JET BROADENING IN EFFECTIVE FIELD THEORY

[GUIDO BELL]

based on: T. Becher, GB, M. Neubert, Phys. Lett. B 704 (2011) 276
T. Becher, GB, Phys. Lett. B 713 (2012) 41
T. Becher, GB, JHEP 1211 (2012) 126
T. Becher, GB, work in progress

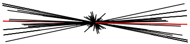
UNIVERSITY OF OXFORD

THE ROYAL SOCIETY

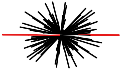
Event shapes

Thrust:

$$T = \frac{1}{Q} \max_{\vec{n}} \left(\sum_i |\vec{p}_i \cdot \vec{n}| \right)$$

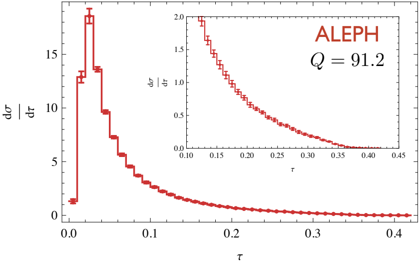


two-jet like: $T \simeq 1$



spherical: $T \simeq 1/2$

High-precision data from LEP, SLD, JADE, ... $(\tau = 1 - T)$



- α_s determination
- testing ground for precision QCD techniques

Event shapes

Other common e^+e^- event shapes:

- heavy jet mass

$$\rho_H = \frac{\max\left(M_L^2, M_R^2\right)}{Q^2}$$

hemisphere jet masses $M_{L/R}^2 = \Big(\sum_{i \in L/R} p_i\Big)^2$

- total and wide jet broadenings

$$b_T = b_L + b_R$$
$$b_W = \max\left(b_L, b_R\right)$$

hemisphere broadenings $b_{L/R} = \frac{1}{2} \sum_{i \in L/R} |\vec{p}_i \times \vec{n}_T|$

- C-parameter

$$C = \frac{3}{2Q^2} \sum_{i,j} |\vec{p}_i|\,|\vec{p}_j|\,\sin^2\theta_{ij}$$

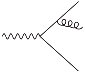
no reference to thrust axis

Precision analysis

Need to control QCD in different regimes:

- fixed-order perturbation theory

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = \frac{\alpha_s}{2\pi}\,A(e) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(e) + \left(\frac{\alpha_s}{2\pi}\right)^3 C(e) + \dots$$



known to NNLO [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 07; Weinzierl 08]

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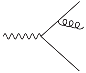

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- resummation of Sudakov logarithms

$$A(e) \overset{e\ll 1}{\simeq} a_2 \frac{\ln e}{e} + a_1 \frac{1}{e} + a_0 + \dots \quad \Rightarrow \quad \int_0^e de \, \frac{1}{\sigma_0}\frac{d\sigma}{de} \simeq c(\alpha_s)\, e^{L\, g_1(\alpha_s L) + g_2(\alpha_s L)}$$

traditionally NLL using coherent branching algorithm [Catani, Trentadue, Turnock, Webber 93]

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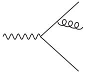

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- non-perturbative (hadronisation) effects

MC estimates + analytic studies based on effective coupling model [Dokshitzer, Webber 97]

$$\frac{d\sigma}{de}(e) = \frac{d\sigma_{\rm pert}}{de}\left(e - c_e\,\frac{\mathcal{A}}{Q}\right) \qquad \mathcal{A}: \text{universal non-perturbative parameter}$$

Soft-collinear effective theory

Advantages of an EFT approach:

- all-order factorisation theorems $d\sigma \simeq H \cdot J \otimes J \otimes S$
- extension to higher log resummation straight-forward
- field-theoretical definiton of non-perturbative parameters

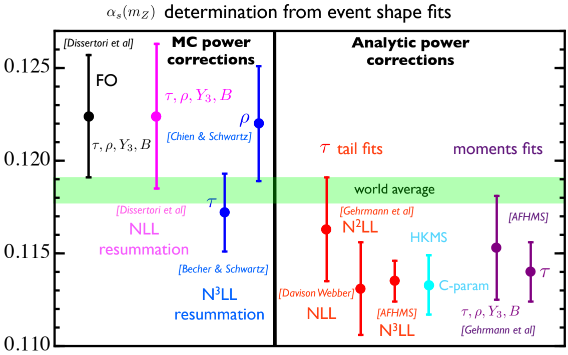
SCET-based event shape studies:

- N^3LL resummation for thrust and heavy jet mass [Becher, Schwartz 08; Chien, Schwartz 10]
- first all-order factorisation for broadenings [Becher, GB, Neubert 11; Chiu, Jain, Neill, Rothstein 11]

  NLLL resummation for broadenings [Becher, GB 12]
- studies of non-perturbative effects [Lee, Sterman 06; Mateu, Stewart, Thaler 12]

also: NNLL resummation for thrust in traditional approach [Gehrmann, Monni, Luisoni 11]

α_s determinations



[compilation from A. Hoang, QCD workshop, Singapore, March 2013]

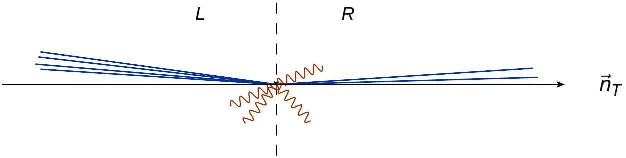
- higher log resummations reduce uncertainties
- lower values from fits based on analytic power corrections
- tension between most precise determinations and world average

[AFHMS: Abbate, Fickinger, Hoang, Mateu, Stewart 10,12]

Factorisation

In the two-jet limit $b_L \sim b_R \to 0$ the broadening distribution factorises

$$\frac{1}{\sigma_0}\,\frac{d^2\sigma}{db_L\,db_R} = H(Q^2,\mu)\int db_L^s \int db_R^s \int d^{d-2}p_L^\perp \int d^{d-2}p_R^\perp \; \mathcal{J}_L(b_L - b_L^s, p_L^\perp, \mu)\; \mathcal{J}_R(b_R - b_R^s, p_R^\perp, \mu)\; \mathcal{S}(b_L^s, b_R^s, -p_L^\perp, -p_R^\perp, \mu)$$



- jet recoils against soft radiation [Dokshitzer, Lucenti, Marchesini, Salam 98]
- relevant scales: $Q^2 \gg b_L^2 \sim b_R^2 \sim (p_L^\perp)^2 \sim (p_R^\perp)^2$

  how can we resum Sudakov logarithms in a two-scale problem?

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Convenient to work in Laplace-Fourier space

- Laplace transform $b_{L,R} \to \tau_{L,R}$
- Fourier transform $p_{L,R}^\perp \to x_{L,R}^\perp$ and define $z_{L,R} = \frac{2|x_{L,R}^\perp|}{\tau_{L,R}}$

$$\frac{1}{\sigma_0}\,\frac{d^2\sigma}{d\tau_L\,d\tau_R} = H(Q^2,\mu)\int dz_L \int dz_R \;\overline{\mathcal{J}}_L(\tau_L, z_L,\mu)\;\overline{\mathcal{J}}_R(\tau_R, z_R,\mu)\;\overline{\mathcal{S}}(\tau_L,\tau_R,z_L,z_R,\mu)$$

$H(Q^2,\mu)$ = square of on-shell vector form factor

Jet function

The broadening jet function reads

$$\mathcal{J}(b, p^\perp) \sim \sum_X \delta(\bar{n} \cdot p_X - Q)\; \delta^{d-2}(p_X^\perp - p^\perp)\; \delta\Big(b - \tfrac{1}{2} \sum_{i \in X} |p_i^\perp|\Big) \; \Big|\Big\langle X \Big| \bar{\psi}(0) W(0) \tfrac{\bar{\not n} \not n}{4} \Big| 0 \Big\rangle\Big|^2$$

- delta-functions ensure that jet has given energy, $p^\perp$ and b
- tree level: $\mathcal{J}(b, p^\perp) = \delta\Big(b - \frac{1}{2} |p^\perp|\Big)$

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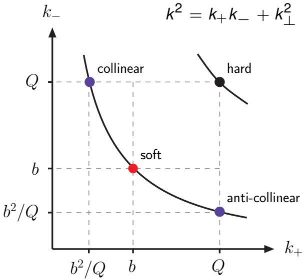
At one-loop the calculation involves

- Wilson-line diagrams are not well-defined in dimensional regularisation!

$$\int_0^Q \frac{dk_-}{k_-}$$ diverges in the soft limit (DR regularises $d^{d-2}k_\perp$)

- same problem in soft function calculation

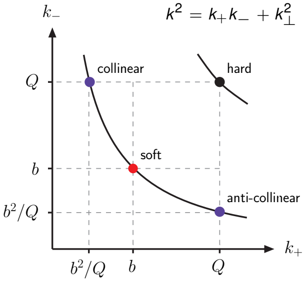
Regularisation in SCET



⇒ cannot distinguish soft from collinear mode if radiated into jet direction

⇒ need an additional regulator that distinguishes modes by their rapidities

Regularisation in SCET



⇒ cannot distinguish soft from collinear mode if radiated into jet direction

⇒ need an additional regulator that distinguishes modes by their rapidities

The regulator can be implemented on the level of phase-space integrals [Becher, GB 11]

$$\int d^d k \;\delta(k^2)\,\theta(k^0) \quad \Rightarrow \quad \int d^d k \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\,\theta(k^0)$$

- regularises ill-defined diagrams + respects the symmetries of the EFT
- analytic, minimal, optimal

Low-scale matching

Let us now put the jet and soft functions together

$$\overline{\mathcal{J}}_L(\tau_L, z_L) = \overline{\mathcal{J}}_L^{(0)}(\tau_L, z_L)$$

$$\Bigg\{1 + \frac{C_F \alpha_s}{\pi} \Bigg[\Bigg(-\frac{1}{\alpha} - \ln\left(Q\nu_+ \bar{\tau}_L^2\right) \Bigg)\Bigg(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_L^2\right) + 2\ln\frac{\sqrt{1+z_L^2}+1}{4}\Bigg)$$

$$+ \Bigg(\Bigg)\Bigg(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_R^2\right) + 2\ln\frac{\sqrt{1+z_R^2}+1}{4}\Bigg) + \dots\Bigg]\Bigg\}$$

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$$\Bigg\{ 1 + \frac{C_F \alpha_s}{\pi} \Bigg[\Bigg(-\frac{1}{\alpha} - \ln\left(Q\nu_+ \bar{\tau}_L^2\right) \Bigg) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_L^2\right) + 2 \ln \frac{\sqrt{1+z_L^2}+1}{4} \right)$$

$$+ \left(+\frac{1}{\alpha} + \ln\left(\frac{\nu_+}{Q}\right) \right) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_R^2\right) + 2 \ln \frac{\sqrt{1+z_R^2}+1}{4} \right) + \dots \Bigg] \Bigg\}$$

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$$\overline{\mathcal{J}}_L(\tau_L, z_L)\ \overline{\mathcal{J}}_R(\tau_R, z_R)\ \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) \ = \ \overline{\mathcal{J}}_L^{(0)}(\tau_L, z_L)\ \overline{\mathcal{J}}_R^{(0)}(\tau_R, z_R)$$

$$\left\{1 + \frac{C_F \alpha_s}{\pi} \left[\left(-\frac{1}{\alpha} - \ln\left(Q\nu_+ \bar{\tau}_L^2\right) + \frac{1}{\alpha} + \ln\left(\nu_+ \bar{\tau}_L\right)\right) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_L^2\right) + 2\ln \frac{\sqrt{1+z_L^2}+1}{4}\right)\right.\right.$$

$$\left.\left. + \left(+\frac{1}{\alpha} + \ln\left(\frac{\nu_+}{Q}\right) - \frac{1}{\alpha} - \ln\left(\nu_+ \bar{\tau}_R\right)\right) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_R^2\right) + 2\ln \frac{\sqrt{1+z_R^2}+1}{4}\right) + \dots\right]\right\}$$

- additional regulator and associated scale ν_+ drop out

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$$\Bigg\{1 + \frac{C_F\alpha_s}{\pi}\Bigg[\Bigg(\quad - \ln\left(Q\bar{\tau}_L\right) \quad \Bigg)\left(\frac{1}{\epsilon} + \ln\left(\mu^2\bar{\tau}_L^2\right) + 2\ln\frac{\sqrt{1+z_L^2}+1}{4}\right)$$

$$+ \Bigg(\quad - \ln\left(Q\bar{\tau}_R\right) \quad \Bigg)\left(\frac{1}{\epsilon} + \ln\left(\mu^2\bar{\tau}_R^2\right) + 2\ln\frac{\sqrt{1+z_R^2}+1}{4}\right) + \dots\Bigg]\Bigg\}$$

- additional regulator and associated scale ν_+ drop out
- generates a large logarithm in a matching calculation

Can show that the rapidity logarithms exponentiate

- collinear anomaly [Becher, Neubert 10]
- rapidity renormalisation group [Chiu, Jain, Neill, Rothstein 11,12]

NLL resummation

First all-order factorisation formula for broadening distributions [Becher, GB, Neubert 11]

$$\frac{1}{\sigma_0}\,\frac{d^2\sigma}{d\tau_L\,d\tau_R} = H(Q^2,\mu)\int_0^\infty dz_L\int_0^\infty dz_R\;(Q^2\bar{\tau}_L^2)^{-F_B(\tau_L,z_L,\mu)}\;(Q^2\bar{\tau}_R^2)^{-F_B(\tau_R,z_R,\mu)}\;W(\tau_L,\tau_R,z_L,z_R,\mu)$$

At NLL the formula reproduces earlier results [Dokshitzer, Lucenti, Marchesini, Salam 98]

$$\frac{1}{\sigma_0}\,\frac{d\sigma}{db_T} = H(Q^2,\mu)\,\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\,\frac{1}{b_T}\left(\frac{b_T}{\mu}\right)^{2\eta} I^2(\eta)$$

$$\frac{1}{\sigma_0}\,\frac{d\sigma}{db_W} = H(Q^2,\mu)\,\frac{2\eta\,e^{-2\gamma_E\eta}}{\Gamma^2(1+\eta)}\,\frac{1}{b_W}\left(\frac{b_W}{\mu}\right)^{2\eta} I^2(\eta)$$

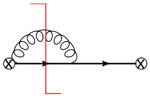
$$\eta = \frac{C_F\alpha_s(\mu)}{\pi}\,\ln\frac{Q^2}{\mu^2}$$

The extension to NNLL requires two new ingredients

- one-loop remainder function W
- two-loop anomaly coefficient F_B

One-loop jet function

The calculation of the one-loop jet function is surprisingly complicated



$$\sim \int d^d q \;\delta(q^2)\;\theta(q^0) \int d^d k \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\;\theta(k^0)\;\; \frac{\bar{n}q\;(\bar{n}k+\bar{n}q)}{\bar{n}k\;(q+k)^2}$$

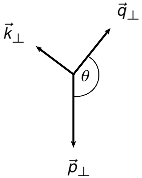
$$\times\;\delta\Big(Q-\bar{n}q-\bar{n}k\Big)\;\;\delta^{d-2}\Big(p_\perp-q_\perp-k_\perp\Big)\;\;\delta\Big(b-\tfrac{1}{2}|q_\perp|-\tfrac{1}{2}|k_\perp|\Big)$$

$$\sim \int_0^1 d\eta\,\eta\,(1-\eta)^{-1+\alpha}\int_{1-y}^{1+y} d\xi\;\frac{\xi(2-\xi)^{1-2\alpha}(\xi(2-\xi)-1+y^2)^{-\frac{1}{2}-\varepsilon}}{(\xi-2y\eta)^2+4\eta(1-y)(1+y-\xi)}$$

- non-trivial angle complicates calculation
- expansion in α and ϵ is subtle

  $\Rightarrow$ have to keep $(2b-p)^{-1-\epsilon}, (2b-p)^{-1-2\epsilon}, \ldots$ to all orders
- computed the integrals in closed form

  $\Rightarrow$ hypergeometric functions of half-integer parameters
- performed Laplace + Fourier transformations analytically



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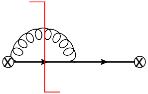


$$\sim \int d^d q \; \delta(q^2)\, \theta(q^0) \int d^d k \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\, \theta(k^0) \; \frac{\bar{n}q\,(\bar{n}k + \bar{n}q)}{\bar{n}k\,(q+k)^2}$$

$$\times\; \delta\Big(Q - \bar{n}q - \bar{n}k\Big)\; \delta^{d-2}\Big(p_\perp - q_\perp - k_\perp\Big)\; \delta\Big(b - \tfrac{1}{2}|q_\perp| - \tfrac{1}{2}|k_\perp|\Big)$$

$$\sim \int_0^1 d\eta\, \eta\, (1-\eta)^{-1+\alpha} \int_{1-y}^{1+y} d\xi \; \frac{\xi(2-\xi)^{1-2\alpha}(\xi(2-\xi) - 1 + y^2)^{-\frac{1}{2}-\varepsilon}}{(\xi - 2y\eta)^2 + 4\eta(1-y)(1+y-\xi)}$$

$$\overline{\mathcal{J}}_L^{(1b)}(\tau, z) = \overline{\mathcal{J}}_L^{(0)}(\tau, z)\, \frac{\alpha_s C_F}{4\pi}\, (\mu^2 \bar{\tau}^2)^{\varepsilon}\, (\nu_+ Q \bar{\tau}^2)^{\alpha}$$

$$\times \Bigg\{ -\frac{2}{\alpha}\left[\frac{1}{\varepsilon} + 2\ln\left(\frac{1+\sqrt{1+z^2}}{4}\right)\right] + \frac{2}{\varepsilon^2} + \frac{2}{\varepsilon} - 8\mathrm{Li}_2\left(-\frac{\sqrt{1+z^2}-1}{\sqrt{1+z^2}+1}\right)$$

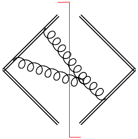
$$+ 8\mathrm{Li}_2(-\sqrt{1+z^2}) - 4\ln^2\left(\frac{1+\sqrt{1+z^2}}{4}\right) + \ln^2(1+z^2) + 2z^2\ln(1+z^2)$$

$$+ 4(1-z^2)\ln(1+\sqrt{1+z^2}) + 4\sqrt{1+z^2} - 8\ln 2 - \frac{\pi^2}{6} \Bigg\}$$

Two-loop anomaly coefficient

Most easily extracted from two-loop soft function

- again two particles in final state $\Rightarrow$ similar integrals
- requires to go one order higher in ϵ-expansion
- encounter harmonic polylogs and elliptic integrals



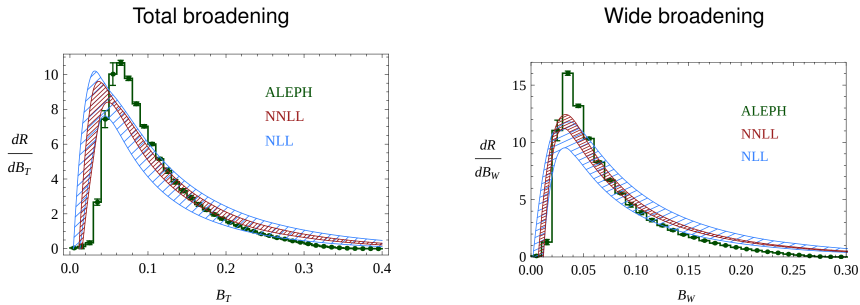
$$d_2^B(z) = C_A \Big\{ -\frac{1+z^2}{9}\, h_1(z) + \frac{67+2z^2}{9}\, h_2(z) - 8\, h_3(z) + 32\, S_{1,2}\Big(-\frac{z_-}{z_+}\Big) - 8\, \mathrm{Li}_3\Big(-\frac{z_-}{z_+}\Big)$$
$$+ 8\, S_{1,2}(-w) - 24\, \mathrm{Li}_3(-w) - 24\, S_{1,2}(1-w) + 8\, \mathrm{Li}_3(1-w) + \dots \Big\}$$
$$+ T_F n_f \Big\{ \frac{2(1+z^2)}{9}\, h_1(z) - \frac{2(13+2z^2)}{9}\, h_2(z) - \frac{4}{3}\ln^2 z_+ - \frac{20}{9}\ln z_+ + \frac{4}{9}z^2 - \frac{82}{27}$$
$$+ \frac{4w(5-z^2)}{9}\ln\Big(\frac{1+w}{w}\Big) + \frac{2w(11+2z^2)}{9} \Big\}$$

$$w = \sqrt{1+z^2}$$
$$z_\pm = (w \pm 1)/4$$

with $\quad h_1(z) = \int_0^1 dt\, \frac{\arcsin t}{\sqrt{1-t^2}}\, \frac{1}{\sqrt{1+t^2 z^2}} = \frac{\pi}{2}\, F(\tfrac{\pi}{2}, -z^2) - \int_0^{\frac{\pi}{2}} d\theta\, F(\theta, -z^2) \quad \dots$

A glimpse at the data

[Becher, GB 12]



- excellent convergence of perturbative predictions
- scale uncertainty significantly reduced in fit region for α_s extraction
- pure resummation result (not yet matched to fixed-order and no hadronisation effects)

Power corrections

Dominant non-perturbative effect is a shift

$$\frac{d\sigma}{de}(e) = \frac{d\sigma_{\rm pert}}{de}\left(e - c_e \frac{\mathcal{A}}{Q}\right)$$

driven by a universal parameter $\mathcal{A}$ that can be fitted to experimental data

The observable-dependent coefficients c_e can be calculated, e. g.

$$c_\tau = 2\,, \qquad c_\rho = 1\,, \qquad c_C = 3\pi$$

Effective coupling model predicts that the broadening distributions get distorted

[Dokshitzer, Marchesini, Salam 98]

$$c_{B_T} = \ln \frac{1}{B_T} + \ldots, \qquad c_{B_W} = \frac{1}{2} \ln \frac{1}{B_W} + \ldots$$

Is this a model-independent statement?

Conclusions

Anomalous factorisation theorems for p_T-dependent observables

- resummation beyond standard RG techniques via collinear anomaly
- jet broadening, p_T resummation, jet veto resummation, ...

Analytic phase-space regularisation in SCET

$$\int d^d k \ \delta(k^2)\, \theta(k^0) \quad \Rightarrow \quad \int d^d k \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\, \theta(k^0)$$

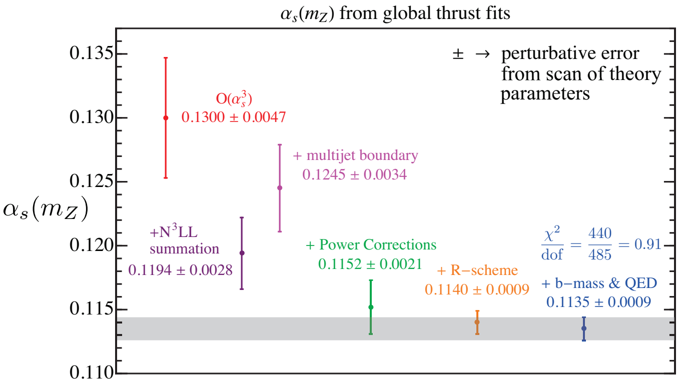
- respects symmetries of EFT and well-suited for efficient calculations

NNLL resummation for jet broadening distributions available

- allows for precision determination of α_s

Backup slides

Precision thrust analysis



distribution: $\alpha_s(M_Z) = 0.1135 \pm 0.0002\,(\text{exp}) \pm 0.0005\,(\text{had}) \pm 0.0009\,(\text{pert})$ [Abbate et al 10]

moment: $\alpha_s(M_Z) = 0.1140 \pm 0.0004\,(\text{exp}) \pm 0.0013\,(\text{had}) \pm 0.0007\,(\text{pert})$ [Abbate et al 12]

NNLO + NNLL: $\alpha_s(M_Z) = 0.1131\,^{+0.0028}_{-0.0022}$ [Monni, Gehrmann, Luisoni 12]

Analytic regularisation in SCET

Our new prescription amounts to

$$\int d^d k \;\delta(k^2)\,\theta(k^0) \quad\Rightarrow\quad \int d^d k \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\,\theta(k^0)$$

- virtual corrections do not need regularisation

  matrix elements of Wilson lines in QCD $\Rightarrow$ the same for thrust and broadening

  technical reason: $\int d^{d-2}k_\perp \; f(k_\perp, k_+) \sim k_+^{-\epsilon}$

- required for observables sensitive to transverse momenta

  $f(k_\perp, k_+) \sim \delta^{d-2}(k_\perp - p_\perp)$ $\Rightarrow$ factor $k_+^{-\epsilon}$ absent $\Rightarrow$ reinstalled as $k_+^{-\alpha}$

  can show that the prescription regularises all LC singularities in SCET [Becher, GB 11]

- not sufficient for cases where virtual corrections are ill-defined

  examples: electroweak Sudakov corrections, Regge limits

Collinear anomaly

Can show that the Q dependence exponentiates using and extending arguments from

- electroweak Sudakov resummation [Chiu, Golf, Kelley, Manohar 07]
- p_T resummation in Drell-Yan production [Becher, Neubert 10]

Start from the logarithm of the product of jet and soft functions

$$\ln P = \ln \overline{\mathcal{J}}_L\Big(\ln\,(Q\nu_+\bar\tau_L^2);\ \tau_L, z_L\Big) + \ln \overline{\mathcal{J}}_R\Big(\ln\,(\tfrac{\nu_+}{Q});\ \tau_R, z_R\Big) + \ln \overline{\mathcal{S}}\Big(\ln\,(\nu_+\bar\tau_L);\ \tau_L, \tau_R, z_L, z_R\Big)$$

collinear: $k_+ \sim \frac{b^2}{Q}$ anticollinear: $k_+ \sim Q$ soft: $k_+ \sim b$

- use that product does not depend on ν_+ and that it is LR symmetric

$$\Rightarrow\ \ln P = \frac{k_2(\mu)}{4}\ln^2\left(Q^2\,\bar\tau_L\bar\tau_R\right) - F_B(\tau_L, z_L, \mu)\ \ln\left(Q^2\bar\tau_L^2\right) - F_B(\tau_R, z_R, \mu)\ \ln\left(Q^2\bar\tau_R^2\right) + \ln W(\tau_L, \tau_R, z_L, z_R, \mu)$$

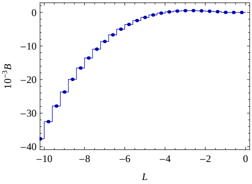
- RG invariance implies $k_2(\mu) = 0$ to all orders

$$\Rightarrow\ \boxed{P(Q^2, \tau_L, \tau_R, z_L, z_R, \mu) = (Q^2\bar\tau_L^2)^{-F_B(\tau_L, z_L, \mu)}\ (Q^2\bar\tau_R^2)^{-F_B(\tau_R, z_R, \mu)}\ W(\tau_L, \tau_R, z_L, z_R, \mu)}$$

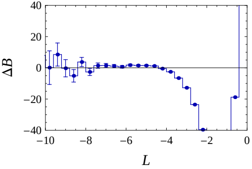
Comparison with fixed order

Confront with output of fixed-order MC generators (EVENT2, EERAD3)

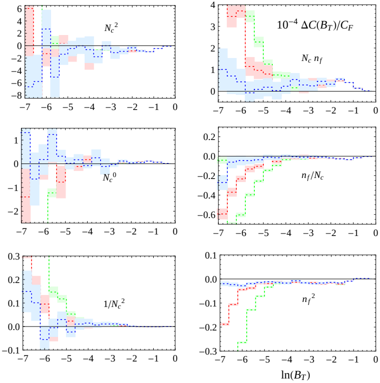
NLO



NLO—NNLL



NNLO—NNLL



$\Rightarrow$ we obtain the right logarithmic terms for small values of $L = \ln B_T$