The Two-Loop Analog of the Passarino-Veltman Result And Beyond

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A Basis for the Virtual Corrections to One-Loop Hard Scattering Processes

$$A^{1-\text{loop}}\left(k_1^{h_1},\ldots,k_n^{h_n}\right)=\sum_{\alpha_5}C_5^{(\alpha_4)}I_5^{(\alpha_5)}+\sum_{\alpha_4}C_4^{(\alpha_4)}I_4^{(\alpha_4)}+\sum_{\alpha_3}C_3^{(\alpha_3)}I_3^{(\alpha_3)}+\sum_{\alpha_2}C_2^{(\alpha_2)}I_2^{(\alpha_2)}+\sum_{\alpha_1}C_1^{(\alpha_1)}I_1^{(\alpha_1)}$$

- It has been known for a long time (G. Passarino and M. J. G. Veltman, Nucl. Phys. **B160**, 151, 1979) that, for most phenomenologically interesting calculations, it is prohibitively inefficient to blindly calculate
- The program of finding an integral basis valid for arbitrary one-loop processes in renormalizable theories was begun by Passarino and Veltman ($2 \to 2$) and completed by Bern, Dixon, and Kosower (Z. Bern, L. J. Dixon, and D. A. Kosower, Phys. Lett. **B302**, 299, 1993)

A Basis for the Virtual Corrections to Two-Loop Hard Scattering Processes?

$$A^{2-\text{loop}}\left(k_1^{h_1},\ldots,k_n^{h_n}\right) \quad = \quad ???$$

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The two-loop integral basis is not known in the generic case, even for $2 \to 2$!

see Anastasiou *et. al.*, Nucl. Phys. **B575**, 416, 2000; Nucl. Phys. **B580**, 577, 2000; Gehrmann *et. al.*, Nucl. Phys. **B580**, 485, 2000; Actis *et. al.*, Nucl. Phys. **B703**, 3, 2004; Gluza *et. al.*, Phys. Rev. **D83**, 045012, 2011 and other recent papers of David A. Kosower and collaborators for partial results in several special cases

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How do we find it?

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How do we find it?

Integration By Parts Relations

K. Chetyrkin and F. Tkachov, Nucl. Phys. B192, 159, 1981

Can We Solve These IBP Relations?

Suppose we want to solve the system of IBP recurrence relations to determine the master integrals for a given multi-loop topology:

- For most interesting examples a highly non-trivial system of recurrence relations results
- Difficult or impossible to solve by hand
- A well-known algorithm due to Laporta (S. Laporta, Int. J. Mod. Phys. **A15**, 5087, 2000) reduces the problem to the solution of (usually) a very large system of linear equations

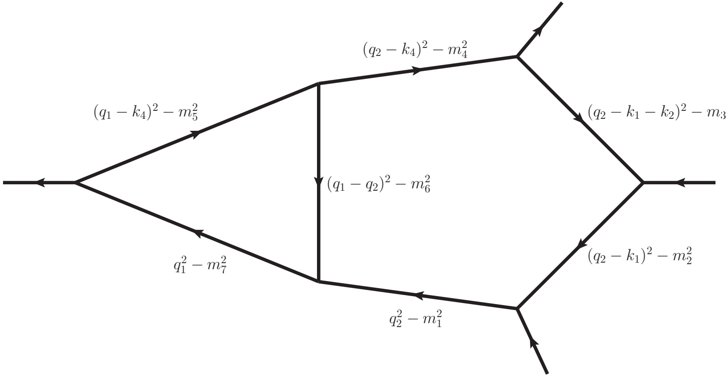
Laporta's algorithm, while very important, requires significant computational resources for most interesting two-loop topologies. What can we do to reduce the computational complexity?

It turns out that determining the master integrals is much simpler than actually solving the IBPs:

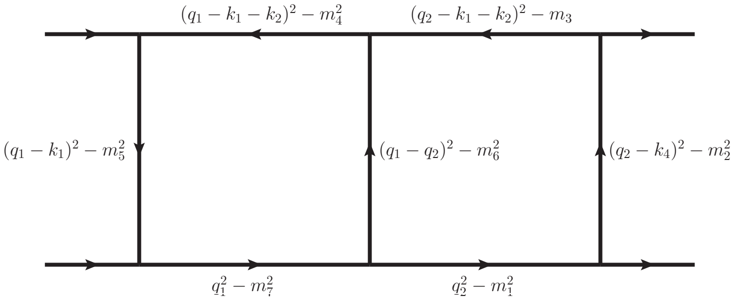
- Find all independent *sectors*, collections of integrals with the same set of propagators, for the Feynman integrals that could potentially arise from Feynman diagrams
- Observe that the correlations between sectors can be ignored if all one wants are the master integrals. This allows for a sector-by-sector divide-and-conquer approach
- Use a phase space point where all internal masses, external masses, and Mandelstam invariants are set to primes. This effectively reduces a many-scale problem to a no-scale problem

It makes sense to work within the framework of Reduze 2 (A. von Manteuffel and C. Studerus, arXiv:1201.4330) because most of the necessary code is already there

The Pentatriangle Topology



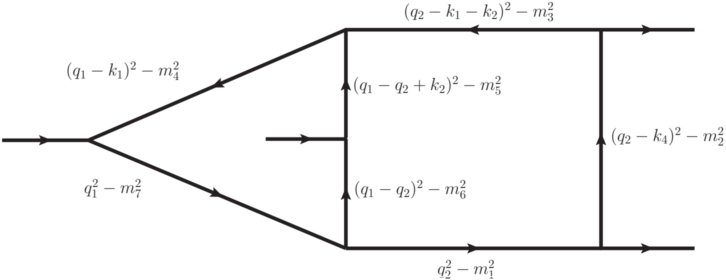
The Planar Double-Box Topology



$$I\left[(q_1 - k_4)^2 - m_8^2\right] \quad I\left[\left((q_1 - k_4)^2 - m_8^2\right)\left((q_2 - k_1)^2 - m_9^2\right)\right]$$

$$I\left[\left((q_1 - k_4)^2 - m_8^2\right)^2\right] \quad I\left[(q_2 - k_1)^2 - m_9^2\right]$$

The Non-Planar Double-Box Topology

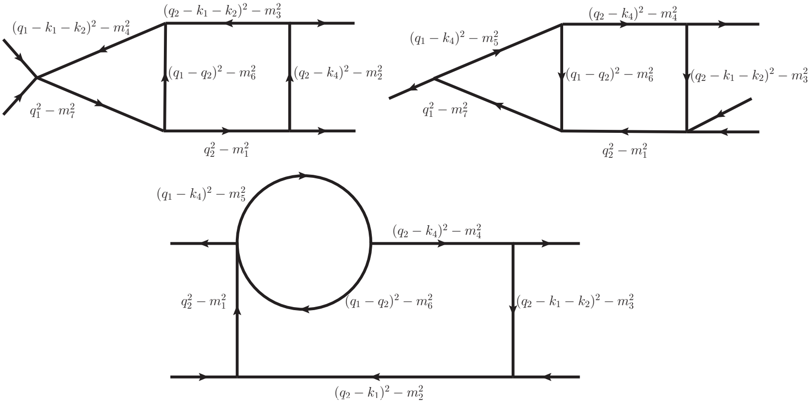


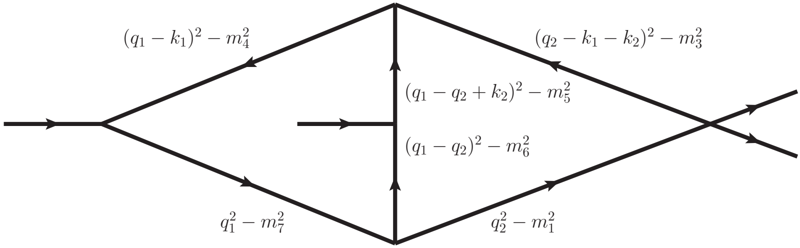
$$I\left[\left((q_1 - k_4)^2 - m_8^2\right)\left((q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right)\right]$$

$$I\left[(q_1 - k_4)^2 - m_8^2\right] \quad I\left[\left((q_1 - k_4)^2 - m_8^2\right)^2\right] \quad I\left[\left((q_1 - k_4)^2 - m_8^2\right)^3\right]$$

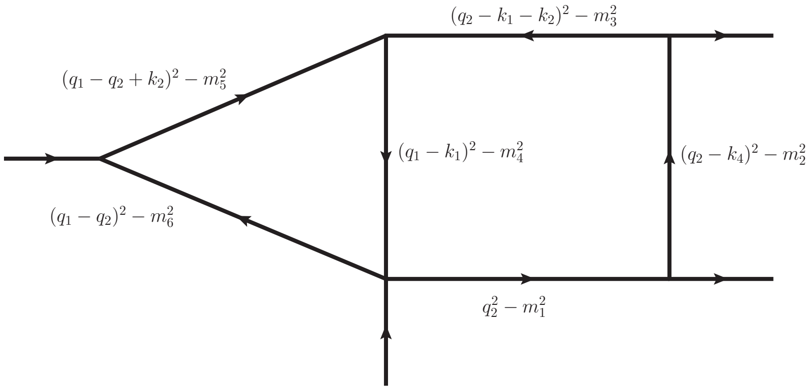
$$I\left[(q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right] \quad I\left[\left((q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right)^2\right]$$

Six-Propagator Topologies

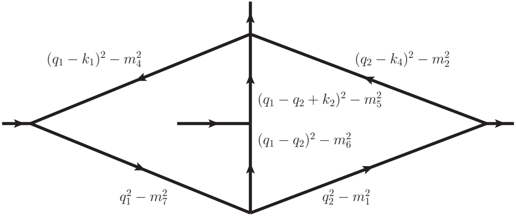




$$I\left[(q_2 - k_4)^2 - m_2^2\right] \qquad I\left[\left((q_2 - k_4)^2 - m_2^2\right)^2\right]$$



$$I\left[q_1^2 - m_7^2\right] \quad I\left[\left(q_1^2 - m_7^2\right)^2\right] \quad I\left[\left((q_1 - k_4)^2 - m_8^2\right)\left((q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right)\right]$$



$$I\left[(q_1 - k_4)^2 - m_8^2\right] \quad I\left[\left((q_1 - k_4)^2 - m_8^2\right)\left((q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right)\right]$$

$$I\left[\left((q_1 - k_4)^2 - m_8^2\right)^2\right] \quad I\left[\left((q_1 - k_4)^2 - m_8^2\right)\left((q_2 - k_1 - k_2)^2 - m_3^2\right)\right]$$

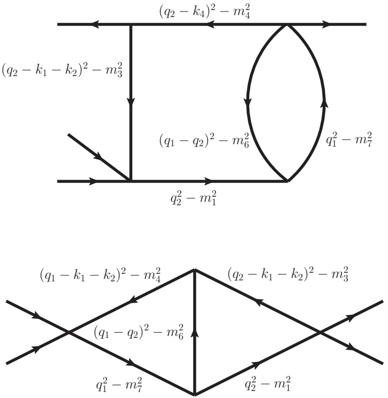
$$I\left[\left((q_2 - k_1 - k_2)^2 - m_3^2\right)\left((q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right)\right] I\left[(q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right]$$

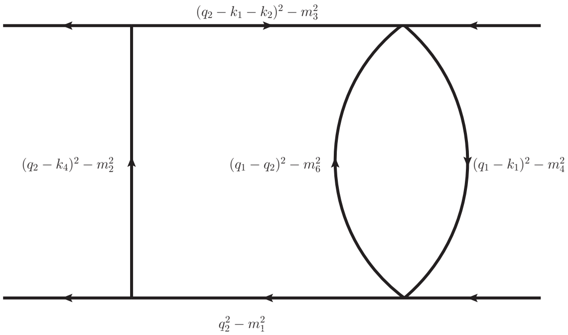
$$I\left[(q_2 - k_1 - k_2)^2 - m_3^2\right] \quad I\left[\left((q_2 - k_1 - k_2)^2 - m_3^2\right)^2\right] \quad I\left[\left((q_2 - k_1 - k_2)^2 - m_3^2\right)^3\right]$$

$$I\left[\left((q_1 - k_4)^2 - m_8^2\right)\left((q_2 - k_1 - k_2)^2 - m_3^2\right)^2\right] \quad I\left[\left((q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right)^2\right]$$

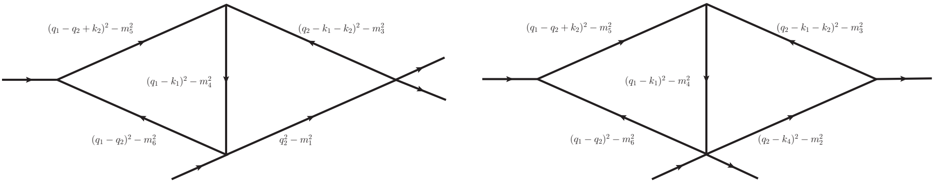
$$I\left[\left((q_2 - k_1 - k_2)^2 - m_3^2\right)^2\left((q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right)\right]$$

Five-Propagator Topologies





$$I\left[(q_1 - q_2 + k_2)^2 - m_5^2\right] \quad I\left[\left((q_1 - q_2 + k_2)^2 - m_5^2\right)^2\right] \quad I\left[q_1^2 - m_7^2\right]$$

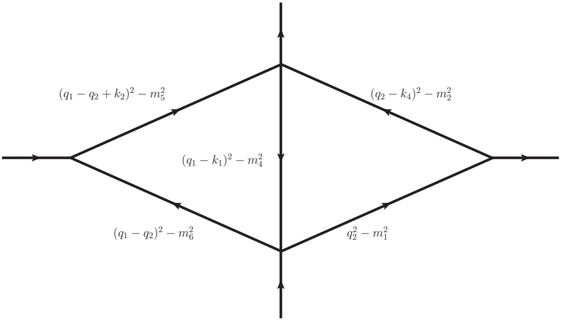


$$I_L\left[(q_2 - k_4)^2 - m_2^2\right] \qquad I_L\left[q_1^2 - m_7^2\right]$$

$$I_L\left[\left((q_2 - k_4)^2 - m_2^2\right)^2\right] \qquad I_L\left[\left((q_2 - k_4)^2 - m_2^2\right)\left(q_1^2 - m_7^2\right)\right]$$

$$I_R\left[q_2^2 - m_1^2\right] \qquad I_R\left[q_1^2 - m_7^2\right]$$

$$I_R\left[\left(q_2^2 - m_1^2\right)^2\right] \qquad I_R\left[\left(q_2^2 - m_1^2\right)\left(q_1^2 - m_7^2\right)\right]$$



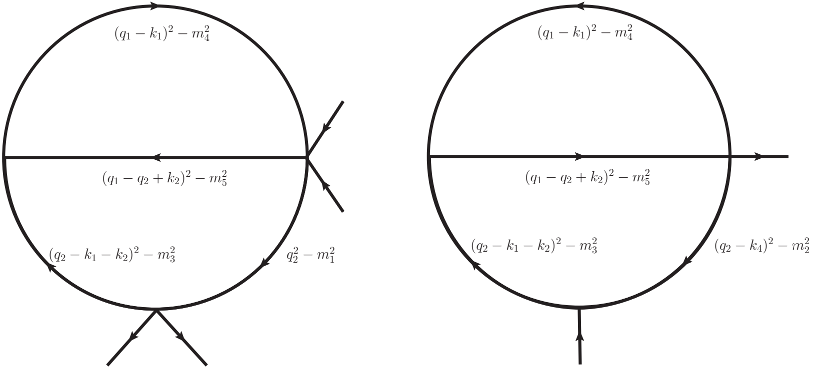
$$I\left[q_1^2 - m_7^2\right] \qquad I\left[(q_1 - q_2 + k_1 + k_2)^2 - m_9^2\right]$$

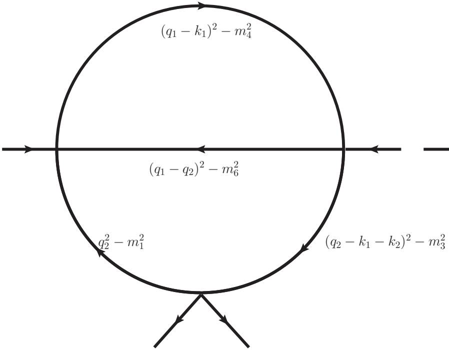
$$I\left[(q_1 - k_4)^2 - m_8^2\right] \qquad I\left[\left((q_2 - k_1 - k_2)^2 - m_3^2\right)\left(q_1^2 - m_7^2\right)\right]$$

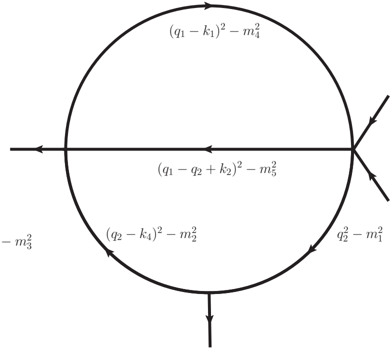
$$I\left[(q_2 - k_1 - k_2)^2 - m_3^2\right] \qquad I\left[\left((q_2 - k_1 - k_2)^2 - m_3^2\right)^2\right]$$

$$I\left[\left((q_2 - k_1 - k_2)^2 - m_3^2\right)\left((q_1 - k_4)^2 - m_8^2\right)\right]$$

Four-Propagator Topologies





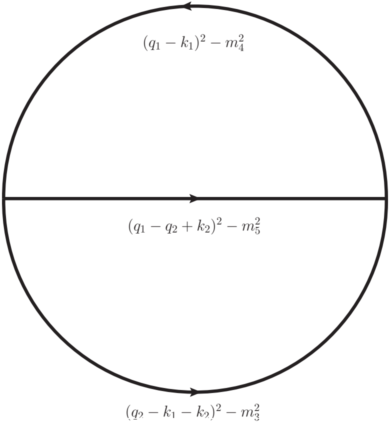


$$I_L\left[(q_2 - k_4)^2 - m_2^2\right] \quad I_L\left[\left((q_2 - k_4)^2 - m_2^2\right)^2\right] \quad I_L\left[(q_1 - q_2 + k_2)^2 - m_5^2\right]$$

$$I_R\left[(q_2 - k_1 - k_2)^2 - m_3^2\right] \quad I_R\left[\left((q_2 - k_1 - k_2)^2 - m_3^2\right)^2\right] \quad I_R\left[(q_1 - q_2)^2 - m_6^2\right]$$

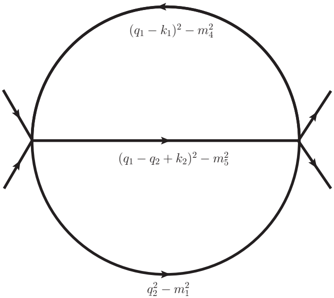
The Non-Trivial Vacuum Topology

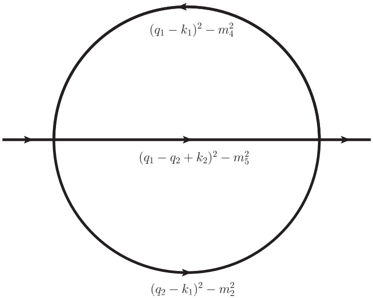
A. I. Davydychev, Phys. Rev. **D61**, 087701, 2000



The Sunrise Topologies

L. Adams, C. Bogner, and S. Weinzierl, arXiv:1302.7004





$$I_L\left[(q_2 - k_4)^2 - m_2^2\right] \quad I_L\left[\left((q_2 - k_4)^2 - m_2^2\right)^2\right] \quad I_L\left[(q_1 - q_2)^2 - m_6^2\right]$$

$$I_R\left[q_2^2 - m_1^2\right] \quad I_R\left[\left(q_2^2 - m_1^2\right)^2\right] \quad I_R\left[(q_1 - q_2)^2 - m_6^2\right]$$

It's Fast!

On my laptop:

- The process $q\bar{q} \to t\bar{t}$ at two loops runs in ~ 4 mins
- The generic two-loop $2 \to 2$ problem runs in ~ 4 mins
- The three-loop gluon-gluon form factor runs in ~ 14 mins
- The generic two-loop $2 \to 3$ problem runs in ~ 8.5 hours

Outlook

Although our code applies to arbitrary scattering processes, limited only by computer time, there is clearly still a very long way to go if the goal is to build a fully automated two-loop program such as those that already exist at one-loop

- Solve the remaining phenomenologically important masters for $2 \to 2$ processes (*e.g.* those needed for the NNLO wishlist)
- Improve the efficiency of the Reduze 2 IBP relation solver
- Rotate to a Henn basis (J. M. Henn, Phys. Rev. Lett. **110**, 251601, 2013) once an algorithm to do so becomes available
- Experiment with other approaches to this problem (*e.g.* that of Lee and Pomeransky, arXiv:1307.4083)

Trivial Topologies