Towards two-loop corrections to ZZ and WW production at LHC

Lorenzo Tancredi

Institute for Theoretical Physics - Zurich University

RADCOR 2013
Durham, 23-27 September 2013

talk based on a work together with *Thomas Gehrmann* and *Erich Weihs*:
JHEP 1202 (2012) 004, JHEP 1304 (2013) 101, JHEP 1308 (2013) 070

Why study vector boson pair production (up to **NNLO**)?

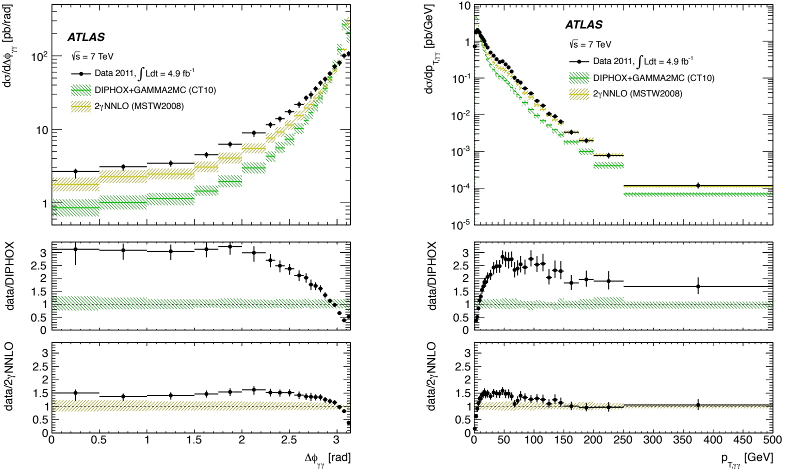
- **Background estimate for Higgs production** at LHC.
  For $m_H = 125$ GeV:
  1. $H \to \gamma\,\gamma$ Branching ratio **very small** - signal **very clear**
  2. $H \to W^+ W^-$ **Larger** branching ratio - **large** missing energy
  3. $H \to Z\,Z$ **Golden** channel

- Study of electroweak symmetry breaking mechanism,
  unitarization of $W\,W$ scattering amplitude.

- Anomalous **triple gauge bosons couplings** $WW\gamma$, WWZ, ...
  Indirect probe for new physics !

Why study vector boson pair production <u>up to **NNLO**</u>?

- **LHC has been running for 3 years!** $\gamma\gamma$ production at ATLAS



How do we get up to **NNLO**? *(in massless QCD!!)*

- **Two-loop** (double-virtual) : $q\bar{q} \to V_1 V_2$ ✗
- **One-loop** (real-virtual) : $q\bar{q} \to V_1 V_2\, g$ ✔
- **Tree-level** (real-real) : $q\bar{q} \to V_1 V_2\, g\, g$ ✔

Plus:

- a **regularisation scheme** for UV and IR divergences ✔
  $\to$ dimensional regularisation
- a **subtraction scheme** for phase-space integration ✔
  $\to$ q_T-subtraction, Antenna subtraction, Sector decomposition...

And a lot of work to put everything together!

Two-loop amplitudes are the **bottleneck** to get to **NNLO**

- How do we proceed? $\rightarrow$ **Diagrammatic approach**

1. Write down **Feynman diagrams**
2. Classify integrals into topologies
   $\rightarrow$ Same set of **denominators** raised to any powers

$$I = \int \Pi_i d^d k_i \, \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}}$$

3. Into every topology **high redundancy**
   $\rightarrow$ Integration-by-parts identities *(d-dimensions!!)*

$$\int \Pi_i d^d k_i \left(\frac{\partial}{\partial k_i^\mu} v_\mu \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}} \right) = 0, \qquad v^\mu = (k_i^\mu, p_j^\mu)$$

   (+ *Symmetry relations, Lorentz identities...)*

4. Solve for **Master Integrals** (MIs) $\rightarrow$ Reduze, AIR, FIRE,...

Two-loop amplitudes are the **bottleneck** to get to **NNLO**

- How do we proceed? → **Diagrammatic approach**

1. Write down **Feynman diagrams**

2. Classify integrals into topologies
   → Same set of **denominators** raised to any powers

$$I = \int \Pi_i d^d k_i \, \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}}$$

3. Into every topology **high redundancy**
   → Integration-by-parts identities *(d-dimensions!!)*

$$\int \Pi_i d^d k_i \left(\frac{\partial}{\partial k_i^\mu} v_\mu \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}} \right) = 0, \qquad v^\mu = (k_i^\mu, p_j^\mu)$$

   (+ *Symmetry relations, Lorentz identities...)*

4. Solve for **Master Integrals** (MIs) → Reduze, AIR, FIRE,...

Two-loop amplitudes are the **bottleneck** to get to **NNLO**

- How do we proceed? → **Diagrammatic approach**

1. Write down **Feynman diagrams**
2. Classify integrals into topologies
   → Same set of **denominators** raised to any powers

$$I = \int \Pi_i d^d k_i \, \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}}$$

3. Into every topology **high redundancy**
   → Integration-by-parts identities *(d-dimensions!!)*

$$\int \Pi_i d^d k_i \left(\frac{\partial}{\partial k_i^\mu} v_\mu \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}} \right) = 0, \qquad v^\mu = (k_i^\mu, p_j^\mu)$$

(+ *Symmetry relations, Lorentz identities...)*

4. Solve for **Master Integrals** (MIs) → Reduze, AIR, FIRE,...

Two-loop amplitudes are the **bottleneck** to get to **NNLO**

- How do we proceed? $\rightarrow$ **Diagrammatic approach**

1. Write down **Feynman diagrams**

2. Classify integrals into topologies
   $\rightarrow$ Same set of **denominators** raised to any powers

$$I = \int \Pi_i d^d k_i \, \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}}$$

3. Into every topology **high redundancy**
   $\rightarrow$ Integration-by-parts identities *(d-dimensions!!)*

$$\int \Pi_i d^d k_i \left(\frac{\partial}{\partial k_i^\mu} v_\mu \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}} \right) = 0, \qquad v^\mu = (k_i^\mu, p_j^\mu)$$

(+ *Symmetry relations, Lorentz identities...)*

4. Solve for **Master Integrals** (MIs) $\rightarrow$ Reduze, AIR, FIRE,...

Two-loop amplitudes are the **bottleneck** to get to **NNLO**

- How do we proceed? $\rightarrow$ **Diagrammatic approach**

1. Write down **Feynman diagrams**

2. Classify integrals into topologies
   $\rightarrow$ Same set of **denominators** raised to any powers

$$I = \int \Pi_i d^d k_i \, \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}}$$

3. Into every topology **high redundancy**
   $\rightarrow$ Integration-by-parts identities *(d-dimensions!!)*

$$\int \Pi_i d^d k_i \left(\frac{\partial}{\partial k_i^\mu} v_\mu \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}} \right) = 0, \qquad v^\mu = (k_i^\mu, p_j^\mu)$$

   (+ *Symmetry relations, Lorentz identities...)*

4. Solve for **Master Integrals** (MIs) $\rightarrow$ Reduze, AIR, FIRE,...

- **Computation** of the MIs $\rightarrow$ differential equation method
  1. PRO: Avoid direct **loop-integration**.
  2. CON: Need to fix a **boundary condition**.
- What makes the MIs **difficult to compute**?
  $\rightarrow$ Analytic structure of the amplitude given by interplay between:
  1. Number of **independent scales**
  2. **Kinematical constraints**
- Directly into the functions needed to represent the result:
  Polylogarithms $\rightarrow$ Multiple Polylogarithms $\rightarrow$ Elliptic Functions $\rightarrow$???

- **Computation** of the MIs $\rightarrow$ differential equation method
  1. PRO: Avoid direct **loop-integration**.
  2. CON: Need to fix a **boundary condition**.
- What makes the MIs **difficult to compute**?
  $\rightarrow$ Analytic structure of the amplitude given by interplay between:
  1. Number of **independent scales**
  2. **Kinematical constraints**
- Directly into the functions needed to represent the result:
  Polylogarithms $\rightarrow$ Multiple Polylogarithms $\rightarrow$ Elliptic Functions $\rightarrow$???

Vector boson pair production - Increasing in **complexity**:

- $q\,\bar{q} \to \gamma\,\gamma$
  1. **2** independent scales: $s + t + u = 0$

  → Nielsen Polylogarithms
  → **(MIs computed in ≈ 2000)** , NNLO ✔ → [Catani et al., 2011]

- $q\,\bar{q} \to Z\,\gamma\,/\,W^{\pm}\,\gamma$
  1. **3** independent scales: $s + t + u = m^2$

  → Multiple Polylogarithms
  → **(MIs computed in ≈ 2001)** , NNLO ✔ → [see D.Rathlev's Talk, ≈ 2013]

- $q\,\bar{q} \to Z\,Z\,/\,W^{\pm}\,W^{\pm}$
  1. Still **3** independent scales: $s + t + u = 2\,m^2$

  → Multiple Polylogarithms → **BUT** much more involved cut structure
  → **(MIs ≈ 2013 still in progress)**

What is special in ZZ/WW production?

- For $V\gamma$ production $\rightarrow$ **3 scales** but **linear kinematics**:

$$s > m^2\,, \quad \text{with} \quad -(s-m^2) < t < 0\,.$$

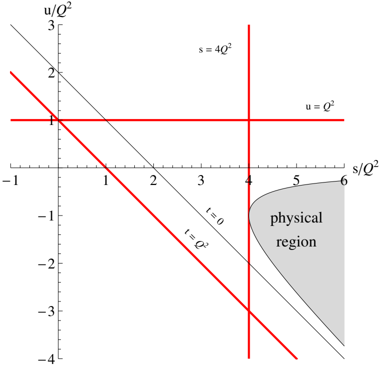
- For VV production $\rightarrow$ **3 scales** but **non-linear kinematics**:

$$p_1 + p_2 \longrightarrow q_1 + q_2\,, \qquad p_1^2 = p_2^2 = 0\,, \qquad q_1^2 = q_2^2 = m^2\,.$$

$$s > 4m^2\,, \quad u_- < u < u_+ \quad \text{with:}$$

$$u_- = \left[m^2 - \frac{s}{2}\left(1 + \sqrt{1 - \frac{4m^2}{s}}\right)\right]$$

$$u_+ = \left[m^2 - \frac{s}{2}\left(1 - \sqrt{1 - \frac{4m^2}{s}}\right)\right]$$



So long as we get only **GHPLs** we are lucky !

- **GHPLs** (or MPLs) - Definition:
  [E.Remiddi, J.Vermaseren; T.Gehrmann, E.Remiddi; A.B.Goncharov; ...]

$$G(0;y) = \ln y\,, \qquad G(a;y) = \ln(1 - y/a)\,,$$
$$G(\vec{0}_n;y) = \frac{1}{n!}\ln^n y$$
$$G(a_z,\vec{b}_z;y) = \int_0^y \frac{dt}{t-a_z}\,G(\vec{b}_z;t)$$

  a_z and $b_z^{(j)}$ are any functions of z.

  The cut structure of the GHPLs is contained in the **indices**!!!
  → *vector of singularities!*

- Many **techniques** have been developed to handle them:
  → **Symbol formalism**, **Co-product**, fast numerical routines
  → [see A.Manteuffel's Talk]

A **closer look** at the two-loop amplitude for **ZZ production**

- **143** Feynman Diagrams
- $\approx$ **3100** PLANAR Integrals
- $\approx$ **1500** NON–PLANAR Integrals

The integrals can be organised into **3 topologies**:

- Topo A: **Planar** Integrals with two **adjacent** massive legs

- Topo B: **Planar** Integrals with two **non-adjacent** massive legs

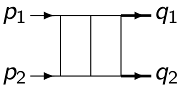
- Topo C: **Non-Planar** Integrals $\rightarrow$ more involved cut structure

A **closer look** at the two-loop amplitude for **ZZ production**

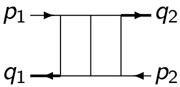
- **143** Feynman Diagrams
- $\approx$ **3100** PLANAR Integrals
- $\approx$ **1500** NON−PLANAR Integrals

The integrals can be organised into **3 topologies**:

- Topo A: **Planar** Integrals with two **adjacent** massive legs



- Topo B: **Planar** Integrals with two **non-adjacent** massive legs



- Topo C: **Non-Planar** Integrals $\rightarrow$ more involved cut structure

- We performed reduction to **MIs** for the three topologies with **Reduze2** [C.Studerus, A.Manteuffel]
  - **Topo A**: **26** 2-loop MIs, **13** new double-boxes
  - **Topo B**: **13** 2-loop MIs, **9** new double-boxes
  - **Topo C**: **16** 2-loop MIs, **13** new double-boxes
- From $\approx$ 5000 **Integrals** $\rightarrow$ $\approx$ 50 **Master Integrals** !
- All **triangles** already known [T.Gehrmann, E.Remiddi; T.G.Birthwright, E.W.N.Glover, P.Marquard; F.Chavez, C.Duhr]
- We computed the <u>double-boxes</u> in **Topo A** and **Topo B**.

Why *start* with planar integrals ?

- **Cut structure easier** $\rightarrow$ function are expected to be easier (*if expressed in the right variables...*)

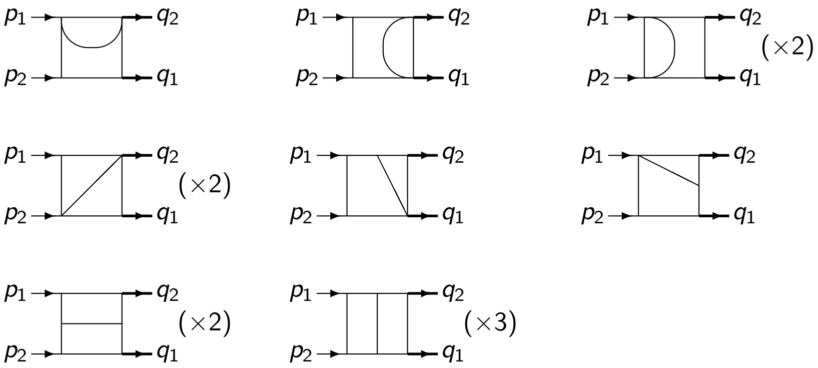
- Defining mandelstam variables

$$s = (p_1 + p_2)^2 \,, \quad t = (p_1 - q_1)^2 \,, \quad u = (p_2 - q_1)^2$$

  1. **Topo A** has cuts in s and u
  2. **Topo B** has cuts in t and u
  3. **Topo C** has cuts in s, t and u !!

- Two variables are independent: $s + t + u = 2m^2$
  Making a choice **breaks** symmetry for **Topo C**

Topo A - Master Integrals



we found compact expressions in **non-physical** region

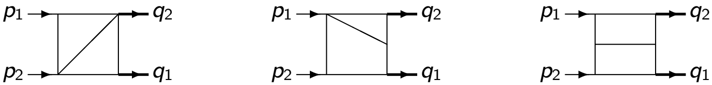
$$s = -m^2 \, \frac{(1+x)^2}{x} < 0 \, , \qquad u = -m^2 \, z < 0 \, , \qquad q_1^2 = q_2^2 = -m^2 < 0$$

All MIs are represented as combinations of **GHPLs** up to **weight 4**

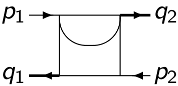
$$G(\vec{f}(x)\,;\,z) \quad \text{with} \quad f_j(x) = \left\{1, 0, -1, -x, -\frac{1}{x}, -\frac{1+x+x^2}{x}, -\frac{x}{1+x+x^2}\right\}$$

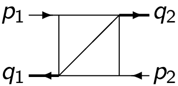
$$G(\vec{a}\,;\,x) \quad \text{with} \quad a_j = \left\{1, 0, -1, -\frac{1+i\sqrt{3}}{2}, -\frac{1-i\sqrt{3}}{2}\right\}$$

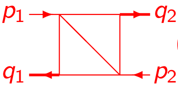
N.B. : The "ugly" indices appear only in **3 topologies** and only at **weight 4**



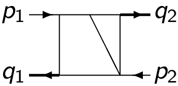
Topo B - Master Integrals

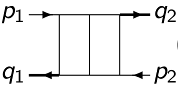






$(\times 4)$





$(\times 2)$

we found compact expressions in **non-physical** region (except one!)

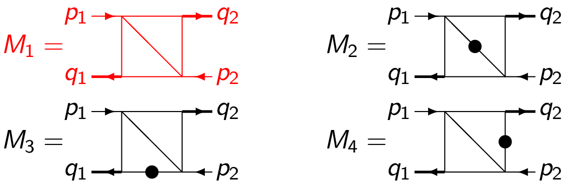
$$t = -m^2\, y < 0\,, \qquad u = -m^2\, z < 0\,, \qquad q_1^2 = q_2^2 = -m^2 < 0$$

All masters **except one** have extremely compact representations as **GHPLs** up to **weight 4** $\rightarrow$ *easy boxes!*

$$G(\vec{f}(z)\,;\,y) \quad \text{with} \quad f_j(z) = \left\{ 1,\, 0,\, 2-z,\, \frac{1}{z} \right\}$$

$$G(\vec{a};\,z) \quad \text{with} \quad a_j = \{\, 1,\, 0,\, 2 \,\}$$

Most complicated topology has 4 MIs



Dots are squared propagators!

Quite surprisingly (?) the **scalar master** is the **most involved**!
→ [See J.Henn's Talk]

System of 4 coupled differential equations:

- The **homogeneous solution** of the DE of M_1 contains a square-root in y, z
- **Nevertheless** with this choice of MIs:
  1. M_1 is **finite** → starts at $\mathcal{O}(1)$
  2. it **decouples** up to $w = 6$ → (also from $t = 6$, $t = 7$ MIs)
- We can integrate all masters without knowing its value!

Quite surprisingly (?) the **scalar master** is the **most involved**!
$\rightarrow$ [See J.Henn's Talk]

System of 4 coupled differential equations:

- The **homogeneous solution** of the DE of M_1 contains a square-root in y, z
- **Nevertheless** with this choice of MIs:
  1. M_1 is **finite** $\rightarrow$ starts at $\mathcal{O}(1)$
  2. it **decouples** up to $w = 6$ $\rightarrow$ (also from $t = 6$, $t = 7$ MIs)
- We can integrate all masters without knowing its value!

- M_1 can still be integrated in terms of GHPLs only

  Going back to $s, u \to$ **Landau variable**:

$$s = m^2 \frac{(1+\xi)^2}{\xi}, \quad \text{and} \quad u = -m^2 \zeta$$

  We find:

$$G(\vec{f}(\xi)\,;\,\zeta) \quad \text{with} \quad f_j(\xi) = \left\{1, 0, -1, \xi, \frac{1}{\xi}, \frac{1+\xi+\xi^2}{\xi}, \frac{1+\xi^2}{\xi}\right\}$$

$$G(\vec{a};\,\xi) \quad \text{with} \quad a_j = \left\{1, 0, -1, +i, -i, \frac{1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2}\right\}$$

- **New indices** are needed to reproduce the cut in t

Conclusions and Outlook

1. We computed all **two-loop planar MIs** for

$$q\,\bar{q} \to VV \qquad g\,g \to V\,V$$

They can all be expressed in GHPLs.

- The results have all been **checked numerically** with:
  FIESTA [A.V.Smirnov, V.A.Smirnov, M.Tentyukov]
  SecDec [S.Borowka, J.Carter, G.Heinrich] .

2. **Next steps** ($\approx$ in parallel):

- Conclude the study of **NPL MIs**
- Compute **leading-colour** two-loop amplitude for $q\bar{q} \to ZZ\;/\;WW$

Thank you !

Back-up slides

Quite surprisingly (?) the scalar master is the **most involved**!
→ [See Henn's Talk]

BUT with this basis the **homogeneous** system reads:

$$\begin{aligned}
\frac{\partial}{\partial y} M_1 &= a_{11}\, M_1 + a_{12}\, M_2 + a_{13}\, M_3 + a_{14}\, M_4 \\
\frac{\partial}{\partial y} M_2 &= a_{22}\, M_2 + (d-4)\, [a_{23}\, M_3 + a_{24}\, M_4] \\
\frac{\partial}{\partial y} M_3 &= (d-4)^2\, [a_{31}\, M_1] + (d-4)\, [a_{32}\, M_2 + a_{33}\, M_3 + a_{34}\, M_4] \\
\frac{\partial}{\partial y} M_4 &= (d-4)^2\, [a_{41}\, M_1] + (d-4)\, [a_{42}\, M_2 + a_{43}\, M_3] + a_{44}\, M_4
\end{aligned}$$

M_1 **decouples** and starts at order $\mathcal{O}(1)$. It can be computed **alone** after all other masters have been computed (up to $t = 7$!!!)

M_1 can influence M_3 and M_4 only starting at $w = 6$.

Homogeneous equation for M_1 reads:

$$\frac{\partial}{\partial y} H_1 = \frac{1}{2}\left[\frac{1}{2-y-z} - \frac{1}{2+y+z}\right] H_1$$

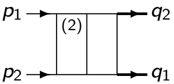
Whose **solution** is:

$$H_1 = \frac{1}{\sqrt{(2-y-z)(2+y+z)}}$$

Going to **Landau variable** we find

$$s = m^2 \frac{(1+\xi)^2}{\xi} \quad \rightarrow \quad H_1 = \frac{\xi}{(1-\xi)(1+\xi)}\,.$$

Example of result: **double-box** $t = 7$ *(up to $w = 3$ fits on 1 slide)*



$$
= \frac{x^2}{(1-x)(1+x)^3} \Bigg\{ + \frac{1}{\epsilon^2}\left[4G(0,-1,x) - 2G(0,0,x) - \frac{\pi^2}{3}\right]
$$

$$
+ \frac{1}{\epsilon}\Bigg[\pi^2\Big(G(-x,z) + G(0,x) - 2G(1,x) - 1/3G(-1/x,z)\Big)
$$

$$
- 2G(-1/x,0,0,z) + 4G(-1/x,1,0,z) + 2G(-x,0,0,z)
$$

$$
- 4G(-x,1,0,z) - 4G(-1,x)G(-1/x,0,z)
$$

$$
+ 4G(-1,x)G(-x,0,z) + 2G(0,x)G(-1/x,0,z)
$$

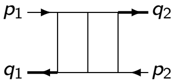
$$
- 2G(0,x)G(-x,0,z) - 4G(0,-1,x)G(-1/x,z)
$$

$$
- 4G(0,-1,x)G(-x,z) - 24G(0,-1,-1,x) + 12G(0,-1,0,x)
$$

$$
+ 2G(0,0,x)G(-1/x,z) + 2G(0,0,x)G(-x,z)
$$

$$
+ 24G(1,0,-1,x) - 12G(1,0,0,x) - 6\zeta_3\Bigg] + \mathcal{O}(\epsilon^0)\Bigg\}
$$

Example of result: **double-box** $t = 7$ *(up to $w = 3$ fits on 1 slide)*



$$= \frac{1}{(1 - y\,z)\,y} \Bigg\{ + \frac{1}{\epsilon^2}\left[\frac{\pi^2}{3} - 2G(1/z,0,y) - 2G(0,z)G(1/z,y)\right]$$

$$+ \frac{1}{\epsilon}\Big[7G(1/z,1/z,0,y) + 3/2G(1/z,y)\pi^2 + 4G(1/z,0,0,y)$$
$$- 8G(1/z,1,0,y) + 2G(0,1/z,0,y) + 7G(0,z)G(1/z,1/z,y)$$
$$+ 2G(0,z)G(0,1/z,y) - 6G(0,z)G(1,1/z,y) - 2/3G(0,y)\pi^2$$
$$+ 4G(0,0,z)G(1/z,y) + 2G(0,1,0,y) - 6G(1,1/z,0,y)$$
$$- 2/3G(1,y)\pi^2 - 8G(1,0,z)G(1/z,y) + 6G(1,0,z)G(1,y)$$
$$- 6G(1,0,0,y) + 4G(1,1,0,y) - 7\zeta_3\Big] + \mathcal{O}(\epsilon^0)\Bigg\}$$