$\mathcal{O}(\alpha\alpha_{\rm s})$ corrections to Drell–Yan processes in the resonance region

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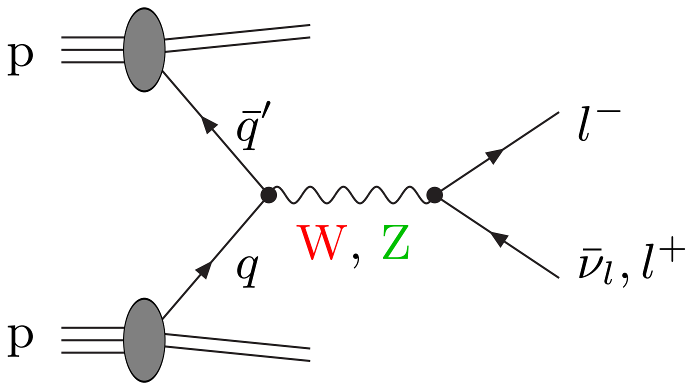
Introduction

W- and Z-boson production at hadron colliders



Physics issues:

- σ $\quad\rightarrow$ standard candle
- M_Z $\quad\rightarrow$ detector calibration by comparing with LEP1 result
- $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$ $\rightarrow$ comparison with results of LEP1 and SLC
- M_W $\quad\rightarrow$ improvement over $\Delta M_\mathrm{W} \sim 15\,\mathrm{MeV}$, strengthen EW precision tests
  ($\mathrm{W/Z}$ shape comparisons even sensitive to $\Delta M_\mathrm{W} \sim 7\,\mathrm{MeV}$ at LHC)
  Besson et al. '08
- decay widths Γ_Z and Γ_W from M_{ll} or $M_{\mathrm{T},l\nu_l}$ tails
- search for Z' and W' at high M_{ll} or $M_{\mathrm{T},l\nu_l}$
- information on PDFs

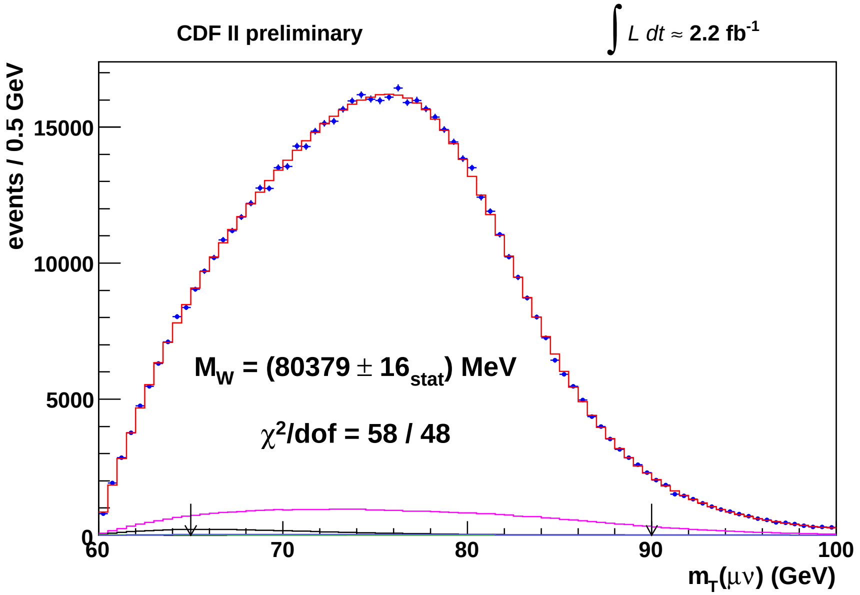
Tevatron example: M_{W} determination @ CDF (2012)

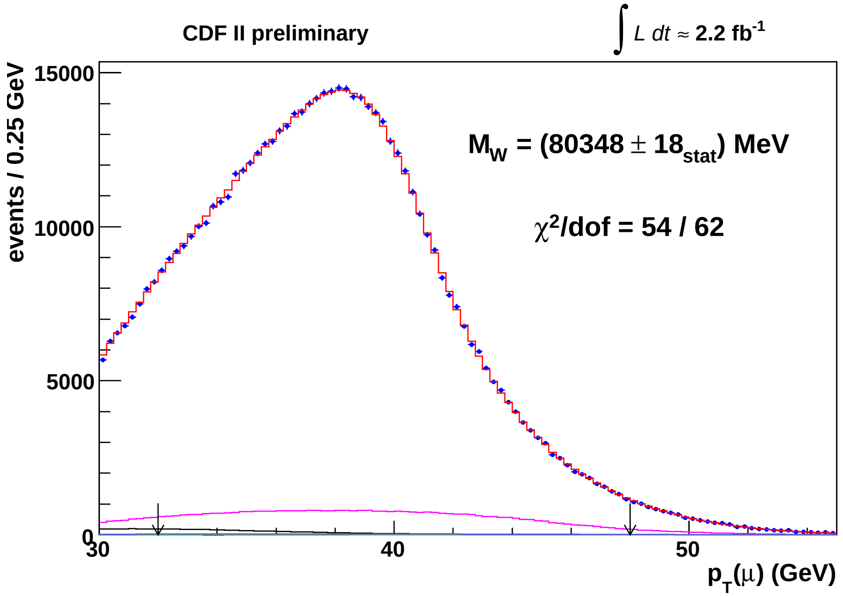
$M_{\mathrm{W}}^{\mathrm{CDF}} = 80.387\,\mathrm{GeV} \pm 19\,\mathrm{MeV}$ from fits to distributions in

a) transverse W-boson mass

$$M_{\mathrm{T},l\nu} = \sqrt{2(E_{\mathrm{T},l}\not{E}_{\mathrm{T}} - \mathbf{p}_{\mathrm{T},l} \cdot \not{\mathbf{p}}_{\mathrm{T}})}$$

b) transverse lepton momentum $p_{\mathrm{T},l}$





Sensitivity to M_{W} via Jacobian peaks from W resonance at

$$M_{\mathrm{T},l\nu} \sim M_{\mathrm{W}} \qquad\qquad p_{\mathrm{T},l} \sim M_{\mathrm{W}}/2$$

$\Rightarrow$ Reduction of ΔM_{W} requires higher theoretical precision in W resonance region !

(for Z resonance as well for reference)

QCD and EW corrections to W/Z production:

- NNLO QCD corrections — Hamberg et al. ’91; Harlander, Kilgore ’02; Anastasiou et al. ’03; Melnikov, Petriello ’06; Catani et al. ’09
- soft + virtual N^{3}LO QCD — Moch, Vogt ’05; Laenen, Magnea ’05; Idilbi et al. ’05; Ravindran, Smith ’07
- QCD resummations — Arnold, Kauffman ’91; Balazs et al. ’95,’97; R.K.Ellis et al. ’97; Qiu, Zhang ’00; Kulesza et al. ’01,’02; Landry et al. ’02; Berge et al. ’05; Bozzi et al. ’08
- MC@NLO matching — Frixione, Webber ’06
- NLO EW correction to W production — S.D., Krämer ’01; Zykunov ’01; Baur, Wackeroth ’04; Arbuzov et al. ’05 Carloni Calame et al. ’06; Brensing et al. ’07
- NLO EW correction to Z production — Baur, Keller, Sakumoto ’97; Baur, Wackeroth ’99 Brein, Hollik, Schappacher ’99; Zykunov ’05; Arbuzov et al. ’06; Carloni Calame et al. ’07; S.D., Huber ’09
- multi-photon radiation via leading logs — Baur, Stelzer ’99; Carloni Calame et al. ’03 Placzek, Jadach ’04; Brensing et al. ’07; S.D., Huber ’09
- photon-induced processes — Arbuzov, Sadykov ’07; Brensing et al. ’07; Carloni Calame et al. ’07; S.D., Huber ’09
- POWHEG matching of QCD/EW corrs. — Bernaciak, Wackeroth ’12; Barze et al. ’13
- NLO SUSY corrections in the MSSM — Brensing et al. ’07; S.D., Huber ’09

Combination of NLO QCD and EW corrections

Issue unambiguously fixed only by calculating the 2-loop $\mathcal{O}(\alpha\alpha_s)$ corrections, until then rely on approximations and estimate the uncertainties:

Comparison of two extreme alternatives:

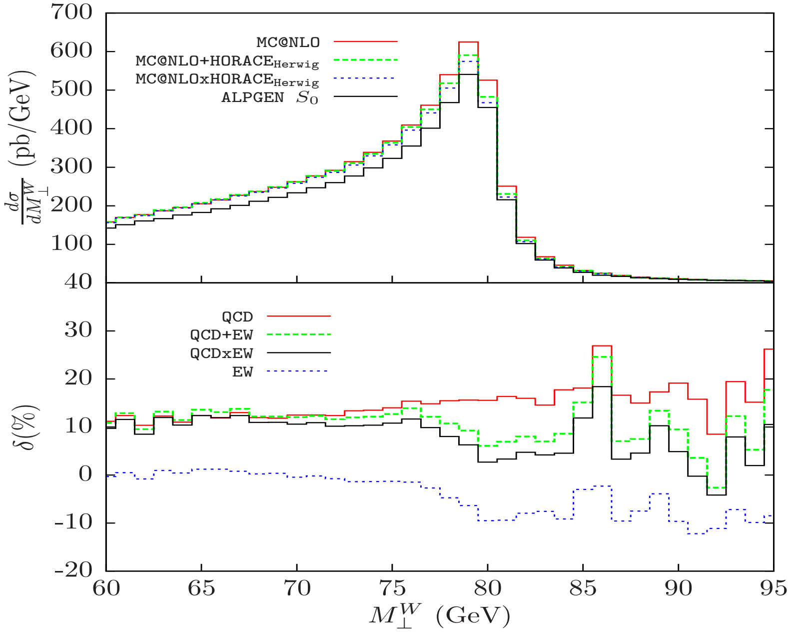
$$(1 + \delta_{\rm QCD}^{\rm NLO} + \delta_{\rm EW}^{\rm NLO})$$

versus

$$(1 + \delta_{\rm QCD}^{\rm NLO}) \times (1 + \delta_{\rm EW}^{\rm NLO})$$

$\hookrightarrow$ difference at %-level with shape distortion

Balossini et al. '09 (HORACE)



$\Rightarrow$ $\mathcal{O}(\alpha\alpha_s)$ corrections should be known at least in resonance region !

Steps towards $\mathcal{O}(\alpha\alpha_s)$ corrections

- NLO EW for $\mathrm{W/Z}$ production with a hard jet
  - $\mathrm{W}+1\,\mathrm{jet}$, stable W boson — Kühn, Kulesza, Pozzorini, Schulze '07; Hollik, Kasprzik, Kniehl '07
  - $\mathrm{W}+1\,\mathrm{jet} \to l\nu_l + 1\,\mathrm{jet}$ — Denner, S.D., Kasprzik, Mück '09
  - $\mathrm{Z}/\gamma+1\,\mathrm{jet}$, stable Z boson — Maina, Moretti, Ross '04; Kühn, Kulesza, Pozzorini, Schulze '04,'05
  - $\mathrm{Z}/\gamma^*+1\,\mathrm{jet} \to l^+l^-/\bar{\nu}_l\nu_l + 1\,\mathrm{jet}$ — Denner, S.D., Kasprzik, Mück '11,'12
- further partial results
  - on-shell $\mathrm{Z}f\bar{f}$ vertex — Kotikov, Kühn, Veretin '07
  - virtual corrections to $q\bar{q}'\to\mathrm{W/Z}\to l\bar{l}'$ — Bonciani '11
  - inclusive $\Gamma_{\mathrm{W}\to q\bar{q}'}$ — Kara '13
- resonance expansion for $q\bar{q}'\to\mathrm{W/Z}\to l\bar{l}'$
  non-factorizable corrections — S.D., Huss, Schwinn '13

This talk !

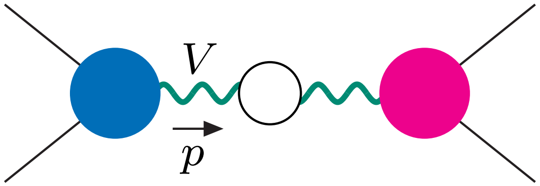
Pole expansion @ $\mathcal{O}(\alpha)$

Pole expansion of loop amplitudes – general idea

Stuart '91; H.Veltman '92
Aeppli, v.Oldenborgh, Wyler '94

Starting point: Dyson-summed matrix element

$$\mathcal{M} = \underbrace{\frac{W(p^2)}{p^2 - M_V^2 + \Sigma(p^2)}} + N(p^2)$$



resonant part with complex pole at $p^2 = \mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V$ gauge invariant

Sirlin '91; Stuart '91; Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01

Isolation of pole structure:

denominator:
$$p^2 - M_V^2 + \Sigma(p^2) = p^2 - \mu_V^2 + \Sigma(p^2) - \Sigma(\mu_V^2)$$
$$= (p^2 - \mu_V^2)[1 + \Sigma'(\mu_V^2)] + \mathcal{O}((p^2 - \mu_V^2)^2)$$

$$\mathcal{M} = \underbrace{\frac{W(\mu_V^2)}{p^2 - \mu_V^2} \frac{1}{1 + \Sigma'(\mu_V^2)}} + \underbrace{\left[\frac{W(p^2)}{p^2 - M_V^2 + \Sigma(p^2)} - \frac{W(\mu_V^2)}{p^2 - \mu_V^2} \frac{1}{1 + \Sigma'(\mu_V^2)}\right] + N(p^2)}$$

resonance pole = gauge invariant
$\hookrightarrow$ "factorizable contributions"

resonant "non-factorizable" corrections
if $W(\mu_V^2)$ contains new IR divergences,
$+$ non-resonant continuum

Note: evaluation of $W(\mu_V^2)$ for complex $p^2 = \mu_V^2$ not straightforward,
but perturbatively calculable from quantities with real momenta

Aeppli et al. '94

Perturbative evaluation of leading pole approximation (PA)

Expansion of matrix element:

($A^{(n)} \equiv n$-loop contribution to A)

$$\mathcal{M} = \mathcal{M}^{(0)}$$

LO:
complete leading order

$$+\frac{W^{(1)}(M_V^2)}{p^2-\mu_V^2} - \frac{W^{(0)}(M_V^2)\Sigma^{(1)\prime}(M_V^2)}{p^2-\mu_V^2} + \mathcal{M}^{(1)}_{\mathrm{non-fact}}$$

NLO:
correction to residue
and
non-factorizable corrections

$+$ higher orders

Comments:

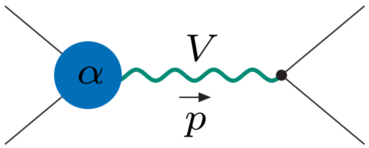
- inclusion of $\mathcal{M}^{(0)}$ is usually easier $+$ better than its expansion
- naive estimate of relative theoretical uncertainty (TU) in NLO:

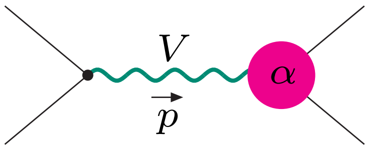
$$\mathrm{TU} \sim \begin{cases} \frac{\alpha}{\pi} \times \frac{\Gamma_V}{M_V} \times \mathrm{const.} & \text{in resonance region } |p^2-M_V^2| \lesssim M_V\Gamma_V \\ \frac{\alpha}{\pi} \times \mathrm{const.} & \text{off resonance } |p^2-M_V^2| \gg M_V\Gamma_V \end{cases}$$

Virtual factorizable corrections

$$\mathcal{M}^{(1)}_{\text{fact}} = \frac{W^{(1)}(M_V^2) - W^{(0)}(M_V^2)\Sigma^{(1)\prime}(M_V^2)}{p^2 - \mu_V^2}$$

$$= \sum_\lambda \frac{\mathcal{M}^{(1)}_{\mathrm{production}}(\lambda)\mathcal{M}^{(0)}_{\mathrm{decay}}(\lambda) + \mathcal{M}^{(0)}_{\mathrm{production}}(\lambda)\mathcal{M}^{(1)}_{\mathrm{decay}}(\lambda)}{p^2 - \mu_V^2}$$





Spin correlations: identical definitions of polarized states $|\phi(\lambda)\rangle$ needed in $\mathcal{M}^{(n)}_{\mathrm{production}}(\lambda)$ and $\mathcal{M}^{(n)}_{\mathrm{decay}}(\lambda)$

Subtlety in kinematics:

gauge invariance of $\mathcal{M}^{(n)}_{\mathrm{production/decay}}$ requires $p^2 = M_V^2$

$\hookrightarrow$ "on-shell projection" of momenta needed (impact beyond PA)

Example: W production $u\bar{d} \to \mathrm{W} \to \nu_l l^+$

$$\mathcal{M}^{(1)}_{\text{fact}} = \delta^{\mathrm{virt}}_{\mathrm{fact}}\,\mathcal{M}^{(0)}, \qquad \delta^{\mathrm{virt}}_{\mathrm{fact}} = \delta_{\mathrm{W}ud}(\hat{s} = M_{\mathrm{W}}^2) + \delta_{\mathrm{W}l\nu}(\hat{s} = M_{\mathrm{W}}^2) = \mathrm{const.}$$

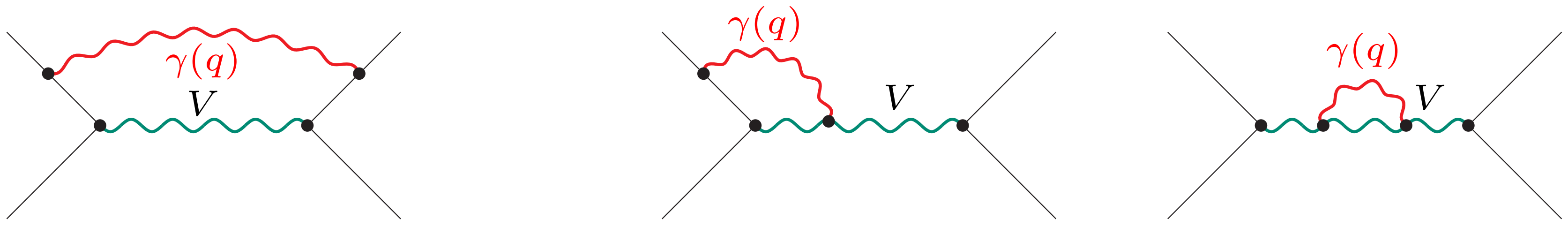
Virtual non-factorizable corrections

Fadin, Khoze, Martin '94; Melnikov, Yakovlev '96; Beenakker, Berends, Chapovsky '97; Denner, Dittmaier, Roth '97,'98

Origin:

on-shell limit ($p^2 \to M_V^2$) and IR regularization (e.g. $m_\gamma \to 0$) do not commute

in diagrams with exchange of γ/g between external and/or resonant lines:



"manifestly non-factorizable"

- no explicit propagator $(p^2 - M_V^2)^{-1}$
- resonant IR-divergent contribution

"not manifestly non-factorizable" diagrams

- explicit propagator $(p^2 - M_V^2)^{-1}$, contribution also to fact. corrections $W^{(1)}(M_V^2)$
- non-factorizable part:
$W^{(1)}_{\mathrm{non-fact}}(p^2) \;\equiv\; \left[W^{(1)}(p^2) - W^{(1)}(M_V^2)\right]_{p^2 \to M_V^2}$

General features:

Fadin, Khoze, Martin '94

- contributions only from soft momenta $|q^\mu| \sim \Gamma_V \ll M_V$
- virtual $+$ real non-fact. corrections cancel in inclusive quantities such as σ_{tot} (integration over virtuality of propagators, KLN argument)

Evaluation of virtual non-factorizable corrections

"Extended soft-photon approximation" $|q^\mu| \sim \Gamma_V \ll M_V$

- q only kept in singular propagators $\quad\to\quad$ only scalar integrals
- complex mass μ_V in resonant propagators
- limits $p^2, \mu_V^2 \to M_V^2$ taken whenever possible

$\hookrightarrow$ Result factorizes from Born amplitude: $\quad \mathcal{M}^{\mathrm{virt}}_{\mathrm{non-fact}} = \delta^{\mathrm{virt}}_{\mathrm{non-fact}}\, \mathcal{M}^{(0)}$

- gauge independence by construction
- IR-divergent terms like $\dfrac{c_\epsilon}{\epsilon}\left(\dfrac{p^2-\mu_V^2}{\mu M_V}\right)^{2\epsilon}$

  from non-commutativity of on-shell and soft-photon limits
- no collinear singularities

Example: W production $u\bar{d} \to \mathrm{W} \to \nu_l l^+$ S.D., Krämer '01

$$\delta^{\mathrm{virt}}_{\mathrm{non-fact}} = -\frac{\alpha}{2\pi}\Bigg\{-2 + Q_d\,\mathrm{Li}\left(1+\frac{M_\mathrm{W}^2}{\hat{t}_\mathrm{res}}\right) - Q_u\,\mathrm{Li}\left(1+\frac{M_\mathrm{W}^2}{\hat{u}_\mathrm{res}}\right)$$
$$- \left[\frac{c_\epsilon}{\epsilon} - 2\ln\left(\frac{M_\mathrm{W}^2 - \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W} - \hat{s}}{\mu M_\mathrm{W}}\right)\right]\left[1 + Q_d\ln\left(-\frac{M_\mathrm{W}^2}{\hat{t}_\mathrm{res}}\right) - Q_u\ln\left(-\frac{M_\mathrm{W}^2}{\hat{u}_\mathrm{res}}\right)\right]\Bigg\}$$

($\hat{t}_\mathrm{res}, \hat{u}_\mathrm{res}$ = on-shell projections of Mandelstam variables $\hat{t}, \hat{u}$; $c_\epsilon = \Gamma(1+\epsilon)(4\pi)^\epsilon$, $D = 4-2\epsilon$)

Pole expansion of real photonic corrections

NLO: 1-particle bremsstrahlung in LO (tree-level diagrams)

$\hookrightarrow$ LO prescriptions for resonances applicable

But: real $|\mathcal{M}_{i\to f+\gamma}|^2$ is related to $2\,\mathrm{Re}\{\mathcal{M}^{(0)*}_{i\to f}\mathcal{M}^{(1)}_{i\to f}\}$ in soft and collinear limits

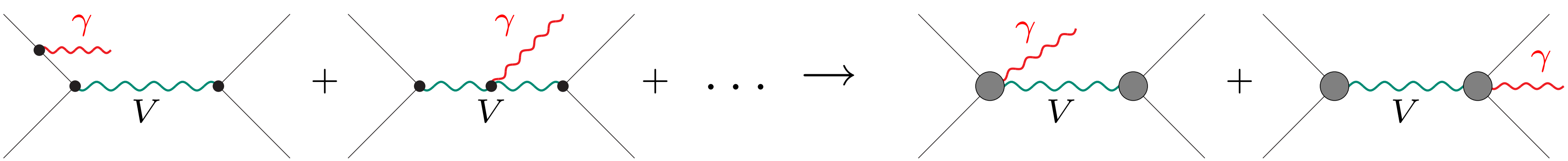
$\hookrightarrow$ matching between resonance descriptions in virtual and real corrections !

Pole expansions for real corrections:

Split diagrams with radiating resonances (2 resonant propagators) as follows:

$$\frac{1}{[(p+k)^2-\mu_V^2](p^2-\mu_V^2)} = \frac{1}{2pk}\left[\frac{1}{p^2-\mu_V^2} - \frac{1}{(p+k)^2-\mu_V^2}\right]$$

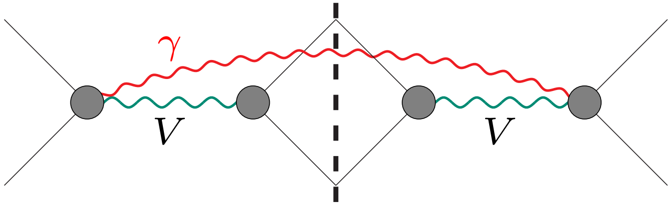
$\hookrightarrow$ decomposition of $\mathcal{M}_{i\to f+\gamma}$ into initial- and final-state radiation:



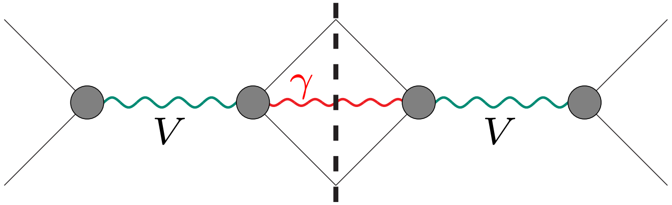
$\bullet$ = on-shell tree-like subamplitudes

Classification of real photonic corrections in PA

Factorizable contributions to $|\mathcal{M}|^2$:

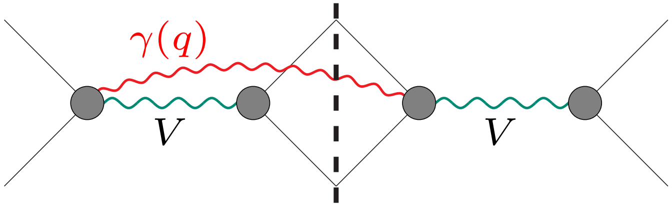


Initial-state radiation



Final-state radiation

Non-factorizable contributions to $|\mathcal{M}|^2$:



$+$ c.c.

Only $q = \mathcal{O}(\Gamma_V)$ relevant !

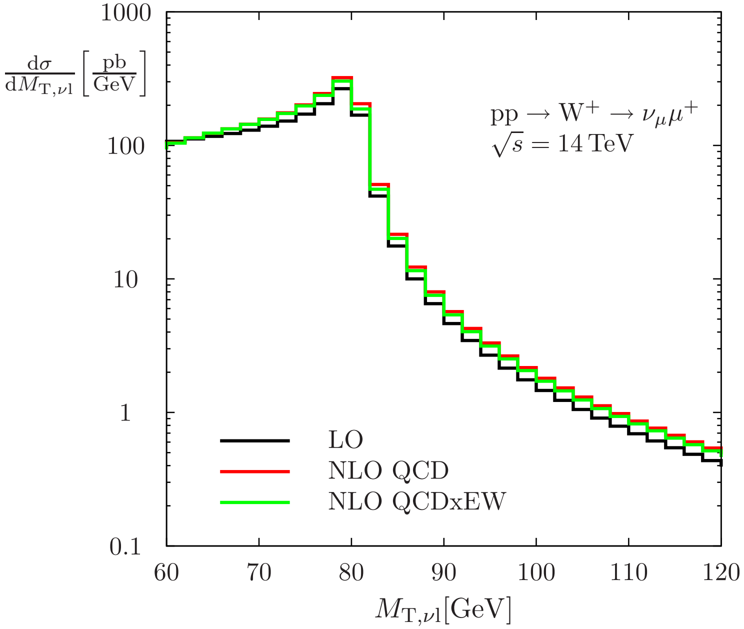
calculable from modified eikonal currents:

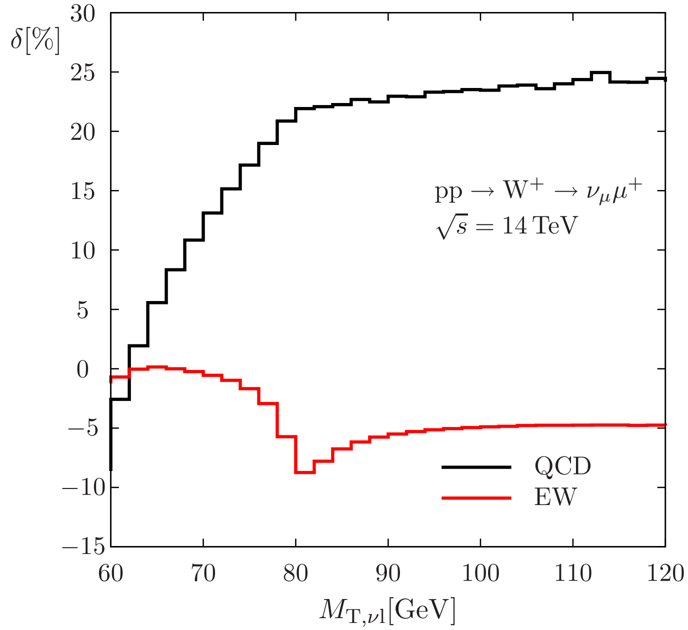
$$\mathrm{d}\sigma_{\mathrm{non-fact}} = \mathrm{d}\sigma_0\, \delta^{\mathrm{real}}_{\mathrm{non-fact}}, \qquad \delta^{\mathrm{real}}_{\mathrm{non-fact}} = \frac{\alpha}{2\pi^2}\int \frac{\mathrm{d}^3\mathbf{q}}{q^0}\, \mathrm{Re}\{\mathcal{J}^\mu_{\mathrm{prod}}\mathcal{J}^*_{\mathrm{dec},\mu}\},$$

$$\mathcal{J}^\mu_{\mathrm{prod}} = Q_1\frac{p_1^\mu}{p_1q} - Q_2\frac{p_2^\mu}{p_2q} - (Q_1-Q_2)\frac{(p_1+p_2)^\mu}{p_1q+p_2q},$$

$$\mathcal{J}^\mu_{\mathrm{dec}} = \left[-Q'_1\frac{k_1^\mu}{k_1q} + Q'_2\frac{k_2^\mu}{k_2q} + (Q'_1-Q'_2)\frac{(k_1+k_2)^\mu}{k_1q+k_2q}\right]\frac{(k_1+k_2)^2-\mu_V^2}{(k_1+k_2+q)^2-\mu_V^2}$$

Transverse-mass distribution for W production





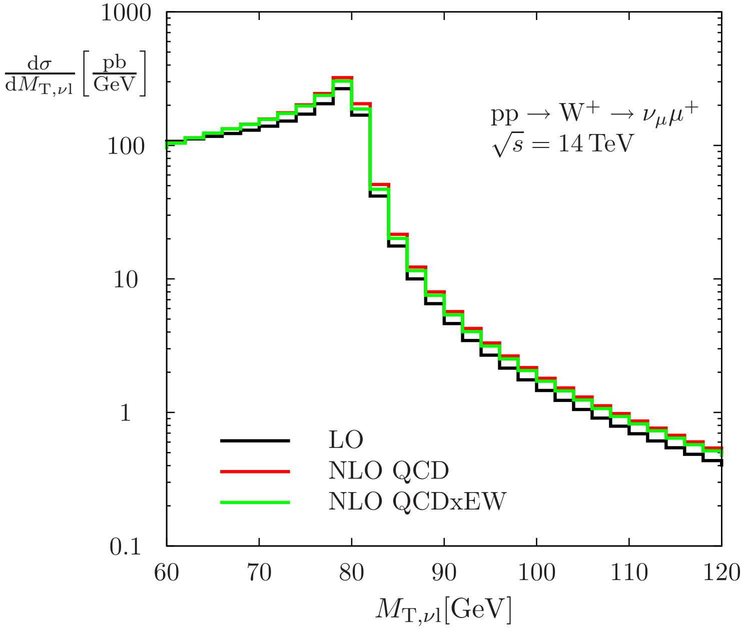
Features of $M_{\mathrm{T},\nu l}$:

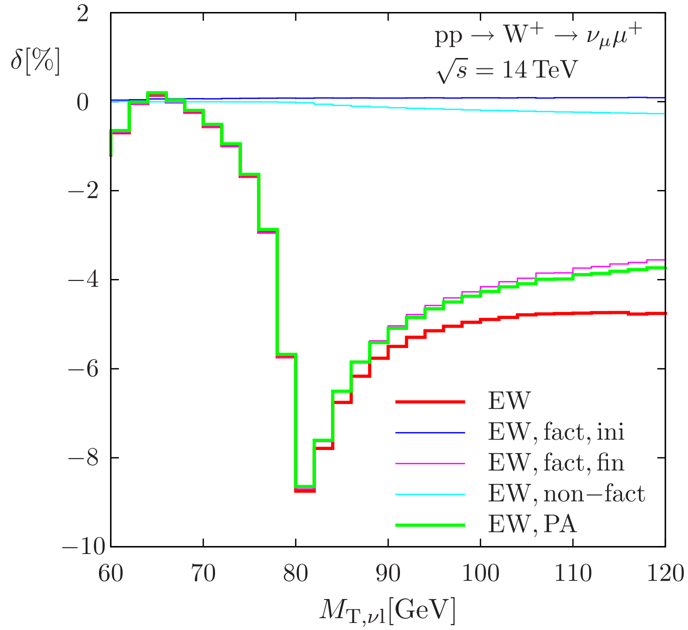
- most important observable for M_{W} det.
- stability wrt QCD corrs/uncertainties (insensitive to jet recoil)
- sensitive to detector effects via $\not{E}_{\mathrm{T}}$

Corrections:

- QCD corrections quite flat near resonance
- EW corrections distort resonance shape

Transverse-mass distribution for W production





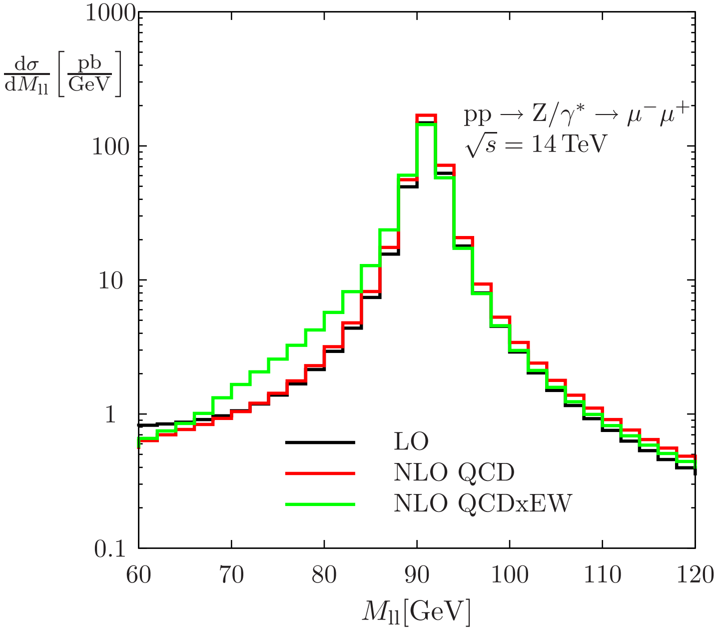
Features of $M_{\mathrm{T},\nu l}$:

- most important observable for M_{W} det.
- stability wrt QCD corrs/uncertainties (insensitive to jet recoil)
- sensitive to detector effects via $\not{E}_{\mathrm{T}}$

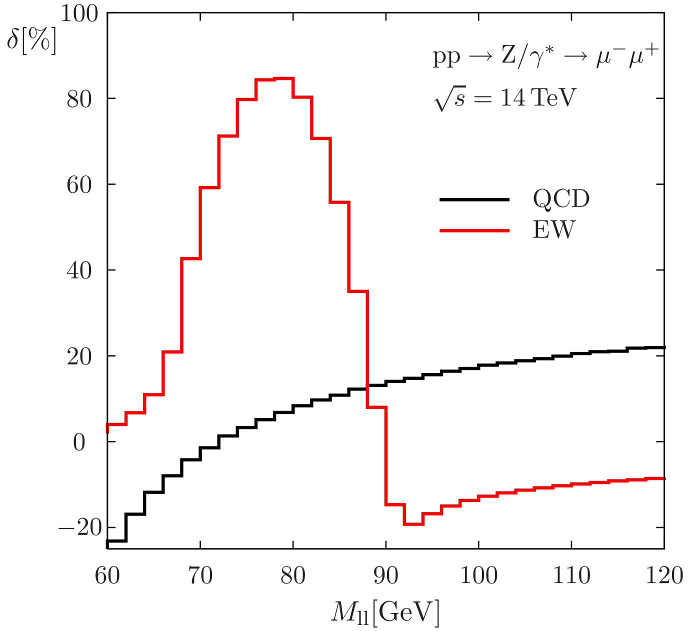
Pole approximation (PA):

- PA reproduces EW corr near resonance
- resonance distortion merely due to factorizable FS correction
- factorizable IS and non-fact. corrections flat (and even negligible)

Invariant-mass distribution for Z production



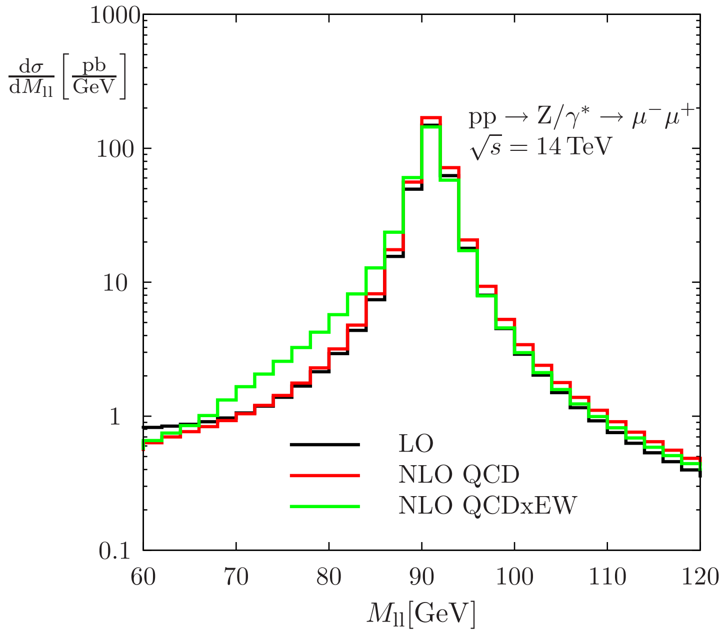
Reference process for M_W measurement

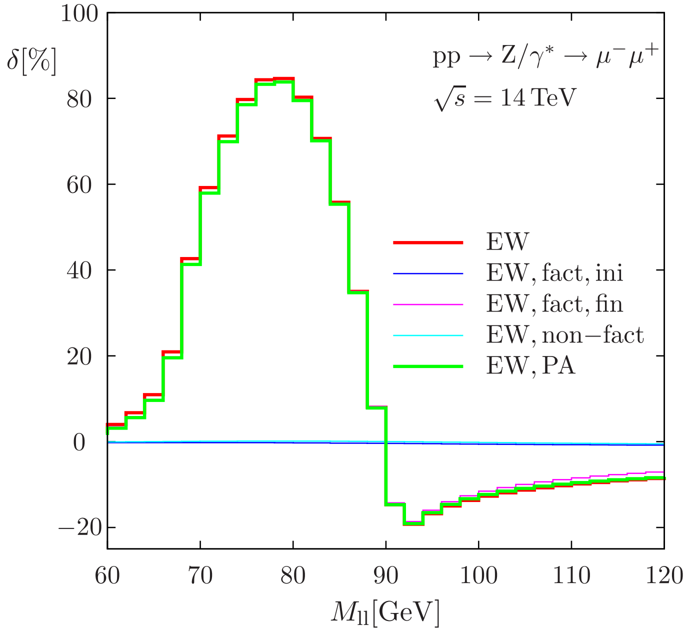


Corrections:

- QCD corrections quite flat near resonance
- EW corrections distort resonance shape

Invariant-mass distribution for Z production



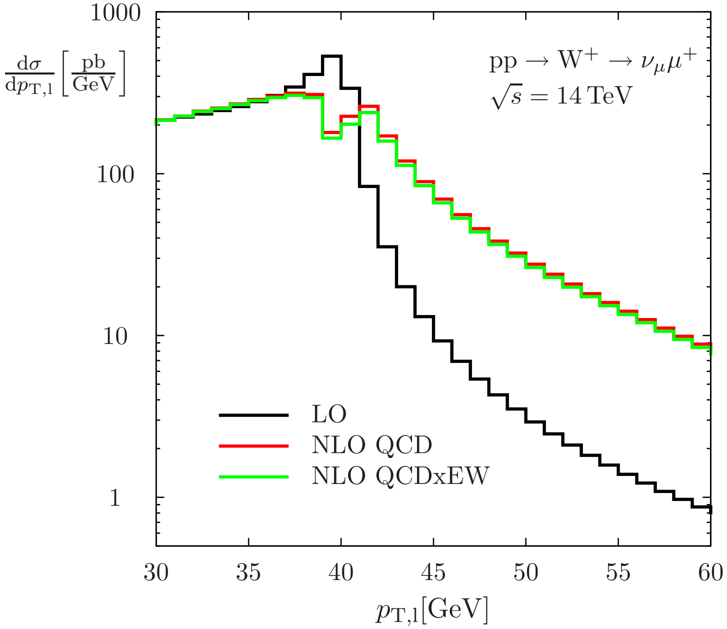


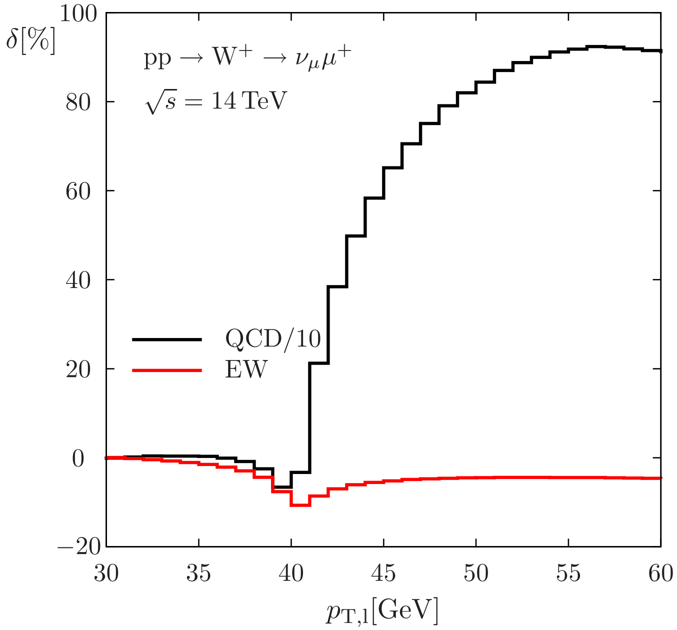
Reference process for M_W measurement

Behaviour of PA analogous to $M_{T,\nu l}$:

- PA reproduces EW corr near resonance
- resonance distortion merely due to factorizable FS correction
- factorizable IS and non-fact. corrections flat (and even negligible)

Transverse-momentum distribution for W production





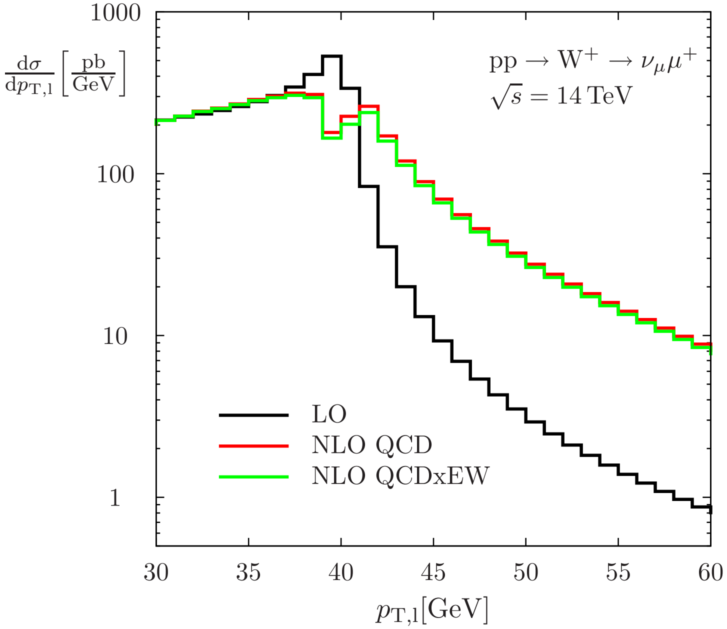
Features of $p_{\mathrm{T},l}$:

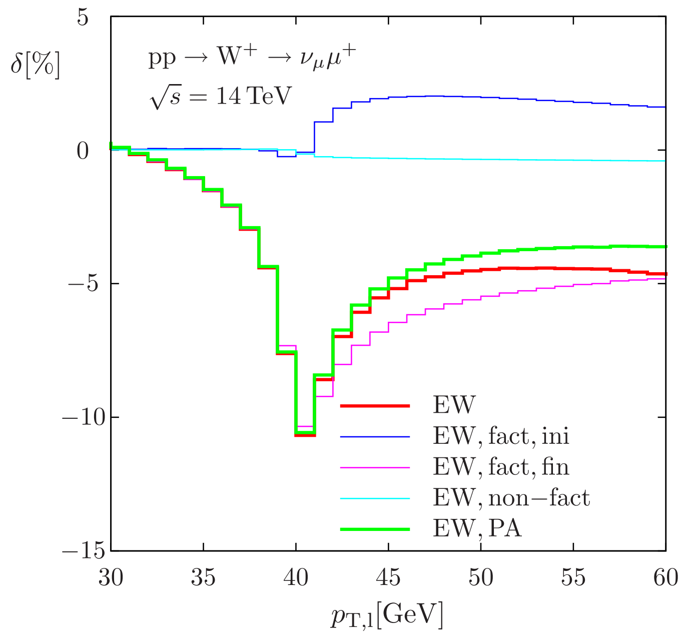
- also relevant for M_{W} measurement
- stability wrt detector effects
- sensitive to QCD effects/modelling/uncertainties

Corrections:

- QCD corrections huge above resonance (jet recoil)
- EW corrections distort resonance shape as well

Transverse-momentum distribution for W production





Features of $p_{\mathrm{T},l}$:

- also relevant for M_W measurement
- stability wrt detector effects
- sensitive to QCD effects/modelling/uncertainties

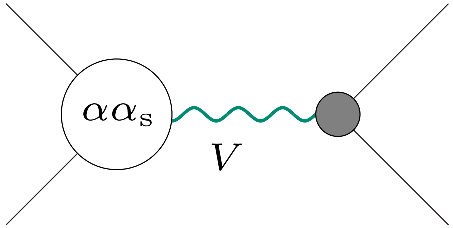
PA works well:

- EW corr reproduced near resonance
- factorizable FS corrs distort resonance shape
- factorizable IS corrs overwhelmed by QCD
- non-fact. corrs flat and negligible

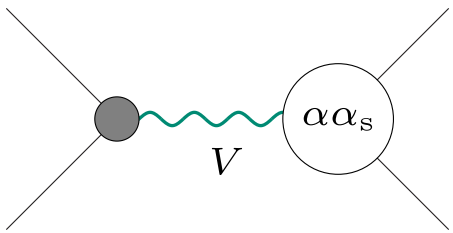
Pole expansion @ $\mathcal{O}(\alpha\alpha_\mathrm{s})$

Classification of $\mathcal{O}(\alpha\alpha_{\rm s})$ corrections in PA

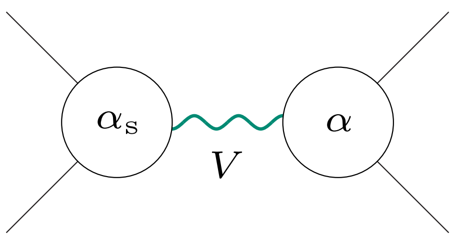
Factorizable contributions: (only virtual contributions indicated)



- no significant resonance distortion expected
- no PDFs with $\mathcal{O}(\alpha\alpha_{\rm s})$ corrections

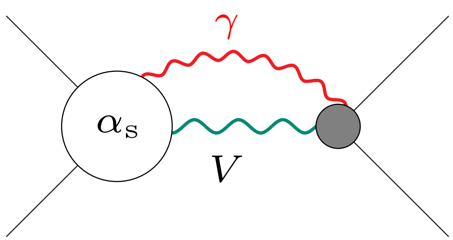


- only $Vl\bar{l}'$ counterterm contributions
  $\hookrightarrow$ uniform rescaling, no distortions



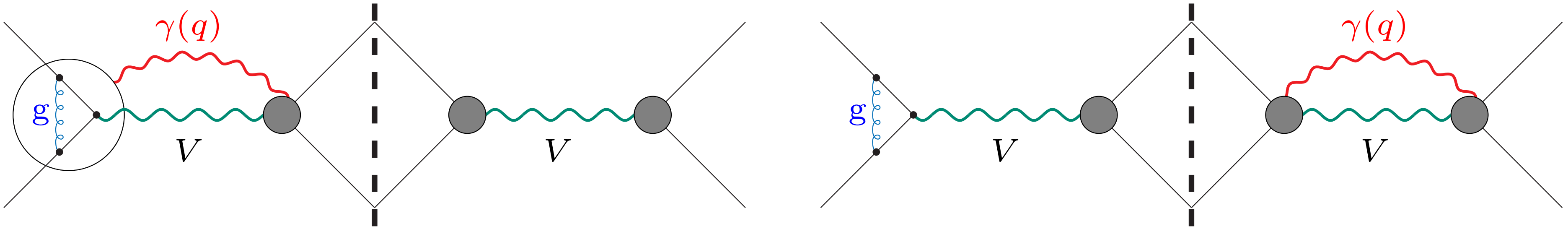
- significant resonance distortions from FSR

Non-factorizable contributions: (only virtual contributions indicated)



- small @ $\mathcal{O}(\alpha)$, but could be enhanced by large $\mathcal{O}(\alpha_{\rm s})$ corrections (jet recoil)
- calculated and discussed in the following

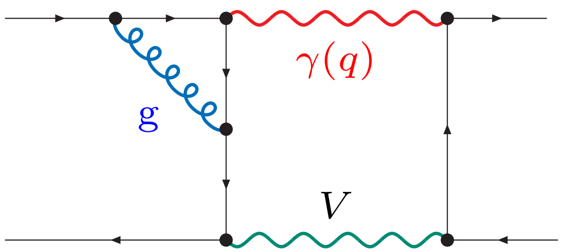
Virtual–virtual contributions to non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections



Result: $$|\mathcal{M}|^2\Big|_{\substack{\mathrm{non-fact}\\ \mathrm{virt-virt}}} = 4\,\mathrm{Re}\{\delta^{(\alpha_s,\mathrm{virt})}_{q\bar{q}'V}\}\;\mathrm{Re}\{\delta^{(\alpha,\mathrm{virt})}_{\mathrm{non-fact}}\}\,|\mathcal{M}_0|^2$$

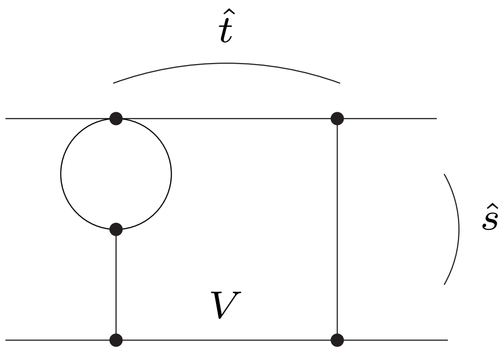
- factorized structure (1-loop) $\times$ (1-loop) after non-trivial cancellations
- expansion of all loops in $q^\mu \sim \Gamma_V \sim (p^2-\mu_V^2)/M_V \to 0$
- issue of overlapping IR singularities
- different methods applied $\to$ results agree
  - ◇ diagrammatic calculation (expansion via Mellin–Barnes technique)
  - ◇ gauge-invariance argument à la Yennie/Frauschti/Suura ’61
    (even holds to any order $\alpha\alpha_s^n$, $n=1,2,\ldots$)
  - ◇ effective field theory for unstable particles Beneke et al. ’03,’04

Example: Two-loop box graph



$$\sim \quad -\frac{C_\mathrm{F}\alpha_\mathrm{s}}{4\pi}\,\frac{Q_q Q_l \alpha}{2\pi}\,\mathcal{M}_0\,(1-\epsilon)\,(-\hat{t})\,(\mu_V^2-\hat{s})\,I(\hat{s},\hat{t})$$

Master integral:

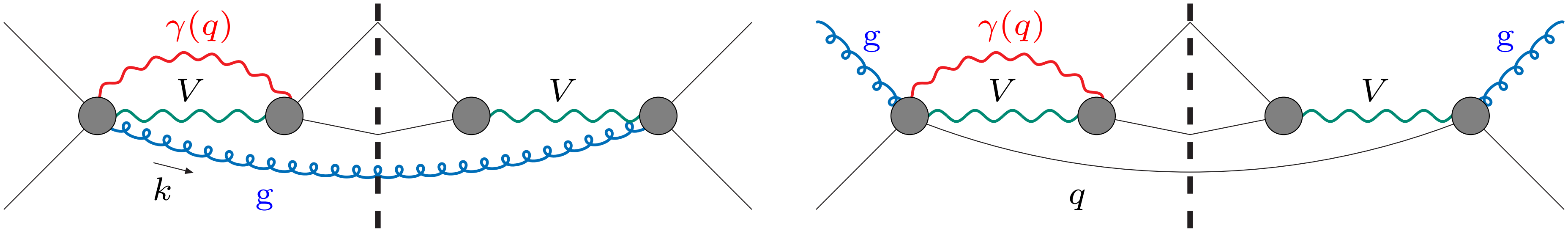


$$= \quad I(\hat{s},\hat{t}) \quad = \quad \left(\frac{(2\pi\mu)^{2\epsilon}}{\mathrm{i}\pi^2}\right)^2 \int \mathrm{d}^D q \int \mathrm{d}^D q' \,\frac{1}{q^2\cdots}$$

$$= \frac{c_\epsilon^2}{(-\hat{t})(\mu_V^2-\hat{s})}\left(\frac{\mu_V^2-\hat{s}}{M_V^2}\right)^{-3\epsilon}\left(\frac{-\hat{t}}{\mu^2}\right)^{-2\epsilon}\left\{\frac{1}{2\epsilon^3}+\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\left[\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)+\frac{5\pi^2}{12}+2\right]\right.$$
$$+\,2\,\mathrm{Li}_3\left(\frac{-\hat{t}}{M_V^2}\right)+\mathrm{Li}_3\left(1+\frac{\hat{t}}{M_V^2}\right)-6\zeta(3)+\ln^2\left(\frac{-\hat{t}}{M_V^2}\right)\ln\left(1+\frac{\hat{t}}{M_V^2}\right)$$
$$\left.-\,2\ln\left(\frac{-\hat{t}}{M_V^2}\right)\left[\frac{\pi^2}{6}-\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)\right]+\frac{5\pi^2}{6}+2\,\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)+4+\mathcal{O}(\hat{s}-\mu_V^2)+\mathcal{O}(\epsilon)\right\}$$

Note: many cancellations in sum over all contributions $(1/\epsilon^4, \mathrm{Li}_3, \zeta(3), \ldots)$

Virtual–real contributions to non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections



Result: $$|\mathcal{M}|^2\Big|_{\substack{\mathrm{non-fact}\\ \mathrm{virt-real}}} = 2\,\mathrm{Re}\{\delta^{(\alpha,\mathrm{virt})}_{\mathrm{non-fact},q\bar{q}'\to l\bar{l}'\mathrm{g}}\}\,|\mathcal{M}_{0,q\bar{q}\to l\bar{l}'\mathrm{g}}|^2,$$ etc.

- From explicit diagrammatic calculation analogous to NLO $\mathcal{O}(\alpha)$ calculation
- New feature in $q\mathrm{g}$ channels: γ exchange between final-state particles
  Structure different from initial–final interferences $\rightarrow$ enhancement ?

Example: W production $u\bar{d} \to \mathrm{W} \to \nu_l l^+\mathrm{g}$

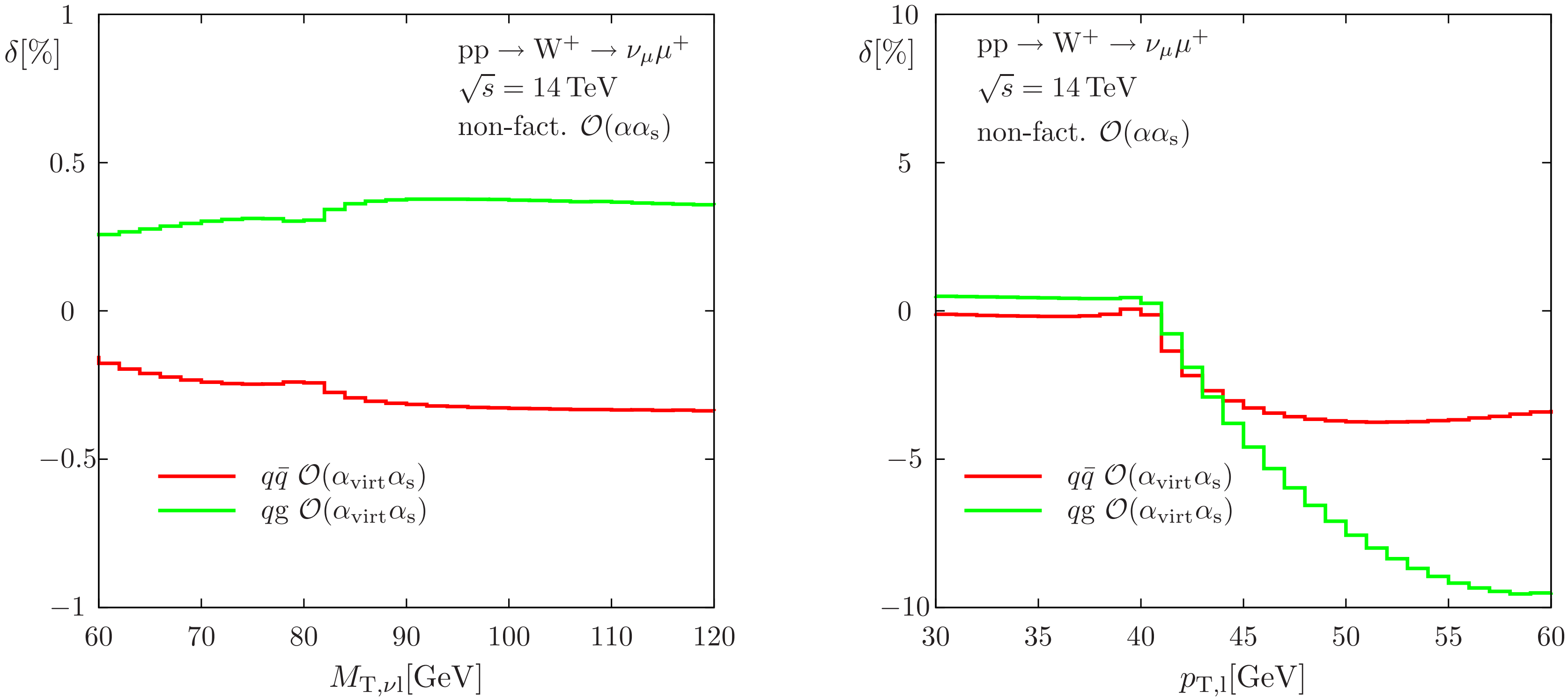
$$\delta^{(\alpha,\mathrm{virt})}_{\mathrm{non-fact},u\bar{d}\to\nu_l l^+\mathrm{g}} = -\frac{\alpha}{2\pi}\Bigg\{-2 + Q_d\,\mathrm{Li}\left(1+\frac{M_\mathrm{W}^2-\hat{t}_{u\mathrm{g}}}{\hat{t}_{dl}}\right) - Q_u\,\mathrm{Li}\left(1+\frac{M_\mathrm{W}^2-\hat{t}_{d\mathrm{g}}}{\hat{t}_{ul}}\right)$$
$$-\left[\frac{c_\epsilon}{\epsilon} - 2\ln\left(\frac{\mu_\mathrm{W}^2-\hat{s}}{\mu M_\mathrm{W}}\right)\right]\left[1 + Q_d\ln\left(\frac{M_\mathrm{W}^2-\hat{t}_{u\mathrm{g}}}{-\hat{t}_{dl}}\right) - Q_u\ln\left(\frac{M_\mathrm{W}^2-\hat{t}_{d\mathrm{g}}}{-\hat{t}_{ul}}\right)\right]\Bigg\}$$

($\hat{t}_{qj} = (p_q - k_j)^2$, on-shell projection for W !)

$$\xrightarrow[k\to 0]{} \delta^{(\alpha,\mathrm{virt})}_{\mathrm{non-fact},u\bar{d}\to\nu_l l^+}$$

Virtual-photon + soft-photon non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

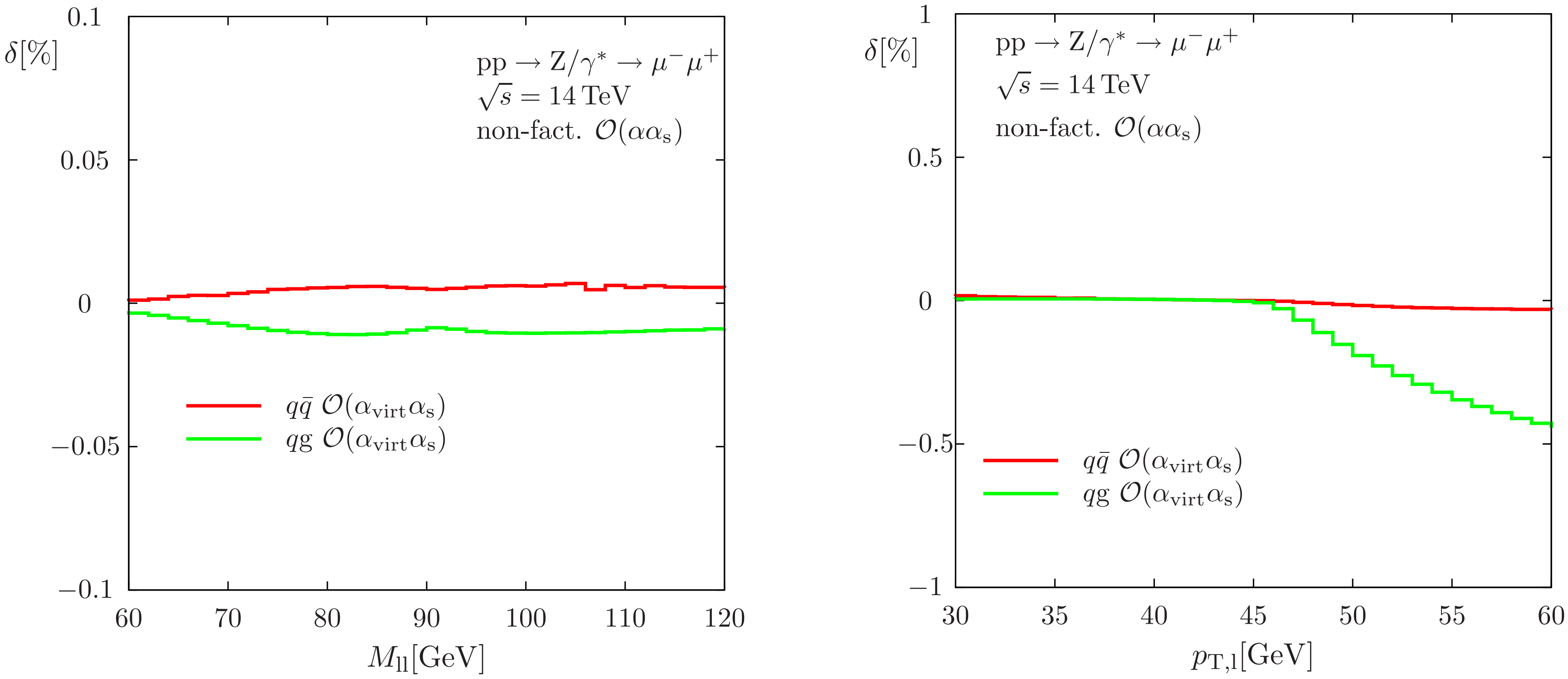
W production:



- $\delta = \delta_{\mathrm{non-fact,virt}\gamma} + \delta_{\mathrm{non-fact,soft}\gamma}(E_\gamma < \Delta E), \quad \Delta E = 10^{-4}\sqrt{\hat{s}}/2 \ll \Gamma_V$
- $M_{\mathrm{T},\nu l}$: corrections and distortion very small
- $p_{\mathrm{T},l}$: corrections several % with distortion
  $\hookrightarrow$ cancellation against real photonic corrections ??

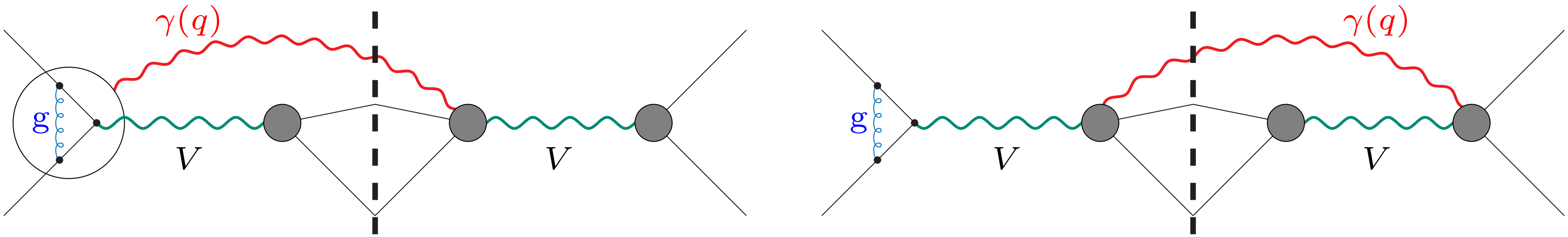
Virtual-photon + soft-photon non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

Z production:



- $\delta = \delta_{\mathrm{non-fact,virt}\gamma} + \delta_{\mathrm{non-fact,soft}\gamma}(E_\gamma < \Delta E), \quad \Delta E = 10^{-4}\sqrt{\hat{s}}/2 \ll \Gamma_V$
- M_{ll}: corrections and distortion tiny
- $p_{\mathrm{T},l}$: corrections and distortion small

Real–virtual contributions to non-factorizable $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrections



Result:

$$|\mathcal{M}|^2\Big|_{\substack{\mathrm{non-fact}\\ \mathrm{real-virt}}} = 2\,\mathrm{Re}\{\delta^{(\alpha_\mathrm{s},\mathrm{virt})}_{q\bar{q}'V}\}\,\delta^{(\alpha,\mathrm{real})}_{\mathrm{non-fact}}\,|\mathcal{M}_0|^2,$$

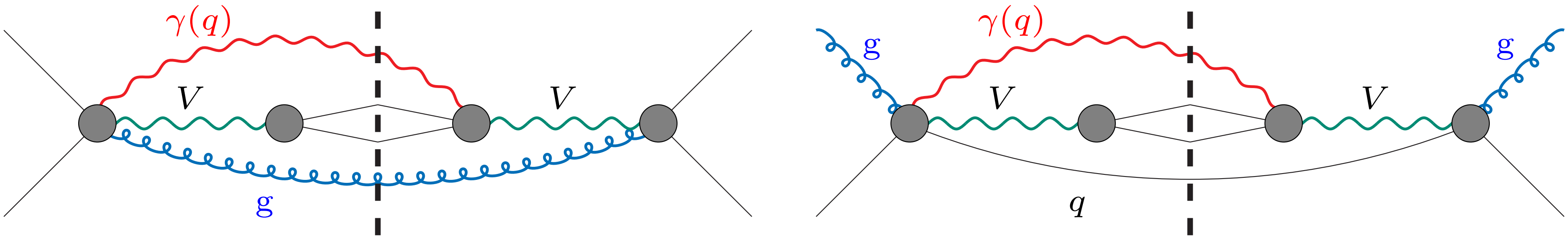
$$\delta^{(\alpha,\mathrm{real})}_{\mathrm{non-fact}} = \frac{\alpha}{2\pi^2}\int\frac{\mathrm{d}^3\mathbf{q}}{q^0}\,\mathrm{Re}\{\mathcal{J}^\mu_{\mathrm{prod}}\mathcal{J}^*_{\mathrm{dec},\mu}\}$$

- factorization, e.g., justified by YFS argument as in virtual–virtual case
- modified eikonal currents $\mathcal{J}_{\mathrm{prod}}, \mathcal{J}_{\mathrm{dec}}$ as in $\mathcal{O}(\alpha)$
- factorization/decoupling of γ phase space in PA accuracy,

  i.e. $\delta(p_q + p_{q'} - k_l - k_{l'} \cancel{-q})$ in phase space

  $\hookrightarrow$ technical simplification

Real–real contributions to non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections



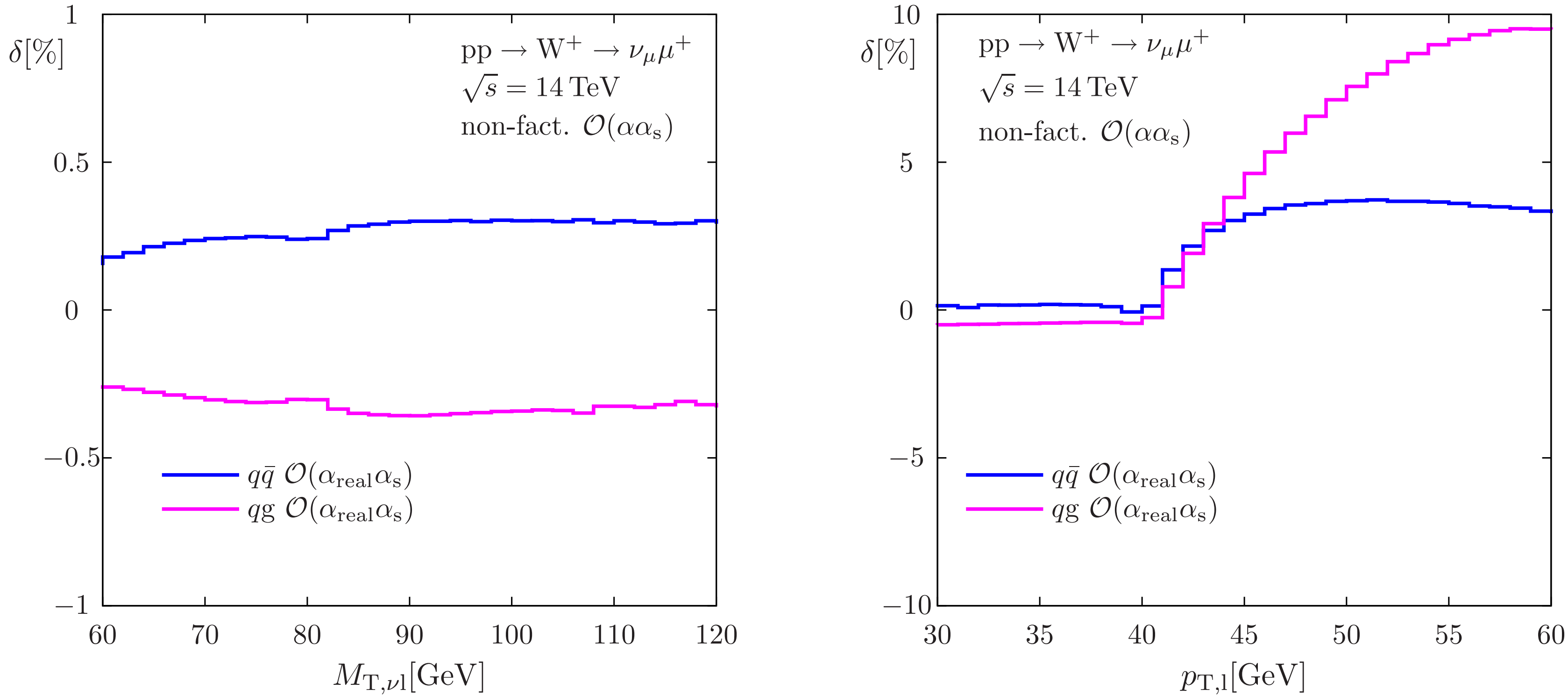
Result: $$\left.|\mathcal{M}|^2\right|_{\substack{\text{non-fact}\\ \text{real-real}}} = \delta^{(\alpha,\text{real})}_{\text{non-fact}}\,|\mathcal{M}_{0,q\bar{q}\to l\bar{l}'\text{g}}|^2, \qquad \text{etc.}$$

$$\delta^{(\alpha,\text{real})}_{\text{non-fact}} = \frac{\alpha}{2\pi^2}\int\frac{\mathrm{d}^3\mathbf{q}}{q^0}\,\mathrm{Re}\{\mathcal{J}^{\mu}_{\text{prod}}\mathcal{J}^{*}_{\text{dec},\mu}\}$$

- use exact momentum conservation $p_{q/\text{g}} + p_{q'} = k_l + k_{l'} + k_{\text{g}/q} + q$ in modified eikonal currents $\mathcal{J}_{\text{prod}}, \mathcal{J}_{\text{dec}}$
- γ phase-space factorization possible in PA

Real-photon and full non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

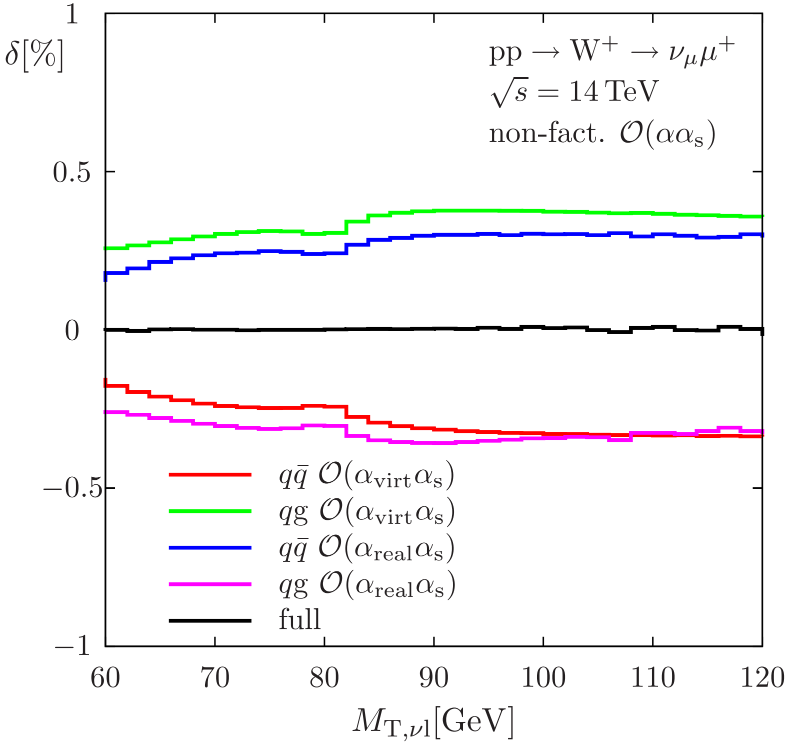
W production:

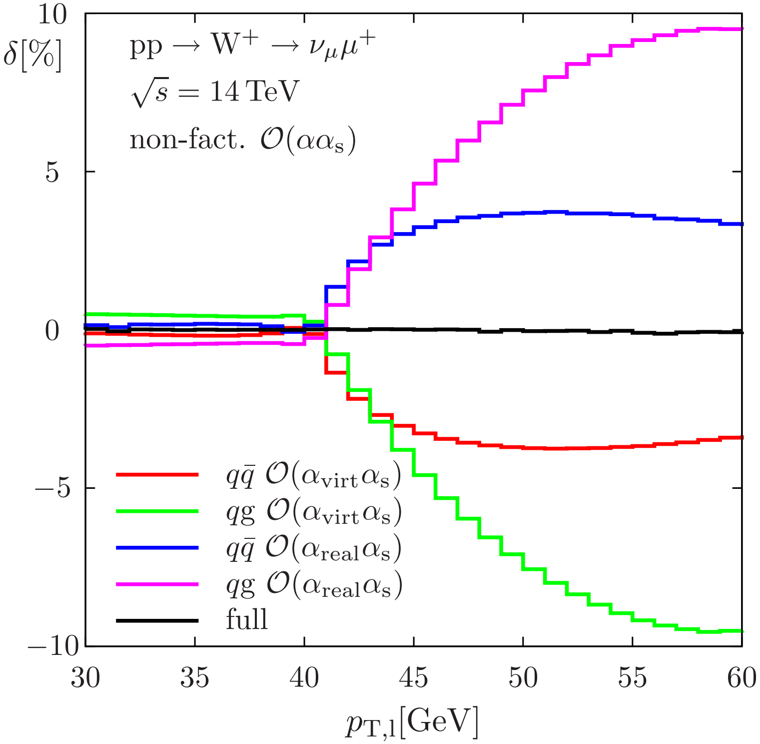


- $\delta \;=\; \delta_{\mathrm{non-fact,real}\gamma}(E_\gamma > \Delta E), \quad \Delta E = 10^{-4}\sqrt{\hat{s}}/2 \ll \Gamma_V$

Real-photon and full non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

W production:

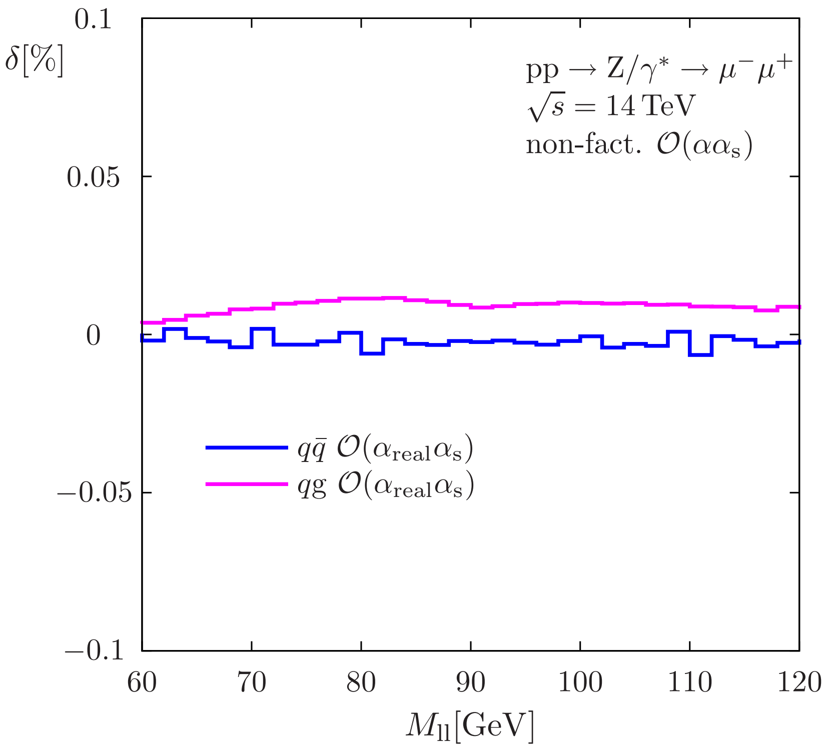


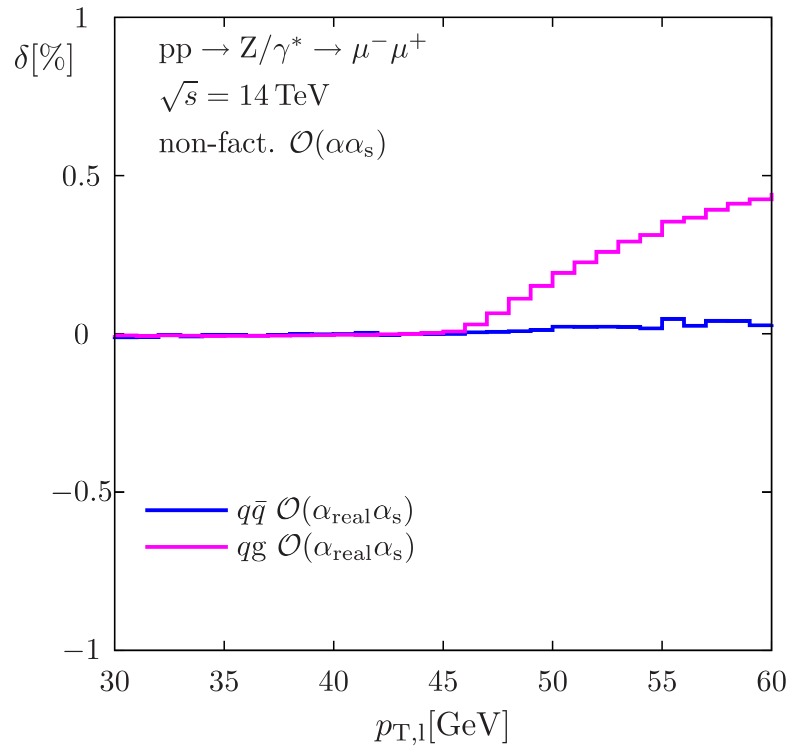


- $\delta \;=\; \delta_{\mathrm{non-fact,real}\gamma}(E_\gamma > \Delta E), \quad \Delta E = 10^{-4}\sqrt{\hat{s}}/2 \ll \Gamma_V$
- Full non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections tiny
  due to complete cancellation between virtual and real corrections

Real-photon and full non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

Z production:

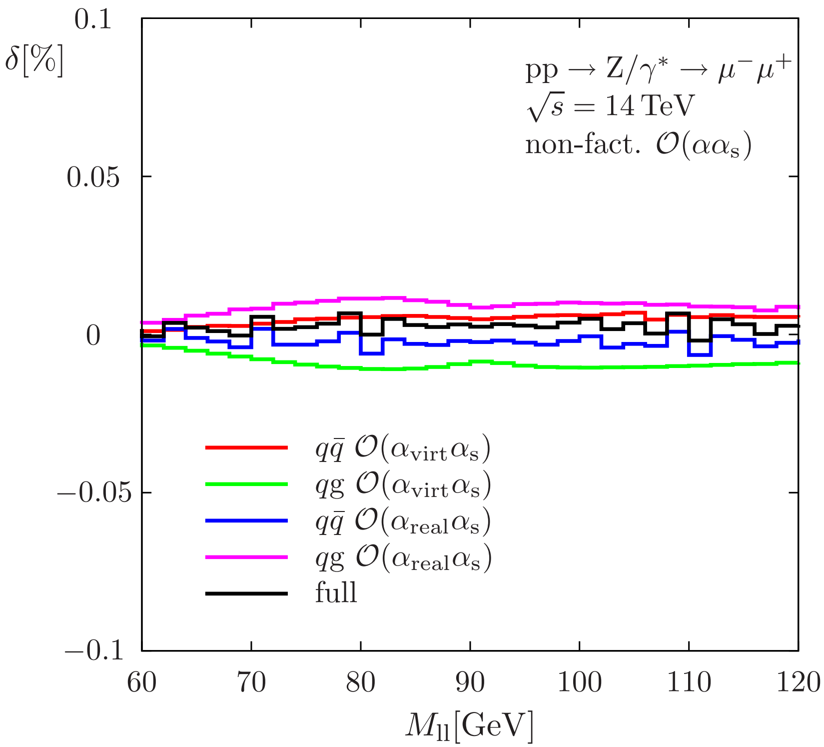


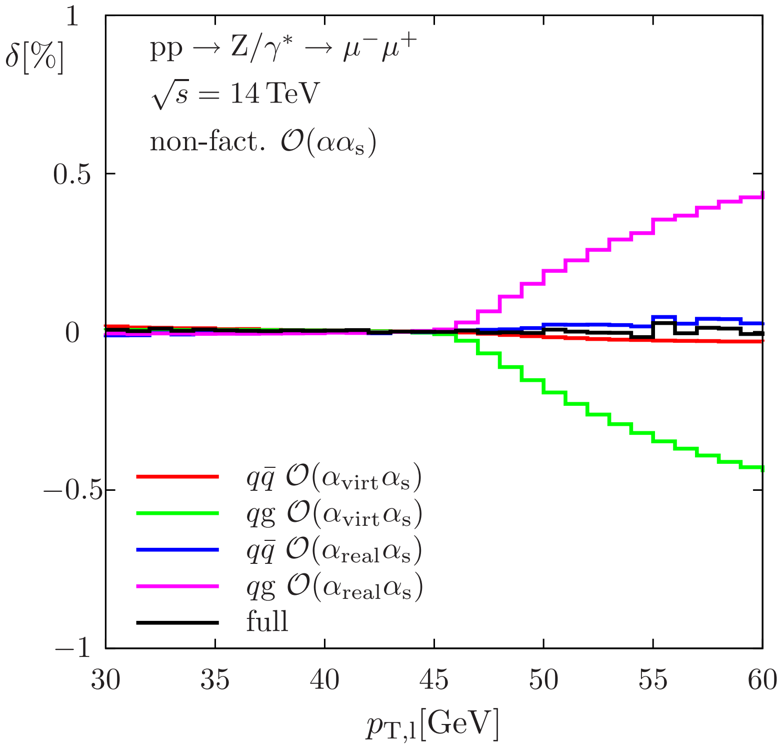


- $\delta \;=\; \delta_{\mathrm{non-fact,real}\gamma}(E_\gamma > \Delta E), \quad \Delta E = 10^{-4}\sqrt{\hat{s}}/2 \ll \Gamma_V$

Real-photon and full non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

Z production:





- $\delta \;=\; \delta_{\mathrm{non-fact,real}\gamma}(E_\gamma > \Delta E), \quad \Delta E = 10^{-4}\sqrt{\hat{s}}/2 \ll \Gamma_V$
- Full non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections tiny

  due to complete cancellation between virtual and real corrections

Summary & outlook

High-precision Drell–Yan physics @ LHC

- promises M_W with accuracy $\Delta M_\mathrm{W} \sim 8\,\mathrm{MeV}$ and $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$ with $\mathcal{O}($LEP precision$)$
- requires highest possible theoretical precision near resonances
  NNLO QCD + NLO EW + QCD resummations etc. known
  $\mathcal{O}(\alpha\alpha_\mathrm{s})$ is biggest unknown correction

$\mathcal{O}(\alpha\alpha_\mathrm{s})$ in pole approximation

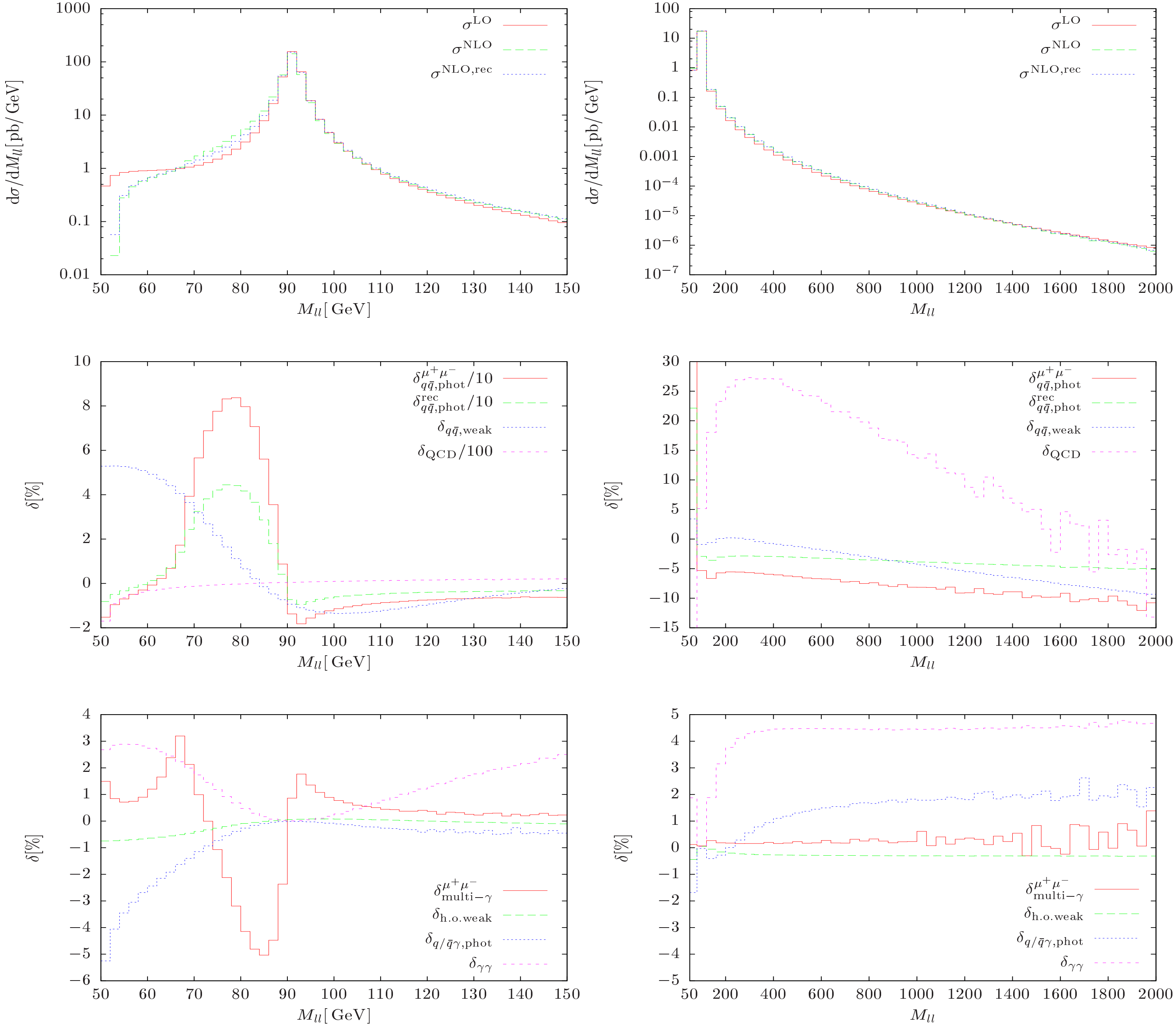
- non-factorizable corrections calculated $\to$ negligible
  $\hookrightarrow$ only factorizable corrections to $2 \to 1$ and/or $1 \to 2$ processes relevant
- $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrections to $q\bar{q}' \to V$ production
  $\hookrightarrow$ no significant resonance distortion expected
- $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrections to $V' \to l\bar{l}'$ decay
  $\hookrightarrow$ only irrelevant rescaling of distributions (only from counterterms)
- $\left[\mathcal{O}(\alpha_\mathrm{s}) \text{ to } q\bar{q}' \to V\right] \otimes \left[\mathcal{O}(\alpha) \text{ to } V' \to l\bar{l}'\right]$
  $\hookrightarrow$ significant resonance distortions expected, but straightforward to calculate

... in progress

Backup slides

Corrections to Z production – overview

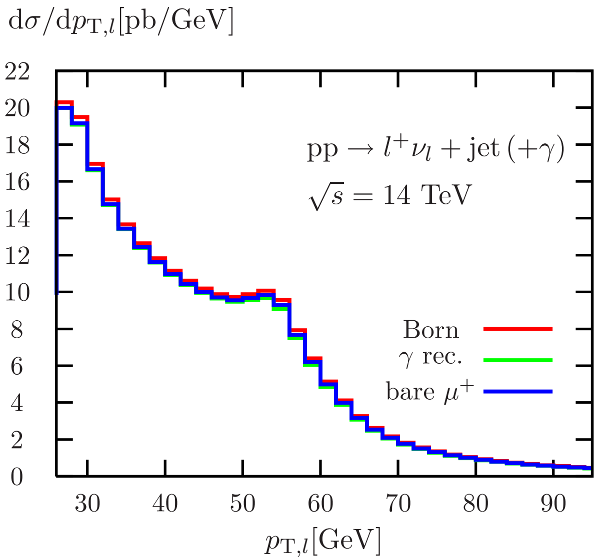
S.D., Huber '09

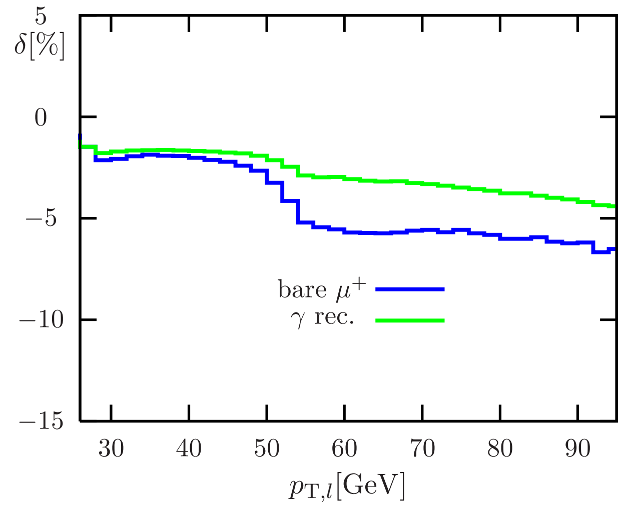


Comparison of EW corrections to W+jet and single (jet-inclusive) W production

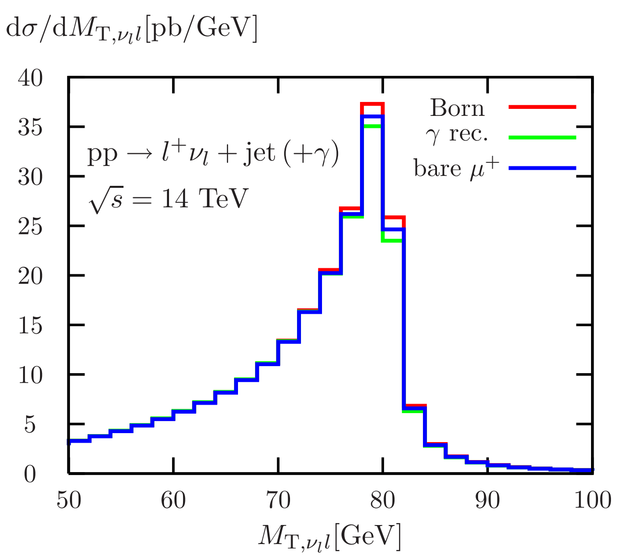
$\hookrightarrow$ argument for factorization QCD$\times$EW if EW corrections coincide

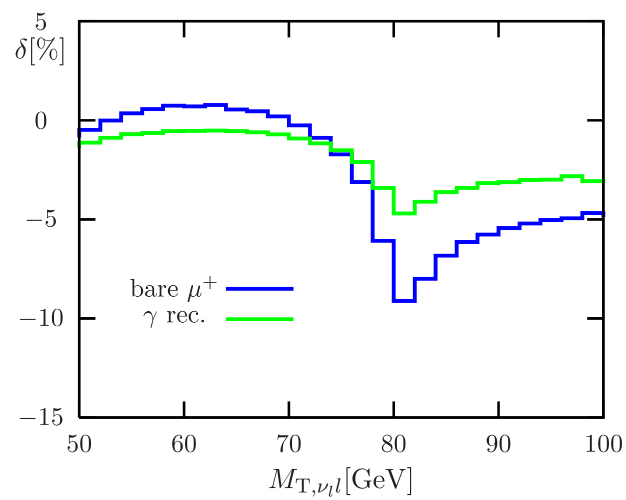
Denner et al. '09





Denner et al. '09





Comparison of EW corrections to W+jet and single (jet-inclusive) W production

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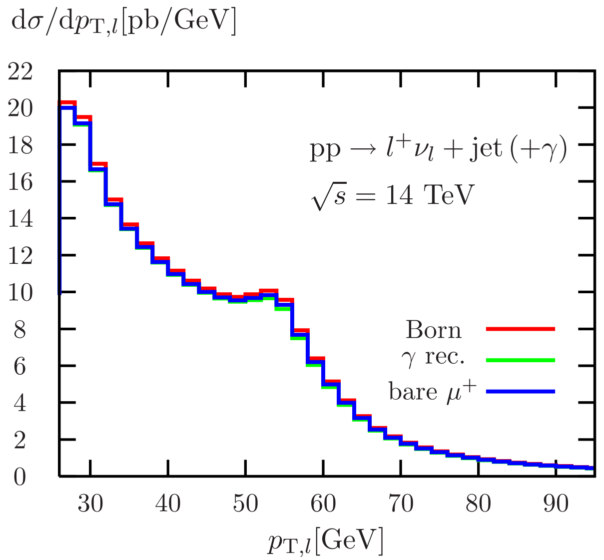
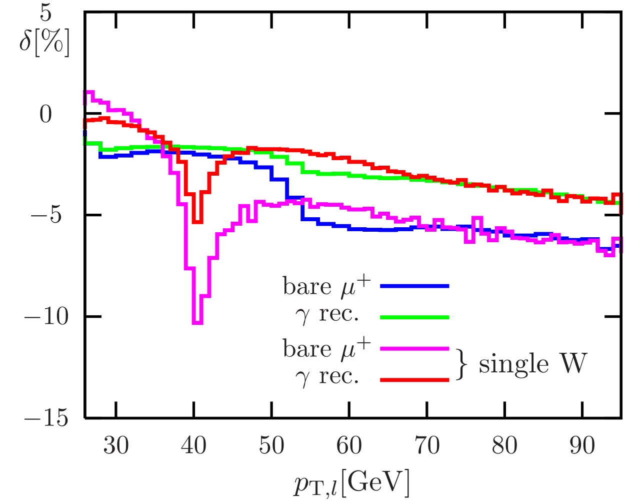
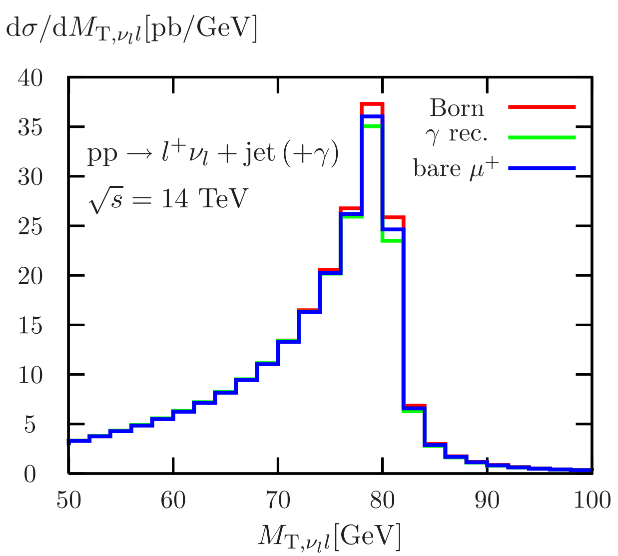


Denner et al. ’09

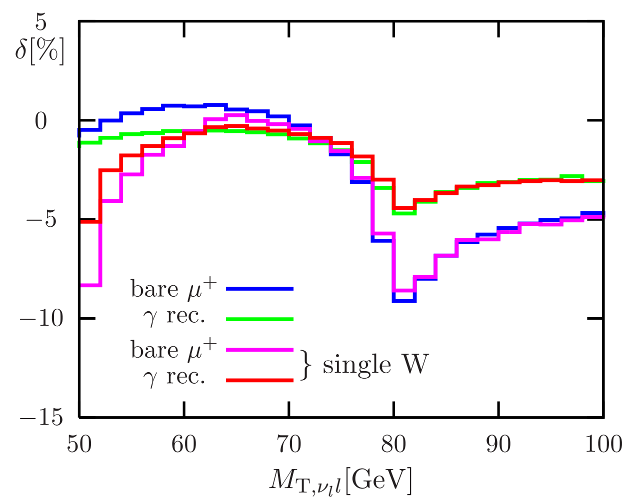


Jet recoil destroys simple factorization !

Single-W results from S.D./Krämer ’01; Brensing et al. ’07



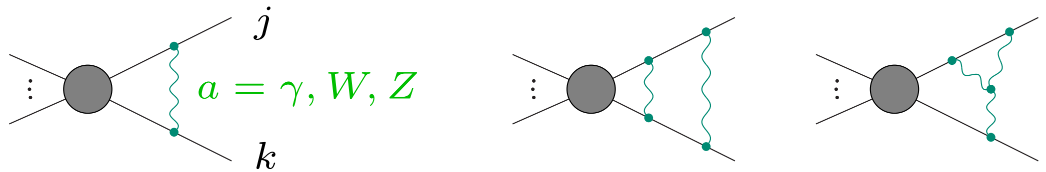
Denner et al. ’09



EW corrections factorize from hard gluon emission near Jacobian peak !

Electroweak radiative corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange



etc.

+ sub-leading logarithms from collinear singularities

Typical impact on $2 \to 2$ reactions at $\sqrt{s} \sim 1\,\mathrm{TeV}$:

$$\delta_{\mathrm{LL}}^{1-\mathrm{loop}} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -26\%, \qquad \delta_{\mathrm{NLL}}^{1-\mathrm{loop}} \sim +\frac{3\alpha}{\pi s_{\mathrm{W}}^2} \ln\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 16\%$$

$$\delta_{\mathrm{LL}}^{2-\mathrm{loop}} \sim +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 3.5\%, \qquad \delta_{\mathrm{NLL}}^{2-\mathrm{loop}} \sim -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -4.2\%$$

$\Rightarrow$ Corrections still relevant at 2-loop level

Note: differences to QED / QCD where Sudakov log's cancel

- massive gauge bosons W, Z can be reconstructed
  $\hookrightarrow$ no need to add "real W, Z radiation"
- non-Abelian charges of W, Z are "open" $\longrightarrow$ Bloch–Nordsieck theorem not applicable

Extensive theoretical studies at fixed perturbative (1-/2-loop) order and suggested resummations via evolution equations

Beccaria et al.; Beenakker, Werthenbach; Ciafaloni, Comelli; Denner, Pozzorini; Fadin et al.; Hori et al.; Melles; Kühn et al., Denner et al. '00–'0

Electroweak radiative corrections at high energies (continued)

- NLO EW high-energy logs – an approximation for full NLO EW ?
  - – miss finite contributions of $\mathcal{O}(\alpha)$
  - – do not include photonic radiation effects
  - + very simple approximation in Sudakov regime:
    s and $|t|$ large for $2 \to 2$ $\Rightarrow$ large p_{T} !
  - – fail in non-Sudakov regime:
    e.g. s large, but $|t|$ NOT large for $2 \to 2$ $\Rightarrow$ e.g. large M_{ll} in Drell–Yan !
  - + generically included in ALPGEN Chiesa, Montagna, Piccinini et al. '13
- Real W and Z emission processes
  - ◇ cannot be fully separated from underlying process
    (e.g. hadronically decaying W/Z's in jet environment)
  - ◇ partially compensate negative EW corrections
    $\hookrightarrow$ strongly dependent on W/Z reconstruction / separation
  - ◇ can be included by multipurpose LO MC's for $\mathcal{O}(\alpha)$
    Note: 2-loop EW high-energy logs require WW/WZ/... emission
    and 1-loop W/Z emission counterparts !

Electroweak radiative corrections at high energies (continued)

Example: Drell–Yan production

Neutral current: $\mathrm{pp} \to \mathrm{l}^+\mathrm{l}^-$ at $\sqrt{s} = 14\,\mathrm{TeV}$ (based on S.D./Huber arXiv:0911.2329)

$M_{\mathrm{ll}}/\mathrm{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	738.733(6)	32.7236(3)	1.48479(1)	0.0809420(6)	0.00679953(3)	0.000303744(1)
$\delta^{\mathrm{rec}}_{\mathrm{q\bar{q},phot}}/\%$	-1.81	-4.71	-2.92	-3.36	-4.24	-5.66
$\delta_{\mathrm{q\bar{q},weak}}/\%$	-0.71	-1.02	-0.14	-2.38	-5.87	-11.12
$\delta^{(1)}_{\mathrm{Sudakov}}/\%$	0.27	0.54	-1.43	-7.93	-15.52	-25.50
$\delta^{(2)}_{\mathrm{Sudakov}}/\%$	-0.00046	-0.0067	-0.035	0.23	1.14	3.38

no Sudakov domination!

Charged current: $\mathrm{pp} \to \mathrm{l}^+\nu_{\mathrm{l}}$ at $\sqrt{s} = 14\,\mathrm{TeV}$ (based on Brensing et al. arXiv:0710.3309)

$M_{\mathrm{T},\nu_{\mathrm{l}}\mathrm{l}}/\mathrm{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	4495.7(2)	27.589(2)	1.7906(1)	0.084697(4)	0.0065222(4)	0.00027322(1)
$\delta^{\mu^+\nu_\mu}_{\mathrm{q\bar{q}}}/\%$	$-2.9(1)$	$-5.2(1)$	$-8.1(1)$	$-14.8(1)$	$-22.6(1)$	$-33.2(1)$
$\delta^{\mathrm{rec}}_{\mathrm{q\bar{q}}}/\%$	$-1.8(1)$	$-3.5(1)$	$-6.5(1)$	$-12.7(1)$	$-20.0(1)$	$-29.6(1)$
$\delta^{(1)}_{\mathrm{Sudakov}}/\%$	0.0005	0.5	-1.9	-9.5	-18.5	-29.7
$\delta^{(2)}_{\mathrm{Sudakov}}/\%$	-0.0002	-0.023	-0.082	0.21	1.3	3.8

Sudakov domination!