}},$$

$$\beta = \sqrt{1-\frac{4(M_\mathrm{W}^2-\mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W})}{\hat{s}}}, \quad \Delta_M = \frac{\left|k_+^2-k_-^2\right|}{\hat{s}}$$

damping factor:

$$g(\bar{\beta}) = \left(1-\bar{\beta}^2\right)^2$$

On-shell projection [Denner et al.]

$$a(p_+) + b(p_-) \to f_1(k_1) + \bar{f}_2(k_2) + f_3(k_3) + \bar{f}_4(k_4)$$

- fix direction of W^+ boson, of fermion f_1, and of fermion f_3

$$\hat{k}_{+0} = \frac{1}{2}\sqrt{s}, \qquad \hat{\mathbf{k}}_+ = \frac{\mathbf{k}_+}{|\mathbf{k}_+|}\beta\frac{\sqrt{s}}{2}, \qquad \hat{k}_-^\mu = p_+^\mu + p_-^\mu - \hat{k}_+^\mu,$$

$$\hat{k}_1^\mu = k_1^\mu \frac{M_\mathrm{W}^2}{2\hat{k}_+ k_1}, \qquad \hat{k}_2^\mu = \hat{k}_+^\mu - \hat{k}_1^\mu,$$

$$\hat{k}_3^\mu = k_3^\mu \frac{M_\mathrm{W}^2}{2\hat{k}_- k_3}, \qquad \hat{k}_4^\mu = \hat{k}_-^\mu - \hat{k}_3^\mu$$

with $\beta = \sqrt{1 - 4M_\mathrm{W}^2/s}$

$$\to \hat{k}_+^2 = \hat{k}_-^2 = M_\mathrm{W}^2$$

Effective collinear factor

$$f(m_q, x, E_q, \theta) = \frac{\sin^2 \frac{\theta}{2}}{\left[\sin^2 \frac{\theta}{2} + \frac{m_q^2 x^2}{4E_q^2(1-x)^2}\right]^2} \left\{ \sin^2 \frac{\theta}{2} + \frac{m_q^2 x^4}{4E_q^2(1-x)^2(1+(1-x)^2)} \right\}$$

where θ is the angle between the incoming and the outgoing quark of mass m_q defined in the partonic centre-of-mass frame, E_q is the energy of the incoming quark in the same frame, and $x = k^0/E_q$ is the fraction of the incoming quark's momentum that is carried by the emitted photon.