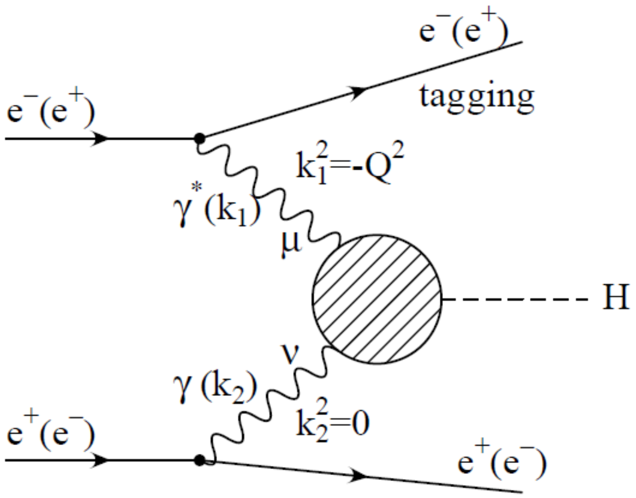
Transition Form Factor in Higgs production through Two-photon Processes



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in collaboration with

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Talk at RADCOR2013

Durham, September 26, 2013

Plan of the talk

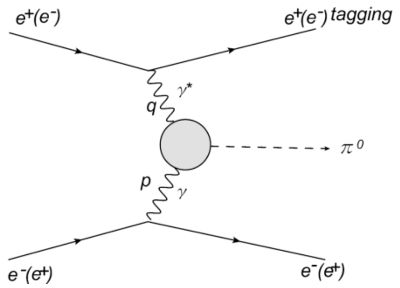
1. Introduction and motivation
2. Higgs Production in 2γ processes
3. Transition Form Factor
4. Numerical Analysis
5. Summary and outlook

1. Introduction and Motivation

How Higgs couples to 2 photons?

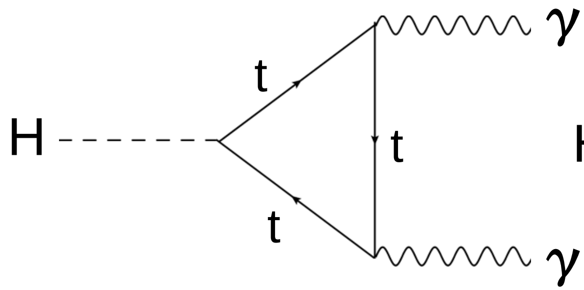
- Di-photon decay mode observed for a Higgs of mass 125-126 GeV at LHC (i.e. $H \to 2\gamma$)
- Here we investigate Higgs production in 2-photon processes of e^+e^- collisions (i.e. $2\gamma \to H$)
- We evaluate transition form factor of Higgs and its Q^2 dependence just like

  a transition form factor of pion π^0

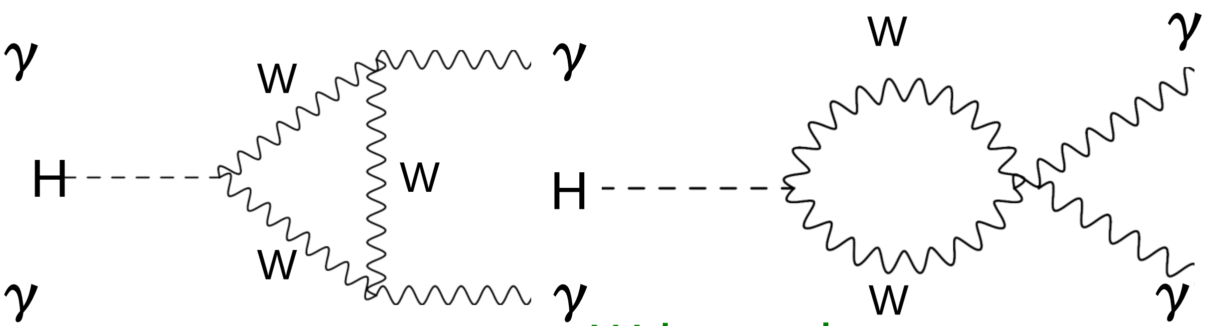


2γ decay of Higgs

γ real photon



Top-quark loop



W-boson-loop

Decay rate

$$\Gamma(H \to 2\gamma) = \frac{\alpha_{\rm em}^2 g^2}{1024\pi^3} \frac{m_H^3}{m_W^2} \left| \sum_i N_{ci} e_i^2 F_i \right|^2$$

N_{ci} number of colors

$$F_{\rm quark} = -2\tau \left[1 + (1-\tau) f(\tau)\right]$$

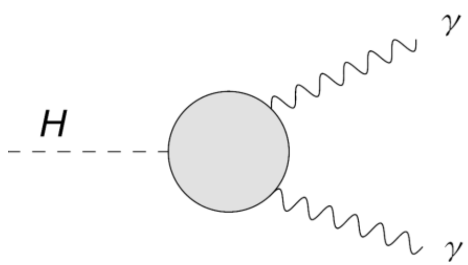
$$F_{\rm W} = 2 + 3\tau + 3\tau(2-\tau) f(\tau)$$

$$f(\tau) = \left[\sin^{-1}\sqrt{\frac{1}{\tau}}\right]^2 \qquad \tau \geq 1$$

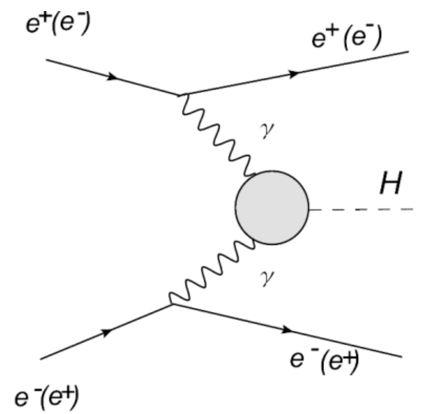
$$\tau = \frac{4m_i^2}{m_H^2}$$

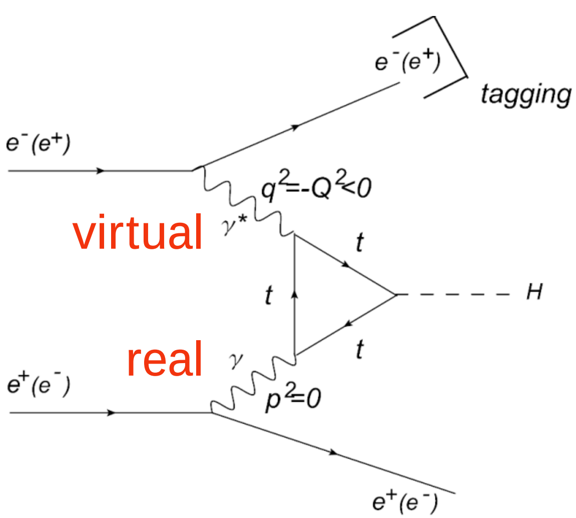
$$= -\frac{1}{4}\left[\ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 \qquad \tau < 1$$

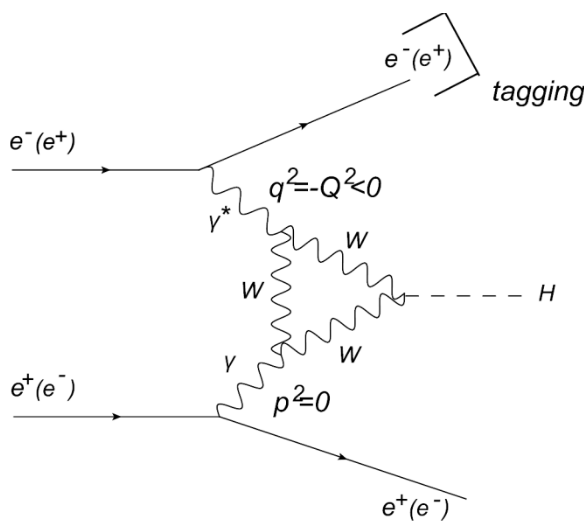
Diagrams for the Higgs Production Process

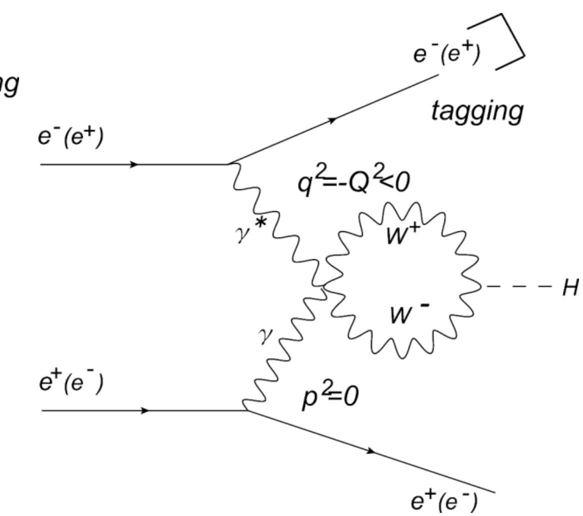


Inverse process



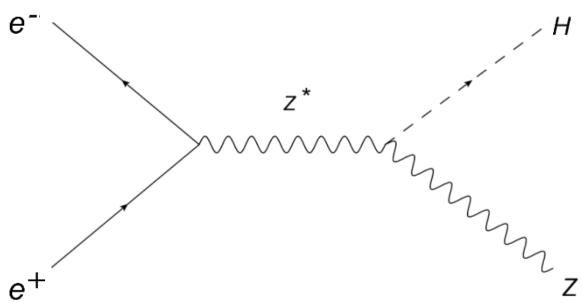




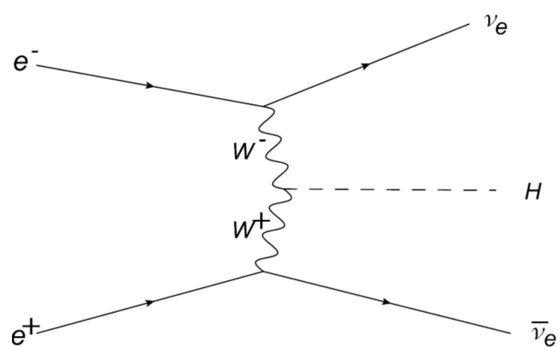


Tree diagrams for Higgs production in e^+e^- collision

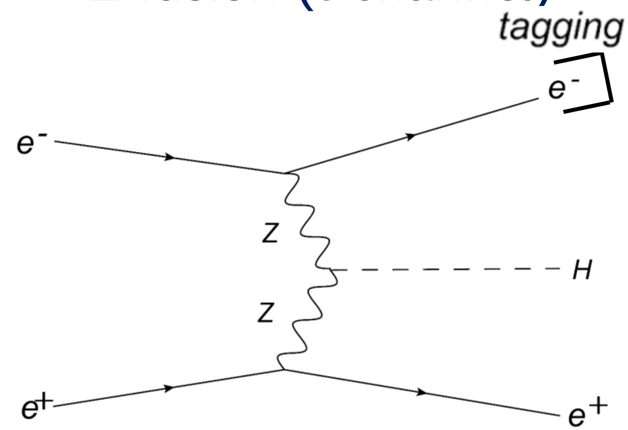
ZH production
(Higgs-strahlung
s-channel)



W-fusion (t-channel)



Z-fusion (t-channel)



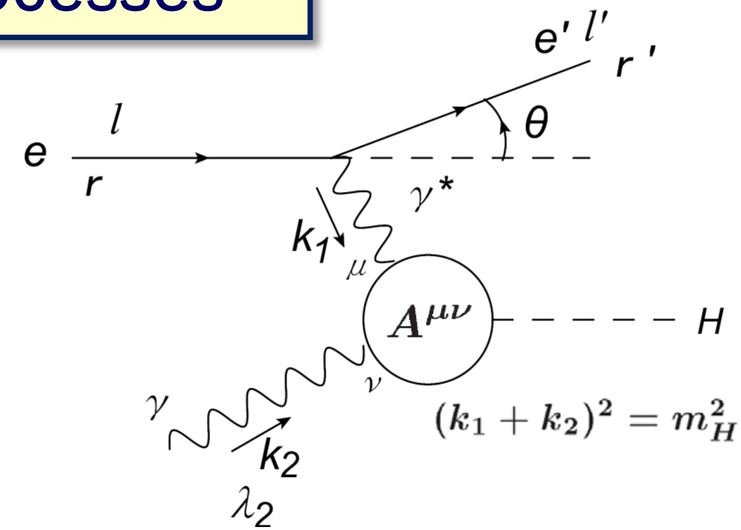
t-channel processes dominate at high-energy

We perform single tagging of scattered electron

The Z-fusion is the tree-level contribution
for the $e^+e^- \rightarrow e^+e^-H$ production process

We consider $e\gamma$ collision to avoid Z-fusion contribution

2. Higgs Production in 2γ processes



One photon is virtual while
the other photon is real

Scattering of electron off real photon

$$e(l) + \gamma(k_2) \to e'(l') + H(q)$$

$$\langle e'H|T|e\gamma\rangle = \overline{u}_{r'}(l')(-ie\gamma_\mu)u_r(l)\frac{-i}{k_1^2+i\epsilon}A^{\mu\nu}\epsilon_\nu(k_2,\lambda_2)$$

scattering amplitude for $\gamma^* + \gamma \to H$ $\qquad k_1^2 = -Q^2 < 0$

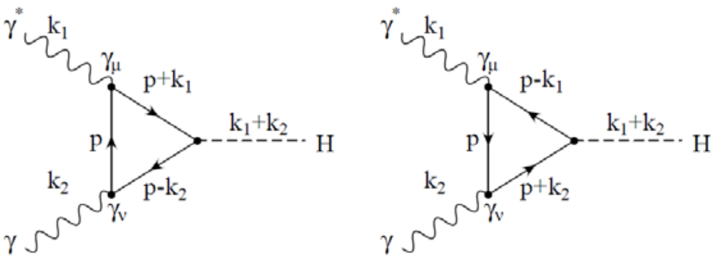
$$M = A^{\mu\nu}\epsilon_\mu(k_1)\epsilon_\nu(k_2) \qquad k_2^\nu\epsilon_\nu(k_2) = 0, \quad k_2^2 = 0$$

where from gauge inv.

$$A^{\mu\nu} = \left[g^{\mu\nu}(k_1\cdot k_2) - k_2^\mu k_1^\nu\right] S_1(m^2,Q^2,m_H^2) + \left[k_1^\mu k_2^\nu - \frac{k_1^2}{k_1\cdot k_2}k_2^\mu k_2^\nu\right] S_2(m^2,Q^2,m_H^2)$$

$$M = \left[g^{\mu\nu}(k_1\cdot k_2) - k_2^\mu k_1^\nu\right] S_1(m^2,Q^2,m_H^2)\epsilon_\mu(k_1)\epsilon_\nu(k_2)$$

Top-quark-loop contribution



$$(k_1+k_2)^2 = m_H^2$$

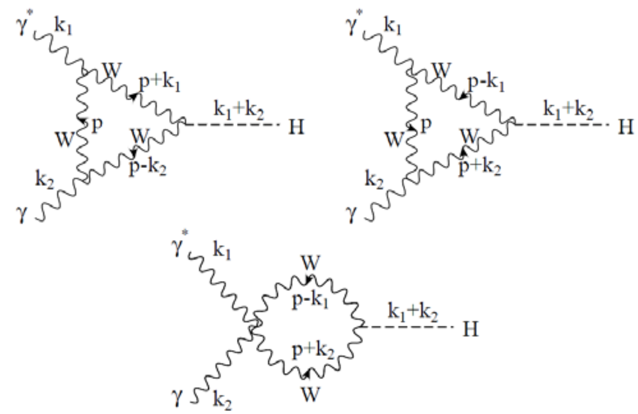
$$S_1(m_t^2, Q^2, m_H^2)$$
$$= -\frac{ige^2}{(4\pi)^2}\frac{1}{m_W}\frac{4m_t^2}{m_H^2+Q^2}\left\{2+\frac{1}{2}\left(1-\frac{4m_t^2}{m_H^2+Q^2}\right)(4f(\tau)+g(\rho))\right.$$
$$\left.+\frac{2Q^2}{m_H^2+Q^2}\left[2\sqrt{\tau-1}\sqrt{f(\tau)}-\sqrt{\frac{1+\rho}{\rho}}\sqrt{g(\rho)}\right]\right\}$$

$$f(\tau) \equiv \left[\sin^{-1}\sqrt{\frac{1}{\tau}}\right]^2, \quad \tau = \frac{4m_t^2}{m_H^2} \quad \text{for} \quad \tau \geq 1$$

same function in the decay rate expression

$$g(\rho) \equiv \left[\log\frac{\sqrt{\rho+1}+\sqrt{\rho}}{\sqrt{\rho+1}-\sqrt{\rho}}\right]^2 \qquad \rho \equiv \frac{Q^2}{4m_t^2} > 0$$

W-boson-loop contribution



$$(k_1 + k_2)^2 = m_H^2$$

$$S_1(m_W^2, Q^2, m_H^2)$$

$$= \frac{ige^2}{(4\pi)^2} \frac{1}{m_W} \frac{m_H^2}{m_H^2 + Q^2} \left\{ \frac{\tau}{1+\rho\tau} \left[4\rho + 8\rho^2\tau + 6(1+\rho\tau) - 3\tau \right] \left[f(\tau) + \frac{1}{4} g(\rho) \right] \right.$$

$$\left. + \left[4\rho + 2(1+\rho\tau) + 3\tau \right] \times \left[1 - \frac{m_H^2 \tau}{m_H^2 + Q^2} \sqrt{\rho(\rho+1)} \sqrt{g(\rho)} + \frac{2Q^2}{m_H^2 + Q^2} \sqrt{\tau - 1} \sqrt{f(\tau)} \right] \right\}$$

$$f(\tau) \equiv \left[\sin^{-1} \sqrt{\frac{1}{\tau}} \right]^2, \quad \tau = \frac{4m_W^2}{m_H^2} \quad \text{for} \quad \tau \geq 1$$

$$g(\rho) \equiv \left[\log \frac{\sqrt{\rho+1} + \sqrt{\rho}}{\sqrt{\rho+1} - \sqrt{\rho}} \right]^2 \qquad \rho \equiv \frac{Q^2}{4m_W^2} > 0$$

3. Transition Form Factor of Higgs

We define the transition form factor F_i as follows

$$S_1(m^2, Q^2, m_H^2) = \frac{ige^2}{(4\pi)^2}\frac{1}{m_W}F_i(m^2, Q^2, m_H^2)$$

where $i = 1/2, 1$ for fermion-loop $F_{1/2}$

and for W-boson loop F_1

m : the mass of the particle going around the loop m_t or m_W

The transition form factor shows a scaling behavior!

$$F_i(m^2, Q^2, m_H^2) \to F_i(\rho, \tau)$$

where $$\tau = \frac{4m^2}{m_H^2}, \quad \rho = \frac{Q^2}{4m^2}$$

The transition form factor in the scaling form

For fermion (top-quark)-loop

$$F_{1/2}(\rho,\tau) = -\frac{1}{\rho+1/\tau}\left\{2+\frac{1}{2}\left(1-\frac{1}{\rho+1/\tau}\right)(4f(\tau)+g(\rho)) + \frac{2}{1+1/\rho\tau}\left(2\sqrt{\tau-1}\sqrt{f(\tau)}-\sqrt{1+1/\rho}\sqrt{g(\rho)}\right)\right\}$$

For W-boson-loop

$$F_1(\rho,\tau) = \frac{1}{1+\rho\tau}\left\{\frac{\tau}{1+\rho\tau}(4\rho+8\rho^2\tau+6(1+\rho\tau)-3\tau)\left(f(\tau)+\frac{1}{4}g(\rho)\right) + (4\rho+2(1+\rho\tau)+3\tau)\left(1-\frac{\tau}{1+\rho\tau}\sqrt{\rho(\rho+1)}\sqrt{g(\rho)}+\frac{2\rho\tau}{1+\rho\tau}\sqrt{\tau-1}\sqrt{f(\tau)}\right)\right\}$$

In the $Q^2 \to 0$ limit or $\rho \to 0$ limit they reduce to the functions appearing in the H→2γ decay-rate expression:

$$F_{1/2}(\rho\to 0,\tau) = F_{1/2}(\tau) = -2\tau[1+(1-\tau)f(\tau)]$$
$$F_1(\rho\to 0,\tau) = F_1(\tau) = 2+3\tau+3\tau(2-\tau)f(\tau)$$

Real Photon limit!

Charged scalar contribution

e.g. charged Higgs in MSSM

$$A^{\mu\nu} = [g^{\mu\nu}(k_1 \cdot k_2) - k_2^\mu k_1^\nu] \frac{ge^2}{(4\pi)^2} \frac{1}{m_W} \frac{4m_H^{\pm 2}}{m_H^2} \frac{m_H^2}{Q^2 + m_H^2}$$

$$\times \left[1 - \frac{\tau}{2(1+\rho\tau)} \left(\frac{1}{2} g(\rho) + 2f(\tau)\right) + \frac{\rho\tau}{1+\rho\tau} \left(2\sqrt{\tau - 1}\sqrt{f(\tau)} - \sqrt{1 + \frac{1}{\rho}}\sqrt{g(\rho)}\right)\right]$$

$$F_0(\rho, \tau)$$

$$= \tau \frac{1}{1+\rho\tau} \left[1 - \frac{\tau}{2(1+\rho\tau)} \left(\frac{1}{2} g(\rho) + 2f(\tau)\right) + \frac{\rho\tau}{1+\rho\tau} \left(2\sqrt{\tau - 1}\sqrt{f(\tau)} - \sqrt{1 + \frac{1}{\rho}}\sqrt{g(\rho)}\right)\right]$$

$Q^2 \to 0$ limit

$$\tau = \frac{4m_H^{\pm 2}}{m_H^2}, \quad \rho = \frac{Q^2}{4m_H^{\pm 2}}$$

$$F_0(\rho \to 0, \tau) = \tau[1 - \tau f(\tau)]$$

also appearing in the H→2γ decay-rate expression:

Property of Transition Form Factor

We noticed that all the transition form factors:

$$F_{1/2}(\rho,\tau) \qquad F_1(\rho,\tau) \qquad F_0(\rho,\tau)$$

can be expressed as linear combinations of the two functions:

$$f(\tau)+\frac{1}{4}g(\rho) \quad \text{and} \quad 2\sqrt{\tau-1}\sqrt{f(\tau)}-\sqrt{1+\frac{1}{\rho}}\sqrt{g(\rho)}$$

which are coming from 2-point and 3-point functions and will be discussed by Ken Sasaki, the next talk.

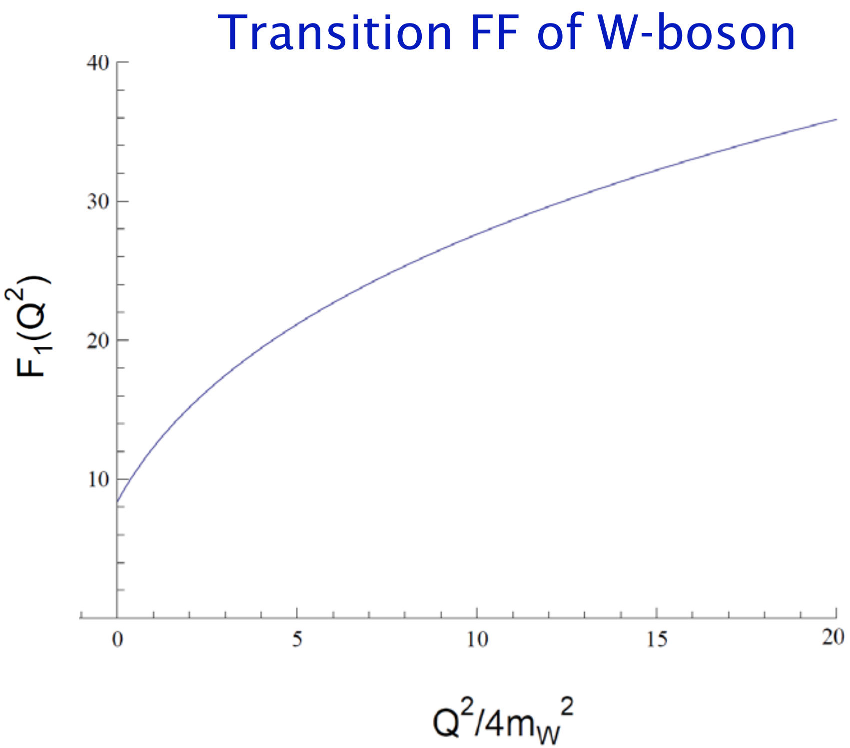
The large Q^2 behavior of the form factors:

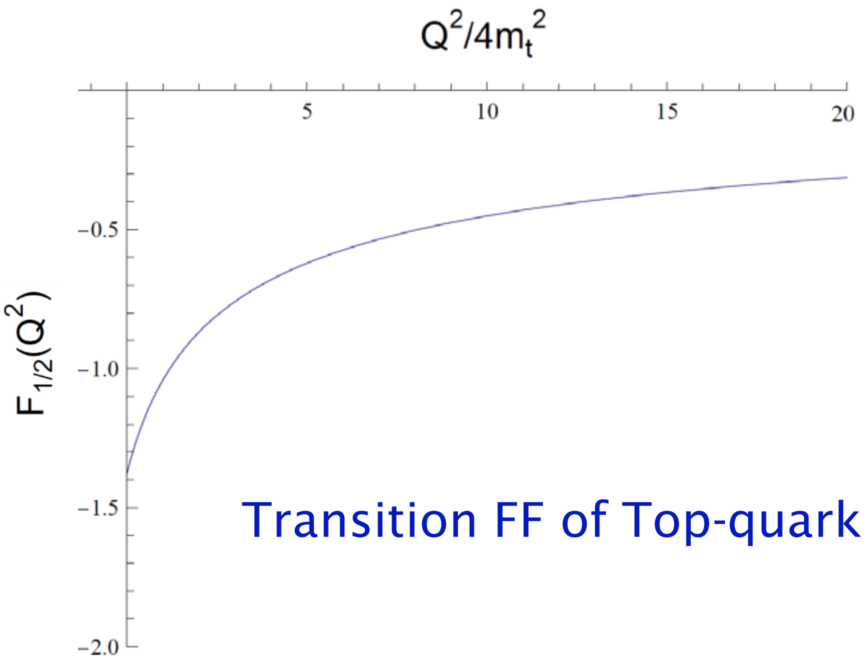
$$F_{1/2}(\rho\to\infty,\tau:\text{fixed})=-\frac{1}{2\rho}g(\rho)=-\frac{2m_t^2}{Q^2}\log^2\frac{Q^2}{m_t^2} \quad \Leftarrow \quad \text{decreasing}$$

$$F_1(\rho\to\infty,\tau:\text{fixed})=2g(\rho)=2\log^2\frac{Q^2}{m_W^2} \quad \Leftarrow \quad \text{increasing}$$

$$g(\rho)\to(4\rho)^2 \quad (\text{as} \quad \rho\to\infty)$$

Q^2 dependence of Transition Form Factor





Transition Form Factor and the Cross Section

In terms of the transition form factor (FF)
the differential cross section reads

$$\frac{d\sigma}{dQ^2} = \frac{\alpha_{\rm em}}{16Q^2}\left[1+\left(\frac{E'}{E}\right)^2\cos^4\frac{\theta}{2}\right]\times|S_1(m^2,Q^2,m_H^2)|^2$$

$$= \frac{\alpha_{\rm em}}{16Q^2}\left[1+\left(\frac{E'}{E}\right)^2\cos^4\frac{\theta}{2}\right]\cdot\frac{g^2}{(4\pi)^2}\alpha_{\rm em}^2\frac{1}{m_W^2}|F_{\rm total}(Q^2)|^2$$

where $Q^2 = 4EE'\sin^2\frac{\theta}{2}$

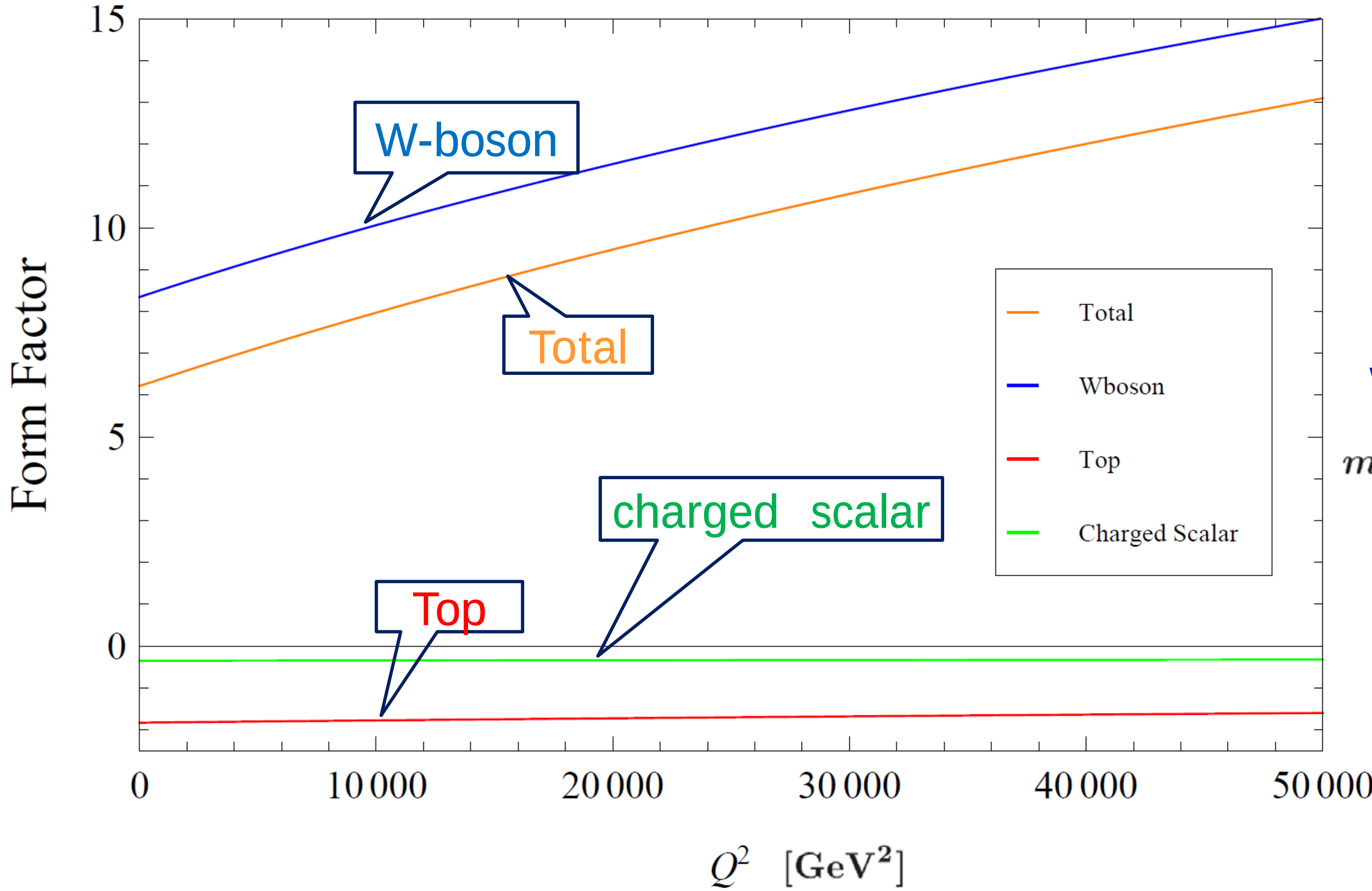
Summing up all the contributions to the transition form factor

$$F_{\rm total}(Q^2) = N_c\sum_i e_i^2 F_{1/2,i}(\tau_i,\rho_i) + F_1(\tau_W,\rho_W)$$

$$\tau_i = \frac{4m_i^2}{m_H^2},\quad \rho_i = \frac{Q^2}{4m_i^2},\quad \tau_W = \frac{4m_W^2}{m_H^2},\quad \rho_W = \frac{Q^2}{4m_W^2}$$

4. Numerical Analysis

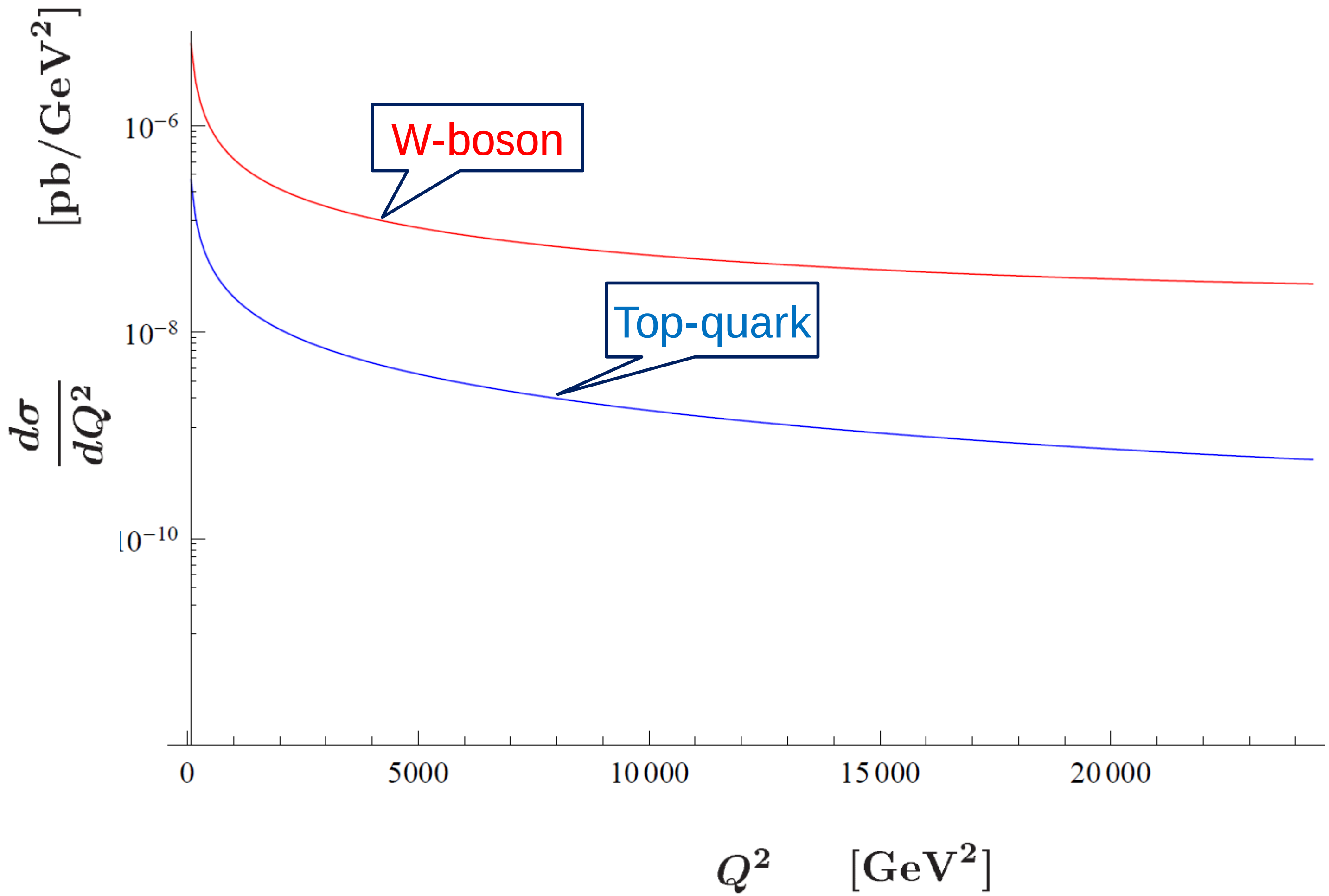
We evaluate the transition form factor
taking account of top-quark as well as W-boson loop



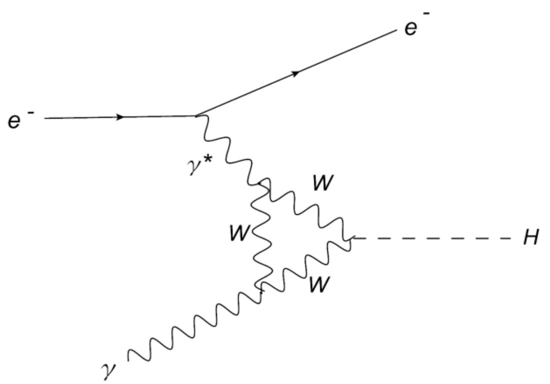
We took

$m_H^{\pm} = 200$ GeV

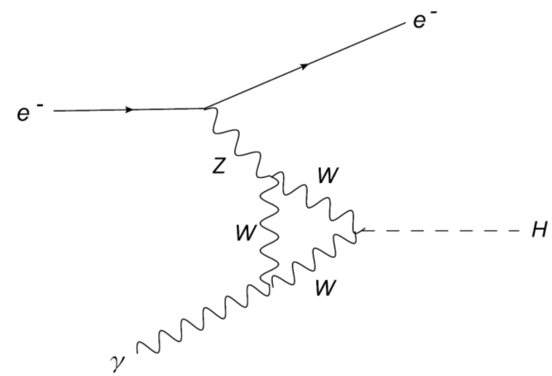
Differential Cross Section



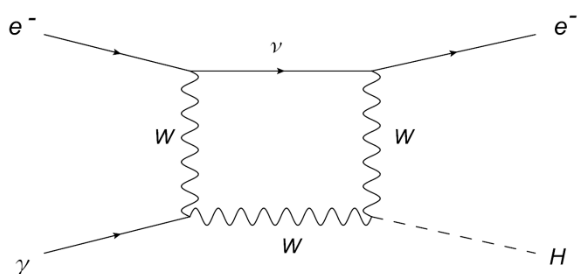
Contributions from various processes



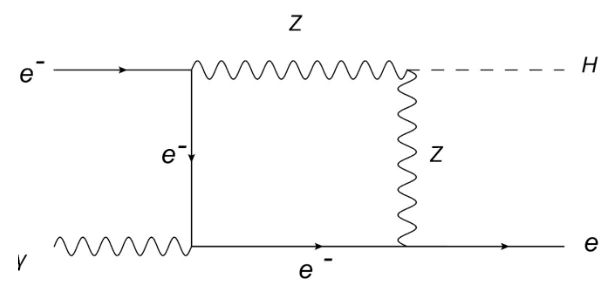
$\gamma^*\gamma$-fusion $\boldsymbol{S_\gamma}$



$Z\gamma$-fusion $\boldsymbol{S_Z}$



$\boldsymbol{W}$ **− related**

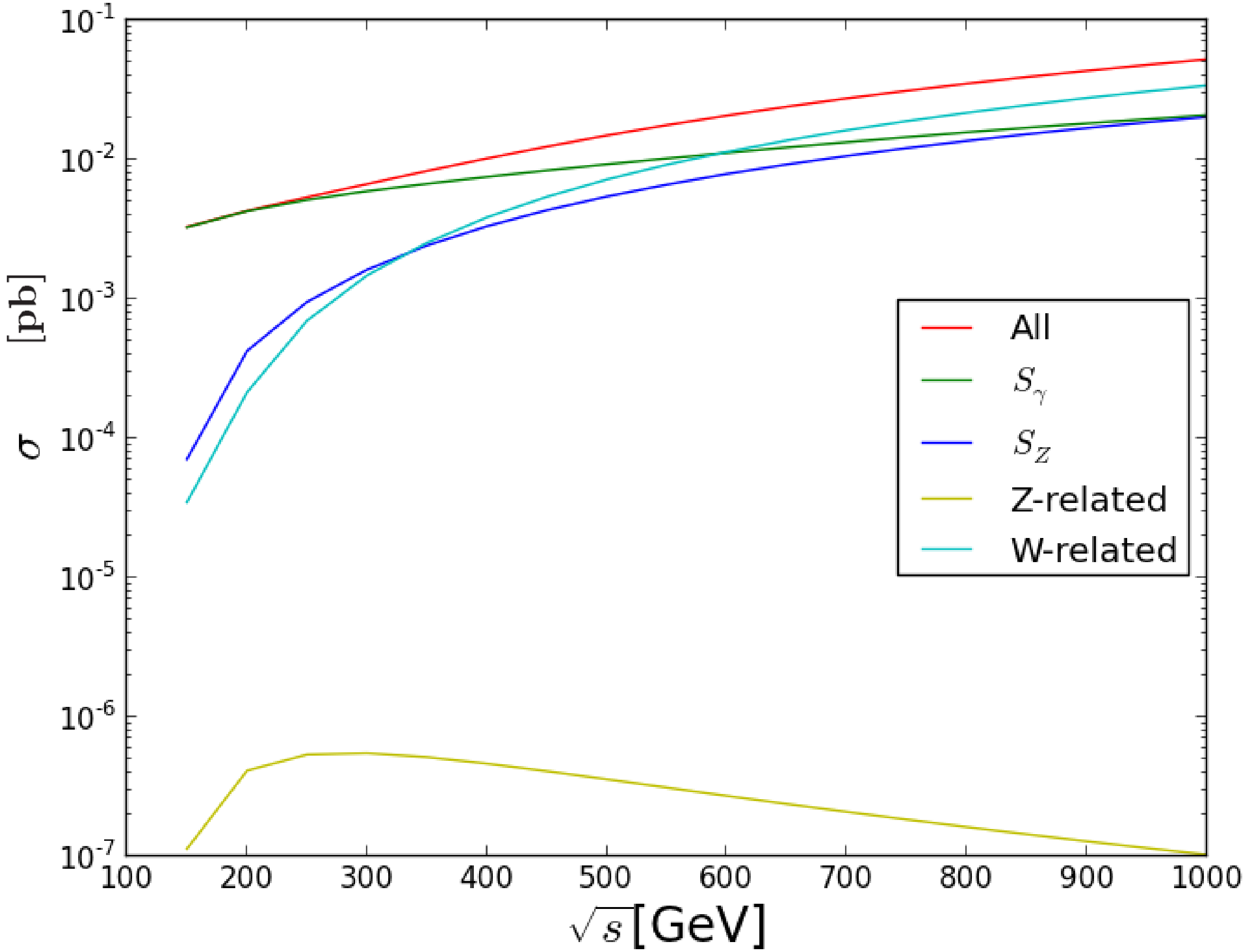


$\boldsymbol{Z}$ **− related**

Ken Sasaki's talk

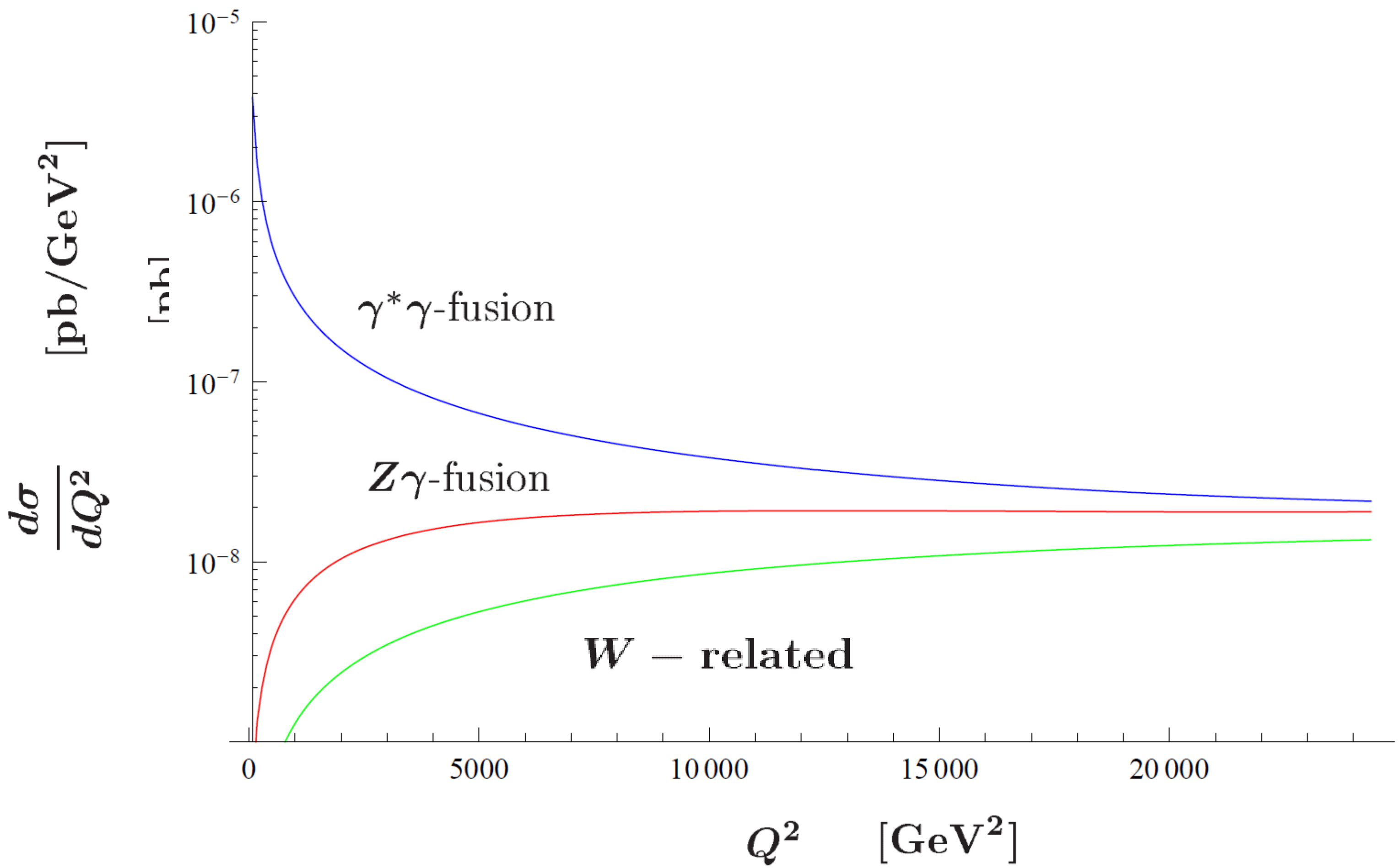
Total Cross Section

Contributions from various processes



Differenatial Cross Section

Contributions from various processes

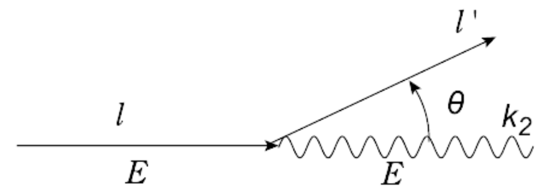


5. Summary and outlook

- We studied the transition form factor of Higgs particle coming from top-quark loop as well as from W-boson loop.
- W-boson loop dominates over top-quark loop for the transition form factor of Higgs
- contribution from light quarks u,d,c,s,b and charged scalars are negligible
- As the future subject we should include the higher order effects of QCD & EW interactions
- We should also investigate the equivalent-photon method in $e^+ e^-$ collision

Back up slides

Electron-photon CM-system



The CM-system of electron-photon collision

$$\frac{d\sigma_{e\gamma\to eH}(\omega)}{dQ^2} = \frac{\alpha_{\rm em}}{16Q^2}\left[1+\left(\frac{E'}{E}\right)^2\cos^4\frac{\theta}{2}\right]\times|S_1(m^2,Q^2,m_H^2)|^2$$

$$|S_1(m^2,Q^2,m_H^2)|^2 = \frac{g^2}{(4\pi)^2}\alpha_{\rm em}^2\frac{1}{m_W^2}|F_{\rm total}(\tau,\rho)|^2$$

Equivalent-photon method

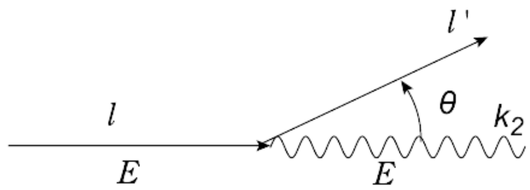
Weizsäcker-Williams method

$$\frac{d\sigma_{ee\to eeH}}{dQ^2} = \int_0^E \frac{d\omega}{\omega}N(\omega)\frac{d\sigma_{e\gamma\to eH}(\omega)}{dQ^2}$$

$$N(\omega) = \frac{\alpha}{\pi}\left[\frac{E^2+E'^2}{E^2}\left(\ln\frac{E}{m_e}-\frac{1}{2}\right)+\frac{(E-E')^2}{2E^2}\left(\ln\frac{2E'}{E-E'}+1\right) + \frac{(E+E')^2}{2E^2}\ln\frac{2E'}{E+E'}\right]$$

Electron-photon CM-system

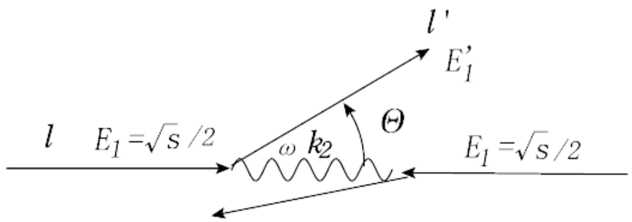
$$l = (E, 0, 0, E)$$
$$l' = (E', E' \sin\theta, 0, E' \cos\theta)$$
$$k_2 = (E, 0, 0, -E)$$



The CM-system of electron-photon collision

Electron-Positron Lab-system

$$l = (E_1, 0, 0, E_1), \quad E_1 = \frac{\sqrt{s}}{2}$$
$$l' = (E_1', E_1' \sin\Theta, 0, E_1' \sin\Theta)$$
$$k_2 = (\omega, 0, 0, -\omega)$$



The Lab-system of electron-positron collision

Relation of both systems

$$E = \sqrt{E_1 \omega}, \qquad E' = E - \frac{m_H^2}{4E} \qquad \sin^2\frac{\theta}{2} = \frac{E_1 E_1'}{E E'} \sin^2\frac{\Theta}{2}$$

Equivalent-photon method

Weizsäcker-Williams method

$$\frac{d\sigma_{ee\to eeH}}{dQ^2} = \int_0^E \frac{d\omega}{\omega} N(\omega) \frac{d\sigma_{e\gamma\to eH}(\omega)}{dQ^2} \qquad \omega = E - E'$$

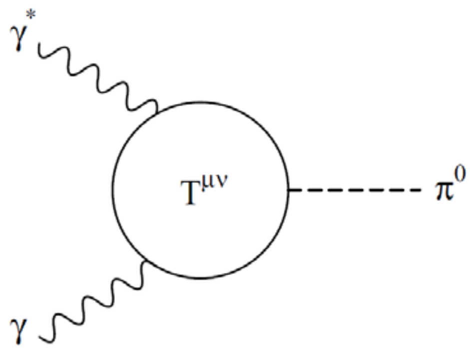
$$\frac{d\sigma_{e\gamma\to eH}(\omega)}{dQ^2} = \frac{\alpha_{\mathrm{em}}}{16Q^2}\left[1 + \left(\frac{E'}{E}\right)^2 \cos^4\frac{\theta}{2}\right] \times |S_1(m^2, Q^2, s)|^2$$

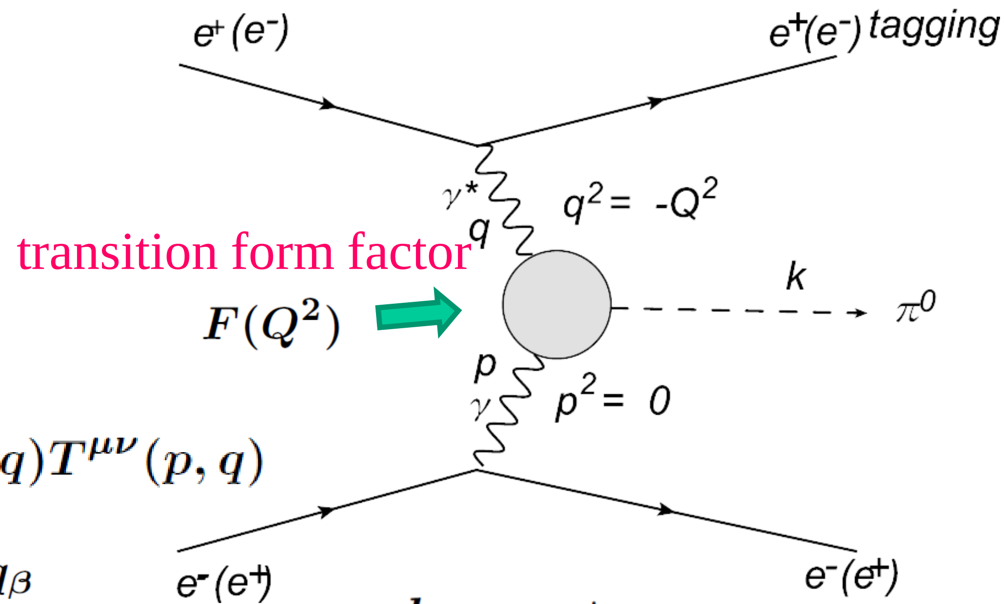
$$|S_1(m^2, Q^2, s)|^2 = \frac{g^2}{(4\pi)^2}\alpha_{\mathrm{em}}^2 \frac{1}{m_W^2} |F_{\mathrm{total}}(\tau, \rho)|^2$$

$$N(\omega) = \frac{\alpha}{\pi}\left[\frac{E^2 + E'^2}{E^2}\left(\ln\frac{E}{m_e} - \frac{1}{2}\right) + \frac{(E-E')^2}{2E^2}\left(\ln\frac{2E'}{E-E'} + 1\right) + \frac{(E+E')^2}{2E^2}\ln\frac{2E'}{E+E'}\right]$$

$$E = \sqrt{\frac{\sqrt{s}\omega}{2}}, \qquad E' = E - \frac{m_H^2}{4E}$$

$\gamma^*\gamma \to \pi^0$ amplitude





$$\langle \pi^0(k)|T|\gamma(p)\gamma^*(q)\rangle = \epsilon_\mu(p)\epsilon_\nu(q)T^{\mu\nu}(p,q)$$

$$T^{\mu\nu}(p,q) = e^2 F(p,q)\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta$$

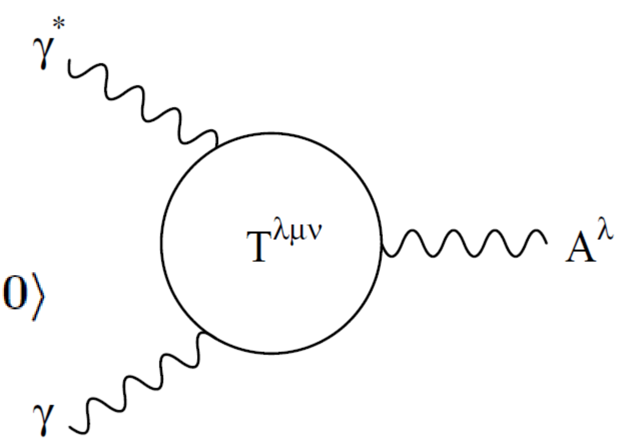
$$k = p + q$$

$$T^{\mu\nu}(p,q) = (m_\pi^2 - k^2)\int d^4x d^4y e^{ip\cdot x + iq\cdot y}\langle 0|T(J^\mu(x)J^\nu(y)\phi_\pi(0))|0\rangle$$

three-current (VVA) amplitude

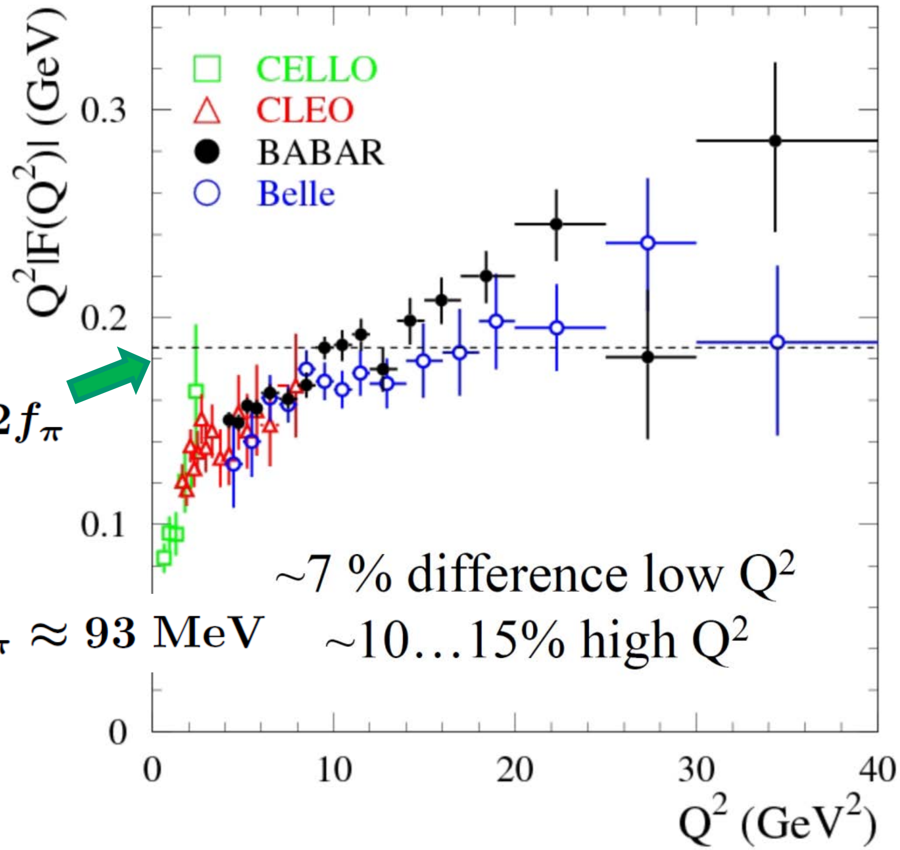
$$T^{\lambda\mu\nu}(p,q) \equiv$$

$$i\int d^4x d^4y e^{ip\cdot x + iq\cdot y}\langle 0|T(J^\mu(x)J^\nu(y)A^\lambda(0))|0\rangle$$

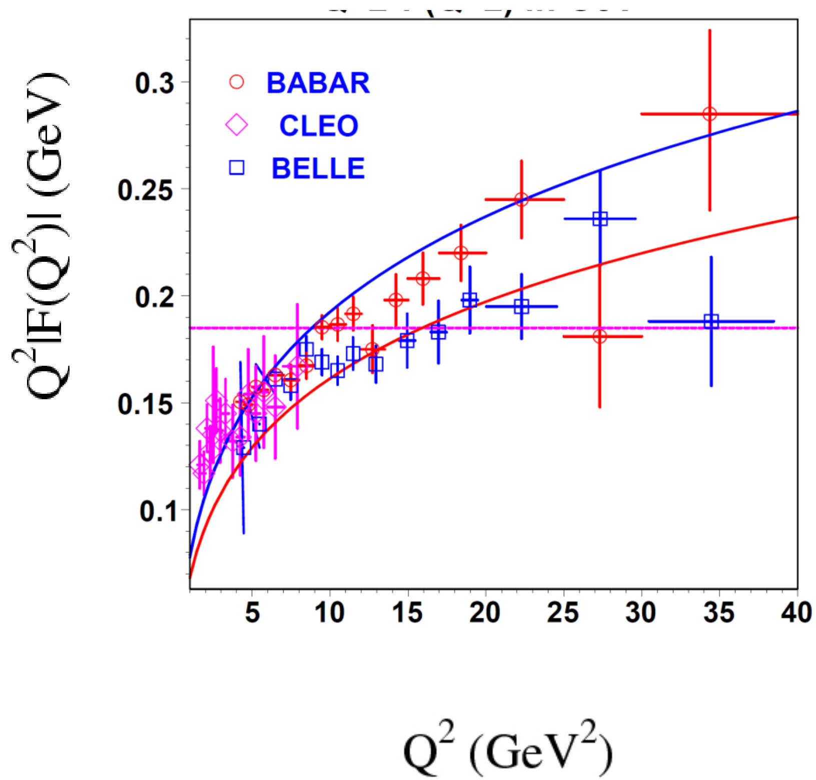


Pion Transition FF
Belle and BaBar data

See also talk by V.Savinov (Belle) at QCD12



$f_\pi \approx 93$ MeV



Talk by A. Denig (BaBar) at QCD12

Talk by T.N.Pham at QCD12

Difference BABAR – BELLE ~$2\sigma_{syst}$