Transition Form Factor in Higgs production through Two-photon Processes

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Plan of the talk

1. Introduction and motivation
2. Higgs Production in $2\gamma$ processes
3. Transition Form Factor
4. Numerical Analysis
5. Summary and outlook
1. Introduction and Motivation

How Higgs couples to 2 photons?

- Di-photon decay mode observed for a Higgs of mass 125-126 GeV at LHC (i.e. $H \rightarrow 2\gamma$)
- Here we investigate Higgs production in 2-photon processes of $e^+e^-$ collisions (i.e. $2\gamma \rightarrow H$)
- We evaluate transition form factor of Higgs and its $Q^2$ dependence just like a transition form factor of pion $\pi^0$
**2γ decay of Higgs**

![Diagram of 2γ decay of Higgs]

**Decay rate**

\[
\Gamma(H \to 2\gamma) = \frac{\alpha_{\text{em}} g_H^2}{1024 \pi^3} m_H^3 \left| \sum_i N_{ci} e_i^2 F_i \right|^2 \\
F_{\text{quark}} = -2\tau \left[ 1 + (1 - \tau)f(\tau) \right] \\
F_{W} = 2 + 3\tau + 3\tau(2 - \tau)f(\tau)
\]

\[
f(\tau) = \begin{cases} 
\sin^{-1} \sqrt{\frac{1}{\tau}} \quad & \tau \geq 1 \\
-\frac{1}{4} \ln \left( \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} - i\pi \right) \quad & \tau < 1
\end{cases}
\]

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Transition From Factor in Higgs Production thru 2-photon processes
The Z-fusion is the tree-level contribution for the $e^+e^- \rightarrow e^+e^-H$ production process.

We consider $e\gamma$ collision to avoid Z-fusion contribution.
2. Higgs Production in $2\gamma$ processes

One photon is virtual while the other photon is real

Scattering of electron off real photon

$$e(l) + \gamma(k_2) \rightarrow e'(l') + H(q)$$

$$\langle e'H|T|e\gamma \rangle = \bar{u}_{r'}(l')(-ie\gamma_\mu)u_r(l)\frac{-i}{k_1^2 + i\epsilon}A^{\mu\nu}\epsilon_\nu(k_2, \lambda_2)$$

scattering amplitude for $\gamma^* + \gamma \rightarrow H$

$$k_1^2 = -Q^2 < 0$$

$$M = A^{\mu\nu}\epsilon_\mu(k_1)\epsilon_\nu(k_2)$$

$$k_2^\nu\epsilon_\nu(k_2) = 0, \quad k_2^2 = 0$$

where from gauge inv.

$$A^{\mu\nu} = \left[ g^{\mu\nu}(k_1 \cdot k_2) - k_2^\mu k_1^\nu \right] S_1(m^2, Q^2, m_H^2) + \left[ k_1^\mu k_2^\nu - \frac{k_1^2}{k_1 \cdot k_2} k_2^\mu k_2^\nu \right] S_2(m^2, Q^2, m_H^2)$$

$$M = \left[ g^{\mu\nu}(k_1 \cdot k_2) - k_2^\mu k_1^\nu \right] S_1(m^2, Q^2, m_H^2)\epsilon_\mu(k_1)\epsilon_\nu(k_2)$$
Transition From Factor in Higgs Production thru 2-photon processes

\[ S_1(m_t^2, Q^2, m_H^2) = \frac{ige^2}{(4\pi)^2} \frac{1}{m_W} \frac{4m_t^2}{m_H^2 + Q^2} \left\{ 2 + \frac{1}{2} \left( 1 - \frac{4m_t^2}{m_H^2 + Q^2} \right) \left( 4f(\tau) + g(\rho) \right) \right. \\
\left. + \frac{2Q^2}{m_H^2 + Q^2} \left[ 2\sqrt{\tau - 1}\sqrt{f(\tau)} - \sqrt{\frac{1 + \rho}{\rho}}\sqrt{g(\rho)} \right] \right\} \]

\[ f(\tau) \equiv \left( \sin^{-1} \sqrt{\frac{1}{\tau}} \right)^2, \quad \tau = \frac{4m_t^2}{m_H^2} \quad \text{for} \quad \tau \geq 1 \]

\[ g(\rho) \equiv \left( \log \frac{\sqrt{\rho + 1} + \sqrt{\rho}}{\sqrt{\rho + 1} - \sqrt{\rho}} \right)^2 \quad \rho \equiv \frac{Q^2}{4m_t^2} > 0 \]

Top-quark-loop contribution
W-boson-loop contribution

\[ S_1(m_W^2, Q^2, m_H^2) \]

\[ = \frac{ige^2}{(4\pi)^2} \frac{1}{m_W} \frac{m_H^2}{m_H^2 + Q^2} \left\{ \frac{\tau}{1 + \rho \tau} \left[ 4\rho + 8\rho^2\tau + 6(1 + \rho \tau) - 3\tau \right] \left[ f(\tau) + \frac{1}{4} g(\rho) \right] \right\} + \left[ 4\rho + 2(1 + \rho \tau) + 3\tau \right] \times \left[ 1 - \frac{m_H^2 \tau}{m_H^2 + Q^2} \sqrt{\rho(\rho + 1)} \sqrt{g(\rho)} + \frac{2Q^2}{m_H^2 + Q^2} \sqrt{\tau - 1} \sqrt{f(\tau)} \right] \}

\[ f(\tau) \equiv \left[ \sin^{-1} \sqrt{\frac{1}{\tau}} \right]^2, \quad \tau = \frac{4m_W^2}{m_H^2} \quad \text{for} \quad \tau \geq 1 \]

\[ g(\rho) \equiv \left[ \log \frac{\sqrt{\rho + 1} + \sqrt{\rho}}{\sqrt{\rho + 1} - \sqrt{\rho}} \right]^2, \quad \rho \equiv \frac{Q^2}{4m_W^2} > 0 \]
3. Transition Form Factor of Higgs

We define the transition form factor $F_i$ as follows

$$S_1(m^2, Q^2, m_H^2) = \frac{ige^2}{(4\pi)^2} \frac{1}{m_W} F_i(m^2, Q^2, m_H^2)$$

where $i = 1/2, 1$ for fermion-loop $F_{1/2}$

and for W-boson loop $F_1$

$m$: the mass of the particle going around the loop $m_t$ or $m_W$

The transition form factor shows a scaling behavior!

$$F_i(m^2, Q^2, m_H^2) \rightarrow F_i(\rho, \tau)$$

where

$$\tau = \frac{4m^2}{m_H^2}, \quad \rho = \frac{Q^2}{4m^2}$$
The transition form factor in the scaling form

For fermion (top-quark)-loop

\[
F_{1/2}(\rho, \tau) = -\frac{1}{\rho + 1/\tau} \left\{ 2 + \frac{1}{2} \left( 1 - \frac{1}{\rho + 1/\tau} \right) (4f(\tau) + g(\rho)) + \frac{2}{1 + 1/\rho\tau} \left( 2\sqrt{\tau - 1}\sqrt{f(\tau)} - \sqrt{1 + 1/\rho}\sqrt{g(\rho)} \right) \right\}
\]

For W-boson-loop

\[
F_1(\rho, \tau) = \frac{1}{1 + \rho\tau} \left\{ \frac{\tau}{1 + \rho\tau} (4\rho + 8\rho^2\tau + 6(1 + \rho\tau) - 3\tau) \left( f(\tau) + \frac{1}{4}g(\rho) \right) + (4\rho + 2(1 + \rho\tau) + 3\tau) \left( 1 - \frac{\tau}{1 + \rho\tau} \sqrt{\rho(\rho + 1)}\sqrt{g(\rho)} + \frac{2\rho\tau}{1 + \rho\tau} \sqrt{\tau - 1}\sqrt{f(\tau)} \right) \right\}
\]

In the \(Q^2 \to 0\) limit or \(\rho \to 0\) limit they reduce to the functions appearing in the \(H \to 2\gamma\) decay-rate expression:

\[
F_{1/2}(\rho \to 0, \tau) = F_{1/2}(\tau) = -2\tau[1 + (1 - \tau)f(\tau)]
\]

\[
F_1(\rho \to 0, \tau) = F_1(\tau) = 2 + 3\tau + 3\tau(2 - \tau)f(\tau)
\]

Real Photon limit!
Charged scalar contribution

e.g. charged Higgs in MSSM

\[ A^{\mu\nu} = [g^{\mu\nu} (k_1 \cdot k_2) - k_2^\mu k_1^\nu] \frac{g e^2}{(4\pi)^2} \frac{1}{m_W} \frac{4m_{\pm}^2}{m_H^2} \frac{m_H^2}{Q^2 + m_H^2} \]

\[ \times \left[ 1 - \frac{\tau}{2(1 + \rho \tau)} \left( \frac{1}{2} g(\rho) + 2 f(\tau) \right) + \frac{\rho \tau}{1 + \rho \tau} \left( 2\sqrt{\tau - 1} \sqrt{f(\tau)} - \sqrt{1 + \frac{1}{\rho} \sqrt{g(\rho)}} \right) \right] \]

\[ F_0(\rho, \tau) \]
\[ = \frac{1}{1 + \rho \tau} \left[ 1 - \frac{\tau}{2(1 + \rho \tau)} \left( \frac{1}{2} g(\rho) + 2 f(\tau) \right) + \frac{\rho \tau}{1 + \rho \tau} \left( 2\sqrt{\tau - 1} \sqrt{f(\tau)} - \sqrt{1 + \frac{1}{\rho} \sqrt{g(\rho)}} \right) \right] \]

\[ Q^2 \rightarrow 0 \text{ limit} \]

\[ F_0(\rho \rightarrow 0, \tau) = \tau [1 - \tau f(\tau)] \]

also appearing in the H \rightarrow 2\gamma decay-rate expression:
We noticed that all the transition form factors:

\[ F_{1/2}(\rho, \tau) \quad F_1(\rho, \tau) \quad F_0(\rho, \tau) \]

can be expressed as linear combinations of the two functions:

\[ f(\tau) + \frac{1}{4} g(\rho) \quad \text{and} \quad 2\sqrt{\tau - 1} \sqrt{f(\tau)} - \sqrt{1 + \frac{1}{\rho} g(\rho)} \]

which are coming from 2-point and 3-point functions and will be discussed by Ken Sasaki, the next talk.

The large \( Q^2 \) behavior of the form factors:

\[ F_{1/2}(\rho \to \infty, \tau : \text{fixed}) = -\frac{1}{2\rho} g(\rho) = -\frac{2m_t^2}{Q^2} \log^2 \frac{Q^2}{m_t^2} \quad \leftarrow \text{decreasing} \]

\[ F_1(\rho \to \infty, \tau : \text{fixed}) = 2g(\rho) = 2 \log^2 \frac{Q^2}{m_W^2} \quad \leftarrow \text{increasing} \]

\[ g(\rho) \to (4\rho)^2 \quad (\text{as} \quad \rho \to \infty) \]
Transition From Factor in Higgs Production thru 2-photon processes

**Q^2 dependence of Transition Form Factor**

- **Transition FF of W-boson**
  
  ![Graph of Transition FF of W-boson](image)

- **Transition FF of Top-quark**
  
  ![Graph of Transition FF of Top-quark](image)
In terms of the transition form factor (FF) the differential cross section reads

\[
\frac{d\sigma}{dQ^2} = \frac{\alpha_{em}}{16Q^2} \left[ 1 + \left( \frac{E'}{E} \right)^2 \cos^4 \frac{\theta}{2} \right] \times |S_1(m^2, Q^2, m_H^2)|^2
\]

\[
= \frac{\alpha_{em}}{16Q^2} \left[ 1 + \left( \frac{E'}{E} \right)^2 \cos^4 \frac{\theta}{2} \right] \cdot \frac{g^2}{(4\pi)^2} \alpha_{em}^2 \frac{1}{m_W^2} |F_{total}(Q^2)|^2
\]

where

\[
Q^2 = 4EE' \sin^2 \frac{\theta}{2}
\]

Summing up all the contributions to the transition form factor

\[
F_{total}(Q^2) = N_c \sum_i e_i^2 F_{1/2,i}(\tau_i, \rho_i) + F_1(\tau_W, \rho_W)
\]

\[
\tau_i = \frac{4m_i^2}{m_H^2}, \quad \rho_i = \frac{Q^2}{4m_i^2}, \quad \tau_W = \frac{4m_W^2}{m_H^2}, \quad \rho_W = \frac{Q^2}{4m_W^2}
\]
4. Numerical Analysis

We evaluate the transition form factor taking account of top-quark as well as W-boson loop.

We took $m_H^\pm = 200$ GeV.
Differential Cross Section

\[ \frac{d\sigma}{dQ^2} \]

\[ \text{pb}/\text{GeV}^2 \]

\[ Q^2 \text{ [GeV}^2] \]

W-boson

Top-quark
Contributions from various processes

\[ \gamma^* \gamma\text{-fusion} \quad S_\gamma \]

\[ Z \gamma\text{-fusion} \quad S_Z \]

\[ \gamma \quad W - \text{related} \]

\[ Z \quad Z - \text{related} \]

Ken Sasaki’s talk
Total Cross Section

Contributions from various processes

\[ \sigma \] [pb]

\[ \sqrt{s} \] [GeV]

- Red: All
- Green: \( S_\gamma \)
- Blue: \( S_Z \)
- Yellow: Z-related
- Cyan: W-related

Transition From Factor in Higgs Production thru 2-photon processes
Differential Cross Section

Contributions from various processes

$\frac{d\sigma}{dQ^2}$ vs. $Q^2$ [GeV$^2$]

- $\gamma^*\gamma$-fusion
- $Z\gamma$-fusion
- $W$ - related
5. Summary and outlook

- We studied the transition form factor of Higgs particle coming from top-quark loop as well as from W-boson loop.
- W-boson loop dominates over top-quark loop for the transition form factor of Higgs
- Contribution from light quarks u,d,c,s,b and charged scalars are negligible
- As the future subject we should include the higher order effects of QCD & EW interactions
- We should also investigate the equivalent-photon method in $e^+ e^-$ collision
Back up slides
Electron-photon CM-system

\[ \frac{d\sigma_{e\gamma\rightarrow eH}(\omega)}{dQ^2} = \frac{\alpha_{em}}{16Q^2} \left[ 1 + \left( \frac{E'}{E} \right)^2 \cos^4 \frac{\theta}{2} \right] \times |S_1(m^2, Q^2, m_H^2)|^2 \]

\[ |S_1(m^2, Q^2, m_H^2)|^2 = \frac{g^2}{(4\pi)^2} \alpha_{em}^2 \frac{1}{m_W^2} |F_{total}(\tau, \rho)|^2 \]

Equivalent-photon method

Weizsäcker-Williams method

\[ \frac{d\sigma_{ee\rightarrow eeH}}{dQ^2} = \int_0^E d\omega \left( \frac{N(\omega) d\sigma_{e\gamma\rightarrow eH}(\omega)}{\omega} \right) \]

\[ N(\omega) = \frac{\alpha}{\pi} \left[ \frac{E^2 + E'^2}{E^2} \left( \ln \frac{E}{m_e} - \frac{1}{2} \right) + \frac{(E - E')^2}{2E^2} \left( \ln \frac{2E'}{E - E'} + 1 \right) \right. \]

\[ \left. + \frac{(E + E')^2}{2E^2} \ln \frac{2E'}{E + E'} \right] \]
Electron-photon CM-system

\[ l = (E, 0, 0, E) \]
\[ l' = (E', E' \sin \theta, 0, E' \cos \theta) \]
\[ k_2 = (E, 0, 0, -E') \]

Electron-Positron Lab-system

\[ l = (E_1, 0, 0, E_1), \quad E_1 = \frac{\sqrt{s}}{2} \]
\[ l' = (E'_1, E'_1 \sin \Theta, 0, E'_1 \sin \Theta) \]
\[ k_2 = (\omega, 0, 0, -\omega) \]

Relation of both systems

\[ E = \sqrt{E_1 \omega}, \quad E' = E - \frac{m^2_H}{4E} \]
\[ \sin^2 \frac{\theta}{2} = \frac{E_1 E'_1}{E E'} \sin^2 \frac{\Theta}{2} \]
\[
\frac{d\sigma_{ee\to eeH}}{dQ^2} = \int_{0}^{E} \frac{d\omega}{\omega} N(\omega) \frac{d\sigma_{e\gamma\to eH}(\omega)}{dQ^2} \quad \omega = E - E' \\
\frac{d\sigma_{e\gamma\to eH}(\omega)}{dQ^2} = \frac{\alpha_{em}}{16Q^2} \left[ 1 + \left( \frac{E'}{E} \right)^2 \cos^4 \frac{\theta}{2} \right] \times |S_1(m^2, Q^2, s)|^2 \\
|S_1(m^2, Q^2, s)|^2 = \frac{g^2}{(4\pi)^2} \alpha_{em}^2 \frac{1}{m_W^2} |F_{total}(\tau, \rho)|^2 \\
N(\omega) = \frac{\alpha}{\pi} \left[ \frac{E^2 + E'^2}{E^2} \left( \ln \frac{E}{m_e} - \frac{1}{2} \right) + \frac{(E - E')^2}{2E^2} \left( \ln \frac{2E'}{E - E'} + 1 \right) + \frac{(E + E')^2}{2E^2} \ln \frac{2E'}{E + E'} \right] \\
E = \sqrt{\frac{s\omega}{2}}, \quad E' = E - \frac{m_H^2}{4E} 
\]
\[ \gamma^* \gamma \rightarrow \pi^0 \text{ amplitude} \]

\[ \langle \pi^0(k) | T | \gamma(p) \gamma^*(q) \rangle = \epsilon_\mu(p) \epsilon_\nu(q) T^{\mu\nu}(p, q) \]

\[ T^{\mu\nu}(p, q) = e^2 F(p, q) \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta \]

\[ T^{\mu\nu}(p, q) = (m_\pi^2 - k^2) \int d^4xd^4ye^{i p \cdot x + i q \cdot y} \langle 0 | T(J^\mu(x)J^\nu(y)\phi_\pi(0)) | 0 \rangle \]

three-current (VVA) amplitude

\[ T^{\lambda\mu\nu}(p, q) \equiv \]

\[ i \int d^4xd^4ye^{i p \cdot x + i q \cdot y} \langle 0 | T(J^\mu(x)J^\nu(y)A^\lambda(0)) | 0 \rangle \]
Pion Transition FF
Belle and BaBar data

Talk by A. Denig (BaBar) at QCD12
Difference BABAR – BELLE $\sim 2\sigma_{syst}$

See also talk by V. Savinov (Belle) at QCD12