

GAUGE BOSON PAIRS at the LHC: ELECTROWEAK CORRECTIONS and MONTE CARLO IMPLEMENTATION

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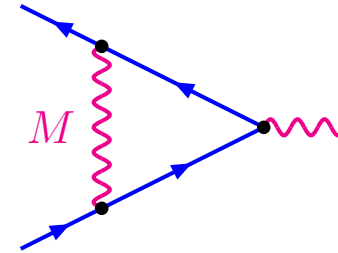
I. Introduction

"Typical" size of electroweak corrections: $\frac{\alpha_{\text{weak}}}{\pi} \approx 10^{-2}$

new aspects at LHC: $\sqrt{\hat{s}} \approx 1\text{-}2\text{TeV} \gg M_{W,Z}$

strong enhancement of negative corrections

one-loop example: massive U(1)



$$\Rightarrow \text{Born} * \left[1 + \frac{\alpha}{4\pi} \left(-\ln^2 \frac{s}{M^2} + 3 \ln \frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3} \right) \right]$$

$\frac{s}{M^2}$	$-\ln^2 \frac{s}{M^2}$	$+3 \ln \frac{s}{M^2}$	$-\frac{7}{2} + \frac{\pi^2}{3}$	Σ	$* 4 \frac{\alpha_{\text{weak}}}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	-25.52	+15.15	-0.21	-10.6	-13%
$\left(\frac{2000}{80}\right)^2$	-41.44	+19.31	-0.21	-22.3	-27%

(four-fermion cross section \Rightarrow factor 4)

- leading \log^2 multiplied by $(\text{charge})^2 = I(I + 1) = \begin{cases} 3/4 & I = 1/2 \\ 2 & I = 1 \end{cases}$
 \Rightarrow further enhancement for W-pairs by nearly factor 2.
- important subleading logarithms (NLL+...)
- One-loop up to $\mathcal{O}(40\%) \rightarrow$ two-loop terms may be relevant
- interplay between electroweak and QCD corrections
- important differences between fermions and electroweak gauge bosons
- important differences between long. and transverse gauge bosons ($I = 1/2$ vs. $I = 1$)
- Small transverse momenta, close to WW threshold!
 \rightarrow complete calculation with masses required

Gauge Boson Pair Production: W^+W^- , $W^\pm Z$, ZZ , $\gamma\gamma$

Two Approaches:

- dominant, logarithmically enhanced terms via evolution equation & separation of QED

⇒ one- and two-loop terms in NNLL

J.H.K., Metzler, Penin, Uccirati: JHEP 1106 (2011) 143

related work based on SCET: Manohar,...

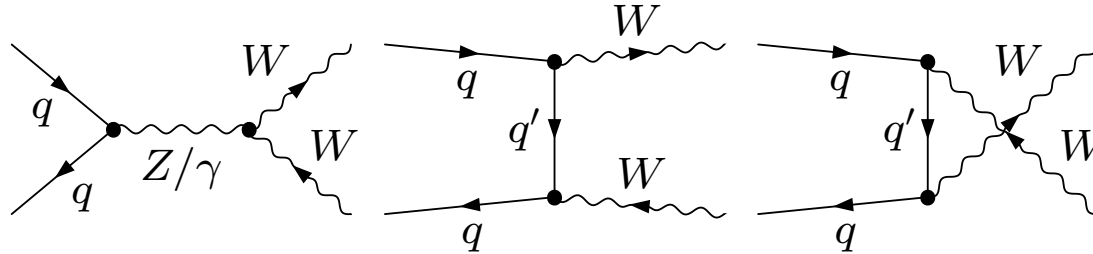
- one-loop calculation, including M_W^2/\hat{s} terms and real radiation: full NLO
Bierweiler, Kasprzik, J.H.K., Uccirati
(JHEP 1211 (2012)093[arXiv:1208.3147], arXiv:1208.3404, arXiv:1305.5402)
earlier work: logarithmically enhanced terms only, including W decays and off-shell effects

Accomando, Denner, Kaiser

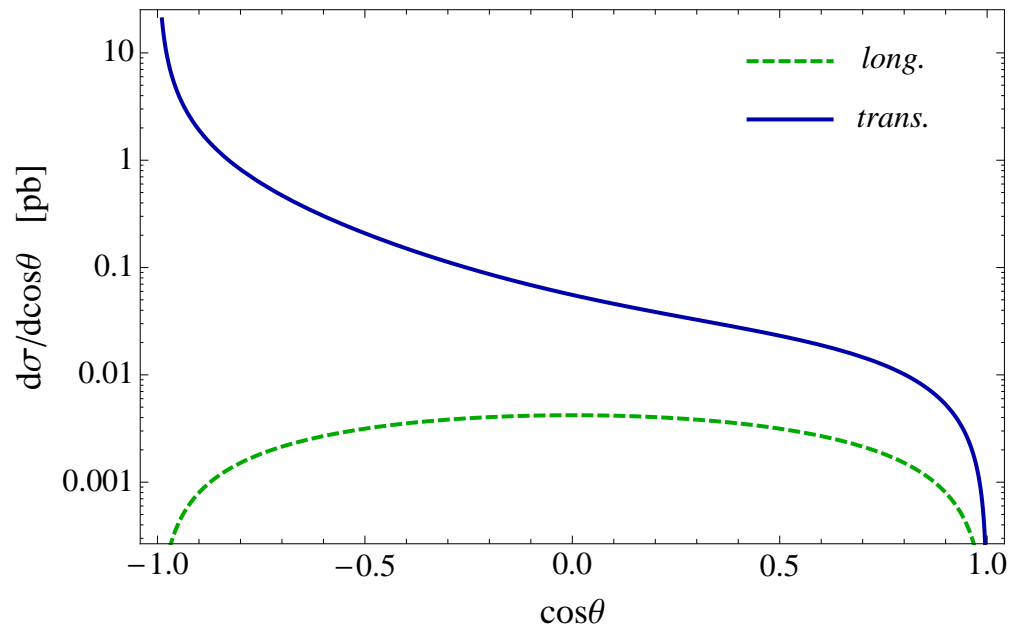
- Aim: Combine the two approaches
→ Implementation in Monte Carlo

II. One-Loop Results

II.1. W-Pairs: Leading Order

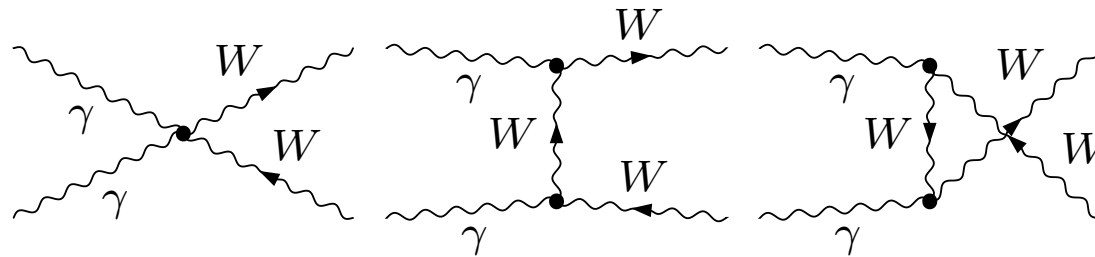


$$u\bar{u} \rightarrow W^+W^-, \sqrt{s} = 1 \text{ TeV}$$

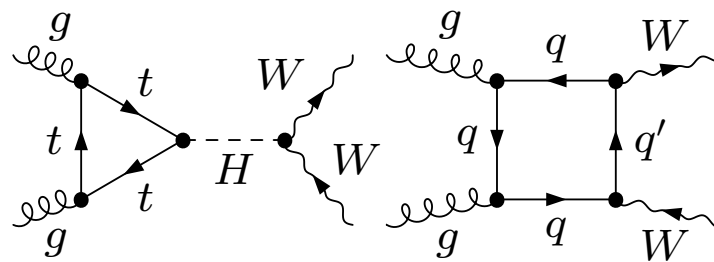


- Strong enhancement for $\Theta \rightarrow 180^\circ$
- dominance of transverse W

Also included: $\gamma\gamma \rightarrow WW$



also included: $gg \rightarrow WW$



One Loop Corrections

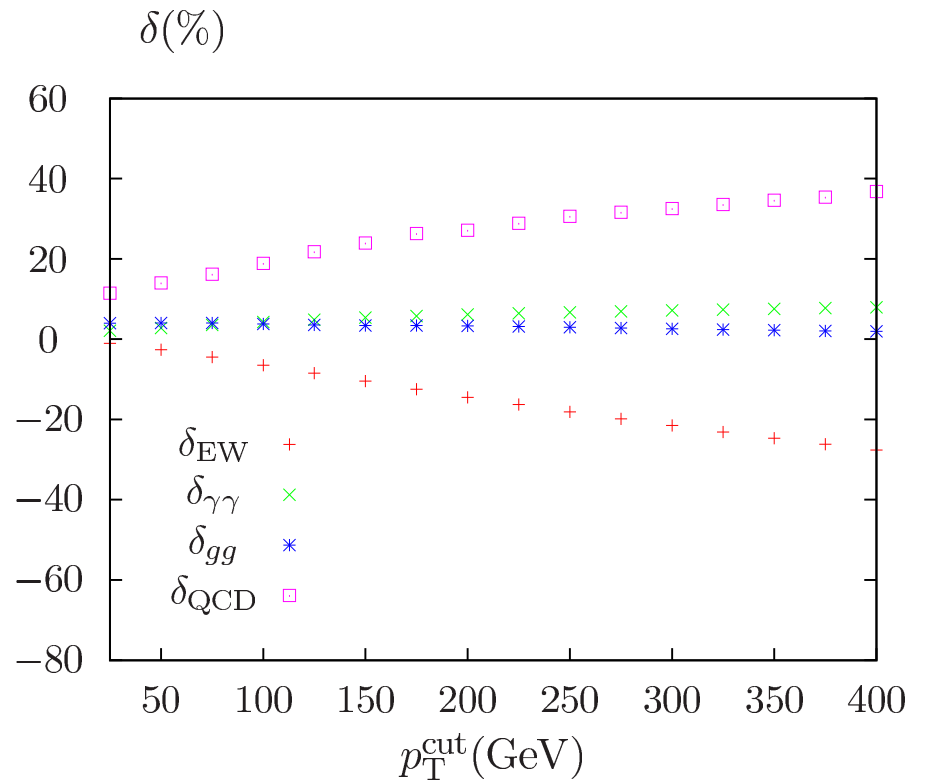
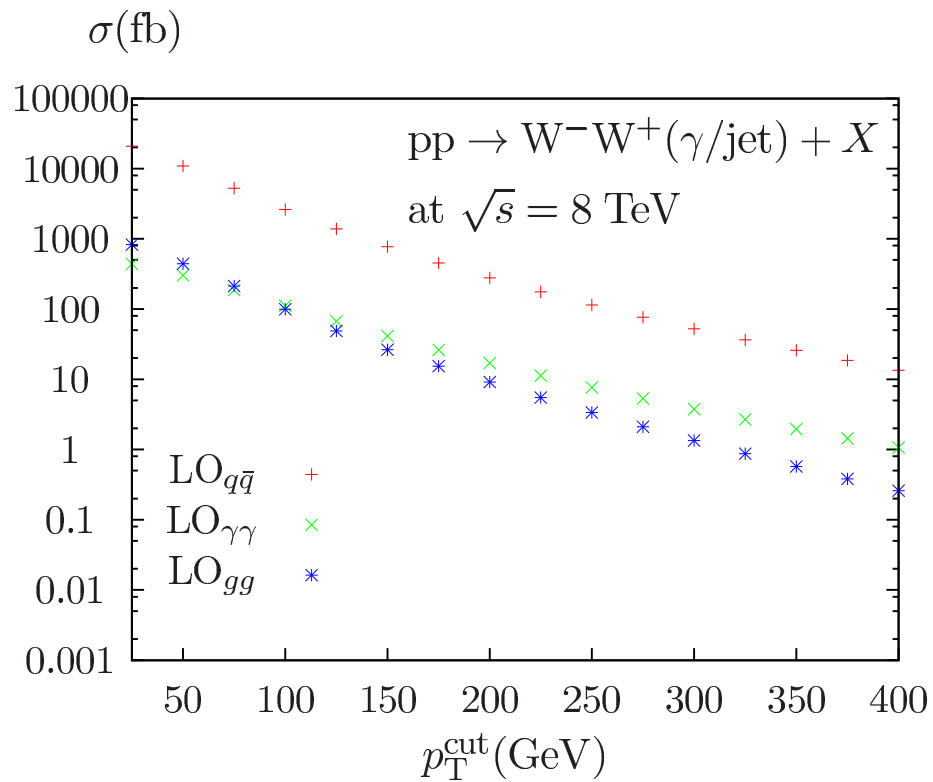
- On-shell, G_μ -scheme (M_W, M_Z, G_μ)
- virtual and real photon radiation included
remaining collinear singularities "absorbed" in PDFs
($q\bar{q}$: MSTW2008LO; $\gamma\gamma$: MRST2004QED/NNPDF2.3QED)
- two independent calculations
- NLO QCD recalculated
→ giant corrections from events with soft W and hard jet: jet or p_T -veto to select "balanced" pairs

(related results: [Baglio](#), [Ninh](#), [Weber](#))

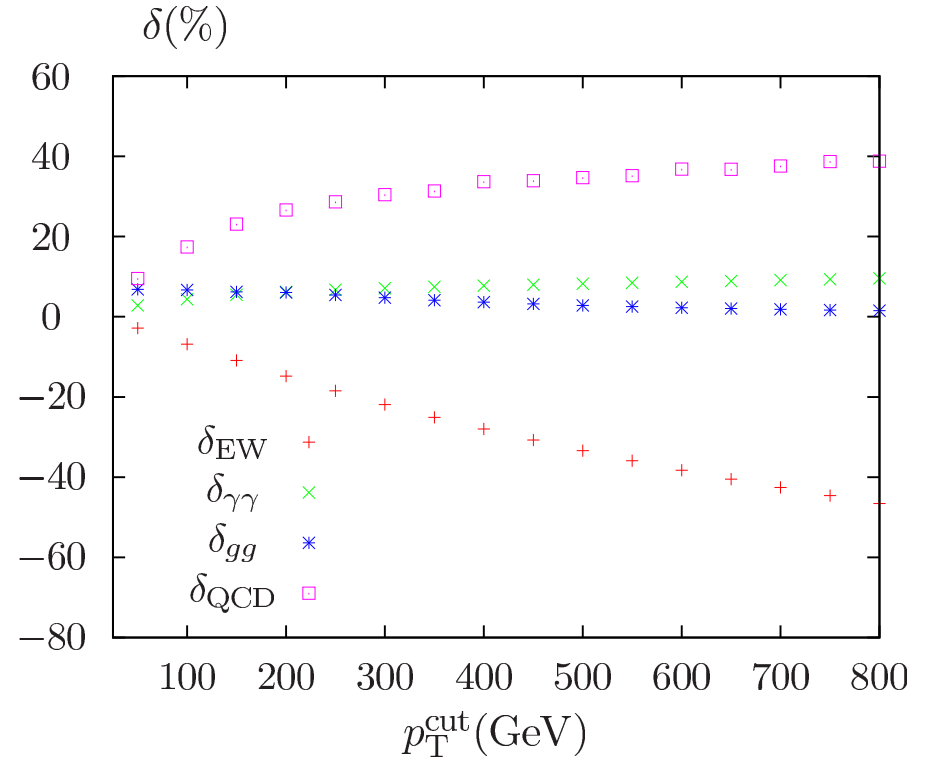
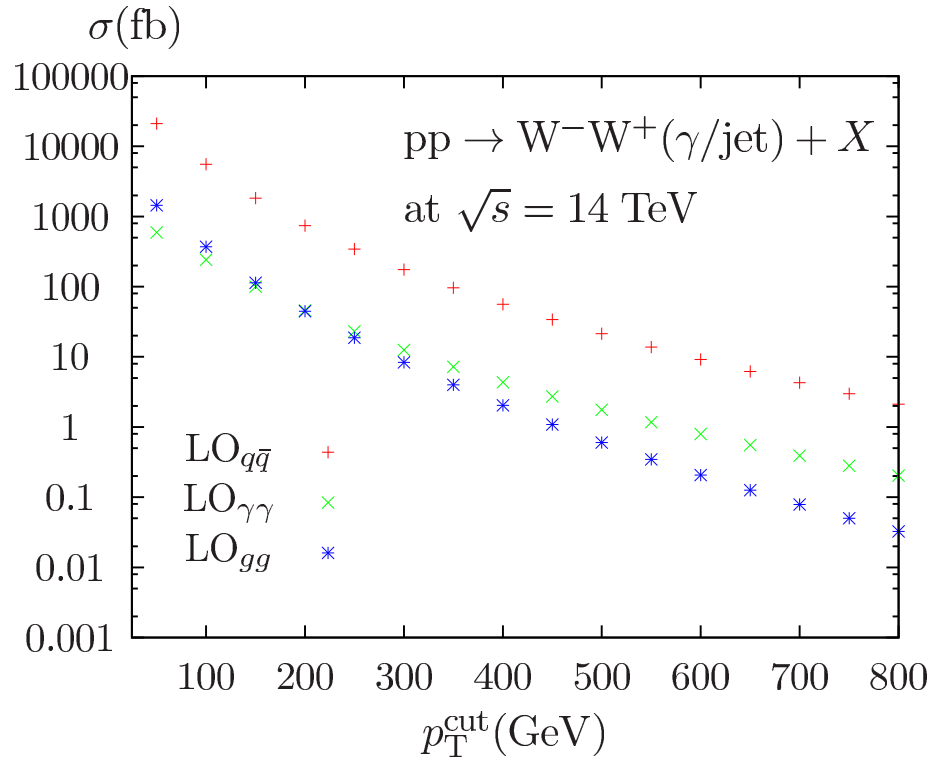
- surprisingly large contribution from $\gamma\gamma \rightarrow WW$ at large \hat{s} and **large** rapidity (small \hat{t})
- large negative corrections at large \hat{s} and small rapidity
- pronounced modification of angular distribution (\rightarrow mimics anomalous couplings) (compare [Accomando, Denner, Kaiser](#))
- sizable QCD corrections
- combination of QCD and EW corrections? additive vs. multiplicative!

One-Loop Results

Total cross sections LHC8, $p_T > p_{T,\text{cut}}$ ($\int \mathcal{L} dt = 25 \text{ fb}^{-1}$ per experiment)

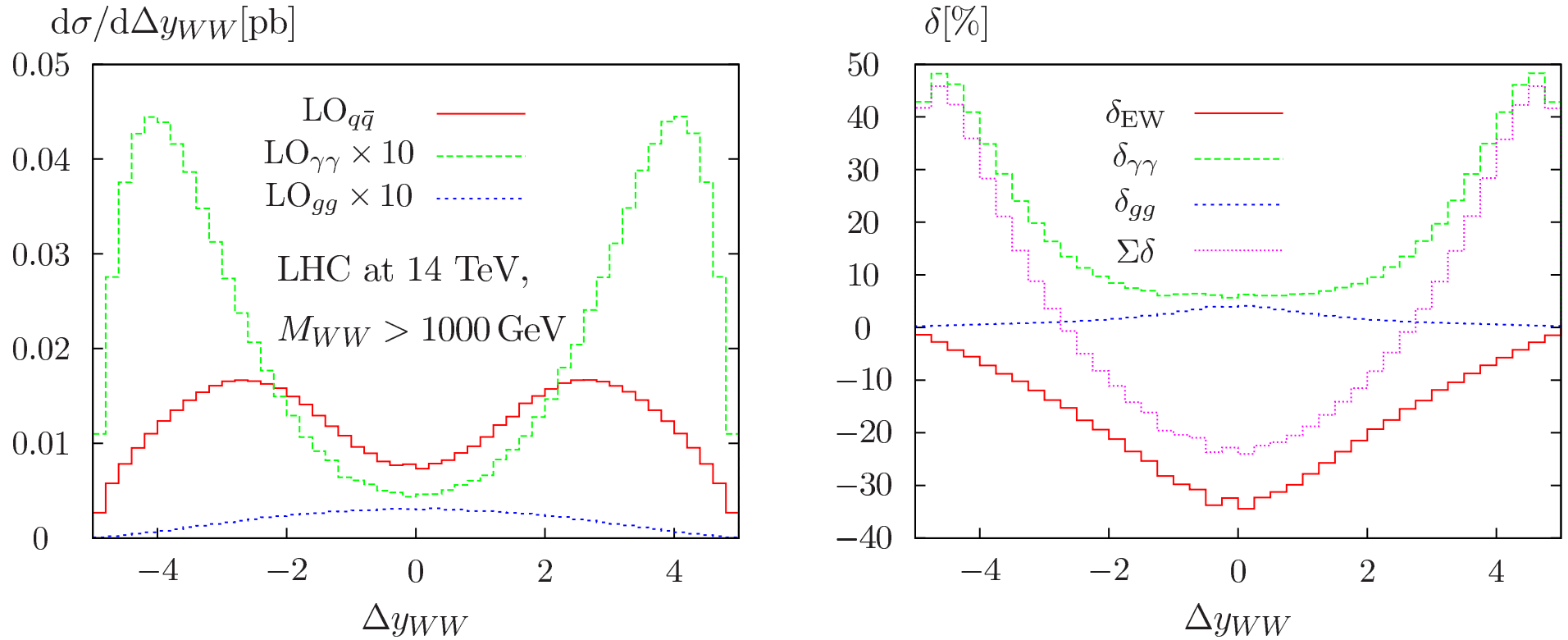


**Total cross sections LHC14, $p_T > p_{T,\text{cut}}$
(Hope for $\int \mathcal{L} dt = 300 \text{ fb}^{-1}$ per experiment)**



assume 10% efficiency

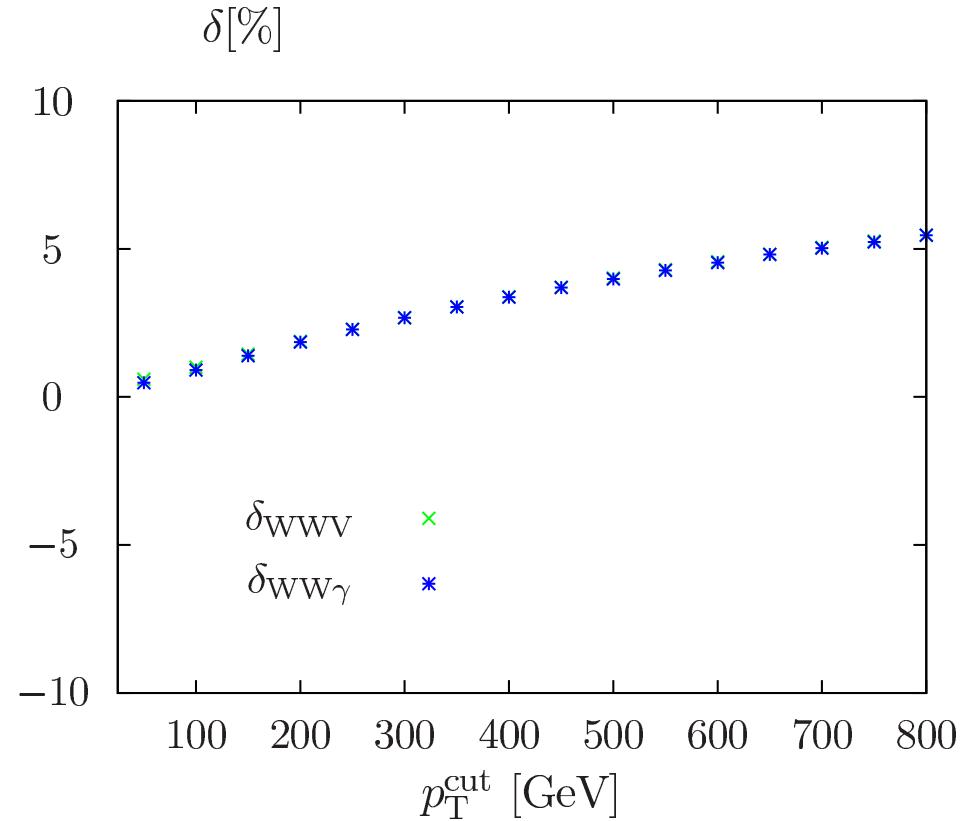
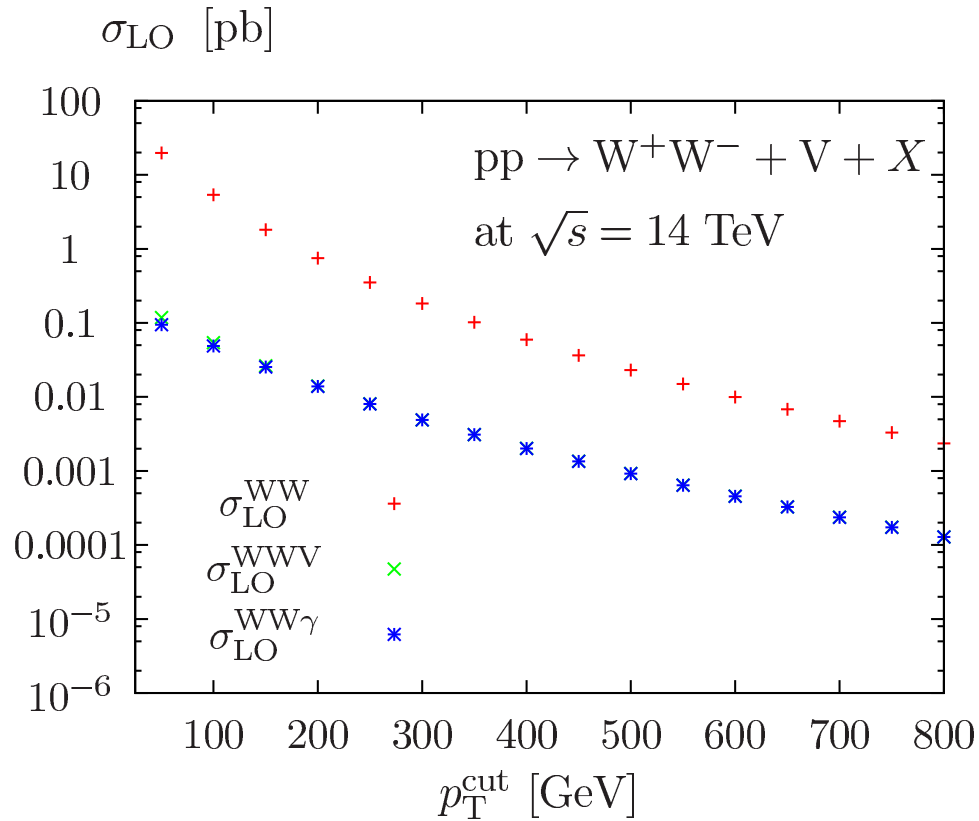
→ up to $p_T^{\text{cut}} \sim 800 \text{ GeV}$



Differential LO cross sections for the W-boson rapidity gap with a minimal invariant mass of 1000 GeV at the LHC14. On the right-hand-side, the corresponding relative rates due to photon- and gluon-induced channels w.r.t. the $q\bar{q}$ -contributions are shown, as well as the EW corrections.

Real W, Z Radiation: Compensation?

- soft and/or collinear radiation may (partly) compensate or overcompensate virtual corrections:
- model study ([Bell, J.K., Rittinger arXiv:1004.4117; EPJC](#))
strong dependence on cuts!
asymptotic energies (multi-TeV)
- semi-realistic evaluation (on-shell W, Z) ([Bierweiler, Kasprzik, J.K.](#))
 $q\bar{q} \longrightarrow W^+W^-(\gamma)$ (Born + one-loop)
vs. $q\bar{q} \longrightarrow W^+W^-Z$
 $q\bar{q} \longrightarrow W^+W^-W^+ + \text{c.c.}$
- request large invariant mass of V-pair
or large p_T of both Vs



Aim: real radiation taken care of by MC \Rightarrow different final states:

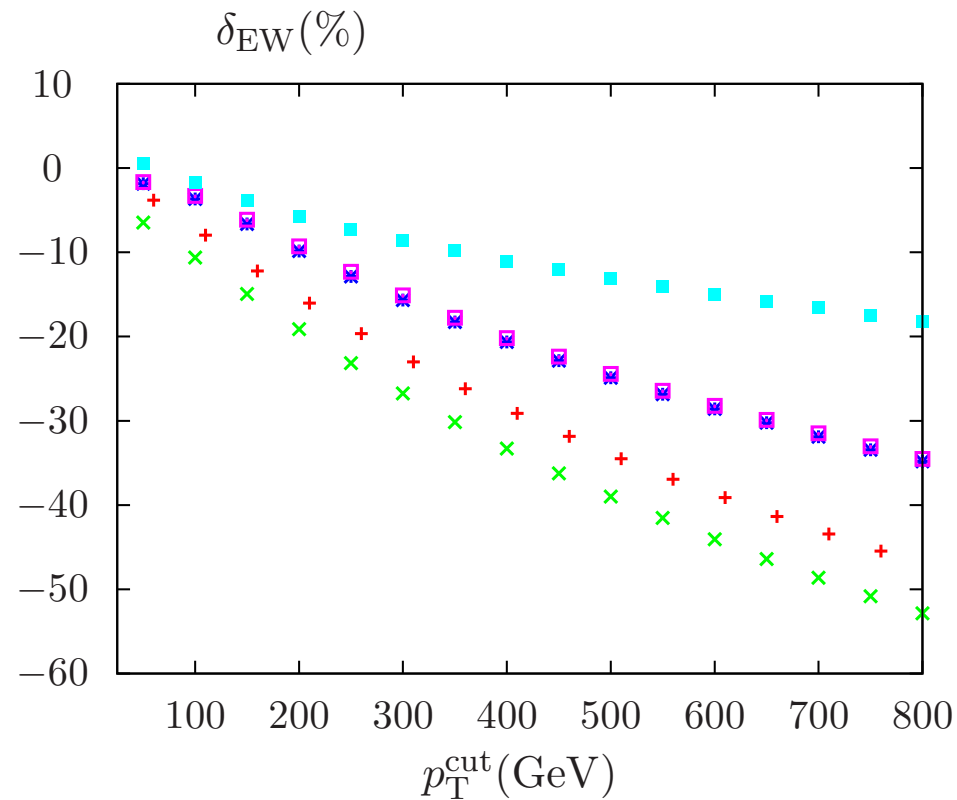
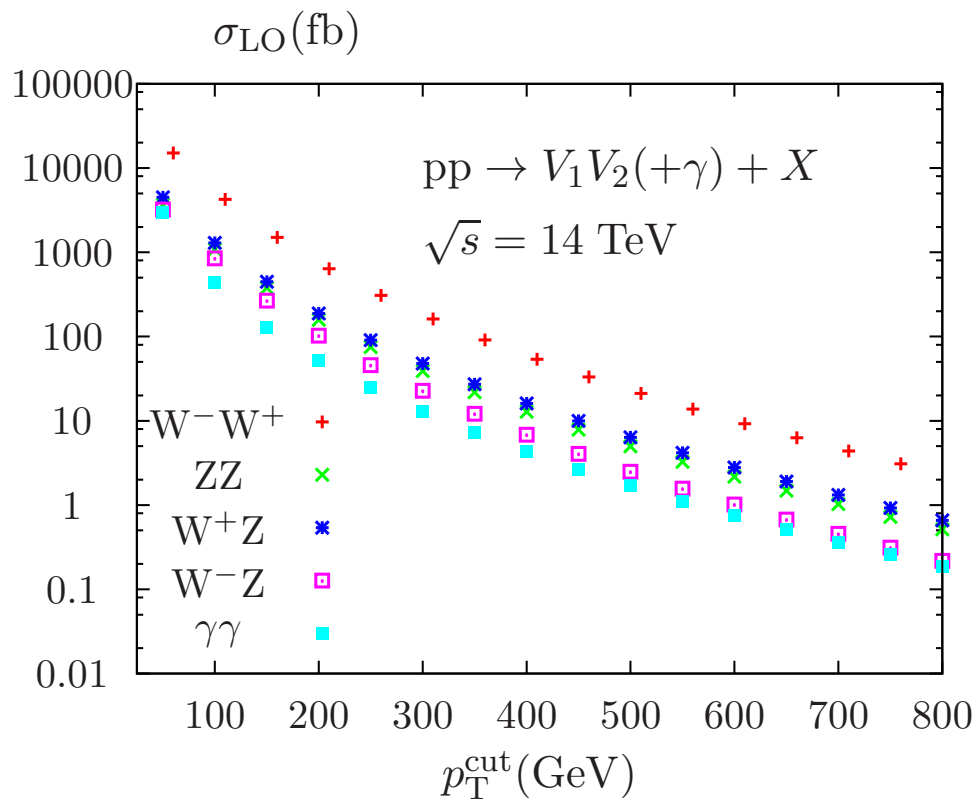
\rightarrow different angular distributions

$W^+W^-: q\bar{q} \rightarrow W^+W^-Z, W^+W^-W^\pm$

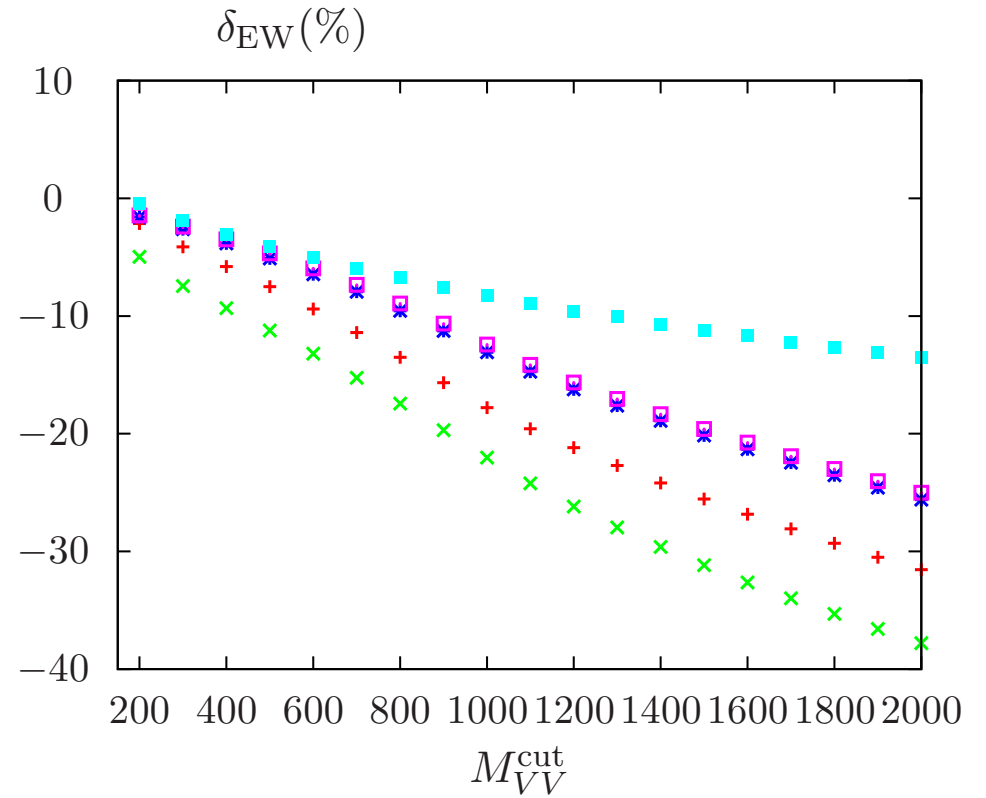
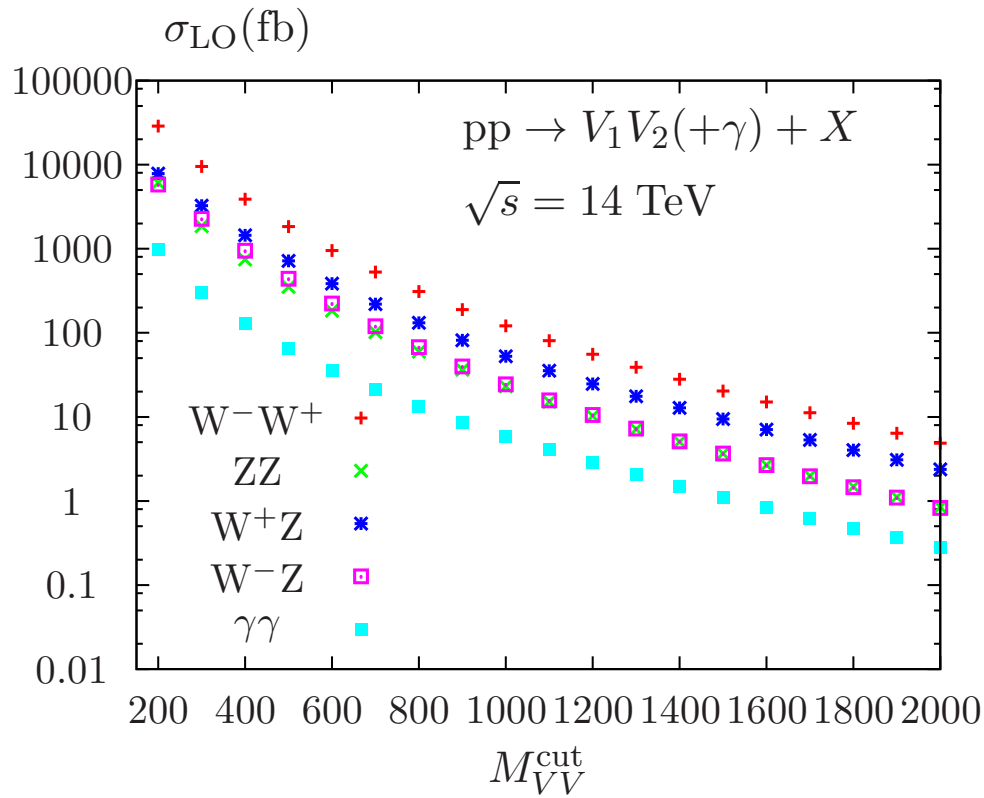
($\delta_{VV\gamma}$ strongly depends on cuts; here: $p_{T,\gamma} > 15$ GeV, $|y_\gamma| < 2.5$)

II.2. ZZ, WZ, $\gamma\gamma$

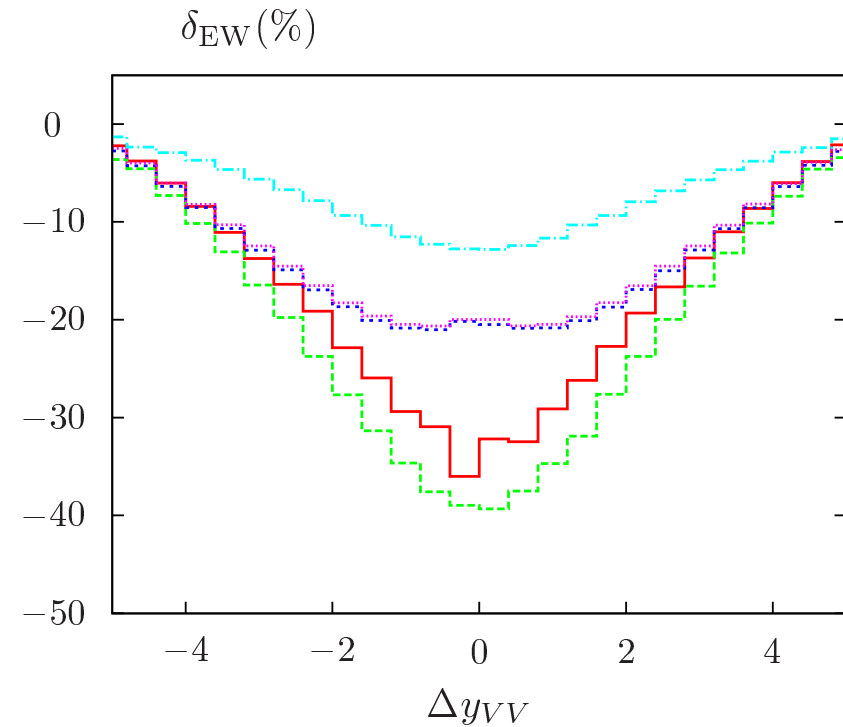
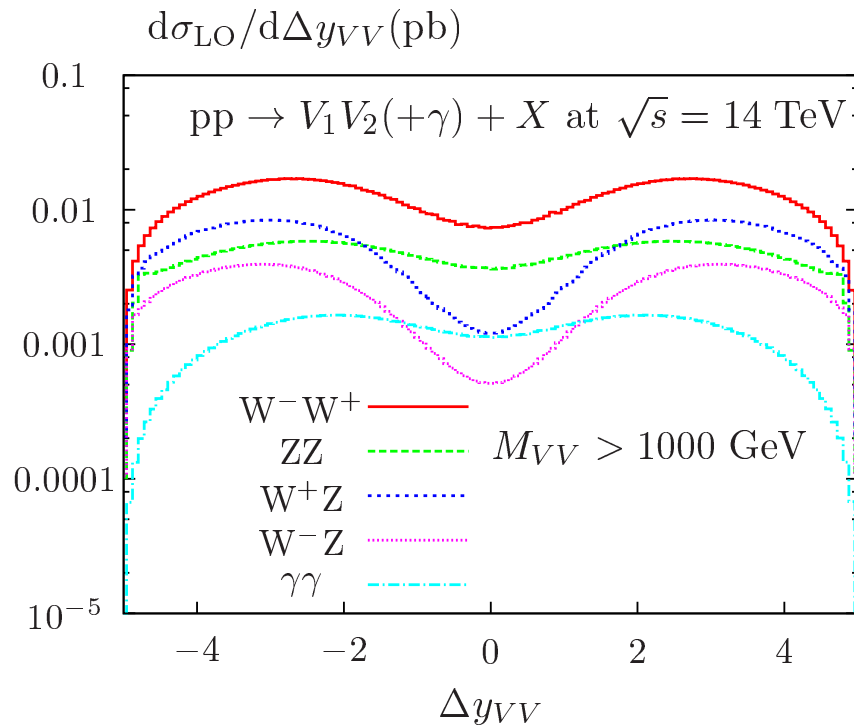
Similar setup, qualitative similar results



particularly large corrections for ZZ at large p_T : $\sim -40\%$
 small p_T : $\sim -4\%$



particularly large corrections for ZZ at large M_{ZZ} : $\sim -40\%$
 small M_{ZZ} : $\sim -4\%$



large effects on $\Delta y \equiv y_{V_1} - y_{V_2}$ distribution for large $M_{V_1 V_2}$

$\frac{d\sigma}{d\Delta y} |_{M_{V_1 V_2}} \hat{=} \text{angular distribution in } V_1 V_2 \text{ restframe}$

→ impact on search for anomalous couplings

ZZ production: polarization and decays

Impact of NLO corrections?

pp → ZZ + X					
ZZ polarizations	summed	LL	L+	++	+−
LHC14					
default cuts					
$\sigma_{\text{LO}}/\text{pb}$	7.067	0.402	0.734	0.100	4.997
$\delta\sigma_{\text{weak}}/\text{pb}$	-0.338(-0.292)	-0.015(-0.014)	-0.029(-0.025)	-0.004(-0.003)	-0.257(-0.223)
$p_{\text{T},Z} > 500$ GeV					
$\sigma_{\text{LO}}/\text{pb}$	$10^{-2} \times [0.499$	$10^{-7} \times [0.921$	$10^{-4} \times [0.334$	$10^{-7} \times [0.230$	$10^{-2} \times [0.492$
$\delta\sigma_{\text{weak}}/\text{pb}$	$-0.195(-0.148)]$	$-4.70(+5.577)]$	$-0.087(-0.067)]$	$-0.426(-0.185)]$	$-0.192(-0.147)]$
$p_{\text{T},Z} > 1000$ GeV					
$\sigma_{\text{LO}}/\text{pb}$	$10^{-3} \times [0.146$	$10^{-9} \times [0.189$	$10^{-6} \times [0.306$	$10^{-10} \times [0.475$	$10^{-3} \times [0.146$
$\delta\sigma_{\text{weak}}/\text{pb}$	$-0.088(-0.062)]$	$-4.319(+30.04)]$	$-0.126(-0.090)]$	$-2.953(+2.295)]$	$-0.088(-0.062)]$

low p_{T} : universal corrections $\sim -4\%$

⇒ angular distribution unchanged

high p_{T} : large corrections, only one combination (+−) survives

⇒ angular distribution unchanged

⇒ **universal correction factor**

III. Implementation into HERWIG++

Assumption: factorization of EW and QCD corrections

$$d\sigma_{\text{QCD}\times\text{EW}} = K_{\text{weak}}(\hat{s}, \hat{t}) \times d\sigma_{\text{QCD}}$$

$d\sigma_{\text{QCD}} \hat{=} \text{best}$ prediction for QCD-corrected cross section

- \hat{s}, \hat{t} from matching of kinematics:

$$\hat{s} = M_{V_1 V_2}^2; \quad \hat{t} = ?$$

– small p_T of $V_1 V_2 \implies \hat{\Theta} =$ scattering angle in $V_1 V_2$ restframe
(collinear or soft QCD corrections)

– large p_T of $V_1 V_2$ (associated with hard jet!)

define directions of incoming protons in $V_1 V_2$ restframe

$$\mathbf{e}_{1,2}^* \equiv \frac{\mathbf{p}_{1,2}^*}{|\mathbf{p}_{1,2}^*|}$$

$$\text{scattering axis: } \hat{\mathbf{e}}^* \equiv \frac{\mathbf{e}_1^* - \mathbf{e}_2^*}{|\mathbf{e}_1^* - \mathbf{e}_2^*|}$$

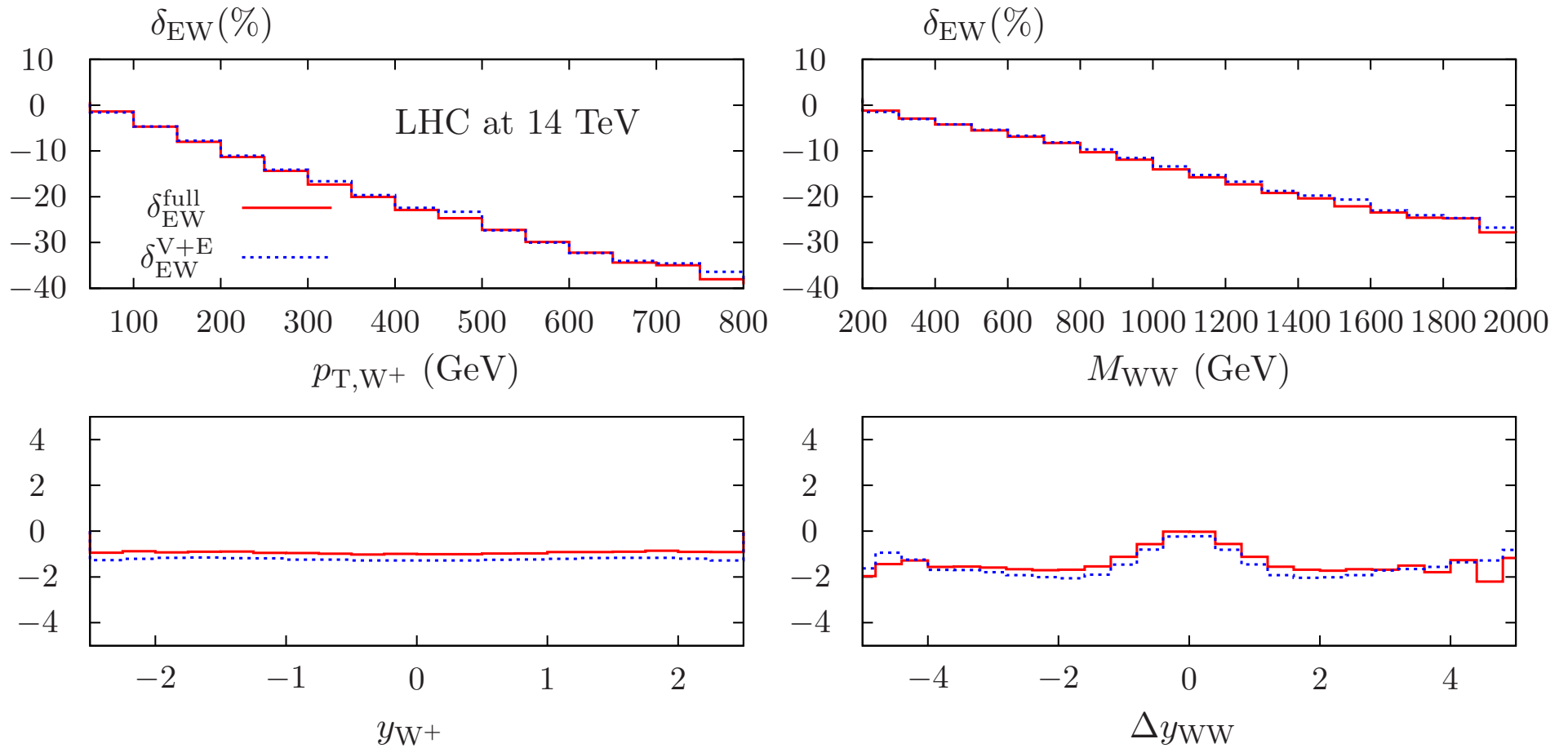
$$\text{scattering angle: } \cos \hat{\Theta} = \frac{\mathbf{p}_{V_1}^*}{|\mathbf{p}_{V_1}^*|} \cdot \hat{\mathbf{e}}^*$$

- or (HERWIG)

\hat{s}, \hat{t} provided by the generator

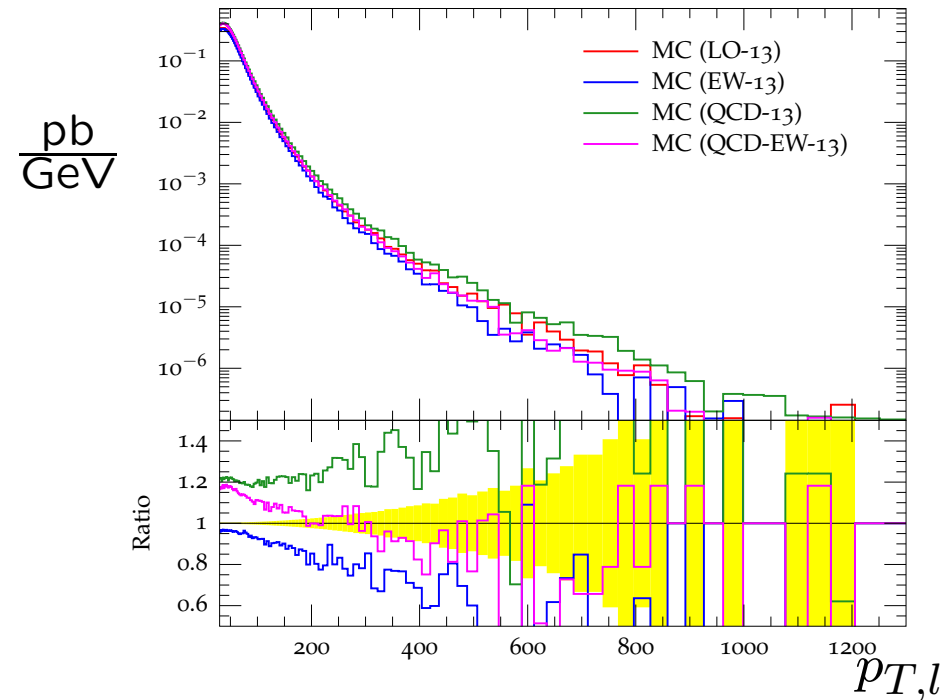
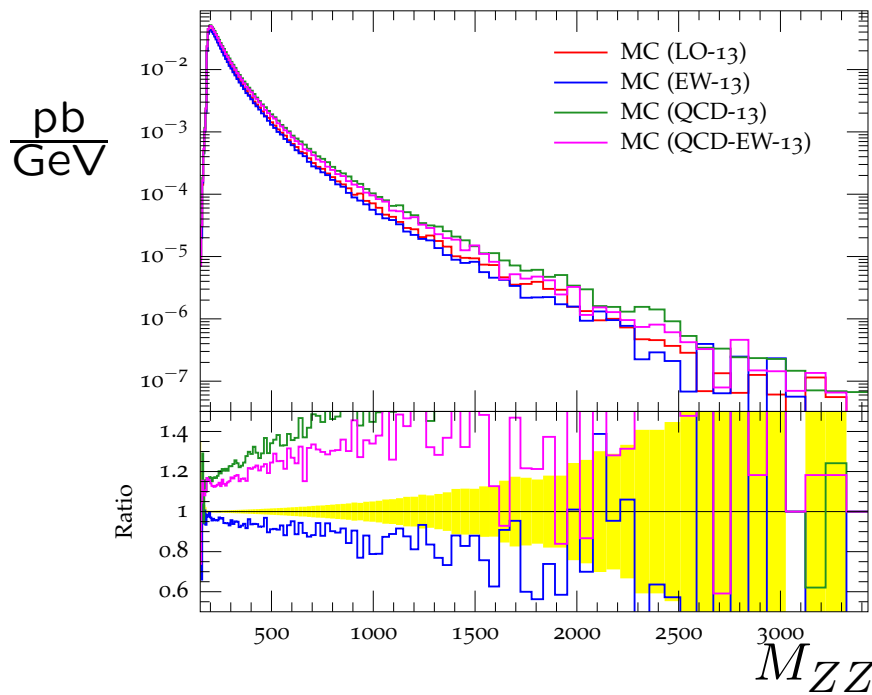
NLO corrections (electroweak) with full QED
replaced by virtual + endpoint subtraction. (Dittmaier)
(remainder = real radiation - endpoint subtraction < 1%)

$pp \rightarrow WW$ at 14 TeV:



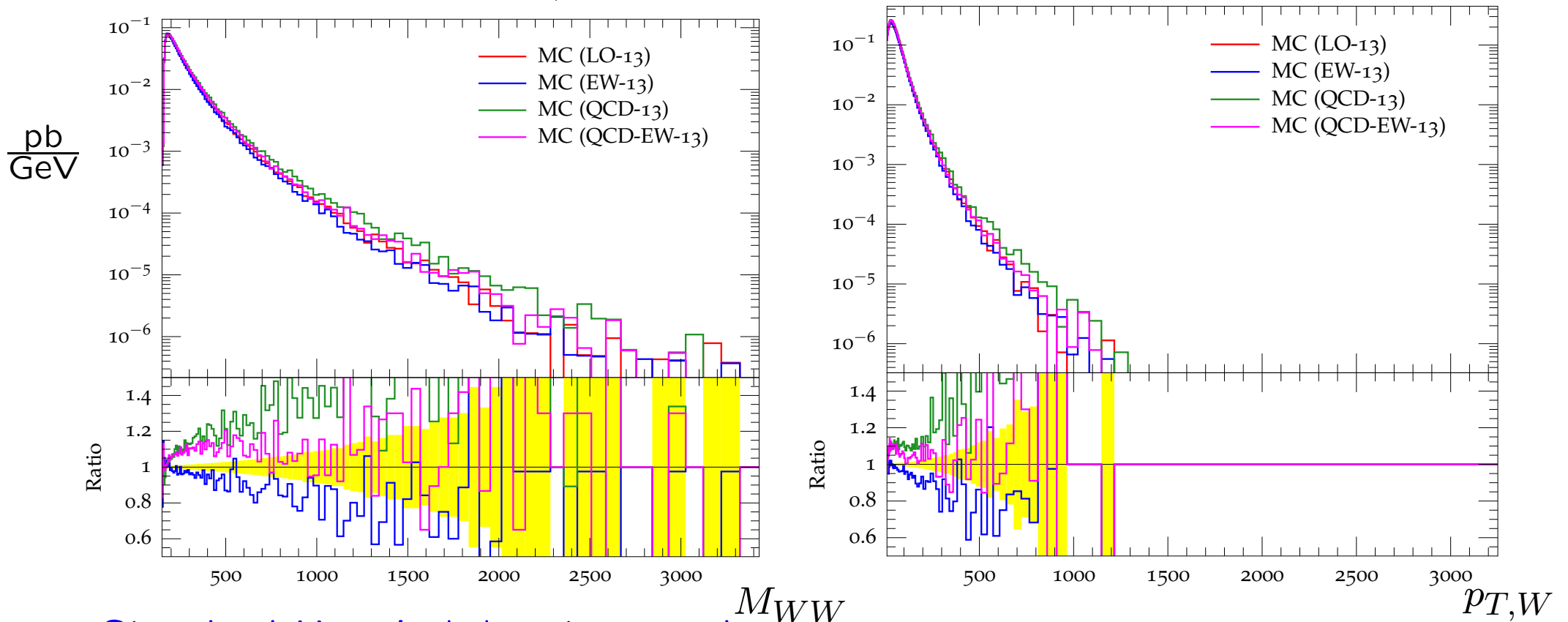
Results (preliminary)

Simulation for $pp \rightarrow ZZ \rightarrow e^+e^-\mu^+\mu^- + X$ at 13 TeV,
 M_{ZZ} and $p_{T,l}$ (average lepton p_T) distributions



- Standard HERWIG++ setup used (v2.6.3, with simple add-on for EW corrections, 2M events), ZZ at NLO QCD matched with parton showers, hadronization included, underlying event switched off
- huge QCD corrections at large $p_{T,Z}$, factorized ansatz not justified
 \rightarrow jet veto, cut on $p_{T,ZZ}$; veto: $|\sum_i \mathbf{p}_{T,l}^i| < 0.3 \sum_i |\mathbf{p}_{T,l}^i|$

Simulation for $pp \rightarrow (W^+ \rightarrow)e^+\nu_e (W^- \rightarrow)\mu^-\bar{\nu}_\mu + X$ at 13 TeV, M_{WW} and leading-W $p_{T,W}$ distributions



- Standard Herwig++ setup used (v2.6.3, with simple add-on for EW corrections, 2M events), WW at NLO QCD \oplus parton shower, hadronization included, underlying event switched off
- V+E approximate results consistent with [arXiv:1208.3147](https://arxiv.org/abs/1208.3147)

IV. Two Loop Results (Sudakov Logarithms)

one-loop: $\sim -40\%$

two-loop: $\sim ?$

(Vast amount of literature since ~ 2000)

Karlsruhe (Jantzen, J.K., Metzler, Penin, Smirnov, Uccirati)

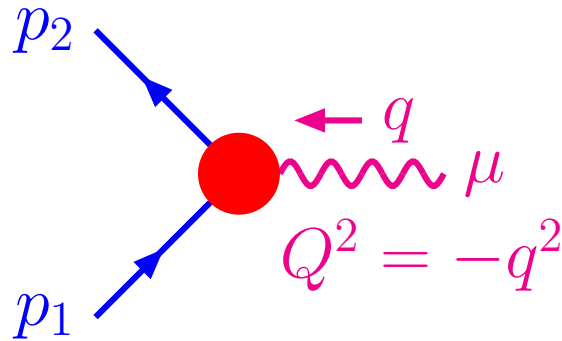
Fadin, Lipatov, Martin, Melles

PSI (Denner, Melles, Pozzorini, ...)

Ciafaloni, ...

Manohar, ...

A) Form Factor and Evolution Equations



Born:

$$\mathcal{F}_{\text{Born}} = \bar{\psi}(p_2) \gamma_\mu \psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2} \mathcal{F} = \left[\int_{M^2}^{Q^2} \frac{dx}{x} \gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2)) \right] \mathcal{F}$$

Collins, Sen

$$\Rightarrow \mathcal{F} = \mathcal{F}_{\text{Born}} F_0(\alpha(M^2)) \exp \left\{ \int_{M^2}^{Q^2} \frac{dx}{x} \left[\int_{M^2}^x \frac{dx'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2)) \right] \right\}$$

aim: N^4LL \Rightarrow corresponds to all terms of the form:

$$\alpha^n \left[\begin{array}{c} \mathcal{L}^{2n} + \mathcal{L}^{2n-1} + \mathcal{L}^{2n-2} + \mathcal{L}^{2n-3} + \mathcal{L}^{2n-4} \\ LL \quad NLL \quad NNLL \quad N^3LL \quad N^4LL \end{array} \right]$$

$$\mathcal{L} \equiv \ln(Q^2/M^2)$$

$NNLL$ requires running of α (i.e. β_0 and β_1) and:

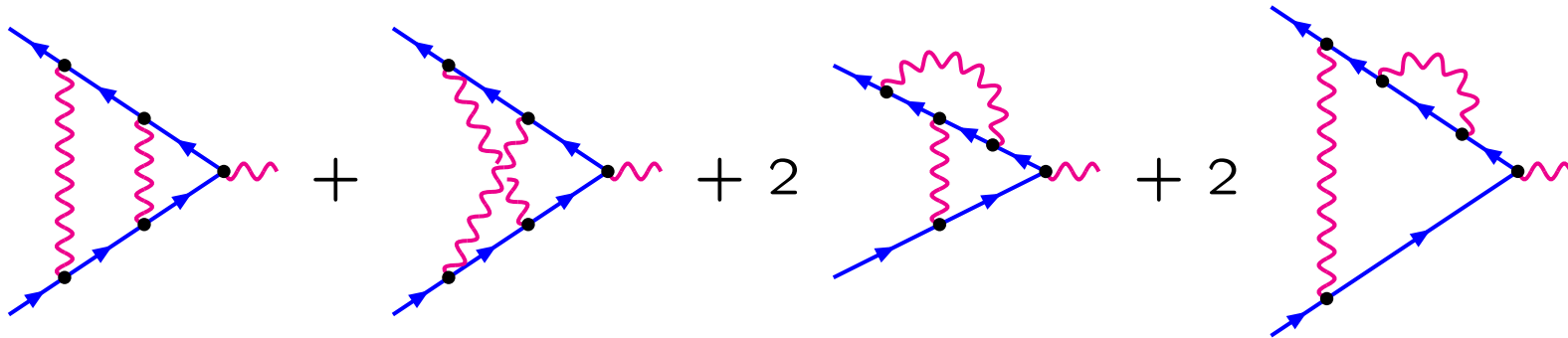
$$\begin{array}{ll} \zeta(\alpha), \xi(\alpha), F_0(\alpha) & \text{up to } \mathcal{O}(\alpha) \quad (\text{one-loop}) \\ \gamma(\alpha) & \text{up to } \mathcal{O}(\alpha^2) \quad (\text{massless two loop}) \end{array}$$

N^3LL requires two-loop calculation in high-energy limit including linear logarithms (available for non-abelian theory)

N^4LL requires complete two-loop calculation in high-energy limit (available for abelian theory)

B) Two-Loop Results: Massive U(1) Model

$$\mathcal{F}_\alpha(M, Q) = \mathcal{F}_{\text{Born}} \left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi} \right)^2 f^{(2)} + \dots \right]$$



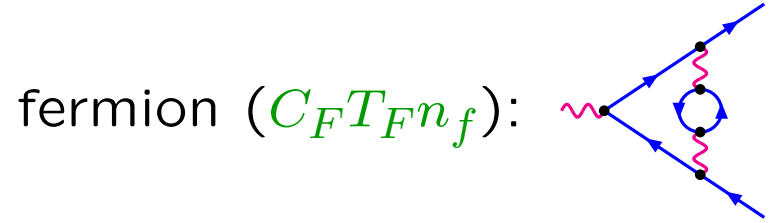
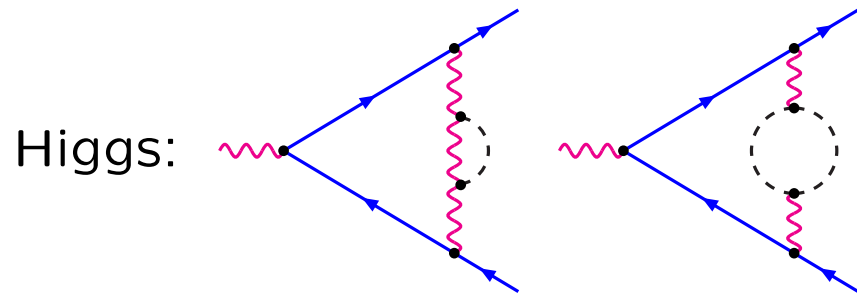
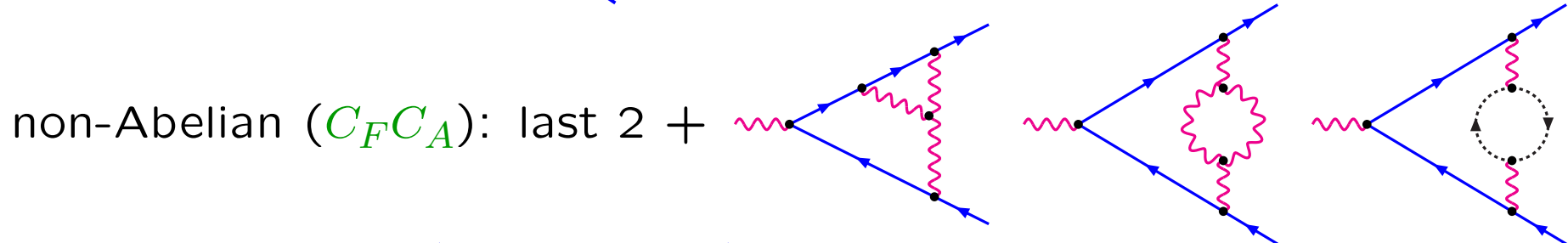
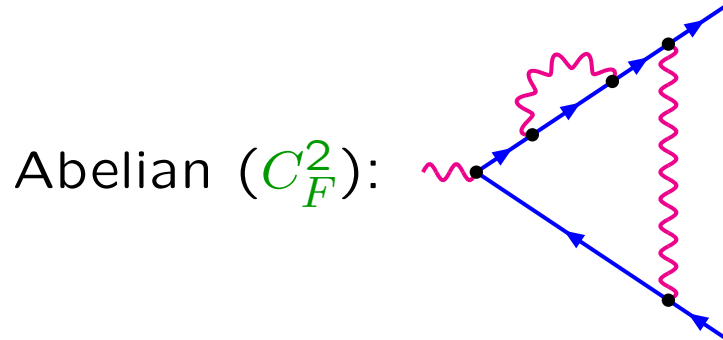
$$f^{(2)} = \frac{1}{2} \mathcal{L}^4 - 3 \mathcal{L}^3 + \left(8 + \frac{2}{3} \pi^2 \right) \mathcal{L}^2 - (9 + 4\pi^2 - 24\zeta_3) \mathcal{L}$$

$$+ \frac{25}{2} + \frac{52}{3} \pi^2 + 80\zeta_3 - \frac{52}{15} \pi^4 - \frac{32}{3} \pi^2 \ln^2 2 + \frac{32}{3} \ln^4 2 + 256 \text{Li}_4 \left(\frac{1}{2} \right)$$

$$\mathcal{L} \equiv \ln(Q^2/M^2)$$

C) Massive SU(2) form factor in 2-loop approximation

2-loop vertex diagrams (massless fermions, massive bosons):



+ 1-loop × 1-loop corrections + renormalization

$$f_2 = +\frac{9}{32} \mathcal{L}^4 - \frac{19}{48} \mathcal{L}^3 - \left(-\frac{7}{8} \pi^2 + \frac{463}{48} \right) \mathcal{L}^2$$

$$+ \left(\frac{39 \operatorname{Cl}_2\left(\frac{\pi}{3}\right)}{2 \sqrt{3}} + \frac{45 \pi}{4 \sqrt{3}} - \frac{61}{2} \zeta_3 - \frac{11}{24} \pi^2 + 29 \right) \mathcal{L}$$

Extensions to four-fermion scattering (Penin et al.) and W-pair production (J.K., Metzler, Penin, Uccirati)

General form (for transverse W) :

$$\mathcal{A} = \alpha(\mu_T) \mathcal{Z}_\Psi \mathcal{Z}_W \tilde{\mathcal{A}}$$

$\mathcal{Z}_{W,\Psi}$ = form factors of quark or W -field

$\tilde{\mathcal{A}}$ = **reduced** amplitude (matrix in isospin space)

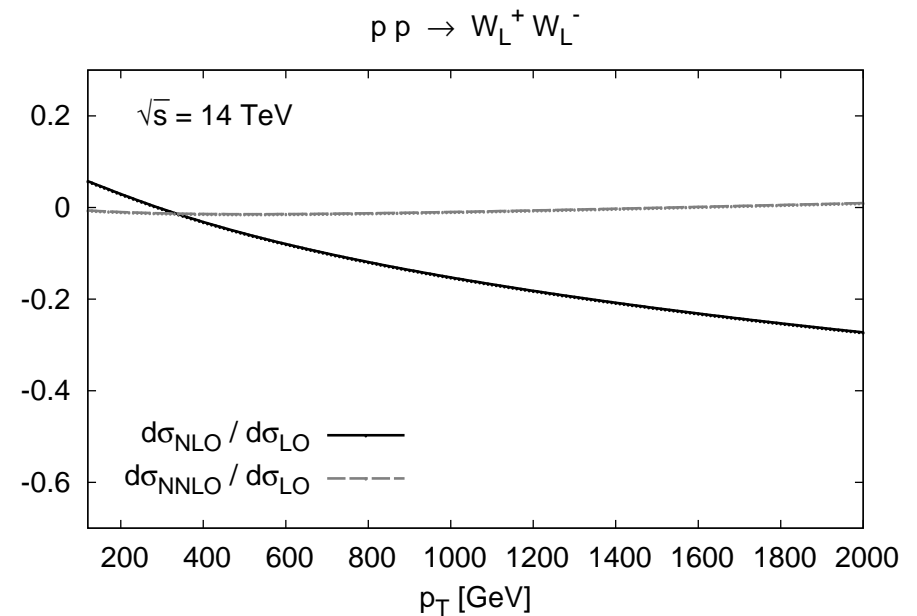
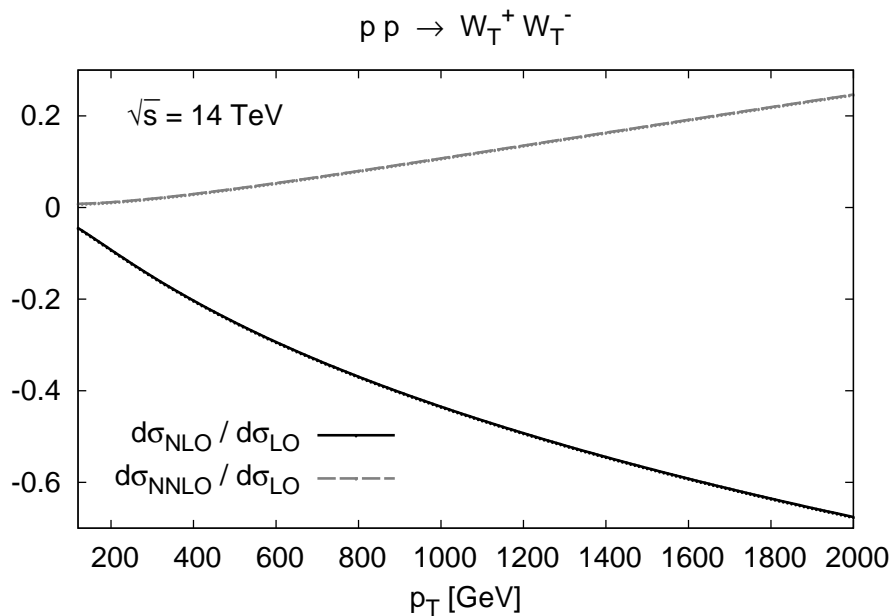
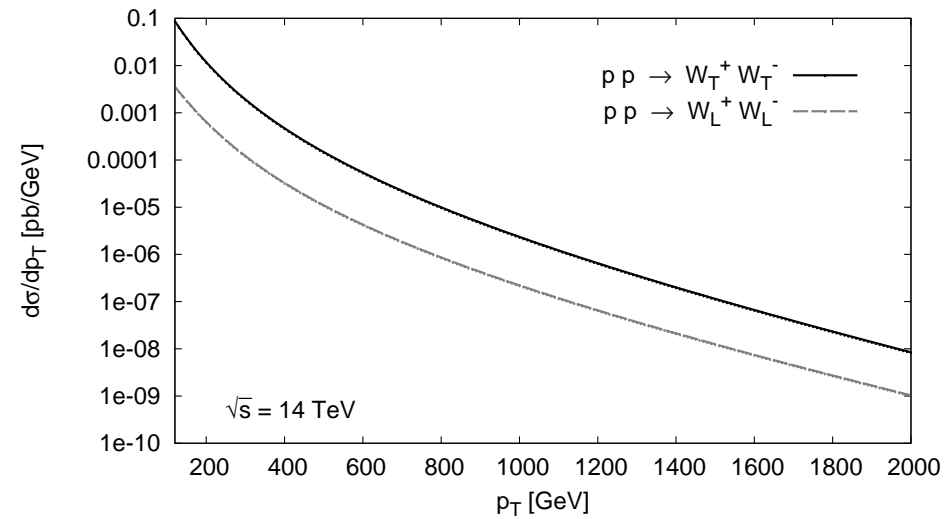
Solution of

$$\frac{\partial}{\partial \ln Q^2} \tilde{\mathcal{A}} = \chi(\alpha(Q^2)) \tilde{\mathcal{A}}$$

($\chi \equiv$ anomalous dimension matrix)

up to quadratic logarithms only one-loop terms (\ln^2, \ln^1, \ln^0) of $q\bar{q} \rightarrow WW$ and the two-loop term $\gamma^{(2)}$ of the anomalous dimension (form factor!) required.

Complication: massless photon! \Rightarrow QED subtracted



($W_{T,L}$: Isospin $I = 1, 1/2$; \Rightarrow relative factor 6/11)

V. Summary

- Weak corrections at LHC14 may reach -40%
- complicated impact on differential distributions
⇒ fake anomalous couplings; sensitive to cuts
- first steps toward combination of QCD and EW corrections in Monte Carlo generator
- dominant two-loop corrections (partially) available