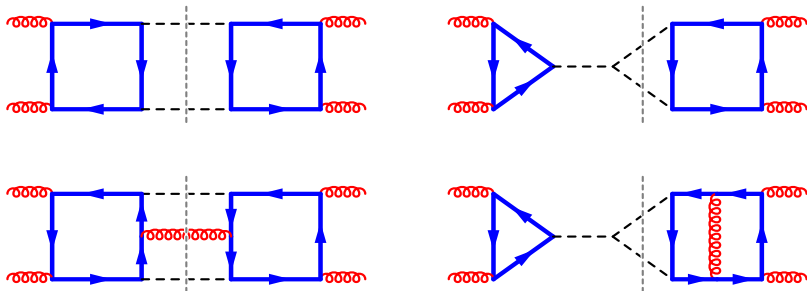


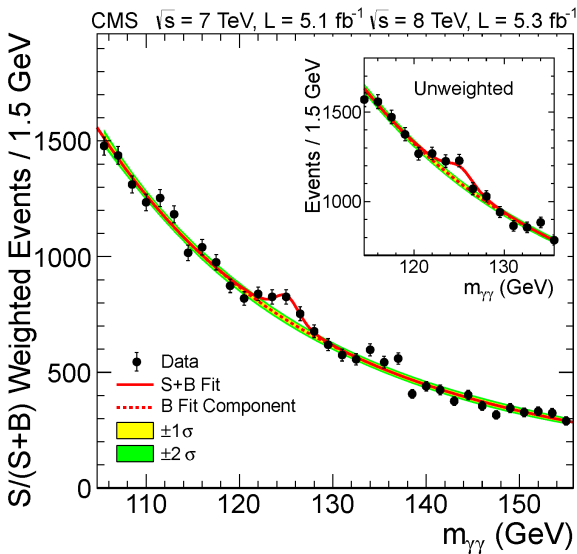
# Higgs boson pair production at the LHC: top quark mass effects at NLO

Radcor 2013, September 22-27, Lumley Castle, Chester-le-Street

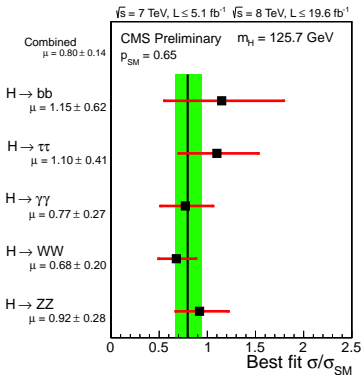
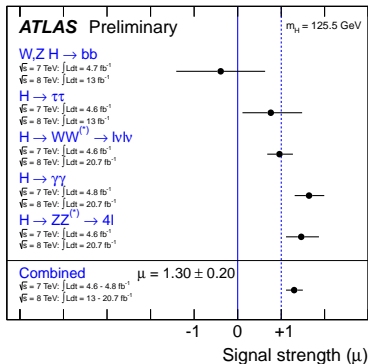
Matthias Steinhauser — TTP Karlsruhe | in collaboration with Jonathan Grigo, Jens Hoff, Kirill Melnikov



# Higgs boson discovery



# Higgs boson discovery

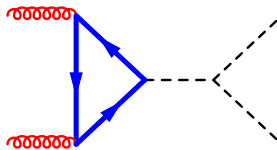
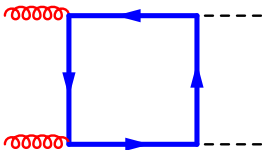


- SM Higgs boson?
- Couplings to fermions?
- Couplings to bosons?
- Self-coupling?

$$\text{■ } V_{\text{Higgs}} = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{1}{4} \lambda H^4$$

$$\lambda^{\text{SM}} = \frac{m_H^2}{2v^2} \approx 0.13$$

$$gg \rightarrow HH$$



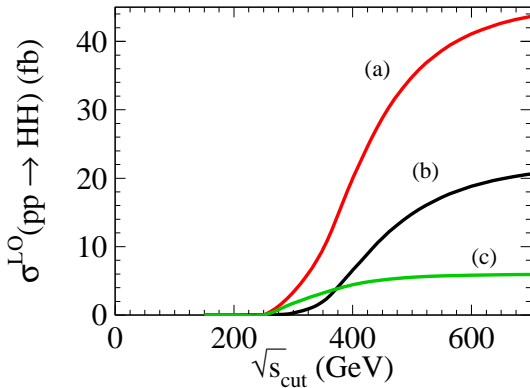
$q\bar{q}' \rightarrow HHq\bar{q}'$ ,  $q\bar{q}' \rightarrow ZHH/WHH$ ,  
 $\Leftrightarrow$  more than 10 times smaller

$q\bar{q}, gg \rightarrow t\bar{t}HH$

# $gg \rightarrow HH$ at LO

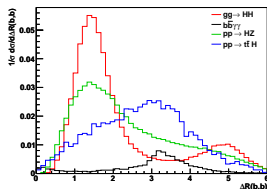
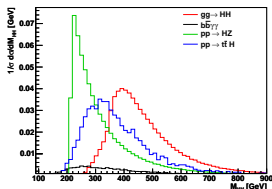
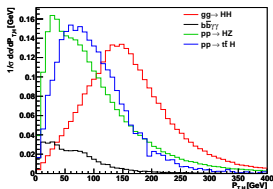


box  
triangle  
box + triangle



■  $gg \rightarrow HH \rightarrow b\bar{b}\gamma\gamma$

[Baglio,Djouadi,Gröber,Mühlleitner,Quevillon,Spira'13]:  $S/\sqrt{B} \approx 16$ , 51 signal events for  $\int \mathcal{L} = 3000 \text{ fb}^{-1}$

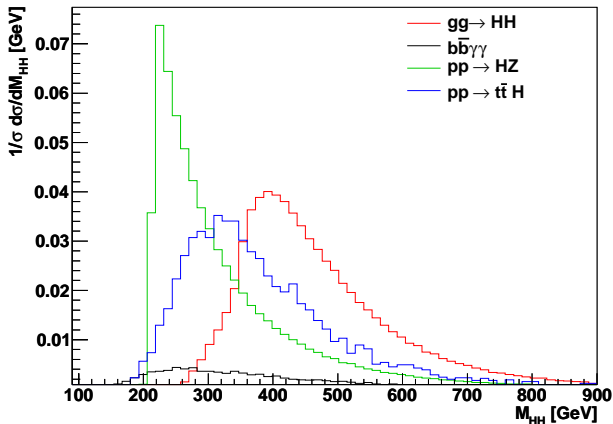


similar results: [Baur,Plehn,Rainwater'04]

# Promising channels/strategies

■  $gg \rightarrow HH \rightarrow b\bar{b}\gamma\gamma$

[Baglio,Djouadi,Gröber,Mühlleitner,Quevillon,Spira'13]:  $S/\sqrt{B} \approx 16$ , 51 signal events for  
 $\int \mathcal{L} = 3000 \text{ fb}^{-1}$

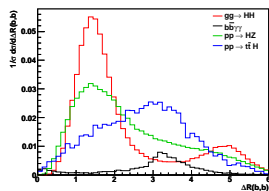
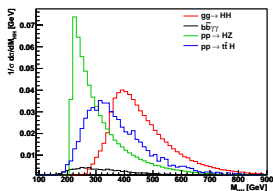
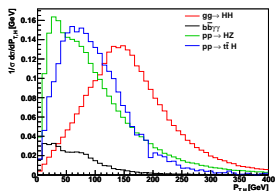


similar results:



- $gg \rightarrow HH \rightarrow b\bar{b}\gamma\gamma$

[Baglio,Djouadi,Gröber,Mühlleitner,Quevillon,Spira'13]:  $S/\sqrt{B} \approx 16$ , 51 signal events for  $\int \mathcal{L} = 3000 \text{ fb}^{-1}$



similar results: [Baur,Plehn,Rainwater'04]

- $gg \rightarrow HH \rightarrow b\bar{b}\tau\tau$ : “promising”;  $gg \rightarrow HH \rightarrow b\bar{b}WW$  ??

- $\frac{\sigma(gg \rightarrow HH)}{\sigma(gg \rightarrow H)}$   $\Leftrightarrow \lambda > 0$  at 95% C.L. with  $600 \text{ fb}^{-1}$  [Goertz,Papaefstathion,Yang,Zurita'13]

[Dolan,Englert,Spannowsky'12; Dawson,Furlan,Lewis'13, ...]

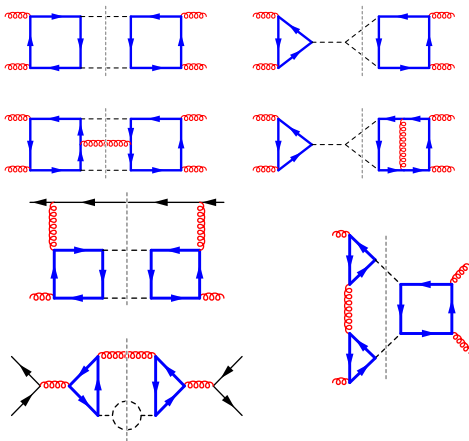
# $gg \rightarrow HH$ : known results

- LO [Glover, van der Bij'88; Plehn, Spira, Zerwas'96]
- NLO for  $m_t \rightarrow \infty$  [Dawson, Dittmaier, Spira'98]  $\approx +100\%$

this talk:  $1/m_t$  terms at NLO [Grigo, Hoff, Melnikov, Steinhauser'13]

- NNLO soft-virtual approx.,  $m_t \rightarrow \infty$  [de Florian, Mazzitelli'13]  $\approx +20\%$
- NNLL resummation [Shao, Li, Li, Wang'13]  $\approx \text{NLO} + 20\%$

- forward scattering amplitude + optical theorem



- forward scattering amplitude + optical theorem

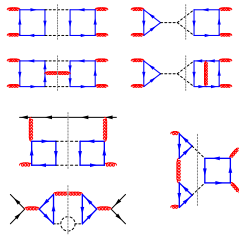
- Asymptotic expansion for  $m_t^2 \gg m_H^2, s$  with

exp [Harlander,Seidensticker,Steinhauser'98]

⇔ expansion in  $\rho = m_H^2/m_t^2$

⇔

$$\hat{\sigma}(gg \rightarrow HH) = \sum_n c(x) \left( \frac{m_H^2}{m_t^2} \right)^n, \quad x = \frac{4m_H^2}{s}$$



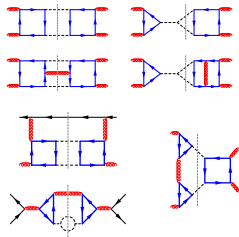
- forward scattering amplitude + optical theorem
- Asymptotic expansion for  $m_t^2 \gg m_H^2$ ,  $s$  with

exp [Harlander,Seidensticker,Steinhauser'98]

⇔ expansion in  $\rho = m_H^2/m_t^2$

⇔

$$\hat{\sigma}(gg \rightarrow HH) = \sum_n c(x) \left( \frac{m_H^2}{m_t^2} \right)^n, \quad x = \frac{4m_H^2}{s}$$



- FORM and TFORM [Vermaseren'90; ...; Tentyukov,Vermaseren'07; Kuipers,Ueda,Vermaseren,Vollinga'13]

- forward scattering amplitude + optical theorem

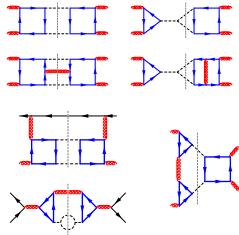
- Asymptotic expansion for  $m_t^2 \gg m_H^2, s$  with

exp [Harlander,Seidensticker,Steinhauser'98]

⇨ expansion in  $\rho = m_H^2/m_t^2$

⇨

$$\hat{\sigma}(gg \rightarrow HH) = \sum_n c(x) \left( \frac{m_H^2}{m_t^2} \right)^n, \quad x = \frac{4m_H^2}{s}$$



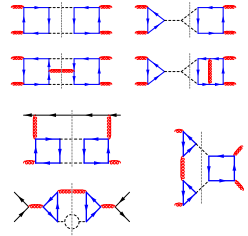
- FORM and TFORM [Vermaseren'90; ...; Tentyukov,Vermaseren'07; Kuipers,Ueda,Vermaseren,Vollinga'13]

- Reduction to MIs with FIRE [Smirnov'08; Smirnov'13]

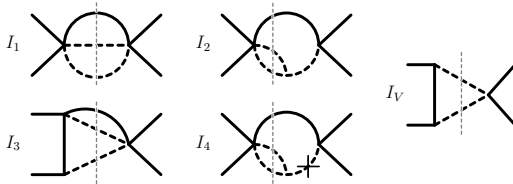
# Technique

- forward scattering amplitude + optical theorem
- Asymptotic expansion for  $m_t^2 \gg m_H^2$ , s with  
 $\exp$  [Harlander,Seidensticker,Steinhauser'98]  
 $\Leftrightarrow$  expansion in  $\rho = m_H^2/m_t^2$   
 $\Leftrightarrow$

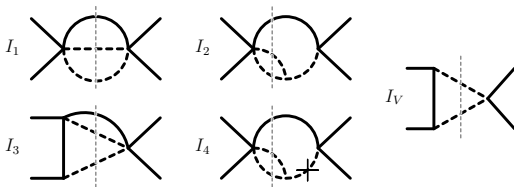
$$\hat{\sigma}(gg \rightarrow HH) = \sum_n c(x) \left( \frac{m_H^2}{m_t^2} \right)^n, \quad x = \frac{4m_H^2}{s}$$



- FORM and TFORM [Vermaseren'90; ...; Tentyukov,Vermaseren'07; Kuipers,Ueda,Vermaseren,Vollinga'13]
- Reduction to MIs with FIRE [Smirnov'08; Smirnov<sup>2</sup>'13]
- Master integrals (+ 2-loop tadpoles, ...)



# Master integrals



- derive 1-dimensional integral representation

- e.g.:  $I_1 = \mathcal{N} s^{1-2\epsilon} \delta^{5/2-3\epsilon} \int_0^1 \frac{d\mu}{\sqrt{1-\mu\delta}} (1-\mu)^{1/2-\epsilon} \mu^{1-2\epsilon}$

- expand in  $\delta = 1 - 4m_H^2/s = 1 - x \iff$  analytic results

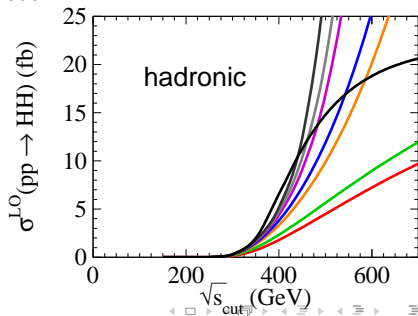
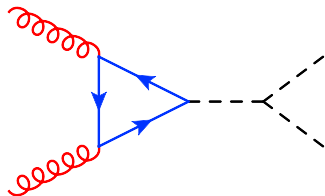
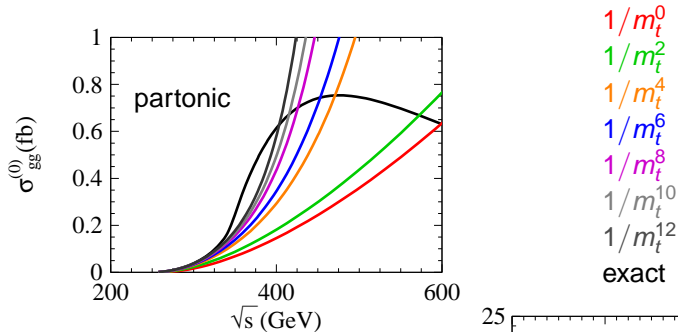


$$I_j = \sum_n^N c_n \delta^n \quad N = 100, 200, \dots$$

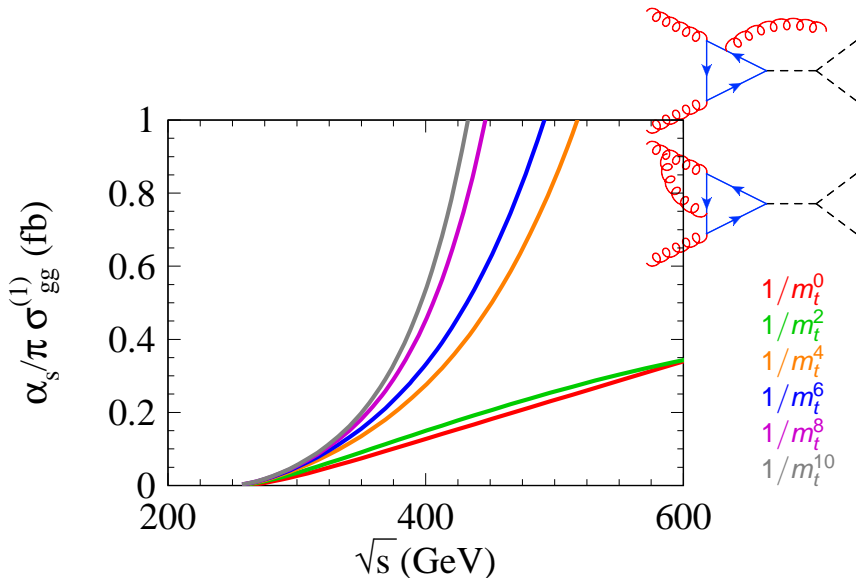
- $\sigma^{\text{partonic}}$  in analytic form



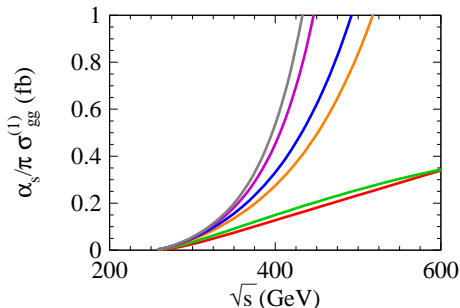
# LO result



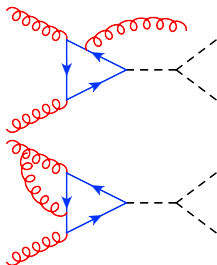
# NLO contribution: gluon channel, partonic



# NLO contribution: gluon channel, partonic

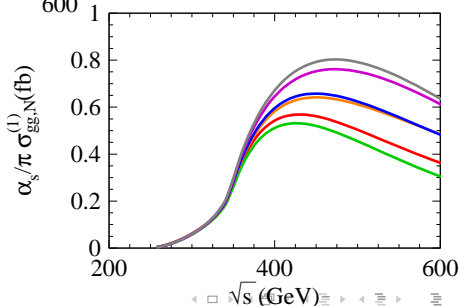


- $1/m_t^0$
- $1/m_t^2$
- $1/m_t^4$
- $1/m_t^6$
- $1/m_t^8$
- $1/m_t^{10}$

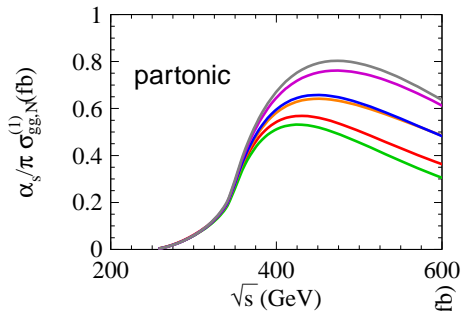


$$\sigma_{ij,N}^{(1)} = \sigma_{gg,exact}^{(0)} \frac{\sigma_{ij,exp}^{(1)}}{\sigma_{gg,exp}^{(0)}} \Leftrightarrow$$

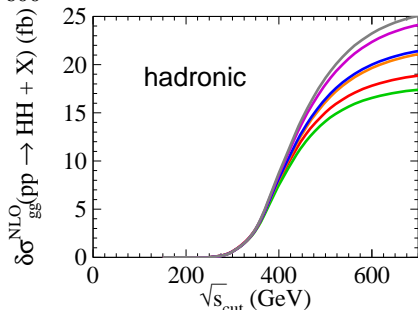
$N$ : use terms up to  $(1/m_t^2)^N$

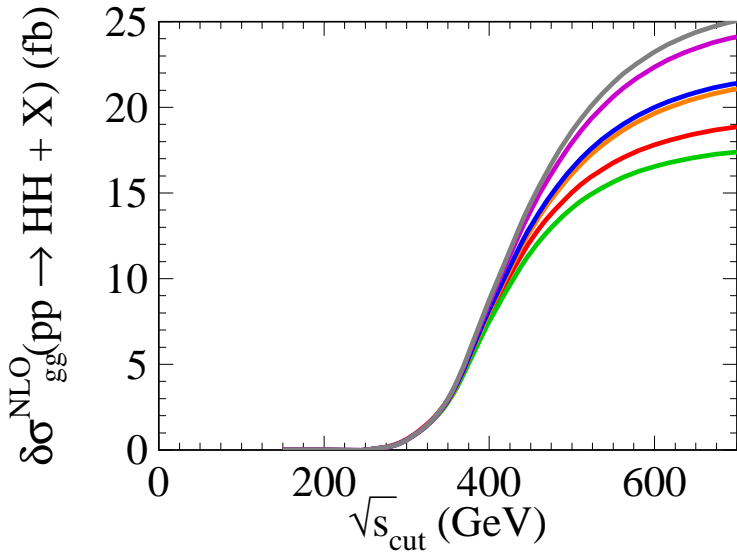


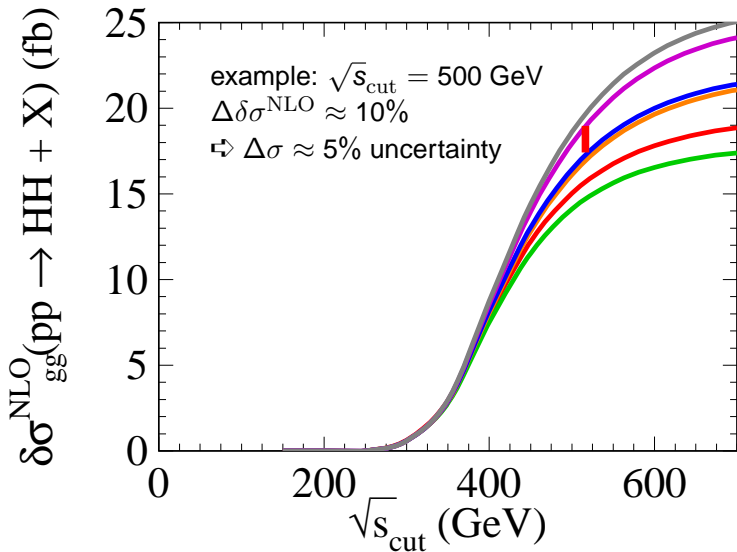
# NLO contribution: gluon channel



$s_{\text{cut}}$ : upper limit on partonic  
center-of-mass energy  
 $\approx m(HH)$  invariant mass

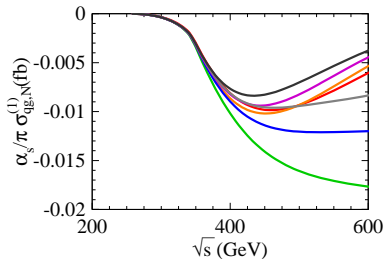






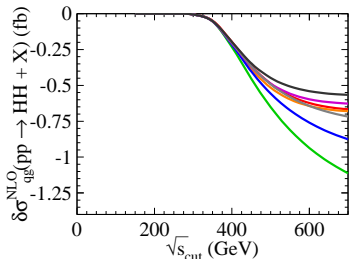
# NLO result: quark channels

partonic

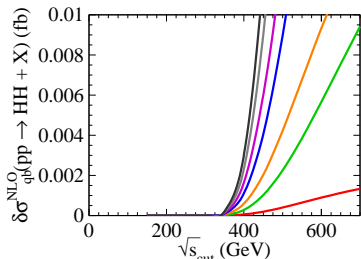
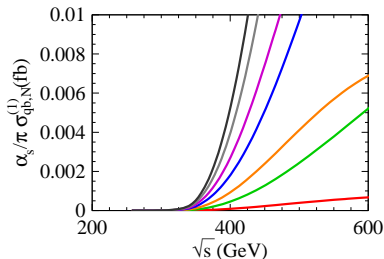


$qg$

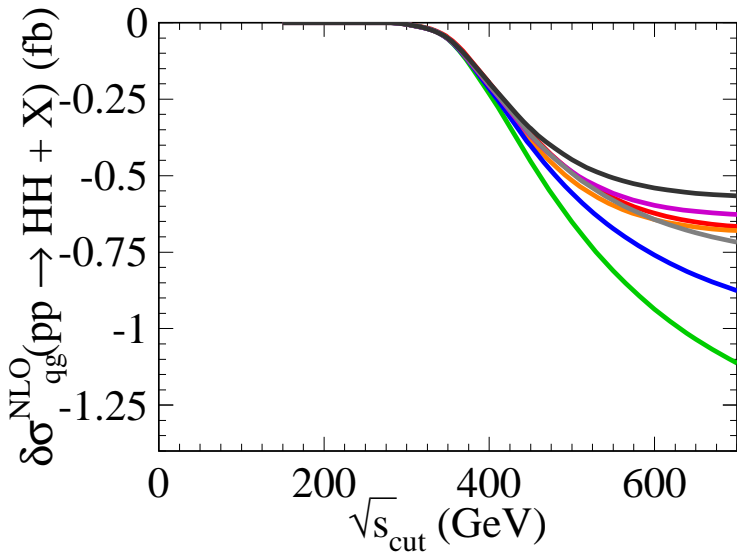
hadronic



$q\bar{q}$



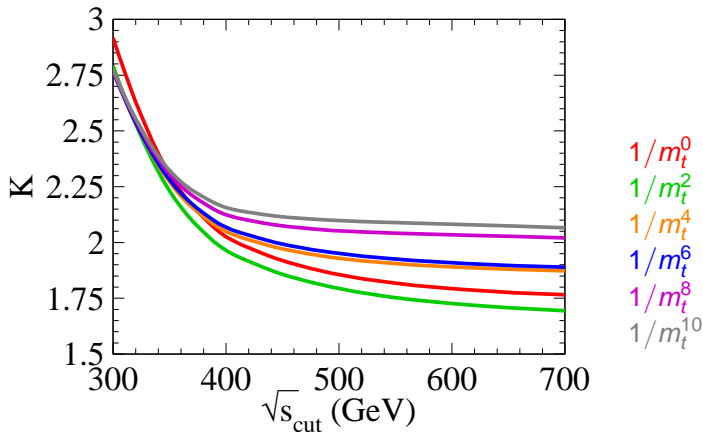
# NLO result: quark channels





# K factor

$$K = \frac{\sigma^{\text{NLO}}}{\sigma^{\text{LO}}}$$



⇒ enhancement close to threshold due to suppression of LO result

# Partonic NLO $gg$ contribution close to threshold

$$\rho = m_H^2/m_t^2, \quad \delta = 1 - x = 1 - 4m_H^2/s$$

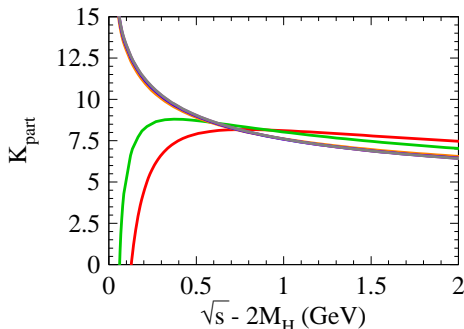
$$\sigma_{gg}^{\text{part}} = \frac{G_F^2 m_H^2}{2\pi} \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ \delta^{5/2} + \# \rho \delta^{3/2} + \rho^2 (\# \delta^{1/2} + \# \delta^{3/2}) \right. \\ \left. + \frac{\alpha_s}{\pi} \left[ \# \delta^{3/2} + \rho (\# \delta^{1/2} + \# \delta^{3/2}) + \rho^2 (\# \delta^{1/2} \ln^2 \delta + \# \delta^{3/2}) \right] \right\} + \dots$$

⇒  $K$  factor close to threshold strongly affected  
by power-suppressed  $1/m_t$  terms

# Partonic NLO $gg$ contribution close to threshold

$$\sigma_{gg}^{\text{part}} = \frac{G_F^2 m_H^2}{2\pi} \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ \delta^{5/2} + \# \rho \delta^{3/2} + \rho^2 (\# \delta^{1/2} + \# \delta^{3/2}) \right. \\ \left. + \frac{\alpha_s}{\pi} \left[ \# \delta^{3/2} + \rho (\# \delta^{1/2} + \# \delta^{3/2}) + \rho^2 (\# \delta^{1/2} \ln^2 \delta + \# \delta^{3/2}) \right] \right\} + \dots$$

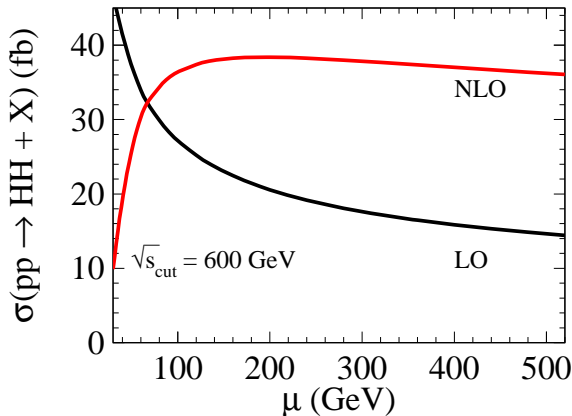
⇒  $K$  factor close to threshold strongly affected by power-suppressed  $1/m_t$  terms



$$K_{\text{part}} = \frac{\sigma_{\text{part}}^{\text{NLO}}}{\sigma_{\text{part}}^{\text{LO}}}$$

# Scale dependence

$$\mu = \mu_f = \mu_r$$

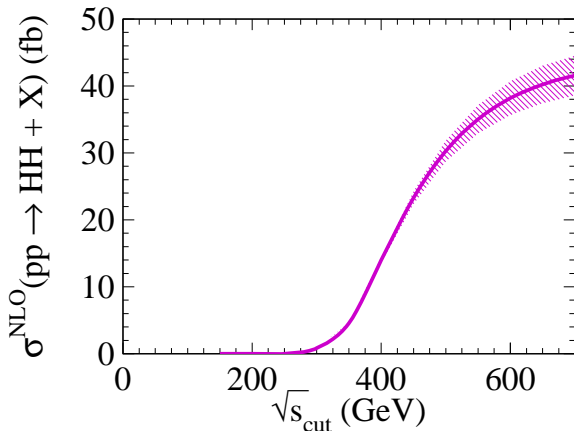


$$\mu^{\text{central}} = 2m_H: \quad \sigma^{\text{LO}} = 18_{-4}^{+6} \text{ fb} \quad \Leftrightarrow \quad \sigma^{\text{NLO}} = 38_{-2}^{+0} \text{ fb}$$

# Implication on extraction of $\lambda_{HHH}$

NLO prediction including  $1/m_t^8$  terms

uncertainty band: compare with  $1/m_t^6$  terms

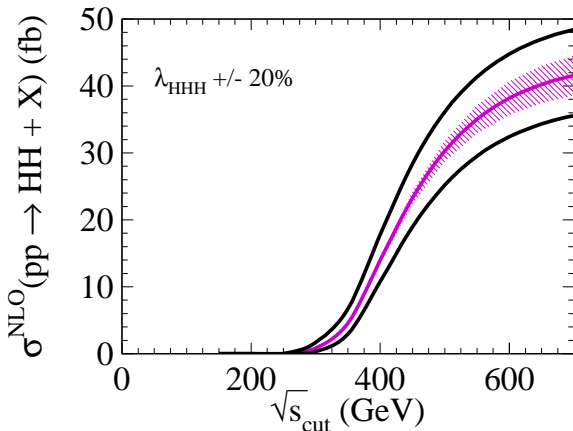


$\Rightarrow \sigma^{\text{NLO}}$  including  $1/m_t$  corrections sufficient to detect  $\mathcal{O}(10\%)$   
deviations of  $\lambda$  from  $\lambda^{\text{SM}}$

# Implication on extraction of $\lambda_{HHH}$

NLO prediction including  $1/m_t^8$  terms

uncertainty band: compare with  $1/m_t^6$  terms



$\Rightarrow \sigma^{\text{NLO}}$  including  $1/m_t$  corrections sufficient to detect  $\mathcal{O}(10\%)$  deviations of  $\lambda$  from  $\lambda^{\text{SM}}$

- $\sigma(pp \rightarrow HH)$  SM cross section
- determination of  $\lambda_{HHH}$
- NLO cross section  
independent check of [Dawson,Dittmaier,Spira'98]
- $1/m_t$  corrections  $\Leftrightarrow$  10% increase
- $K \nearrow$  for  $s \rightarrow$  threshold ( $1/m_t$  terms important!)  
( $\longleftrightarrow$  extraction of  $\lambda_{HHH}$ )
- reliable error estimate at NLO