

# Comparisons of Predictions from Exact Amplitude-Based Resummation Methods with LHC and Cosmological Data

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# Forward

\*\* NEW PHYSICS/PRECISION ‘H’ AT LHC

Must Distinguish from Higher  
Order SM Processes AND Must  
Probe Precisely to Specify Uniquely  
⇒ Precision QCD for the LHC

\*\* UV LIMIT OF EINSTEIN’S THEORY

Can QFT Handle It?  
⇒ Exact, Gauge Invariant Residual Control in  
Resummation for UV



# The Archetype:

\*Approach to 1% Precision QCD for  
LHC Physics via MC Realized  
Amplitude-Based QED $\otimes$ QCD  
Resummation --

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) d\hat{\sigma}_{res}$$

$$\Rightarrow \Delta\sigma_{th} = \Delta F \oplus \Delta\hat{\sigma}_{res} = \Delta\sigma_{th}(tech) \oplus \Delta\sigma_{th}(phys)$$

$\Rightarrow$



$$\begin{aligned}
d\hat{\sigma}_{res} &= \sum_n d\hat{\sigma}_n \\
&= e^{SUM_{IR}(QCED)} \sum_{m,n=0}^{\infty} \frac{1}{m!n!} \int \prod_{j_1=1}^m \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^n \frac{d^3 k_{j_2}}{k_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_{j_1} k_{j_1}-\sum_{j_2} k_{j_2})+D_{QCED}} \\
&\quad * \tilde{\beta}_{m,n}(k_1, \dots, k_m; k_1^{'}, \dots, k_n^{'}) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0} \tag{1}
\end{aligned}$$



# The Archetype: \*Approach to Feynman's Formulation of Einstein's Theory : Amplitude-Based Resummation of the Feynman Propagators therein

$$\begin{aligned} i\Delta_F'(k) &= \frac{i}{k^2 - m^2 - \Sigma_s(k) + i\varepsilon} \\ &= \frac{ie^{B_g''(k)}}{k^2 - m^2 - \Sigma_s'(k) + i\varepsilon} \\ &\equiv i\Delta_F'(k)|_{\text{Re summed}} \end{aligned}$$



## Precision QCD for the LHC

- Contact with Standard Resummations,  
SCT, SCET -- see Phys. Rev. D81(2010)076008

Observations:

- EXACT – Compare:

Sterman-Catani-Trentadue Threshold Resummation

As for any  $f(z)$ ,

$$\left| \int_0^1 dz z^{n-1} f(z) \right| \leq \left( \frac{1}{n} \right) \max |f(z)|,$$

drop non-singular contributions to cross section at  $z \rightarrow 1$

- SCET:

drop  $O(\lambda)$  terms,  $\lambda = \sqrt{\Lambda/Q}$ ,

$\Lambda \sim .3 \text{ GeV}, Q \sim 100 \text{ GeV} \Rightarrow \lambda \approx 5.5\%$

- These methods give approximations to our  $\beta_{n,m}$

## Precision QCD for the LHC

- Contact with Standard Resummations,  
CSS– RESBOS, etc.: (see 1305.0023 for details)

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} \sim \frac{4\pi^2 \alpha^2}{9Q^2 s} \left\{ \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{Q}_T \cdot \vec{b}} \sum_j e_j^2 \widetilde{W}_j(b^*; Q, x_A, x_B) e^{\{-\ln(Q^2/Q_0^2)g_1(b) - g_{j/A}(x_A, b) - g_{j/B}(x_B, b)\}} \right. \\ \left. + Y(Q_T; Q, x_A, x_B) \right\}$$

Dropped terms  $O(Q_T/Q)$  in all orders of  $\alpha_s$ :  
at 5GeV,  $Q=M_Z$ , 5.5% Physical Precision Error(PPE)  
Errors on the NP functions  $g_i$  also yield ~1.5% PPE,...



- Shower/ME Matching:

Remove double counting between

$\tilde{\bar{\beta}}_{m,n} \rightarrow \hat{\tilde{\beta}}_{m,n}$ , shower - subtracted residuals

$$e^{SUM_{IR}}, e^D, F_1(x_1)F_2(x_2) |_{ShowerRealization}$$



- IR-Improved DGLAP-CS Theory(PRD81(2010)076008):  
**New resummed scheme for  $P_{AB}$ , reduced cross section derived from (1) applied to splitting process --**

$F_j, \hat{\sigma} \rightarrow F'_j, \hat{\sigma}'$  for

$$P_{qq} \rightarrow P_{qq}^{\text{exp}} = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+z^2}{1-z} (1-z)^{\gamma_q}, \text{etc.},$$

giving the same value for  $\sigma$ , with improved MC stability  
-- no need for IR cut - off ( $k_0$ ) parameter



- Complete Set ( $\gamma_A$  are  $O(\hbar)$ ) :

$$\begin{aligned}
 P_{qq}^{exp}(z) &= C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[ \frac{1+z^2}{1-z} (1-z)^{\gamma_q} - f_q(\gamma_q) \delta(1-z) \right], \\
 P_{Gq}^{exp}(z) &= C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}, \\
 P_{GG}^{exp}(z) &= 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \left\{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z} (1-z)^{\gamma_G} \right. \\
 &\quad \left. + \frac{1}{2} (z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G) \delta(1-z) \right\}, \\
 P_{qG}^{exp}(z) &= F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \{ z^2 (1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \},
 \end{aligned}$$

$$\begin{aligned}
 \gamma_q &= C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0}, & \delta_q &= \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi} \left( \frac{\pi^2}{3} - \frac{1}{2} \right), \\
 f_q(\gamma_q) &= \frac{2}{\gamma_q} - \frac{2}{\gamma_q+1} + \frac{1}{\gamma_q+2}, \\
 \gamma_G &= C_G \frac{\alpha_s}{\pi} t = \frac{4C_G}{\beta_0}, & \delta_G &= \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi} \left( \frac{\pi^2}{3} - \frac{1}{2} \right), \\
 f_G(\gamma_G) &= \frac{n_f}{6C_G F_{YFS}(\gamma_G)} e^{-\frac{1}{2}\delta_G} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} + \frac{1}{(1+\gamma_G)(2+\gamma_G)} \\
 &\quad + \frac{1}{2(3+\gamma_G)(4+\gamma_G)} + \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)}, \\
 F_{YFS}(\gamma) &= \frac{e^{-C\gamma}}{\Gamma(1+\gamma)}, & C &= 0.57721566...
 \end{aligned}$$



- KEY PART OF APPROACH:  
BUILD ON EXISTING PLATFORMS



IR-Improved DGLAP-CS Theory

U

NLO Parton Shower MC's:  
MC@NLO, POWHEG, ...



- Illustration:

$$d\sigma_{MC@NLO} = \left[ B + V + \int (R_{MC} - C) d\Phi_R \right] d\Phi_B [\Delta_{MC}(0) + \int (R_{MC}/B) \Delta_{MC}(k_T) d\Phi_R] \\ + (R - R_{MC}) \Delta_{MC}(k_T) d\Phi_B d\Phi_R$$

$\implies$  Sudakov FF

$$\Delta_{MC}(p_T) = e^{[- \int d\Phi_R \frac{R_{MC}(\Phi_B, \Phi_R)}{B} \theta(k_T(\Phi_B, \Phi_R) - p_T)]}$$



$$\frac{1}{2} \hat{\bar{\beta}}_{0,0} = \bar{B} + (\bar{B}/\Delta_{MC}(0)) \int (R_{MC}/B) \Delta_{MC}(k_T) d\Phi_R \\ \frac{1}{2} \hat{\bar{\beta}}_{1,0} = R - R_{MC} - B \tilde{S}_{QCD}$$

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$$\bar{B} = B(1 - 2\alpha_s \Re B_{QCD}) + V + \int (R_{MC} - C) d\Phi_R$$

- Rigorous Contact with Wilson's OPE:  
Repeat the Gross-Wilczek-Politzer analysis of DIS.  
From

$$\begin{aligned} W_{\alpha\beta}^F(p_F, q) &= \frac{1}{2\pi} \int d^4y e^{iqy} \langle p_F | [J_\beta(y), J_\alpha(0)] | p_F \rangle \\ &= (2\pi)^3 \sum_X \delta(q + p_F - p_X) \langle p_F | J_\beta(0) | p_X \rangle \langle p_X | J_\alpha(0) | p_F \rangle \end{aligned}$$

(1) allows us to note that



$$\langle p_X | J_\alpha(0) | p_F \rangle = e^{\alpha \cdot B_{\text{QCD}}} \langle p_X | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}} \Rightarrow$$

$$W_{\alpha\beta}^F(p_F, q) = (2\pi)^3 \sum_x \delta(q + p_F - p_X) e^{2\alpha \cdot \Re B_{\text{QCD}}} \langle p_F | J_\beta(0) | p_X \rangle_{\text{IRI-virt}} \\ \langle p_X | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}} \Rightarrow$$

$$_{\text{IRI-virt}} \langle p_F | J_\beta(0) | p_X \rangle \langle p_X | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}}$$

$$= \tilde{S}_{\text{QCD}}(k_1) \cdots \tilde{S}_{\text{QCD}}(k_n) \langle p_F | J_\beta(0) | p_{X'} \rangle \langle p_{X'} | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}} + \cdots +$$

$$_{\text{IRI-virt\&real}} \langle p_F | J_\beta(0) | p_{X'}, k_1, \dots, k_n \rangle \langle p_{X'}, k_1, \dots, k_n | J_\alpha(0) | p_F \rangle_{\text{IRI-virt\&real}}$$

$\Rightarrow$



$$W_{\beta\alpha}^F(p_F, q) = (2\pi)^3 \sum_X \delta(q + p_F - p_X) e^{2\alpha, \Re B_{\text{QCD}}} [\tilde{S}_{\text{QCD}}(k_1) \cdots \tilde{S}_{\text{QCD}}(k_n)]$$

$${}_{\text{IRI-virt}} \langle p_F | J_\beta(0) | p_{X'} \rangle \langle p_{X'} | J_\alpha(0) | p_F \rangle {}_{\text{IRI-virt}} + \cdots$$

$$+ {}_{\text{IRI-virt\&real}} \langle p_F | J_\beta(0) | p_{X'}, k_1, \dots, k_n \rangle \langle p_{X'}, k_1, \dots, k_n | J_\alpha(0) | p_F \rangle {}_{\text{IRI-virt\&real}}$$

$$= \frac{1}{2\pi} \int d^4y \sum_{X'} \sum_n \frac{1}{n!} \int \prod_{j=1}^n \frac{d^3k_j}{k_j^0} e^{\text{SUM}_{\text{IR}}(\text{QCD})} e^{iy(q+p_F-p_{X'}-\sum_j k_j)+D_{\text{QCD}}}$$

$${}_{\text{IRI-virt\&real}} \langle p_F | J_\beta(0) | p_{X'}, k_1, \dots, k_n \rangle \langle p_{X'} | k_1, \dots, k_n | J_\alpha(0) | p_F \rangle {}_{\text{IRI-virt\&real}}$$

$$= \frac{1}{2\pi} \int d^4y e^{iy} e^{\text{SUM}_{\text{IR}}(\text{QCD})+D_{\text{QCD}}} {}_{\text{IRI-virt\&real}} \langle p_F | [J_\beta(y), J_\alpha(0)] | p_F \rangle {}_{\text{IRI-virt\&real}},$$

$$\text{SUM}_{\text{IR}}(\text{QCD}) = 2\alpha_s \Re B_{\text{QCD}} + 2\alpha_s \tilde{B}_{\text{QCD}}(\text{Kmax}), \quad 2\alpha_s \tilde{B}_{\text{QCD}}(\text{Kmax}) = \int \frac{d^3k}{k^0} \tilde{S}_{\text{QCD}}(k),$$

$$D_{\text{QCD}} = \frac{\int d^3k}{k} \tilde{S}_{\text{QCD}}(k) [e^{-iy\cdot k} - \theta(\text{Kmax} - k)]$$



We use

$$W_{\beta\alpha} = \sum_a \int_0^1 \frac{dx}{x} F_a(x) W_{\beta\alpha}^a$$

to get

$$\int_0^1 dx x^n F_1(x, q^2) = \sum_j \tilde{\tilde{C}}_{j,1}^{(n+1)}(q^2) \tilde{M}_j^{n+1},$$

$$\int_0^1 dx x^n F_2(x, q^2) = \sum_j \tilde{\tilde{C}}_{j,2}^{(n)}(q^2) \tilde{M}_j^{n+2},$$

where

$$\tilde{\tilde{C}}_{j,k}^{(n)}(q^2) = \frac{1}{2} i (q^2)^{n+1} \left( \frac{-\partial}{\partial q^2} \right)^n \int d^4 y e^{iqy + \text{SUM}_{\alpha}(\text{QCD}) + D_{\text{QCD}}} \frac{\tilde{C}_{j,k}^{(n)}(y^2)}{y^2 - i\epsilon y_0}$$

and

$$\langle p | \tilde{O}_{\mu_1 \cdots \mu_s}^j(0) | p \rangle|_{\text{spin averaged}} \equiv$$

$$\Big. \Big. \Big. \langle p | O_{\mu_1 \cdots \mu_s}^j(0) | p \rangle \Big|_{\text{IRI-virt&real}} \Big|_{\text{spin averaged}} = i^n \frac{1}{m_p} p_{p\mu_1} \cdots p_{p\mu_s} \tilde{M}_j^n + \cdots$$

Still have Callan-Symanzik Eqn:

$$\left[ \left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) \delta_{ij} - \tilde{Y}_{ij}^{(n)}(g) \right] \tilde{\bar{C}}_{j,k}^{(n)} = 0$$

for new matrix  $\tilde{Y}_{ij}^{(n)}(g)$

We follow Curci, Furmanski and Petronzio(NPB175(1980)27):

For the NS operator  ${}^N O^{F,b}(y) = \frac{1}{2} i^{N-1} S \bar{\psi}(y) \gamma_{\mu_1} \nabla_{\mu_2} \cdots \nabla_{\mu_N} \lambda^b \psi(y) - \text{trace terms}$ ,

where  $\nabla_\mu = \partial_\mu + ig \tau^a A_\mu^a$ ,  $S$  denotes symmetrization

$$\langle p | {}^N O^{F,b}(y) | p \rangle = {}^{F,b} O^N(\alpha_s, \epsilon) p_{\mu_1} \cdots p_{\mu_N} - \text{trace terms}, \quad {}^{F,b} O^N(\alpha_s, \epsilon) \equiv M_{F,b}^N \Rightarrow$$

$${}^{F,b} O^N(\alpha_s, \epsilon, p^2/\mu^2) = Z_O^{-1}(\alpha_s, \frac{1}{\epsilon}) {}^{F,b} O_{\text{bare}}^N((\alpha_s)(\mu^2/p^2)^\epsilon, \epsilon)$$



Regulate collinear div. with  $p^2 \neq 0$ ,  $d=4-\epsilon \Rightarrow$

- Use  $\Delta$ , with  $\Delta^2 = 0$ ,

$${}^{F,b}O^N(\alpha_s, \epsilon) = \langle p \rangle^{-N} {}^{F,b}O_{\mu_1 \dots \mu_N}(y) \langle p \rangle \Delta^{\mu_1} \dots \Delta^{\mu_N} / (\Delta p)^N$$

Set  $\Delta = n$ ,  $x_{\mu_j} = np/np_{\mu_j}$ ,  $x = nk/np$ ,  $nA^\alpha = 0 \Rightarrow$

$${}^{F,b}O^N(\alpha_s, \epsilon) = \int_{-1}^1 dx x^{N-1} {}^{F,b}O(x, \alpha_s, \epsilon)$$

where

$${}^{F,b}O(x, \alpha_s, \epsilon) = Z_F [\delta(x-1) + x \frac{\int d^d k}{(2\pi)^d} \delta(x - \frac{kn}{pn}) \left[ \frac{n}{4kn} T(p, k) \right] p]$$

$T(p, k)$  = fully connected 4-pt fn.,  $Z_F$  = field renorm., and

[**b**] B denotes  $b_{\alpha\alpha'} B^{\alpha\alpha'}{}_{\beta\beta'}$ , etc.

Analytic continuation to  $d=4+\epsilon$ ,  $\epsilon>0$ ,  $p^2 \rightarrow 0$  gives (CFP)

$$Z_0^{-1}(\alpha_s, \frac{1}{\epsilon}) = \int_{-1}^1 dx x^{N-1} \left[ \Gamma_{qq}(x, \alpha_s, \frac{1}{\epsilon}) \theta(x) - \Gamma_{q\bar{q}}(-x, \alpha_s, \frac{1}{\epsilon}) \theta(-x) \right]$$

$\Gamma_{qq}$  ( $\Gamma_{q\bar{q}}$ )  $\Leftrightarrow$  respective parton density for a quark(anti-quark) in a quark  $\Rightarrow$  coefficients of  $\frac{1}{\epsilon}$  give

$$\begin{aligned} -\gamma^{(N)}(\alpha_s) &= 2 \int_{-1}^1 dx x^{N-1} [P_{qq}(x, \alpha_s) \theta(x) - P_{q\bar{q}}(-x, \alpha_s) \theta(-x)] \\ &= 2 [P_{qq}(N, \alpha_s) + (-1)^N P_{q\bar{q}}(N, \alpha_s)], \end{aligned}$$

$$F(N) = \int_0^1 dx x^{N-1} F(x), \text{ and}$$

$P_{BA}$   $\Leftrightarrow$  usual DGLAP-CS kernels in CFP convention,

with  $\gamma^{(N)}(\alpha_s)$   $\Leftrightarrow$  anomalous dimension of  ${}^N O^{F,b}$

## Application to new IR-improved anomalous dimension matrix:

We IR-improve each step –

$$\langle p | {}^N O^{F,b}(y) | p \rangle \Rightarrow \langle p | {}^N \tilde{O}^{F,b}(y) | p \rangle$$

$$\text{and } {}^{F,b} O^N(\alpha_s, \epsilon) \Rightarrow {}^{F,b} \tilde{O}^N(\alpha_s, \epsilon) \Rightarrow$$

$${}^{F,b} \tilde{O}^N(\alpha_s, \epsilon, p^2/\mu^2) = Z_0^{-1}(\alpha_s, \frac{1}{\epsilon}) {}^{F,b} \tilde{O}_{\text{bare}}^N((\alpha_s)(\mu^2/p^2)^\epsilon, \epsilon) \Rightarrow$$

$${}^{F,b} \tilde{O}^N(\alpha_s, \epsilon) = \int_{-1}^1 dx x^{N-1} {}^{F,b} \tilde{O}(x, \alpha_s, \epsilon) \text{ where}$$

$${}^{F,b} \tilde{O}(x, \alpha_s, \epsilon) = Z_F [\delta(x-1) + x \int \frac{d^d k}{(2\pi)^d} \delta(x - \frac{kn}{pn}) \left[ \frac{n}{4kn} \tilde{T}(p, k) \mathbf{p} \right]]$$

$\tilde{T}(p, k)$  is IR-improved  $T(p, k) \rightarrow$

## IR-improved result:

$$Z_\delta^{-1}(\alpha_s, \frac{1}{\epsilon}) = \int_{-1}^1 dx x^{N-1} [\Gamma_{q\bar{q}}^{\text{exp}}(x, \alpha_s, \frac{1}{\epsilon}) \theta(x) - \Gamma_{q\bar{q}}^{\text{exp}}(-x, \alpha_s, \frac{1}{\epsilon}) \theta(-x)]$$

where  $\Gamma_{q\bar{q}}^{\text{exp}}$ ,  $\Gamma_{q\bar{q}}^{\text{exp}}$  are the respective IR-improved parton densities  $\Rightarrow$

$$-\tilde{\gamma}^{(N)}(\alpha_s) = 2 \frac{\alpha_s}{2\pi} [P_{q\bar{q}}^{\text{exp}}(N, \alpha_s) + (-1)^N P_{q\bar{q}}^{\text{exp}}(N, \alpha_s)]$$

where  $P_{q\bar{q}}^{\text{exp}}$ ,  $P_{q\bar{q}}^{\text{exp}}$   $\Leftrightarrow$  respective IR-improved kernels  $\Rightarrow$

## IR-improved one-loop identifications

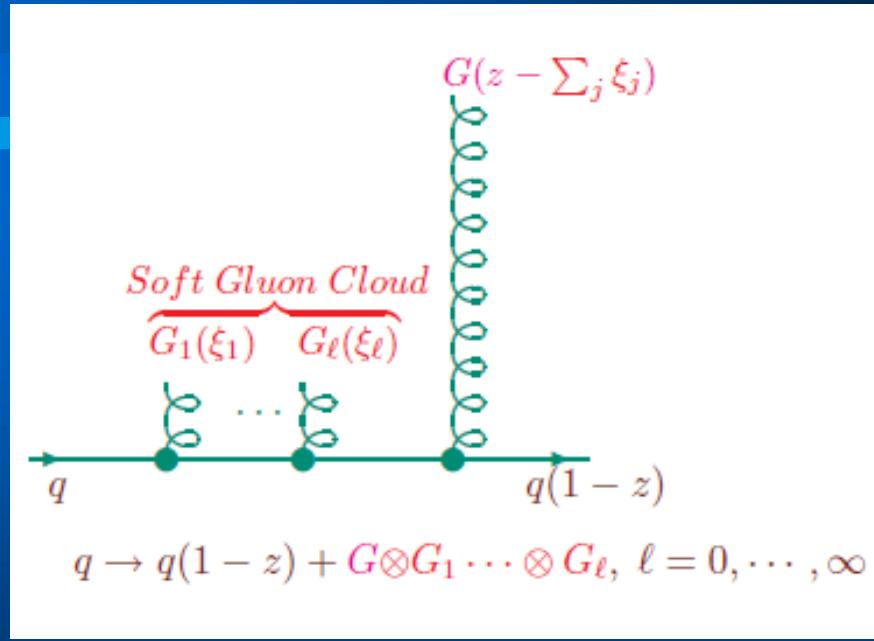
$$-\tilde{\gamma}^{(N)}(\alpha_s)_j = 2 \frac{\alpha_s}{2\pi} P_j^{\text{exp}}(N),$$

rigorous contact with new IR-improved DGLAP-CS theory

# Basic Physical Idea: Bloch-Nordsieck –

Accelerated Charge  $\Rightarrow$  Coherent State of Soft  
Gluons (Photons)

$\Rightarrow$  More Physical View of Splitting Process ( $O(\hbar^n)$ ,  $n \geq 1$ , corrections):



Basic Physical Idea:  
Bloch-Nordsiek –

More Physical View of Splitting Process in Practice:

$$P_{AB}(z) \longrightarrow P_{AB}^{\text{exp}}(z)$$

Resum terms  $O((\alpha_s \ln(q^2/\Lambda^2) \ln(1-z))^n)$   
for IR limit  $z \rightarrow 1 \Rightarrow$   
Generate Gribov-Lipatov exponents  $\gamma_A$ .



## •Example Direct Calculation

$$\begin{aligned}
 \int \frac{\alpha_s(t)}{2\pi} P_{BA} dt dz &= e^{\text{SUM}_{\text{IR}}(\text{QCD})(z)} \int \left\{ \bar{\beta}_0 \int \frac{d^4 y}{(2\pi)^4} e^{(iy \cdot (p_1 - p_2) + \int^{k < K_{\max}} \frac{d^3 k}{k} \tilde{S}_{\text{QCD}}(k) [e^{-iy \cdot k} - 1])} \right. \\
 &\quad + \int \frac{d^3 k_1}{k_1} \bar{\beta}_1(k_1) \int \frac{d^4 y}{(2\pi)^4} e^{(iy \cdot (p_1 - p_2 - k_1) + \int^{k < K_{\max}} \frac{d^3 k}{k} \tilde{S}_{\text{QCD}}(k) [e^{-iy \cdot k} - 1])} \\
 &\quad \left. + \dots \right\} \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0} \\
 &= e^{\text{SUM}_{\text{IR}}(\text{QCD})(z)} \int \left\{ \bar{\beta}_0 \int_{-\infty}^{\infty} \frac{dy}{(2\pi)} e^{(iy \cdot (E_1 - E_2) + \int^{k < K_{\max}} \frac{d^3 k}{k} \tilde{S}_{\text{QCD}}(k) [e^{-iy \cdot k} - 1])} \right. \\
 &\quad + \int \frac{d^3 k_1}{k_1} \bar{\beta}_1(k_1) \int_{-\infty}^{\infty} \frac{dy}{(2\pi)} e^{(iy \cdot (E_1 - E_2 - k_1^0) + \int^{k < K_{\max}} \frac{d^3 k}{k} \tilde{S}_{\text{QCD}}(k) [e^{-iy \cdot k} - 1])} \\
 &\quad \left. + \dots \right\} \frac{d^3 p_2}{p_2^0 q_2^0}
 \end{aligned}$$



$$\begin{aligned}
I_{YFS}(zE, 0) &= \int_{-\infty}^{\infty} \frac{dy}{2\pi} e^{[iy(zE) + \int^{k < zE} \frac{d^3 k}{k} \tilde{S}_{QCD}(k)(e^{-iyk} - 1)]} \\
&= F_{YFS}(\gamma_q) \frac{\gamma_q}{zE} \\
I_{YFS}(zE, k_1) &= \int_{-\infty}^{\infty} \frac{dy}{2\pi} e^{[iy(zE - k_1) + \int^{k < zE} \frac{d^3 k}{k} \tilde{S}_{QCD}(k)(e^{-iyk} - 1)]} \\
&= \left( \frac{zE}{zE - k_1} \right)^{1-\gamma_q} I_{YFS}(zE, 0)
\end{aligned}$$

$$\int \left( \bar{\beta}_0 \frac{\gamma_q}{zE} + \int dk_1 k_1 d\Omega_1 \bar{\beta}_1(k_1) \left( \frac{zE}{zE - k_1} \right)^{1-\gamma_q} \frac{\gamma_q}{zE} \right) \frac{d^3 p_2}{E_2 q_2^0} = \int dt \frac{\alpha_s(t)}{2\pi} P_{BA}^0 dz + \mathcal{O}(\alpha_s^2).$$

so that differentiation yields

$$P_{BA} = P_{BA}^0 z^{\gamma_q} F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q}$$



- Herwiri1.031(PRD81(2010)076008):  
w consultation from Bryan Webber, Stefano Frixione, and Mike Seymour, implementation of IR-improved kernels in Herwig 6.5 environment to get Herwiri1.031,  
MC@NLO/Herwiri1.031.

Observations:

1.  $\text{SUM}_{\text{IR}}$  is an IR effect – It contains as designed only the IR part of the LL, the rest of the LL is in D and the residuals  $\hat{\tilde{\beta}}_m$ , as we show in PRD81(2010)076008.
2. Herwiri is just as general as Herwig6.5, as they run the same set of processes



- Herwiri++, Herwiri++/Powheg, Herwiri++/MC@NLO  
-- running but still undergoing check-out
- Pythia8(Tjorborn Sjostrand, Peter Skands) , in progress
- Sherpa(JanWinter), in progress



- Important Technical Point on MC@NLO vs POWHEG  
Hardest Emission Sudakov in POWHEG uses full  $O(\alpha_s)$  emission result (we follow 0803.0883)

$$\Delta_{\hat{R}}(p_T) = \exp \left[ - \int d\Phi_R \frac{\hat{R}(\Phi_B, \Phi_R)}{B(\Phi_B)} \theta(k_T(\Phi_B, \Phi_R) - p_T) \right]$$

⇒ must synthesize this with (1) as well

- Some **initial** illustrative results follow first.



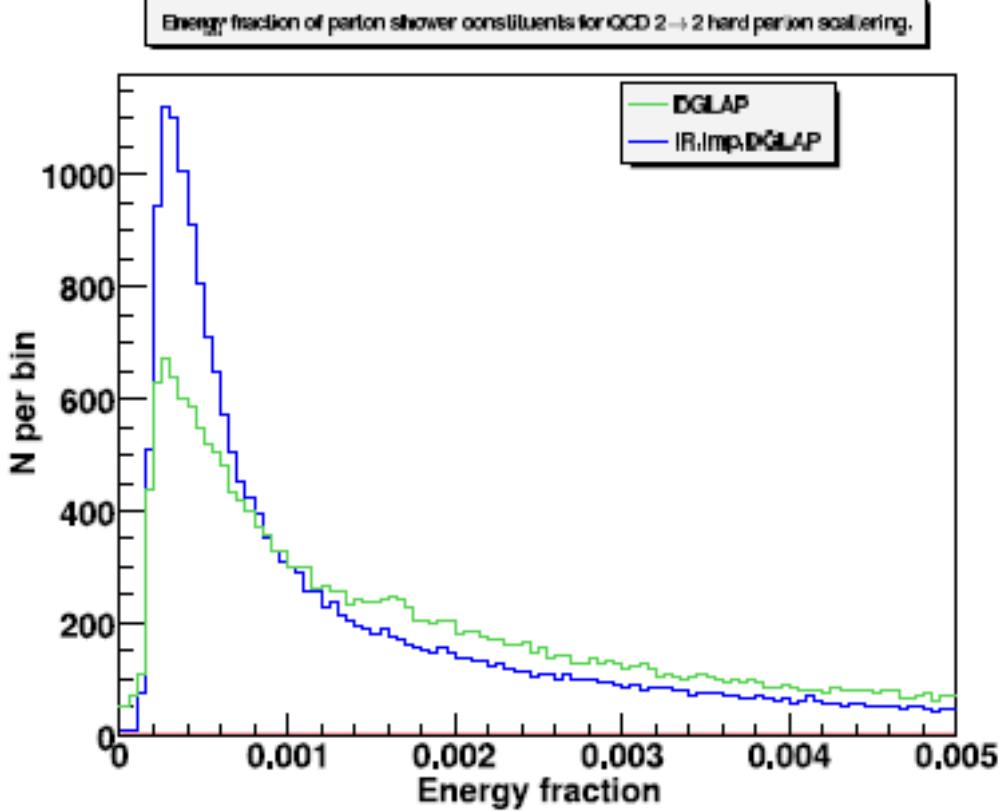


Figure 3: The  $z$ -distribution(ISR parton energy fraction) shower comparison in HER-WIG6.5.

Histogram of  $P_T^2$  for QCD parton shower in Herwig6.5 for  $2 \rightarrow 2$  hard parton scattering.

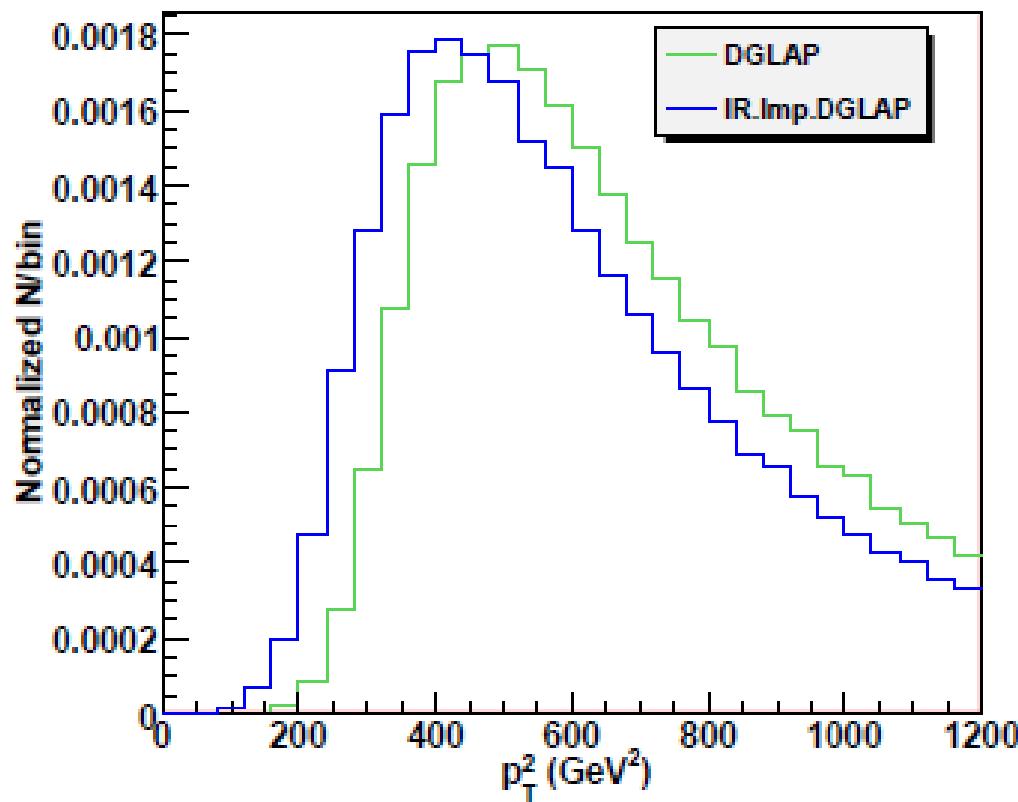


Figure 4: The  $p_T^2$ -distribution (ISR parton) shower comparison in HERWIG6.5.

Energy fraction of  $\pi^+$  In HERWIG6.5 for QCD  $2 \rightarrow 2$  hard parton scattering.

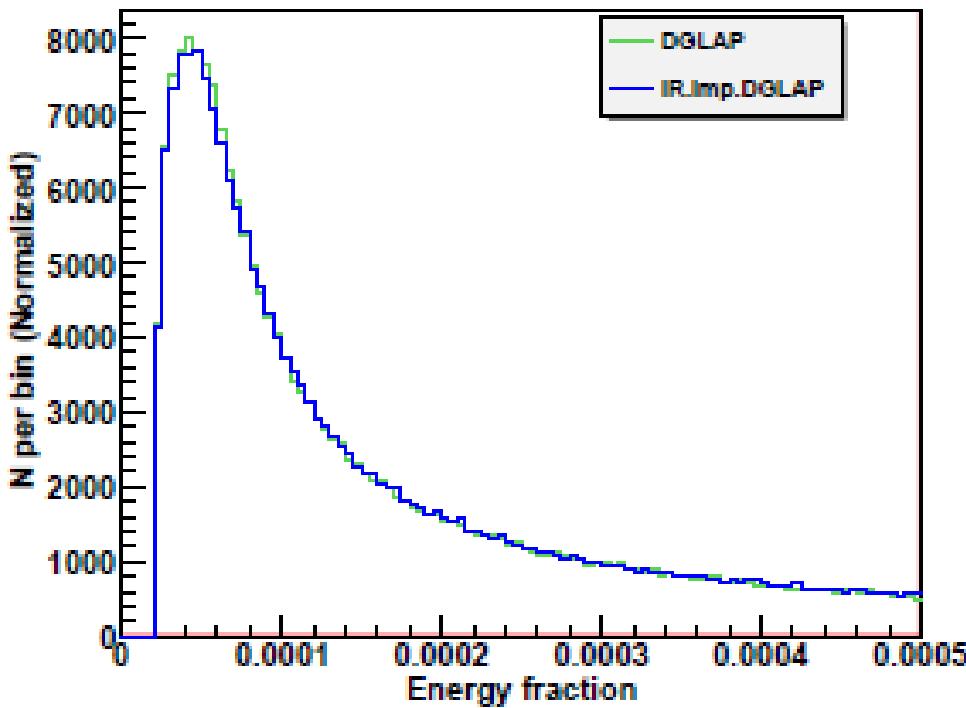


Figure 5: The  $\pi^+$  energy fraction distribution shower comparison in HERWIG6.5.

Histogram of  $P_T^2$  for  $\pi^+$  for QCD 2 $\rightarrow$ 2 hard parton scattering.

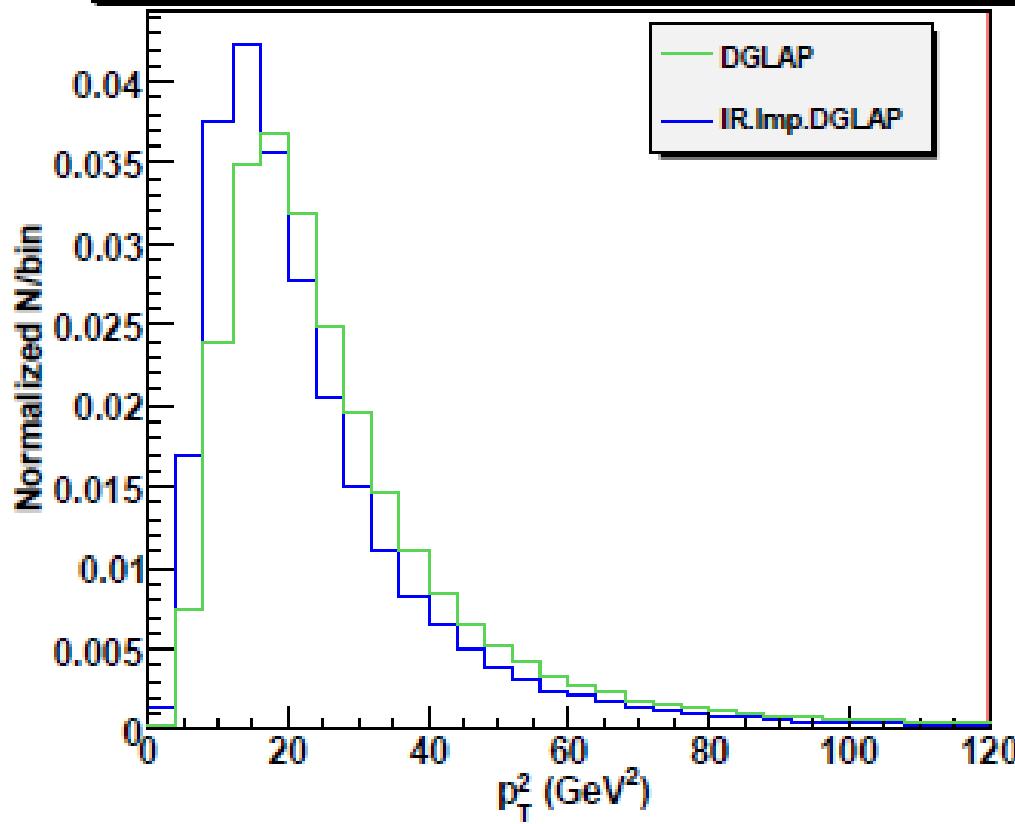


Figure 6: The  $\pi^+$   $p_T^2$ -distribution shower comparison in HERWIG6.5.

Energy fraction distribution of parton shower for single Z production.

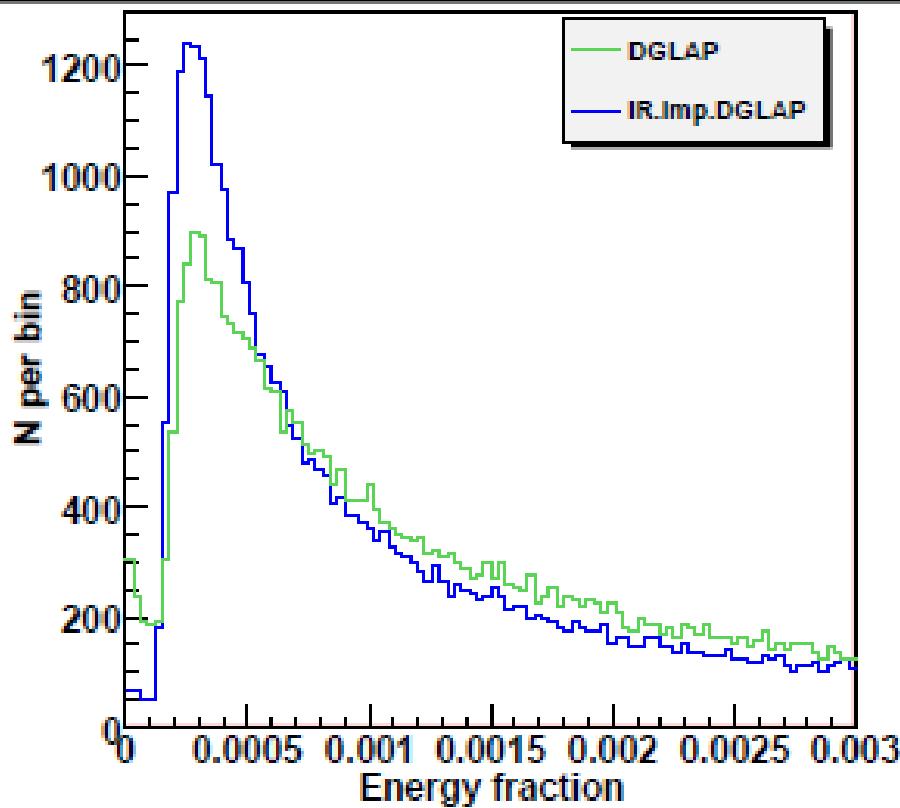


Figure 7: The  $z$ -distribution(ISR parton energy fraction) shower comparison in HER-WIC6.5.

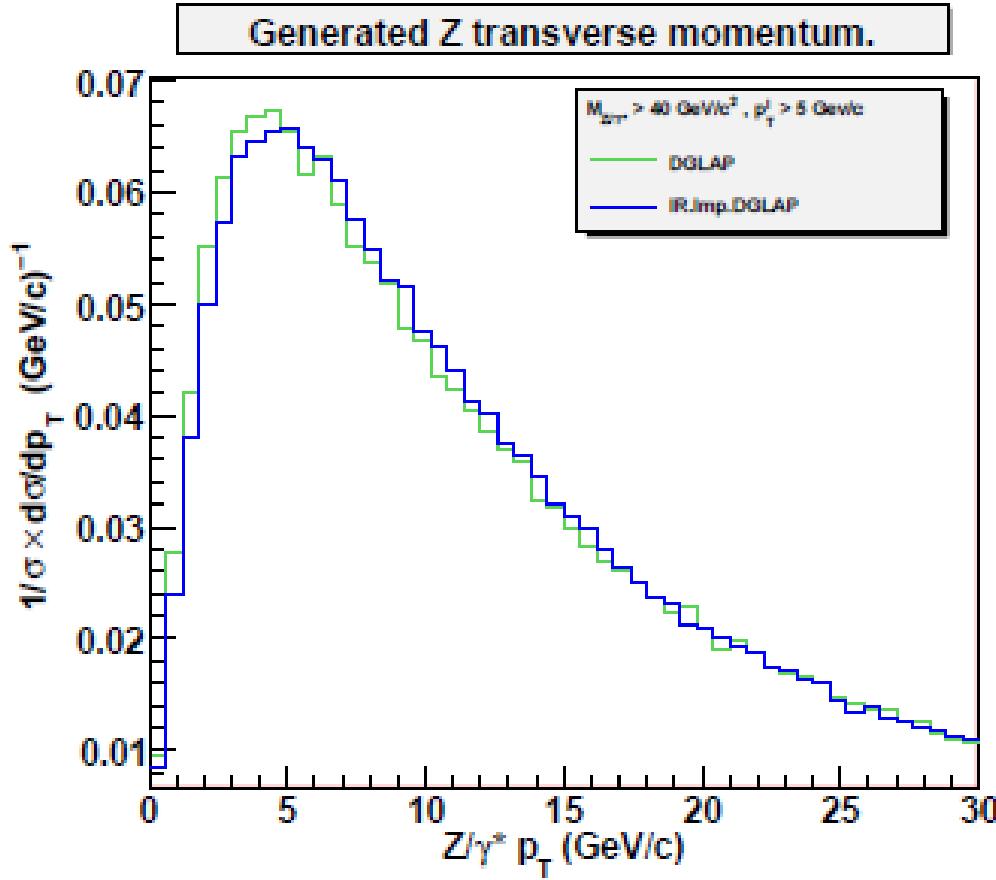


Figure 8: The  $Z$   $p_T$ -distribution(ISR parton shower effect) comparison in HERWIG6.5.



The unit normalized differential cross section for Z production as a function of vector boson rapidity.

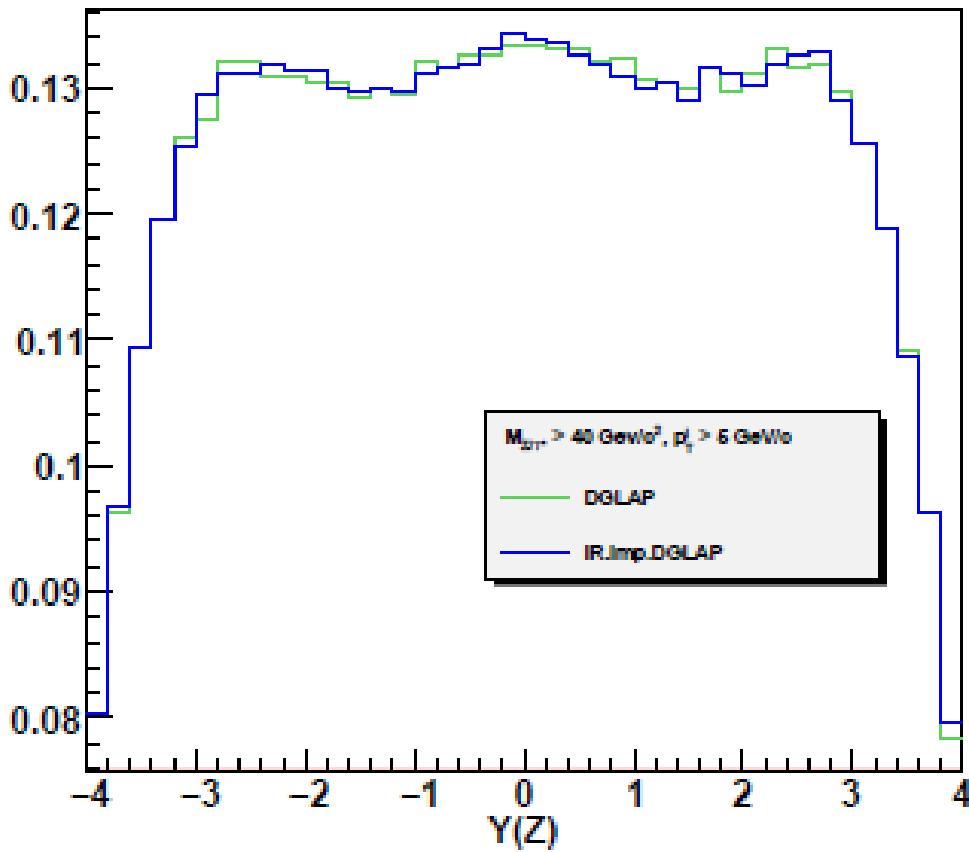
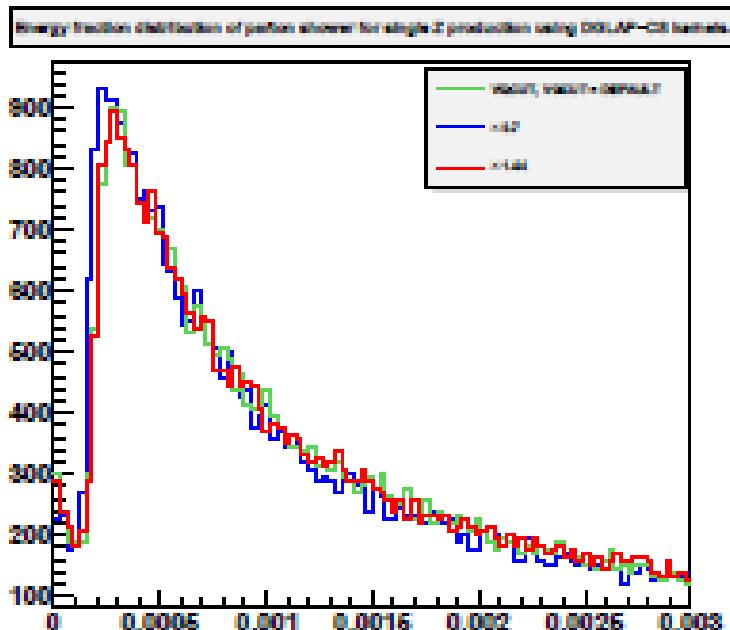


Figure 9: The  $Z$  rapidity-distribution(ISR parton shower) comparison in HERWIG6.5.

(a)



(b)

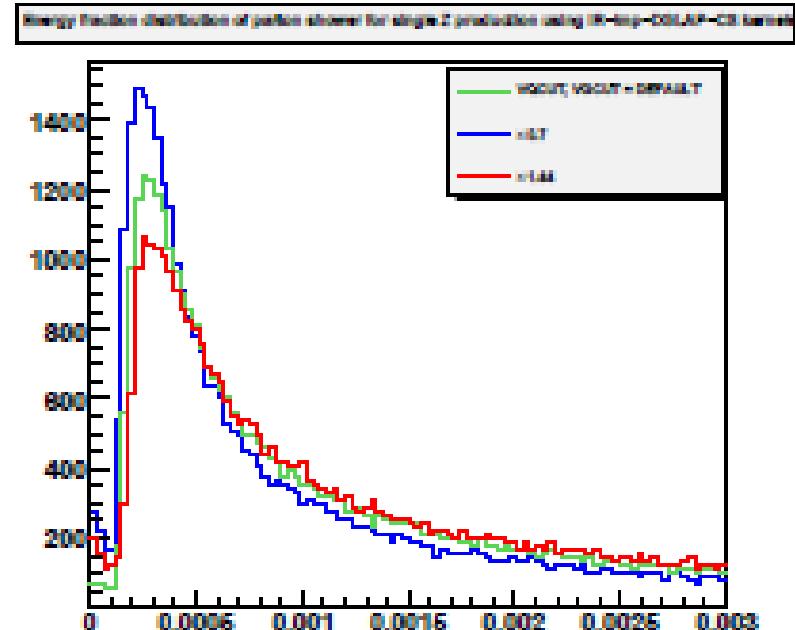


Figure 10: IR-cut-off sensitivity in  $z$ -distributions of the ISR parton energy fraction: (a), DGLAP-CS (b), IR-I DGLAP-CS – for the single  $Z$  hard sub-process in HERWIG-6.5 environment.

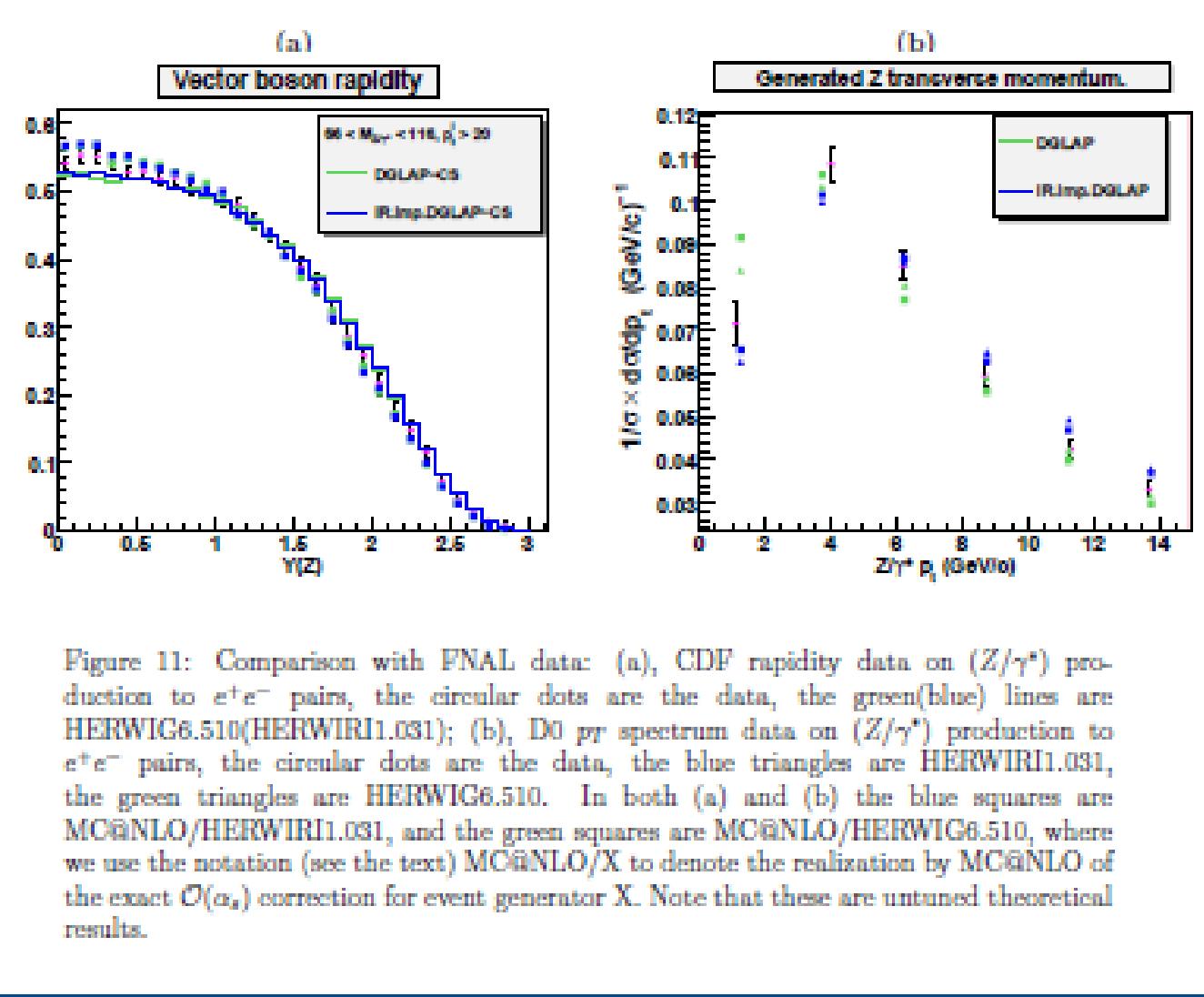


Figure 11: Comparison with FNAL data: (a), CDF rapidity data on  $(Z/\gamma^*)$  production to  $e^+e^-$  pairs, the circular dots are the data, the green(blue) lines are HERWIG6.510(HERWIRI1.031); (b), D0  $p_T$  spectrum data on  $(Z/\gamma^*)$  production to  $e^+e^-$  pairs, the circular dots are the data, the blue triangles are HERWIRI1.031, the green triangles are HERWIG6.510. In both (a) and (b) the blue squares are MC@NLO/HERWIRI1.031, and the green squares are MC@NLO/HERWIG6.510, where we use the notation (see the text) MC@NLO/X to denote the realization by MC@NLO of the exact  $\mathcal{O}(\alpha_s)$  correction for event generator X. Note that these are untuned theoretical results.

- HERWIRI1.031 -- PRD81 (2010) 076008:

- For the CDF rapidity data, HERWIRI1.031 is closer to the data than is HERWIG6.510 (1.54 vs 1.77 for  $\chi^2/\text{d.o.f.}$  resp.);  
for MC@NLO/HERWIRI1.031 and MC@NLO/HERWIG6.510  
the  $\chi^2/\text{d.o.f}$  are 1.42 and 1.40 resp., both are within 10% of the data  
⇒ Need NNLO level, in progress.
- For the D0  $p_T$  data, HERWIRI1.031 gives a better fit  
to the data compared to HERWIG6.5 for low  $p_T$ ,  
for  $p_T < 12.5\text{GeV}$ , the  $\chi^2/\text{d.o.f.}$  are  $\sim 2.5$  and 3.3 respectively  
- we add the statistical and systematic errors,  
showing that the IR-improvement makes a better representation  
of QCD in the soft regime for a given fixed order in perturbation theory.

- LHC DATA: CMS Rapidity & ATLAS PT Spectrum for  $Z/\gamma^*$  Production

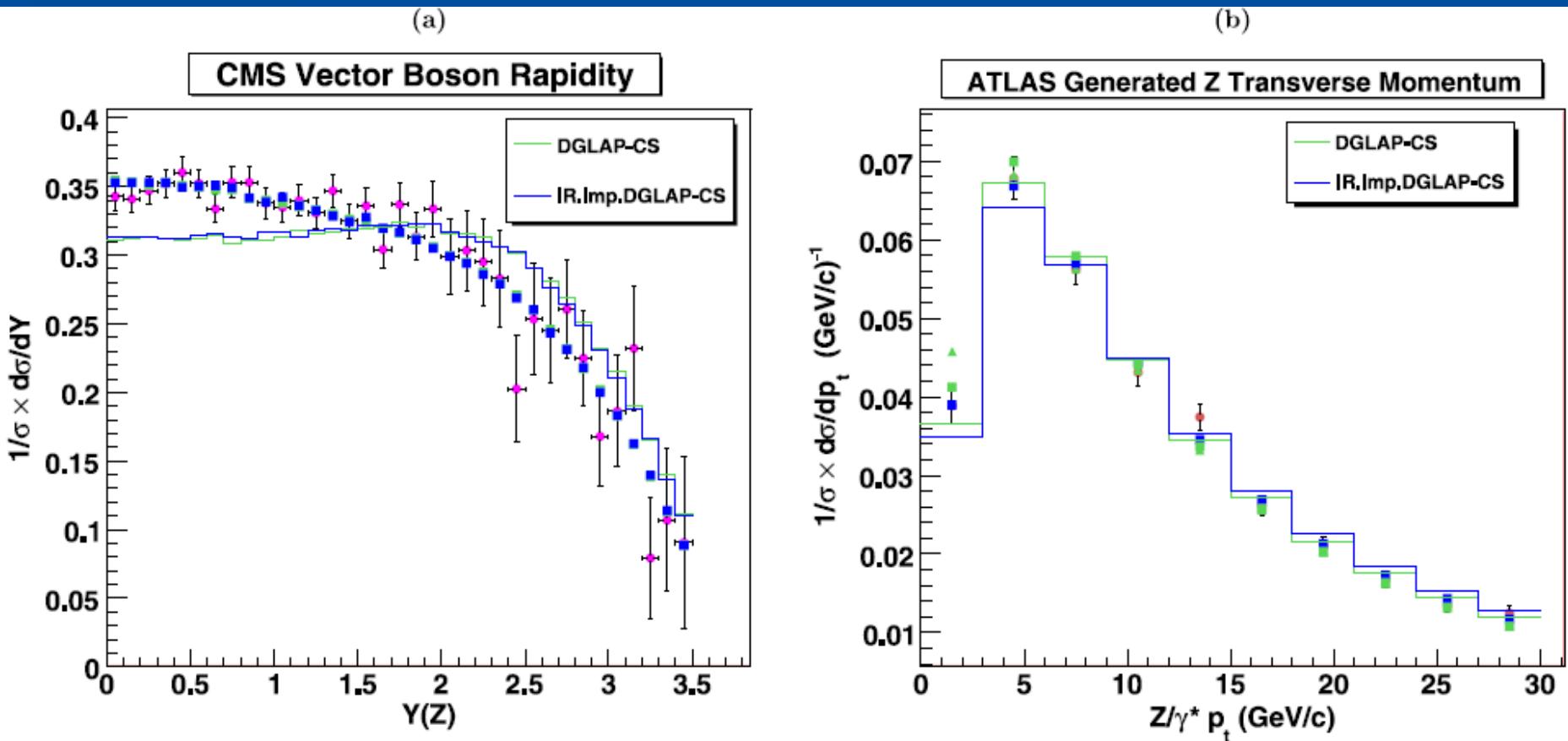


Fig. 2. Comparison with LHC data: (a) CMS rapidity data on  $(Z/\gamma^*)$  production to  $e^+e^-$ ,  $\mu^+\mu^-$  pairs, the circular dots are the data, the green (blue) lines are HERWIG6.510 (HERWIRI1.031); (b) ATLAS  $p_T$  spectrum data on  $(Z/\gamma^*)$  production to (bare)  $e^+e^-$  pairs, the circular dots are the data, the blue (green) lines are HERWIRI1.031 (HERWIG6.510). In both (a) and (b) the blue (green) squares are MC@NLO/HERWIG6.510 (HERWIG6.510 (PTRMS = 2.2 GeV)). In (b), the green triangles are MC@NLO/HERWIG6.510 (PTRMS = 0). These are otherwise untuned theoretical results. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

- Observations

1. For the unimproved case, the data suggest that we need a GAUSSIAN (intrinsic)  
 $\text{PTRMS} \cong 2.2 \text{ GeV}$

[ Herwiri1.031(blue line), Herwig6510(green line(PTRMS=2.2GeV)), MC@NLO/Herwiri1.031(blue squares), MC@NLO/Herwig6510(green squares (PTRMS=2.2GeV), green triangles(PTRMS=0))]  
( similar to what holds at FNAL)

2. This same shape is provided from fundamental principles by the MC@NLO/Herwiri1.031 with  
 $\text{PTRMS} \cong 0 \text{ GeV}$  ( similar to what holds at FNAL)

- Observations (Quantitative)
  1. Unimproved case, the respective  $\chi^2 / \text{d.o.f.}$ 's are 1.37, 0.70 (MC@NLO/Herwig6510(PTRMS=2.2GeV)) for the  $p_T$  and rapidity data
  2. IR-improved case, the respective  $\chi^2 / \text{d.o.f.}$ 's are 0.72, 0.72 (MC@NLO/Herwiri1.031) for the  $p_T$  and rapidity data
  3. Unimproved case, the respective  $\chi^2 / \text{d.o.f.}$ 's are 2.23, 0.70 (MC@NLO/Herwig6510(PTRMS=0)) for the  $p_T$  and rapidity data

- Which is Better for Precision QCD Theory?
  1. Precocious Bjorken Scaling in SLAC-MIT Experiments: already at  $Q^2 \cong 1_+ \text{ GeV}^2$   
⇒ PTRMS<sup>2</sup> small compared to  $1_+ \text{ GeV}^2$

See R.P. Feynman, M. Kislinger and F. Ravndal, Phys. Rev. D**3** (1971) 2706; R. Lipes, *ibid.* **5** (1972) 2849; F.K. Diakonas, N.K. Kaplis and X.N. Mawita, *ibid.* **78** (2008) 054023; K. Johnson, Proc. Scottish Summer School Phys. **17** (1976) p. 245; A. Chodos *et al.*, Phys. Rev. D**9** (1974) 3471; *ibid.* **10** (1974) 2599; T. DeGrand *et al.*, *ibid.* **12** (1975) 2060; .... – all have PTRMS<sup>2</sup> <<  $1_+ \text{ GeV}^2$

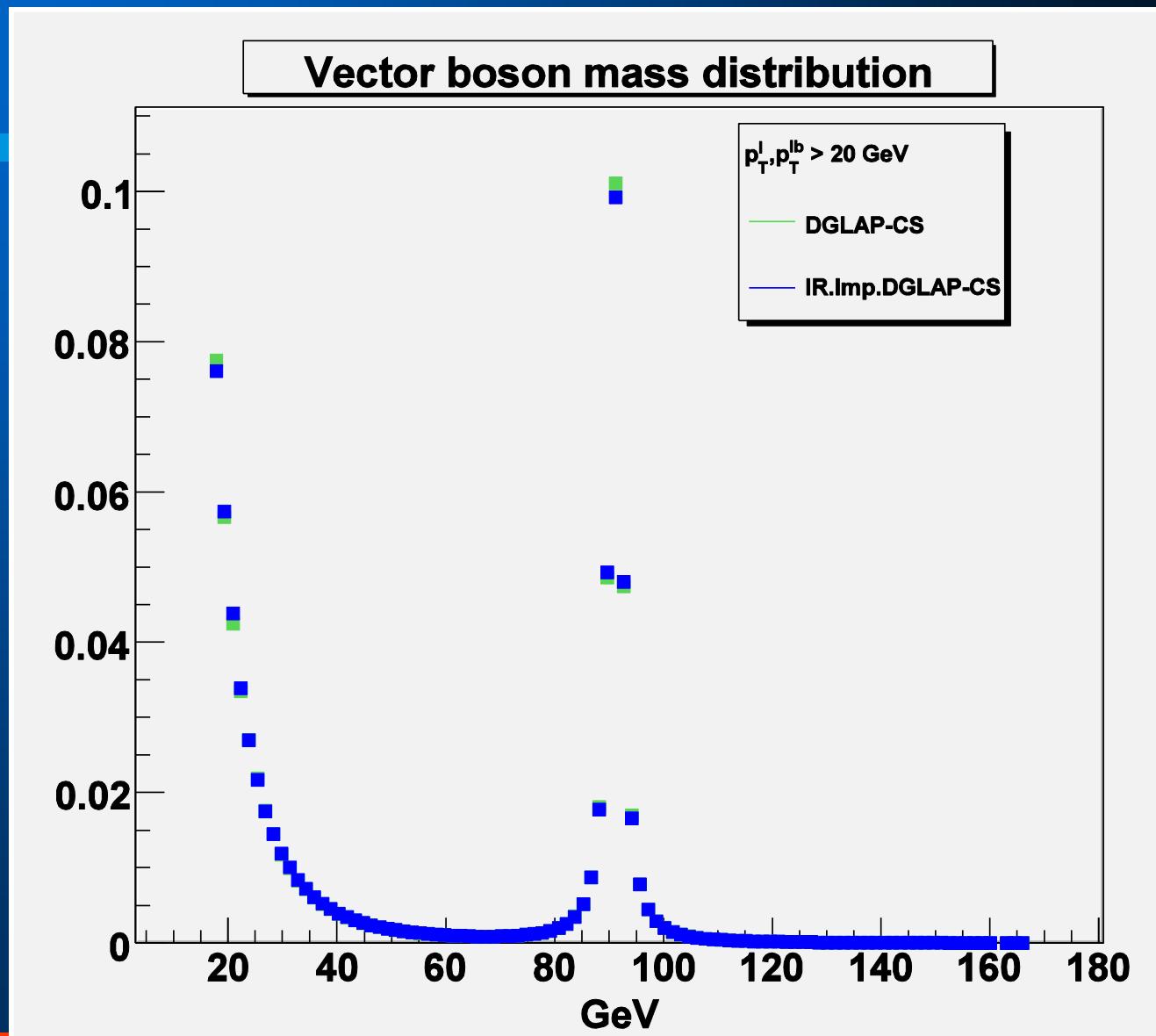


- Which is Better for Precision QCD Theory?
  2. The first principles approach is not subject to arbitrary functional variation to determine its  $\Delta\sigma_{\text{th}}$
  3. Experimentally, **in principle**, events in the two cases should look different in terms of the properties of the rest of the particles in the events – **this is under study.**

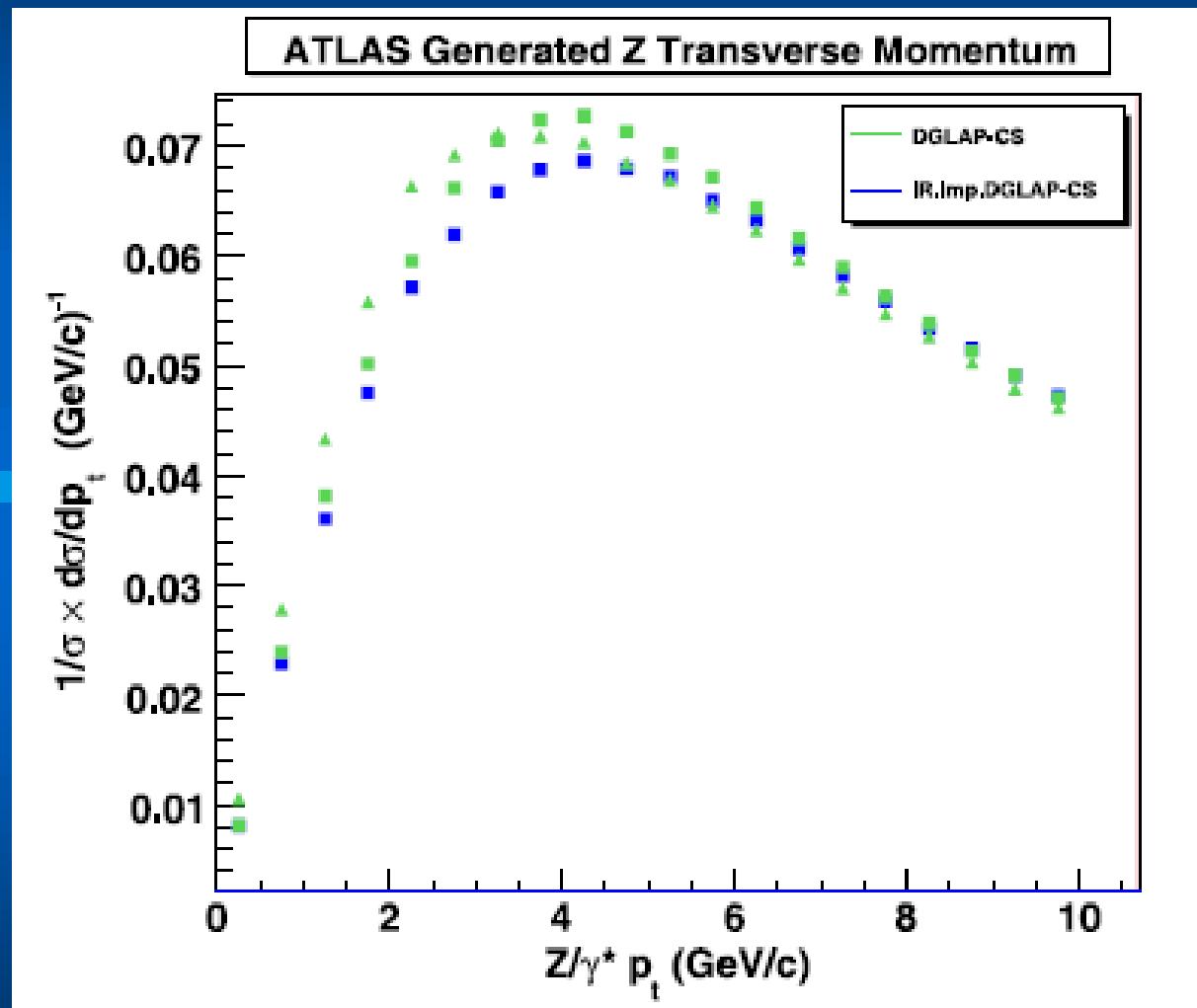
Here, we show the following.



# For example: 2.2% Peak Effect



# THEORY COMPARISON: FINER BINS -- 0.5GeV vs 3.0GeV

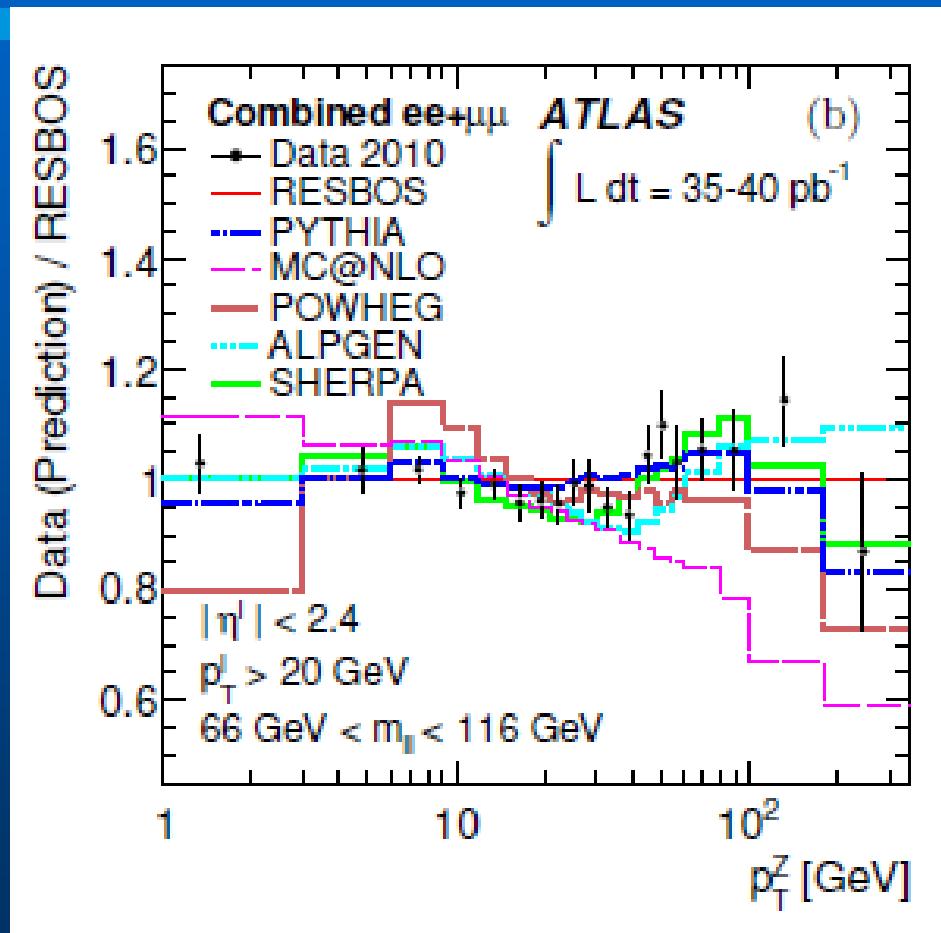


# OBSERVATIONS

- \* IR-Improved DGLAP-CS Theory Increases Definiteness of Precision Determination of NLO Parton Shower MC's and Improves Such.
- \* More Potential Checks Against Experiment Are Being Pursued.

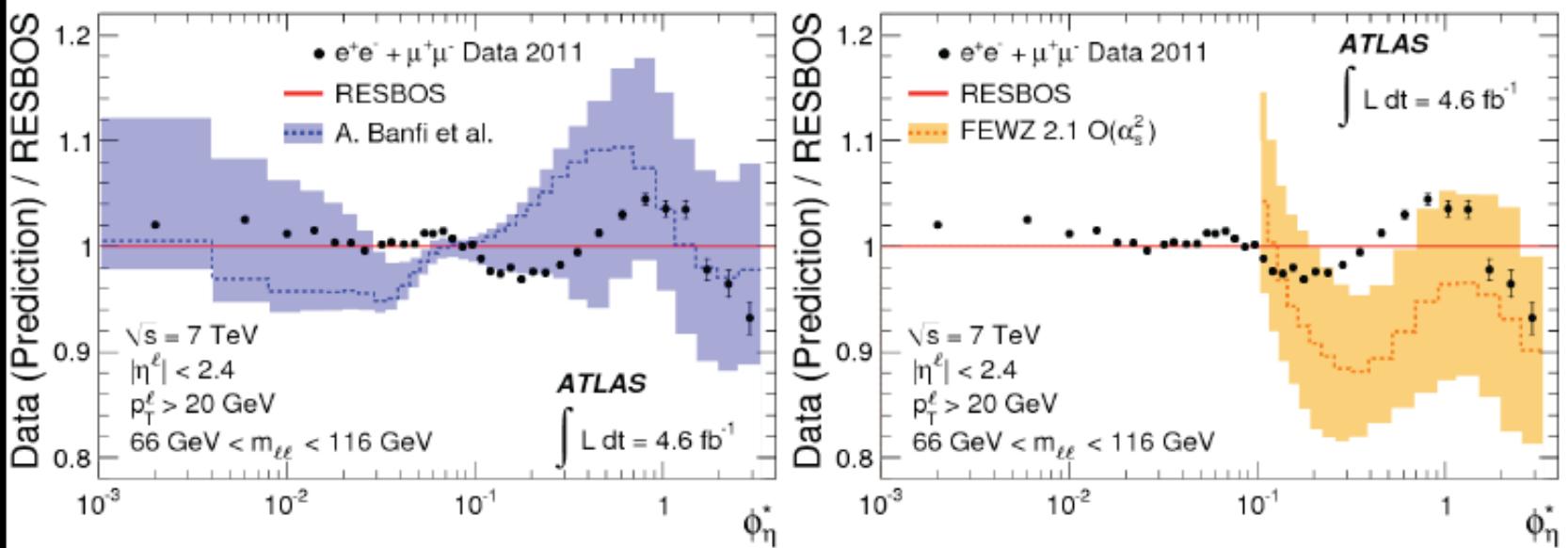


- MORE THEORY COMPARISONS:ATLAS(1107.2381)



- MORE THEORY COMPARISONS:Hassani(EW Moriond, 2013) – {  $\phi_\eta^* = \tan(\frac{1}{2}(\pi - \Delta\phi)) \sin\theta^*$  }

## Z/ $\gamma^*$ transverse momentum ( $d\sigma/d\phi_{\eta}^*(||)$ )



- ↗ Calculations from A. Banfi et al. (resummed QCD predictions+fixed-order pQCD) is less good than ResBos
- ↗ Measurement precision about one order of magnitude lower than the present theoretical uncertainties
- ↗ FEWZ predictions undershoot the data by ~10% which confirm previous CDF observation (PRD 86,052010)

## Near Future

- \* Herwig++(soon, running , under cross checks)
- \* Pyhtia 8,6 (w consultation from Peter Skands and Torbjorn Sjostrand)
- \* Sherpa (w consultation from Jan Winter)



# Near Future

\*New Observables:  $\phi_\eta^*$  (w p<sub>T</sub> cuts, etc.)  
\*New Data: ATLAS & CMS,  
EACH > 10<sup>7</sup> lepton pairs

⇒ COMPLETE INTRINSIC P<sub>T</sub> TESTS

\*HERWIRI2.0 (w S.Yost, M. Hjena, V. Halyo)

HERWIG6.5 ∪ KK MC 4.22

KK MC 4.22 (w S. Jadach, Z.Was), 1307.4037

... \*MC@NNLO



# KK MC 4.22

$v_{\max}$	<i>KKsem Ref.</i>	$\mathcal{O}(\alpha^3)_{\text{EEEX3}}$	$\mathcal{O}(\alpha^3)_{\text{CEEX intOFF}}$	$\mathcal{O}(\alpha^3)_{\text{CEEX}}$
$\sigma(v_{\max}) \text{ [pb]}$				
0.01	$0.9145 \pm 0.0000$	$0.9150 \pm 0.0004$	$0.9150 \pm 0.0004$	$0.9323 \pm 0.0004$
0.10	$1.0805 \pm 0.0000$	$1.0807 \pm 0.0004$	$1.0808 \pm 0.0004$	$1.0920 \pm 0.0004$
0.30	$1.1612 \pm 0.0000$	$1.1615 \pm 0.0004$	$1.1616 \pm 0.0004$	$1.1691 \pm 0.0004$
0.50	$1.1974 \pm 0.0000$	$1.1977 \pm 0.0004$	$1.1981 \pm 0.0004$	$1.2036 \pm 0.0004$
0.70	$1.2310 \pm 0.0000$	$1.2312 \pm 0.0004$	$1.2317 \pm 0.0004$	$1.2357 \pm 0.0004$
0.90	$1.6104 \pm 0.0000$	$1.6128 \pm 0.0003$	$1.6114 \pm 0.0004$	$1.6148 \pm 0.0004$
0.99	$1.6218 \pm 0.0000$	$1.6254 \pm 0.0003$	$1.6244 \pm 0.0004$	$1.6277 \pm 0.0004$
$A_{\text{FB}}(v_{\max})$				
0.01	$0.5883 \pm 0.0000$	$0.5883 \pm 0.0005$	$0.5883 \pm 0.0005$	$0.6033 \pm 0.0005$
0.10	$0.5882 \pm 0.0000$	$0.5881 \pm 0.0004$	$0.5881 \pm 0.0004$	$0.5966 \pm 0.0004$
0.30	$0.5879 \pm 0.0000$	$0.5879 \pm 0.0004$	$0.5879 \pm 0.0004$	$0.5932 \pm 0.0004$
0.50	$0.5875 \pm 0.0000$	$0.5874 \pm 0.0004$	$0.5875 \pm 0.0004$	$0.5912 \pm 0.0004$
0.70	$0.5848 \pm 0.0000$	$0.5845 \pm 0.0004$	$0.5846 \pm 0.0004$	$0.5868 \pm 0.0004$
0.90	$0.4736 \pm 0.0000$	$0.4722 \pm 0.0003$	$0.4728 \pm 0.0003$	$0.4748 \pm 0.0003$
0.99	$0.4710 \pm 0.0000$	$0.4691 \pm 0.0003$	$0.4697 \pm 0.0003$	$0.4716 \pm 0.0003$

TABLE II. Study of total cross section  $\sigma(v_{\max})$  and charge asymmetry  $A_{\text{FB}}(v_{\max})$ ,  $d\bar{d} \rightarrow \mu^- \mu^+$ , at  $\sqrt{s}=189\text{GeV}$ . See Table I for definition of the energy cut  $v_{\max}$ , scattering angle and M.E. type,



v=1- s'/s (EW lib. = DIZET6.21)

# Resummed Quantum Gravity

- Recent Progress: Cosmological Constant  $\Lambda$

In Phys.Dark Univ. 2(2013)97, using

$$B_g''(k) = \frac{\kappa^2 |k^2|}{8\pi^2} \ln \left( \frac{m^2}{m^2 + |k^2|} \right)$$

we show that we get the UV limit  
 $k^2 G_N(k) \rightarrow .0442$



and the scalar contribution to  $\Lambda$  as

$$\begin{aligned}\Lambda_s &= -8\pi G_N \frac{\int d^4k}{2(2\pi)^4} \frac{(2k_0^2)e^{-\lambda_c(k^2/(2m^2))\ln(k^2/m^2+1)}}{k^2 + m^2} \\ &\cong -8\pi G_N \left[ \frac{1}{G_N^2 64\rho^2} \right], \quad \rho = \ln \frac{2}{\lambda_c}\end{aligned}$$

for  $\lambda_c = \frac{2m^2}{M_{Pl}^2}$ .

A Dirac fermion gives -4 times  $\Lambda_s$ .  
 $\Rightarrow$  UV limit

$$\begin{aligned}\Lambda(k) &\xrightarrow{k^2 \rightarrow \infty} k^2 \lambda_* \\ \lambda_* &= -\frac{c_{2eff}}{2880} \sum_j (-1)^{F_j} n_j / \rho_j^2 \\ &\cong 0.0817\end{aligned}$$



Comparison with EFRG(Reuter et al., Percacci et al,  
Litim ,....):  
Illustration(Laucsher&Reuter(PRD65(2002)025013))--

## UV Fixed Point:

$$\begin{aligned}\beta_\lambda(\lambda_k, g_k; \alpha, d) = & -2\lambda_k + \nu_d d g_k + \left[ 2d(d-1+2\alpha)(4\pi)^{1-\frac{d}{2}} \Phi_{d/2}^2(0) - (d-2)\omega_d \right] \lambda_k g_k \\ & + \frac{1}{2} d(d+1)(d-2)(4\pi)^{1-\frac{d}{2}} \omega_d \Phi_{d/2}^1(0) g_k^2 + \mathcal{O}(g^3) ,\end{aligned}$$

$$\beta_g(\lambda_k, g_k; \alpha, d) = (d-2) g_k - (d-2)\omega_d g_k^2 + \mathcal{O}(g^3)$$



For  $d=4$ , cut-off profile

$$R^{(0)}(y) = y/(e^y - 1),$$

$$g_* \cong \pi/(13\pi^2/144 + 55/24 + \alpha)$$

$$\lambda_* \cong 3\zeta(3)/(13\pi^2/144 + 19/24)$$

Evidently, for appropriate  $\alpha$  and  $R^{(0)}(y)$  we can have qualitative agreement with our pure gravity results

$$g_* \cong 0.0533$$

$$\lambda_* \cong -0.000189$$



# An Estimate of $\Lambda$ : Planck Scale Cosmology -- (Bonanno&Reuter(J.Phys.Conf.Series140(2008)012008)) Transition between Planck regime and classical FRW regime at

$$t_{\text{tr}} \simeq 25 t_{\text{Pl}}$$

⇒

$$\begin{aligned}\rho_{\Lambda}(t_{\text{tr}}) &\equiv \frac{\Lambda(t_{\text{tr}})}{8\pi G_N(t_{\text{tr}})} \\ &= \frac{-M_{\text{Pl}}^4(k_{\text{tr}})}{64} \sum_j \frac{(-1)^F n_j}{\rho_j^2}\end{aligned}$$



For  $t_{\text{eq}}$  = time of radiation matter equality

we get (see Branchina&Zappala (G.R.Grav.42(2010)141))

$$\begin{aligned}\rho_{\Lambda}(t_0) &\cong \frac{-M_{Pl}^4(1 + c_{2,\text{eff}}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64} \sum_j \frac{(-1)^F n_j}{\rho_j^2} \\ &\quad \times \frac{t_{tr}^2}{t_{eq}^2} \times \left(\frac{t_{eq}}{t_0^{2/3}}\right)^3 \\ &\cong \frac{-M_{Pl}^2(1.0362)^2(-9.197 \times 10^{-3})}{64} \frac{(25)^2}{t_0^2} \\ &\cong (2.400 \times 10^{-3} \text{eV})^4.\end{aligned}$$

Compare:



$$\rho_{\Lambda}(t_0)|_{\text{expt}} \cong (2.368 \times 10^{-3} \text{eV}(1 \pm 0.023))^4.$$

# CONSISTENCY CHECKS

- \* What About EW, QCD, GUT Symmetry Breaking Scales?

Consider GUT symmetry breaking:

It gives a  $M_{\text{GUT}}^4 / (.01 M_{\text{Pl}}^4 / 64) < 10^{-6}$

correction, which we drop here.

The other breaking scales are even smaller and hence their corrections are even less significant in our result for

$\rho_\Lambda$ .



# CONSISTENCY CHECKS

## \* What About BBN Constraint?

B-R BDY CONDITION:  $H(t_{tr-})=H(t_{tr+})$

$\Rightarrow$  Gauge Transformation Between Planck Scale Regime and usual FRW Regime

B-R:  $t \rightarrow t' = t - t_{as} \Rightarrow \alpha/t_{tr} = 1/(2(t_{tr} - t_{as}))$

$\Rightarrow t_{as} = (1 - 1/(2\alpha))t_{tr}$ , with  $t_{tr} = \alpha/M_{Pl}$  ,  
 $\alpha = 25$ .

RQG:  $t \rightarrow t' = \gamma t$ , as part of a dilatation  $\Rightarrow$   
 $\alpha/t_{tr} = 1/(2\gamma t_{tr}) \Rightarrow \gamma = 1/(2\alpha)$

$\Rightarrow$

$$\Omega_A(t_{BBN}) = \frac{M_{Pl}^2 (1.0362)^2 9.194 \times 10^{-3} (25)^2 / (64 t_{BBN}^2)}{(3/(8\pi G_N)) (1/(2\gamma t_{BBN})^2)}$$
$$\cong \frac{\pi 10^{-2}}{24}$$
$$= 1.31 \times 10^{-3}.$$



# CONSISTENCY CHECKS

## \* What About SUSY GUTS?

Note

$$\langle 0 | \mathcal{H} | 0 \rangle \sim \int^{M_{Pl}} \frac{d^3 k}{(2\pi)^3} \frac{1}{2} \omega(k) = \int^{M_{Pl}} \frac{d^3 k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m_i^2}$$

Raises the question of GUTS: Use SO(10)  
SUSY GUT Approach of Dev & Mohapatra  
(PRD82(2010)035014):

Intermediate Stage:

$SU_{2L} \times SU_{2R} \times U_1 \times SU(3)^c$

SM Stage at  $\sim 2\text{TeV} = M_R$ :

$SU_{2L} \times U_1 \times SU(3)^c$

SUSY Breaking at EW scale  $M_S$ :  
 $U_1 \times SU(3)^c$

# CONSISTENCY CHECKS

## \* What About SUSY GUTS?

- Possible spectrum

$$m_{\tilde{g}} \cong 1.5(10) \text{TeV}$$

$$m_{\tilde{G}} \cong 1.5 \text{TeV}$$

$$m_{\tilde{q}} \cong 1.0 \text{TeV}$$

$$m_{\tilde{\ell}} \cong 0.5 \text{TeV}$$

$$m_{\tilde{\chi}_i^0} \cong \begin{cases} 0.4 \text{TeV}, & i = 1 \\ 0.5 \text{TeV}, & i = 2, 3, 4 \end{cases}$$

$$m_{\tilde{\chi}_i^\pm} \cong 0.5 \text{TeV}, \quad i = 1, 2$$

$$m_S = .5 \text{TeV}, \quad S = A^0, \quad H^\pm, \quad H_2,$$



$$\Delta_{\text{GUT}} = \sum_{j \in \{\text{MSSM low energy susy partners}\}} \frac{(-1)^F n_j}{\rho_j^2}$$
$$\cong 1.13(1.12) \times 10^{-2}$$

# CONSISTENCY CHECKS

## \* What About SUSY GUTS?



- Compensate by either (A) adding new susy families with scalars lighter than fermions or (B) allowing the gravitino mass to go to  $\sim .05 M_{\text{GUT}} \sim 2 \times 10^{15} \text{ GeV}$ .
- For approach (A),  
new quarks and leptons at  
 $M_{\text{High}} \sim 3.4(3.3) \times 10^3 \text{ TeV}$ ,  
scalar partners at  $\sim .5 \text{ TeV} = M_{\text{Low}}$

# CONCLUSIONS

- \* Herwiril.031 Just as General Herwig6.5, No Tweaking, Should Be Better in IR Due to Bloch-Nordsiek Effect – Await New Data
- \* Real Progress on  $\Lambda$  in QFT  
(Resummed Quantum Gravity Realization of Feynman's Approach Einstein-Hilbert Theory)

