

Threshold Production of Unstable Top

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- Top-antitop threshold production
 - *brief introduction*

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 - *beyond the complex energy shift*
 - *effective theory of unstable particles “ ρ NRQCD”*
 - *unstable top production in NLO and NNLO*

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 - *brief introduction*
- Top width effect:
 - *beyond the complex energy shift*
 - *effective theory of unstable particles “ ρ NRQCD”*
 - *unstable top production in NLO and NNLO*
- **Based on: A. Penin, J. Piclum, JHEP 1201 (2012) 034**

Why top threshold scan at a LC?

● Theory

- *top quark width is a natural infrared cutoff*
- ➔ *first principle QCD predictions*

● Experiment

- *as clean as possible for a strongly interacting particle*

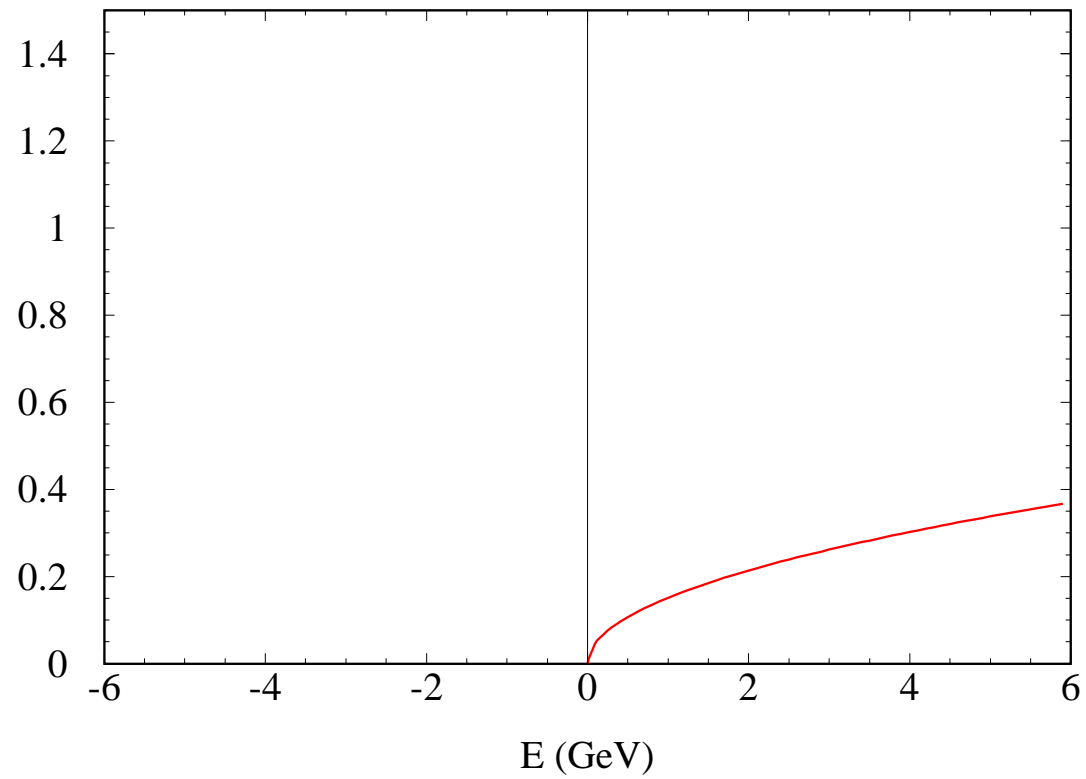
● Phenomenology

- *most precise determination of top quark*

mass, width, vector couplings

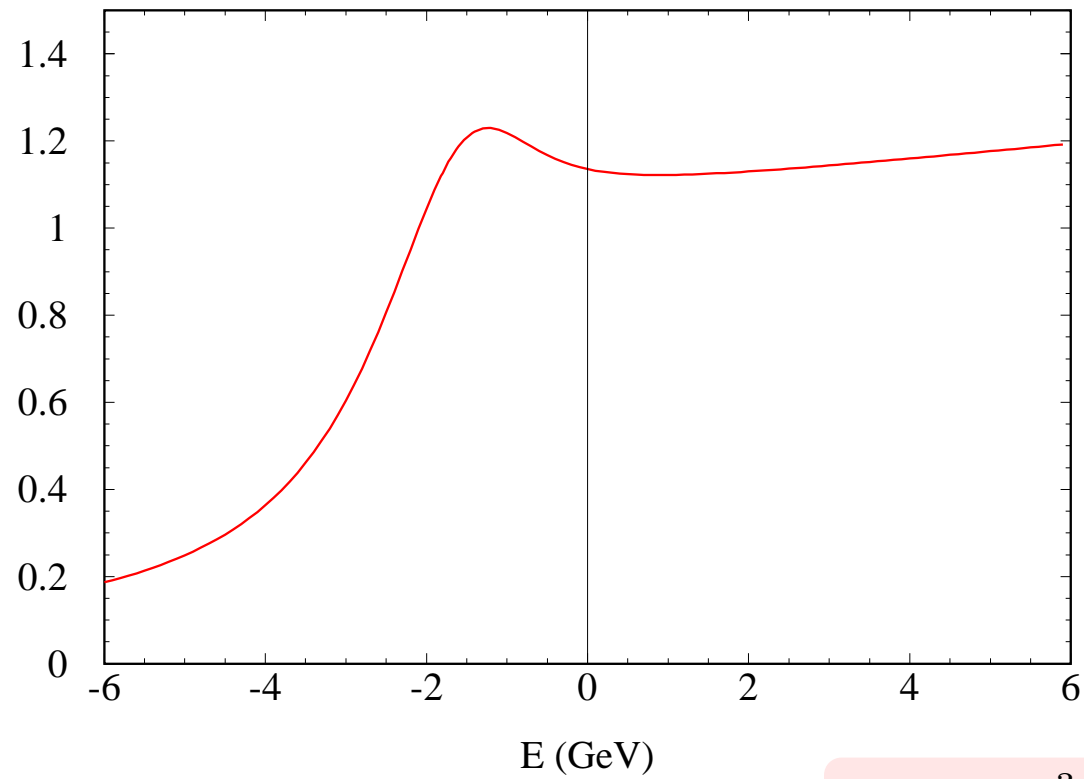
Born cross section

$$R(e^+e^- \rightarrow t\bar{t})$$



Coulomb and finite width effects

$R(e^+e^- \rightarrow t\bar{t})$



$$R_{\text{res}} \sim \frac{\alpha_s^3}{m_t \Gamma_t}, \quad E_{\text{res}} \sim \alpha_s^2 m_t$$

Perturbation theory for heavy quarkonium

- Apparent slow convergence

➔ *Full N^3LO analysis is mandatory*

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- ➔ *Full N^3LO analysis is mandatory*

- ✓ *resonance energy*

- (A. Penin, M. Steinhauser Phys.Lett. B538 (2002) 335-345)

- ✗ *resonance cross section (work in progress)*

- (big collaboration, work in progress)

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- **Fine effects of top instability become important**

Finite top lifetime

- Resonant approximation

- *complex energy shift* $E \rightarrow E + i\Gamma_t$

(V.Fadin, V.Khoze, JETP Lett. 46 (1987) 525)

- *not consistent in pNRQCD beyond LO!*

Finite top lifetime

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● Nonresonant contribution

- *Phase space matching (tight cuts on top invariant mass)*

(A. Hoang, C. Reißer, P. Ruiz-Femenía, Phys. Rev. D82 (2010) 014005)

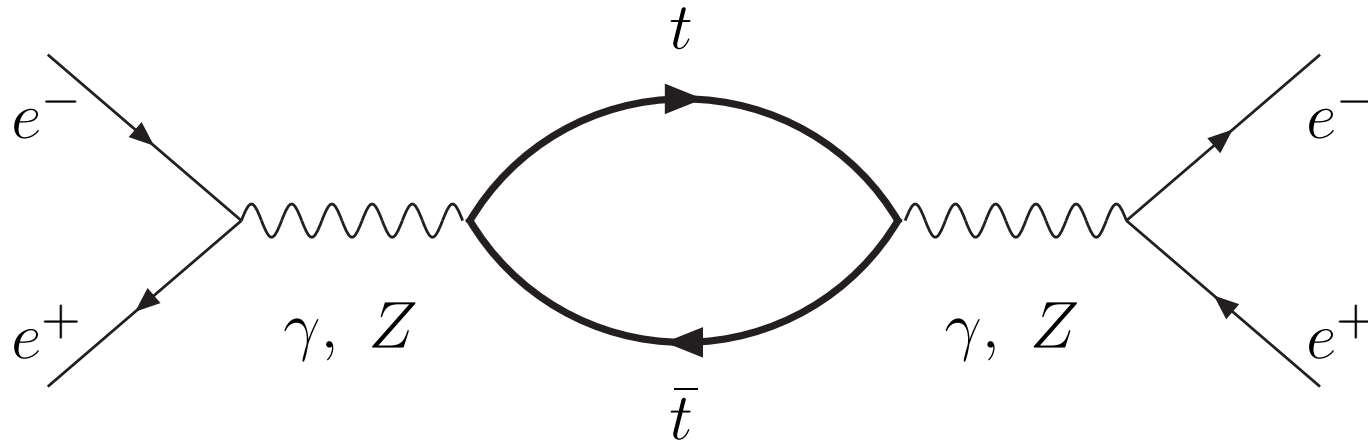
- *QCD effective theory of unstable particles to NLO*

(M. Beneke, B. Jantzen, P. Ruiz-Femenía, Nucl. Phys. B840 (2010) 186)

- *NRQCD effective theory of unstable particles to NNLO*

(A. Penin, J. Piclum, JHEP 1201 (2012) 034)

Stable top



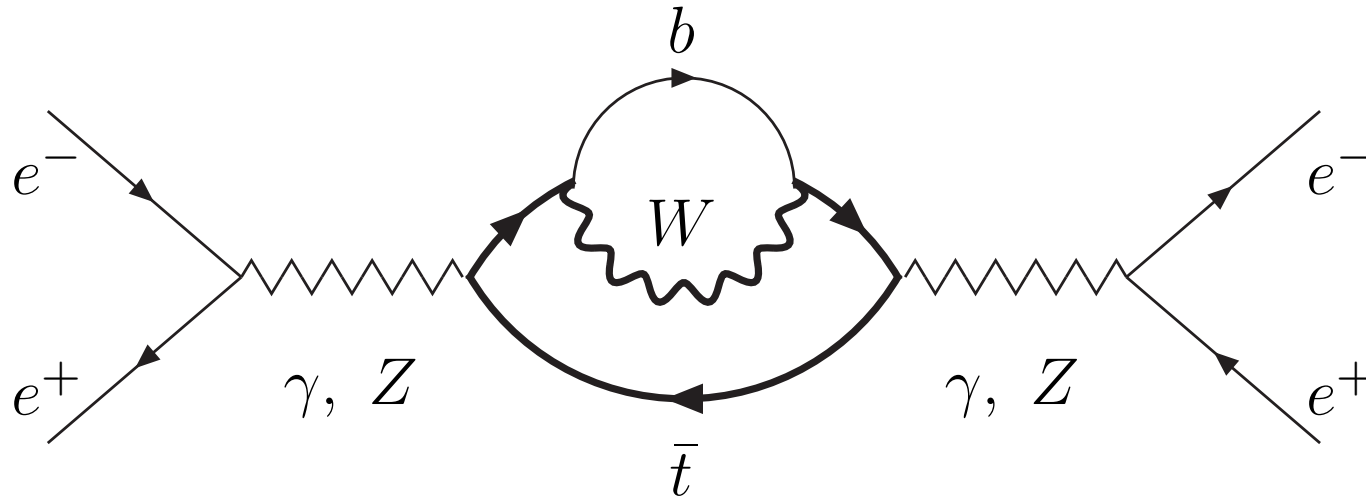
Optical theorem:

$$R_{res}^{Born} \sim \Im \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{1}{\mathbf{p}^2 - m_t E - i\epsilon} \sim \Im \sqrt{-E - i\epsilon},$$

On-shell top:

$$\Im \left[\frac{1}{\mathbf{p}^2 - m_t E - i\epsilon} \right] \sim \delta(\mathbf{p}^2 - m_t E),$$

Unstable top

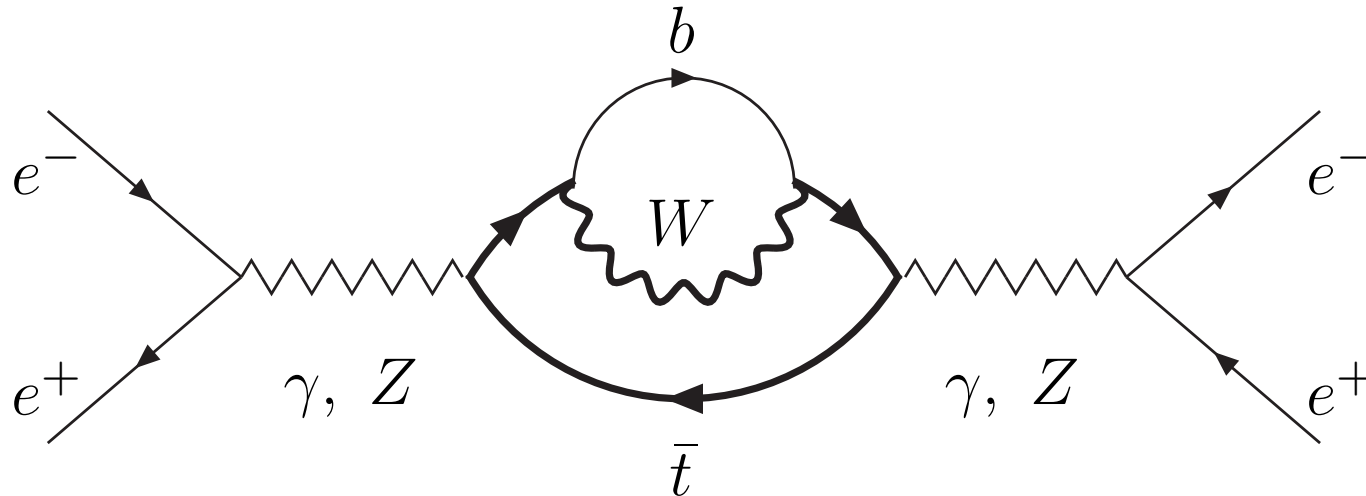


Imaginary part of mass operator:

here $\rho = 1 - M_W/m_t$, $z = (\mathbf{p}^2 - m_t E)/m_t^2 \ll 1$

$$\Im[\Sigma(z)] = \frac{\Gamma_t}{2} - \frac{\Gamma_t}{2} \left[\theta(z - \rho) + \left(\frac{2z}{\rho} - \frac{z^2}{\rho^2} \right) \theta(\rho - z) + \mathcal{O}(\rho, z) \right]$$

Unstable top



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Resonant contribution

Nonresonant contribution

Resonant contribution

- Complex energy shift:

- *Dyson resummation*

$$\frac{1}{\mathbf{p}^2 - m_t E - i\epsilon} \rightarrow \frac{1}{\mathbf{p}^2 - m_t E - im_t \Gamma_t}$$

- *Breit-Wigner resonance*

$$\delta(\mathbf{p}^2 - m_t E) \rightarrow \frac{1}{\pi} \frac{\Gamma_t}{(\mathbf{p}^2 - m_t E)^2 + m_t^2 \Gamma_t^2},$$

- *Born cross section*

$$R_{res}^{Born} \sim \Im \left[\sqrt{-E - i\Gamma_t} \right]$$

- Invariant mass distribution:

- $2\mathbf{p}^2 \approx m_t^2 - (p_W + p_b)^2$

Nonresonant contribution

● On-shell $t \Leftrightarrow$ on-shell W and b

● *kinematical constraint* $M_W^2 < (p_W + p_b)^2 < m_t^2$

● *natural cutoff on spatial momentum* $0 < \mathbf{p}^2 < \rho m_t^2$

● $\Im[\Sigma] - \Gamma_t/2 \neq 0$ for $\mathbf{p}^2 \neq 0 \Leftrightarrow$ "nonresonant"

Nonresonant contribution

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- Approximation $\rho \ll 1$

- *nonrelativistic t and W , ultrarelativistic b*

- *expansion in ρ similar to pNRQCD expansion in $v^2 \sim E/m_t$*

- *actual value $\rho = 0.53 \dots$*

Nonrelativistic effective theory of unstable top

• Scales

• $pNRQCD$:

hard m_t *soft* $v m_t$ *ultrasoft* $v^2 m_t$

• $\rho NRQCD$:

hard m_t ρ -*soft* $\rho^{1/2} m_t$ ρ -*ultrasoft* ρm_t

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• Scale hierarchy and power counting

• $pNRQCD$ scaling: $\alpha_{ew}^{1/2} \sim \alpha_s \sim v \ll 1$, $\Gamma_t/m_t \sim \alpha_{ew}$

• complimentary expansion in ρ with $v \ll \rho^{1/2} \ll 1$

• ρ -Coulomb terms $\alpha_s/\rho^{1/2} \ll 1$

Nonrelativistic effective theory of unstable top

- How to expand?
 - ρ - p NRQCD Feynman rules
 - expansion by regions

Nonrelativistic effective theory of unstable top

● How to expand?

✗ ρ - p NRQCD Feynman rules

✓ expansion by regions

NLO nonresonant contribution

- Power counting

- *resonant contribution*
- *nonresonant contribution*

$$\Im\sqrt{-E - i\Gamma_t} \sim v$$

$$\Gamma_t \sim v^2$$

NLO nonresonant contribution

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- *resonant contribution*
- *nonresonant contribution*

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● Calculation steps

- *treat $\Im[\Sigma] - \Gamma_t/2$ as a perturbation*
 - *add all the two-loop diagrams with t - W - b cut*
 - *expand in $E/\rho m_t$, expand in $\rho \Leftrightarrow$ single region left:
 t and W are ρ -potential, b is ρ -ultrasoft*
- ➔ *recover nonrelativistic propagators and vertices*

NLO nonresonant contribution

● Power counting

- *resonant contribution*
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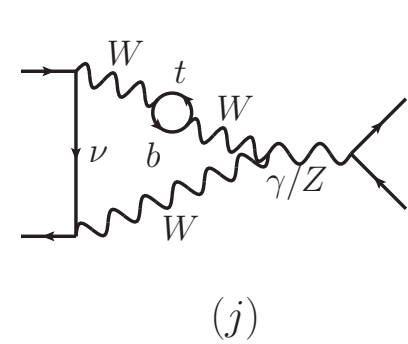
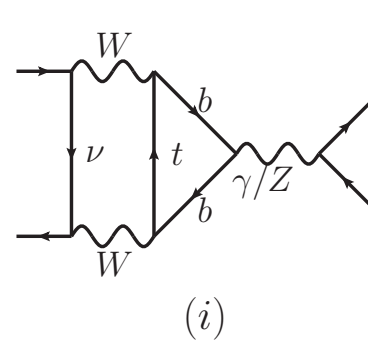
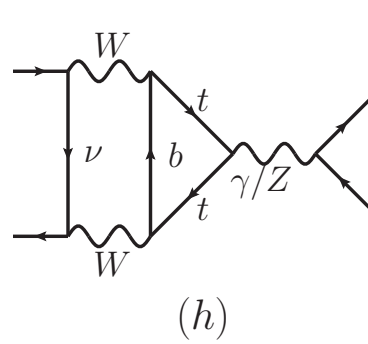
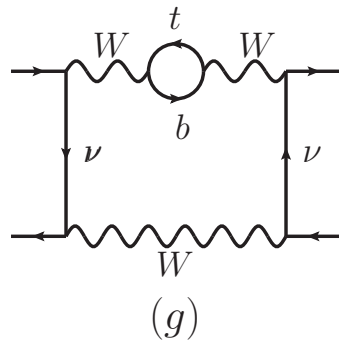
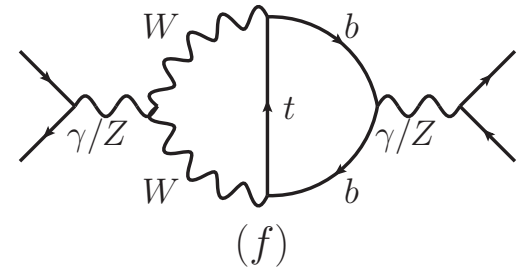
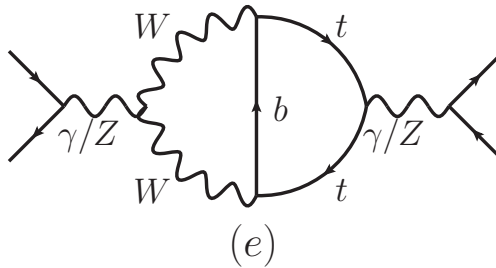
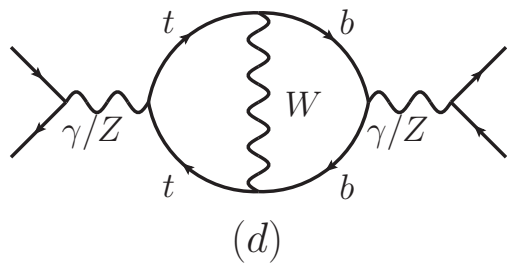
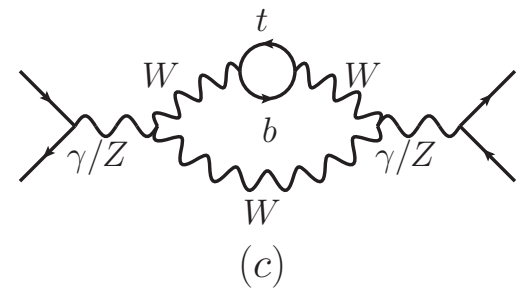
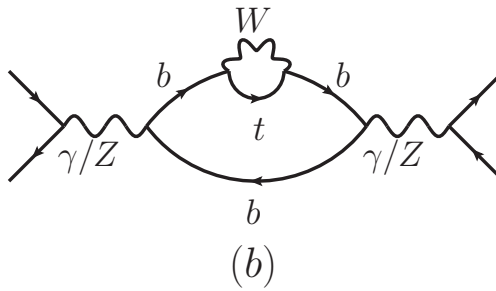
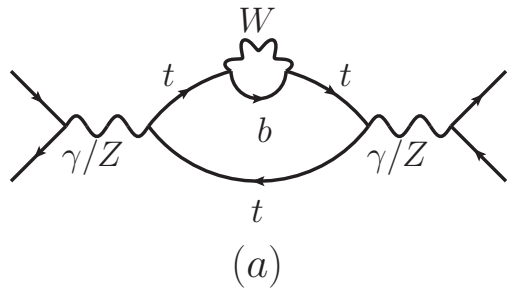
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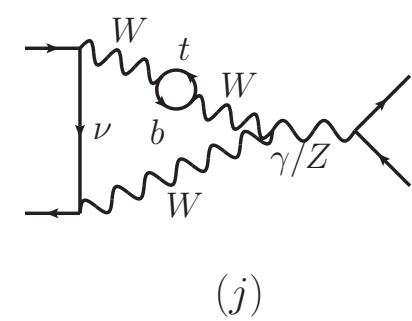
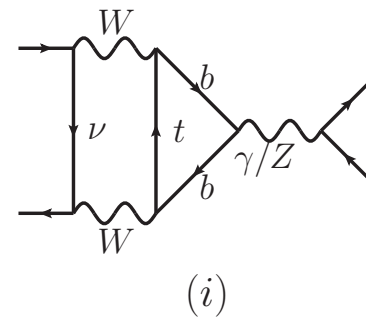
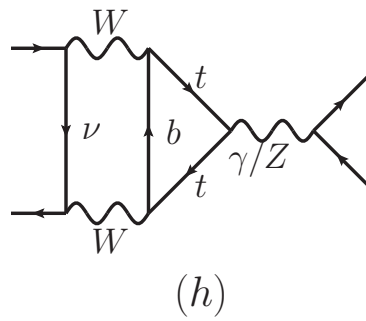
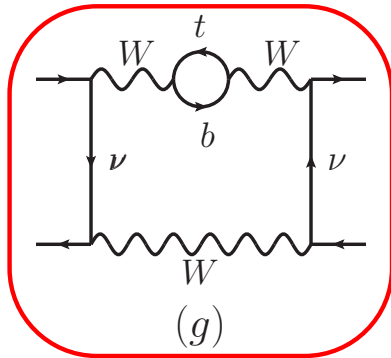
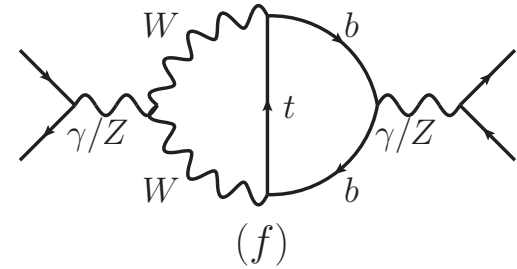
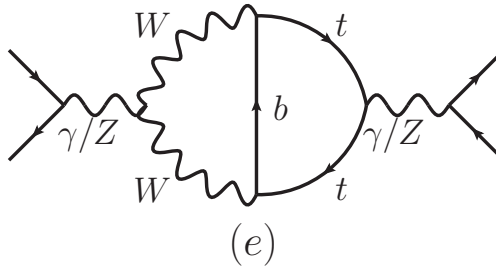
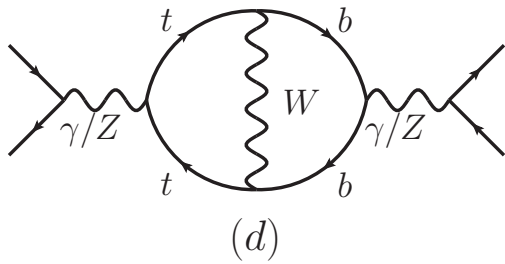
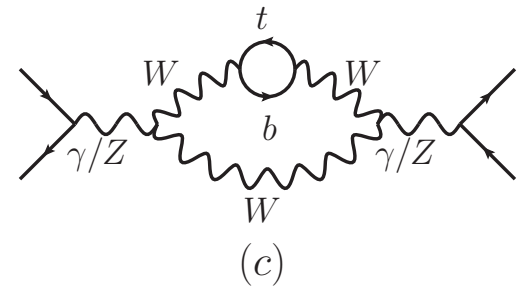
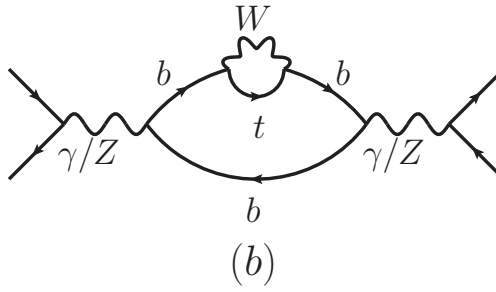
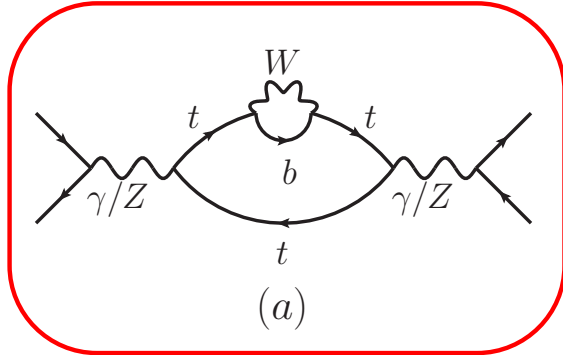
➔ *recover nonrelativistic propagators and vertices*

- *no expansion in $\rho \Leftrightarrow$ fully relativistic calculation (M. Beneke et al.)*

NLO diagrams



NLO diagrams



NLO result

- leading term of ρ -expansion

$$R_{nr}^{NLO} = -\frac{24}{\pi\rho^{1/2}} \frac{\Gamma_t}{m_t} \left[\frac{4}{9} + \text{“Z”} - \frac{1}{\sin^4 \theta_W} \left(\frac{17}{48} - \frac{9\sqrt{2}}{32} \ln(1 + \sqrt{2}) \right) \right]$$

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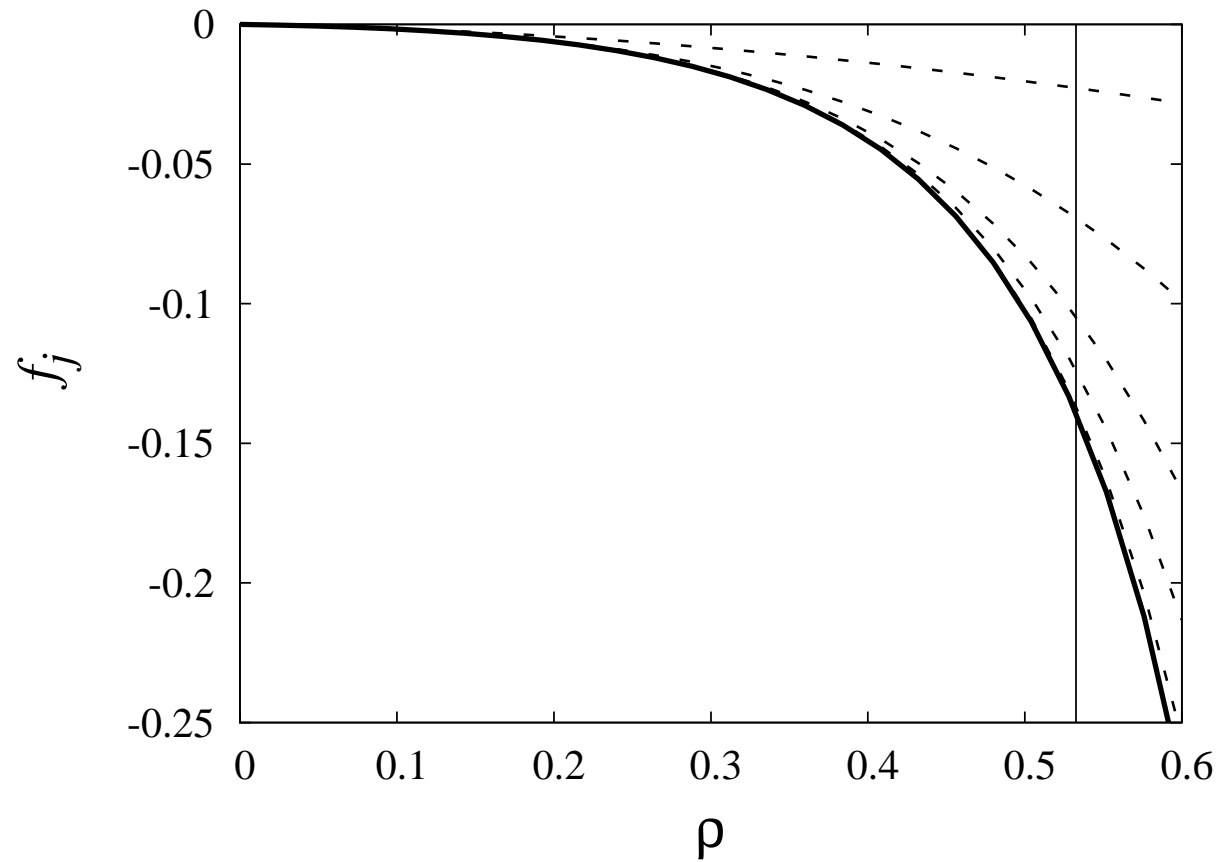
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- Convergence?

- *generally not bad*
- *for some diagrams Padé is necessary*

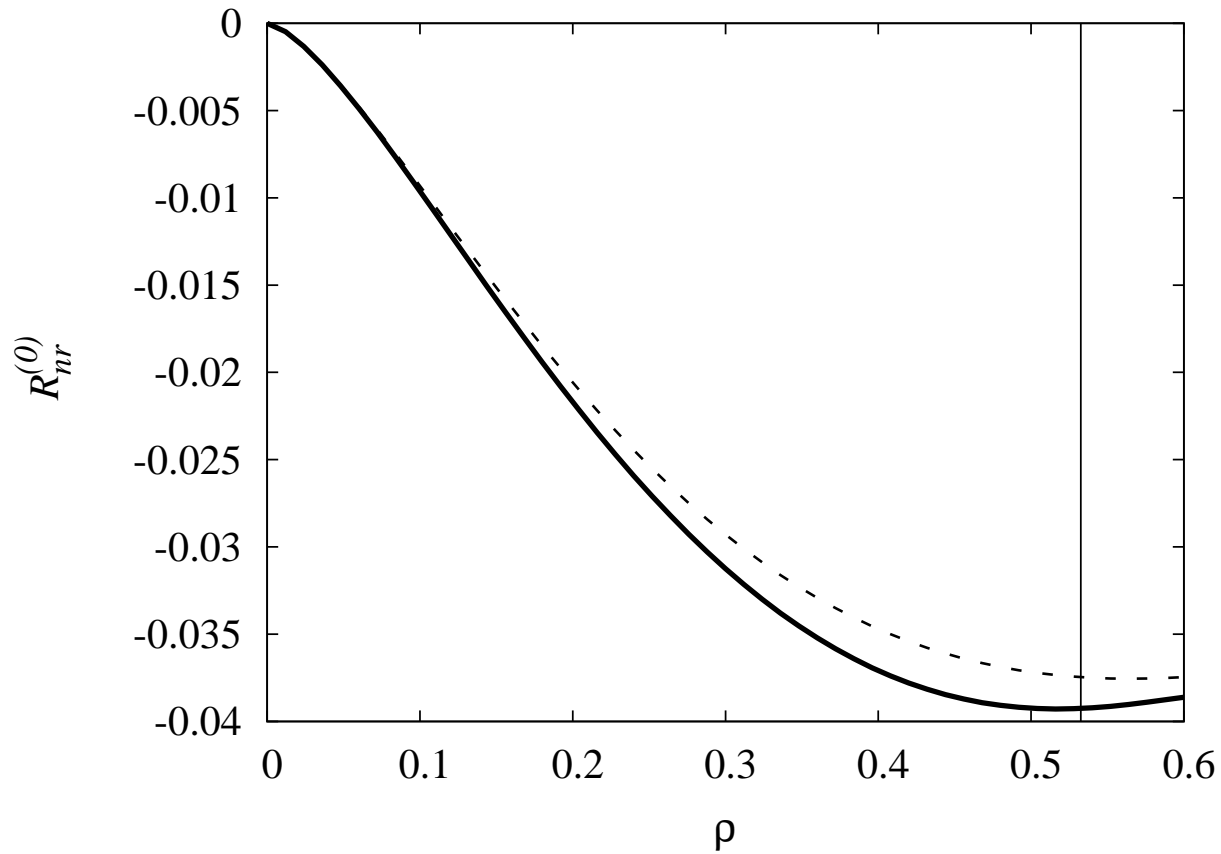
Convergence

diagram "j"



Convergence

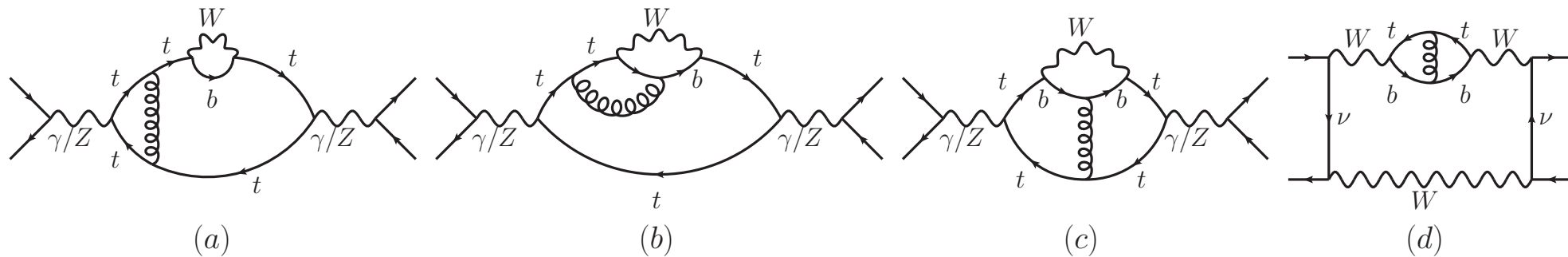
all diagrams



dash line - leading ρ -dependence
solid line - exact ρ -dependence

NNLO nonresonant contribution

ρ -leading diagrams



- Regions of gluon momentum
 - (a) and (d) - hard, potential, ρ -potential
 - (b) and (d) - hard, ρ -soft
- Suppression of nonfactorizable contribution
 - (c) vanishes in NLO

(K. Melnikov, O. Yakovlev, Phys. Lett. B324 (1994) 217)

NNLO result

● leading term of ρ -expansion

$$\begin{aligned} R_{nr}^{N^2LO} = & \frac{3C_F\alpha_s}{\pi^2\rho^{1/2}} \frac{\Gamma_t}{m_t} \left\{ \left[\frac{4}{9} + \text{“Z”} \right] \left[\frac{\pi^2}{\rho^{1/2}} \left(3 \ln \left(\sqrt{E^2 + \Gamma_t^2/\rho m_t} \right) + \frac{3}{2} + 6 \ln 2 \right) + (18 + 24 \ln 2) \right] \right. \\ & + \frac{1}{\sin^4 \theta_W} \left[\frac{22}{3} + \frac{17\pi^2}{6} - \frac{17}{2} \ln 2 + (2 - 3\pi^2 + 9 \ln 2) \frac{3\sqrt{2}}{4} \ln (1 + \sqrt{2}) \right. \\ & \left. \left. - \frac{27\sqrt{2}}{8} \left(\ln^2 (1 + \sqrt{2}) + \text{Li}_2 (2\sqrt{2} - 2) \right) \right] \right\}. \end{aligned}$$

NNLO result

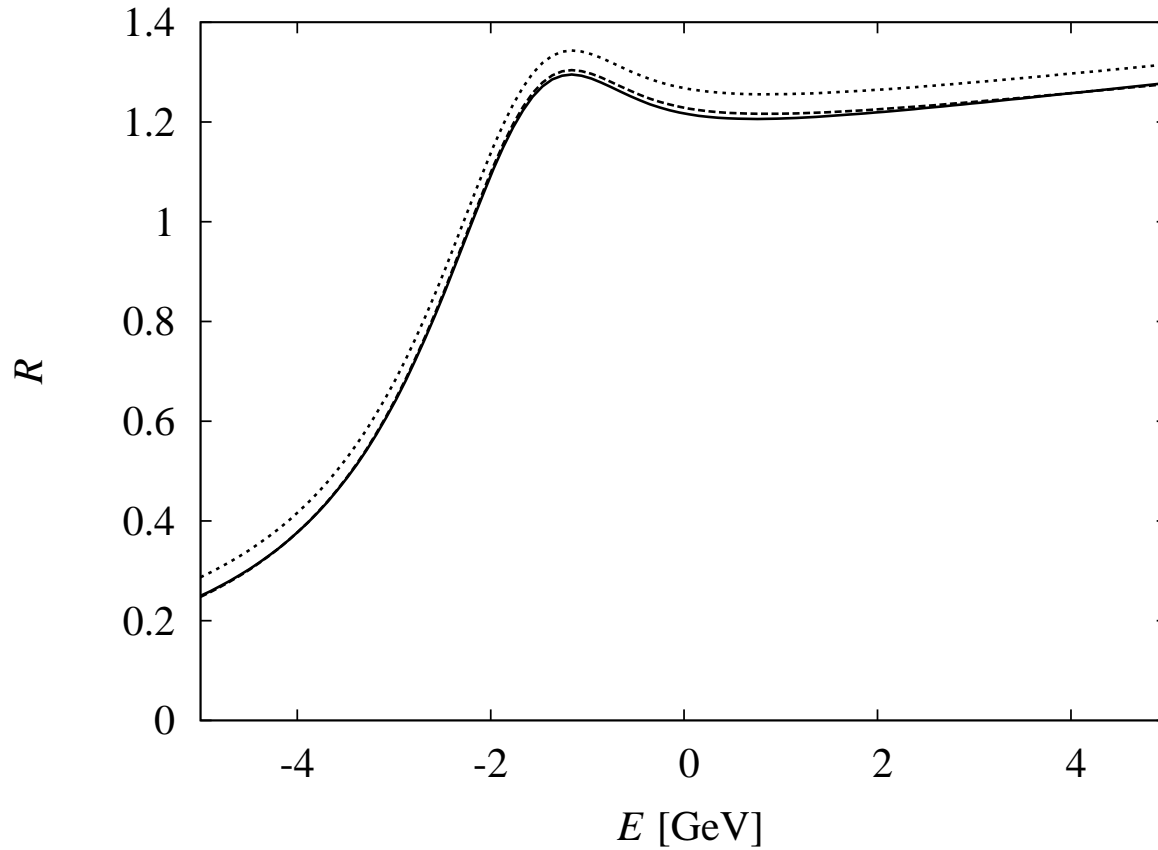
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● ρ -Coulomb term $\alpha_s/\rho^{1/2}$

● new type of logs $\ln(E/\rho m_t) \sim \ln(v^2/\rho)$

Numerics



dot line - LO

dash line - LO+NLO nonresonant

solid line - LO+NNLO nonresonant

Summary

- Effective theory of ρ NRQCD

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- *based on nonrelativistic expansion in $\rho = 1 - m_t/M_W$*
- *systematically accounts for finite width effects in threshold top-antitop production*
- *optimized for high-order calculations*

Summary

- Effective theory of ρ NRQCD
 - *based on nonrelativistic expansion in $\rho = 1 - m_t/M_W$*
 - *systematically accounts for finite width effects in threshold top-antitop production*
 - *optimized for high-order calculations*
 - *conceptually clear and aesthetically appealing*