New results for Higgs+jet at NNLO

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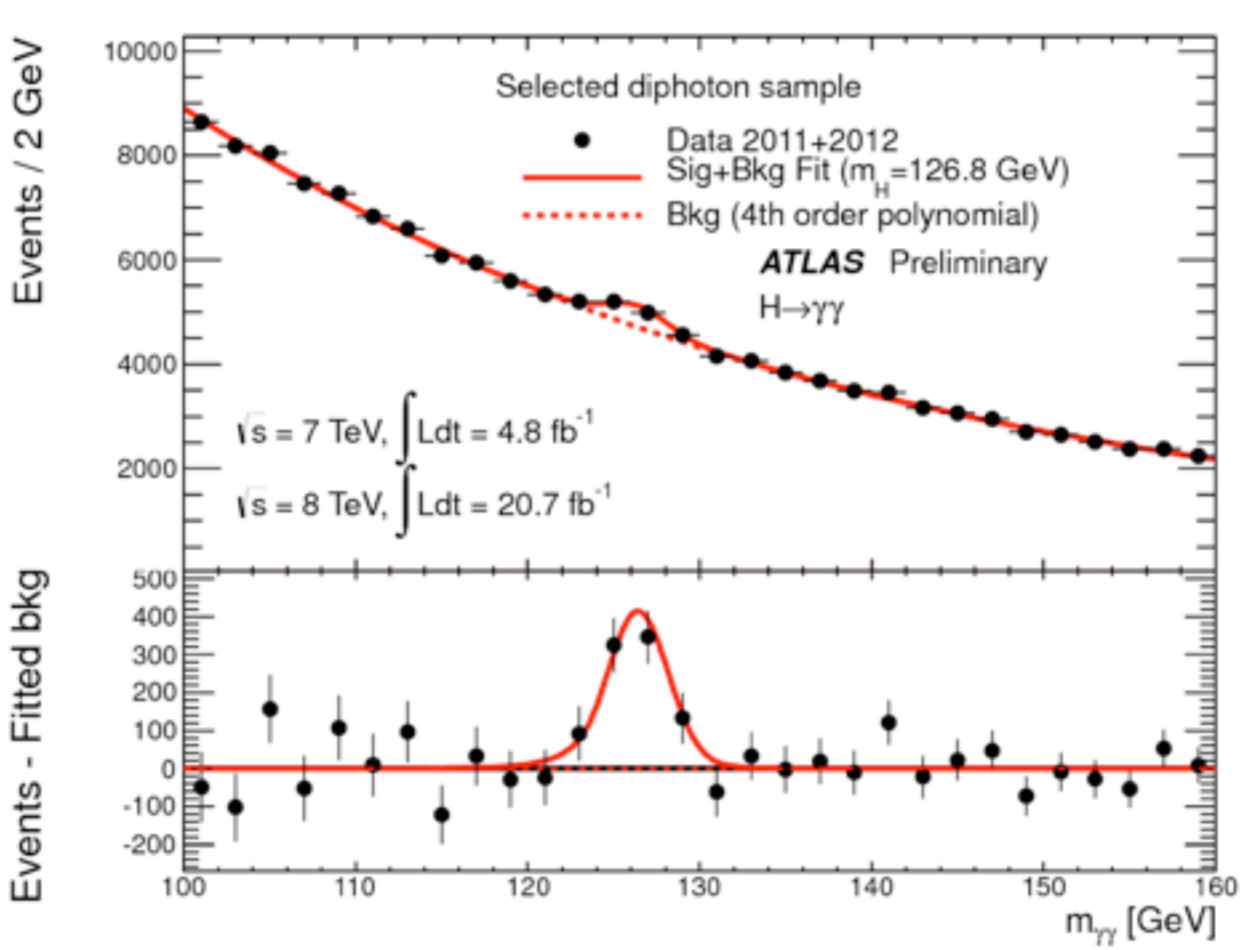
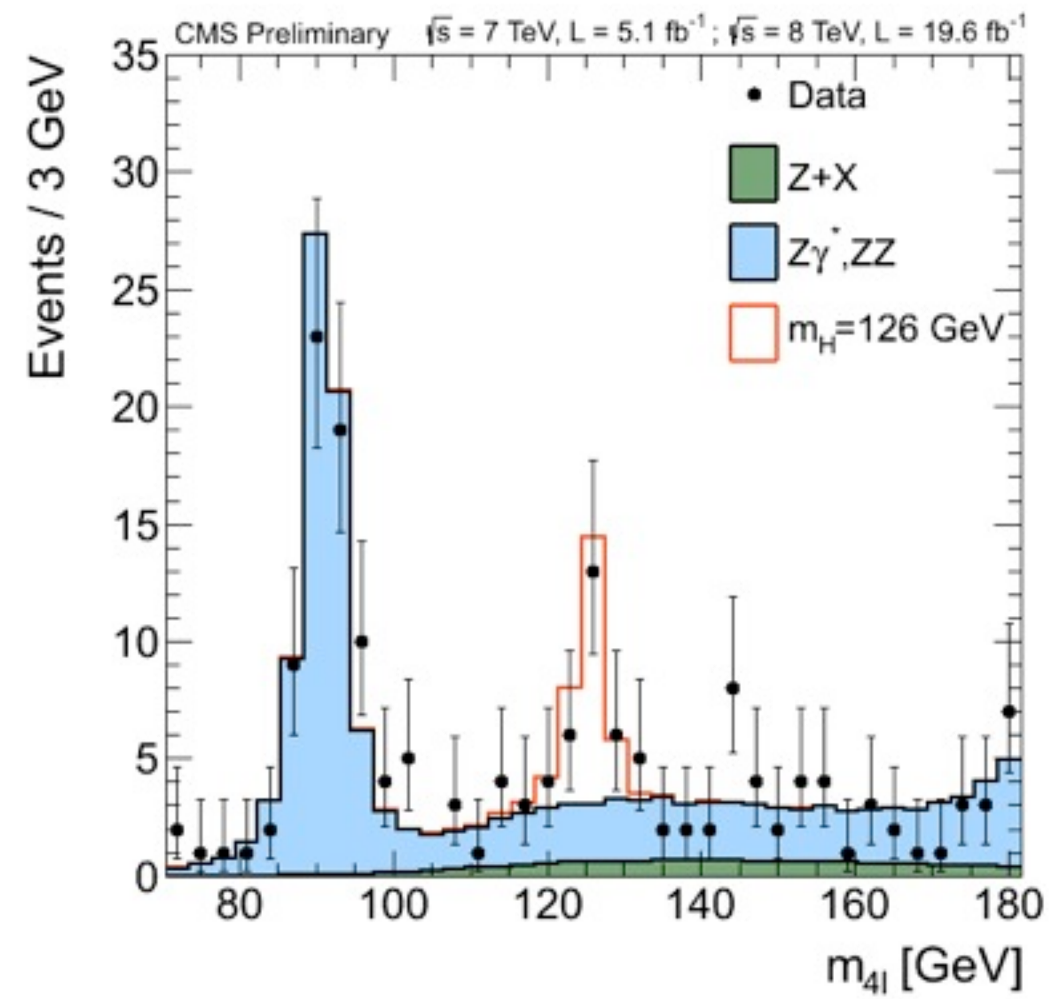
Outline

- Brief overview of Higgs measurements at the LHC
- A description of sector-improved subtraction
- The calculation of H+j at NNLO
- Numerical results for the gluon-initiated contributions

The Higgs discovery

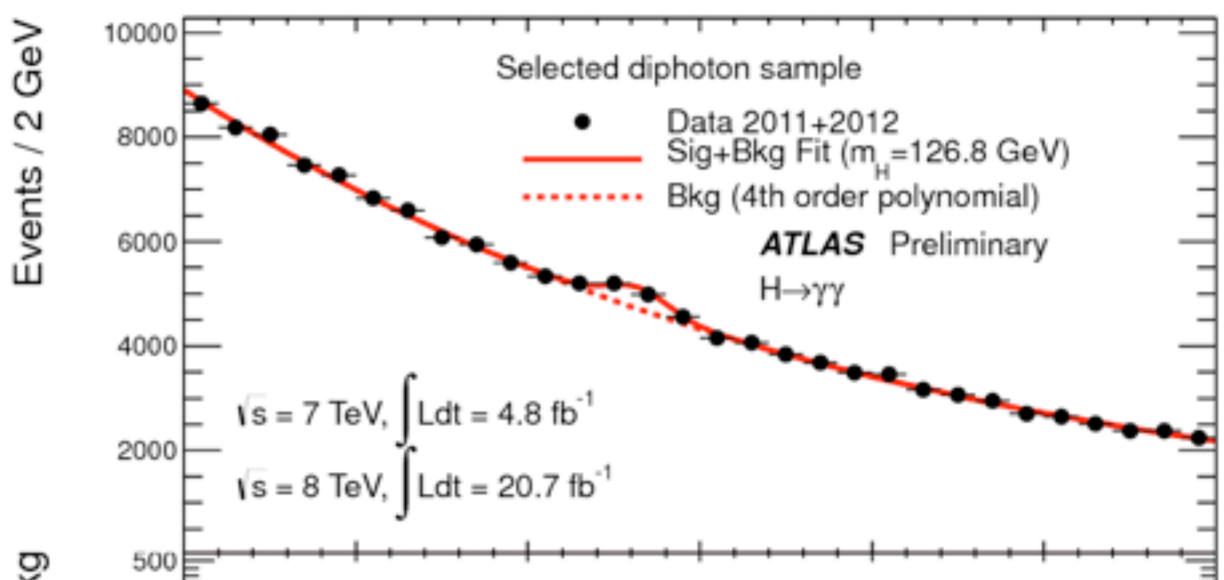
- The landscape of high energy physics has changed since RADCOR 2011:

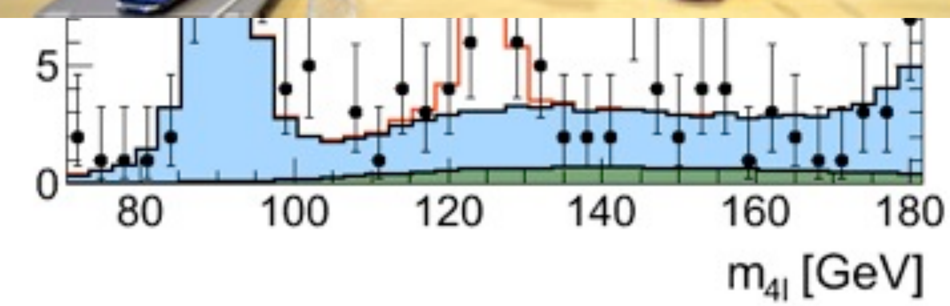




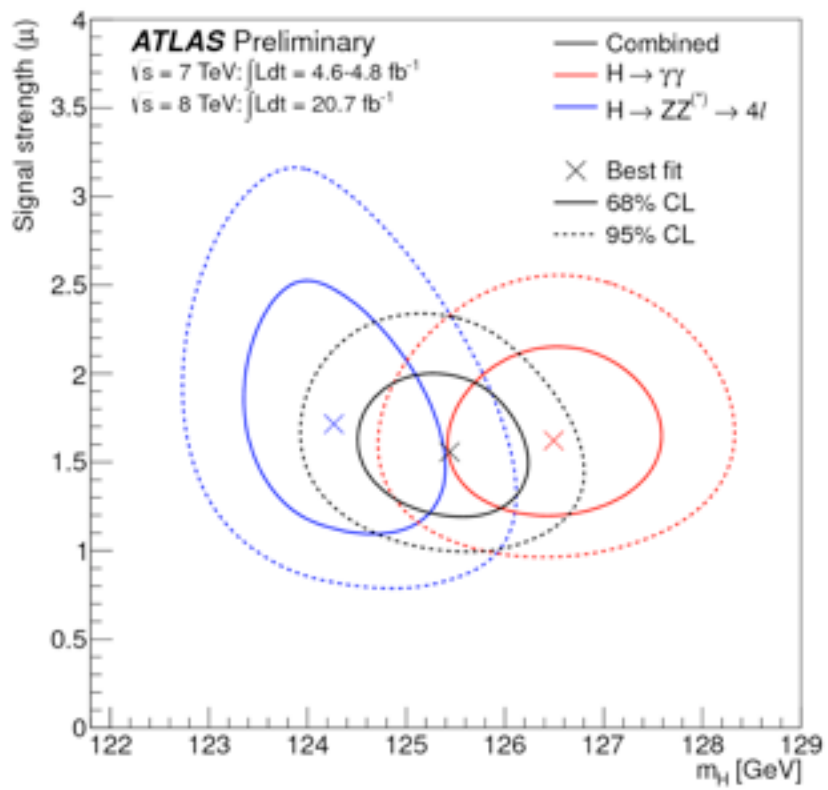
The Higgs discovery

- The landscape of high energy physics has changed since RADCOR 2011:

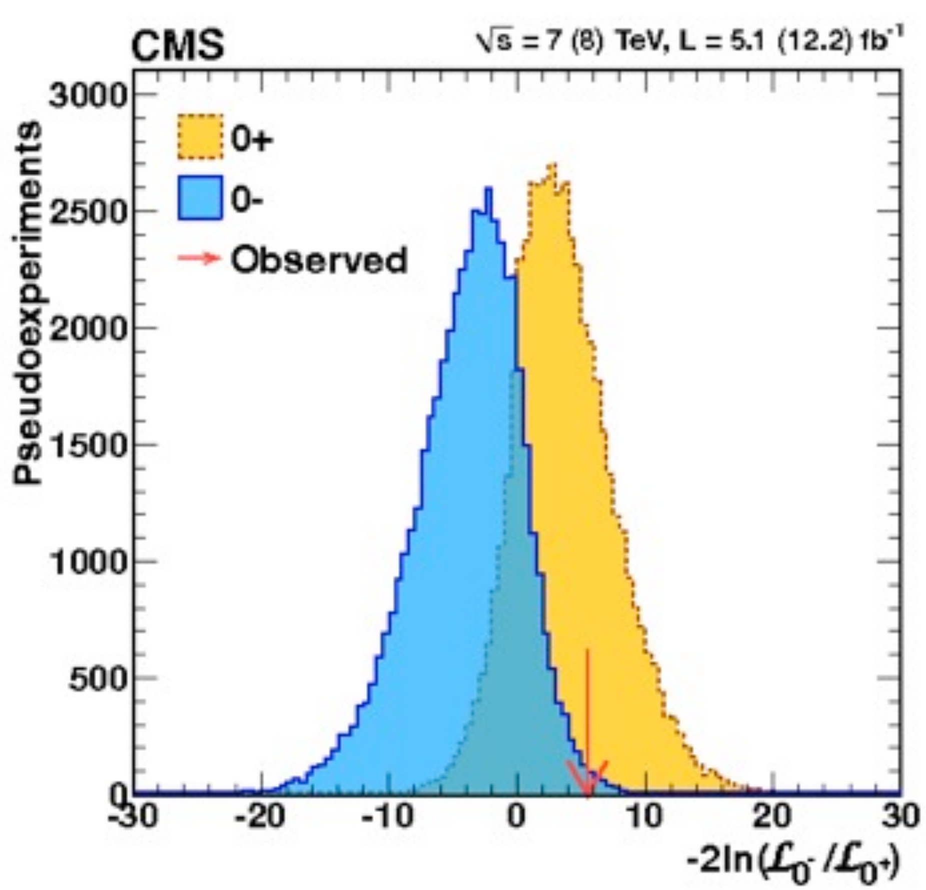


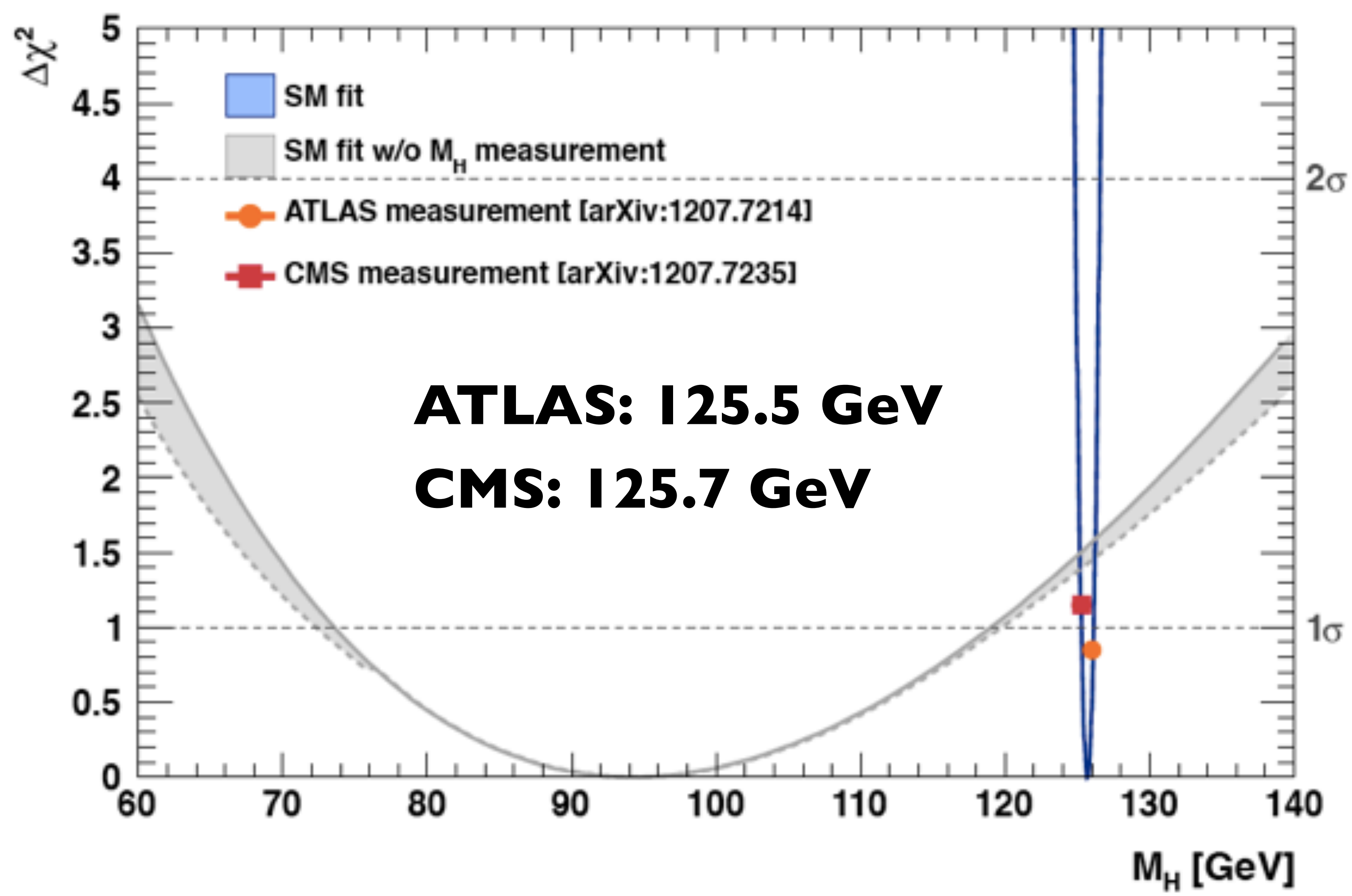


Mass and spin-parity measurement

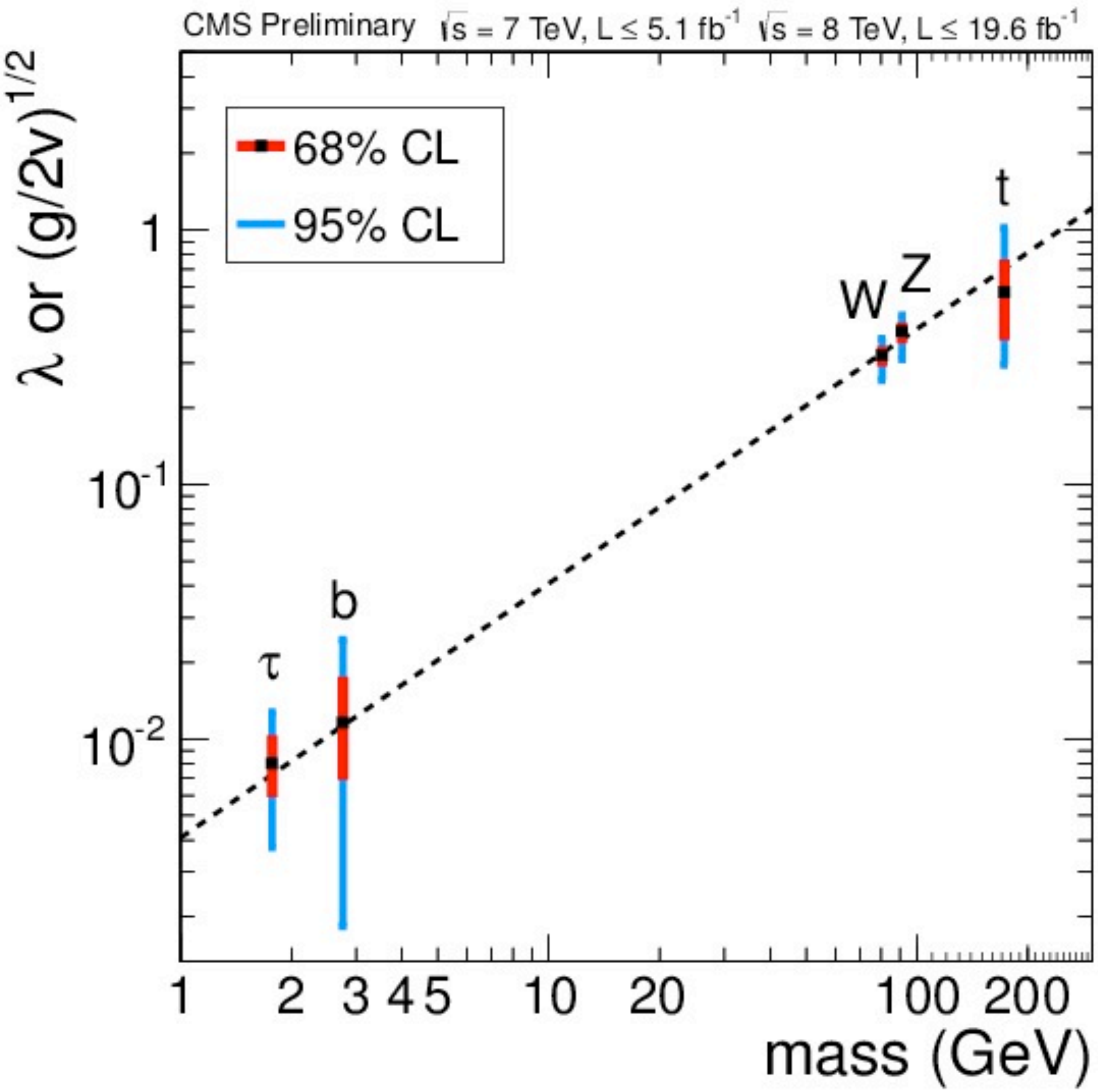


- Mass consistent with the global EW fit without the LHC measurement to better than 2σ
- Kinematic distributions in γγ, ZZ, and WW final states prefer a 0^+ state
- ATLAS example: 0^-, 1^+, 1^-, and 2^+ excluded at or above 97.8% C.L.

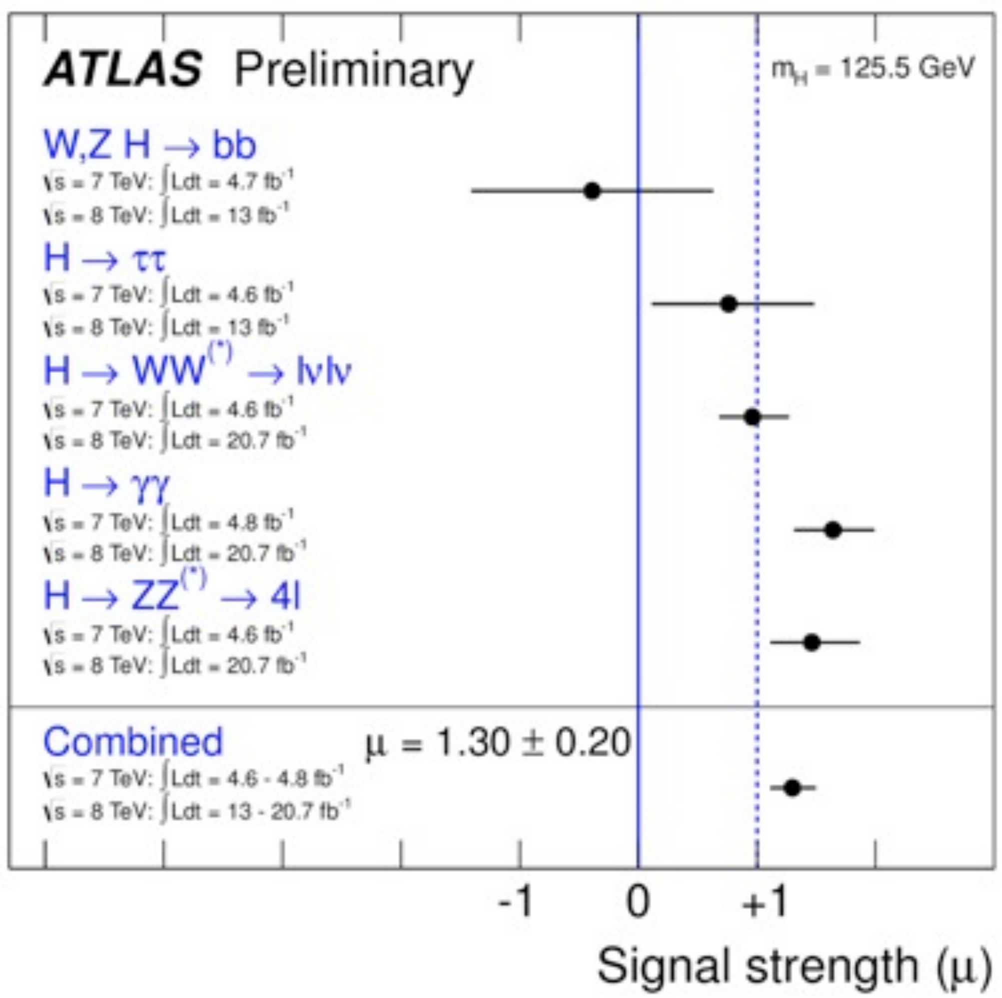




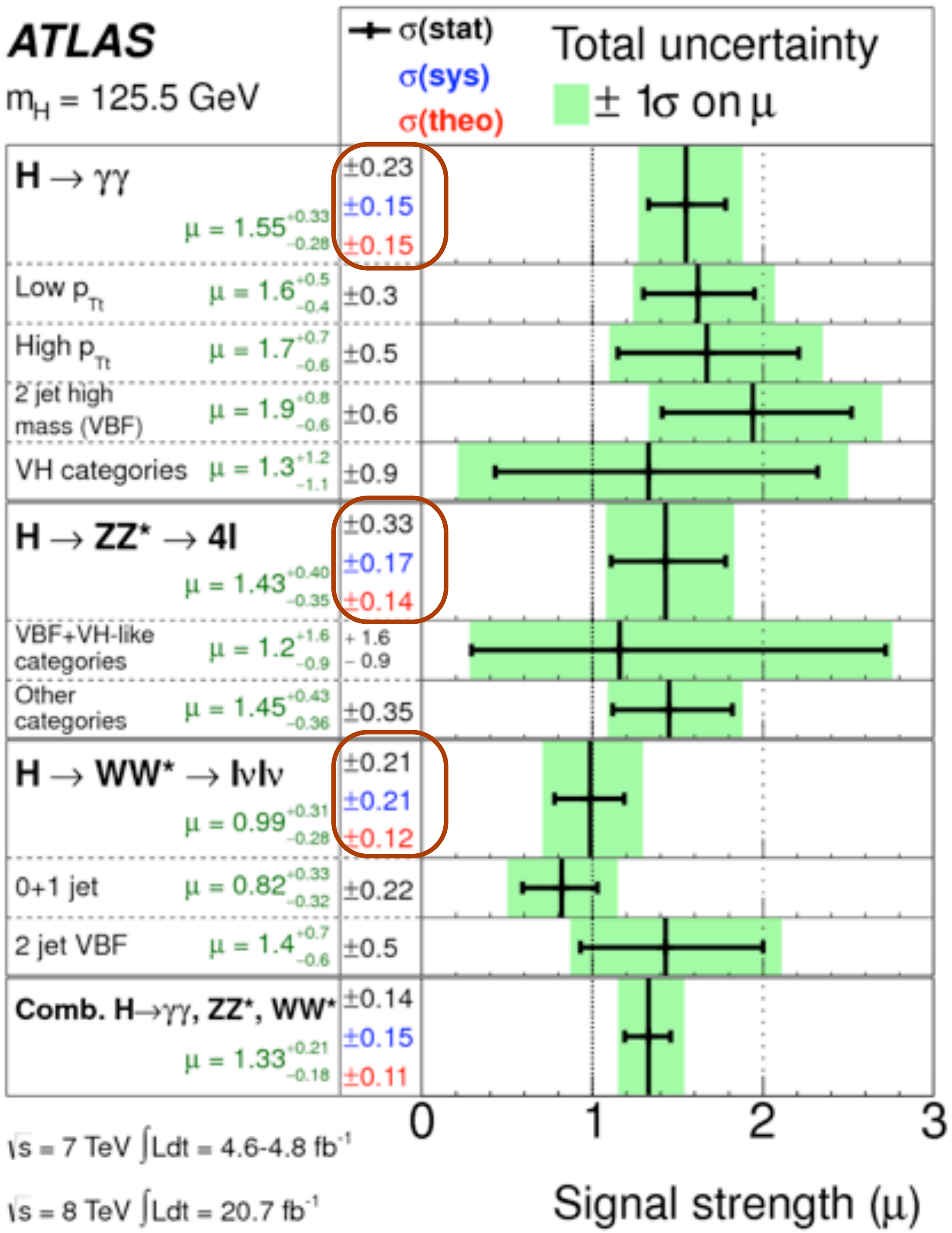
Coupling measurement



•Couplings proportional to mass, as predicted in the SM



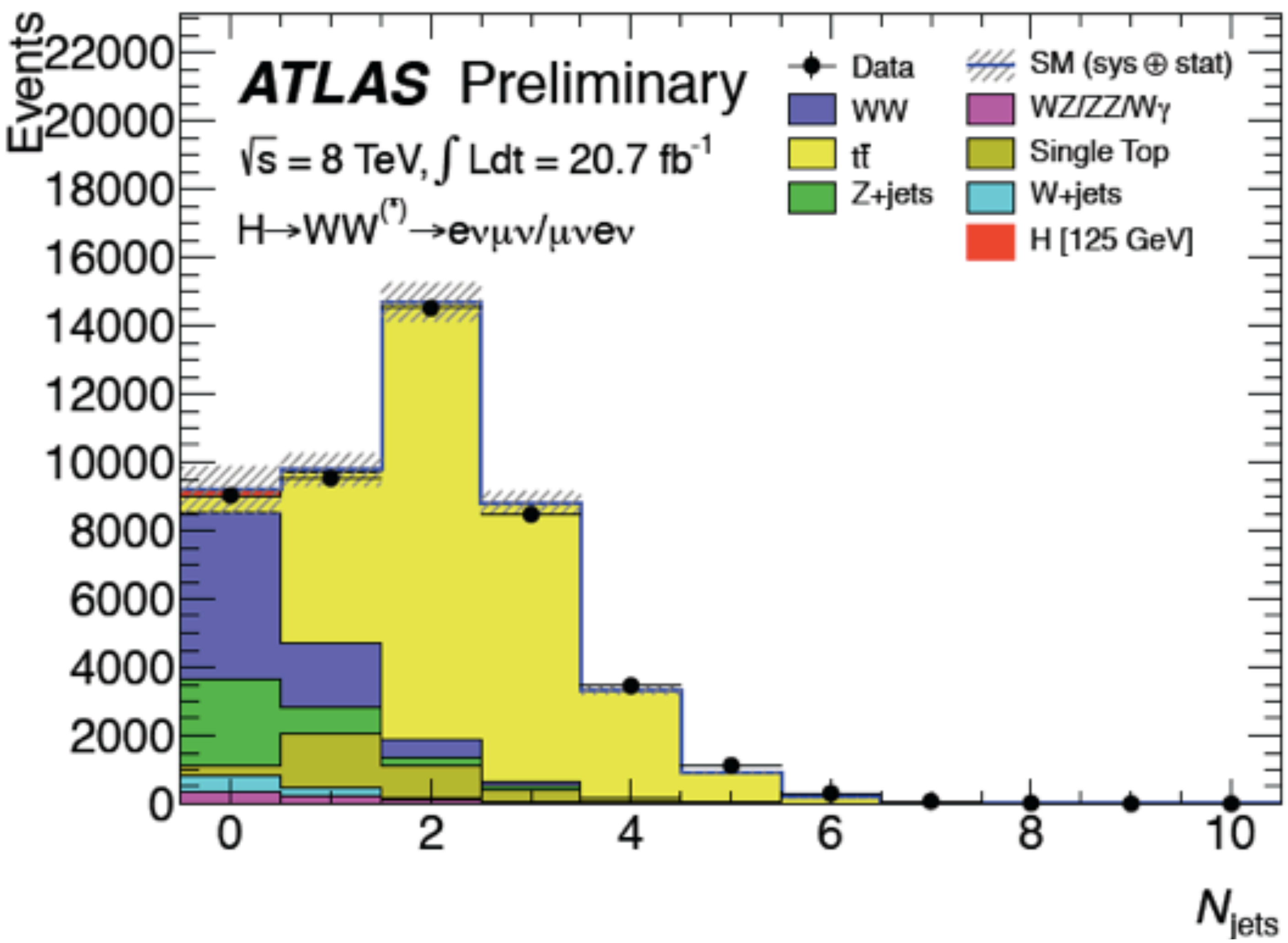
Sharpening our image



- Systematic errors already approaching statistical; will overtake with 14 TeV data
- Systematic error shown is the combination of experimental and theoretical systematics; theory already dominates this
- A particular issue, especially in the WW channel, is the division into bins of exclusive jet multiplicity

Source	$N_{jet} = 0$	$N_{jet} = 1$	$N_{jet} \geq 2$
Theoretical uncertainties on total signal yield (%)			
QCD scale for ggF, $N_{jet} \geq 0$	+13	-	-
QCD scale for ggF, $N_{jet} \geq 1$	+10	−27	-
QCD scale for ggF, $N_{jet} \geq 2$	-	−15	+4
QCD scale for ggF, $N_{jet} \geq 3$	-	-	+4
Parton shower and underlying event	+3	−10	±5
QCD scale (acceptance)	+4	+4	±3
Experimental uncertainties on total signal yield (%)			
Jet energy scale and resolution	5	2	6
Uncertainties on total background yield (%)			
WW transfer factors (theory)	±1	±2	±4
Jet energy scale and resolution	2	3	7
b-tagging efficiency	-	+7	+2
f_{recoil} efficiency	±4	±2	-

Exclusive jet bins

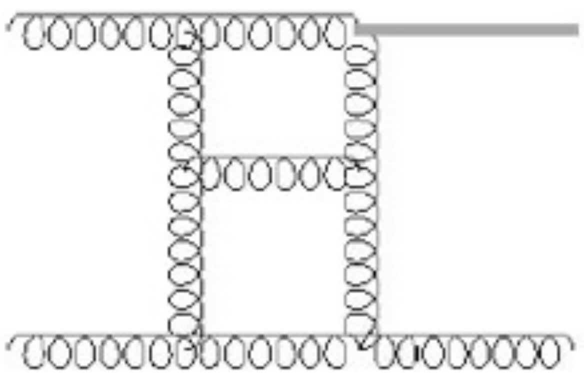


Category	$N_{\text{jet}} = 0$	$N_{\text{jet}} = 1$	$N_{\text{jet}} \geq 2$
Pre-selection	Two isolated leptons ($\ell = e, \mu$) with opposite charge Leptons with $p_{\text{T}}^{\text{lead}} > 25$ and $p_{\text{T}}^{\text{sublead}} > 15$ $e\mu + \mu e$: $m_{\ell\ell} > 10$ $ee + \mu\mu$: $m_{\ell\ell} > 12$, $\lvert m_{\ell\ell} - m_Z \rvert > 15$		
Missing transverse momentum and hadronic recoil	$e\mu + \mu e$: $E_{\text{T,rel}}^{\text{miss}} > 25$ $ee + \mu\mu$: $E_{\text{T,rel}}^{\text{miss}} > 45$ $ee + \mu\mu$: $p_{\text{T,rel}}^{\text{miss}} > 45$ $ee + \mu\mu$: $f_{\text{recoil}} < 0.05$	$e\mu + \mu e$: $E_{\text{T,rel}}^{\text{miss}} > 25$ $ee + \mu\mu$: $E_{\text{T,rel}}^{\text{miss}} > 45$ $ee + \mu\mu$: $p_{\text{T,rel}}^{\text{miss}} > 45$ $ee + \mu\mu$: $f_{\text{recoil}} < 0.2$	$e\mu + \mu e$: $E_{\text{T}}^{\text{miss}} > 20$ $ee + \mu\mu$: $E_{\text{T}}^{\text{miss}} > 45$ $ee + \mu\mu$: $E_{\text{T,STVF}}^{\text{miss}} > 35$ -
General selection	- $\lvert \Delta\phi_{\ell\ell,MET} \rvert > \pi/2$ $p_{\text{T}}^{\ell\ell} > 30$	$N_{b\text{-jet}} = 0$ - $e\mu + \mu e$: $Z/\gamma^* \to \tau\tau$ veto	$N_{b\text{-jet}} = 0$ $p_{\text{T}}^{\text{tot}} < 45$ $e\mu + \mu e$: $Z/\gamma^* \to \tau\tau$ veto

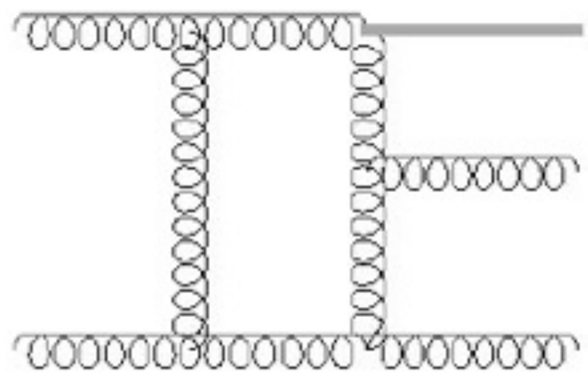
- Required experimentally in the WW channel due to the background composition
- Continuum WW production in the 0-jet bins; both WW and ttbar in the 1-jet bin; ttbar in the 2-jet bin
- Need different cuts as a function of jet multiplicity
- Typical jet-p_T choices: 25-30 GeV
- Similar divisions used in some ττ analyses

Higgs plus one-jet @NNLO

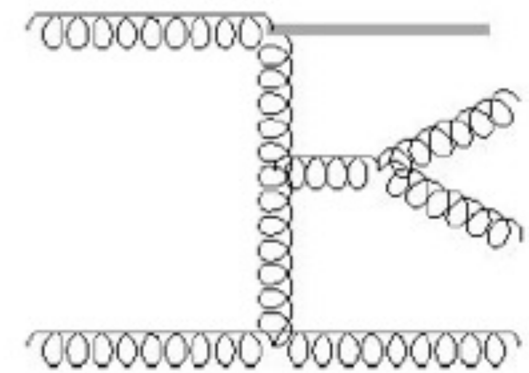
- Higgs plus zero-jets known in fixed-order through NNLO
- Until recently, Higgs plus one-jet known only through NLO
- The following ingredients are needed for H+1-jet @NNLO:



Gehrmann, Jaquier, Glover, Koukoutsakis (2011)



Badger, Glover, Mastrolia, Williams (2009)



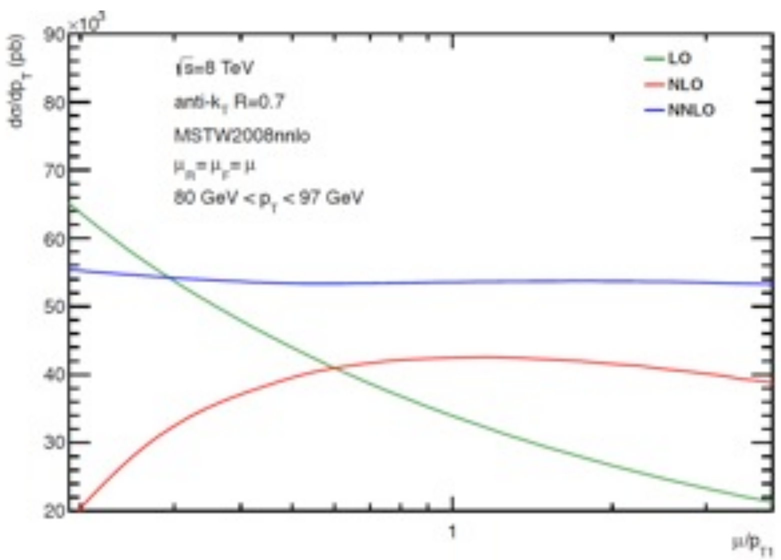
Del Duca, Frizzo, Maltoni;
Dixon, Glover, Khoze (2004)

- What prevented us from performing this calculation before now?
- IR singularities cancel only after phase space integration for real radiation

Sufficiently powerful subtraction schemes for extracting IR singularities from RR and RV before phase-space integration were unknown until very recently

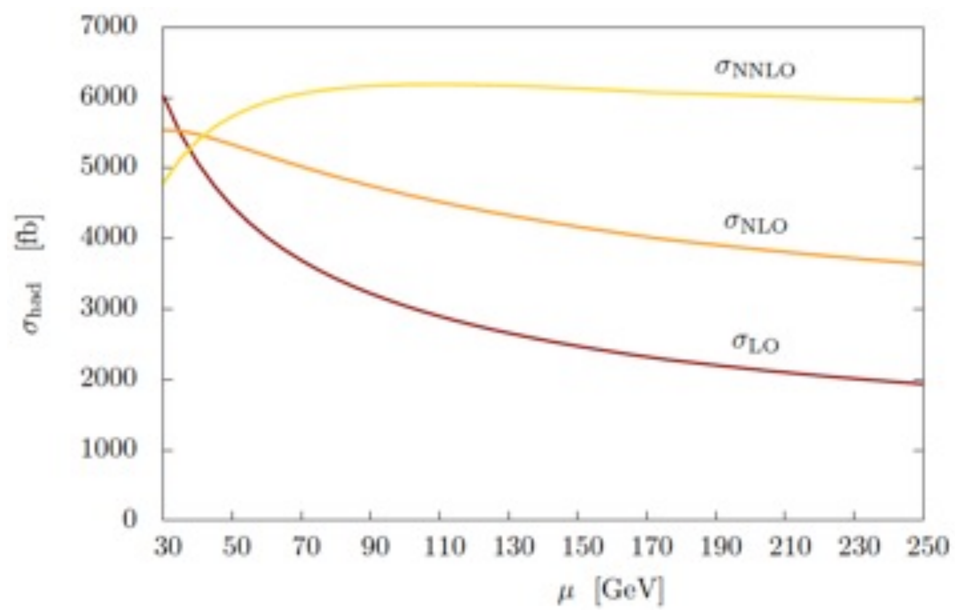
NNLO QCD at the LHC

- After more than a decade of research we finally know how to generically handle NNLO QCD corrections to processes with both colored initial and final states



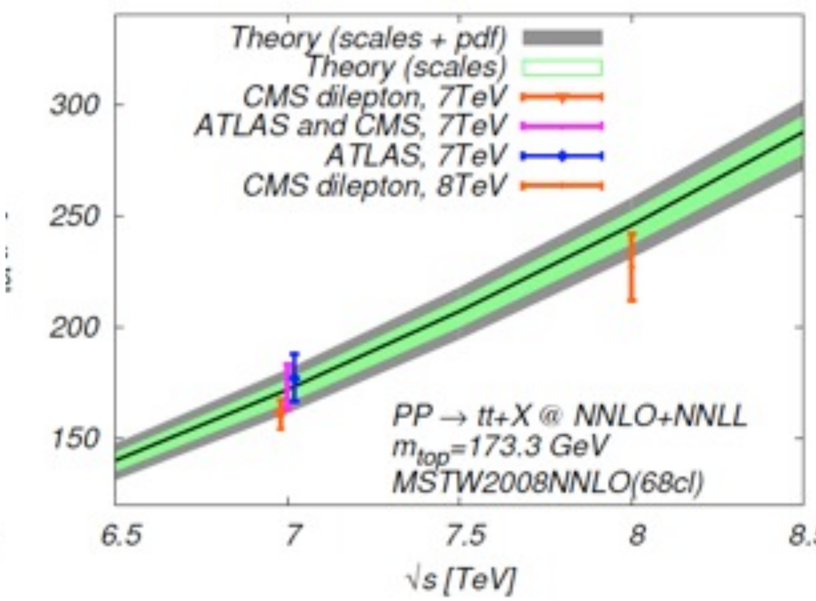
Gehrmann-de Ridder, Gehrmann, Glover, Pires (2013)

dijet: gg-channel



Boughezal, Caola, Melnikov, FP, Schulze (2013)

H+1j:gg-channel



Czakon, Fiedler, Mitov (2013)

ttbar: all-channels

Based on Antenna subtraction scheme

Based on sector-improved subtraction scheme

Subtraction at NNLO

- The generic form of an NNLO subtraction scheme is the following:

$$
\begin{aligned}
d\sigma_{NNLO} = & \int_{d\Phi_{m+2}} \left(d\sigma^{R}_{NNLO} - d\sigma^{S}_{NNLO}\right) \\
& + \int_{d\Phi_{m+1}} \left(d\sigma^{V,1}_{NNLO} - d\sigma^{VS,1}_{NNLO}\right) \\
& + \int_{d\Phi_{m+2}} d\sigma^{S}_{NNLO} + \int_{d\Phi_{m+1}} d\sigma^{VS,1}_{NNLO} \\
& + \int_{d\Phi_{m}} d\sigma^{V,2}_{NNLO} \,,
\end{aligned}
$$

- Maximally singular configurations at NNLO can have two collinear, two soft singularities
- Subtraction terms must account for all of the many possible singular configurations: triple-collinear ($p_1||p_2||p_3$), double-collinear ($p_1||p_2, p_3||p_4$), double-soft, single-soft, soft +collinear, etc.

- The factorization of the matrix elements in all singular configurations is known in the literature

The triple-collinear example

- To illustrate the problems that occur when trying to use these formulae, consider the triple-gluon collinear limit. The factorization of the matrix element squared in this limit is the following.

$$|\mathcal{M}(\ldots,p_1,p_1,p_3)|^2 \approx \frac{4g_s^4}{s_{123}^2}\mathcal{M}^\mu(\ldots,p_1+p_2+p_3)\mathcal{M}^{\nu *}(\ldots,p_1+p_2+p_3)\, P^{\mu\nu}_{g_1g_2g_3}$$

$z_i = E_i/(\sum E_j)$

$$\begin{aligned}
\hat{P}^{\mu\nu}_{g_1g_2g_3} &= C_A^2 \left\{ \frac{(1-\epsilon)}{4s_{12}^2}\left[-g^{\mu\nu}t_{12,3}^2 + 16 s_{123}\frac{z_1^2 z_2^2}{z_3(1-z_3)}\left(\frac{\widetilde{k}_2}{z_2}-\frac{\widetilde{k}_1}{z_1}\right)^\mu \left(\frac{\widetilde{k}_2}{z_2}-\frac{\widetilde{k}_1}{z_1}\right)^\nu\right]\right. \\
&- \frac{3}{4}(1-\epsilon)g^{\mu\nu} + \frac{s_{123}}{s_{12}}g^{\mu\nu}\frac{1}{z_3}\left[\frac{2(1-z_3)+4z_3^2}{1-z_3} - \frac{1-2z_3(1-z_3)}{z_1(1-z_1)}\right] \\
&+ \frac{s_{123}(1-\epsilon)}{s_{12}s_{13}}\left[2z_1\left(\widetilde{k}_2^\mu \widetilde{k}_2^\nu \frac{1-2z_3}{z_3(1-z_3)} + \widetilde{k}_3^\mu \widetilde{k}_3^\nu \frac{1-2z_2}{z_2(1-z_2)}\right)\right. \\
&+ \frac{s_{123}}{2(1-\epsilon)}g^{\mu\nu}\left(\frac{4z_2z_3+2z_1(1-z_1)-1}{(1-z_2)(1-z_3)} - \frac{1-2z_1(1-z_1)}{z_2z_3}\right) \\
&+ \left.\left.\left(\widetilde{k}_2^\mu \widetilde{k}_3^\nu + \widetilde{k}_3^\mu \widetilde{k}_2^\nu\right)\left(\frac{2z_2(1-z_2)}{z_3(1-z_3)} - 3\right)\right]\right\} + (5 \text{ permutations}) \;.
\end{aligned}$$

Catani, Grazzini 1999

Entangled singularities

- To illustrate the problems that occur when trying to use these formulae, consider the triple-gluon collinear limit. The factorization of the matrix element squared in this limit is the following.

$$|\mathcal{M}(\ldots,p_1,p_1,p_3)|^2 \approx \frac{4g_s^4}{s_{123}^2}\mathcal{M}^{\mu}(\ldots,p_1+p_2+p_3)\mathcal{M}^{\nu *}(\ldots,p_1+p_2+p_3)\,P^{\mu\nu}_{g_1 g_2 g_3}$$

- When one introduces an explicit parameterization:

$s_{123} \sim E_1E_2(1\text{-}c_{12})+E_1E_3(1\text{-}c_{13})+E_2E_3(1\text{-}c_{23})$

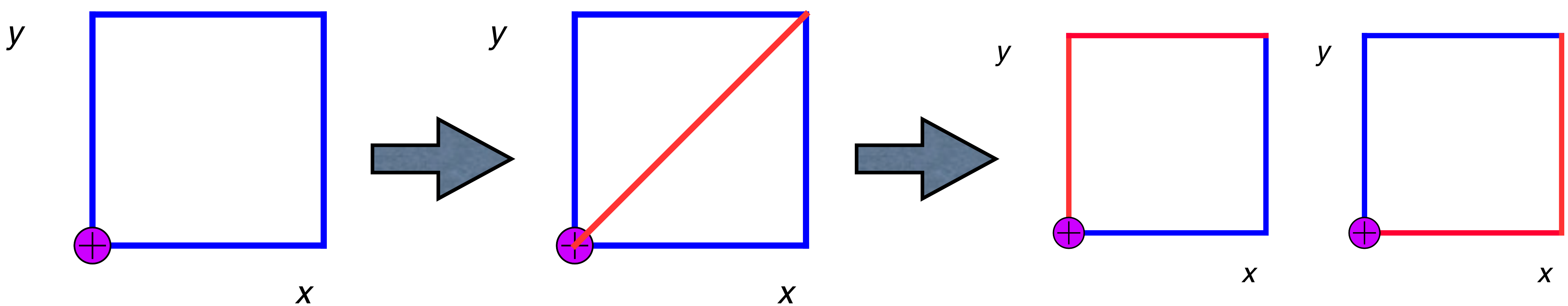
- What goes to zero quicker? E_1,E_2,E_3,$(1\text{-}c_{12})$,$(1\text{-}c_{13})$, or $(1\text{-}c_{23})$?
- Need to order the limits, since singularities must be extracted from integrals of the schematic form:

$$\int_0^1 dx dy \frac{x^{\epsilon} y^{\epsilon}}{(x+y)^2} F_J(x,y)$$

- Need a systematic technique for ordering limits, too many of such issues appear

Sector decomposition

- Can define a systematic procedure to order limits



$$I = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^\epsilon y^\epsilon}{(x+y)^2}$$

$$I_1 = \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \frac{x^\epsilon y^\epsilon}{(x+y)^2} \qquad I_1 = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^{-1+2\epsilon} y^\epsilon}{(1+y)^2}$$

$$I_2 = \int_0^1 \mathrm{d}y \int_0^y \mathrm{d}x \frac{x^\epsilon y^\epsilon}{(x+y)^2} \qquad I_2 = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{y^{-1+2\epsilon} x^\epsilon}{(1+x)^2}$$

$$y^{-1-\epsilon} = -\frac{\delta(y)}{\epsilon} + \left[\frac{1}{y}\right]_+ - \epsilon \left[\frac{\ln y}{y}\right]_+ + \mathcal{O}(\epsilon^2)$$

Binoth, Heinrich; Anastasiou, Melnikov, FP 2003-2005

Sector decomposition for simple processes

$$\int |M|^2 d\phi = \frac{F(0)}{\epsilon} + \int dx \frac{F(x) - F(0)}{x} + \ldots$$

Subtraction and integrated subtraction terms are for free

(no need for analytic PS integrations)

Successfully applied for NNLO differential cross sections, but for "special" processes

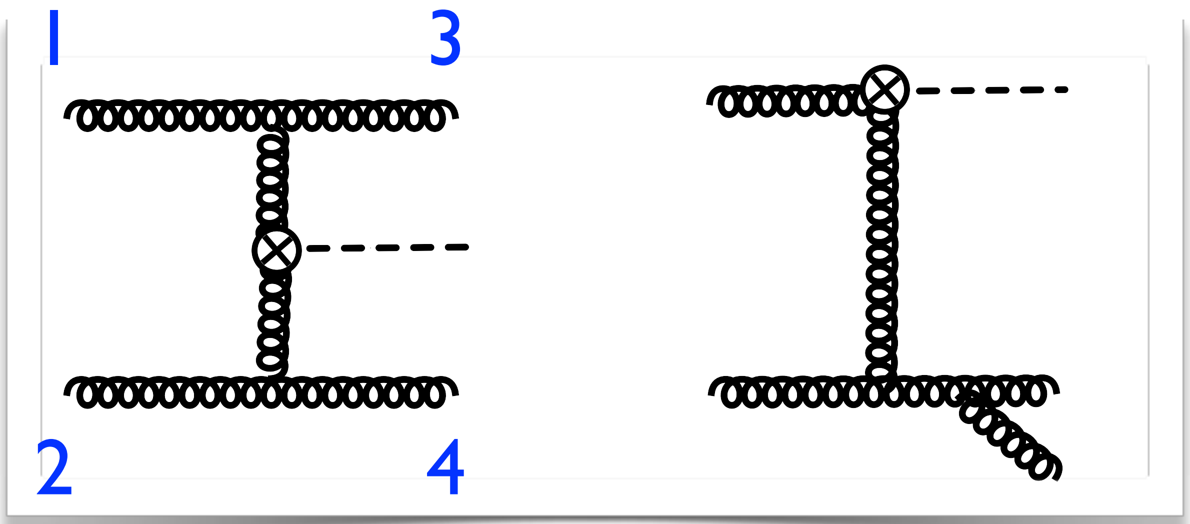
- $e^+e^- \to 2$ jets (Anastasiou, Melnikov, FP 2004)
- Higgs production (Anastasiou, Melnikov, FP 2005)
- Electroweak gauge boson production (Melnikov, FP 2006)

Note that:

- Parametrization known only for one collinear direction
- In its original version, sector decomposition is a highly process-dependent framework

Higgs production

•To illustrate the drawbacks, use Higgs production as an example. Consider one of the diagrammatic contributions to the double-real radiation correction.



• Invariants that occur in this topology : s_{13}, s_{24}, s_{134}, s_{34}. These contain collinear singularities $p_1||p_3$, $p_2||p_4$, $p_3||p_4$, $p_1||p_3||p_4$

•The structure of these singularities makes it difficult to find a suitable global parameterization amenable to sector decomposition.

•Would need to start over with entirely new parameterization for Higgs+jet

•However, can only have $p_1||p_3$ & $p_2||p_4$ or $p_1||p_3||p_4$ in a given phase space region. Not all invariants above can have collinear singularities simultaneously.

Higgs plus jet singularity structure

- Much more complicated singularity structure, in particular three collinear directions:

Potential troubles: $s_{1g}, s_{2g}, s_{3g}, s_{gg}, s_{1gg}, s_{2gg}, s_{3gg}$ and combinations

Finding a 'good' global parametrization is (very) hard

Sector-improved subtraction

- Key improvement : A combination of sector decomposition and FKS (Frixione,Kunszt,Signer) ideas makes the extraction of singularities systematic Czakon (2010)

  - @ NNLO the elementary building block is the double unresolved phase space where two unresolved particles can become soft or collinear to one or two hard directions

  - partition the phase space such that in each partition only a subset of particles leads to singularities: only two soft singularities can occur, and only one triple collinear or one double collinear singularity can occur.

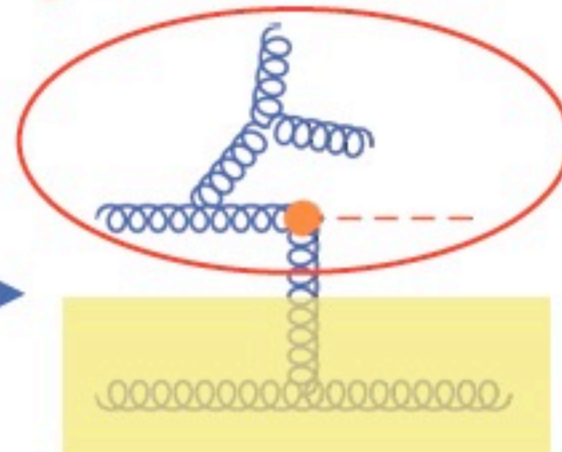
  - we can now pick a ***local*** parametrization for each partition

  - the partitioning is done using energies and angles of the unresolved particles w.r.t. the hard parton(s) emitting them

Sector-improved subtraction

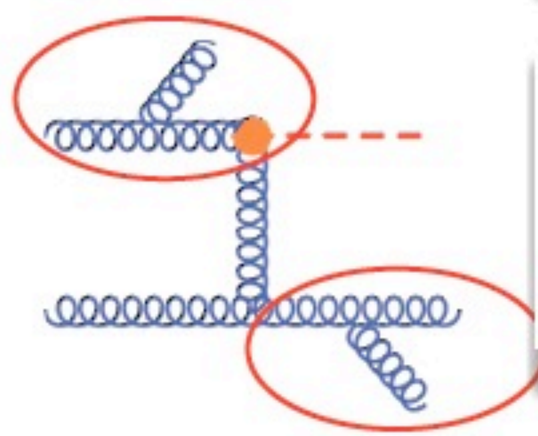
$$\int |M|^2 d\phi = \sum_s \int |M|^2 d\phi \Delta_s^{g_1||i, g_2||j}$$

$$\int |M|^2 d\phi \Delta^{g_1||1, g_2||1} \longrightarrow$$



Single collinear direction
~ parametrization of
ggH, DY, $e^+e^- \rightarrow$ dijets

$$\int |M|^2 d\phi \Delta^{g_1||1, g_2||3} \longrightarrow$$



Two (~uncorrelated) dir.
~ NLO^2

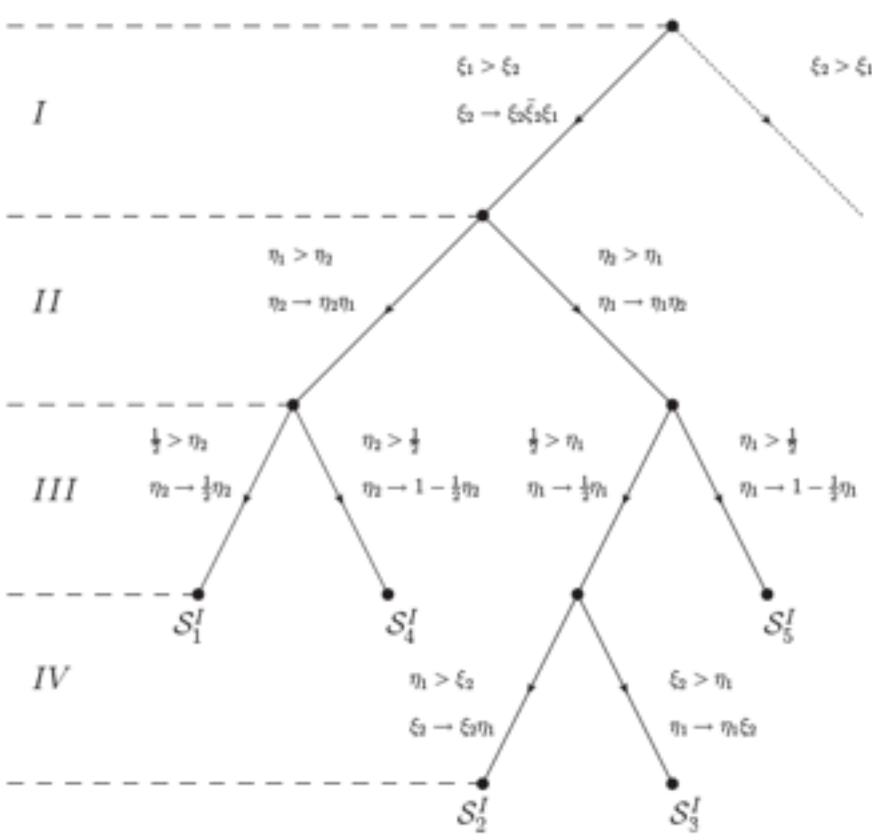
No matter how complicated the process is,
it can be reduced to the sum of individual contributions. For each of them, we know a sector decomposition-friendly PS parametrization

Partitioning for Higgs+jet

- First introduce a transverse-momentum partitioning to ensure that at least one hard parton is in the final state:

$$\Delta = \frac{p_{T3}}{p_{T3} + p_{T4} + p_{T5}}$$

- Left with the angular partitions: $p_5||p_4||p_1$, $p_5||p_4||p_2$, $p_5||p_4||p_3$, $p_5||p_1\&p_4||p_2$, $p_5||p_2\&p_4||p_1$, $p_5||p_1\&p_4||p_3$, $p_5||p_3\&p_4||p_1$, $p_5||p_2\&p_4||p_3$, $p_5||p_3\&p_4||p_2$



from Czakon,1005.0274

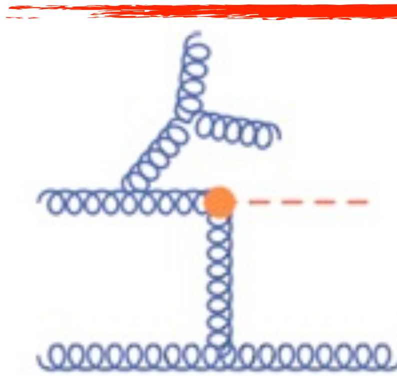
- Divide each partition into the necessary sectors; shown here for triple-collinear
- This same sector tree applies to ***all*** triple-collinear partitions
- Very helpful to use rotational invariance to use different reference frames in each partition. For $p_5||p_4||p_1$ set $p_1=E_1(1,0,0,1)$. For $p_5||p_4||p_3$, rotate and set $p_3=E_3(1,0,0,1)$.

Building blocks for Higgs+jet

Recall the general structure: $F(x) = \int [|M|^2 x]\{dy\}$

$$\int |M|^2 d\phi = \frac{F(0)}{\epsilon} + \int dx \frac{F(x) - F(0)}{x} + \ldots$$

The Subtraction terms are constructed from reduced matrix elements using QCD factorization of soft and collinear singularities



$$\sim \frac{P_{ggg} \otimes |M_j|^2}{s_{igg}}, \quad \frac{P_{gg} \otimes |M_{jj}|^2}{s_{gg}}$$

We need to provide

- $F(\vec{x};\{y\})$: fully-resolved matrix element (RR and RV)
- $\lim_{x_i \to 0} F(\vec{x};\{y\})$: matrix element in a singular configuration

↓

$\lim_{x_i \to 0} F(\vec{x};\{y\})$: reduced (=lower multiplicity) matrix element times universal eikonals / splitting functions

[Catani, Grazzini (1998, 2000); Kosower, Uwer (1999)]

Real-virtual

- Treatment of the real-virtual corrections possible with same technique
- Phase-space is that of an NLO real-emission correction, so FKS@NLO is suitable.
- However, the amplitudes now have branch cuts, which change the overall fractional powers appearing in the integral we must perform.

$$\mathrm{RV}_i = \int \{dy\} \frac{dx_1}{x_1^{1+2\epsilon}} \frac{dx_2}{x_2^{1+\epsilon}} \left(F_{i,1} + (x_1^2 x_2)^{-\epsilon} F_{i,2} + x_1^{-2\epsilon} F_{i,3} \right)$$

energy variable (x_1)

angular variable (x_2)

must keep track of the different fractional powers which can appear, to properly expand in plus distributions

Building blocks for Higgs+jet

- The use of d-dimensional rotational invariance necessitates CDR

  - tree-level H+3j
  - tree-level H+2j up to $O(\varepsilon^2)$
  - tree-level H+1j up to $O(\varepsilon)$
  - one-loop H+2j Badger, Glover, Mastrolia, Williams (2009)
  - one-loop H+1j up to $O(\varepsilon^2)$
  - two-loop H+1j Gehrmann, Jaquier, Glover, Koukoutsakis (2011)
  - renormalization, collinear subtraction

- Since the amplitudes have to be evaluated near singular configurations, numerical stability of all the above amplitudes is very important
- Extremely grateful to MCFM and Gehrmann, Glover et al. for providing the H+2-jet@1-loop and H+1-jet@2-loops amplitudes in a user-friendly format!

Checks

Two entirely independent computations (JHU/ANL-Northwestern)

Phase space parametrization and partitioning

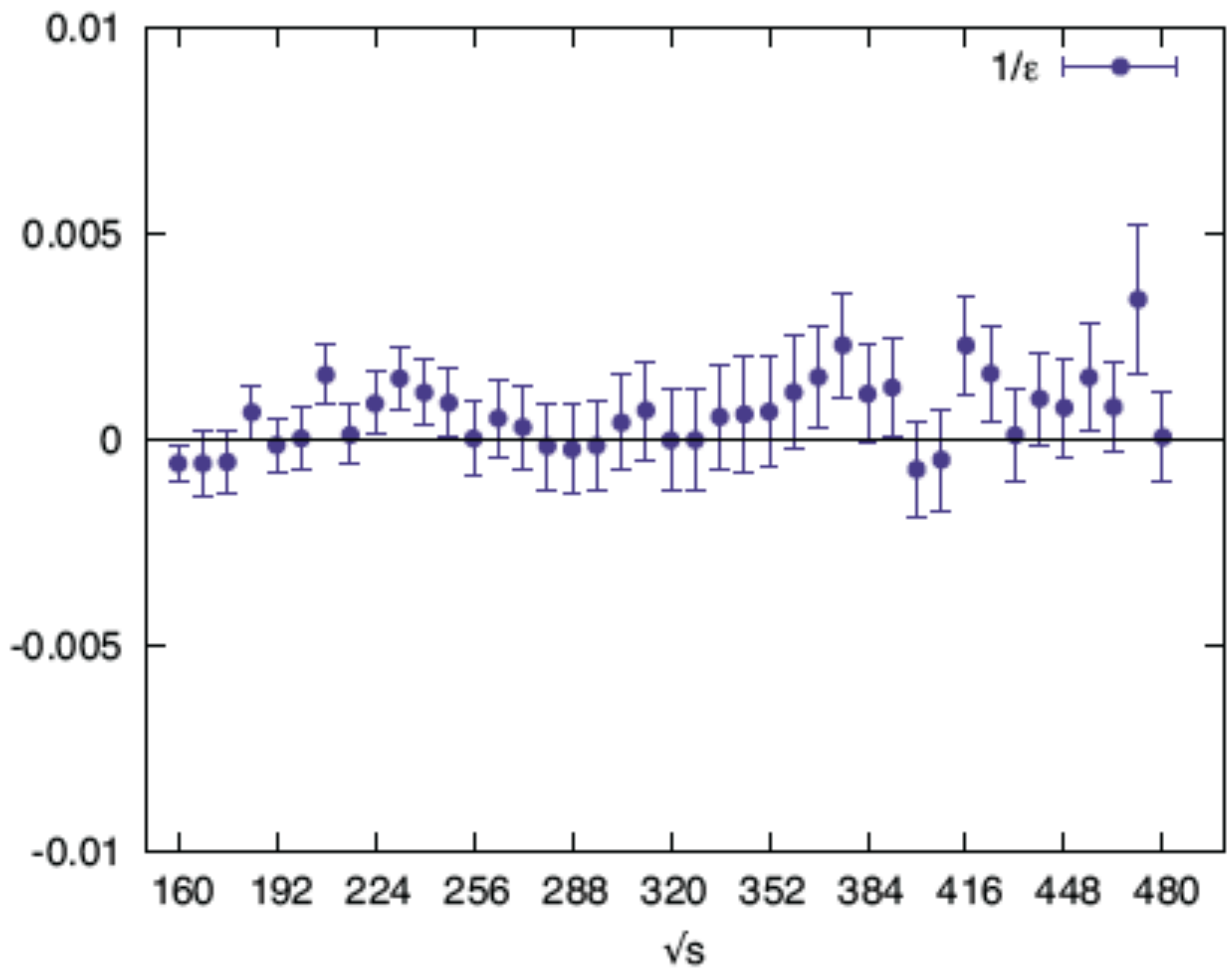
- correct D-dimensional PS volume in each partition
- rotational invariance in D-dimensions (spin-correlations)

Amplitudes

- tree-level amplitudes tested against MadGraph
- loop-amplitudes implementation checked against original MCFM
- singular limits
- D-dimensional helicity amplitudes checked against brute-force computation for $\sum_{pol} |M|^2$

Pole cancellation

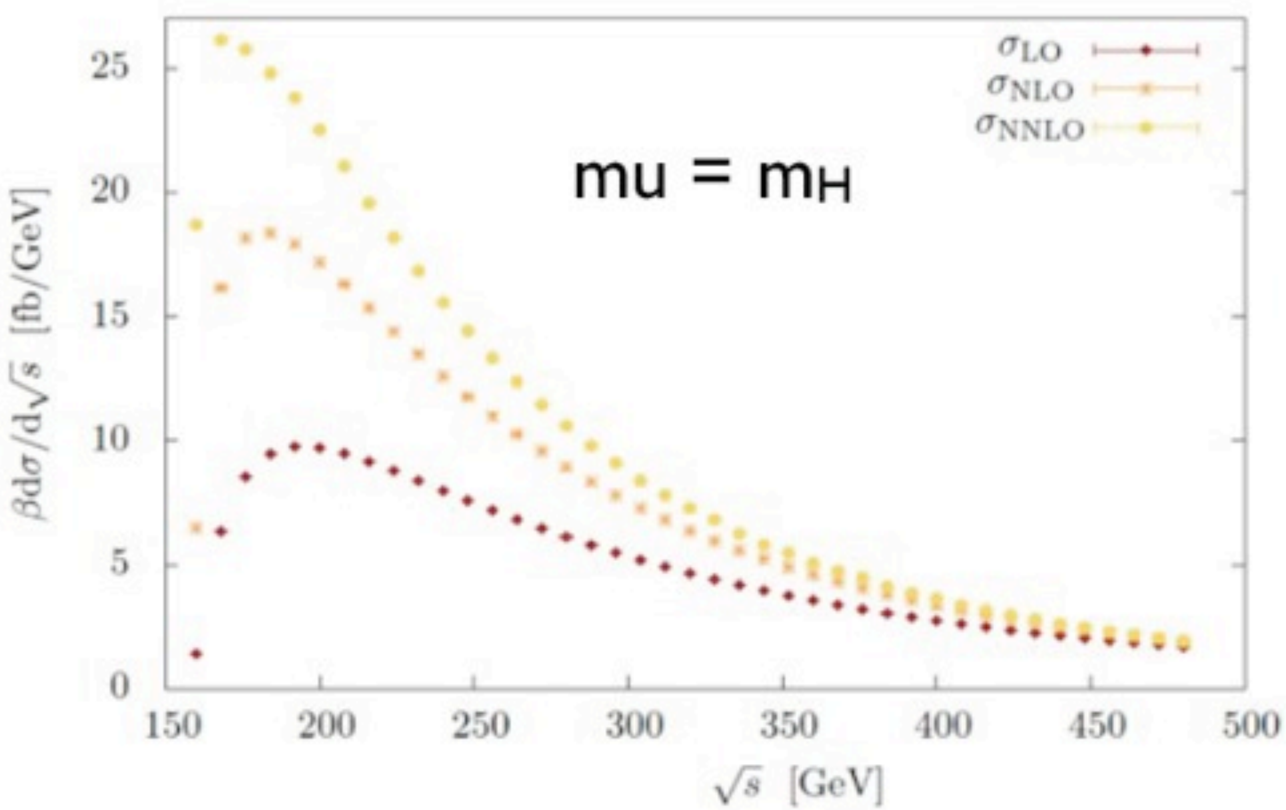
Numerical Cancellation between
renormalization and coll. couterterms, RR, RV, VV

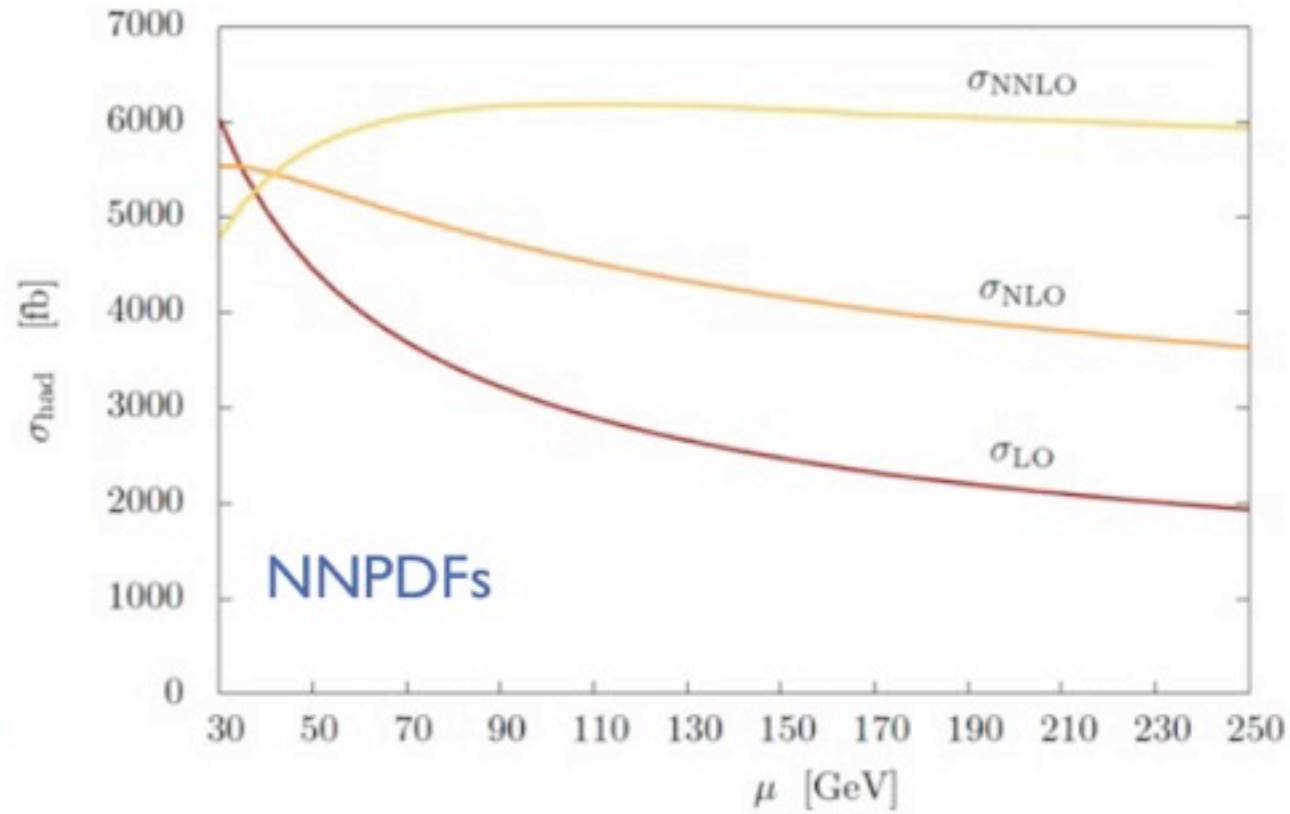


$1/\epsilon$ poles, degree of cancellation $\sim 10^{-3}$

$(1/\epsilon^2 : \ \sim 10^{-4})$

Numerical results (gg only)

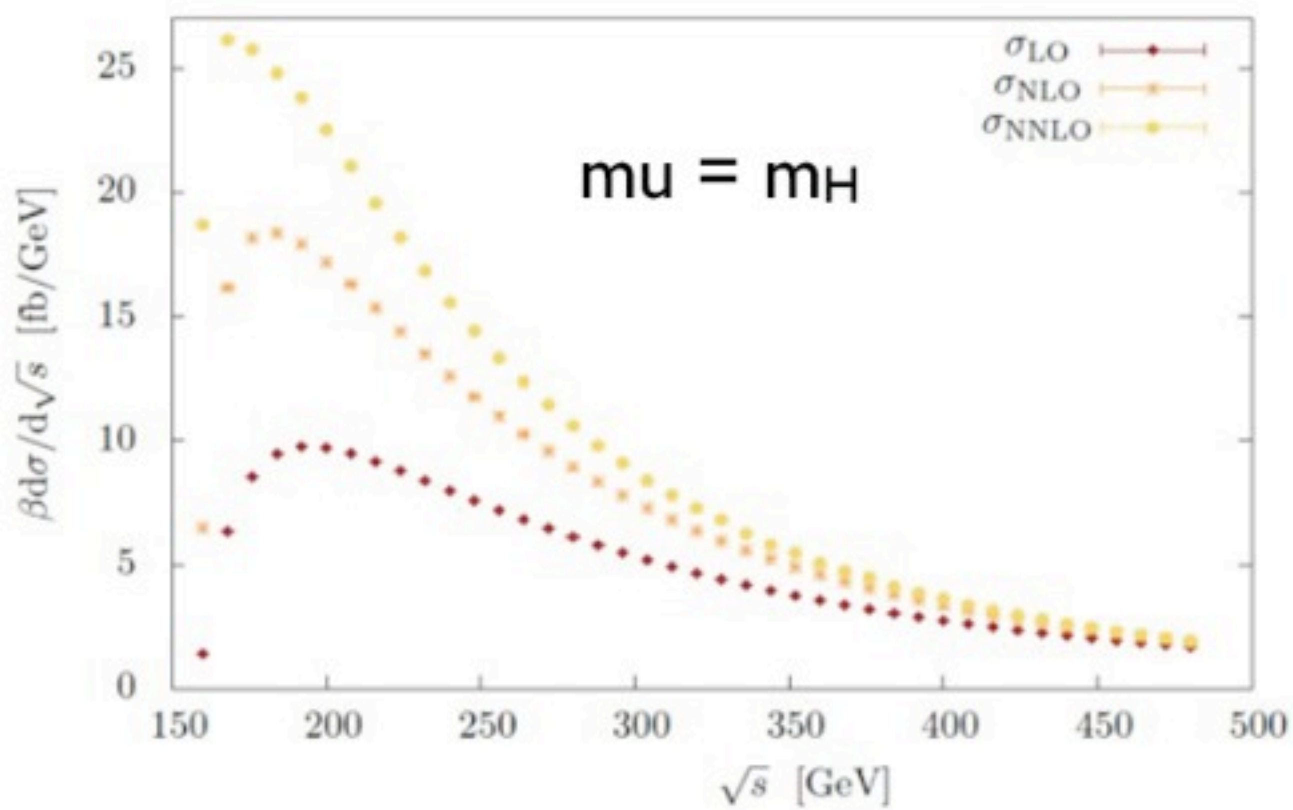


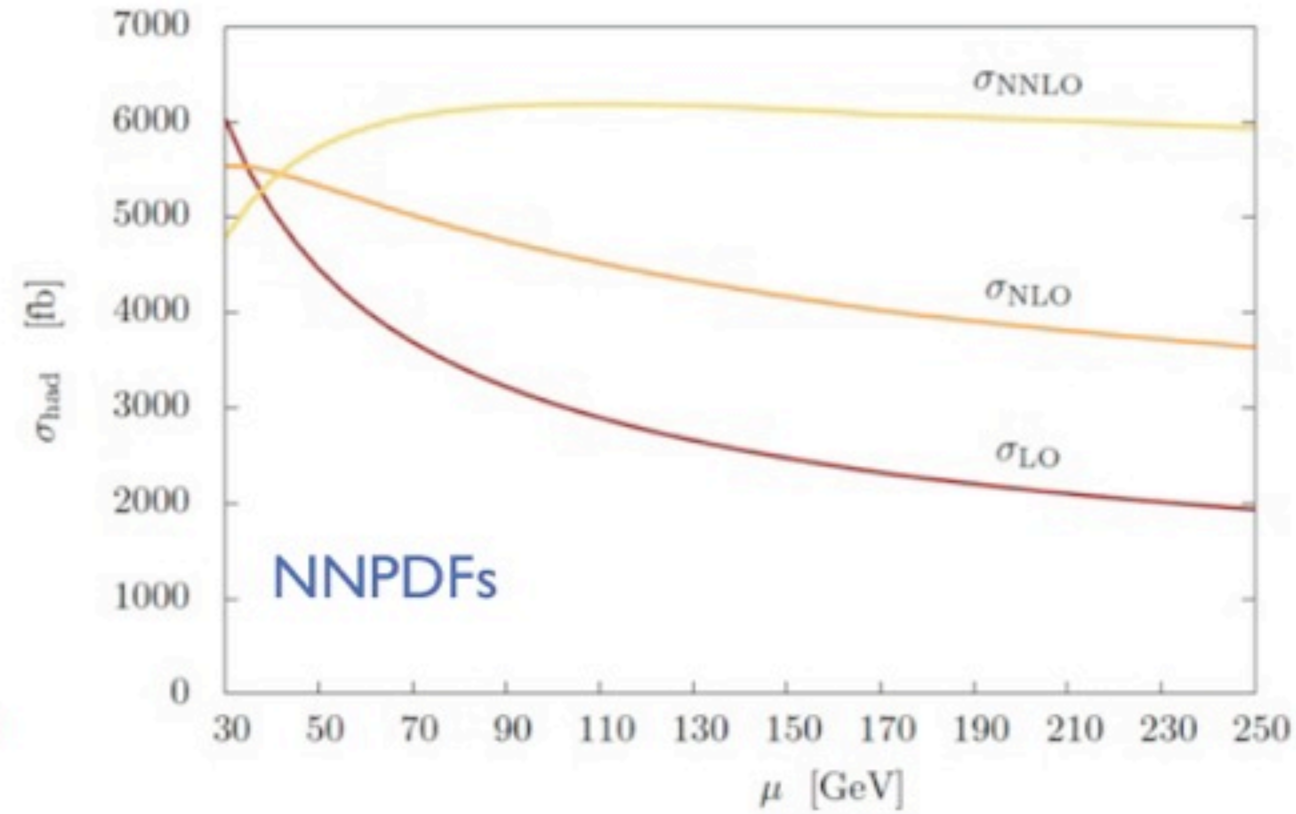


Boughezal, Caola, Melnikov, FP, Schulze 1302.6216

- Partonic cross section for $gg \to Hj$ @ LO, NLO, NNLO
- Realistic jet algorithm, k_T with R=0.5, $p_T > 30$ GeV
- Hadronic cross-section $pp \to Hj$ using latest NNPDF sets
- Scale variation in the range $m_H/2 < \mu < 2\, m_H$, $m_H = 125$ GeV

Numerical results (gg only)





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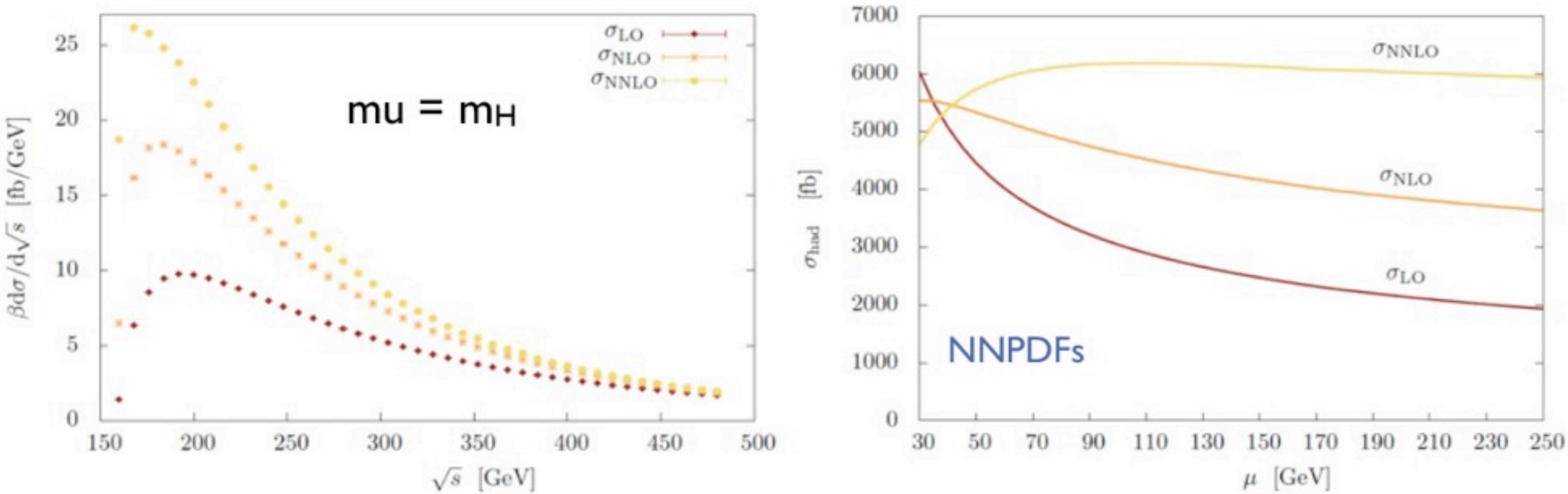
Significantly reduced scale dependence O(4%)

$\sigma_{NLO}/\sigma_{LO} = 1.6$

$\sigma_{NNLO}/\sigma_{NLO} = 1.3$

Large K-factor

Numerical results (gg only)



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- gg-channel is the dominant one for phenomenological studies: at NLO gg (70%), qg(30%)
- quark channels necessary for achieving the desired precision: ongoing work

Conclusions

- Increasingly precise experimental Higgs results demand improved theory predictions to unravel the origin of EWSB
- Work on the quark channels for H+jet at NNLO ongoing in order to provide phenomenological results for LHC experiments
- Looking forward to future applications of these techniques at the LHC!