Event by Event Weighting at NLO

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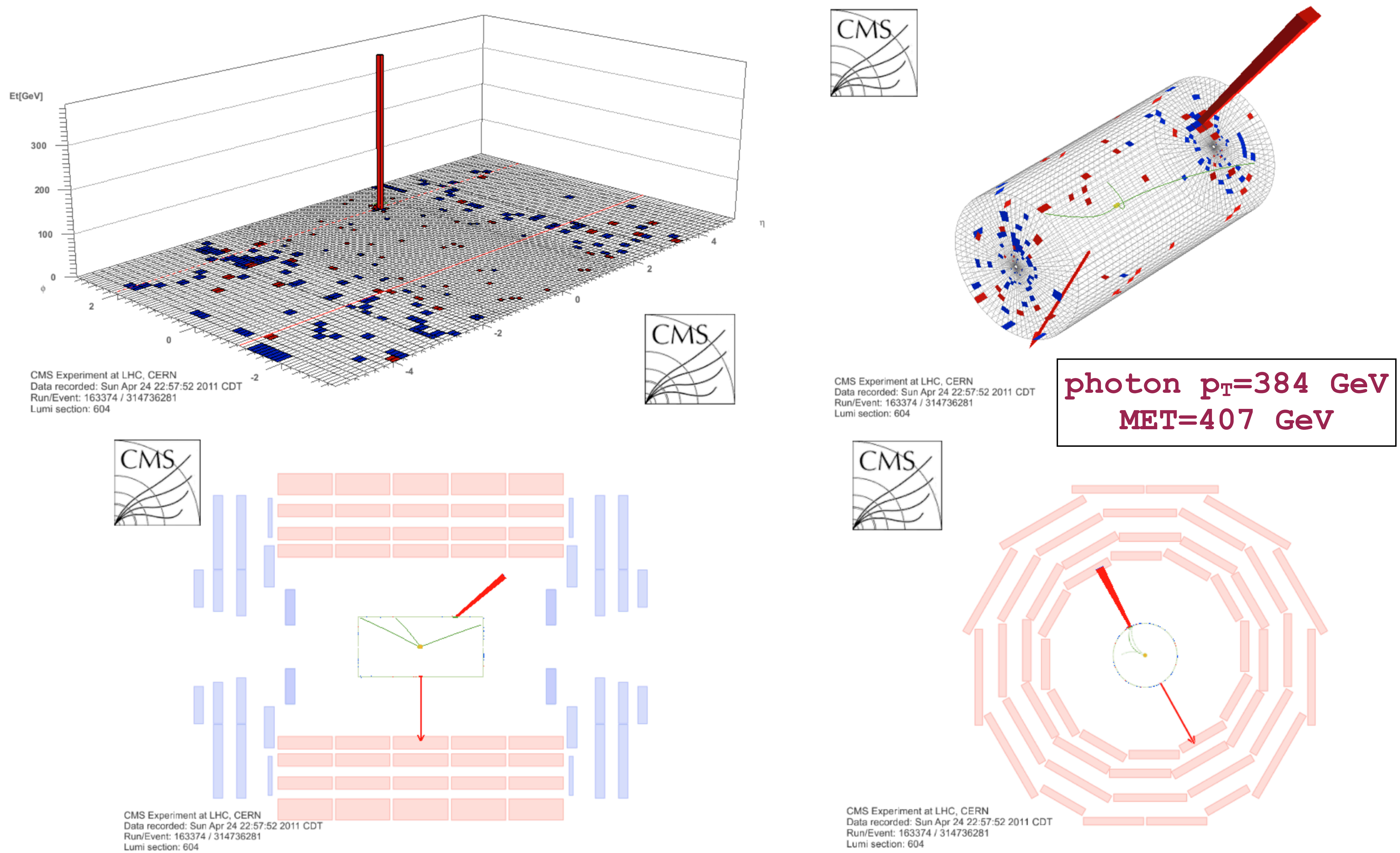
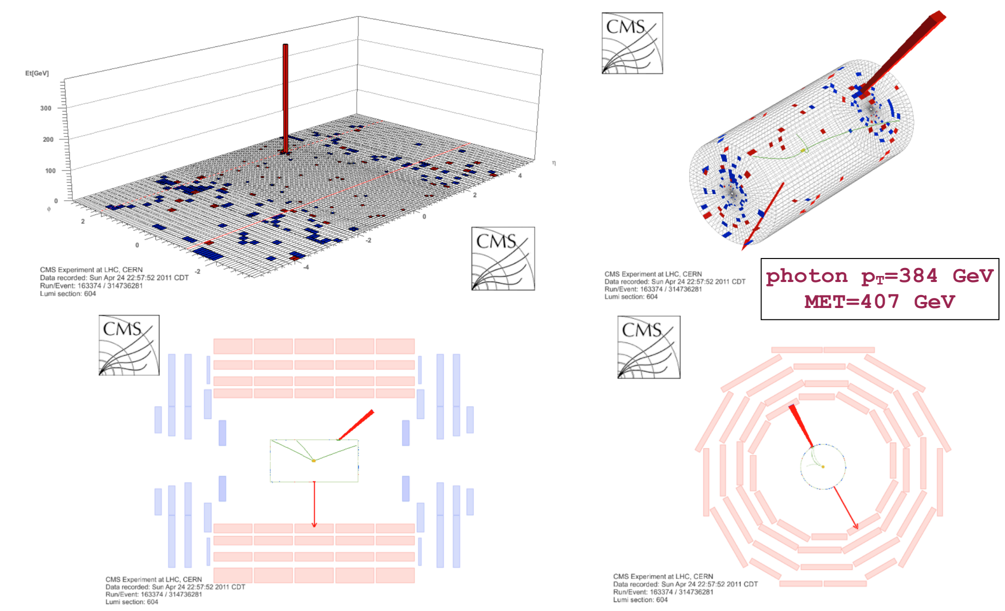
Motivation



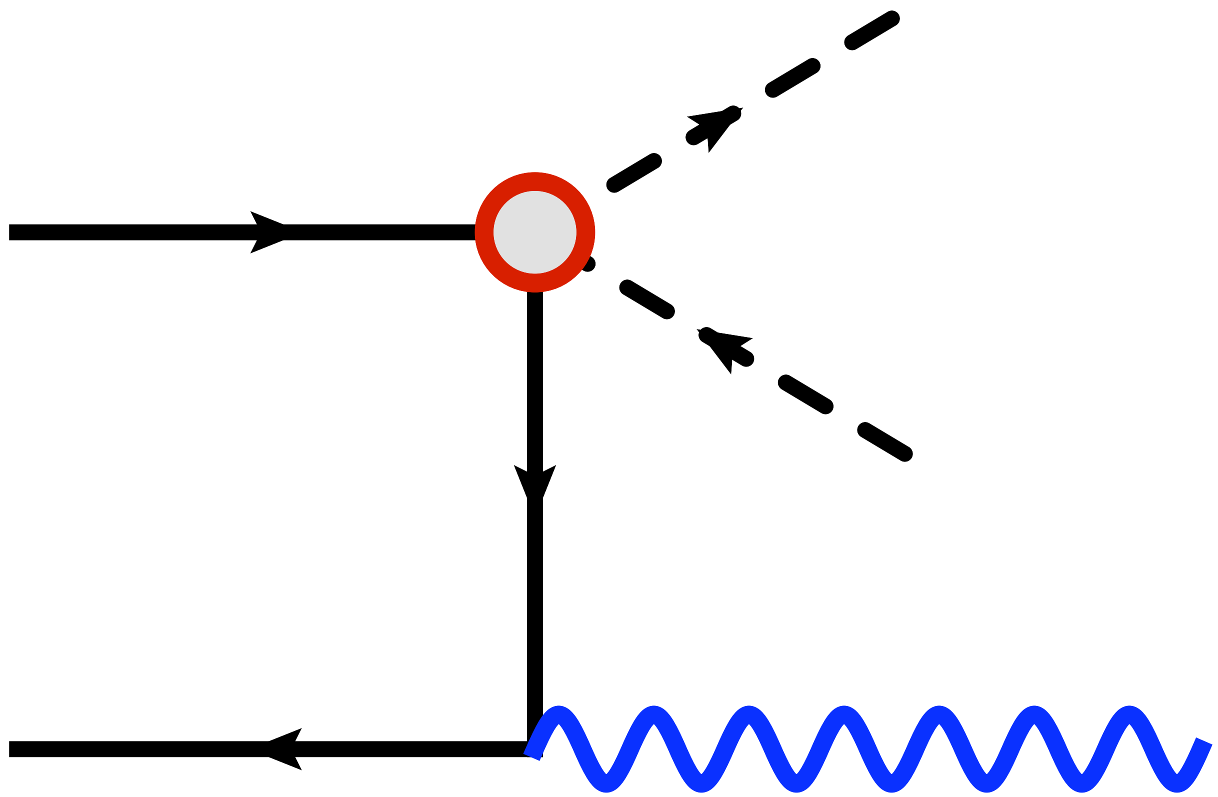
Suppose we see an interesting event at the LHC, how can we describe how likely it is to be from the SM or something new?

Motivation



Et[GeV]
300
200
100
0
CMS
CMS Experiment at LHC, CERN
Data recorded: Sun Apr 24 22:57:52 2011 CDT
Run/Event: 163374 / 314736281
Lumi section: 604
photon p_T=384 GeV
MET=407 GeV

Motivation



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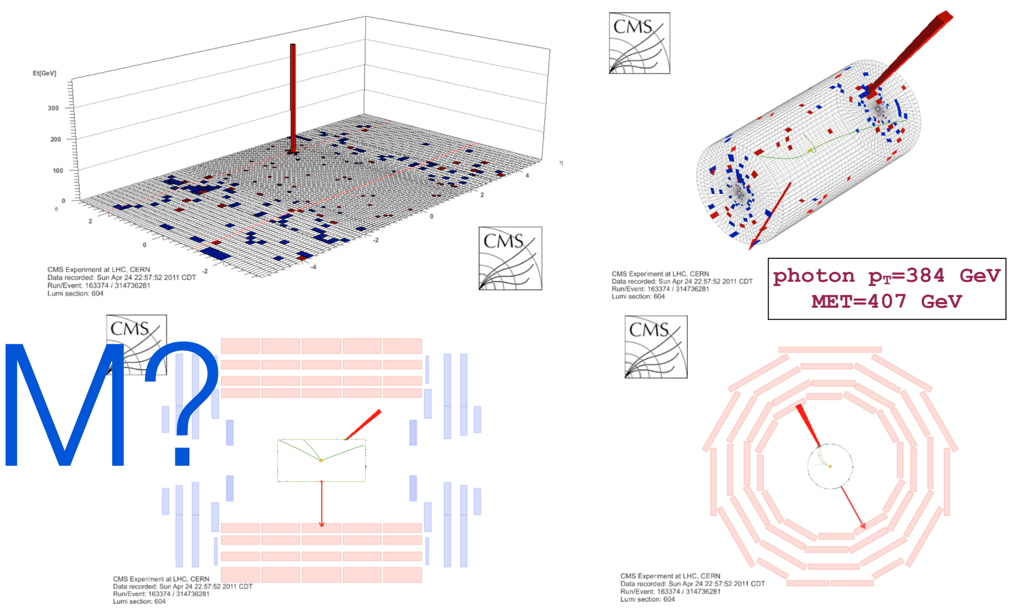
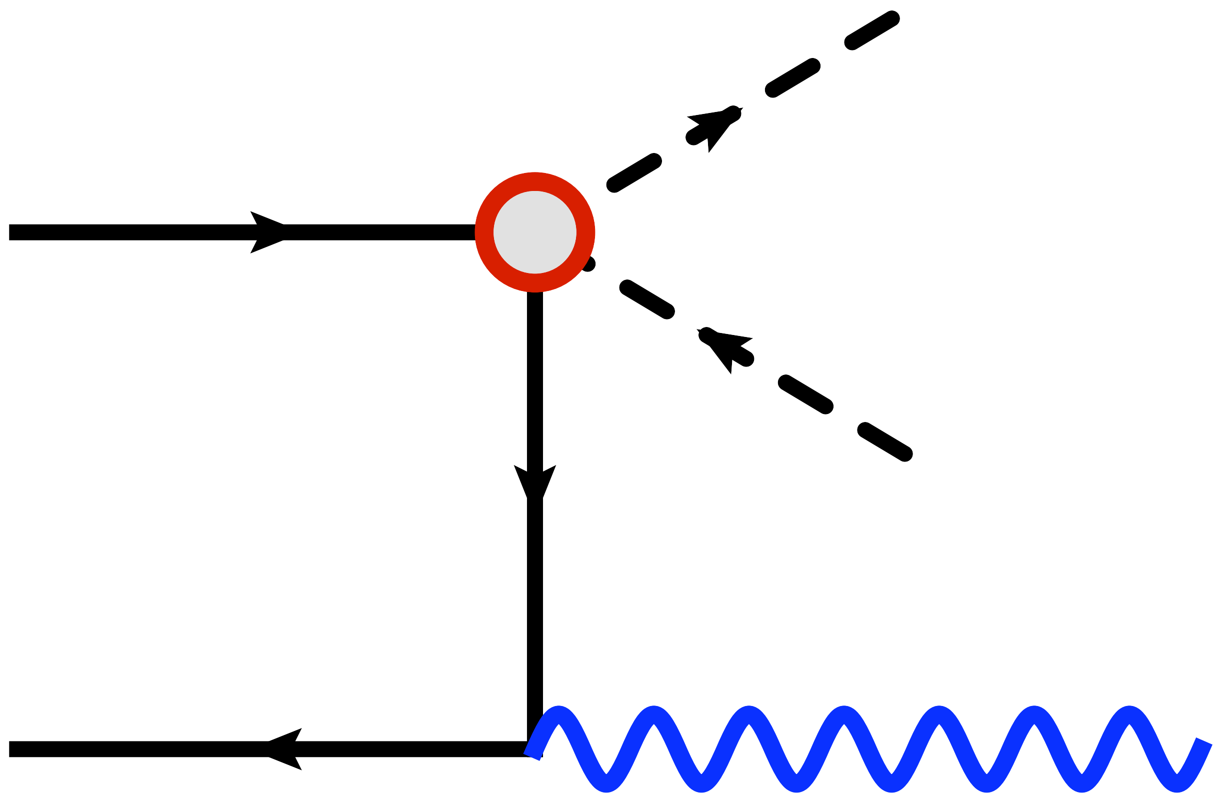
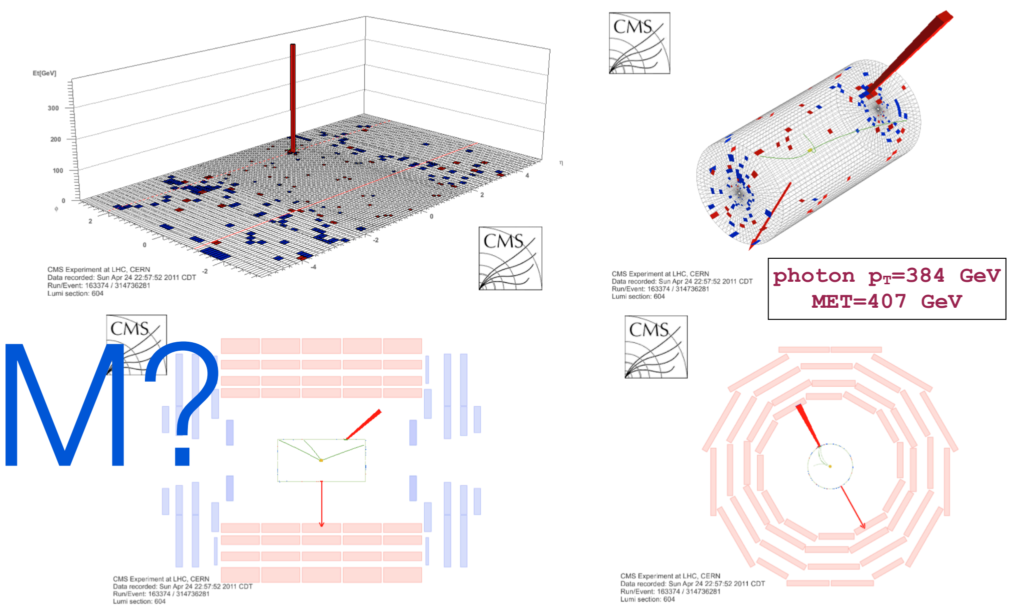


Motivation





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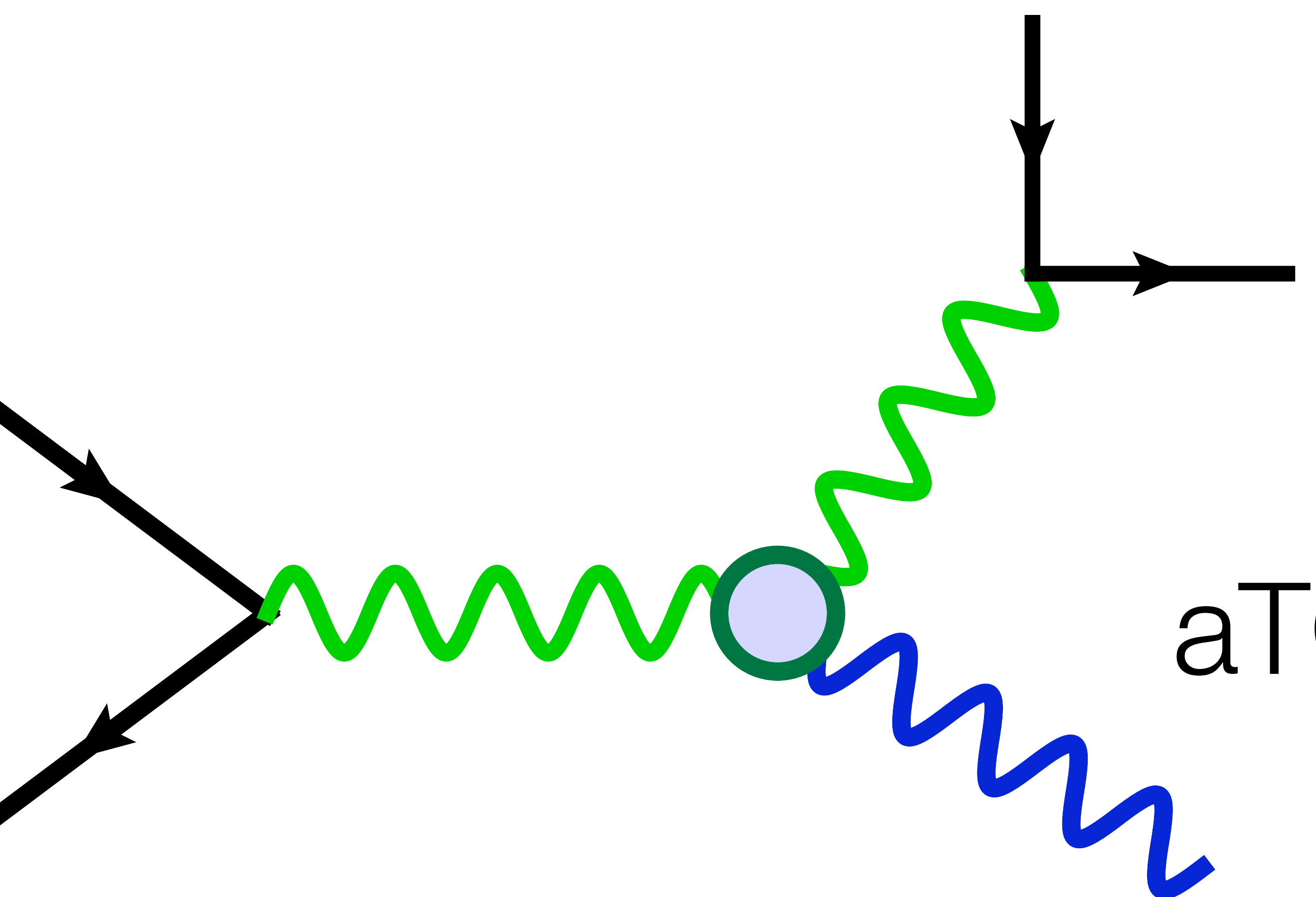

aTGC??

Motivation

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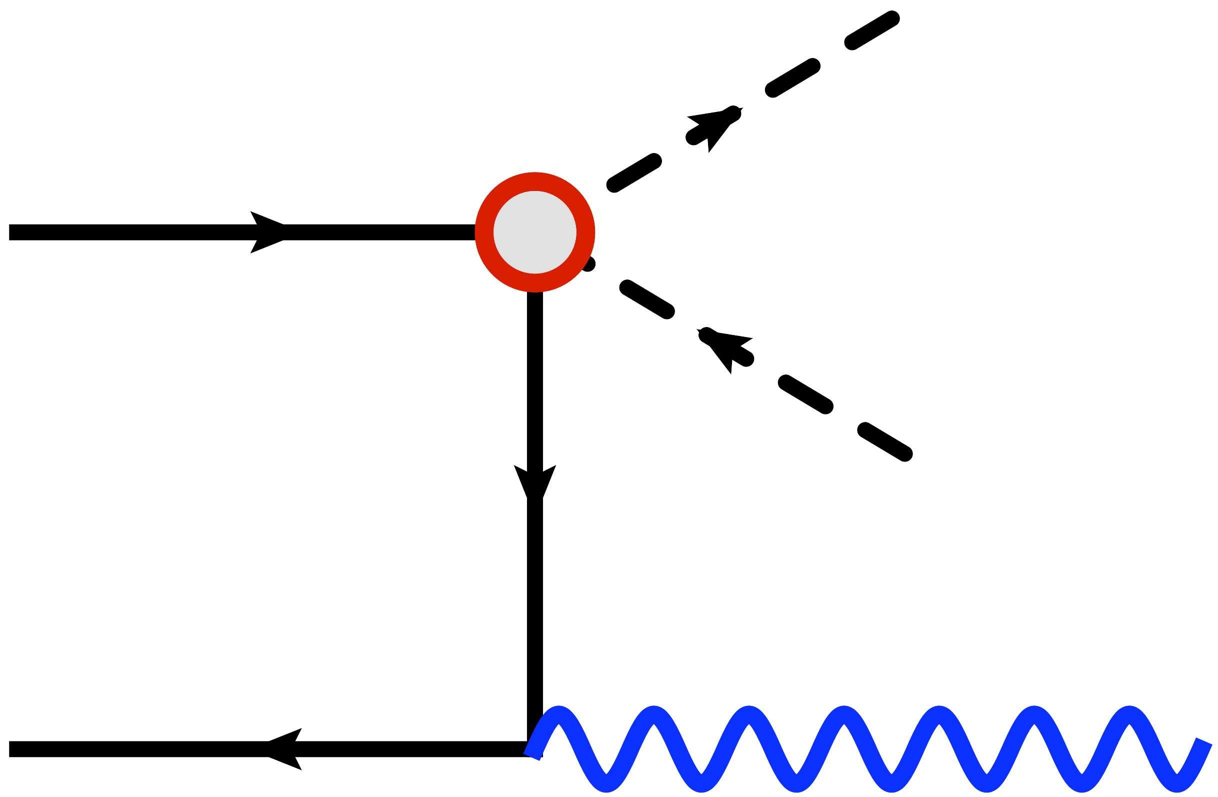

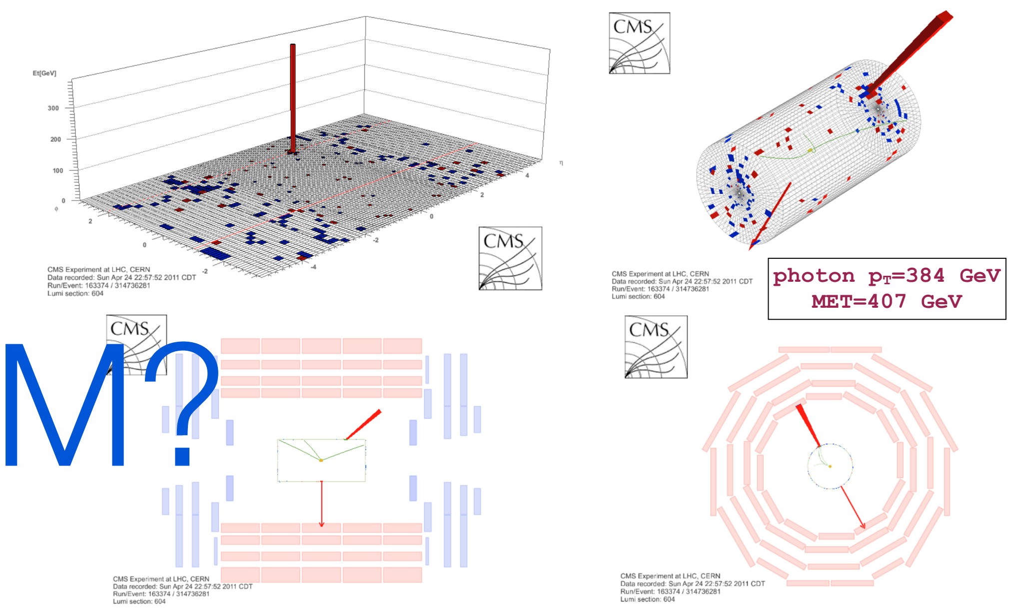

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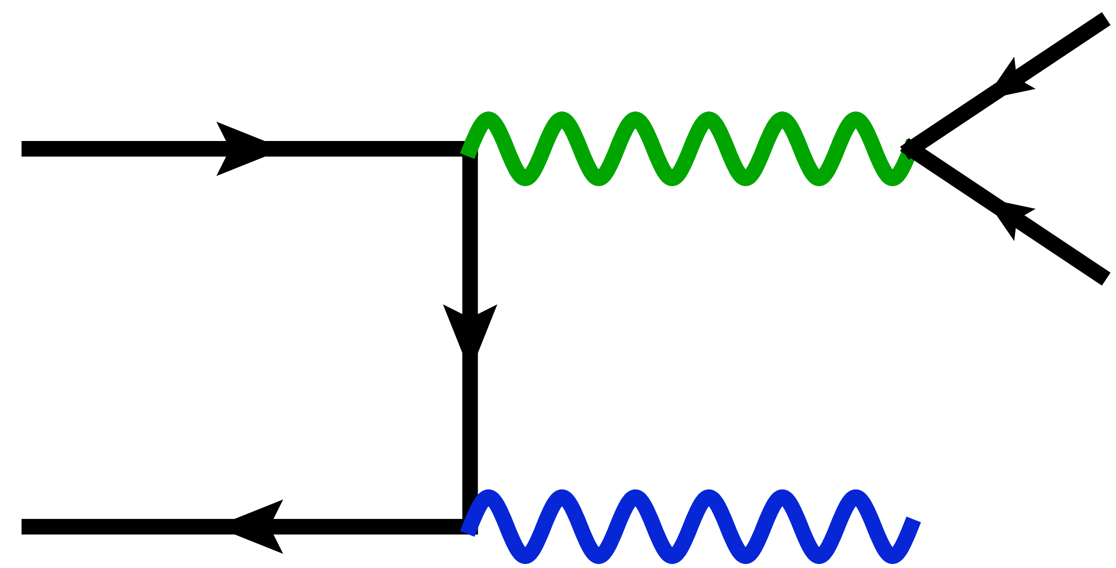
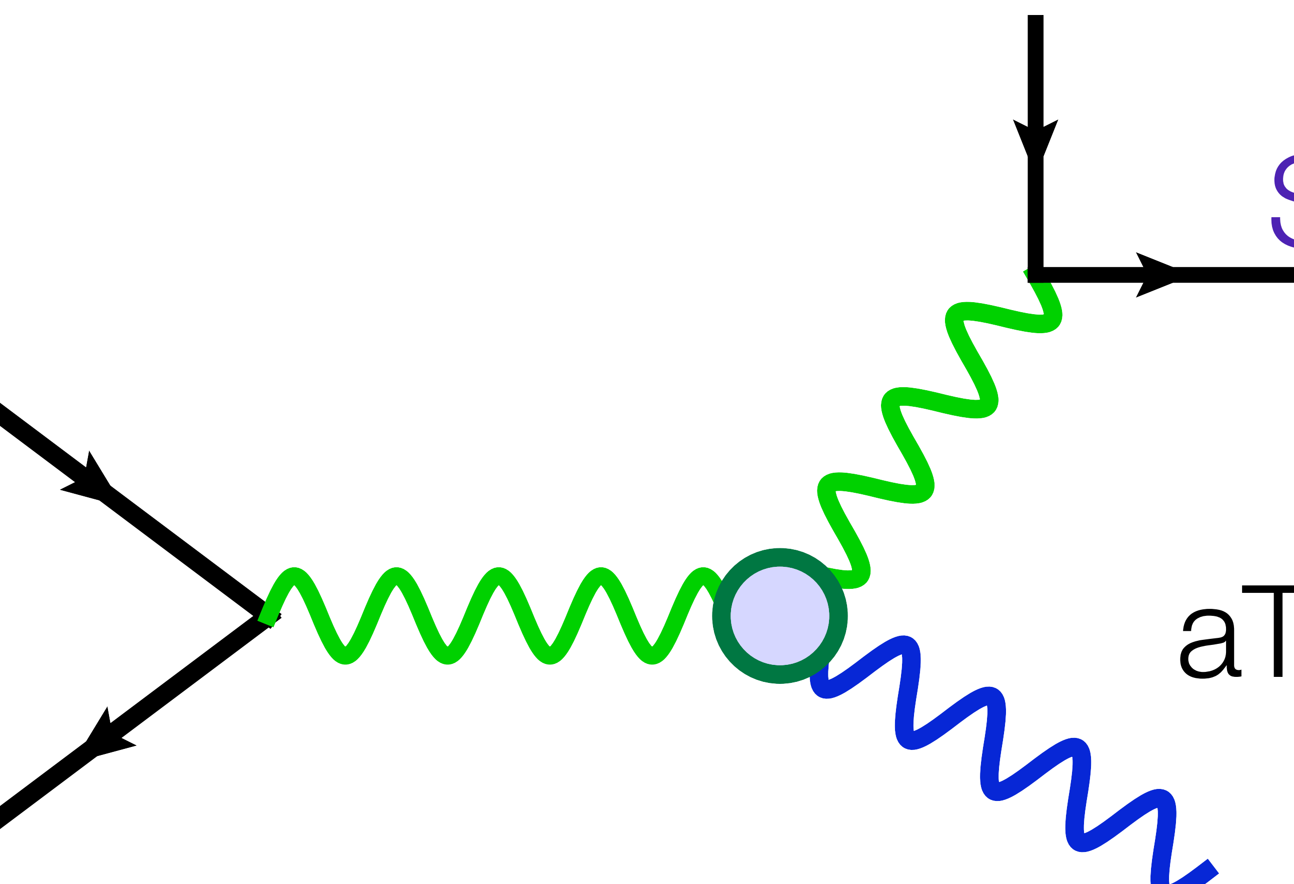


SM?

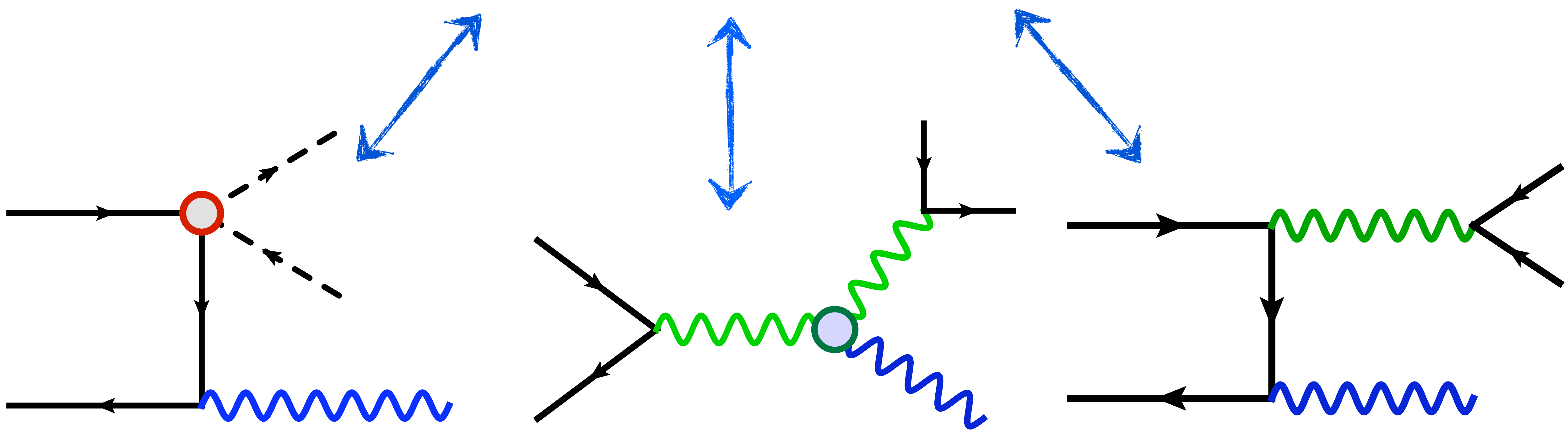


aTGC??

Motivation

Kinematic discriminants attempt to quantitively answer this question by providing each event with a probabilistic weight associated with a particular hypothesis

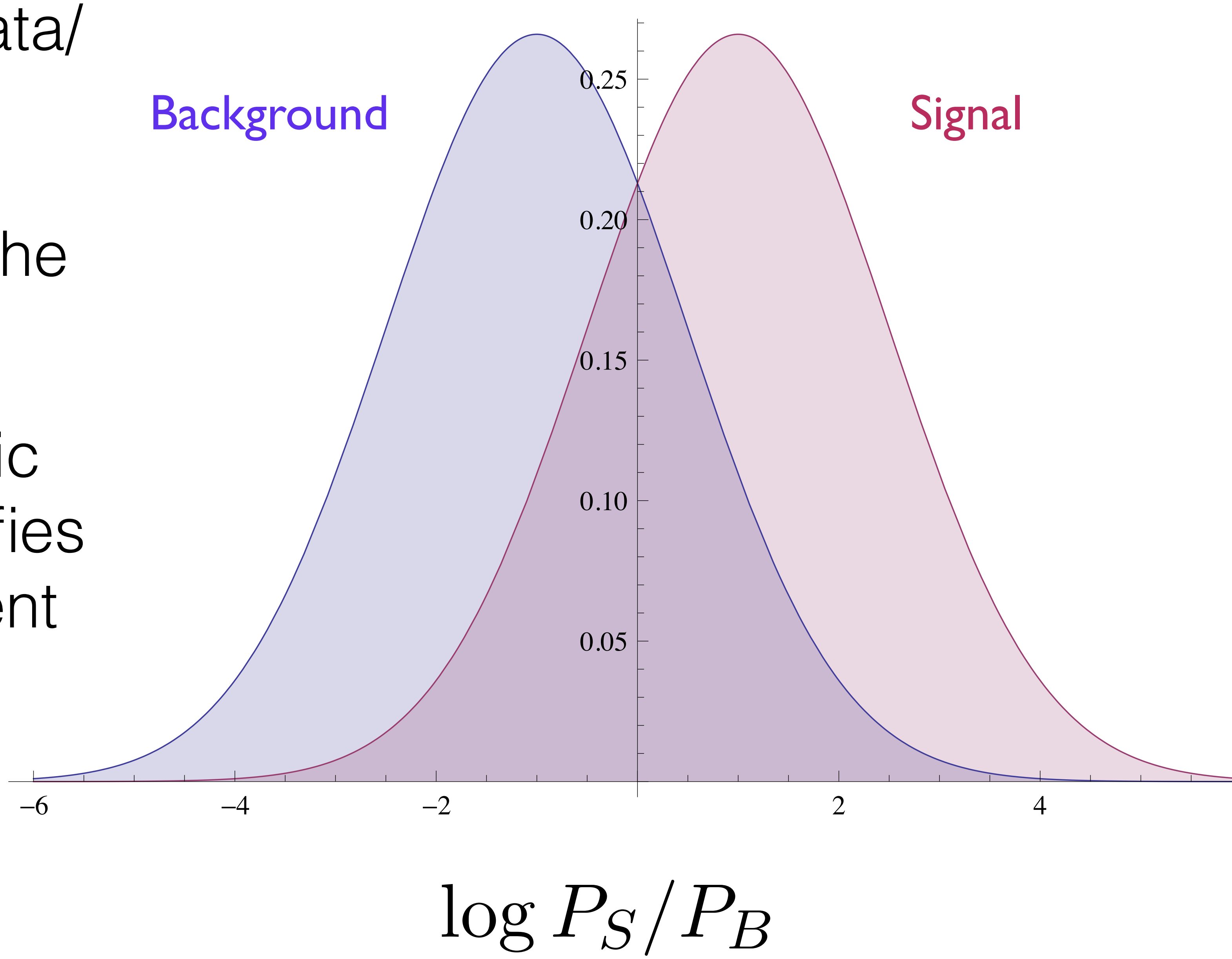
$$\mathcal{P}(\{x\}|\Omega_S)$$



How are the weights used?

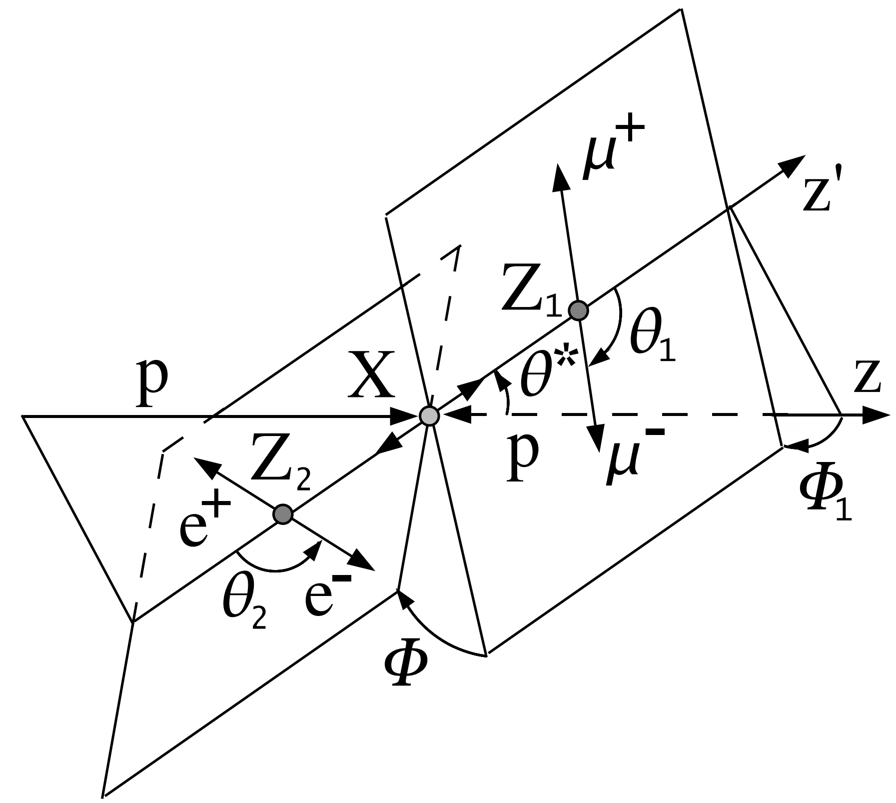
- ★ Once each event in the data/MC sample has received signal and background weights the properties of the ensemble can be studied.
- ★ One can define a kinematic discriminant, which classifies how “signal like” each event is.

$$KD = \frac{P_S}{P_S + P_B}$$

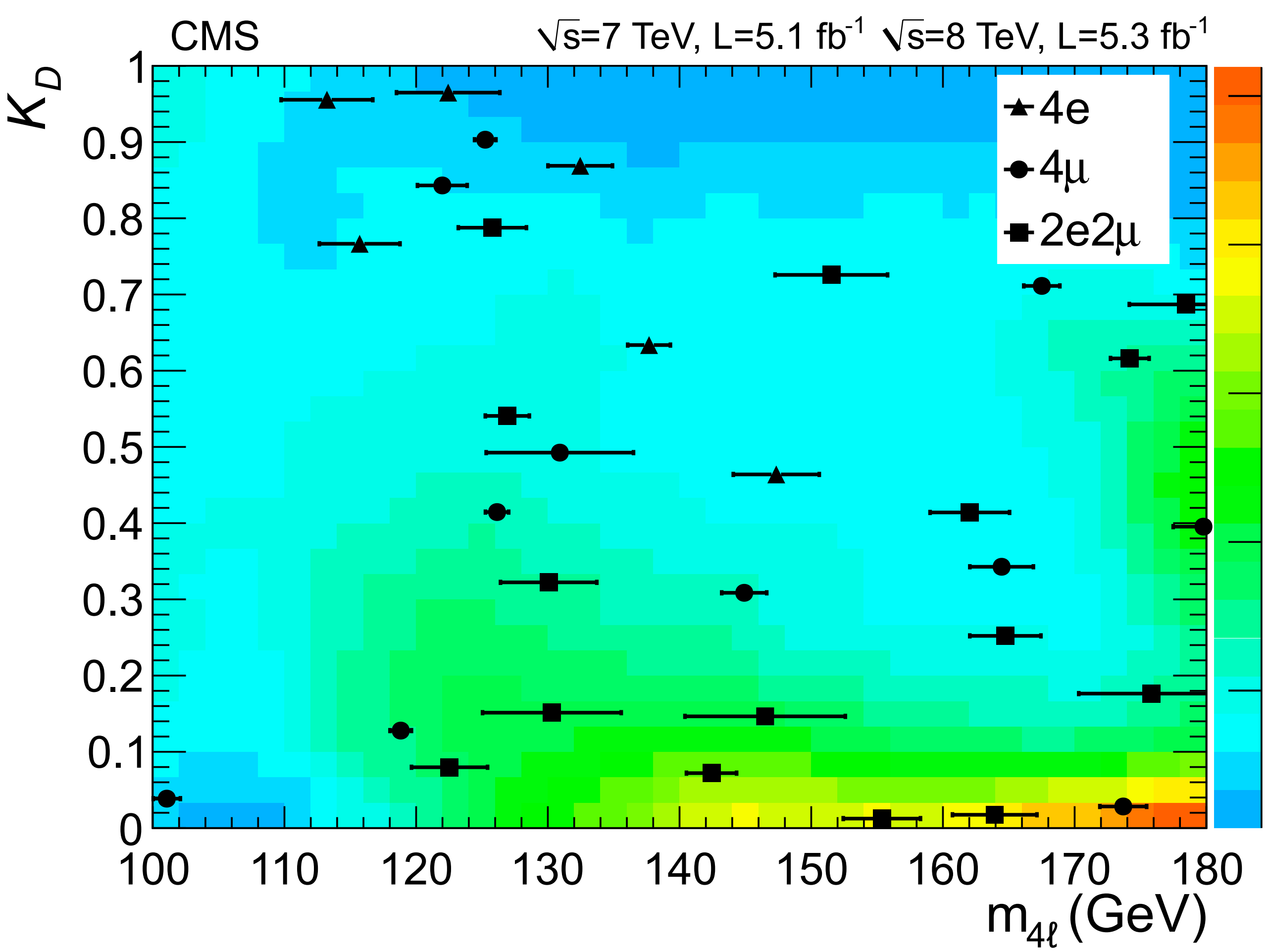


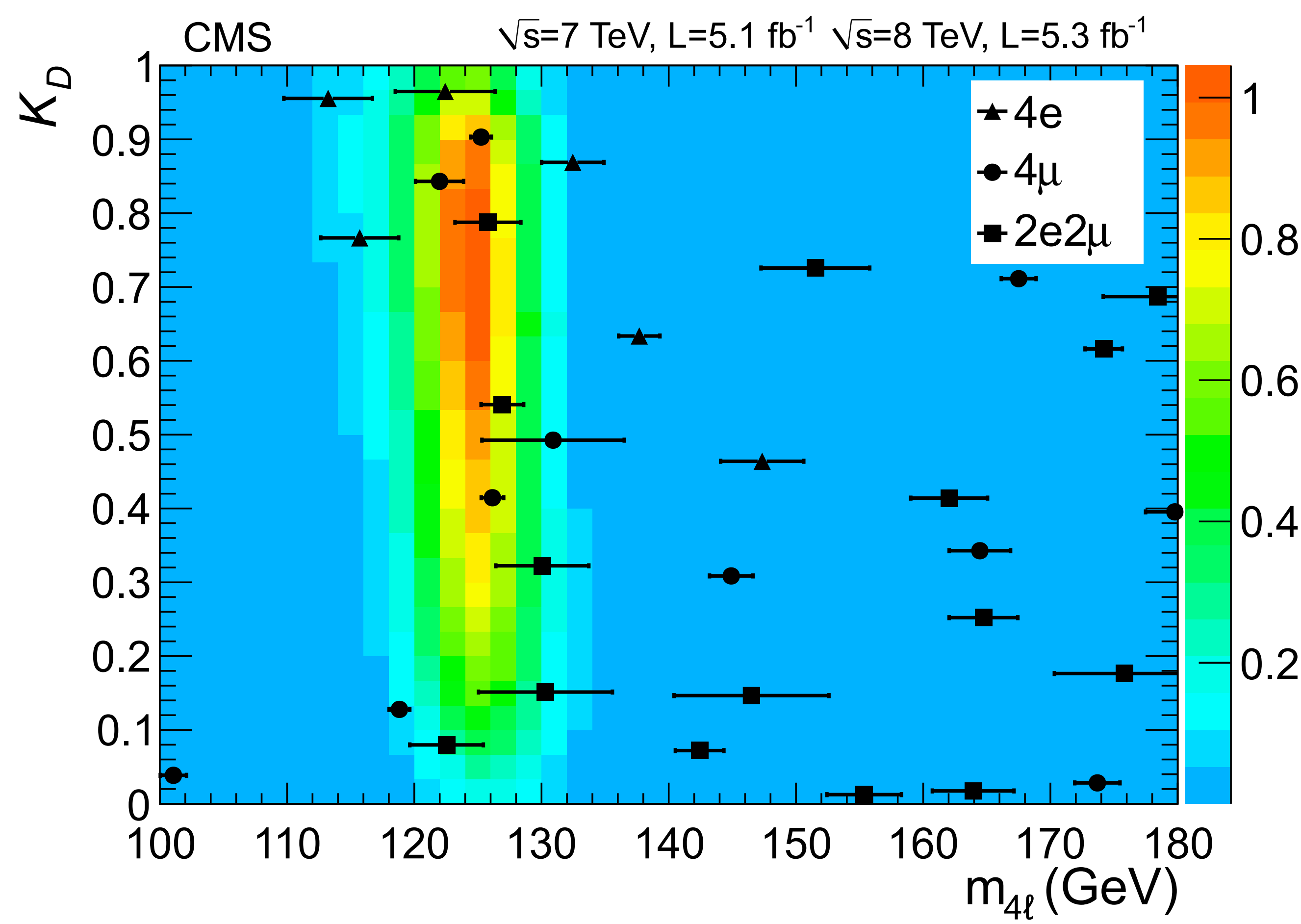
A KD example : MELA

MELA, (Gao, Gritsan, Guo, Melnikov, Schulze and Tran) is a nice example of a KD.



It uses Lorentz invariant final state information to discriminant between different types of Lorentz structures in ZZ=>4l (i,e. Spin-1 couplings of background versus Spin-0 signal).





Motivation for extensions

We would like to build algorithms for event by event kinematic discrimination which satisfy the following criteria :

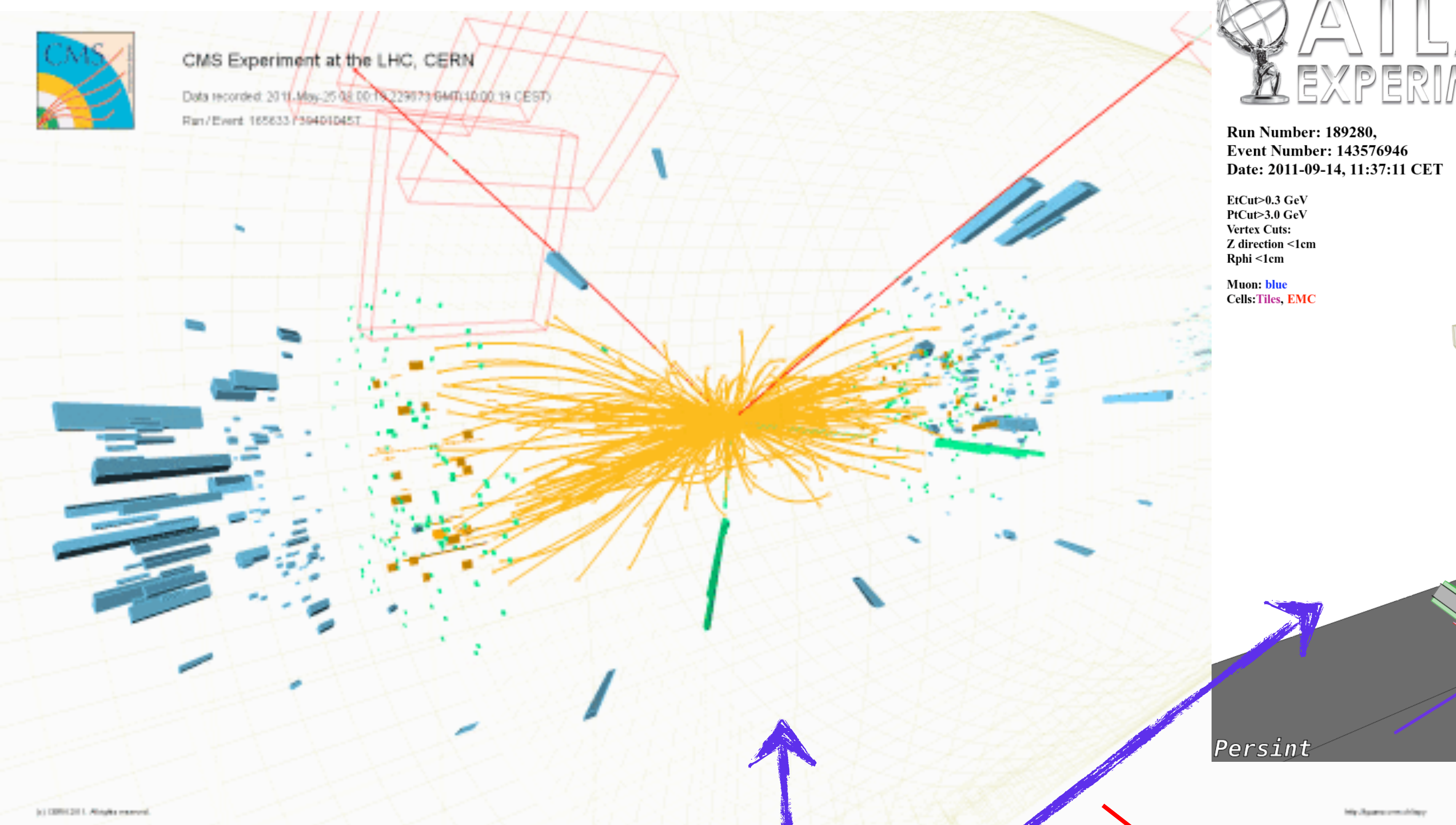
- ★ Full generality, we should be able to include all final states in the discriminant, the production mechanism should naturally be included. Each weight should be unique, and well-defined.
- ★ A well defined theoretical accuracy, which can be systematically improved (in perturbation theory).

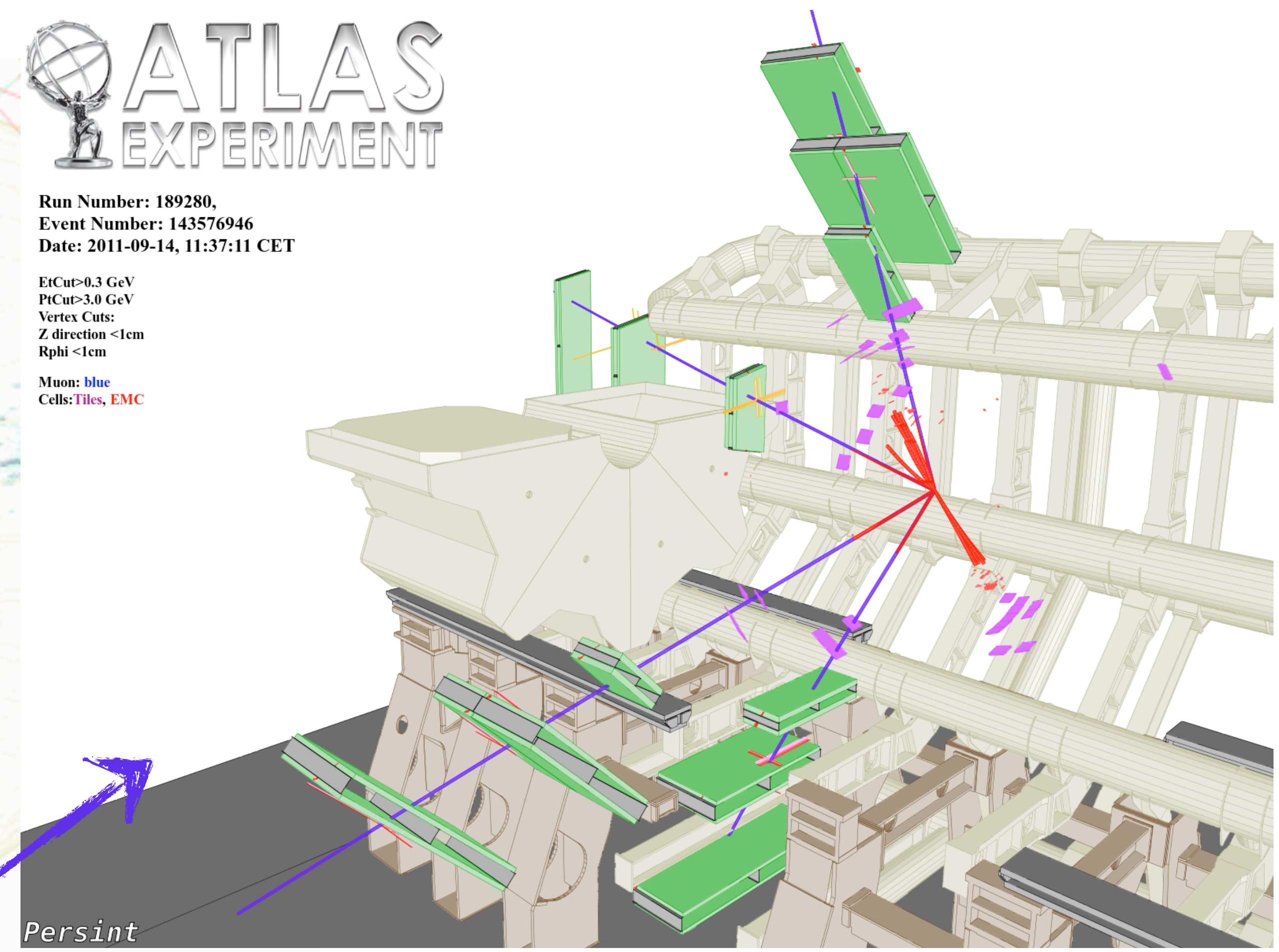
I will address each of these issues in this talk.

What to use as KD?

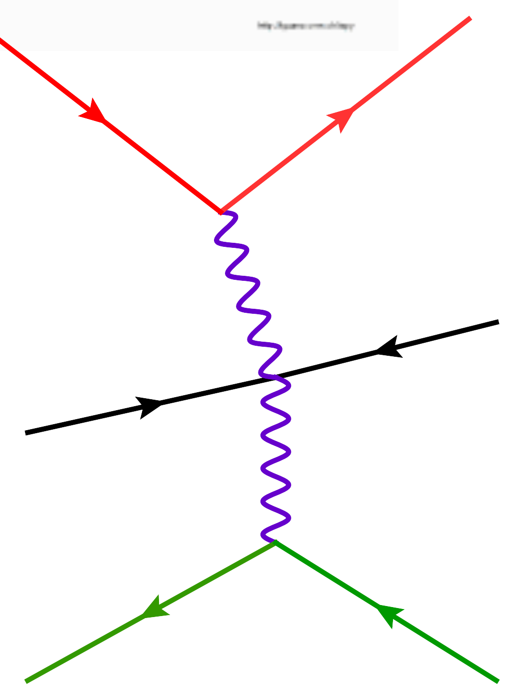
- We will use the Matrix Element to build our KDs.
- The ME contains a huge amount of theoretical information.
- The ME contains production information.
- And has a rather large range of potential applications..

The MEM





Have to map this



to this

Momentum conservation

- Define the sum over final state momenta

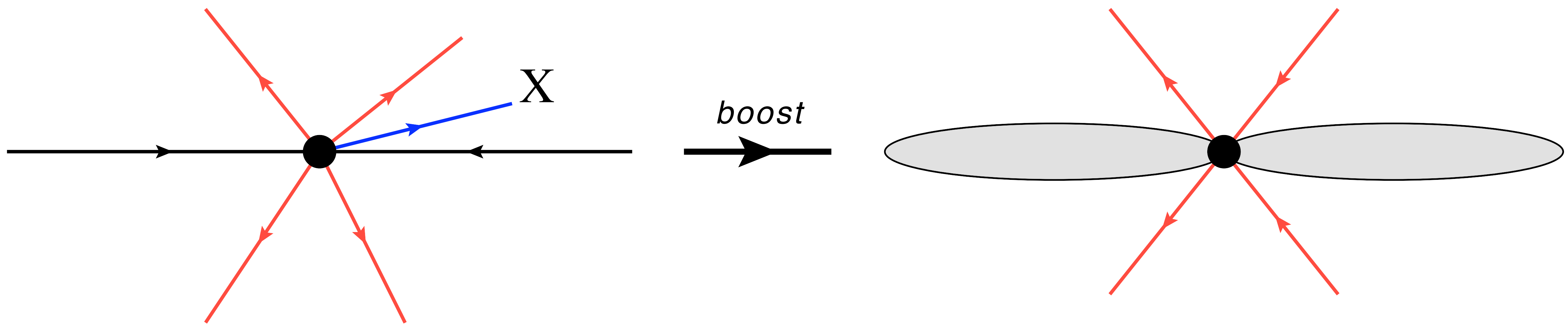
$$X = -\sum_{i=1}^{n} \tilde{p}_i.$$

- A LO phase space point requires (for momentum conservation and regular PDFs)

$$X^x = X^y = 0,$$

- Clearly this is not usually the case in a data (or full simulation MC) event.

Making the event well-defined



We need to balance momenta in the transverse plane so that we can calculate a ME with usual PDFs.

This can be done by boosting the event, so that the final state for the MEM has no p_T.

However this boost actually introduces the uniqueness problem....

What about unique?

★ Can perform a Lorentz transformation on the final state particles,

$$p_i^\mu = \Lambda^\mu{}_\nu(X)\,\tilde{p}_i^\nu \qquad \text{with} \qquad \sum_{i=1}^n p_i^x = \sum_{i=1}^n p_i^y = 0\,.$$

★ This transformation is not unique, there is freedom in the definition of the longitudinal components

★ Recall that the longitudinal components specify the parton fractions,

$$x_a - x_b = \frac{2}{\sqrt{s}}\left(\sum_{i=1}^n p_i^z\right)\ , \qquad x_a + x_b = \frac{2}{\sqrt{s}}\left(\sum_{i=1}^n E_i\right)$$

★ Our boosts do not fix x uniquely only the product.

$$x_a x_b s = Q^2$$

Boosts and uniqueness

Choosing only one boost, means that the weight is not unique.
We need to integrate over all longitudinally equivalent boosts.

The Matrix Element is a Lorentz scalar, so can be evaluated once for any boost, however the PDFs are not.

$$
\begin{aligned}
\mathcal{L}_{ij}(s_{ab}, x_l, x_u) &= \int dx_a dx_b \, \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \, \delta(x_a x_b s - s_{ab}) \\
&= \int_{x_l}^{x_u} dx_a \, \frac{f_i(x_a) f_j(s_{ab}/(s x_a))}{s x_a s_{ab}} \, ,
\end{aligned}
$$

The boost function, describes the integration over longitudinally equivalent boosts.

The MEM@LO

LO matrix element

The following defines the probability distribution for a given LO final state.

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_\Omega^{LO}} \mathcal{L}_{ij}(s_{ab}, x_l, x_u) \mathcal{B}_\Omega^{ij}(p_a, p_b, \mathbf{x}) \ .$$

In practical applications, one normally also needs to integrate over detector response functions

$$\mathcal{P}^{Full}(\{x\}|\Omega_1) = \int \mathcal{P}^{LO}(\{y\}|\Omega_1) W(x,y) d\,y$$

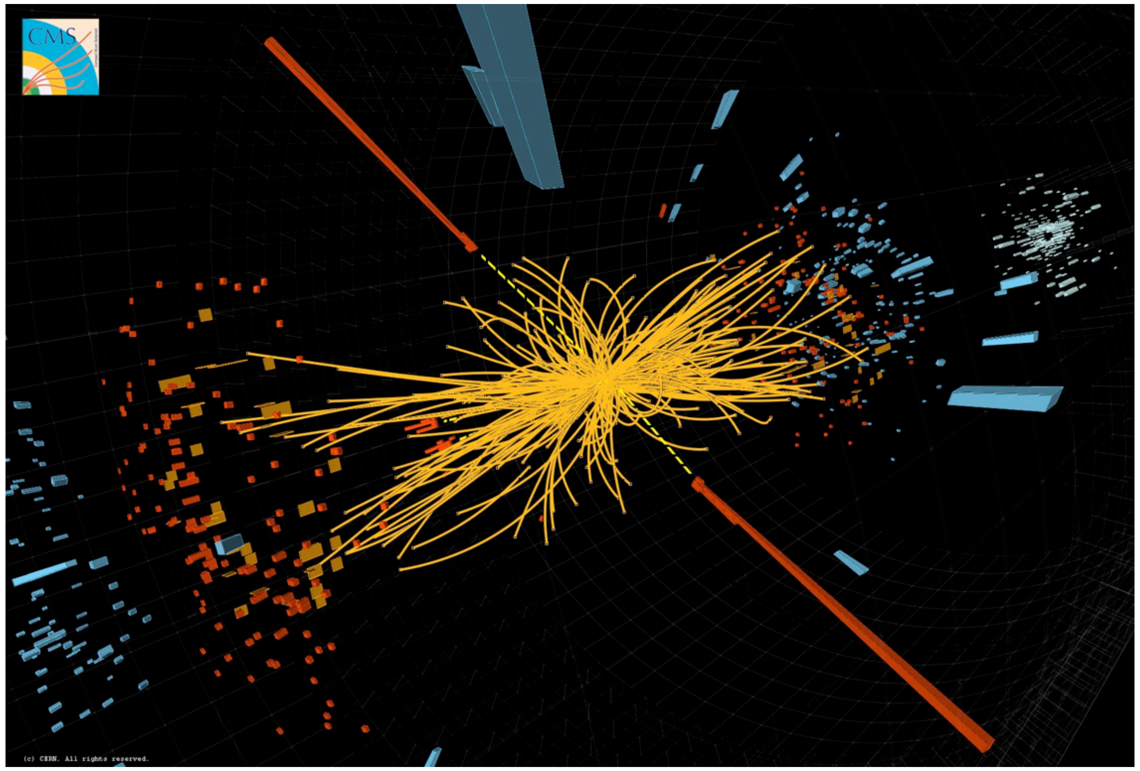
As a mere theorist I will mostly neglect these, but they are vital for a real analysis.

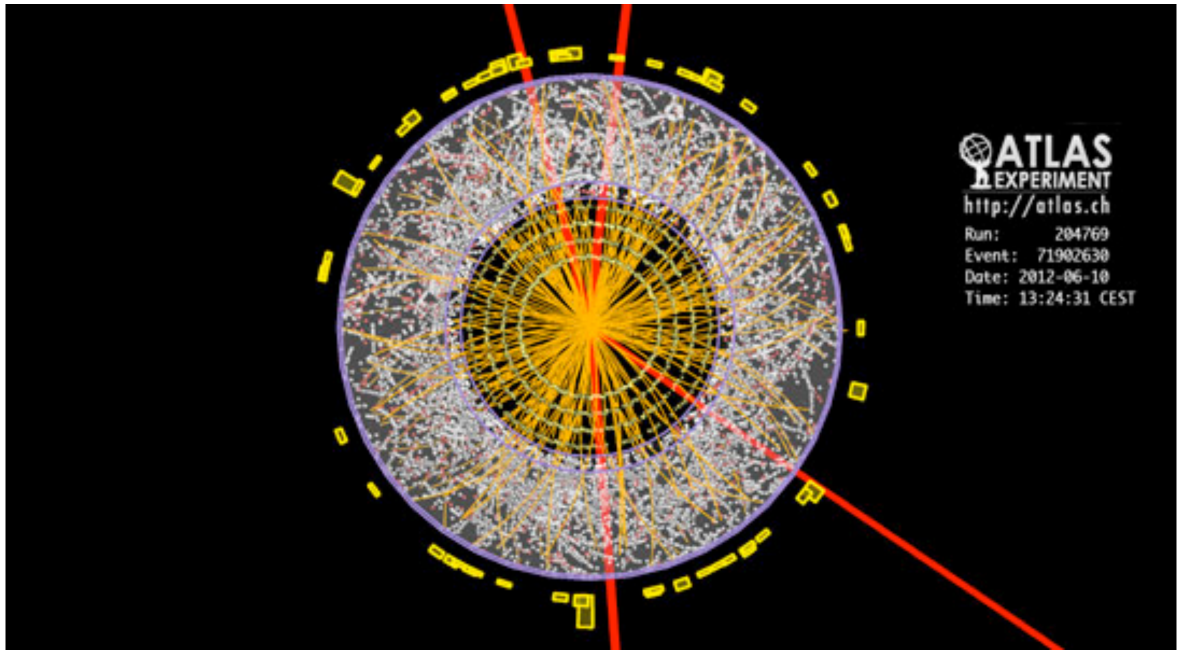
Event by Event weighting @ NLO

- ★ An experimental event is about the most exclusive quantity you can think of.

- ★ We will also need to think very exclusively at NLO, i.e. we want to define our NLO calculation in the following format

$$\mathcal{P}_{NLO}(\Phi_B) = K(\Phi_B)\mathcal{P}_{LO}(\Phi_B)$$

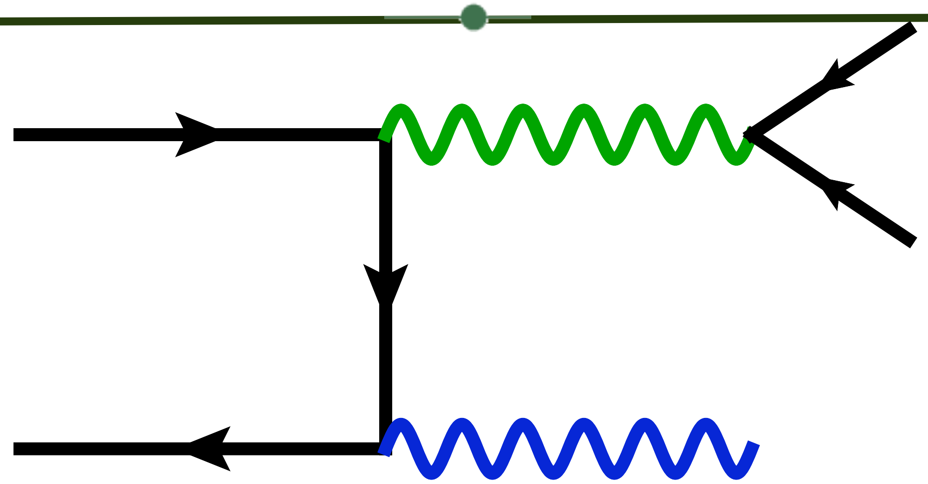




- ★ Once we have done this we can use our LO tools to define NLO event by event weights.

- ★ For the next few slides we will focus on the problem of re-weighting the Born phase space point to NLO.

Re-weighting Born

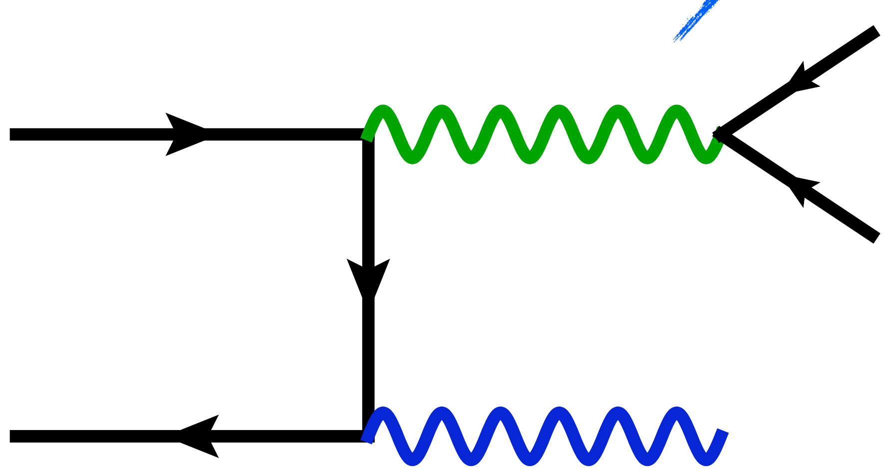


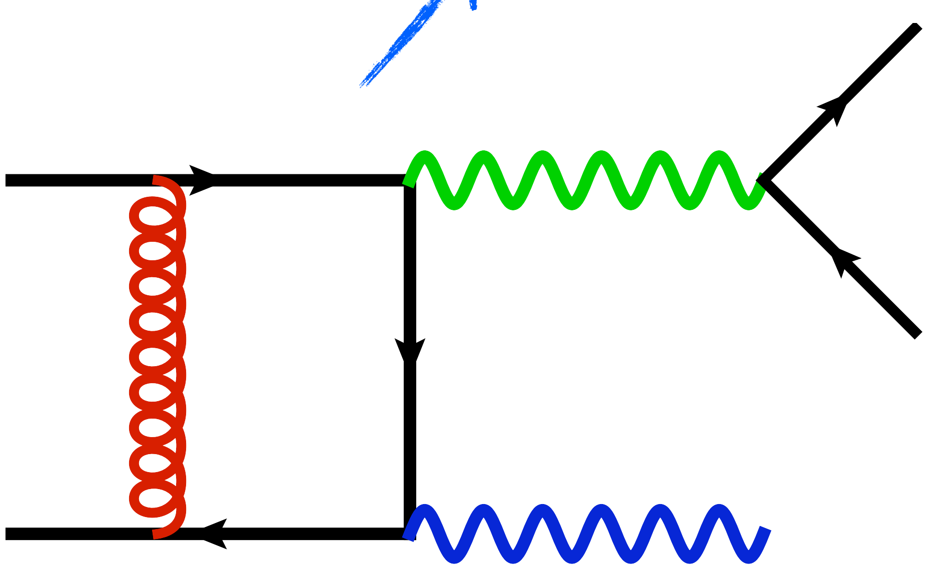
Imagine a LO event, containing only EW final state particles.

$$\Phi_B = (x_1, x_2, \{Q_n\}).$$

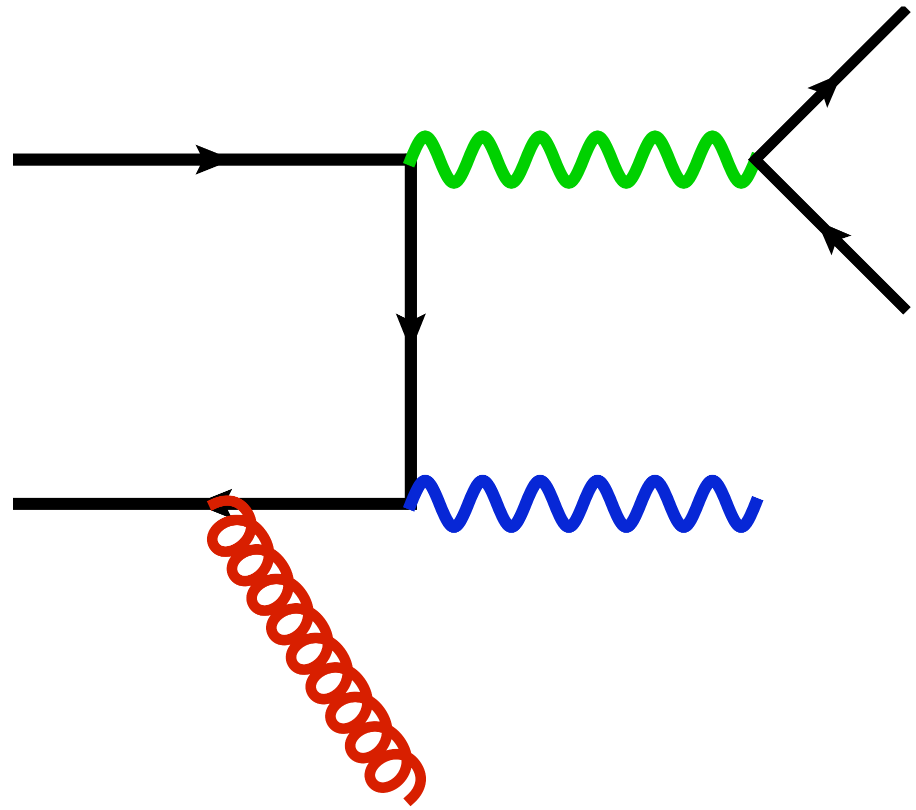
This completely determines the LO and virtual corrections.

$$\tilde{\mathcal{P}}_V(\Phi_B) = \frac{f(x_1)f(x_2)}{2x_1x_2s}\left(|\mathcal{M}^{(0)}(\Phi_B)|^2 + 2\mathrm{Re}\left\{\mathcal{M}^{(0)}\mathcal{M}^{(1)^\dagger}(\Phi_\mathrm{B})\right\}\right)$$





What about the real pieces?



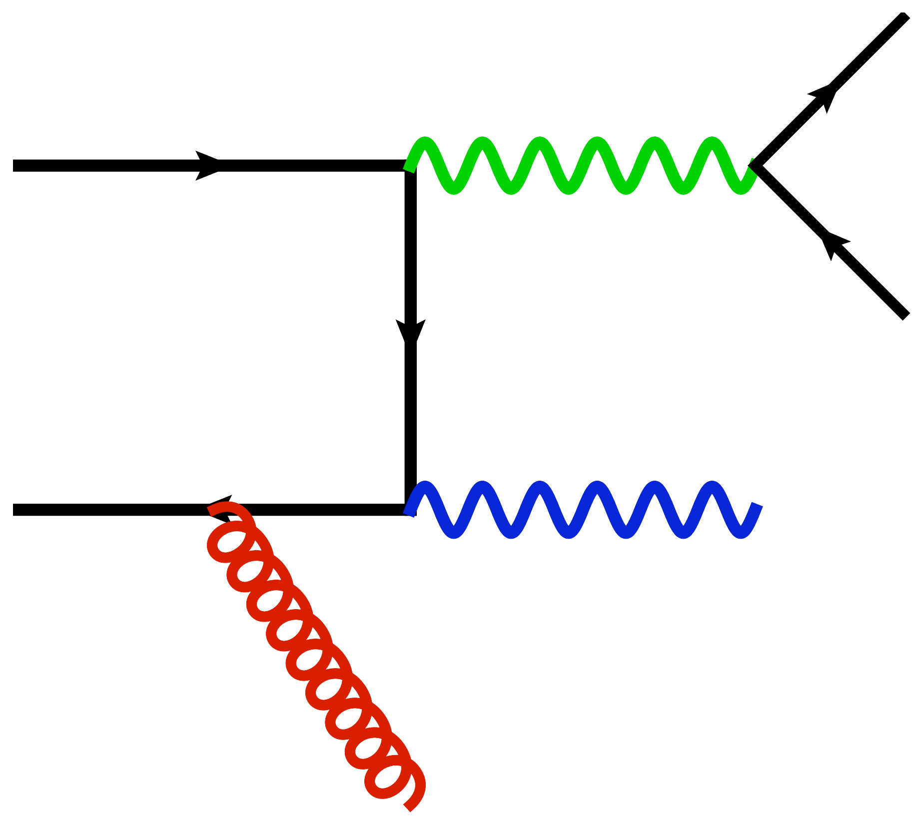
The real corrections naturally live in a larger (by one parton) phase space.

$$\hat{\Phi}_R = (\hat{x}_1, \hat{x}_2, \{\hat{Q}_n\}, \hat{p}_r)$$

We need to define a many to one map which defines the real phase space as a parameter of the Born phase space. This can be done by collecting all real points which share the same final state EW particles.

$$\Phi_R(\Phi_B) = (x_a, x_b, \{Q_n\}, p_r).$$

What about the real pieces?



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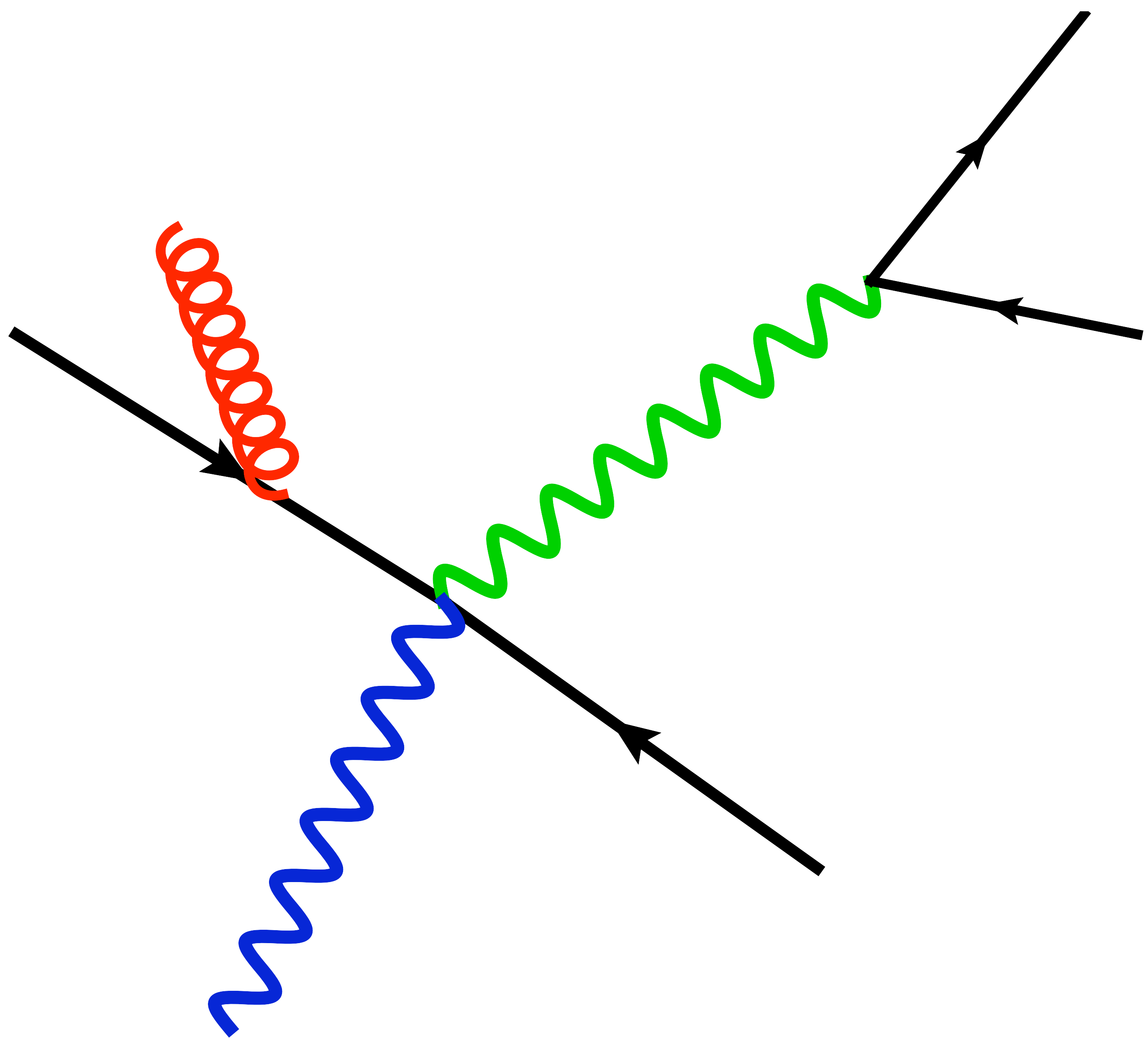
Recall that $\quad \Phi_B = (x_1, x_2, \{Q_n\}).$

Real pieces cont...

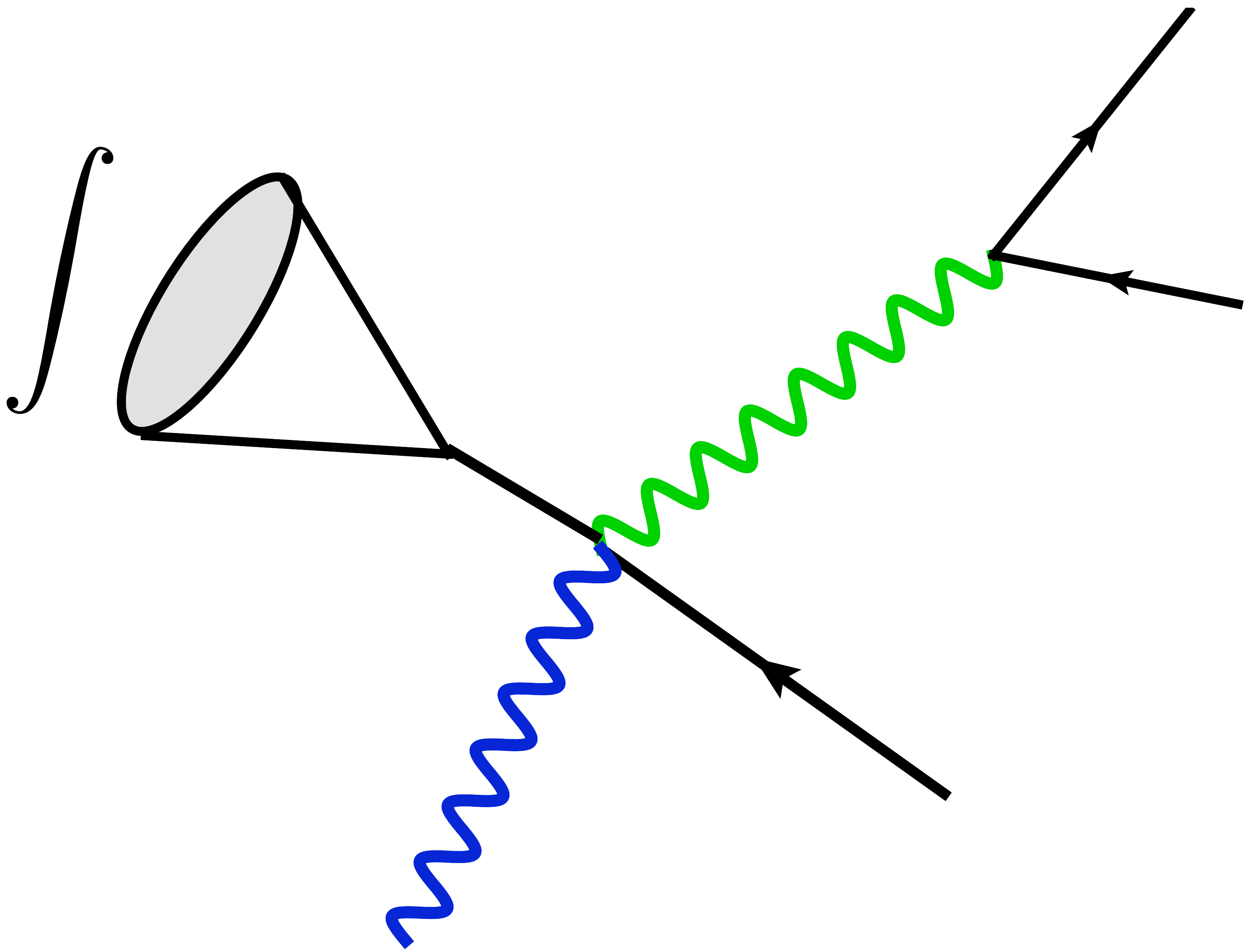


Remember that Q defines a LO final state phase space, so the EW particles balance in transverse momentum.

That means that in the frame in which the EW particles are held fixed the beam is moved away from the z-axis.

This is most sensible if we only allow small departures from the LO topology, i.e. we veto emissions if they become too hard.

Real pieces cont..



Since the real phase space is larger we integrate out all the emissions for each final state Born topology. (Over both beams!)

If one integrates over the LO final state phase space then the NLO exclusive cross section (governed by the emission veto) is recovered.

But what about the singularities?

Regularization issues....

A subtlety arises when we attempt to regulate the IR divergences. In the usual Catani-Seymour framework (which we were using since the code is based on MCFM) one introduces multiple dipole transformations, each with a different LO phase space point

$$\Phi_R \to \{\Phi^{ii}_{LO}, \Phi^{fi}_{LO}, \Phi^{if}_{LO}\Phi^{ff}_{LO}\}$$

This breaks our required factorization

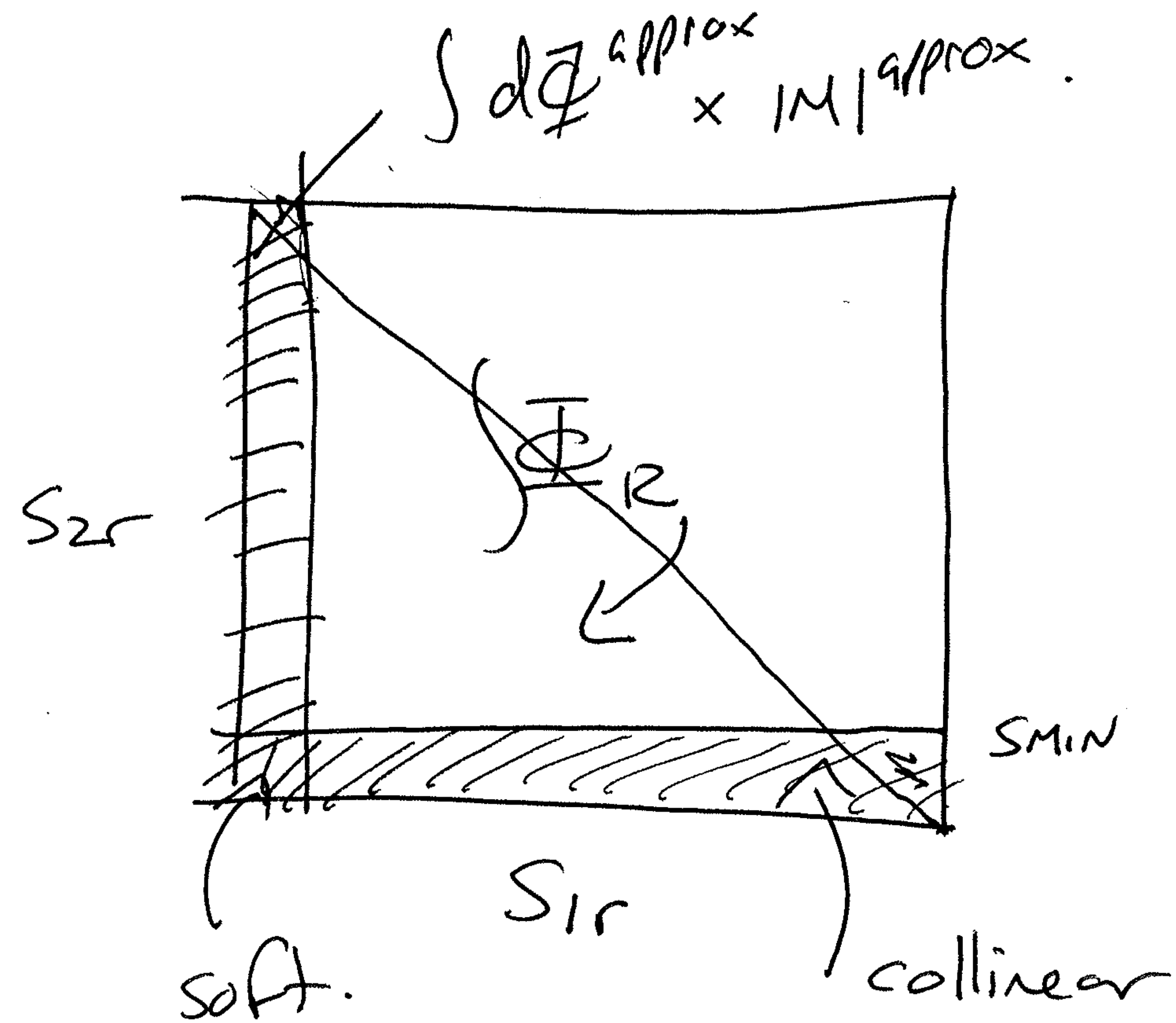
$$\mathcal{P}_{NLO}(\Phi_B) = K(\Phi_B)\mathcal{P}_{LO}(\Phi_B)$$

So that individual events depend on the regularization (α parameters)

So we need to use a different formalism which doesnt introduce new Borns.....

Phase space slicing

- ★ Need our regularizing functions to be defined at the Born phase space point
- ★ Simplest possible scheme is to use phase space slicing (Giele, Glover, Kosower), which naturally maps all of the singularities to the identified Born phase space point.
- ★ In the future, we will likely move to FKS (Frixione, Kunszt, Signer) subtractions. Which have advantages (no smin dependence)



Jets in the final state

The inclusion of jets introduces a couple of new problems.

1) MC/Data jets will have mass

2) Jets at NLO can be identified with multiple parton configurations.

The solution to 1) can be achieved by re-writing the jet co-ordinates.

$$J = (p_x, p_y, p_z, E) = (\overline{p}_T, \phi, \eta, m)$$

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Niels Bohr Institutet

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$$J = (p_x, p_y, p_z, E) = (\overline{p}_T, \phi, \eta, m)$$

Then we define our data=> LO map as

$$p_T^{data} \to p_T^{LO} \quad \phi^{data} \to \phi^{LO} \quad p_Z^{data} \to \gamma p_Z^{LO} \quad E^{data} \to \gamma E^{LO}$$

$$\gamma = \sqrt{\frac{p_T^2}{p_L^2}}$$

So that m=0, with other jet kinematics fixed.

Final state jets

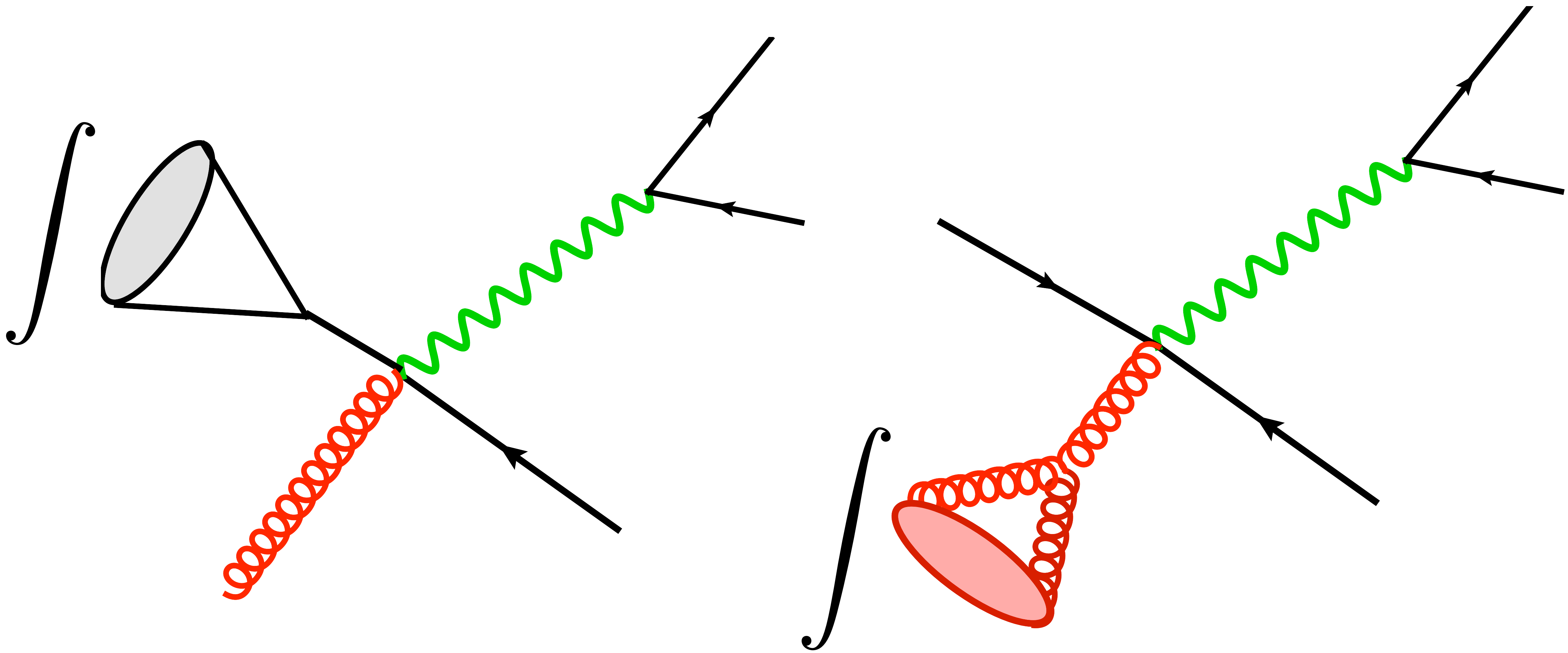
Its straightforward to map a LO parton level event to our observed final state jets (with massless definition).

$$C^{LO}(\{p_m\}|\{J_m\}) = \sum_{perms}\prod_{i=1}^{m}\delta(p_{T,i}-J_{T,i})\delta(\phi_i-\phi_i^J)\delta(\eta_i-\eta_i^J)$$

At NLO there are two types of contributions, depending on whether or not the partons cluster to form the observed jet or not.

$$\begin{aligned}C^{NLO}(\{p_{m+1}\}|\{J_m\}) &= \sum_{perms}\sum_{j}\prod_{i=1,i\neq j}^{m+1}\delta(p_{T,i}-J_{T,i})\delta(\phi_i-\phi_i^J)\delta(\eta_i-\eta_i^J)\\ &+\sum_{perms}\sum_{j}\prod_{i=1,i\neq j,j+1}^{m+1}\Big(\delta(p_{T,i}-J_{T,i})\delta(\phi_i-\phi_i^J)\delta(\eta_i-\eta_i^J)\Big)\\ &\times\delta(p_{T,j+(j+1)}-J_{T,i})\delta(\phi_{j+(j+1)}-\phi_i^J)\delta(\eta_{j+(j+1)}-\eta_i^J)\end{aligned}$$

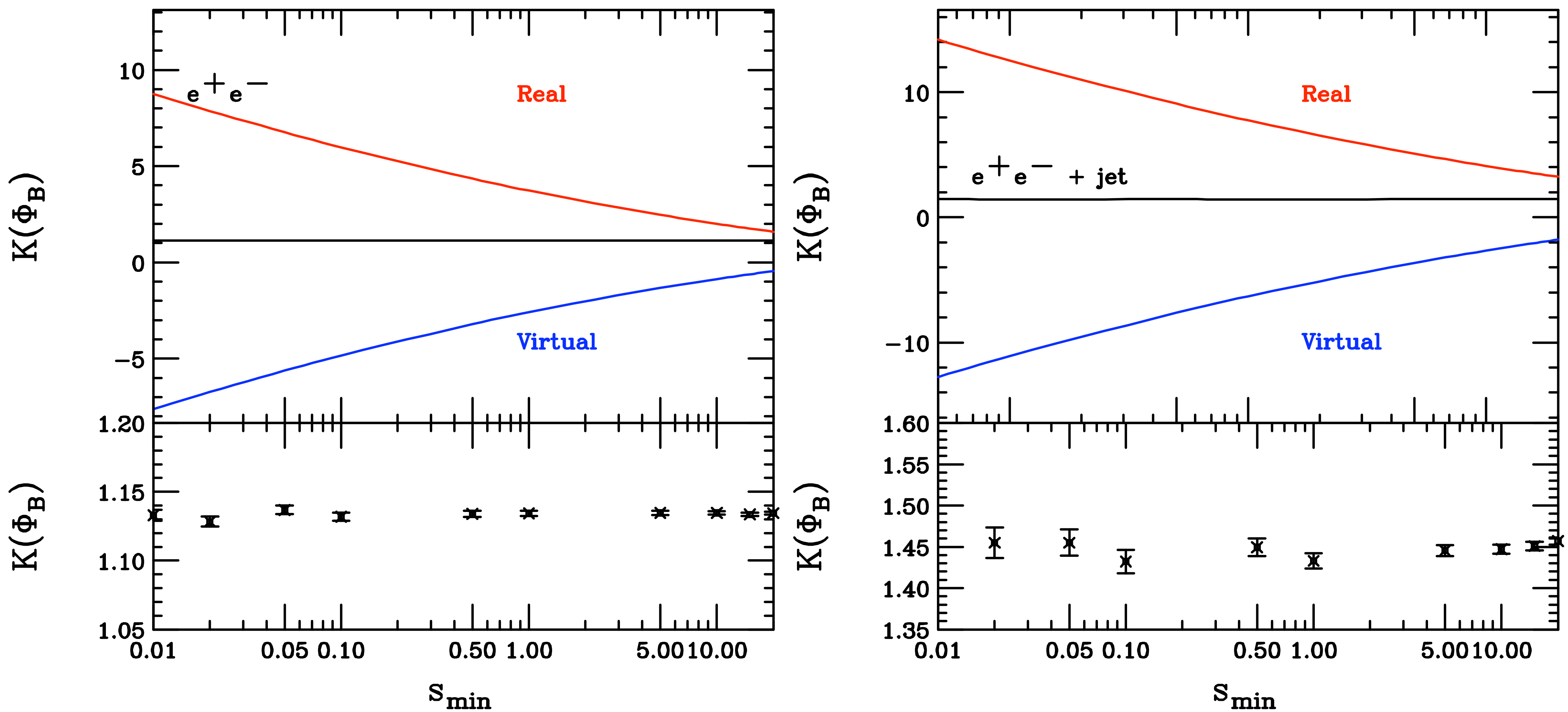
Jets summary



With jets in the final state we now have two types of contributions. The first is very similar to those found in EW only calculations, where we emitted from the beam (and veto hard radiation).

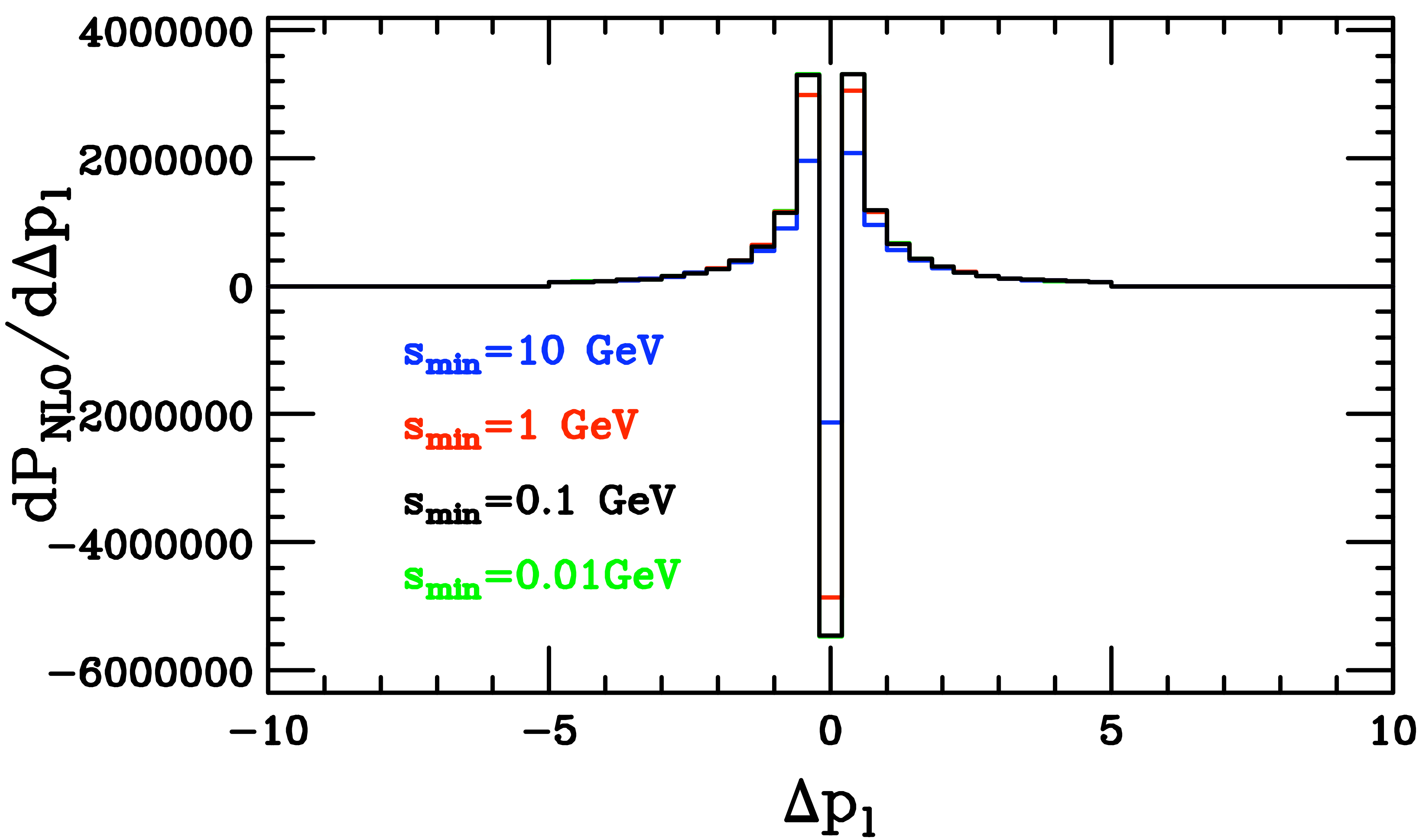
The second contribution is new, and occurs when two partons cluster to make the Born jet.

s_{min} cancellation



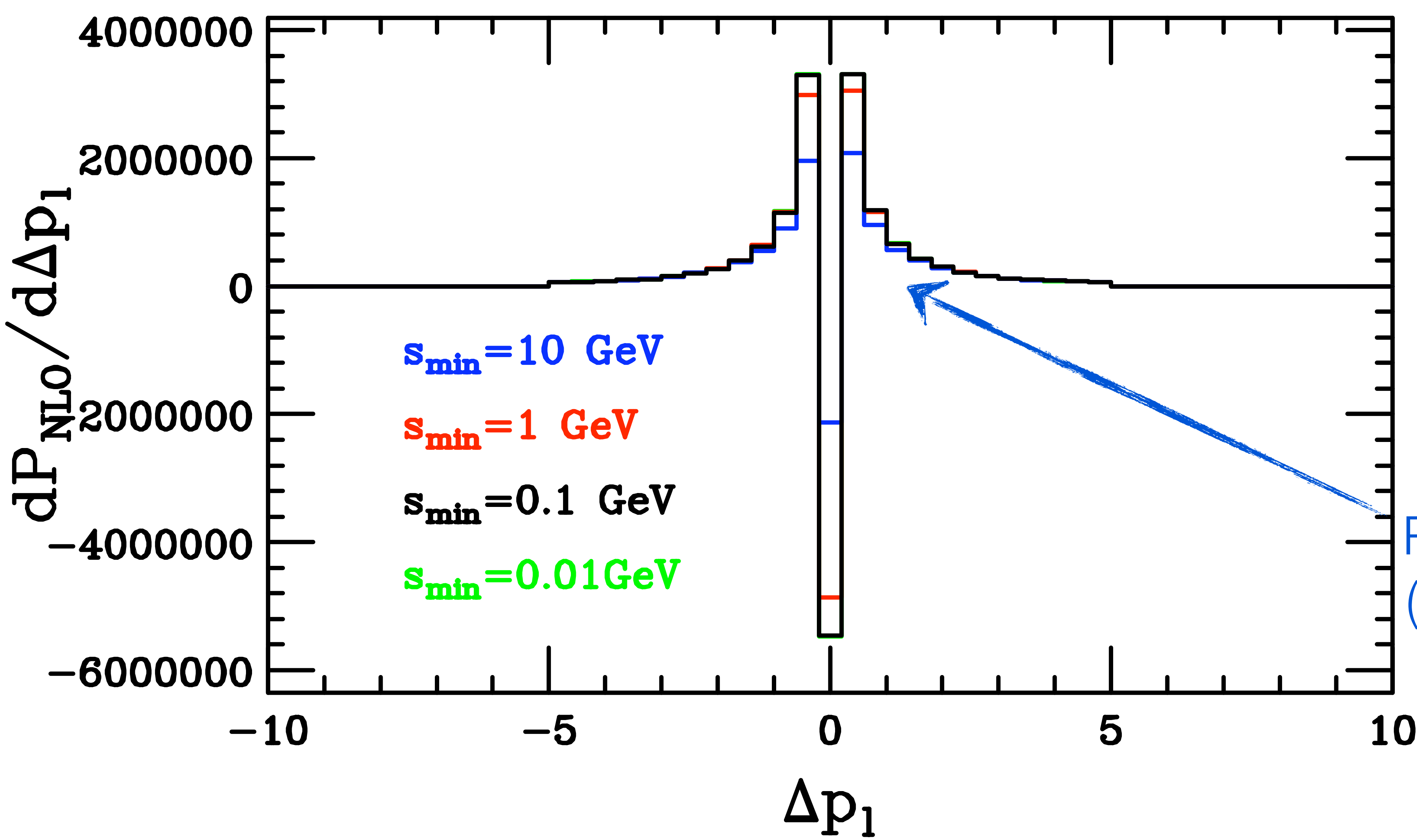
The crucial test that the method is working is that each Born event is the logarithmic cancellation of s_{min}. Here I plot the dependence on s_{min} for a single Born phase space point for Z+0 and Z+1 jet final states.

The recoil of an event.



It is also interesting to look at the difference between the LO pT and recoil p_T for a single Born phase space point. (This is unphysical but illustrative).

The recoil of an event.



Real corrections (note ~< 5 GeV)

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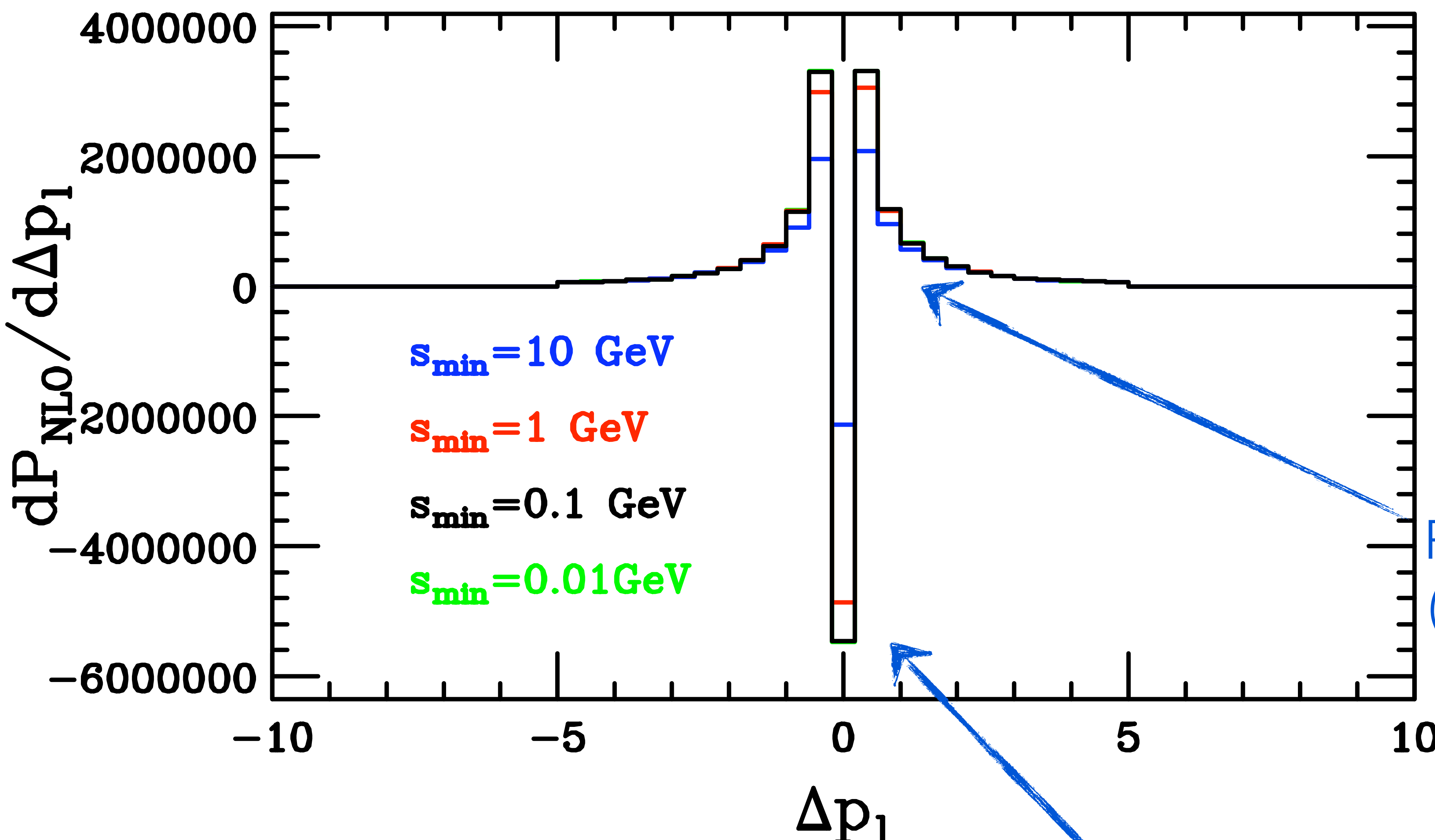
The recoil of an event.



Real corrections
(note ~< 5 GeV)

Virtual + Born

It is also interesting to look at the difference between the LO pT and recoil p_T for a single Born phase space point. (This is unphysical but illustrative).

The master formula!

The following formula describes the NLO weight as a function of a single Born phase space point.

$$\mathcal{P}_{NLO} = \frac{f(x_1)f(x_2)}{2x_1x_2s}\left((1+\mathcal{R}_v(s_{min}))|\mathcal{M}^{(0)}(\Phi_B)|^2 + 2\mathrm{Re}\left\{\mathcal{M}^{(0)}\mathcal{M}^{(1)^\dagger}(\Phi_\mathrm{B})\right\}\right)$$
$$+\sum_{i=1}^{n_{jets+1}}\int_{s_{min}} d\,\Phi^{IS}_{\mathrm{FBPS}}(\Phi_B)J_x\frac{f(x_a)f(x_b)}{2x_ax_bs}|M_R^{(0)}(\Phi_R(\Phi_B))|^2C_{IS}(i)$$
$$+\sum_{i\neq j,i>j}\int_{s_{min}} d\,\Phi^{FS}_{\mathrm{FBPS}}(\Phi_B)J_x\frac{f(x_a)f(x_b)}{2x_ax_bs}|M_R^{(0)}(\Phi_R(\Phi_B))|^2C_{FS}(i,j) + \mathcal{O}(s_{min})$$

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Real pieces where two partons cluster to form a jet.

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$$+\sum_{i=1}^{n_{jets+1}}\int_{s_{min}} d\,\Phi^{IS}_{\mathrm{FBPS}}(\Phi_B)J_x\frac{f(x_a)f(x_b)}{2x_ax_bs}|M_R^{(0)}(\Phi_R(\Phi_B))|^2 C_{IS}(i)$$

$$+\sum_{i\neq j, i>j}\int_{s_{min}} d\,\Phi^{FS}_{\mathrm{FBPS}}(\Phi_B)J_x\frac{f(x_a)f(x_b)}{2x_ax_bs}|M_R^{(0)}(\Phi_R(\Phi_B))|^2 C_{FS}(i,j) + \mathcal{O}(s_{min})$$

Real pieces, where the initial state branches.

Real pieces where two partons cluster to form a jet.

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$$\mathcal{P}_{NLO} = \frac{f(x_1)f(x_2)}{2x_1x_2s}\left((1+\mathcal{R}_v(s_{min}))|\mathcal{M}^{(0)}(\Phi_B)|^2 + 2\mathrm{Re}\left\{\mathcal{M}^{(0)}\mathcal{M}^{(1)^\dagger}(\Phi_\mathrm{B})\right\}\right)$$

$$+\sum_{i=1}^{n_{jets+1}}\int_{s_{min}} d\,\Phi^{IS}_{\mathrm{FBPS}}(\Phi_B)J_x\frac{f(x_a)f(x_b)}{2x_ax_bs}|M_R^{(0)}(\Phi_R(\Phi_B))|^2C_{IS}(i)$$

$$+\sum_{i\neq j,i>j}\int_{s_{min}} d\,\Phi^{FS}_{\mathrm{FBPS}}(\Phi_B)J_x\frac{f(x_a)f(x_b)}{2x_ax_bs}|M_R^{(0)}(\Phi_R(\Phi_B))|^2C_{FS}(i,j)+\mathcal{O}(s_{min})$$

Integrated slice over approximate phase space, cancels divergences in virtual

Real pieces, where the initial state branches.

Real pieces where two partons cluster to form a jet.

The MEM@NLO

- We now have a procedure to perform the MEM@NLO,
- Take an input event, perform the usual MEM@LO algorithm but reweight each point using the following dynamic K-factor

$$\mathcal{P}_{NLO}(\Phi_B) = K(\Phi_B)\mathcal{P}_{LO}(\Phi_B)$$

Phenomenological applications

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- ★ I've focussed on a theoretical overview, but the potential for phenomenology with KD's is rich. The method is computationally expensive, so is best applied in searches/measurements where advanced tools are needed.
- ★ Some applications/ongoing projects are :
- ★ H=>ZZ (find off-shell Higgs/gg events for Width measurements, based on idea by Caola and Melnikov)
- ★ VBF production, motivated by LO study (Andersen, Englert and Spannowsky)
- ★ Anomalous couplings of Higgs
- ★ Searches for EW Chargino/neutralino production
- ★ Top mass, Higgs self coupling..........

Summary

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- ★ I have discussed algorithms for event by event weighting which can be used for any final state, and extended to higher orders in perturbation theory.
- ★ Ultimately we would like to release a modified version of MCFM (codenamed MemCFM), which can be run in fully exclusive mode (appropriate for the MEM).
- ★ Primary applications would be to Higgs characterization and self-coupling studies. Secondary applications could include further BSM searches/ SM measurements, where advanced tools are needed.