Helicity amplitudes for high-energy scattering processes

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presented at
RADCOR 2013
23-09-2013, Lumley Castle, Chester le Street

This research was partially supported by
NCBiR grant LIDER/02/35/L-2/10/NCBiR/2011

Narodowe Centrum Badań i Rozwoju

LIDER

High–energy factorization

Gribov, Levin, Ryskin 1983
Catani, Ciafaloni, Hautmann 1991

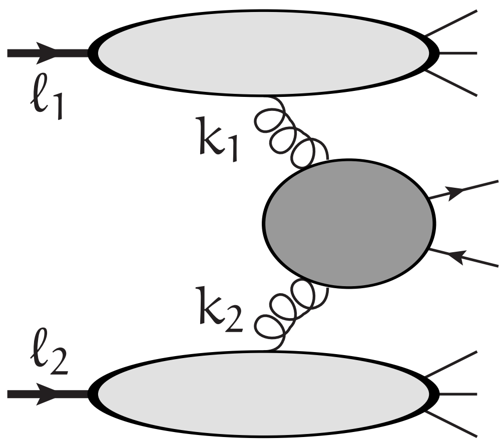
$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

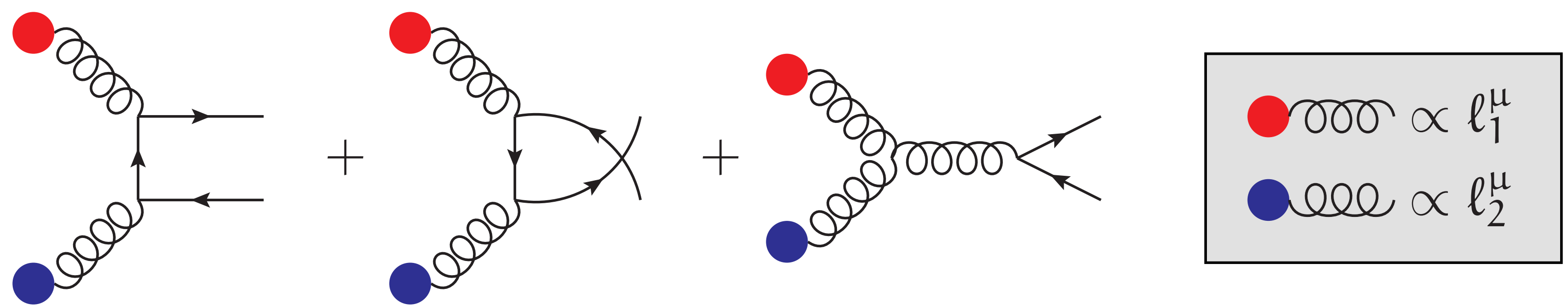
- to be applied in the 3-scale regime $s \gg m^2 \gg \Lambda^2_{QCD}$
- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$,
  but holds also for $s \gg m^2 \sim k_\perp^2$
- *unintegrated pdf* $\mathcal{F}$ may satisfy BFKL-eqn, CCFM-eqn, BK-eqn, KGBJS-eqn, ...
- typically associated with small-x physics
- relevant for forward physics, saturation physics, heavy-ion physics...
- $k_\perp$ gives a handle on the size of the proton
- allows for higher-order kinematical effects at leading order

High–energy factorization

Catani, Ciafaloni, Hautmann 1991

$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\;d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\;\hat{\sigma}_{gg}\!\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$





Imposing high-energy kinematics,

$$k_1^\mu = x_1\ell_1^\mu + k_{1\perp}^\mu \quad , \quad k_2^\mu = x_2\ell_2^\mu + k_{2\perp}^\mu \qquad \text{with} \quad \ell_{1,2}\cdot k_{1\perp,2\perp} = 0 \quad ,$$

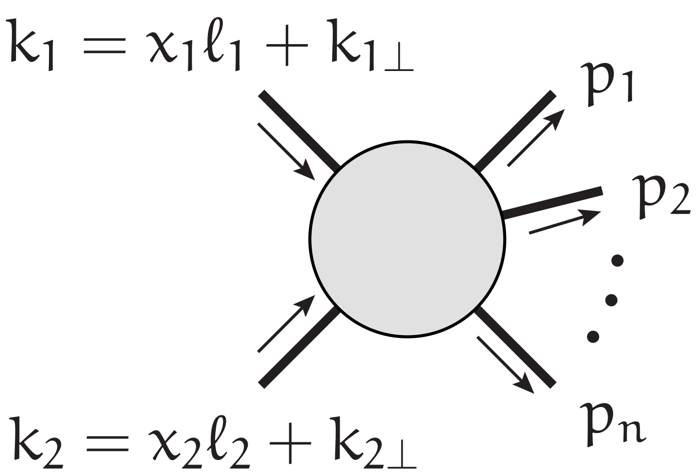
the amplitude for $g^*g^* \to Q\bar{Q}$ is gauge invariant.

Can this be generalized to arbitrary processes?

Matrix elements

<u>The issue:</u>

High-energy factorization requires matrix elements for parton-level scattering process with off-shell initial states



where ℓ_1, ℓ_2 are light-like momenta associated with the scattering hadrons, and $k_{1\perp}, k_{2\perp}$ are perpendicular to both ℓ_1 and ℓ_2.

Matrix elements, squared and summed over final-state spins, may be calculated using spin amplitudes.

Amplitudes must be gauge invariant

- must be calculable in any gauge
- must satisfy Ward identities.
- must preferably be practical.

We cannot just take a prescription to calculate on-shell matrix elements and keep initial-state momenta off-shell, because we won't have gauge invariance. Using projectors

won't be enough.

Lipatov's effective action

Lipatov 1995, Antonov, Lipatov, Kuraev,Cherednikov 2005

Effective action in terms of quarks $\psi, \bar{\psi}$ gluons v_μ and reggeized gluons $A_\pm$.

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{ind}}$$

$$\mathcal{L}_{\mathrm{QCD}} = i\bar{\psi}\not{D}\psi + \frac{1}{2}\mathrm{Tr}\,G_{\mu\nu}^2 \qquad D_\mu = \partial_\mu + g v_\mu \qquad G_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu]$$

$$\mathcal{L}_{\mathrm{ind}} = -\mathrm{Tr}\left\{\frac{1}{g}\partial_+\left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^+} v_+(y)\,dy^+\right)\right]\cdot\partial_\sigma^2 A_-(x)\right.$$
$$\left. + \frac{1}{g}\partial_-\left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^-} v_-(y)\,dy^-\right)\right]\cdot\partial_\sigma^2 A_+(x)\right\}$$

$$k_\pm = \frac{1}{E}\,(\ell_\mu^\pm)k^\mu \qquad (\ell^-)^2 = (\ell^+)^2 = 0 \qquad \ell^+\cdot\ell^- = 2E^2$$

Reggeized gluon $\rightarrow$ gluon with momentum $x_\pm\ell^\pm + k_\perp$.

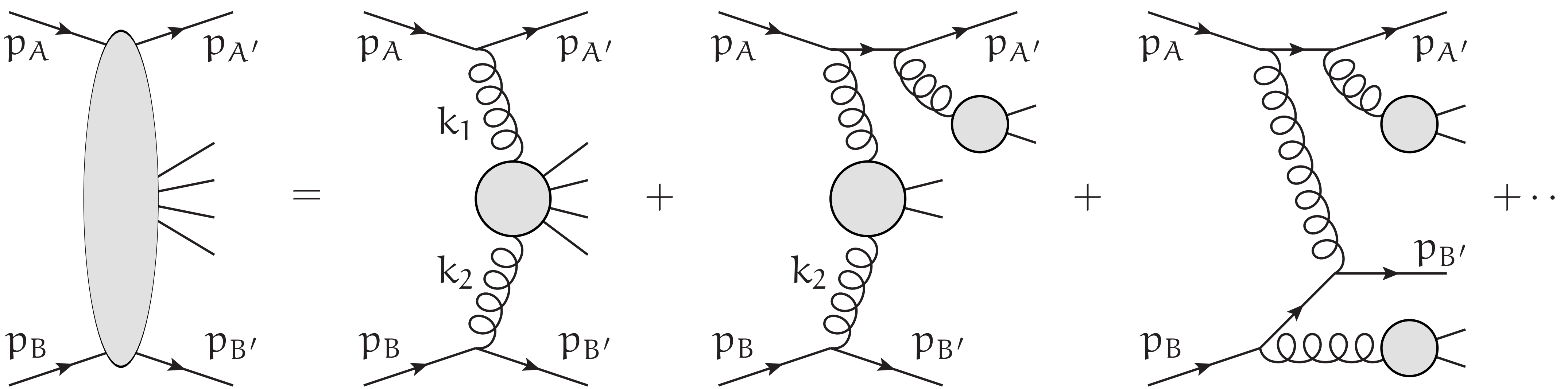
Effective action $\Rightarrow$ vertices of arbitrary order. Can this be avoided?

Embedding

Strategy: embed the off-shell process in an on-shell process. For off-shell gluons:

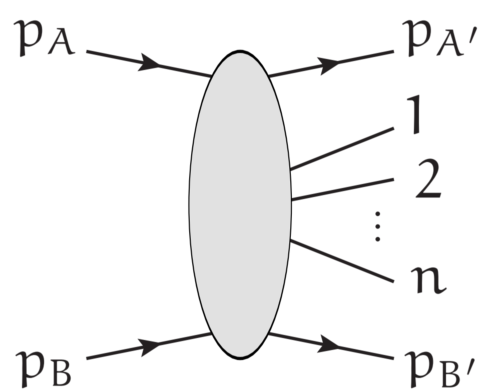
$$g^*(k_1)g^*(k_2) \to X(k_1+k_2 = p_1+p_2+\cdots+p_n)$$

$$q_A(p_A)q_B(p_B) \to q_A(p_{A'})q_B(p_{B'})X(k_1+k_2 = p_1+p_2+\cdots+p_n)$$
$$p_A - p_{A'} = k_1 \quad , \quad p_B - p_{B'} = k_2$$



On-shellness of the auxiliary external quarks guarantees gauge invariance.

Kinematics



High-energy factorization dictates that

$$p_A - p_{A'} = x_1 \ell_1 + k_{1\perp} \quad , \quad p_B - p_{B'} = x_2 \ell_2 + k_{2\perp}$$

Now there is a freedom in the choice of the momenta $p_A, p_{A'}, p_B, p_{B'}$.
High-energy factorization suggests that

$$p_A \cdot \ell_1 = 0 \quad , \quad p_B \cdot \ell_2 = 0$$

but this, together with on-shellness of $p_A, p_{A'}, p_B, p_{B'}$, is kinematically not possible.

Maybe it is possible in a certain limit...

Complex momenta

$$\ell_3^\mu = \tfrac{1}{2}\langle\ell_2|\gamma^\mu|\ell_1]$$
$$\ell_4^\mu = \tfrac{1}{2}\langle\ell_1|\gamma^\mu|\ell_2]$$

$$\ell_1^2 = \ell_2^2 = 0$$
$$\ell_3^2 = \ell_4^2 = 0$$
$$\ell_{1,2}\cdot\ell_{3,4} = 0$$

$$\ell_1\cdot\ell_2 = -\ell_3\cdot\ell_4$$

$$p_A^\mu = (\Lambda + x_1)\ell_1^\mu - \frac{\ell_4\cdot k_{1\perp}}{\ell_1\cdot\ell_2}\,\ell_3^\mu \qquad p_{A'}^\mu = \Lambda\ell_1^\mu + \frac{\ell_3\cdot k_{1\perp}}{\ell_1\cdot\ell_2}\,\ell_4^\mu$$

$$p_B^\mu = (\Lambda + x_2)\ell_2^\mu - \frac{\ell_3\cdot k_{2\perp}}{\ell_1\cdot\ell_2}\,\ell_4^\mu \qquad p_{B'}^\mu = \Lambda\ell_2^\mu + \frac{\ell_4\cdot k_{2\perp}}{\ell_1\cdot\ell_2}\,\ell_3^\mu$$

Now we have both the high-energy limit and on-shellness:

$$p_A^\mu - p_{A'}^\mu = x_1\ell_1^\mu + k_{1\perp}^\mu \qquad p_B^\mu - p_{B'}^\mu = x_2\ell_2^\mu + k_{2\perp}^\mu$$

$$p_A^2 = p_{A'}^2 = p_B^2 = p_{B'}^2 = 0$$

for any value of the dimensionless parameter Λ.

External spinors:

$$|p_A] = \frac{\Lambda + x_1 + \kappa_{13}}{\sqrt{|\Lambda + x_1 + \kappa_{13}|}}\,|\ell_1] \quad , \quad \langle p_{A'}| = \sqrt{|\Lambda - \kappa_{14}|}\,\langle\ell_1|$$

$$|p_B] = \frac{\Lambda + x_2 + \kappa_{24}}{\sqrt{|\Lambda + x_2 + \kappa_{24}|}}\,|\ell_2] \quad , \quad \langle p_{B'}| = \sqrt{|\Lambda - \kappa_{23}|}\,\langle\ell_2|$$

Prescription for off–shell gluons

Take the limit $\Lambda \to \infty$ to remove imaginary momentum components.

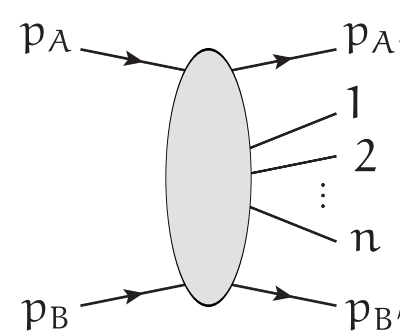
- Auxiliary quark spinors give an overall factor Λ^2
- Further only auxiliary quark line propagators are affected: $\dfrac{i\not{p}}{p^2} \xrightarrow{\Lambda\to\infty} \dfrac{i\not{\ell}_1}{2\,\ell_1\cdot p}$

<u>Prescription to calculate scattering amplitudes with off-shell gluons:</u>

1. Consider the embedding $q_A q_B \to q_A q_B X$ with momentum flow as if the momenta p_A, p_B of the initial-state quarks and $p_{A'}, p_{B'}$ of the final-state quarks are given by

$$p_A^\mu = k_1^\mu \quad , \quad p_B^\mu = k_2^\mu \quad , \quad p_{A'}^\mu = p_{B'}^\mu = 0 \,.$$

2. Assign the spinors $|\ell_1]$, $\langle\ell_1|$ the external A-quarks, and assign $i\not{\ell}_1/(2\ell_1\cdot p)$ instead of $i\not{p}/p^2$ to the propagators on the A-quark line.
3. Do the same with the B quark line, using ℓ_2 instead of ℓ_1.
4. Multiply the amplitude with $g_s^{-1}x_1\sqrt{-k_{1\perp}^2/2} \times g_s^{-1}x_2\sqrt{-k_{2\perp}^2/2}$.
5. For the rest, normal Feynman rules apply.



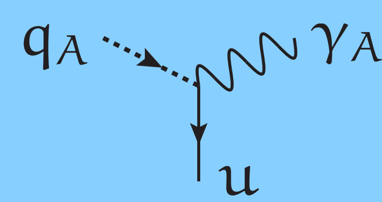
In agreement with Lipatov's effective action.

Off–shell initial–state quarks

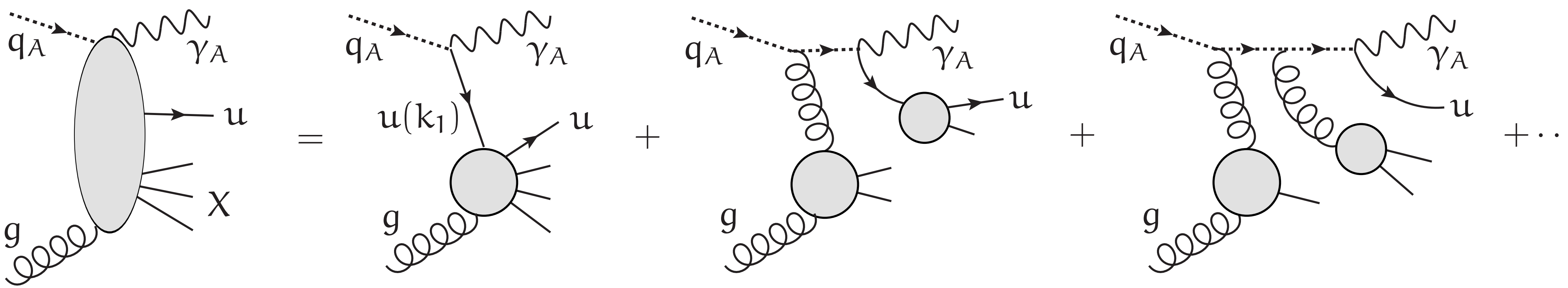
Existing work: Lipatov, Vyazovsky, 2001
Ermolaev, Greco, Troyan 2011 Nefedov, Saleev, Shipilova 2013

Our approach: embed the proces

$$u^* g \to u\, X \quad \Longrightarrow \quad q_A\, g \to \gamma_A u\, X$$

auxiliary photon: 

$= \gamma^\mu$



Use same momentum decomposition for $p_A, p_{A'}$ as for off-shell gluon.
What polarization vector for auxiliary photon?

$$\text{negative helicity } u\text{-quark} \;\to\; \langle p_u| \,, \quad q_A \;\leftarrow\; |\ell_1] \,, \quad \varepsilon^\mu_{A'} = \frac{\langle \ell_1|\gamma^\mu|\ell_2]}{\sqrt{2}\,[\ell_1|\ell_2]}$$

Prescription for off–shell quarks

1. Consider the embedding of the process, in which the off-shell u-quark is replaced by an auxiliary quark q_A, and an auxiliary photon γ_A is added in final state.

2. The momentum flow is as if q_A carries momentum k_1 and the momentum of γ_A is identical to 0.

3. γ_A only interacts via $q_A\gamma_A u$-vertex, and q_A further only interacts with gluons via normal quark-gluon vertices.

4. q_A-line propagators are interpreted as $i\not{\ell}_1/(2\ell_1{\cdot}p)$, and are diagonal in color space.

5. Sum the squared amplitude over helicities of the auxiliary photon. For one helicity, simultaneously assign to the external q_A-quark and to γ_A the spinor and polarization vector

$$|\ell_1] \quad , \quad \frac{\langle\ell_1|\gamma^\mu|\ell_2]}{\sqrt{2}\,[\ell_1|\ell_2]} \quad \text{and for the other helicity assign} \quad |\ell_1\rangle \quad , \quad \frac{\langle\ell_2|\gamma^\mu|\ell_1]}{\sqrt{2}\,\langle\ell_2|\ell_1\rangle}$$

6. Multiply the amplitude with $\sqrt{-x_1 k_1^2/2}$.

In agreement with Lipatov-Vyazovsky.

Analytic result for g* g –> g g

$$0 \to g(p_1 + k_T)\, g(p_2)\, g(p_3)\, g(p_4)$$

$$\mathcal{M}^{a_1 a_2 a_3 a_4}(1,2,3,4) = \frac{4g_s^2}{\sqrt{2}} \sum_{\text{non-cyclic}} \mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4})\,\mathcal{A}(1,2,3,4)$$

$$\mathcal{A}(2^-,3^-,4^-) = 0 \qquad\qquad \mathcal{A}(2^+,3^+,4^+) = 0$$

$$\mathcal{A}(2^-,3^-,4^+) = \frac{[3|\not{k}_T|1\rangle}{|k_T|[31]}\,\frac{[41]^4}{[12][23][34][41]} \qquad \mathcal{A}(2^+,3^+,4^-) = \frac{\langle 1|\not{k}_T|3]}{|k_T|\langle 13\rangle}\,\frac{\langle 41\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

$$\mathcal{A}(2^+,3^-,4^-) = \frac{[3|\not{k}_T|1\rangle}{|k_T|[31]}\,\frac{[12]^4}{[12][23][34][41]} \qquad \mathcal{A}(2^-,3^+,4^+) = \frac{\langle 1|\not{k}_T|3]}{|k_T|\langle 13\rangle}\,\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

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$$\left|\frac{[i|\not{k}_T|1\rangle}{|k_T|[i1]}\right| = \left|\frac{\langle 1|\not{k}_T|i]}{|k_T|\langle 1i\rangle}\right| = 1$$

Analytic result for u* g –> u g

$$0 \to g(p_1)\, g(p_2)\, u(p_u)\, \bar{u}^*(p_{\bar{u}} + k_T)$$

$$\mathcal{M}^{a_1,a_2}_{j_u,j_{\bar{u}}}(1,2,u,\bar{u}) = 2ig_S^2\Big[\big(T^{a_1}T^{a_2}\big)_{j_u,j_{\bar{u}}}\mathcal{A}(1,2,u,\bar{u}) + \big(T^{a_2}T^{a_1}\big)_{j_u,j_{\bar{u}}}\mathcal{A}(2,1,u,\bar{u})\Big]$$

$$\mathcal{A}(1^+,2^-,u^+,\bar{u}^+) = -\frac{[\bar{u}|\not{k}_T|1\rangle}{|k_T|\langle\bar{u}1\rangle}\,\frac{\langle\bar{u}1\rangle^3\langle u1\rangle}{\langle u1\rangle\langle 12\rangle\langle 2\bar{u}\rangle\langle\bar{u}u\rangle}$$

$$\mathcal{A}(1^-,2^+,u^+,\bar{u}^+) = -\frac{[\bar{u}|\not{k}_T|2\rangle}{|k_T|\langle\bar{u}2\rangle}\,\frac{\langle\bar{u}2\rangle^3\langle u2\rangle}{\langle u1\rangle\langle 12\rangle\langle 2\bar{u}\rangle\langle\bar{u}u\rangle}$$

$$\mathcal{A}(1^+,2^-,u^-,\bar{u}^-) = \frac{\langle\bar{u}|\not{k}_T|1]}{|k_T|[\bar{u}1]}\,\frac{[\bar{u}1]^3[u1]}{[u1][12][2\bar{u}][\bar{u}u]}$$

$$\mathcal{A}(1^-,2^+,u^-,\bar{u}^-) = \frac{\langle\bar{u}|\not{k}_T|2]}{|k_T|[\bar{u}2]}\,\frac{[\bar{u}2]^3[u2]}{[u1][12][2\bar{u}][\bar{u}u]}$$

$$\mathcal{A}(1^+,2^+,u^-,\bar{u}^-) = -|k_T|\,\frac{\langle\bar{u}u\rangle^3}{\langle u1\rangle\langle 12\rangle\langle 2\bar{u}\rangle\langle\bar{u}u\rangle}$$

$$\mathcal{A}(1^-,2^-,u^+,\bar{u}^+) = |k_T|\,\frac{[\bar{u}u]^3}{[u1][12][2\bar{u}][\bar{u}u]}$$

Forward 3–jet production

AvH, Kutak, Kotko 2013

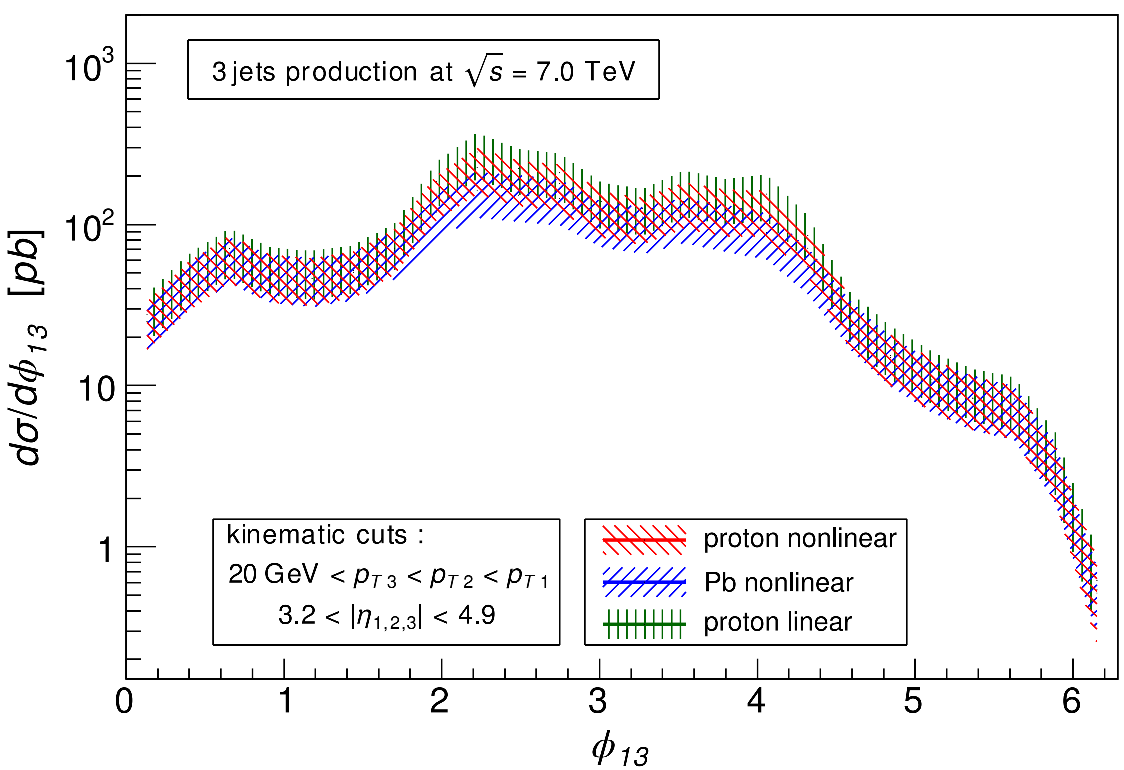
$$d\sigma_{AB\to 3j} = \sum_b \int d^2k_{A\perp} \int \frac{dx_A}{x_A} \mathcal{F}(x_A, k_{A\perp}) \int dx_B\, f_{b/B}(x_B)\, d\sigma_{g^*b\to 3j}(x_A, k_{A\perp}, x_B)$$

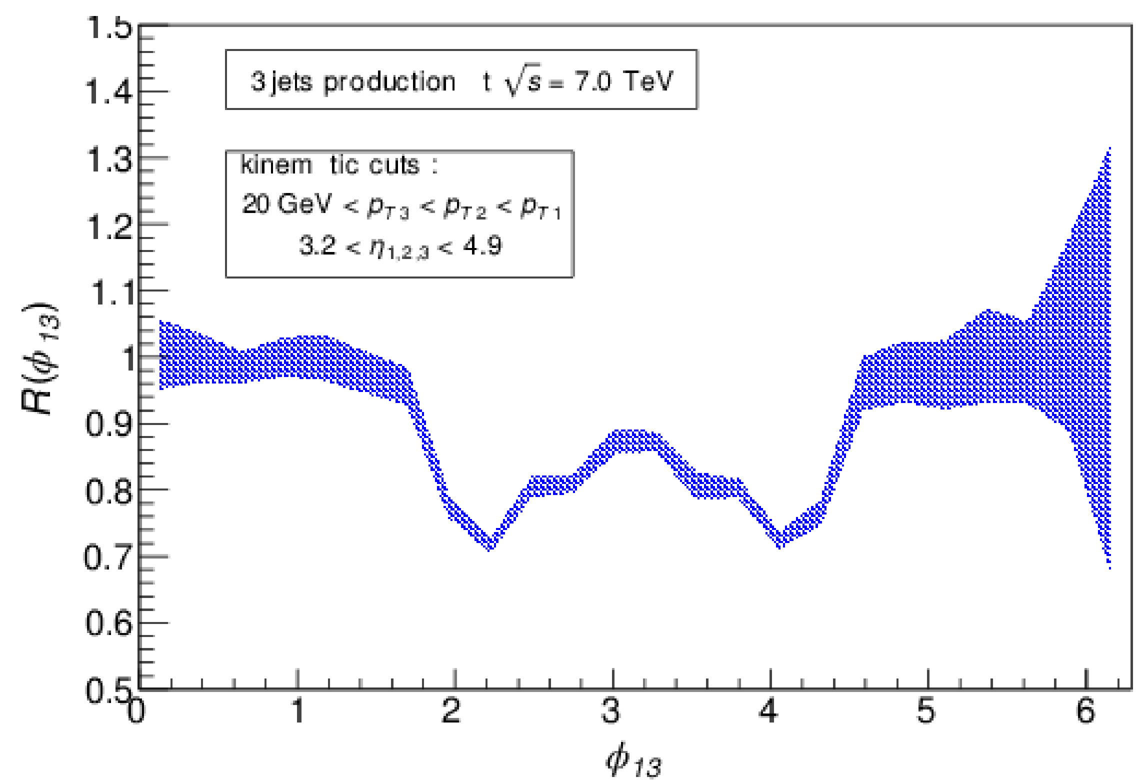
Unintegrated pdfs $\mathcal{F}$ from Kutak, Sapeta 2012:

- (linear) BFKL
- non-linear unified BK/DGLAP for proton
- non-linear unified BK/DGLAP for Pb

angular decorrelation:
$\phi_{13} = |\phi_{\text{hardest}} - \phi_{\text{softest}}|$

nuclear modification factor:
$$\left(\frac{d\sigma}{d\phi_{13}}\right)_{\text{Pb}} \left(\frac{d\sigma}{d\phi_{13}}\right)^{-1}_{\text{proton}}$$





Summary

- We presented a prescription to evaluate tree-level scattering amplitudes with off-shell initial-state partons, that can be readily applied in automatic calculations for arbitrary final states.

- Has been implemented into

  - a `C++` program `LxJet` (Kotko) for $g^* x_b \to x_1 x_2 x_3$ with x_i arbitrary partons. Uses the fact that almost all gauge contributions vanish in a suitable axial gauge.
  - a `Fortran` program (AvH) for arbitrary processes.

  to be public soon.

- Has been applied to a study of forward jets. More studies to follow.