Fixed order total cross section results for heavy quark pair production

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Outline

Research Directions

- Resummation
- Next-to-next-to-leading order
- Differential description
- Decay modelling

Total cross section

- Factorization theorem

$$\sigma_{h_1h_2}(s,m_t) = \sum_{ij}\int dx_1 dx_2 \phi_{i/h_1}(x_1,\mu_F)\phi_{j/h_2}(x_2,\mu_F)\hat{\sigma}_{ij}(x_1x_2s,m_t,\alpha_s(\mu_R),\mu_R,\ \mu_F)$$

σ_{h_1,h_2}	hadronic cross section	$\phi_{i/h}$	PDF for parton i in hadron h
$h_{1,2}$	hadrons	$\hat{\sigma}_{ij}$	partonic cross section
s	square of collider energy	μ_R	renormalization scale
m_t	top quark mass	μ_F	factorization scale

- Scale dependence at fixed order of perturbation theory can be derived from Renormalization Group invariance

- The minimal object to calculate: $\hat{\sigma}_{ij}(\beta)$

$$\hat{\sigma}_{ij}(\hat{s},m_t,\alpha_s(m_t),m_t,m_t) = \frac{\alpha_s^2(m_t)}{m_t^2}\hat{\sigma}_{ij}(\beta)\ , \quad \beta = \sqrt{1-\frac{4m_t^2}{\hat{s}}}\ , \quad \hat{s} = x_1x_2s$$

β heavy quark velocity , $\hat{s}$ partonic energy squared

$$\hat{\sigma}_{ij}(\beta) = \hat{\sigma}_{ij}^{(0)}(\beta) + \alpha_s(m_t)\hat{\sigma}_{ij}^{(1)}(\beta) + \alpha_s^2(m_t)\hat{\sigma}_{ij}^{(2)}(\beta) + \ldots$$

Status theory

Before NNLO:

Beneke, Falgari, Klein, Schwinn `09-`11
Ahrens, Ferroglia, Neubert, Pecjak, Yang `10-`11
Kidonakis `04-`11
Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann ’10
Cacciari, MC, Mangano, Mitov, Nason ‘11

NNLO:

Bärnreuther, MC, Mitov, Phys. Rev. Lett., April ‘12
MC, Mitov, JHEP, July ‘12
MC, Mitov, JHEP, October ‘12
MC, Fiedler, Mitov, Phys. Rev. Lett., March ’13

Publicly available software:

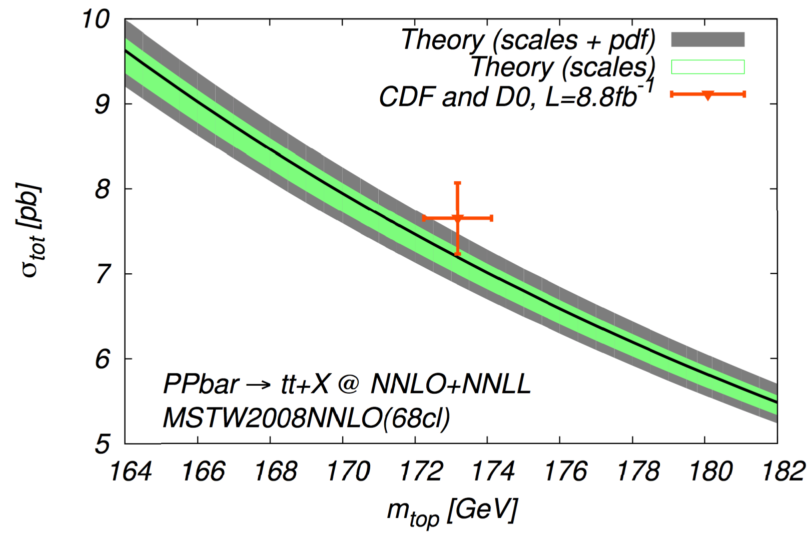
- **HATHOR**
  Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedemann `10
  NLO + approximations for NNLO (recently switched to complete NNLO)
- **Top++**
  Czakon, Mitov `11
  NNLO + NNLL soft gluon resummation in Mellin-space
- **TOPIXS**
  Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan `12
  NLO + approximations for NNLO + NNLL soft and Coulomb resummation in x-space

Predictions for hadron colliders

MC, Fiedler, Mitov `13

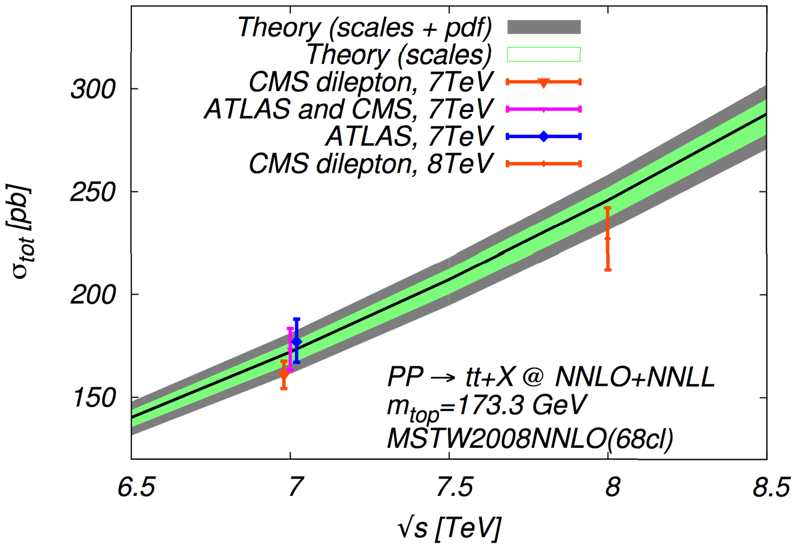
NNLO + NNLL

Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$^{+0.110(1.5\%)}_{-0.200(2.8\%)}$	$^{+0.169(2.4\%)}_{-0.122(1.7\%)}$
LHC 7 TeV	172.0	$^{+4.4(2.6\%)}_{-5.8(3.4\%)}$	$^{+4.7(2.7\%)}_{-4.8(2.8\%)}$
LHC 8 TeV	245.8	$^{+6.2(2.5\%)}_{-8.4(3.4\%)}$	$^{+6.2(2.5\%)}_{-6.4(2.6\%)}$
LHC 14 TeV	953.6	$^{+22.7(2.4\%)}_{-33.9(3.6\%)}$	$^{+16.2(1.7\%)}_{-17.8(1.9\%)}$

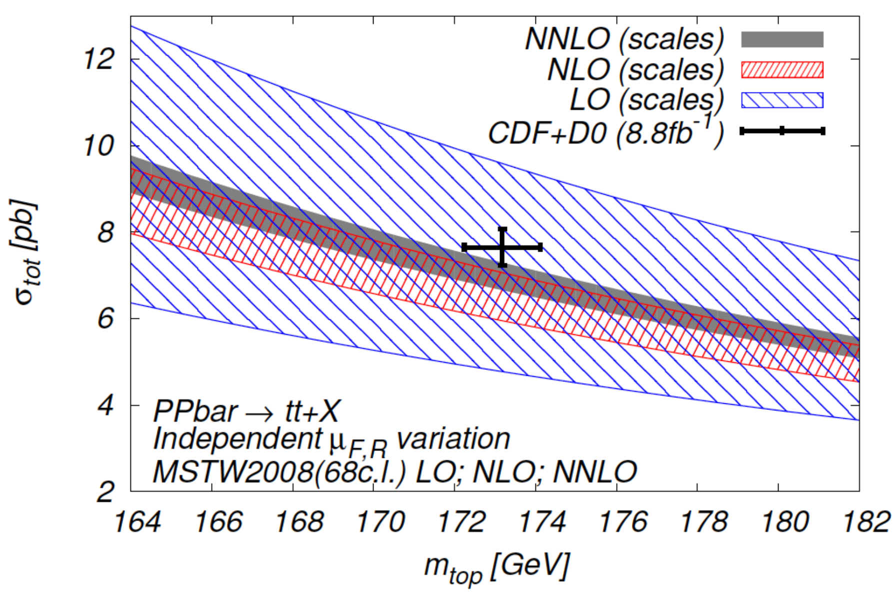


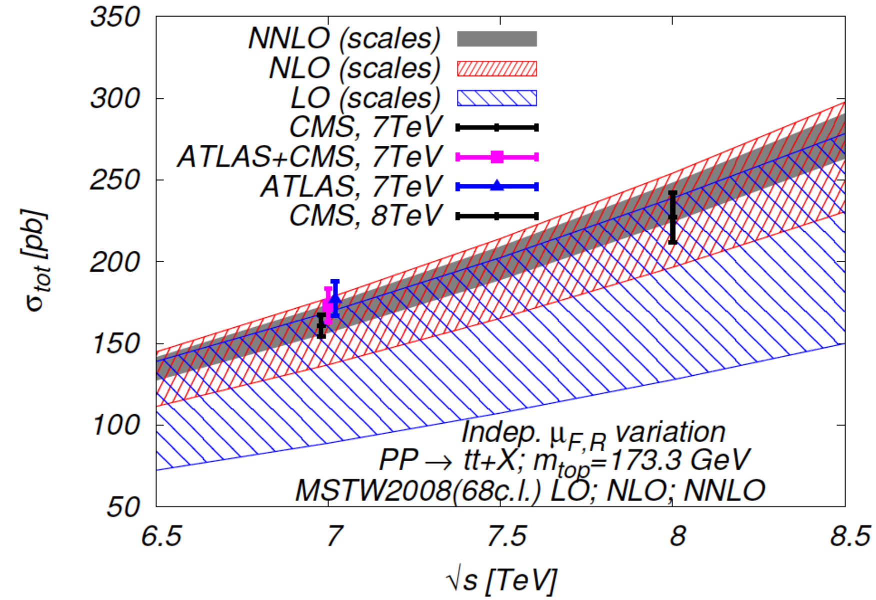
NNLO

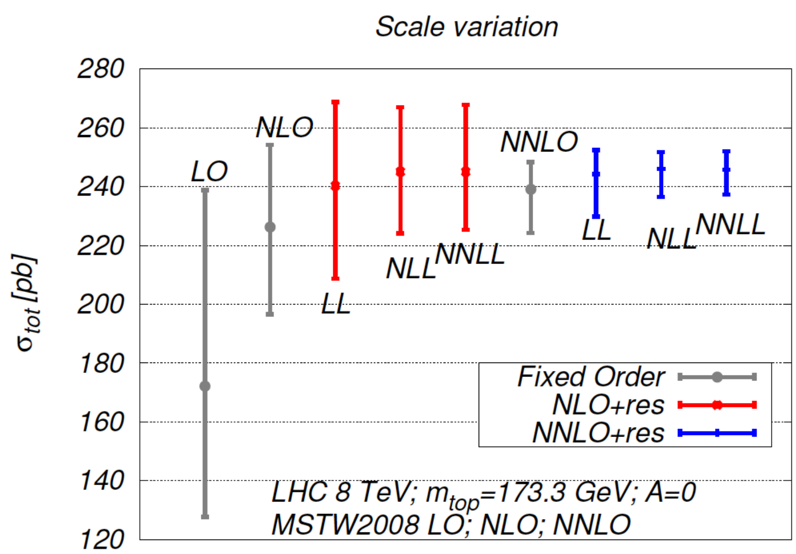
Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$^{+0.259(3.7\%)}_{-0.374(5.3\%)}$	$^{+0.169(2.4\%)}_{-0.121(1.7\%)}$
LHC 7 TeV	167.0	$^{+6.7(4.0\%)}_{-10.7(6.4\%)}$	$^{+4.6(2.8\%)}_{-4.7(2.8\%)}$
LHC 8 TeV	239.1	$^{+9.2(3.9\%)}_{-14.8(6.2\%)}$	$^{+6.1(2.5\%)}_{-6.2(2.6\%)}$
LHC 14 TeV	933.0	$^{+31.8(3.4\%)}_{-51.0(5.5\%)}$	$^{+16.1(1.7\%)}_{-17.6(1.9\%)}$



Perturbative convergence







Concurrent uncertainties:

Scales	~ 3%
pdf (at 68%cl)	~ 2-3%
α_S (parametric)	~ 1.5%
m_{top} (parametric)	~ 3%

Soft gluon resummation makes a difference:

5% -> 3%

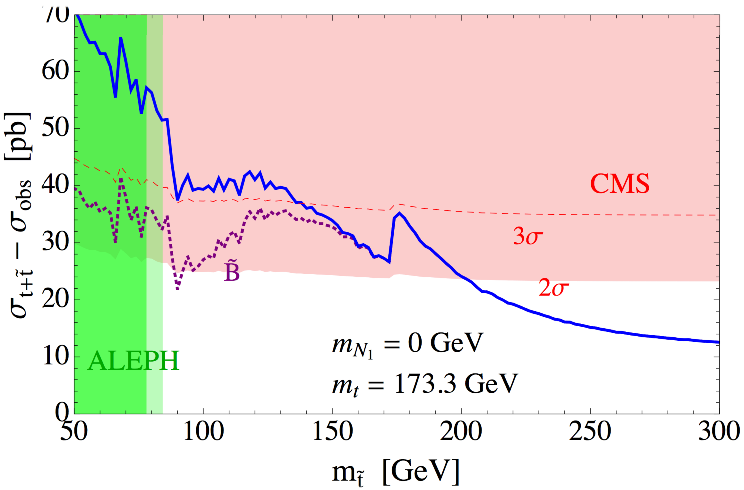
Status experiment

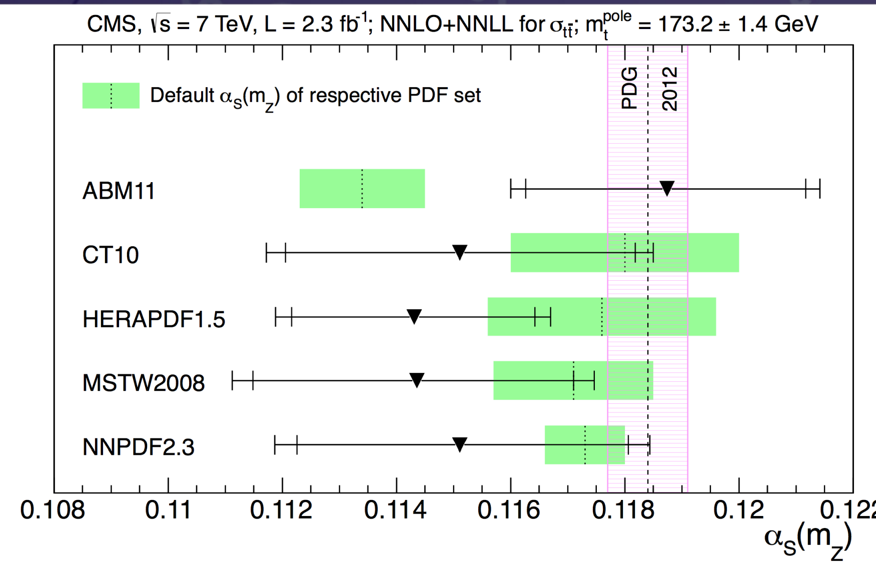
Summary of combinations of total cross section measurements

- **Combining measurements from CDF and DØ gives a Tevatron cross section at 1.96 TeV c-o-m of $7.60 \pm 0.41(5.4\%)$ pb , to be compared with the theoretical calculation (NNLO+NNLL) $7.24^{+0.23}_{-0.27}(3.4\%)$pb (Czakon et. al).**
- **Combining measurements from ATLAS and CMS gives a LHC cross section at 7 TeV c-o-m of $173 \pm 10(5.8\%)$ pb, to be compared to the theoretical calculation (NNLO+NNLL) of $172.0^{+6.4}_{-7.5}(4.1\%)$ pb.**
- **The most precise measurements at 8 TeV are from the ATLAS and CMS dilepton channel: $238 \pm 11(4.6\%)$ pb and $227 \pm 15(6.6\%)$ pb.
The NNLO+NNLL SM prediction is $245.8^{+8.8}_{-10.6}(4.0\%)$ pb.**

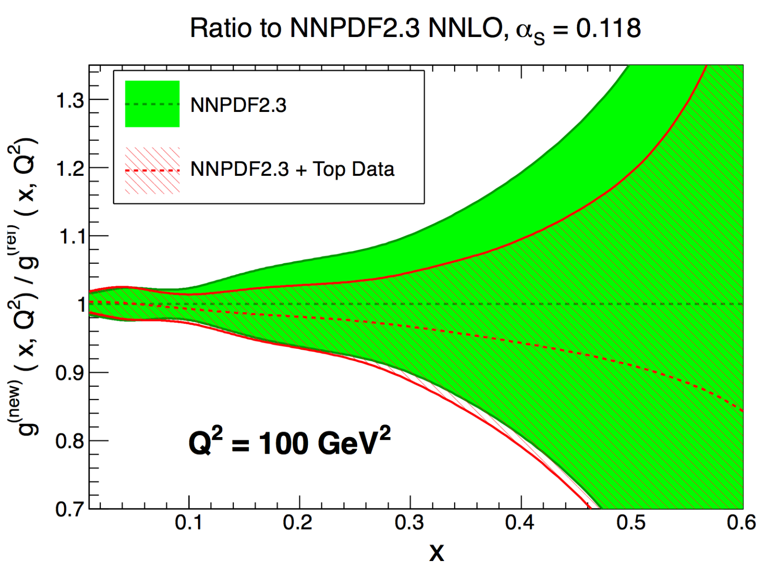
S. Protopopescu, TOP 2013, 15th September 2013

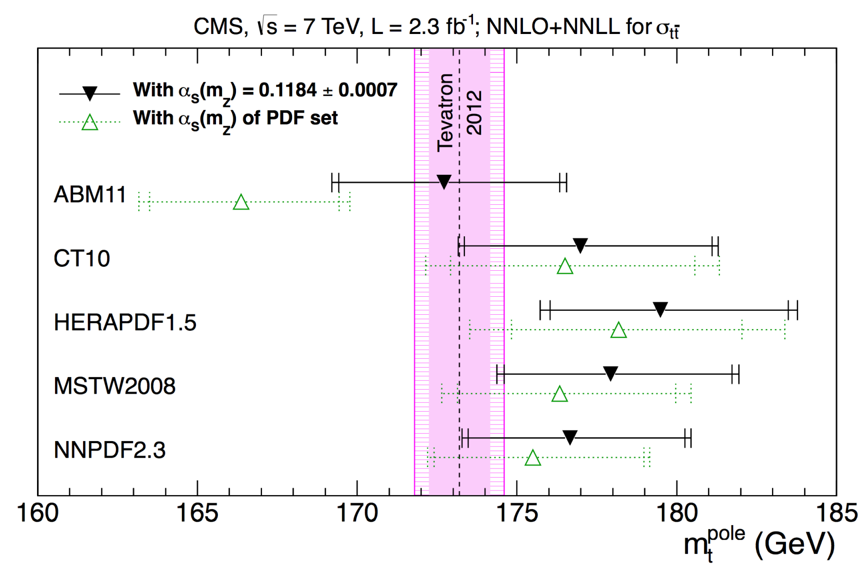
Applications





MC, Mitov, Papucci, Ruderman, Weiler, in preparation





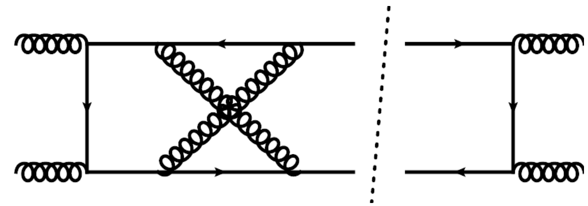
MC, Mangano, Mitov, Rojo `13

arXiv:1307.1907 (CMS-TOP-12-022)

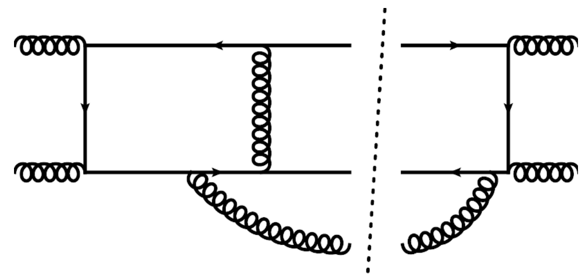
NNLO methods

There are 3 principal contributions:

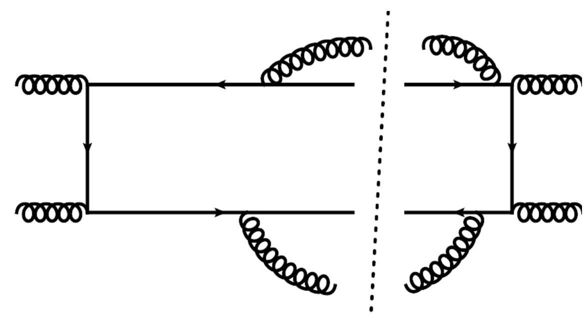
- 2-loop virtual corrections (V-V)
  MC `07 (quark annihilation)
  Bärnreuther, MC, Fiedler, in preparation (gluon fusion)



- 1-loop virtual with one extra parton (R-V)
  code by Stefan Dittmaier
  new subtraction terms: Bierenbaum, Czakon, Mitov `11



- 2 extra emitted partons at tree level (R-R)
  MC `10 `11 invention of a new subtraction scheme called STRIPPER



And 2 secondary contributions:

- Collinear subtraction for the initial state — Known, in principle. Done numerically. (the only non-differential contribution)
- One-loop squared amplitudes — Körner, Merebashvili, Rogal `07 (quark annihilation) done from scratch for gluon fusion

Additionally: divergences of two-loop amplitudes in quark annihilation: Ferroglia, Neubert, Pecjak, Yang '09

Partonic results: qQ -> tT + X

Partonic cross-section through NNLO:

$$\sigma_{ij}\left(\beta,\frac{\mu^2}{m^2}\right)=\frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)}+\alpha_S\left[\sigma_{ij}^{(1)}+L\,\sigma_{ij}^{(1,1)}\right]+\alpha_S^2\left[\sigma_{ij}^{(2)}+L\,\sigma_{ij}^{(2,1)}+L^2\sigma_{ij}^{(2,2)}\right]+\mathcal{O}(\alpha_S^3)\right\},$$

The NNLO term:

$$\sigma_{q\bar{q}}^{(2)}(\beta)=F_0(\beta)+F_1(\beta)N_L+F_2(\beta)N_L^2$$

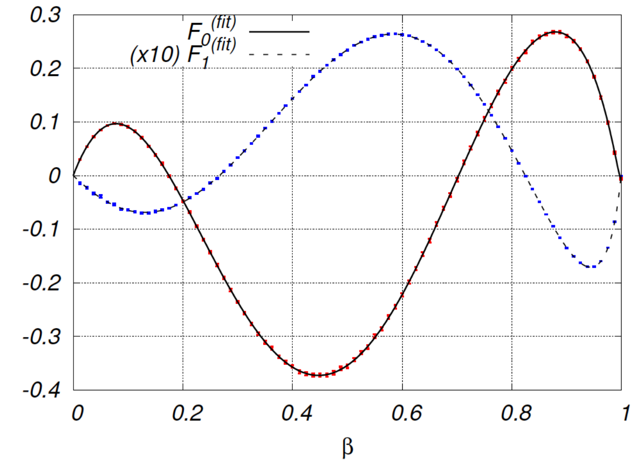
Numeric

Analytic

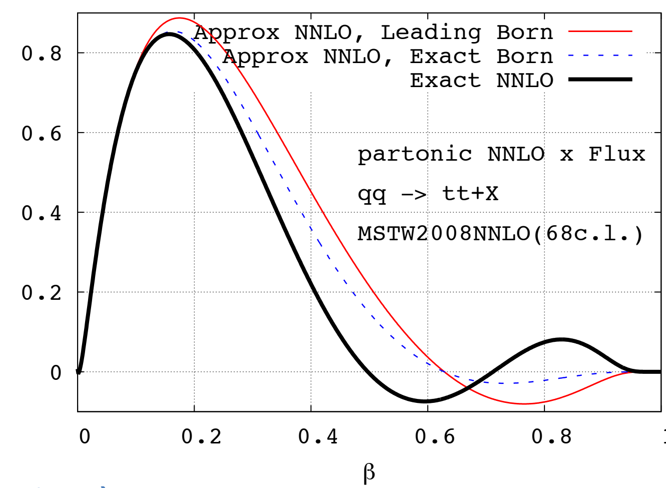
$$F_i\equiv F_i^{(\beta)}+F_i^{(\mathrm{fit})},i=0,1$$

The known threshold approximation

- Small numerical errors, agrees with limits

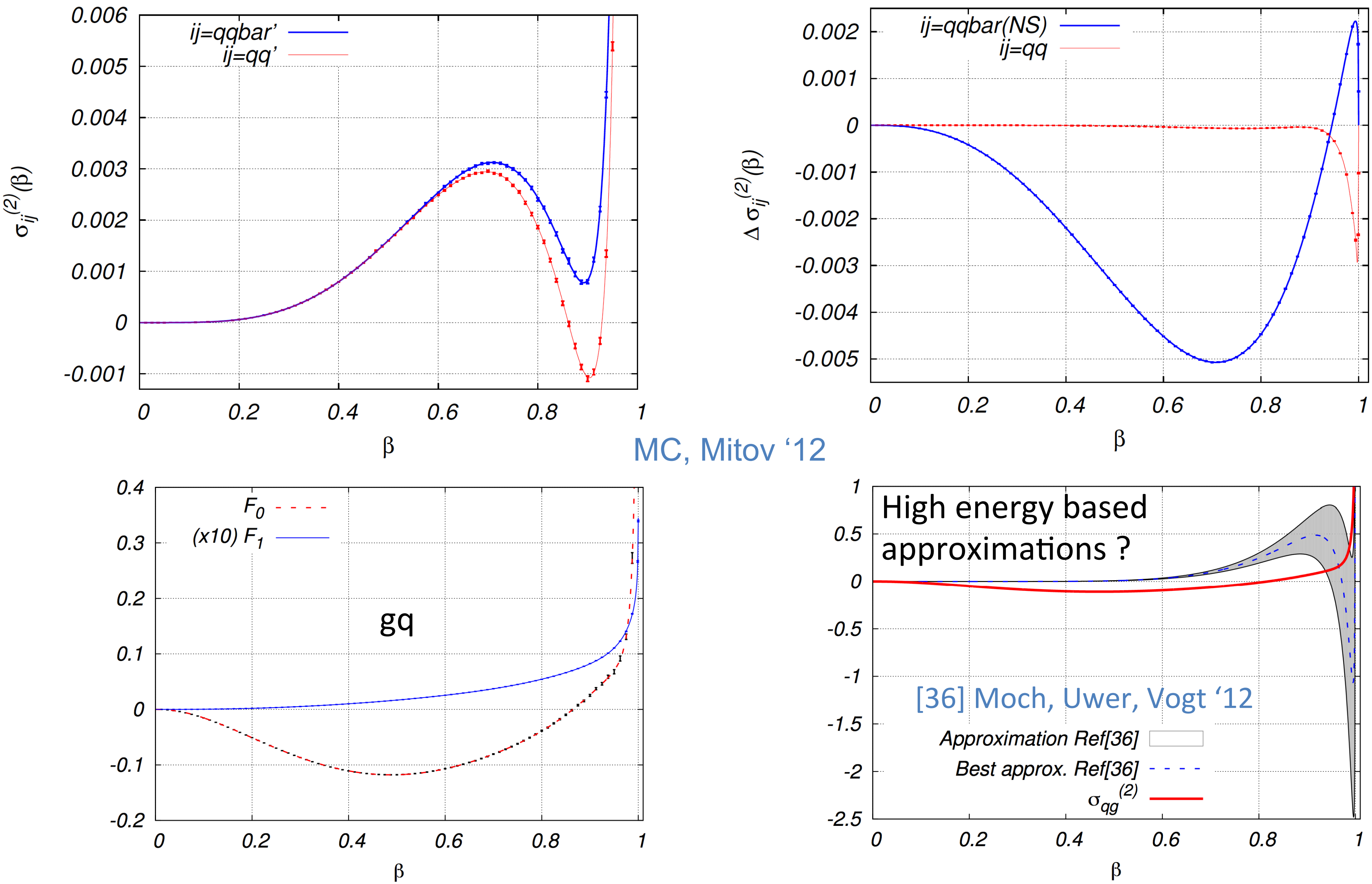


Bärnreuther, MC, Mitov '12



Beneke, MC, Falgari, Mitov, Schwinn `09

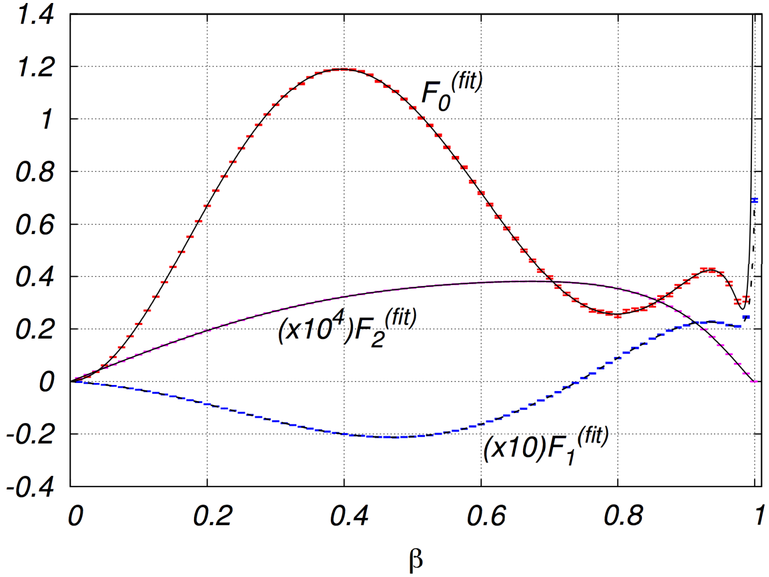
Partonic results: qq’ & gq -> tT + X

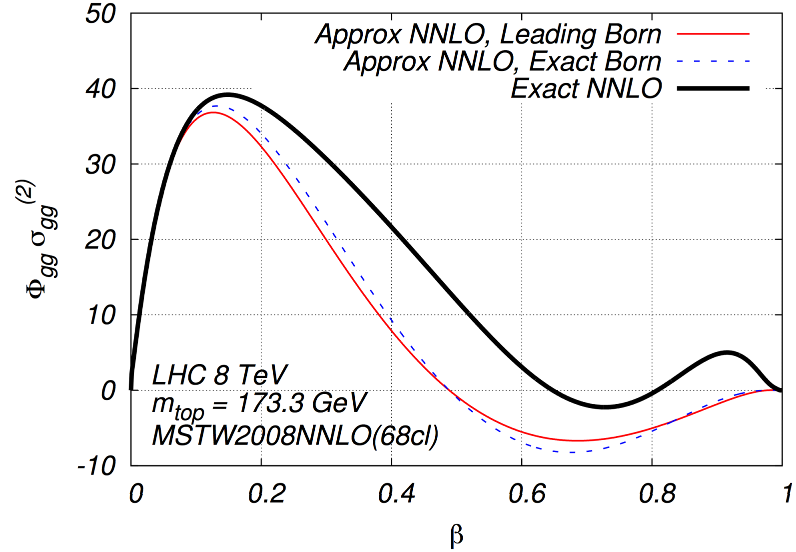


Partonic results: gg -> tT + X

MC, Fiedler, Mitov `13

$$\sigma_{gg}^{(2)}(\beta) = F_0(\beta) + F_1(\beta)N_L + F_2(\beta)N_L^2$$





Top++ MC, Mitov

Version 1.4 | Version 2.0

Precision of
the gluon channel

NNLO$_{approx}$ $\xrightarrow{5\ \%}$ NNLO

NNLO $\xrightarrow{2.8\ \%}$ NNLO + NNLL

NLO + NNLL $\xrightarrow{11.5\ \%}$ NNLO + NNLL

Two-loop amplitudes

Analytic calculations by Bonciani, Ferroglia, Gehrmann, Maitre, von Manteuffel, Studerus '08 – '13
Numerical calculations by Bärnreuther, MC, Fiedler '08 – '13

Quark channel MC '08

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
A	0.22625	1.391733154	-2.298174307	-4.145752449	17.37136599
B	-0.4525	-1.323646320	8.507455541	6.035611156	-35.12861106
C	0.22625	-0.06808683395	-18.00716652	6.302454931	3.524044913
D_l		-0.22625	0.2605057339	-0.7250180282	-1.935417247
D_h			0.5623350684	0.1045606449	-1.704747998
E_l		0.22625	-0.3323207300	7.904121951	2.848697837
E_h			-0.5623350684	4.528240788	12.73232424
F_l					-1.984228442
F_{lh}					-2.442562819
F_h					-0.07924540546

No analytic results for these coefficients

For practical purposes:

1) Renormalization with heavy quark decoupling
2) Results as finite remainders

Ferroglia, Neubert, Pecjak, Yang '09

$$\boldsymbol{Z}^{-1}(\epsilon,\{\underline{p}\},\{\underline{m}\},\mu)\,|\mathcal{M}_n(\epsilon,\{\underline{p}\},\{\underline{m}\})\rangle\big|_{\alpha_s^{\rm QCD}\to\xi\alpha_s} = \text{finite}$$

$$\boldsymbol{Z}^{-1}(\epsilon,\{\underline{p}\},\{\underline{m}\},\mu)\,\frac{d}{d\ln\mu}\,\boldsymbol{Z}(\epsilon,\{\underline{p}\},\{\underline{m}\},\mu) = -\boldsymbol{\Gamma}(\{\underline{p}\},\{\underline{m}\},\mu)$$

$$\begin{aligned}\boldsymbol{\Gamma}(\{\underline{p}\},\{\underline{m}\},\mu) &= \sum_{(i,j)}\frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)\\ &\quad - \sum_{(I,J)}\frac{\boldsymbol{T}_I\cdot\boldsymbol{T}_J}{2}\,\gamma_{\rm cusp}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s) + \sum_{I,j}\boldsymbol{T}_I\cdot\boldsymbol{T}_j\,\gamma_{\rm cusp}(\alpha_s)\\ &\quad + \sum_{(I,J,K)} i f^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_K^c\,F_1(\beta_{IJ},\beta_{JK},\beta_{KI})\\ &\quad + \sum_{(I,J)}\sum_k i f^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_k^c\,f_2\Big(\beta_{IJ},\ln\frac{-\sigma_{Jk}\,v_J\cdot p_k}{-\sigma_{Ik}\,v_I\cdot p_k}\Big) + \mathcal{O}(\alpha_s^3)\,.\end{aligned}$$

no contribution to colored averaged (real) amplitudes at NNLO

Gluon channel Bärnreuther, MC '11

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
A_{LC}	10.74942557	18.69389337	-156.8237244	262.1482588	12.72180680
A	10.74942557	18.69389337	-156.8237244	262.1482588	12.72180680
B	-21.28599123	-55.99039551	-235.0412564	1459.833288	-509.6019155
C		-6.199051597	-68.70297402	-268.1060373	804.0981895
D			94.08660818	-130.9619794	-283.3496755
E_l		-12.54099650	18.20646589	27.95708293	-112.6060988
E_h			0.012907497	11.79259573	-47.68412574
F_l		24.83365643	-26.60868620	-50.75380859	125.0537955
F_h			0.0	-23.32918072	132.5618962
G_l			3.099525798	67.04300456	-214.1081462
G_h				0.0	-179.3374874
H_l			2.388761238	-5.452031425	3.632861953
H_{lh}				-0.004302499	-3.945712447
H_h					0.00439856
I_l			-4.730220272	10.81032548	-7.182940516
I_{lh}				0.0	7.780900470
I_h					0.0

Two-loop amplitudes

Illustration: gluon fusion case

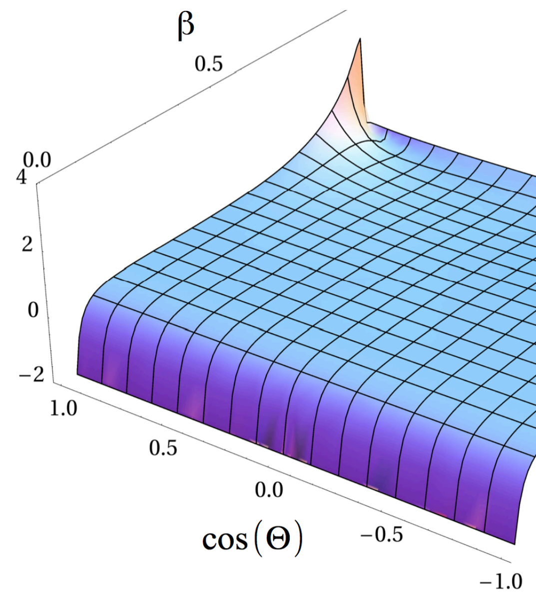
- 422 master integrals
- Solution by numerical differential equations & semi-analytic expansions
- Boundaries with Mellin-Barnes techniques
- About 1h per phase space point

$$M_i(\beta, x, \epsilon) = \sum_{k=Min}^{Max} C_{ik}(\beta, x)\, \epsilon^k$$

$M_i(\beta, x, \epsilon)$ Master integral

$C_{ik}(\beta, x)$ Coefficient of Laurent series

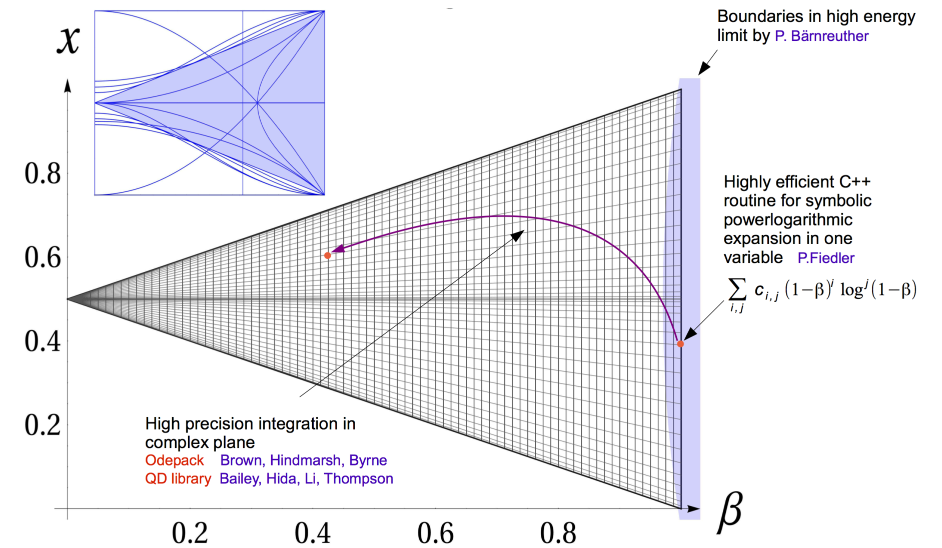
$$x \equiv \frac{m^2 - \hat{t}}{\hat{s}} = \frac{1}{2}(1 - \beta\cos(\Theta))$$

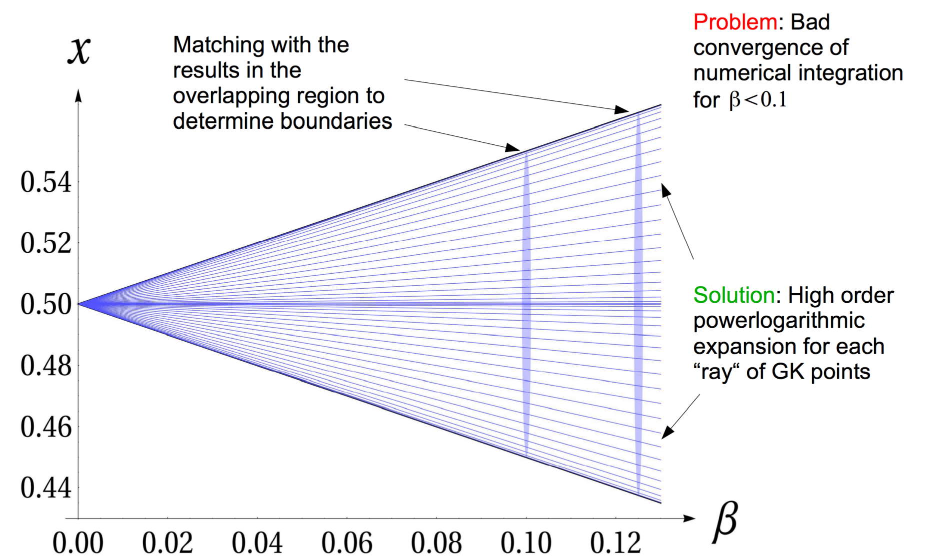


$$\frac{d\vec{C}(\beta, x)}{d\beta} = J_\beta(\beta, x)\, \vec{C}(\beta, x)$$

$$\frac{d\vec{C}(\beta, x)}{dx} = J_x(\beta, x)\, \vec{C}(\beta, x)$$

Finite remainder





Bärnreuther, MC, Fiedler, in preparation

Real radiation

General subtraction scheme invented as consequence of difficulties in treating double-real radiation for top quark pair production MC ’10

Real radiation for tops in most channels calculated as first application in MC ‘11

Subsequently applied by others to several non-trivial problems:

- $Z \to e^+e^-$ (as a warmup) Boughezal, Melnikov, Petriello ’11
- top quark decay Brucherseifer, Caola, Melnikov ’13
- $b \to X_u e\nu$ Brucherseifer, Caola, Melnikov ‘13
- H + jet Boughezal, Caola, Melnikov, Petriello, Schulze ’13

Main ideas:

1. parameterization of the massless system with energies and angles modified to allow for a description of all collinear singular configurations with only two variables

2. level 1 decomposition into sectors allowing for only one type of collinear singularities

3. level 2 decomposition into sectors defining the order of singular limits

4. Subtraction at the endpoint derived from known soft and collinear limits of QCD amplitudes

5. No analytic integration of the subtraction terms

Phase Space

$$d\Phi_4 = \frac{d^{d-1}k_1}{(2\pi)^{d-1}2k_1^0}\frac{d^{d-1}k_2}{(2\pi)^{d-1}2k_2^0}\frac{d^{d-1}q_1}{(2\pi)^{d-1}2q_1^0}\frac{d^{d-1}q_2}{(2\pi)^{d-1}2q_2^0}(2\pi)^d\delta^{(d)}(k_1+k_2+q_1+q_2-p_1-p_2)$$

$$\begin{aligned}
p_1^\mu &= \frac{\sqrt{s}}{2}(1,0,0,1)\ , \\
p_2^\mu &= \frac{\sqrt{s}}{2}(1,0,0,-1)\ , \\
n_1^\mu &= \frac{\sqrt{s}}{2}\beta^2(1,0,\sin\theta_1,\cos\theta_1)\ , \\
n_2^\mu &= \frac{\sqrt{s}}{2}\beta^2(1,\sin\phi\sin\theta_2,\cos\phi\sin\theta_2,\cos\theta_2)\ , \\
k_1^\mu &= \hat{\xi}_1\, n_1^\mu\ , \\
k_2^\mu &= \hat{\xi}_2\, n_2^\mu\ ,
\end{aligned}$$

$$\begin{aligned}
\hat{\eta}_{1,2} &= \frac{1}{2}(1-\cos\theta_{1,2})\ , \\
\eta_3 &= \frac{1}{2}(1-\cos\theta_3) \\
&= \frac{1}{2}(1-\cos\phi\sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2) \\
&= \frac{1}{2}(1-\cos(\theta_1-\theta_2)+(1-\cos\phi)\sin\theta_1\sin\theta_2)\ ,
\end{aligned}$$

$$\eta_3 = \frac{(\hat{\eta}_1-\hat{\eta}_2)^2}{\hat{\eta}_1+\hat{\eta}_2-2\hat{\eta}_1\hat{\eta}_2-2(1-2\zeta)\sqrt{\hat{\eta}_1(1-\hat{\eta}_1)\hat{\eta}_2(1-\hat{\eta}_2)}}$$

all collinear limits with only two variables

$$\zeta = \frac{1}{2}\frac{(1-\cos(\theta_1-\theta_2))(1+\cos\phi)}{1-\cos(\theta_1-\theta_2)+(1-\cos\phi)\sin\theta_1\sin\theta_2}$$

$$\begin{aligned}
d\Phi_3(p_1+p_2;k_1,k_2) &= \frac{\pi^{2\epsilon}}{8(2\pi)^5\Gamma(1-2\epsilon)}s^{2-2\epsilon}\beta^{8-8\epsilon}\,(\zeta(1-\zeta))^{-\frac{1}{2}-\epsilon} \\
&\times\quad (\hat{\eta}_1(1-\hat{\eta}_1))^{-\epsilon}(\hat{\eta}_2(1-\hat{\eta}_2))^{-\epsilon}\frac{\eta_3^{1-2\epsilon}}{|\hat{\eta}_1-\hat{\eta}_2|^{1-2\epsilon}}\,\hat{\xi}_1^{1-2\epsilon}\hat{\xi}_2^{1-2\epsilon} \\
&\times\quad d\zeta\ d\hat{\eta}_1\ d\hat{\eta}_2\ d\hat{\xi}_1\ d\hat{\xi}_2\ .
\end{aligned}$$

$d\mu_{\eta\xi}$

$$d\Phi_4 = d\Phi_3(p_1+p_2;k_1,k_2)\ d\Phi_2(Q;q_1,q_2)$$

Level 1 Decomposition

most difficult

non-trivial only because
of soft-collinear divergences

$$
\begin{aligned}
1 = {}& \\
&\left.\begin{array}{l} +\theta_1(k_1)\theta_1(k_2) \\ +\theta_2(k_1)\theta_2(k_2) \end{array}\right\} \text{triple-collinear sector} \\
&\left.\begin{array}{l} +\theta_1(k_1)\theta_2(k_2)(1-\theta_3(k_1,k_2)) \\ +\theta_2(k_1)\theta_1(k_2)(1-\theta_3(k_1,k_2)) \end{array}\right\} \text{double-collinear sector} \\
&\left. +(\theta_1(k_1)\theta_2(k_2)+\theta_2(k_1)\theta_1(k_2))\theta_3(k_1,k_2)\right\} \text{single-collinear sector}
\end{aligned}
$$

trivial, because NLO type
attach to first sector (contains same divergences)

top quark pair production

general case

$$1 = \sum_{\substack{\text{pairs} \\ i,j\in\mathcal{F}}} \sum_{\substack{k\in\mathcal{I}\cup\mathcal{F} \\ k\notin\{i,j\}}} \left[\theta_{ij,k} + \sum_{\substack{l\in\mathcal{I}\cup\mathcal{F} \\ l\notin\{i,j,k\}}} \theta_{ij,kl} \right]$$

$$d_{ij} = \left[\left(\frac{2E_i}{\sqrt{s}}\right)\left(\frac{2E_j}{\sqrt{s}}\right)\right]^\alpha (1-\cos\theta_{ij})^\beta,$$

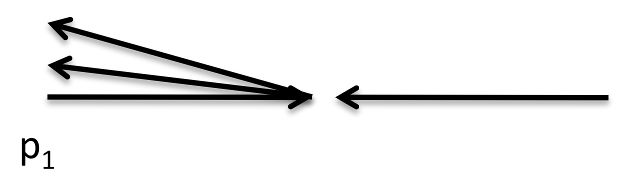
$$d_{ijk} = \left[\left(\frac{2E_i}{\sqrt{s}}\right)\left(\frac{2E_j}{\sqrt{s}}\right)\left(\frac{2E_k}{\sqrt{s}}\right)\right]^\alpha \left[(1-\cos\theta_{ij})(1-\cos\theta_{ik})(1-\cos\theta_{jk})\right]^\beta$$

$$\theta_{ij,k} = \frac{1}{\mathcal{D}}\frac{h_{ij,k}}{d_{ijk}}, \qquad \theta_{ij,kl} = \frac{1}{\mathcal{D}}\frac{h_{i,k}}{d_{ik}}\frac{h_{j,l}}{d_{jl}}$$

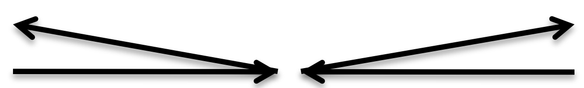
$$\mathcal{D} = \sum_{\substack{\text{pairs} \\ i,j\in\mathcal{F}}} \sum_{\substack{k\in\mathcal{I}\cup\mathcal{F} \\ k\notin\{i,j\}}} \left[\frac{h_{ij,k}}{d_{ijk}} + \sum_{\substack{l\in\mathcal{I}\cup\mathcal{F} \\ l\notin\{i,j,k\}}} \frac{h_{i,k}}{d_{ik}}\frac{h_{j,l}}{d_{jl}} \right]$$

collinear-singular
configurations at NNLO

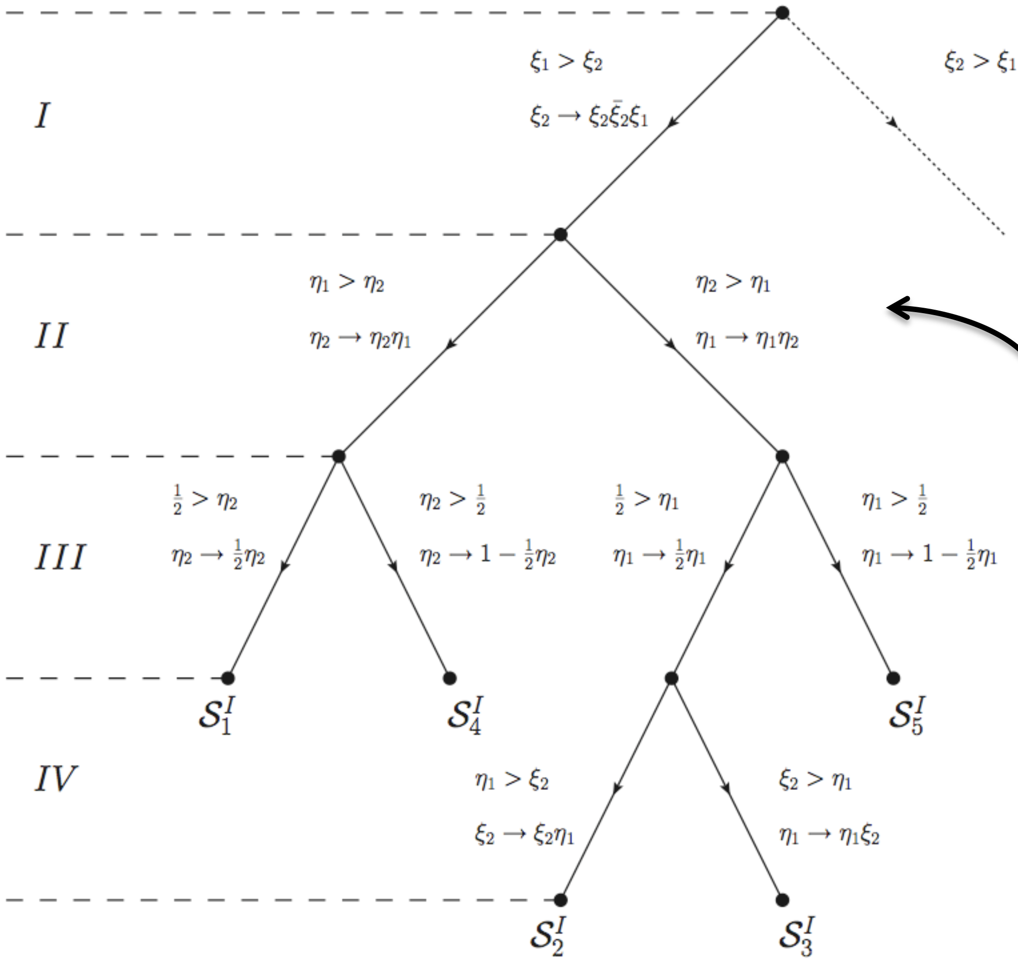
triple collinear



double collinear



Level 2 Decomposition



I) factorization of the soft singularities;

II, III) factorization of the collinear singularities;

IV) factorization of the soft-collinear singularities.

triple-collinear sector

Example from S_1^{I}

$$s_{156} = -\beta^2(\hat{\eta}_1\hat{\xi}_1 + \hat{\eta}_2\hat{\xi}_2 - \beta^2\hat{\xi}_1\hat{\xi}_2\eta_3)$$

$$-\frac{1}{2}\beta^2\eta_1\xi_1\left(2 + \eta_2\xi_2\bar{\xi}_2 - 2\beta^2\xi_1\xi_2\eta_{31}\bar{\xi}_2\right)$$

Subtraction Terms

$$d\mu_{\eta\xi} = \eta_1^{a_1+b_1\epsilon}\eta_2^{a_2+b_2\epsilon}\xi_1^{a_3+b_3\epsilon}\xi_2^{a_4+b_4\epsilon}\ \mu_{\mathcal{S}}^{\rm reg}\ d\eta_1 d\eta_2 d\xi_1 d\xi_2$$

$$\sigma_{\mathcal{O}} = \sum_{\mathcal{S}} \sigma_{\mathcal{O}}^{(\mathcal{S})} \qquad \sigma_{\mathcal{O}}^{(\mathcal{S})} = \int d\zeta\ d\eta_1\ d\eta_2\ d\xi_1\ d\xi_2\ d\cos\theta_Q\ d\phi_Q\ d\cos\rho_Q\ \Sigma_{\mathcal{O}}^{(\mathcal{S})}$$

$$\Sigma_{\mathcal{O}}^{(\mathcal{S})} = \frac{1}{2s}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{3\epsilon} \mu_\zeta\ \mu_{\mathcal{S}}^{\rm reg}\ \mu_2\ \theta_{\mathcal{S}}\ F_J \frac{1}{\eta_1^{1-b_1\epsilon}}\frac{1}{\eta_2^{1-b_2\epsilon}}\frac{1}{\xi_1^{1-b_3\epsilon}}\frac{1}{\xi_2^{1-b_4\epsilon}}\ \mathfrak{M}_{\mathcal{S}}$$

$$\mathfrak{M}_{\mathcal{S}} = \eta_1^{1+a_1}\eta_2^{1+a_2}\xi_1^{1+a_3}\xi_2^{1+a_4}\ \overline{|\mathcal{M}_4|^2}$$

$$\int_0^1 \frac{d\lambda}{\lambda^{1-b\epsilon}} f(\lambda) \longrightarrow \int_0^1 d\lambda \left[\frac{f(0)}{b\epsilon} + \frac{f(\lambda)-f(0)}{\lambda^{1-b\epsilon}}\right]$$

apply four times

$$\Sigma_{\mathcal{O}}^{(\mathcal{S})} \longrightarrow \left[\Sigma_{\mathcal{O}}^{(\mathcal{S})}\right]$$

Subtraction Terms

$$\mathbf{X} \subseteq \{\eta_1, \eta_2, \xi_1, \xi_2\}$$

$$\lim_{\mathbf{X}\to\mathbf{0}} \mathfrak{M}_{\mathcal{S}} = g^2 \, \langle \mathcal{M}_3|\mathbf{V}|\mathcal{M}_3\rangle \quad \text{or} \quad \lim_{\mathbf{X}\to\mathbf{0}} \mathfrak{M}_{\mathcal{S}} = g^4 \, \langle \mathcal{M}_2|\mathbf{V}|\mathcal{M}_2\rangle$$

Example:

$$\mathfrak{R}_{\mathcal{S}} = \eta_1^{1+a_1}\eta_2^{1+a_2}\xi_1^{1+a_3}\xi_2^{1+a_4}$$

$$\boxed{\hat{\eta}_1 = \hat{\eta}_2 = 0}$$

$$\mathbf{V}^{ss'}_{a_1 a_5 a_6} = \lim_{\mathbf{X}\to\mathbf{0}} \mathfrak{R}_{\mathcal{S}} \frac{4\hat{P}^{ss'}_{a_1 a_5 a_6}}{s^2_{156}}$$

$$x_1 = -1\,, \quad x_5 = \beta^2\hat{\xi}_1\,, \quad x_6 = \beta^2\hat{\xi}_2\,,$$

$$k^\mu_{\perp 1} = 0\,, \quad k^\mu_{\perp 5} = \beta^2\hat{\xi}_1\sqrt{\hat{\eta}_1}\,\bar{k}^\mu_{\perp 5}\,, \quad k^\mu_{\perp 6} = \beta^2\hat{\xi}_2\sqrt{\hat{\eta}_2}\,\bar{k}^\mu_{\perp 6}(\hat{\eta}_1,\hat{\eta}_2)\,,$$

$$\begin{aligned}
\bar{k}^\mu_{\perp 5} &= (0,0,1,0)\,, \\
\bar{k}^\mu_{\perp 6}(\hat{\eta}_1,\hat{\eta}_2) &= \frac{1}{\hat{\eta}_1+\hat{\eta}_2-2(1-2\zeta)\sqrt{\hat{\eta}_1\hat{\eta}_2}} \\
&\times \left(0,\ 2|\hat{\eta}_1-\hat{\eta}_2|\sqrt{\zeta(1-\zeta)},\ 2\sqrt{\hat{\eta}_1\hat{\eta}_2}-(\hat{\eta}_1+\hat{\eta}_2)(1-2\zeta),\ 0\right)
\end{aligned}$$

Comparison of schemes

Sector improved residue subtraction MC ‘10	Antenna subtraction Gehrmann-DeRidder, Gehrmann, Glover ‘05
Phase space dependent	Phase space independent
Based on limits of amplitudes	Based on simplified matrix elements
Local	Non-local

Matching coefficients

Beneke, MC, Falgari, Mitov, Schwinn `09

$$\sigma_{q\bar{q}}^{(2)} = \frac{3.60774}{\beta^2} + \frac{1}{\beta}\Big(- 140.368 \ln^2\beta + 32.106 \ln\beta + 3.95105\Big) \\ +910.222 \ln^4\beta - 1315.53 \ln^3\beta + 592.292 \ln^2\beta + 528.557 \ln\beta + C_{qq}^{(2)}$$

$$\sigma_{gg}^{(2)} = \frac{68.5471}{\beta^2} + \frac{1}{\beta}\Big(496.3 \ln^2\beta + 321.137 \ln\beta - 8.62261\Big) \\ +4608 \ln^4\beta - 1894.91 \ln^3\beta - 912.349 \ln^2\beta + 2456.74 \ln\beta + C_{gg}^{(2)}$$

- $C^{(2)}_{qq}$ & $C^{(2)}_{gg}$ become matching coefficients in the resummation formula

$$\frac{\hat{\sigma}^{N}_{ij,I}(m_t{}^2,\mu_f^2,\mu_r^2)}{\hat{\sigma}^{(0),N}_{ij,I}(m_t{}^2,\mu_f^2,\mu_r^2)} = g^0_{ij,I}(m_t{}^2,\mu_f^2,\mu_r^2)\cdot\exp\left(G^{N+1}_{ij,I}(m_t{}^2,\mu_f^2,\mu_r^2)\right) + \mathcal{O}(N^{-1}\ln^n N)$$

- It is possible to obtain them from the numerical fitting formulae, but with large error, e.g.

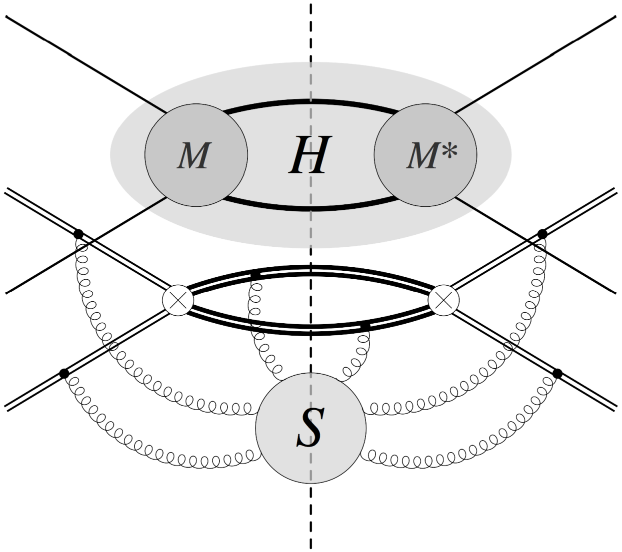
$$C^{(2)}_{gg} = 338.179 - 26.8912 N_L + 0.142848 N_L^2$$

where we “guesstimate” that the purely bosonic coefficient has an error up to 50%

Matching coefficients

- How to obtain the matching coefficient from factorization?
- Consider cross section $d\sigma_{h_1h_2} = \sum_{ab} d\hat{\sigma}^0_{ab} \otimes \phi^0_{a/h_1} \otimes \phi^0_{b/h_2}$ (no collinear renormalization)
- Close to threshold there are only soft gluons in real radiation, which factorize into soft gluon emission from eikonal lines

$$\hat{\sigma}^0 = \sum_I H^0_I \otimes S^0_I$$



- sum over color configurations from hard matrix element singlet and octet to be considered
- hard function is the virtual amplitude, because
  1) no hard external momenta
  2) purely virtual eikonal corrections vanish
- convolution in energy due to loss through radiation from the hard cross section

Subtle point: same color projector left & right
Beneke, Falgari, Schwinn ‘09

- finite hard function is the square of the finite remainders from the virtual corrections $H^0_I = |Z_I|^2 H$

Matching coefficients

- Renormalized soft function (in Mellin space to avoid convolutions) given as

$$S_I = |Z_I|^2 Z_\phi^2 S_I^0$$

- Singlet case known from Belitsky '98
- Color octet production at rest MC, Fiedler, in preparation

$$\begin{aligned} S_R^{(2)}(L) = C_A^2 & \left(\frac{17L^2}{12} + \left(\frac{\zeta(3)}{2} + \frac{151}{36} - \frac{\pi^2}{12} \right) L - \frac{5\zeta(3)}{8} + \frac{13\pi^4}{2880} + \frac{\pi^2}{24} + \frac{223}{54} \right) \\ & + C_R\, C_A \left(\frac{29L^3}{9} + \left(\frac{103}{18} - \frac{\pi^2}{6} \right) L^2 + \left(-\frac{7\zeta(3)}{2} + \frac{101}{27} + \frac{\pi^2}{12} \right) L \right. \\ & \left. - \frac{11\zeta(3)}{72} - \frac{\pi^4}{48} + \frac{139\pi^2}{864} + \frac{607}{324} \right) \\ & + C_R^2 \left(2L^4 + \frac{\pi^2 L^2}{6} + \frac{\pi^4}{288} \right) \\ & + C_A\, T_F\, n_l \left(-\frac{L^2}{3} - \frac{11L}{9} - \frac{40}{27} \right) \\ & + C_R\, T_F\, n_l \left(-\frac{4L^3}{9} - \frac{10L^2}{9} - \frac{28L}{27} + \frac{\zeta(3)}{18} - \frac{5\pi^2}{216} - \frac{41}{81} \right) \end{aligned}$$

$$L = \ln\left(\frac{\mu N}{Q} \right)$$

Matching coefficients

- Threshold expansion of the cross section including constants given by

$$\hat{\sigma}^{(2)} \approx \sum_I H_I^{(2)} + H_I^{(1)} \otimes S_I^{(1)} + H_I^{(0)} \otimes S_I^{(2)}$$

2-loop finite remainder &
1-loop squared finite remainder

Coulomb potential (color dependent)
spin dependent potentials
s- p- & d-wave contributions

1-loop finite remainder

Coulomb potential
s-wave only

s-wave Born -> soft function @ rest

- no problems with Coulomb factorization
  (general case discussed in Beneke, Falgari, Schwinn '09)

- performing expansions of the finite remainders of the virtual corrections we obtain
  for example (PRELIMINARY by Bärnreuther, MC, Fiedler, in preparation)

$$C_{gg}^{(2)} = 484.845 - 29.9249 N_L + 0.142857 N_L^2$$

- to be compared with the fitted value $C_{gg}^{(2)} = 338.179 - 26.8912 N_L + 0.142848 N_L^2$

Outlook

Current work on: matching coefficients for resummation

Next project: calculation of the Forward-Backward asymmetry

Current technical status: description of on-shell top quark pair production in a fully differential Monte Carlo

What is possible without new concepts?
NNLO including decays in the Narrow Width Approximation

BACKUP

Two-Particle Phase Space

$$
\begin{aligned}
q_i^0 &= \frac{Q^0 q_i'^0 + \vec{Q}\cdot\vec{q_i'}}{\sqrt{Q^2}}\,, \\
\vec{q_i} &= \vec{q_i'} + \left(q_i'^0 + \frac{\vec{Q}\cdot\vec{q_i'}}{Q^0+\sqrt{Q^2}} \right) \frac{\vec{Q}}{\sqrt{Q^2}}
\end{aligned}
$$

Boost into CM system

$$
\begin{aligned}
{q_1'}^0 \;&=\; {q_2'}^0 \;=\; \tfrac{1}{2}\sqrt{Q^2}\,, \\
\vec{q_1'} \;&=\; -\vec{q_2'} \;=\; \tfrac{1}{2}\sqrt{Q^2+\beta^2-1} \\
&\times\; \left(\sin\rho_Q \sin\phi_Q \sin\theta_Q\; \vec{n}^{(d-4)}, \cos\rho_Q \sin\phi_Q \sin\theta_Q, \cos\phi_Q \sin\theta_Q, \cos\theta_Q\right)
\end{aligned}
$$

five dimensional vectors

$$\vec{n}^{(d-4)} = (\vec{0}^{(d-5)}, 1)$$

$$
\begin{aligned}
d\Phi_2(Q;q_1,q_2) &= \frac{(4\pi)^\epsilon \Gamma(1-\epsilon)}{8(2\pi)^2\Gamma(1-2\epsilon)} \left(Q^2\right)^{-\epsilon} \left(\sqrt{1-\frac{4m^2}{Q^2}}\right)^{1-2\epsilon} \left(1-\cos^2\theta_Q\right)^{-\epsilon} \left(\sin^2\phi_Q\right)^{-\epsilon} \\
&\times \frac{4^{1+\epsilon}\Gamma(-2\epsilon)}{\Gamma^2(-\epsilon)(1-\cos^2\rho_Q)^{1+\epsilon}}\, d\cos\theta_Q\, d\phi_Q\, d\cos\rho_Q
\end{aligned}
$$

ordinary four-vectors

$$\frac{4^{1+\epsilon}\Gamma(-2\epsilon)}{\Gamma^2(-\epsilon)(1-\cos^2\rho_Q)^{1+\epsilon}} = \delta(1-\cos\rho_Q) + \frac{4^{1+\epsilon}\Gamma(-2\epsilon)}{\Gamma^2(-\epsilon)} \left[\frac{1}{(1-\cos^2\rho_Q)^{1+\epsilon}}\right]_+$$