NNLO QCD corrections to dijet production at hadron colliders

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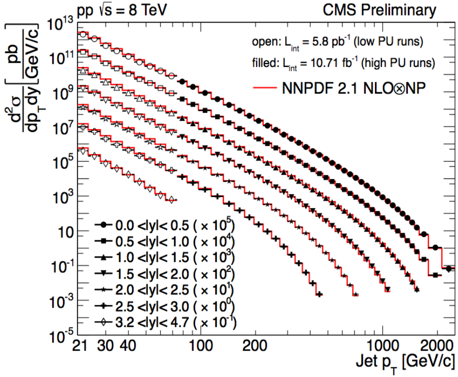
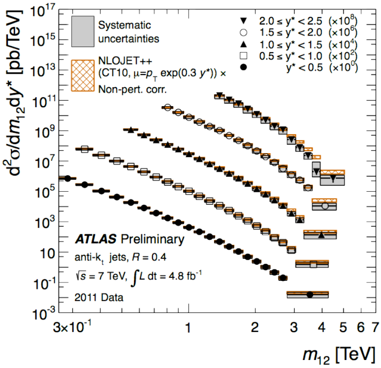
Inclusive jet and dijet cross sections

- look at the production of jets of hadrons with large transverse energy in
  - inclusive jet events $pp \to j + X$
  - exclusive dijet events $pp \to 2j$

- cross sections measured as a function of the jet p_T, rapidity y and dijet invariant mass m_{jj} in double differential form

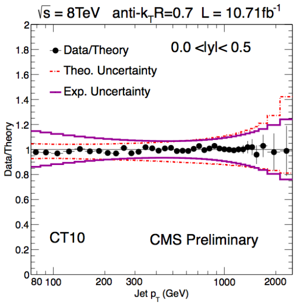


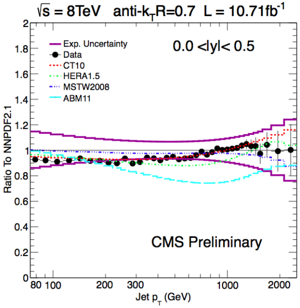
(CMS-PAS-SMP-12-012)



(ATLAS-CONF-2012-021)

Inclusive jet cross section

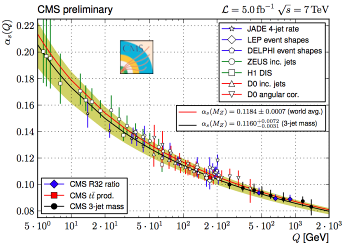


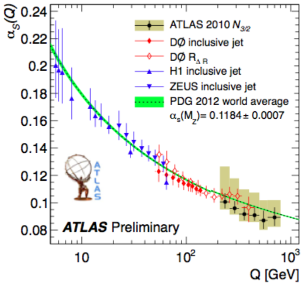


Motivation for NNLO

- experimental uncertainties at high-p_T smaller than theoretical → need pQCD predictions to NNLO accuracy
- collider jet data can be used to constrain parton distribution functions
- size of NNLO correction important for precise determination of PDF's
- inclusion of jet data in NNLO parton distribution fits requires NNLO corrections to jet cross sections

Inclusive jet cross section





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- size of NNLO correction important for precise determination of PDF's
- inclusion of jet data in NNLO parton distribution fits requires NNLO corrections to jet cross sections
- α_s determination from hadronic jet observables limited by theoretical uncertainty due to scale choice

inclusive jet and dijet cross sections

State of the art:

- dijet production is completely known in NLO QCD [Ellis, Kunszt, Soper '92], [Giele, Glover, Kosower '94], [Nagy '02]
- NLO+Parton shower [Alioli, Hamilton, Nason, Oleari, Re '11]
- threshold corrections [Kidonakis, Owens '00]

Goal:

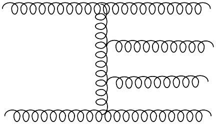
- obtain the jet cross sections at NNLO accuracy in double differential form

$$\frac{d^2\sigma}{dp_T d|y|} \qquad \frac{d^2\sigma}{dm_{jj} dy^*}$$

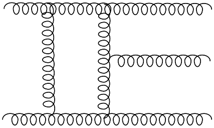
This talk:

- present IR structure of the gluons only subleading colour NNLO calculation
- present NNLO inclusive jet and dijet cross sections (gluons only full colour)

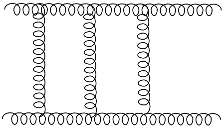
$pp \to 2j$ at NNLO: gluonic contributions



$A_6^{(0)}(gg \to gggg)$



$A_5^{(1)}(gg \to ggg)$



$A_4^{(2)}(gg \to gg)$

[Berends, Giele '87], [Mangano, Parke, Xu '87], [Britto, Cachazo, Feng '06]
[Bern, Dixon, Kosower '93]
[Anastasiou, Glover, Oleari, Tejeda-Yeomans '01],[Bern, De Freitas, Dixon '02]

$$\mathrm{d}\hat{\sigma}_{NNLO} = \int_{\mathrm{d}\Phi_4} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_3} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_2} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

- explicit infrared poles from loop integrations
- implicit poles in phase space regions for single and double unresolved gluon emission
- procedure to extract the infrared singularities and assemble all the parts in a parton-level generator

NNLO antenna subtraction

$$\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} &= \int_{\mathrm{d}\Phi_4} \left(\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_3} \left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_2} \left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right)
\end{aligned}$$

- $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$: real radiation subtraction term for $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$
- $\mathrm{d}\hat{\sigma}^{T}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$
- $\mathrm{d}\hat{\sigma}^{U}_{NNLO}$: two-loop virtual subtraction term for $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$
- subtraction terms constructed using the antenna subtraction method at NNLO for hadron colliders → presence of initial state partons to take into account
- contribution in each of the round brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically

Double-real contribution

- gluons only double-real colour summed cross section

$$d\hat{\sigma}^{RR}_{NNLO} = \left(\frac{\alpha_s}{2\pi}\right)^2 N^4(N^2-1)\, d\Phi_4(p_3,\ldots,p_6;p_1;p_2)\, J_2^{(4)}(p_3,\ldots,p_6)\,\frac{1}{4!}$$
$$\sum_{\sigma\in S_6/Z_6}\left[|A_6^{(0)}(\sigma)|^2 + \frac{2}{N^2}A_6^0(\sigma)\Big(A_6^{\dagger 0}(\sigma\cdot\rho_1) + A_6^{\dagger 0}(\sigma\cdot\rho_2) + A_6^{\dagger 0}(\sigma\cdot\rho_3)\Big)\right]$$

- double-real sub-leading colour contribution written as three interferences summed over permutations. For $\sigma = (1,2,3,4,5,6)$

$$(\sigma\cdot\rho_1) = (1,3,5,2,6,4) \qquad (\sigma\cdot\rho_2) = (1,3,6,4,2,5) \qquad (\sigma\cdot\rho_3) = (1,4,2,6,3,5)$$

- $(\sigma\cdot\rho_{1,2,3})$ are the only three independent orderings that exist for six gluon scattering which have no common two or three particle poles in common with σ

$\Rightarrow$ no single, double or triple collinear singularities at subleading colour ✓

- subtract divergences associated with single and double soft gluons only ✓

Single unresolved subtraction term

- in the single soft limit the interferences factorize in the following way,

$$
\begin{aligned}
&\mathcal{M}^{0,\dagger}_{n+1}(\cdots,a,i,b,\cdots)\mathcal{M}^{0}_{n+1}(\cdots,c,i,d,\cdots) \xrightarrow{i\to 0} \\
&\qquad J_\mu(p_a,p_i,p_b)\epsilon^\mu(p_j)J_\nu(p_c,p_i,p_d)\epsilon^\nu(p_i)\mathcal{M}^{0,\dagger}_{n}(\cdots,a,b,\cdots)\mathcal{M}^{0}_{n}(\cdots,c,d,\cdots) \\
= &\quad \frac{1}{2}(S_{aid}+S_{bic}-S_{aic}-S_{bid})\mathcal{M}^{0,\dagger}_{n}(\cdots,a,b,\cdots)\mathcal{M}^{0}_{n}(\cdots,c,d,\cdots)
\end{aligned}
$$

where $S_{ajc} = \frac{2s_{ac}}{s_{aj}s_{jc}}$ is the squared eikonal factor

- eikonal factor with uniquely identified radiators and unresolved momenta is mapped to a three parton antenna function

- $\mathrm{d}\sigma^{S,a}_{NNLO}$ single unresolved subtraction term constructed from a product of differences of tree-level three parton antenna functions and reduced colour ordered matrix element interferences

Double unresolved subtraction term

- in the double soft limit the interferences factorize in the following way,

$$\mathcal{M}^{0,\dagger}_{n+1}(\cdots,a,i,j,b,\cdots)\mathcal{M}^{0}_{n+1}(\cdots,c,i,d,\cdots,e,j,f,\cdots) \xrightarrow{i,j\to 0}$$
$$J_{\mu 1\mu 2}(p_a,p_i,p_j,p_b)\epsilon^{\mu 1}(p_i)\epsilon^{\mu 2}(p_j)J_{\nu 1}(p_c,p_i,p_d)J_{\nu 2}(p_e,p_j,p_f)\epsilon^{\nu 1}(p_i)\epsilon^{\nu 2}(p_j)$$
$$\times\,\mathcal{M}^{0,\dagger}_{n}(\cdots,a,b,\cdots)\mathcal{M}^{0}_{n}(\cdots,c,d,\cdots,e,f,\cdots)$$

- when summing the subleading colour contribution explicitly over all colour permutations the tree level double soft current drops out from the limit using its symmetric properties
- the double soft limit at subleading colour can be written completely in terms of contractions of tree level single soft currents

$$\begin{aligned}
\mathbf{A}^0_6(p)|_{slc} \quad \overset{5,6\to 0}{\propto} \quad & \left(S_{153}+S_{254}-S_{154}-S_{254}\right)\left(S_{163}+S_{264}-S_{162}-S_{364}\right)|A^0_4(1,2,3,4)|^2 \\
+ \quad & \left(S_{153}+S_{254}-S_{154}-S_{254}\right)\left(S_{162}+S_{364}-S_{164}-S_{263}\right)|A^0_4(1,2,4,3)|^2 \\
+ \quad & \left(S_{152}+S_{354}-S_{154}-S_{253}\right)\left(S_{162}+S_{364}-S_{163}-S_{264}\right)|A^0_4(1,3,2,4)|^2
\end{aligned}$$

- at subleading colour no contribution from the colour connected double soft function in the limit ⇒ no four parton tree level antennae needed
- contribution produces $1/\epsilon^2$ poles only

Real-virtual contribution

- gluons only real-virtual colour summed cross section

$$d\hat{\sigma}^{RV}_{NNLO} = \left(\frac{\alpha_s}{2\pi}\right)^2 N^4(N^2-1)\, d\Phi_3(p_3,\ldots,p_5;p_1;p_2)\, J_2^{(3)}(p_3,\ldots,p_5)\,\frac{1}{3!}$$
$$\sum_{\sigma\in S_5/Z_5} 2\Re\left[A_5^{\dagger 0}(\sigma)A_5^1(\sigma) + \frac{12}{N^2}A_5^{\dagger 0}(\sigma)A_{5,1}^1(\sigma\cdot\rho)\right]$$

- real-virtual subleading colour contribution written as a single interference summed over permutations

$$\text{For } \sigma = (1,2,3,4,5) \qquad (\sigma\cdot\rho) = (1,4,2,5,3)$$

- $(\sigma\cdot\rho)$ is the only independent ordering that exist for five gluon scattering which have no common neighbouring partons with σ

$\Rightarrow$ no single collinear singularities in the subleading colour real-virtual contribution✓

- subtract divergences associated with single soft gluons only ✓

Single unresolved subtraction term

- in the single soft limit the one-loop amplitude factorizes in the following way

$$A^1_{5,1}(\cdots,a,i,b,\cdots) \quad \xrightarrow{i\to 0} \quad \mathcal{S}^0(a,i,b)A^1_{4,1}(\cdots,a,b,\cdots) \\ + \quad \mathcal{S}^1(a,i,b)A^0_4(\cdots,a,b,\cdots)$$

- using symmetry properties the one-loop soft functions cancel in the total real-virtual cross section at subleading colour for five gluon scattering
- obtain the following single unresolved subtraction term for a generic one-loop interference at subleading colour

$$\begin{aligned}
&\mathcal{A}^{0\dagger}_5(\cdots,a,i,b,\cdots)\mathcal{A}^1_5(\cdots,c,i,d,\cdots) \stackrel{i\to 0}{\approx} \\
&\quad + \quad X^0_3(a,i,d)\ \mathcal{M}^{0\dagger}_4(\cdots,\widetilde{(ai)},b,\cdots)\mathcal{M}^1_{4,1}(\cdots,c,\widetilde{(id)},\cdots) \\
&\quad + \quad X^0_3(b,i,c)\ \mathcal{M}^{0\dagger}_4(\cdots,a,\widetilde{(bi)},\cdots)\mathcal{M}^1_{4,1}(\cdots,\widetilde{(ic)},d,\cdots), \\
&\quad - \quad X^0_3(a,i,c)\ \mathcal{M}^{0\dagger}_4(\cdots,\widetilde{(ai)},b,\cdots)\mathcal{M}^1_{4,1}(\cdots,\widetilde{(ic)},d,\cdots) \\
&\quad - \quad X^0_3(b,i,d)\ \mathcal{M}^{0\dagger}_4(\cdots,a,\widetilde{(bi)},\cdots)\mathcal{M}^1_{4,1}(\cdots,c,\widetilde{(id)},\cdots)
\end{aligned}$$

- no three parton one-loop antennae needed at subleading colour

NNLO antenna subtraction

Implementation checks (gluons only channel full colour in $pp \to 2j$):

- subtraction terms correctly approximate the matrix elements in all unresolved configurations of partons j, k

$$\boxed{\mathrm{d}\hat{\sigma}^{RR,RV}_{NNLO} \xrightarrow{\forall\{j,k\},\{j\}\to 0} \mathrm{d}\hat{\sigma}^{S,T}_{NNLO}}$$

- local (pointwise) analytic cancellation of all infrared explicit ϵ-poles when integrated subtraction terms are combined with one, two-loop matrix elements

$$\boxed{\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right) = 0}$$

$$\boxed{\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right) = 0}$$

- process independent NNLO subtraction scheme
- allows the computation of multiple differential distributions in a single program run

Numerical setup $(gg \to gg + X)$

- jets identified with the anti-k_T jet algorithm with resolution parameter $R = 0.7$
- jets accepted at rapidities $|y| < 4.4$
- leading jet with transverse momentum $p_t > 80$ GeV
- subsequent jets required to have at least $p_t > 60$ GeV
- MSTW2008nnlo PDF
- dynamical factorization and renormalization scales equal to the leading jet p_T $(\mu_R = \mu_F = \mu = p_{T1})$

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Integrated cross section results

$$\sigma^{8TeV-LO}_{incl.jet} = (9.6495 \pm 0.001) \times 10^5 \text{pb}$$

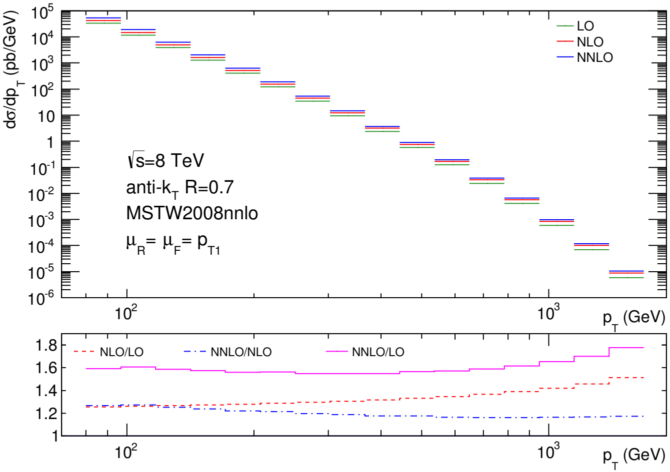
$$\sigma^{8TeV-NLO}_{incl.jet} = (12.152 \pm 0.001) \times 10^5 \text{pb}$$

$$\sigma^{8TeV-NNLO}_{incl.jet} = (15.20 \pm 0.02) \times 10^5 \text{pb}$$ ← leading colour

$$\sigma^{8TeV-NNLO}_{incl.jet} = (12.40 \pm 0.01) \times 10^5 \text{pb}$$ ← subleading colour

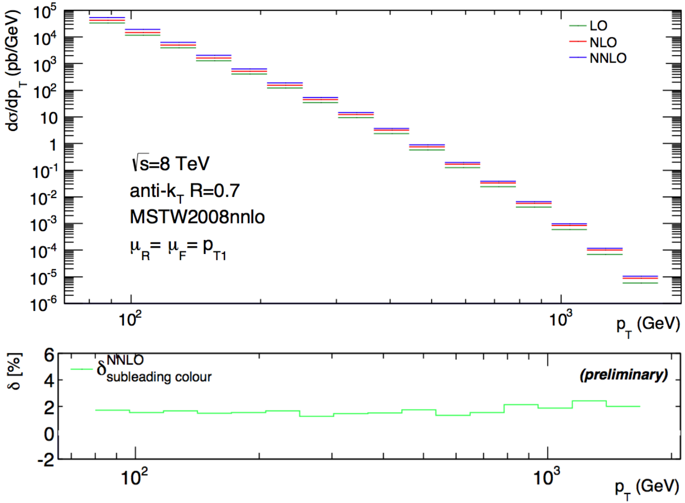
- NNLO result increased by about 26% with respect to the NLO cross section
- subleading colour contribution $\sim 8\%$ to the full colour NNLO coefficient

inclusive jet p_T distribution at NNLO $(gg \to gg + X)$



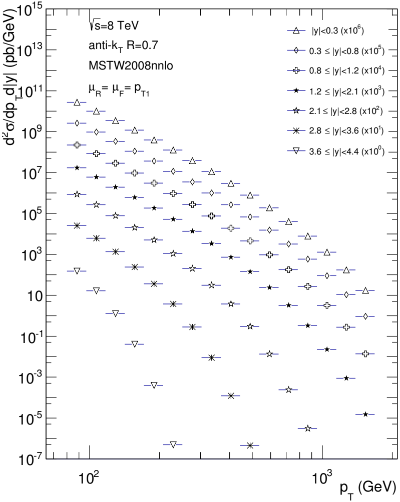
- $gg \to gg$ subprocess in full colour with same PDF for all fixed order predictions
- all jets in an event are binned
- NNLO correction stabilizes the NLO k-factor growth with p_T

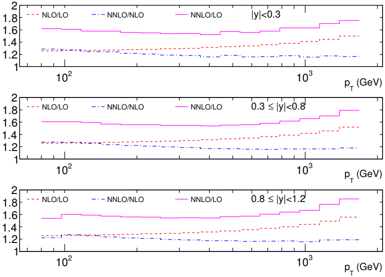
inclusive jet p_T distribution at NNLO $(gg \to gg + X)$



- $gg \to gg$ subprocess in full colour with same PDF for all fixed order predictions
- all jets in an event are binned
- subleading colour contribution to the NNLO cross section is at the 2% level

double differential inclusive jet p_T distribution at NNLO

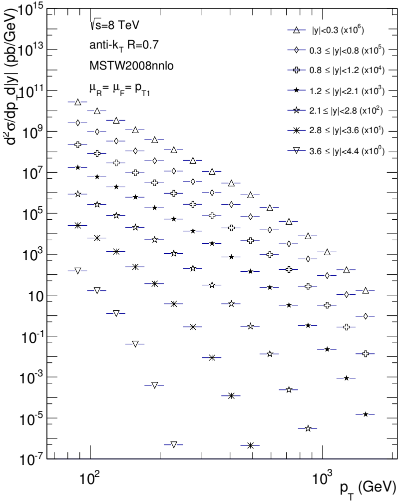


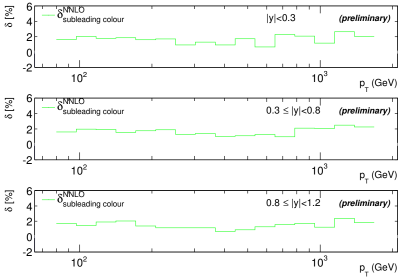


double differential k-factors

- $gg \to gg$ subprocess in full colour
- NNLO result varies between 26% to 12% with respect to the NLO cross section
- similar behaviour between the rapidity slices

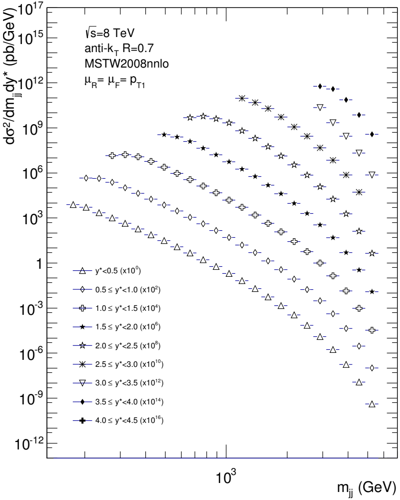
double differential inclusive jet p_T distribution at NNLO

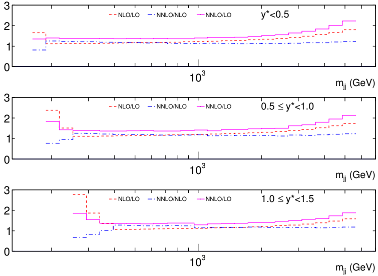




- subleading colour contribution to the NNLO cross section is at the 2% level
- similar behaviour between the rapidity slices

double differential exclusive dijet mass distribution at NNLO

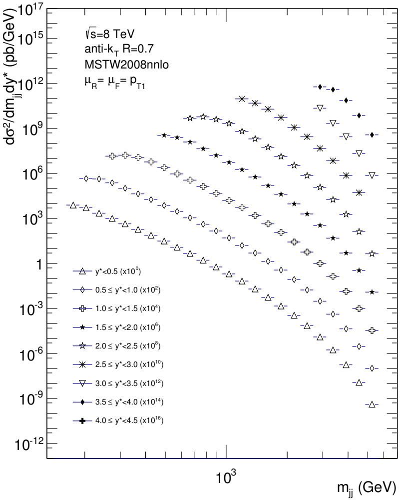


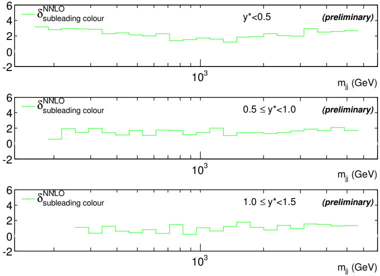


double differential k-factors

- $gg \to gg$ subprocess in full colour
- NNLO corrections up to 20% with respect to the NLO cross section
- similar behaviour between the $y^* = 1/2|y_1 - y_2|$ slices

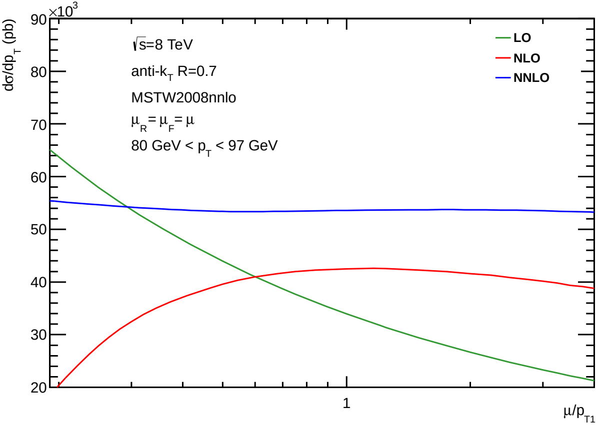
double differential exclusive dijet mass distribution at NNLO





- subleading colour contribution to the NNLO cross section is at the 2% level
- similar behaviour between the rapidity slices

inclusive jet p_T scale dependence $(gg \to gg + X)$



- scale dependence study at leading colour
- dynamical scale choice: leading jet p_T
- same PDF and α_s for all fixed order predictions
- flat scale dependence at NNLO

Conclusions

- antenna subtraction method generalised for the calculation of NNLO QCD corrections for exclusive collider observables with partons in the initial-state
- explicit ϵ-poles in the matrix elements are analytically cancelled by the ϵ-poles in the subtraction terms
- non-trivial check of analytic cancellation of infrared singularities between double-real, real-virtual and double-virtual corrections
- successful treatment of colour-correlated matrix elements in the subleading colour calculation with the antenna subtraction method
- subleading colour NNLO corrections in the gluons only channel at 2% level
- proof-of principle implementation of the $gg \to gg$ full colour contribution to $pp \to 2j$ at NNLO in the new NNLOJET parton-level generator

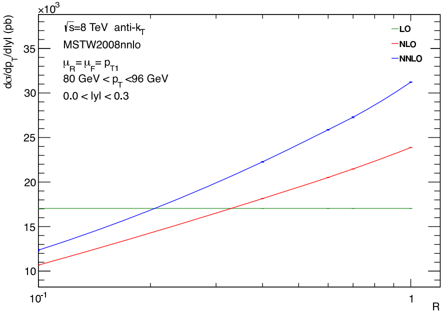
Future work:

- develop colour space approach for NNLO calculations within the antennae method $\to$ James Currie talk
- include remaining channels
  - 4g2q processes
  - 2g4q processes
  - 6q processes

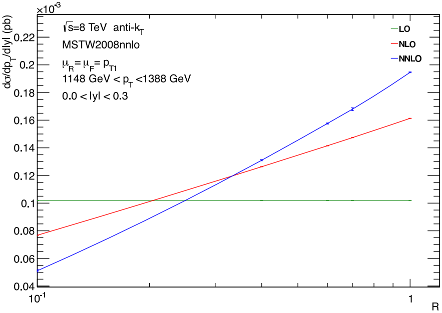
Back-up slides

inclusive jet p_T distribution

$80\text{GeV} < p_T < 96\text{GeV}$



$1148\text{GeV} < p_T < 1388\text{GeV}$



- inclusive jet cross section versus R
- NNLO corrections smaller for small R but p_T dependent