Recent progress with GENEVA

or

casting perturbative calculation as event generators

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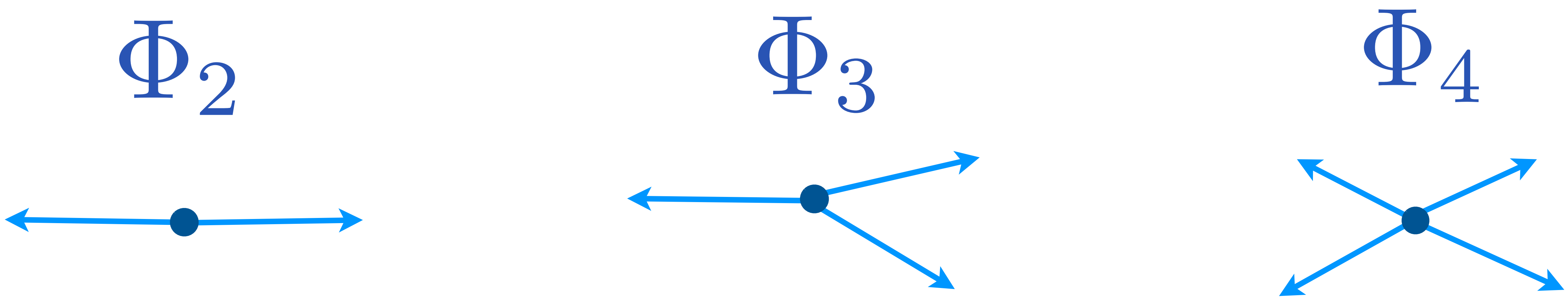
There are two very different ways of making theoretical predictions: perturbative* calculations and event generators

* Can mean either fixed order or resummed

Perturbative calculations	Event generators
Can typically be performed with higher accuracy	Are fully differential, more similar to experimental data
Typically, observables have to be chosen before running code	Can just generate events, define observables later
Intrinsically, has only information on partonic final states	By attaching hadronization model, provides fully hadronized final state

To cast perturbative calculations as event generators, separate the total hadronic event into different jet multiplicities



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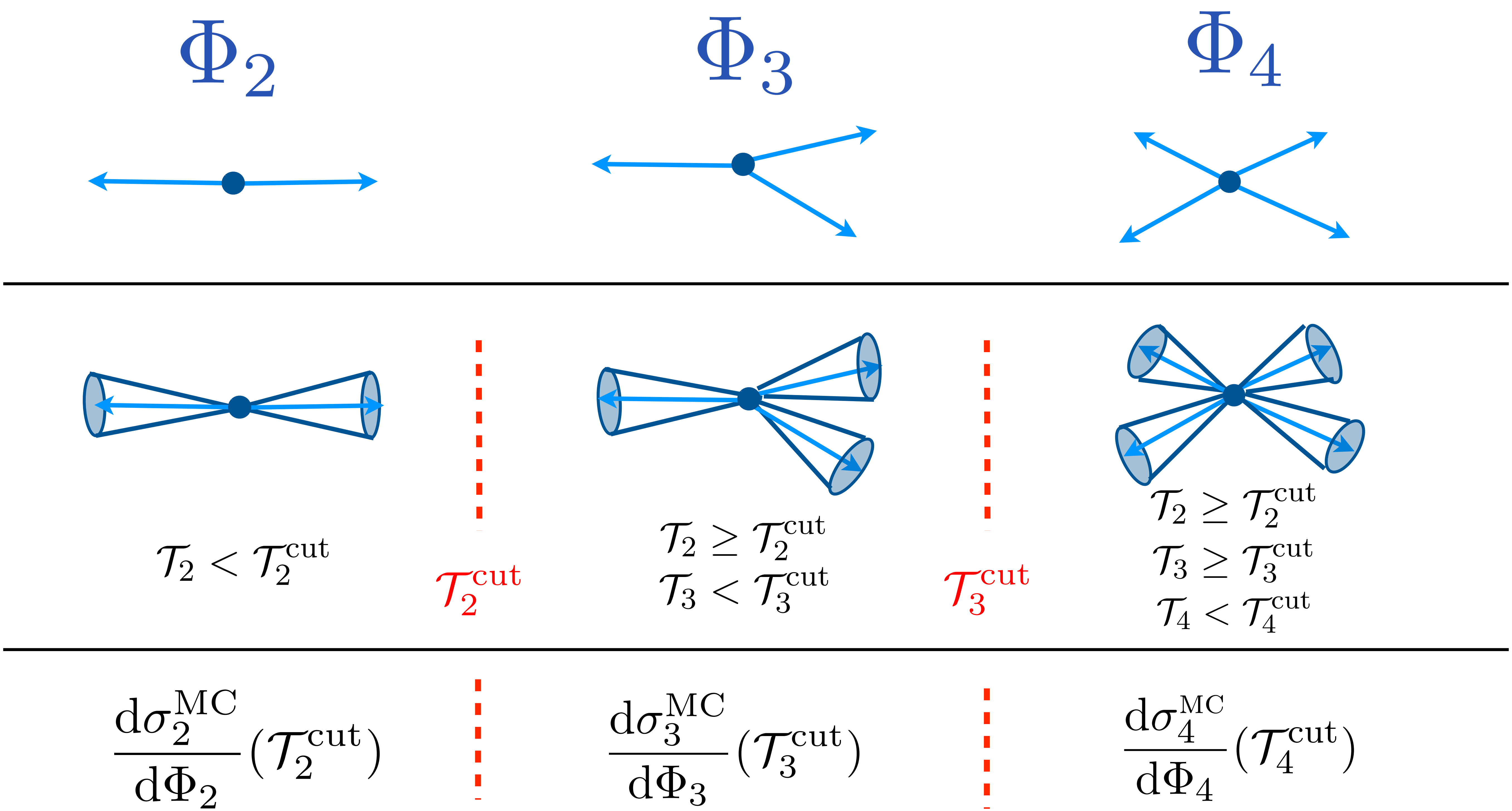
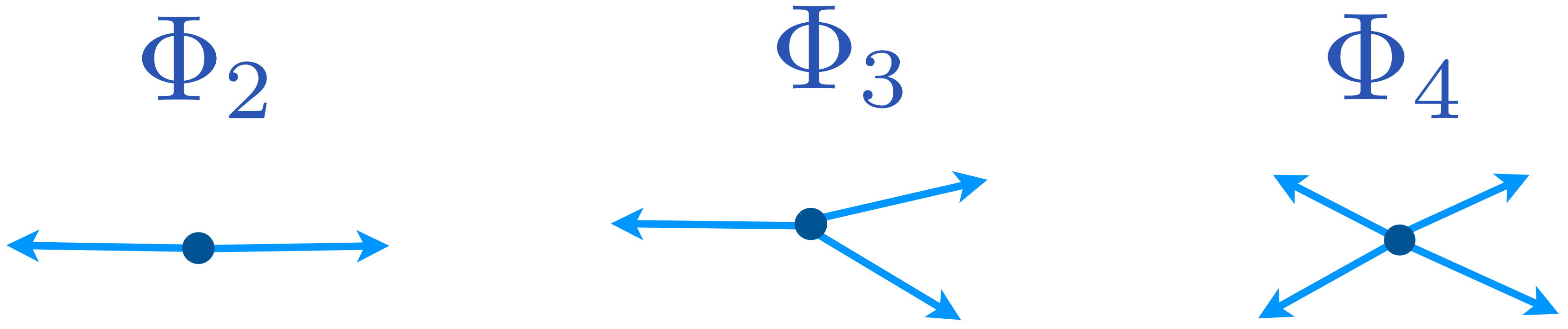


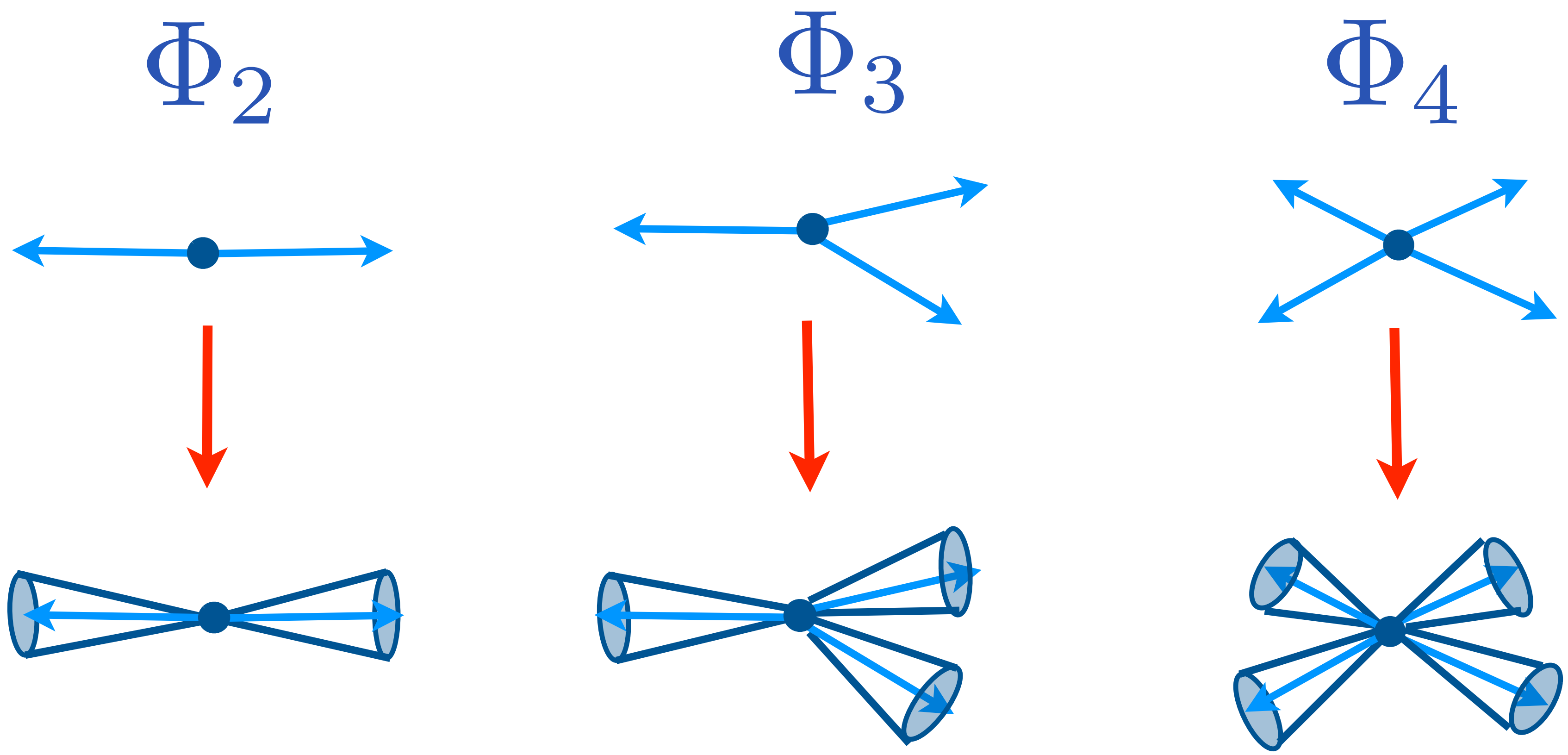
Dealing with these MC cross-sections makes the interface with a Parton shower straightforward

- Create phase space for jet event



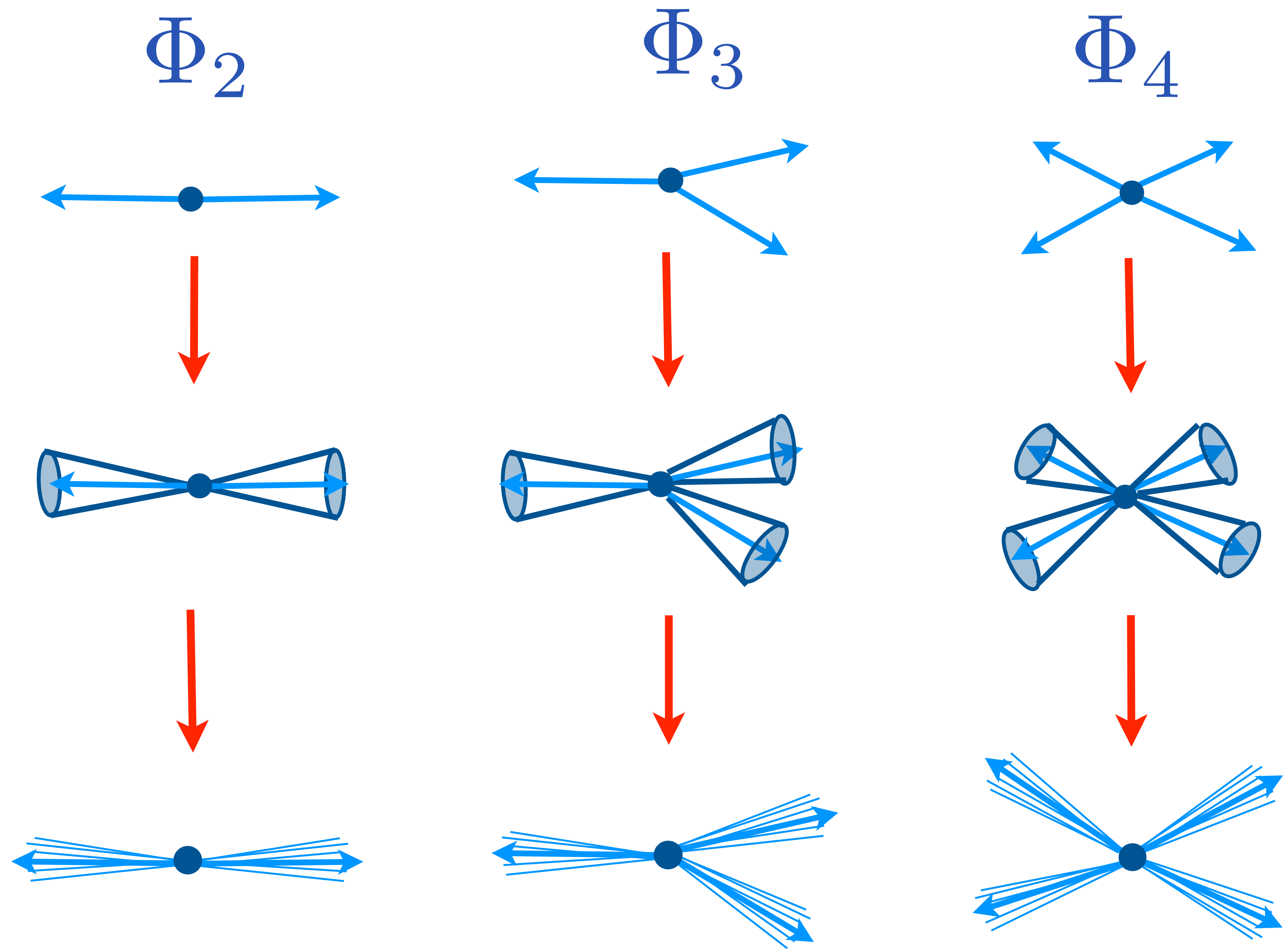
Dealing with these MC cross-sections makes the interface with a Parton shower straightforward

- Create phase space for jet event
- Calculate cross section and assign to partonic event



Dealing with these MC cross-sections makes the interface with a Parton shower straightforward

- Create phase space for jet event
- Calculate cross section and assign to partonic event
- Let parton shower fill jets with radiation



The main question is what expression to use for the differential MC jet cross-section

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}?$$

Before one starts any calculations, a clear idea of the desired accuracy is required

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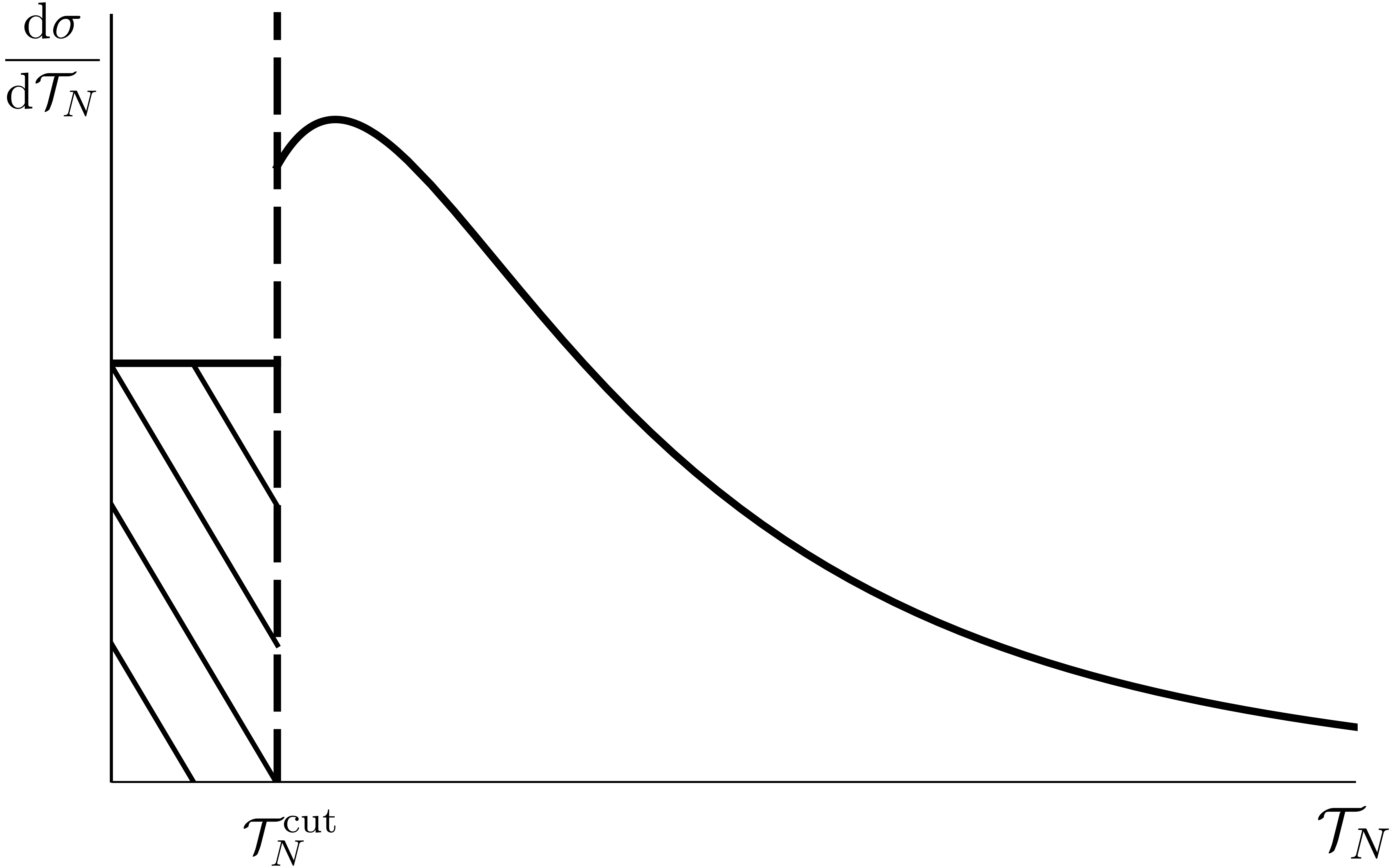

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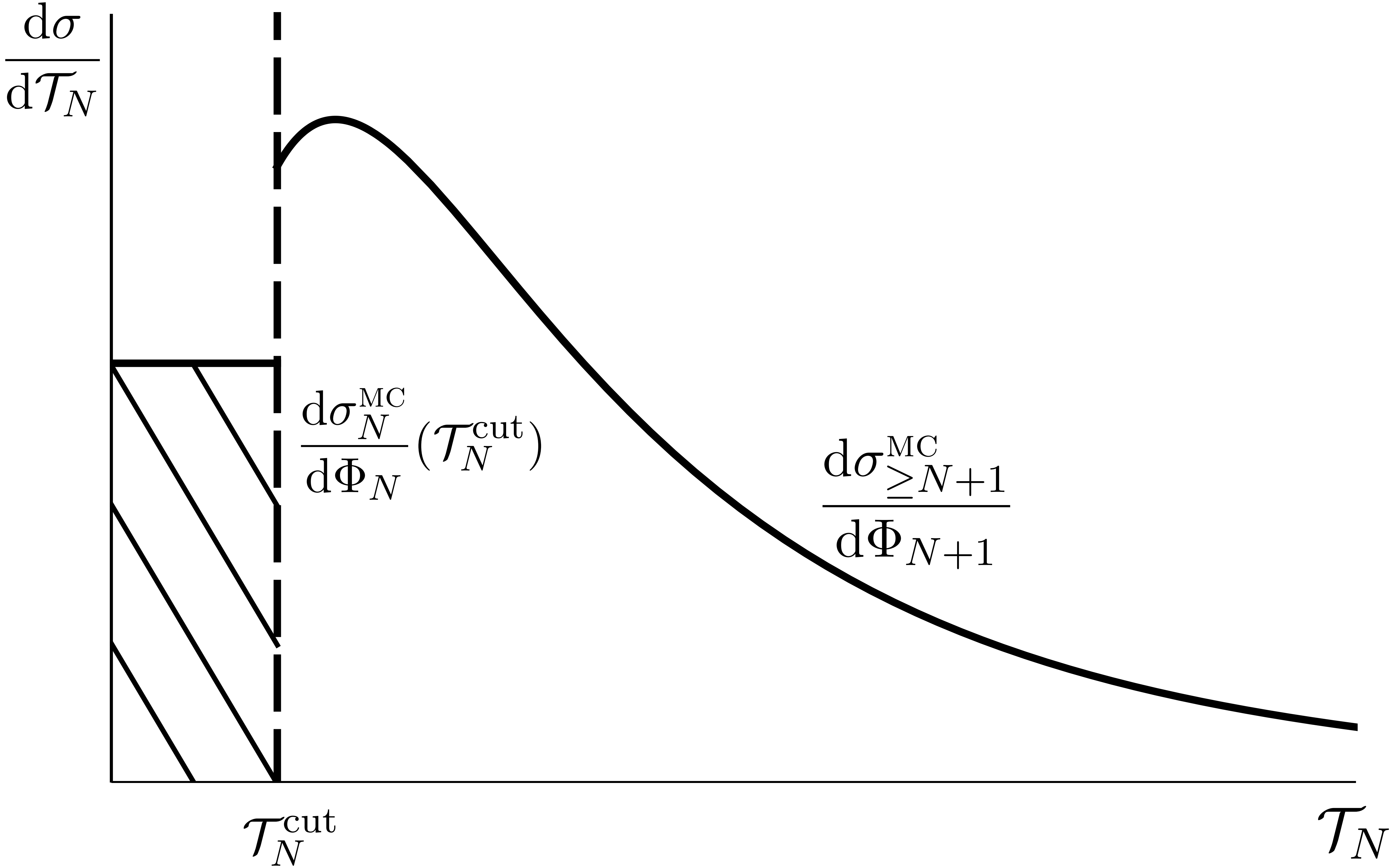

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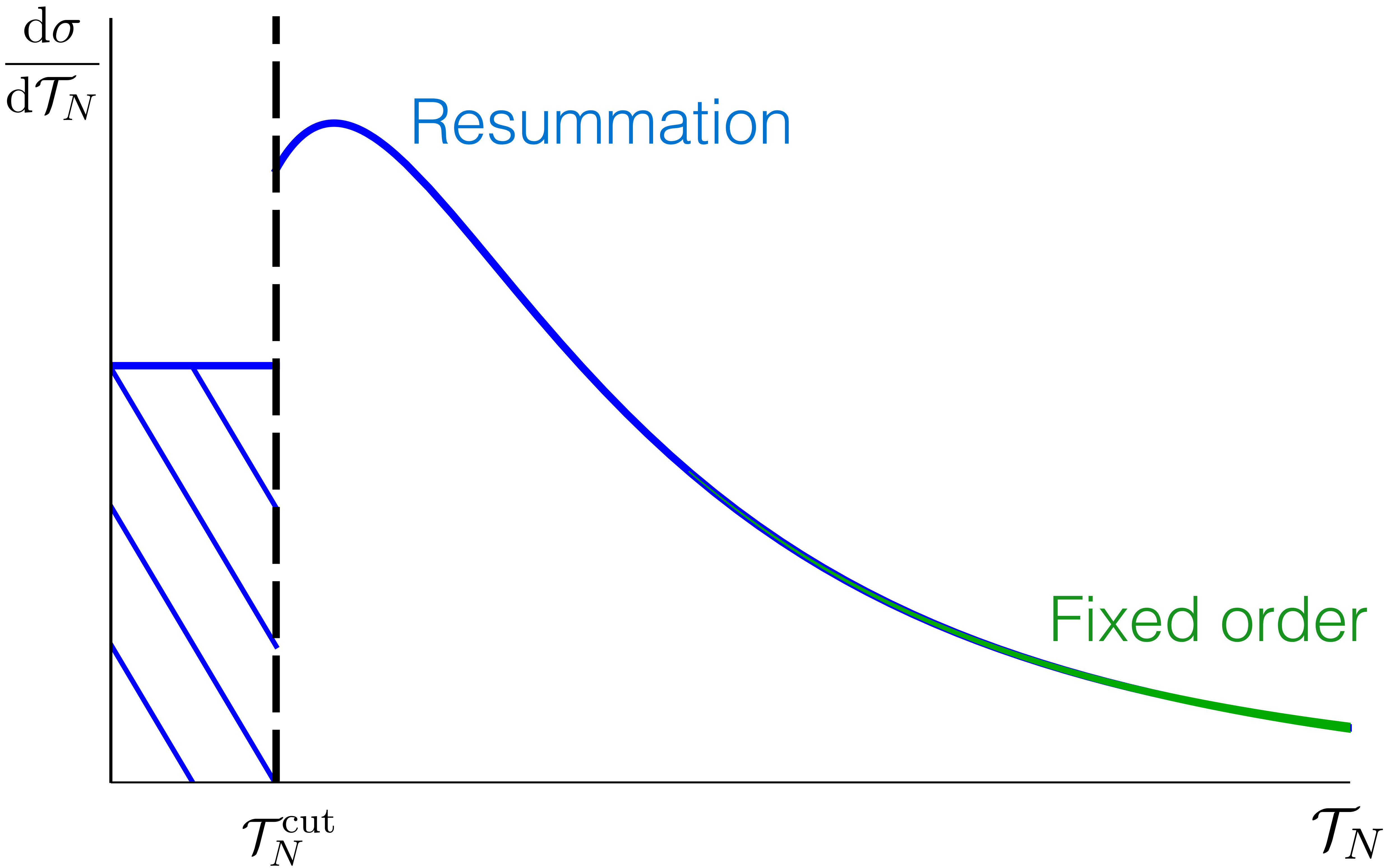

Before one starts any calculations, a clear idea of the desired accuracy is required

Relative accuracy if α_s Log^2 ~ 1 α_s^k	FO Accuracy	Resummation Accuracy
k = 0	LO	LL
k = 1	NLO	NLL'
k = 2	NNLO	NNLL'

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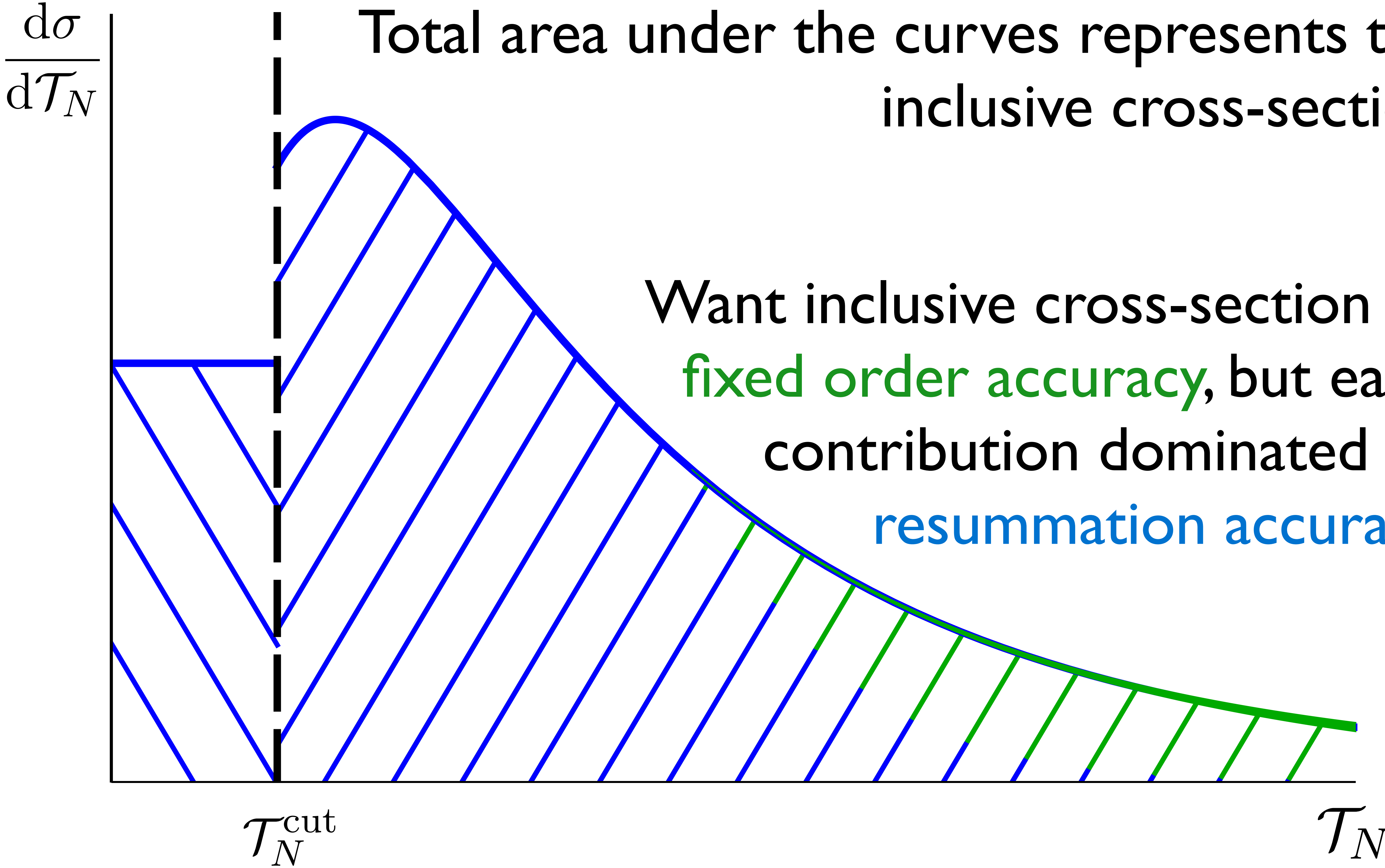
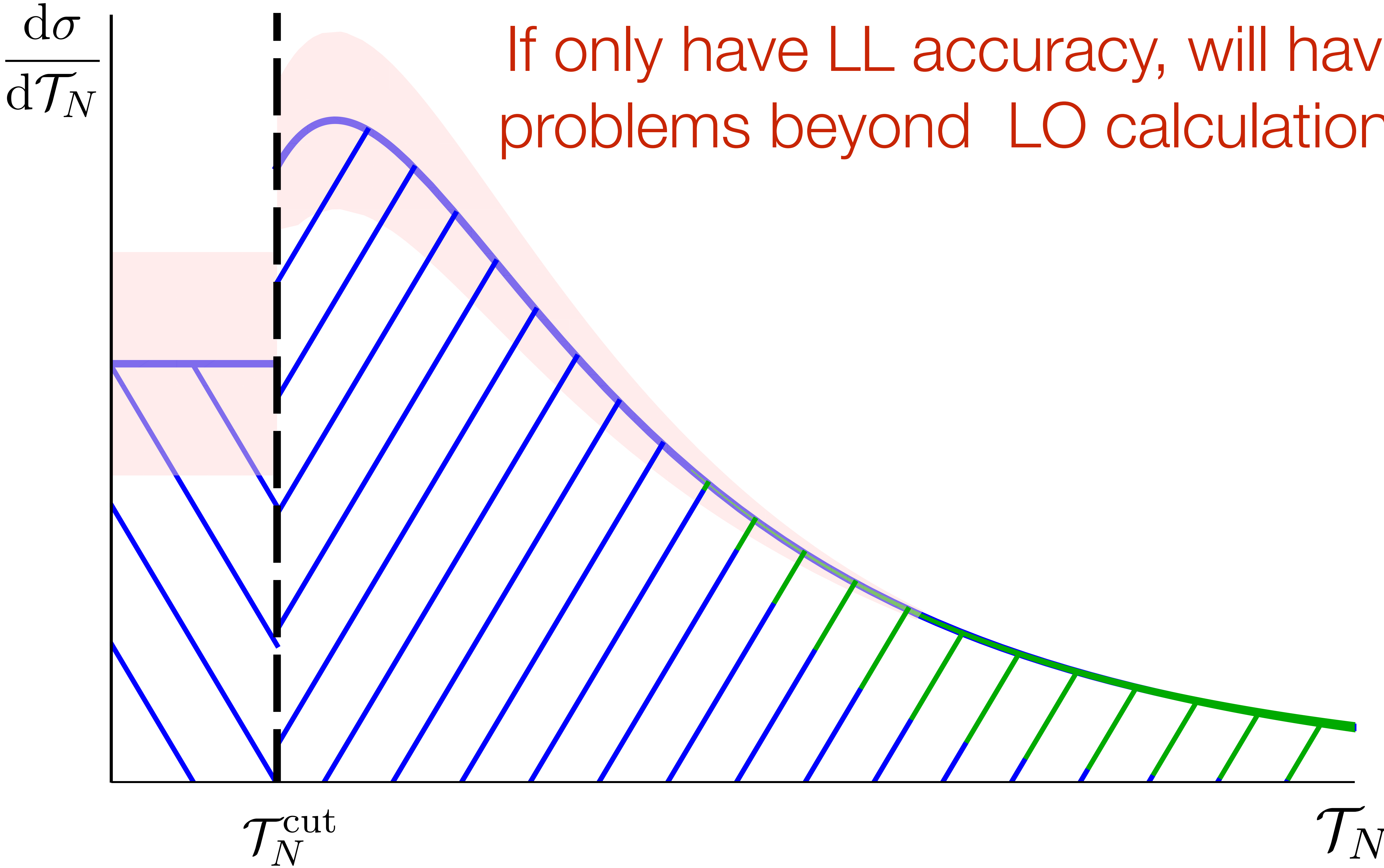


Total area under the curves represents the inclusive cross-section

Want inclusive cross-section to fixed order accuracy, but each contribution dominated by resummation accuracy

Before one starts any calculations, a clear idea of the desired accuracy is required

If only have LL accuracy, will have problems beyond LO calculations



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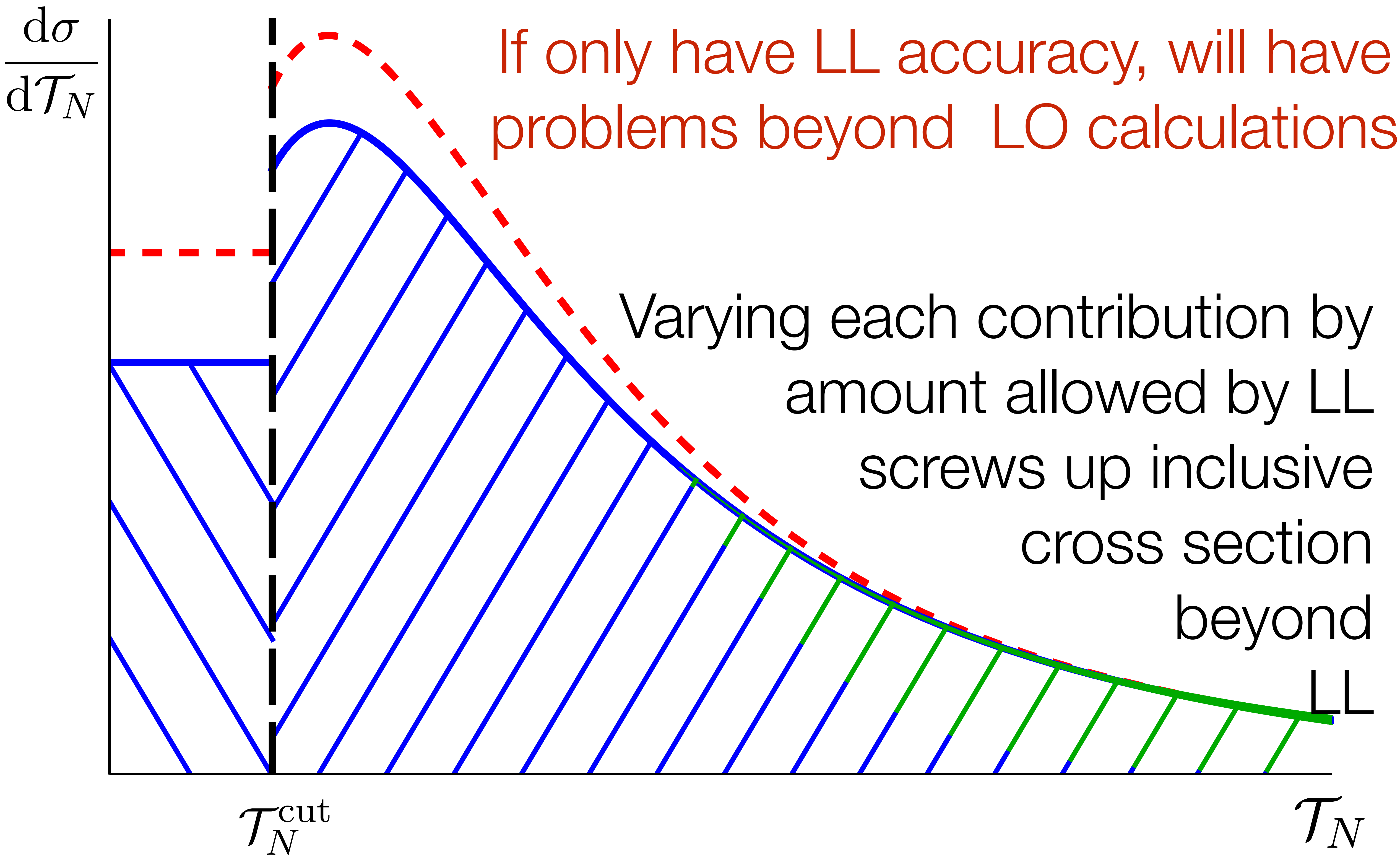

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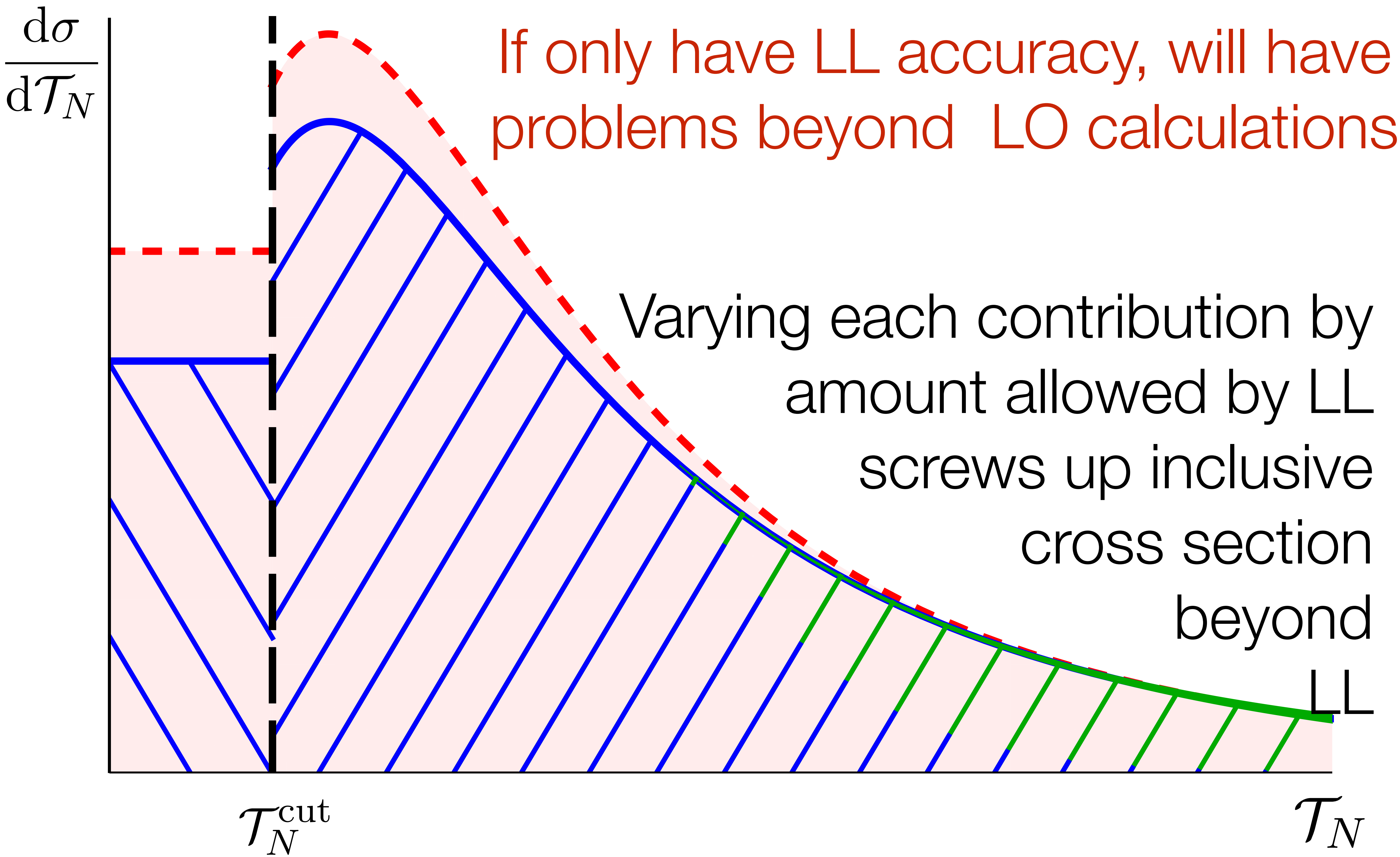

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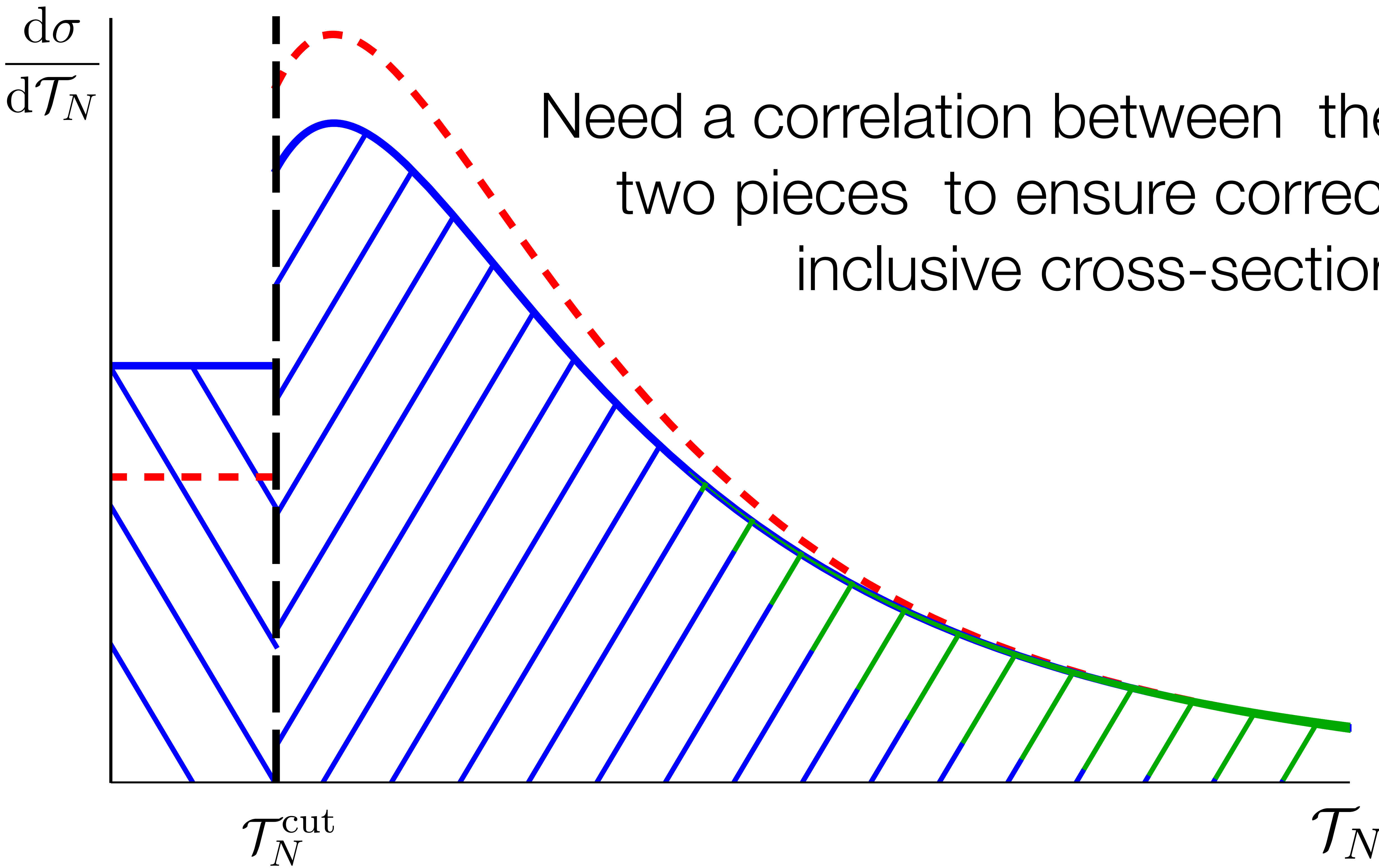

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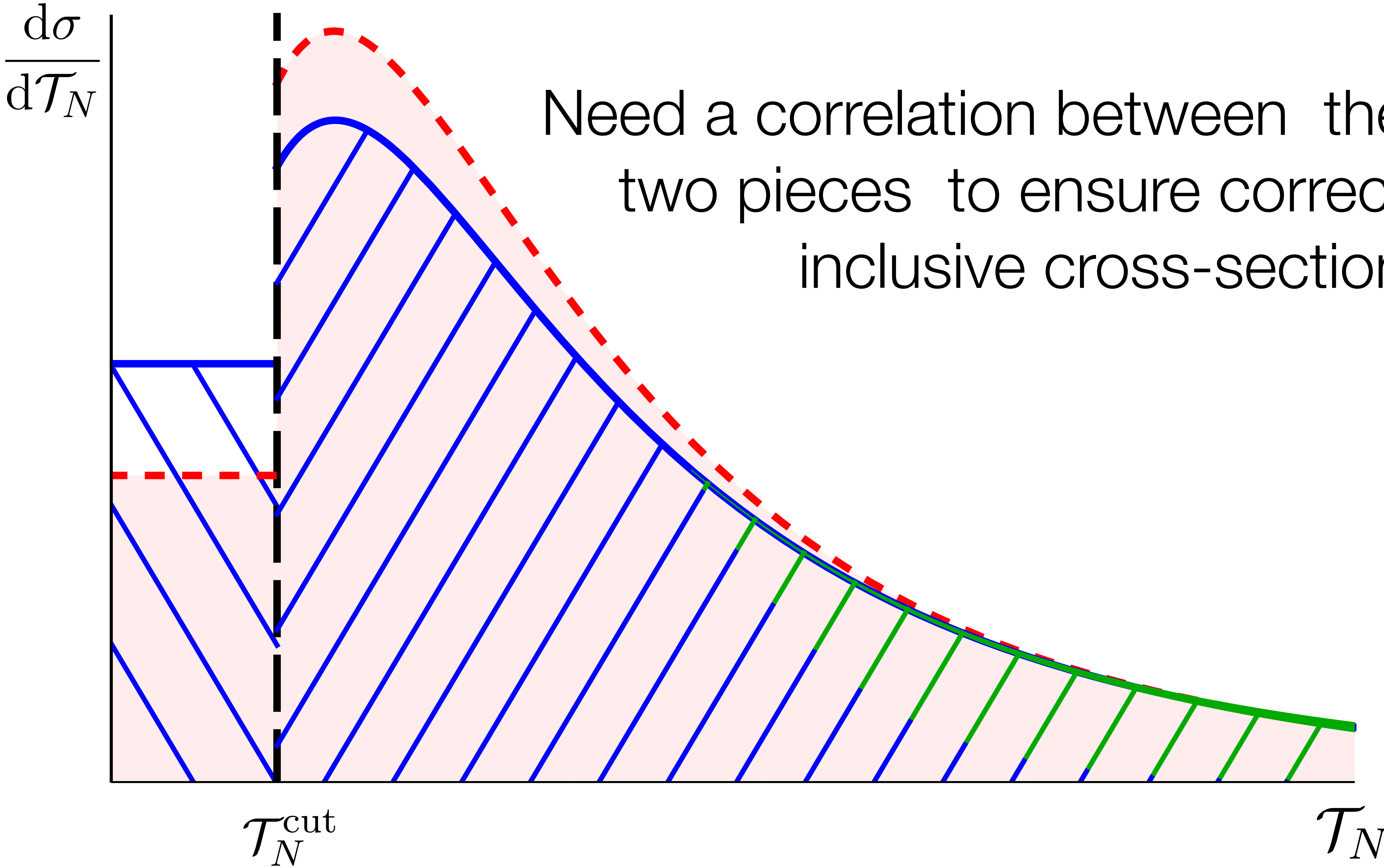


Need a correlation between the two pieces to ensure correct inclusive cross-section

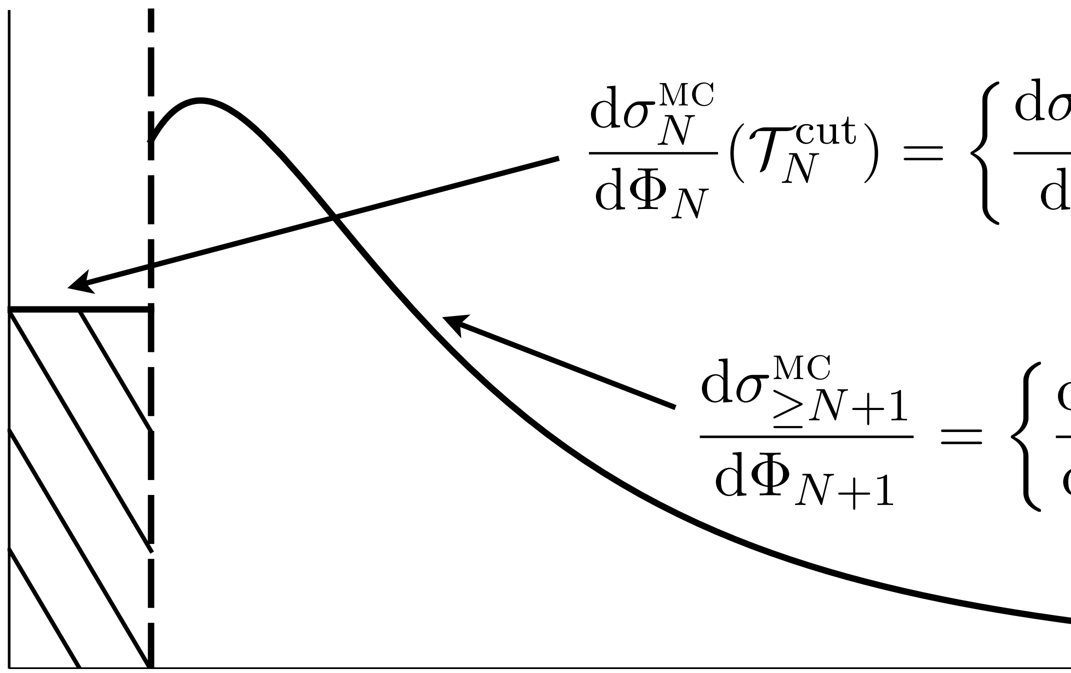
Before one starts any calculations, a clear idea of the desired accuracy is required

To summarize:

- For each $d\sigma_N^{MC}$, need accuracy in at both FO and Res $(N^nLO\ /\ N^kLL)_N$
- Need to ensure that inclusive cross-sections are still correct to N^nLO_N

Careful with second condition, since integrals individually are determined by Res accuracy k

Commonly used generators (Powheg, MC@NLO) are accurate to (NLO / LL)$_N$ and (LO / LL)$_{N+1}$



$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \left\{ \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N} - \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N} \Big[B_{N+1} - S_{N+1} \Big] \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) \right\} \Delta_N(\mathcal{T}_N^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}} = \left\{ \frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N) \frac{S_{N+1}}{B_N} \Delta_N(\mathcal{T}_N) + (B_{N+1} - S_{N+1})(\mathcal{T}_N) \Delta_N(\mathcal{T}_N) \right\}$$

Spectrum is total derivative of cumulant

Inclusive cross-section exactly equal to $\frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N}$

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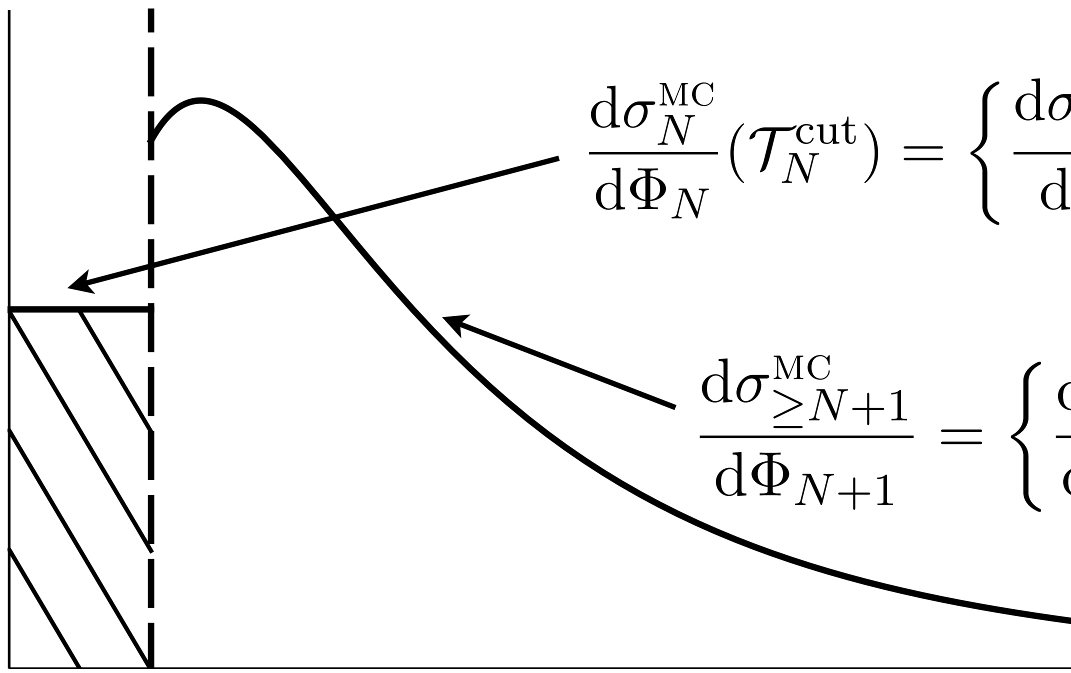

$$\frac{d\sigma_N^{\rm MC}}{d\Phi_N}(\mathcal{T}_N^{\rm cut}) = \left\{ \frac{d\sigma_{\geq N}^{\rm NLO}}{d\Phi_N} - \int \frac{d\Phi_{N+1}}{d\Phi_N}\Big[B_{N+1} - S_{N+1}\Big]\theta(\mathcal{T}_N > \mathcal{T}_N^{\rm cut})\right\}\Delta_N(\mathcal{T}_N^{\rm cut})$$

$$\frac{d\sigma_{\geq N+1}^{\rm MC}}{d\Phi_{N+1}} = \left\{ \frac{d\sigma_N^{\rm MC}}{d\Phi_N}(\mathcal{T}_N)\frac{S_{N+1}}{B_N}\Delta_N(\mathcal{T}_N) + (B_{N+1} - S_{N+1})(\mathcal{T}_N)\big(\Delta_N(\mathcal{T}_N)\big)\right\}$$

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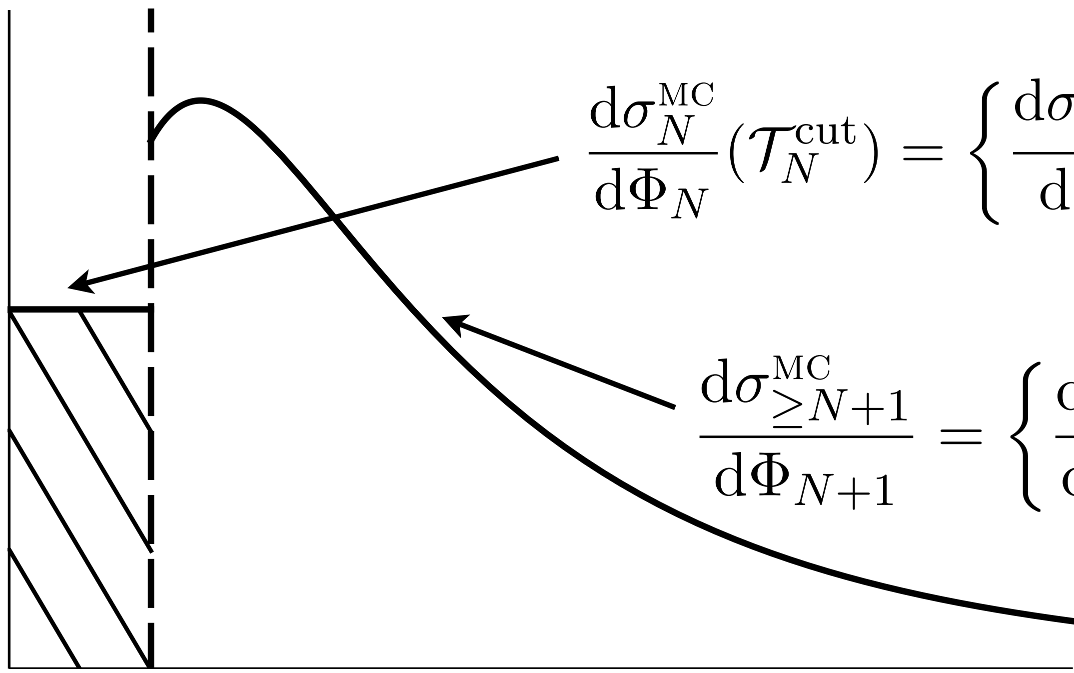

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \left\{ \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N} - \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N} \Big[B_{N+1} - S_{N+1} \Big] \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) \right\} \Delta_N(\mathcal{T}_N^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}} = \left\{ \frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N) \frac{S_{N+1}}{B_N} \Delta_N(\mathcal{T}_N) + (B_{N+1} - S_{N+1})(\mathcal{T}_N)\big(\Delta_N(\mathcal{T}_N)\big) \right\}$$

Spectrum is total derivative of cumulant

Inclusive cross-section exactly equal to $\frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N}$

Powheg chooses
$S_{N+1} = B_{N+1}$

MC@NLO chooses
$S_{N+1} =$ Herwig

Two obvious ways to improve the current situation: More fixed order, or more logarithms

Powheg and MC@NLO have

$N^{1}LO$ / $N^{0.5}LL$

Two obvious ways to go beyond:

Go beyond (N)LL in resummation

Go beyond NLO in fixed order

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GENEVA

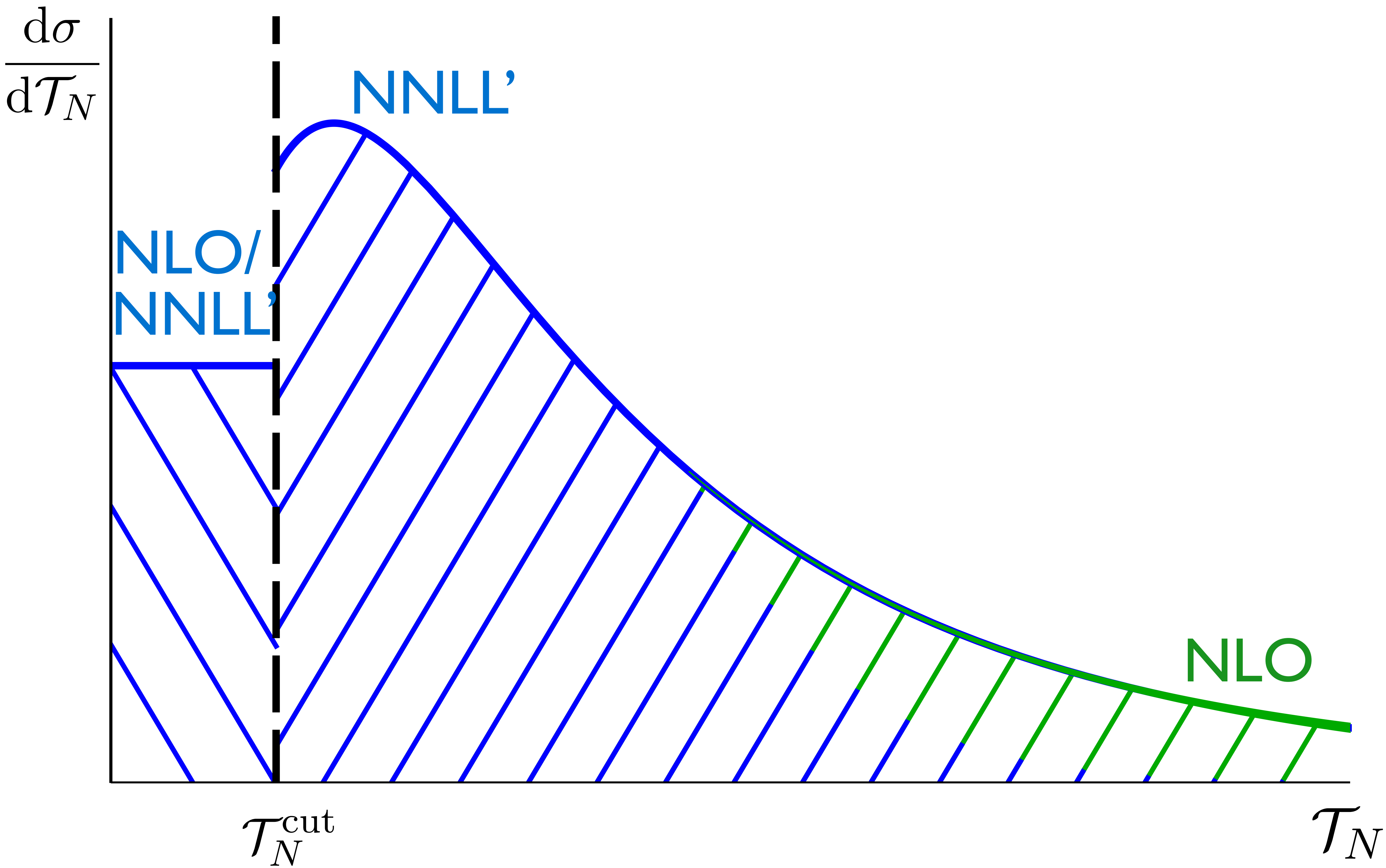
Go beyond NLO in fixed order

First results of GENEVA have accuracy to $(\text{NLO / NNLL'})_N$ $(\text{NLO / NNLL'})_{N+1}$

Main new calculational ingredient is higher logarithmic resummation

Combination of multiple NLO’s almost a side-effect of the resummation (albeit a very important one)

First results of GENEVA have accuracy to (NLO / NNLL')$_N$ (NLO / NNLL')$_{N+1}$



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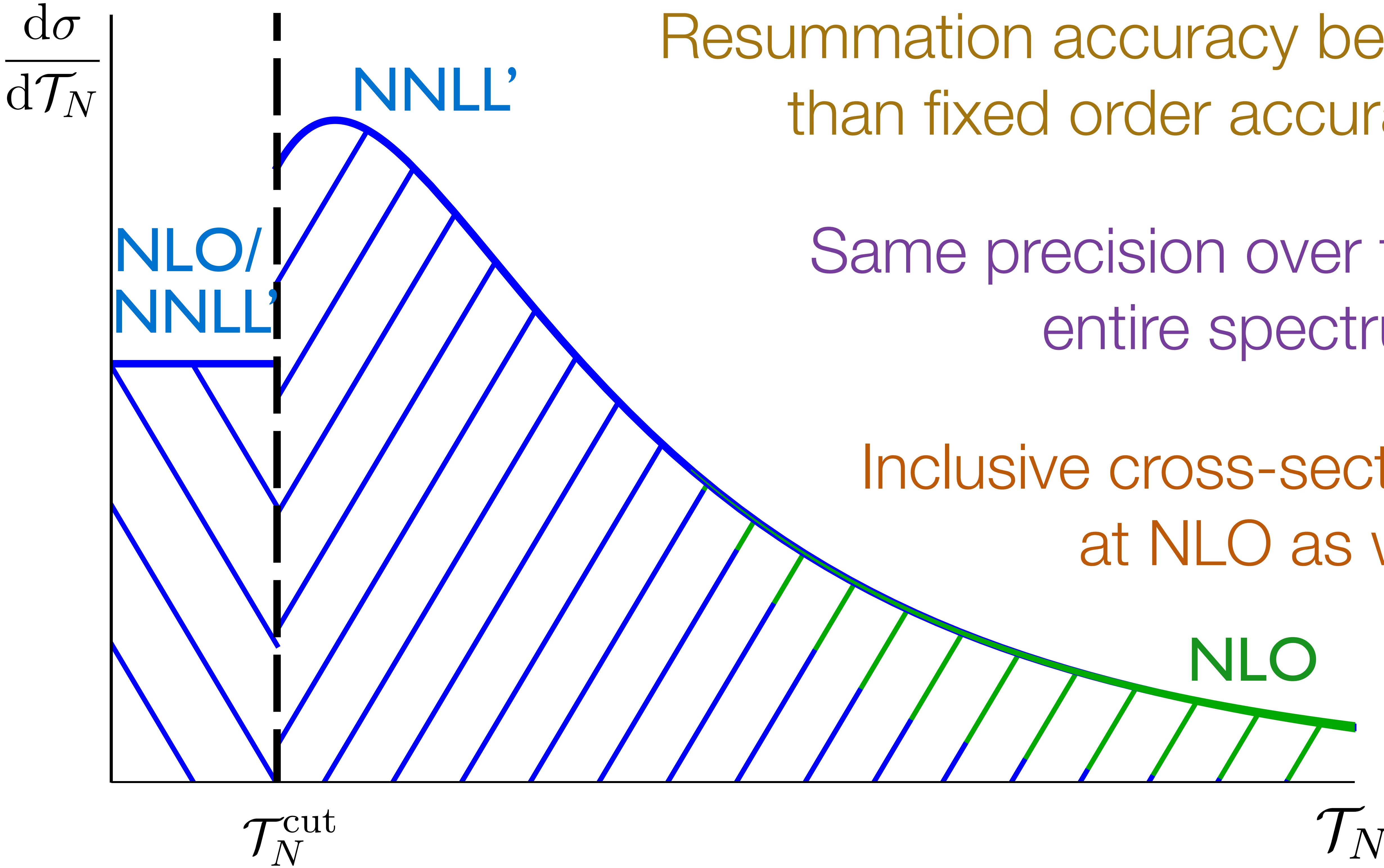

Resummation accuracy better than fixed order accuracy

Same precision over the entire spectrum

Inclusive cross-section at NLO as well

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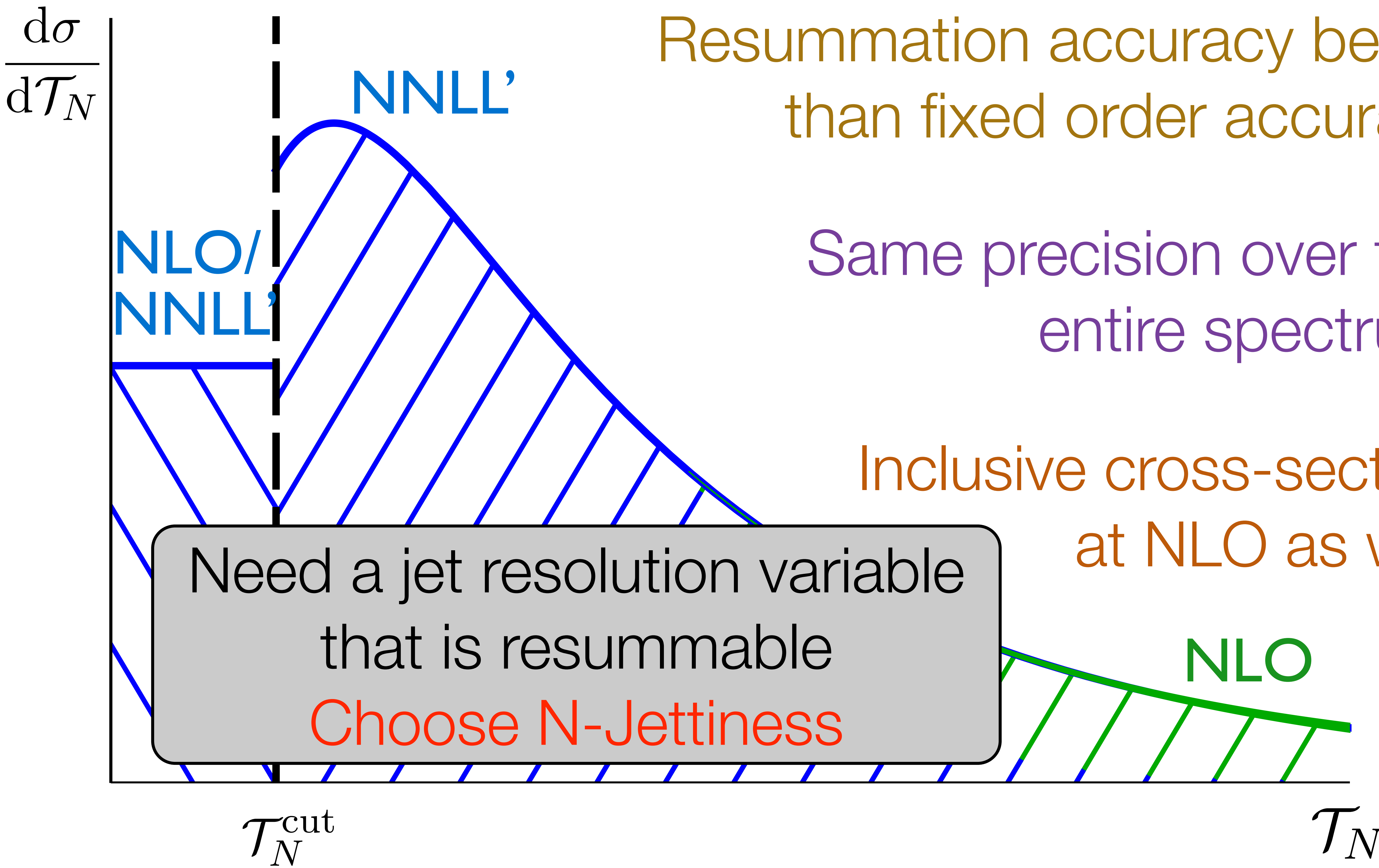


Resummation accuracy better than fixed order accuracy

Same precision over the entire spectrum

Inclusive cross-section at NLO as well

Need a jet resolution variable that is resummable
Choose N-Jettiness

Required expressions can be obtained relatively straightforwardly

Fully differential fixed order result can be obtained using standard techniques

$$\frac{d\sigma_2^{\rm MC}}{d\Phi_2}(\mathcal{T}_2^{\rm cut}) = \left[B_2 + V_2\right](\Phi_2) + \int \frac{d\Phi_3}{d\Phi_2} B_3(\Phi_3)\, \theta\left[\mathcal{T}_2(\Phi_3) \le \mathcal{T}_2^{\rm cut}\right]$$

$$\frac{d\sigma_{\ge 3}^{\rm MC}}{d\Phi_3} = \left[B_3 + V_3\right](\Phi_2) + \int \frac{d\Phi_4}{d\Phi_3} B_4(\Phi_4)$$

Most easily done with FKS subtractions, which we choose in Geneva

Required expressions can be obtained relatively straightforwardly

Fully differential resummed result can not be obtained easily

Expression that can be resummed is the 2-jettiness distribution and cumulant

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq 3}}{\mathrm{d}\Phi_2 \mathrm{d}\mathcal{T}_2} = \int \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2 \mathrm{d}\mathcal{T}_2} \frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq 3}}{\mathrm{d}\Phi_3}$$

Procedure to resum to arbitrary accuracy is known using SCET and results up to (N)NNLL are available

Required expressions can be obtained relatively straightforwardly

Combine fixed and resummed order

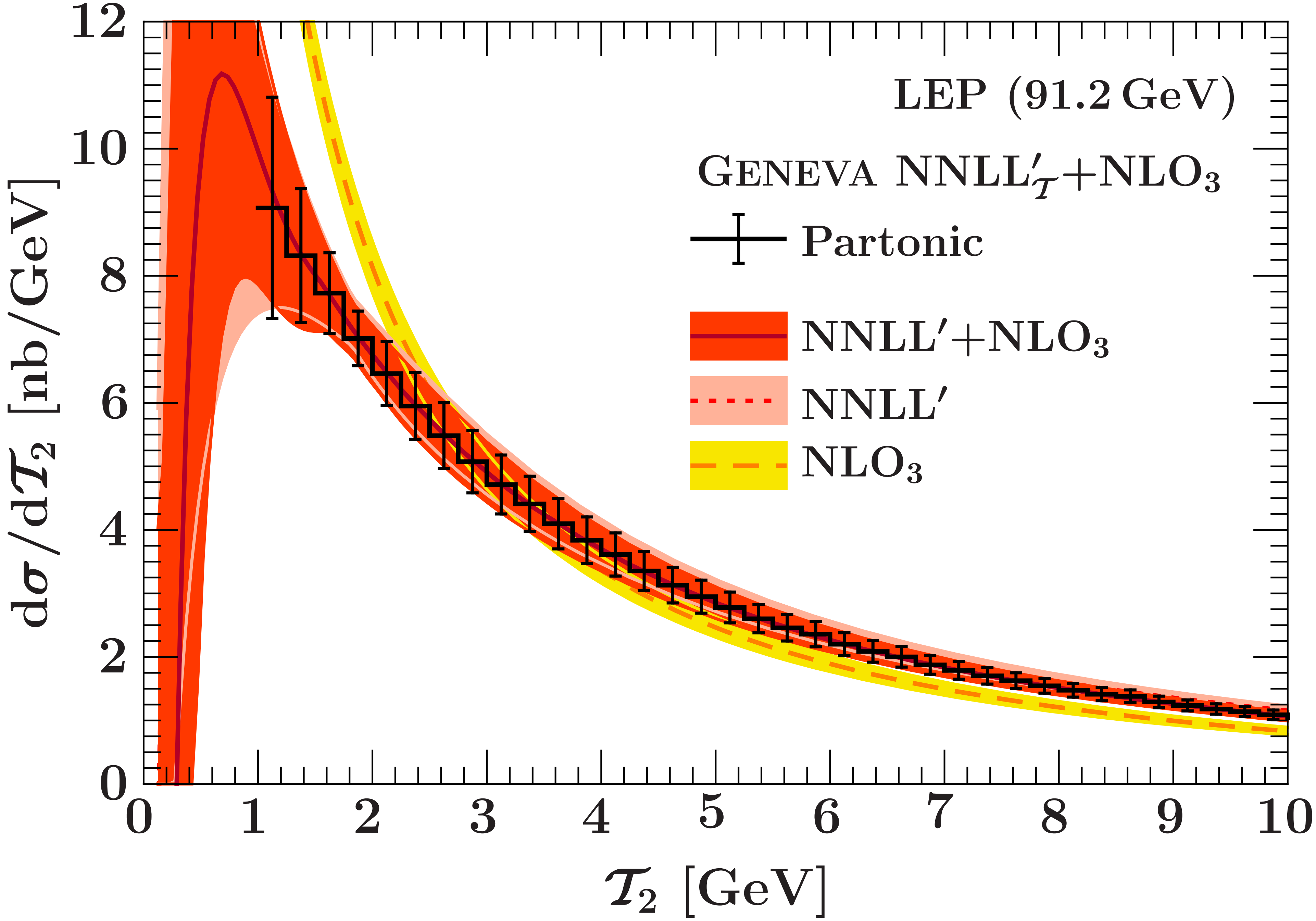
$$\frac{d\sigma_2(\tau^{\mathrm{cut}})}{d\Phi_2} = \left[\frac{d\sigma_2^{\mathrm{R}}(\tau^{\mathrm{cut}})}{d\Phi_2} \Big/ \frac{d\sigma_2^{\mathrm{RE}}(\tau^{\mathrm{cut}})}{d\Phi_2}\right] \frac{d\sigma_2^{\mathrm{F}}(\tau^{\mathrm{cut}})}{d\Phi_2}$$

$$\frac{d\sigma_3}{d\Phi_3} = \left[\frac{d\sigma_3^{\mathrm{R}}}{d\tau d\Phi_2} \Big/ \frac{d\sigma_3^{\mathrm{RE}}}{d\tau d\Phi_2}\right] \frac{d\sigma_3^{\mathrm{F}}}{d\Phi_3}$$

R: Resummed   RE: Expansion of R   F: Fixed order

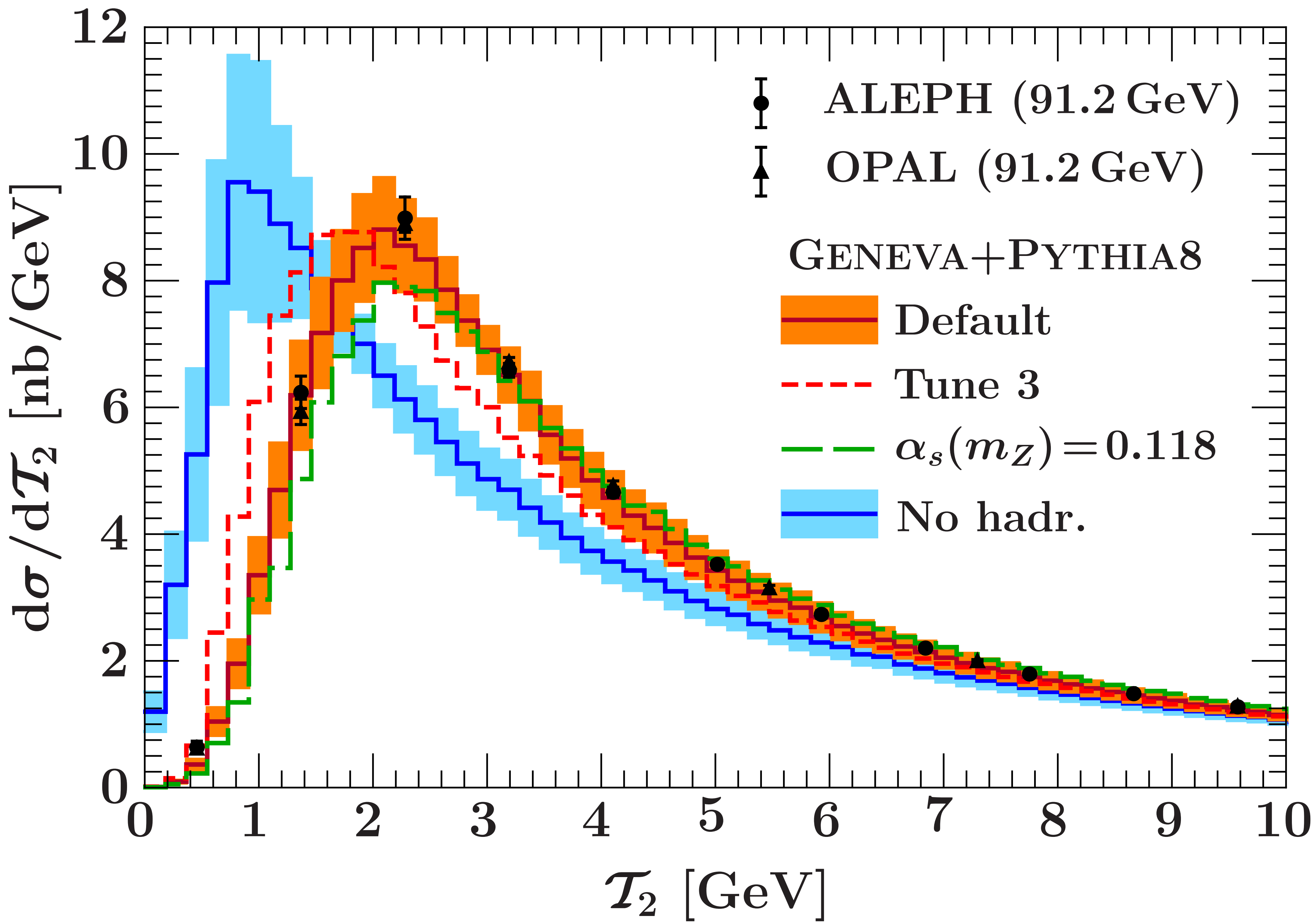
Properly interpolates between fixed order result for large τ and resummed result for small τ

As a validation, check that GENEVA reproduces correctly the thrust distribution at NLO and NNLL'



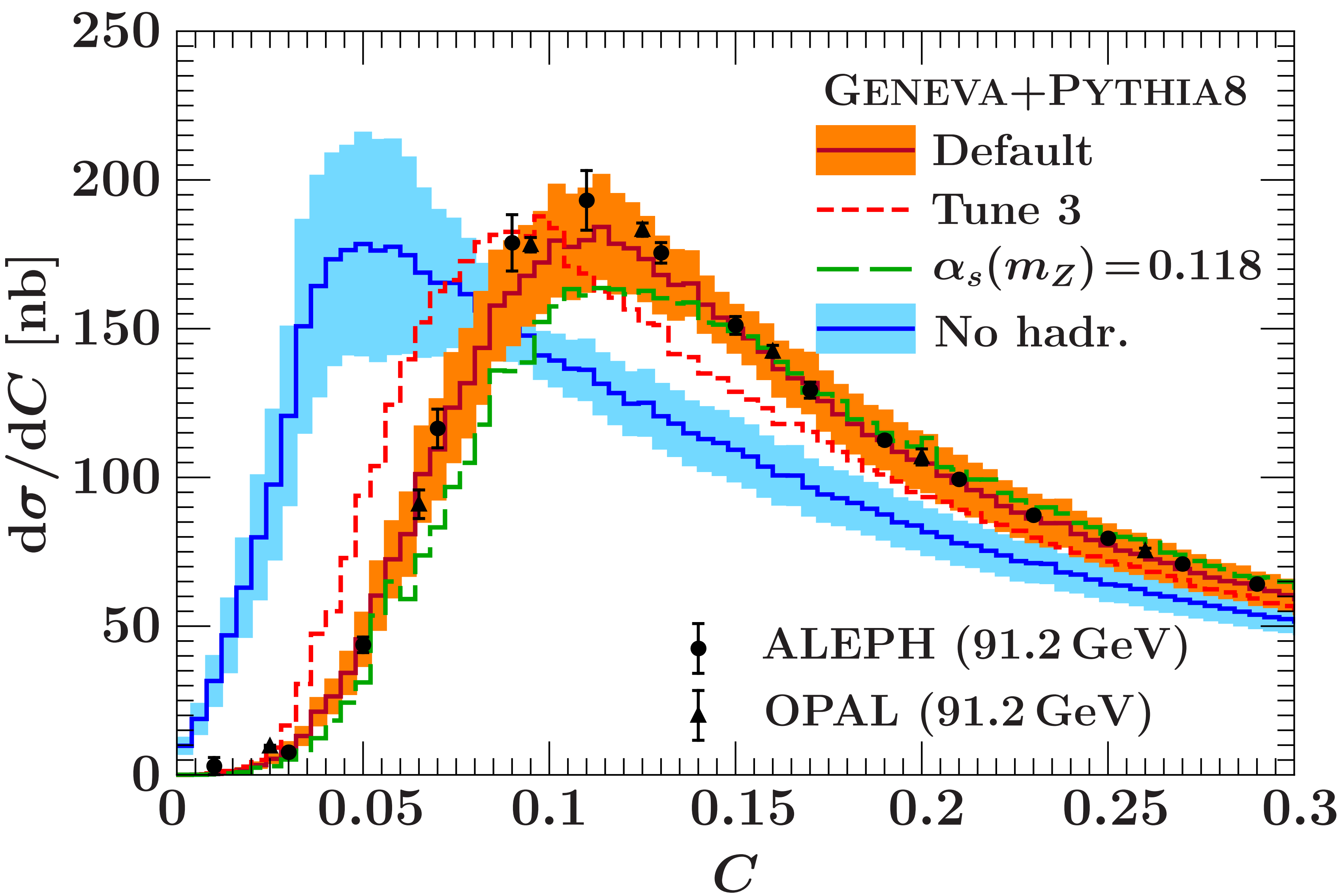
GENEVA exactly reproduces analytical results

To match onto parton shower, need to fill jets with radiation, without changing the thrust distribution we carefully worked out



After hadronization, spectrum in agreement with data

Other distributions are predicted with same level of accuracy



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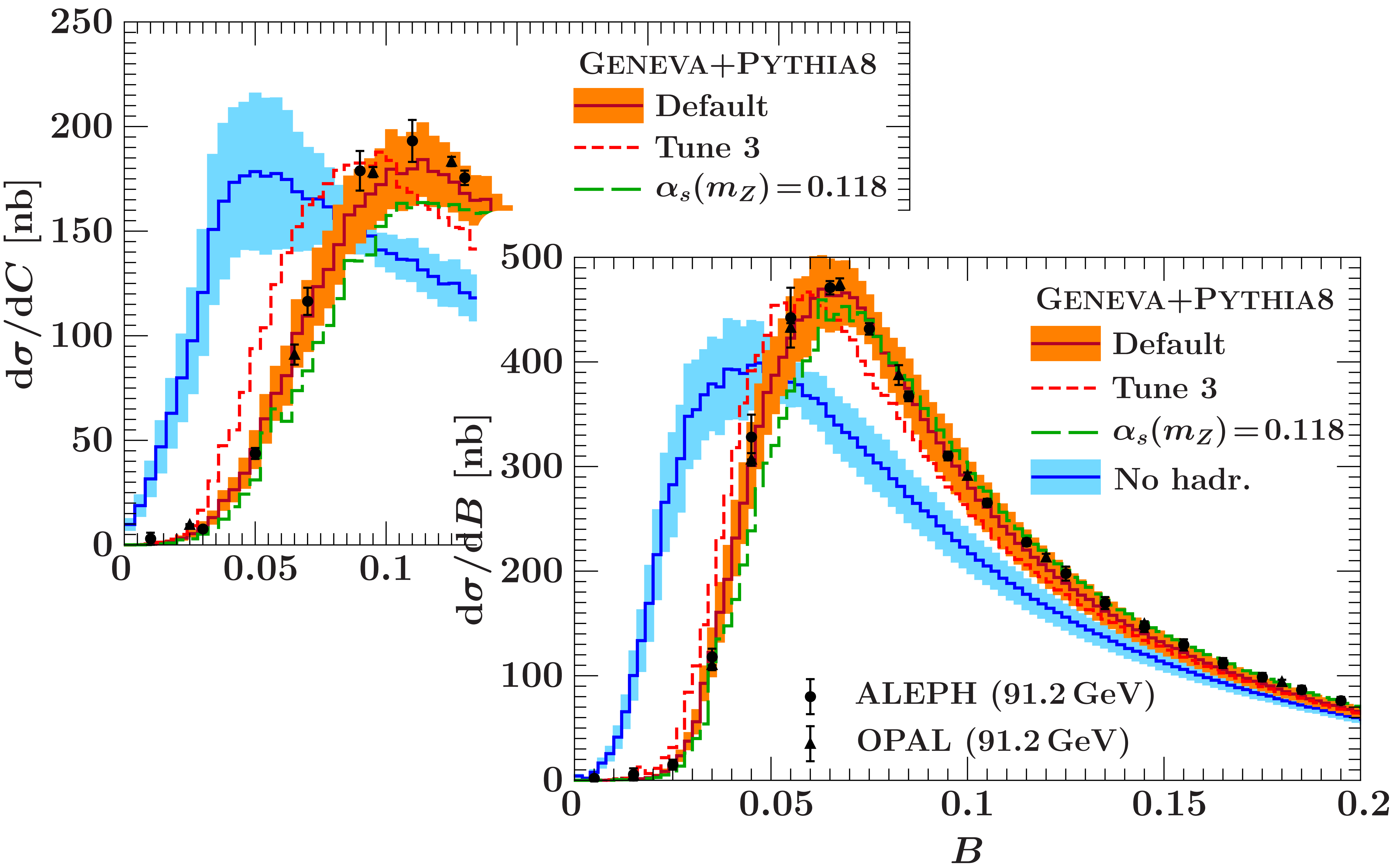


From e^+e^- to pp Collisions

So far,we have shown complete results only for ee collisions

Extension to pp is theoretically straightforward, but requires lots of code improvements

Currently finishing the implementation for Drell-Yan production

Two obvious ways to improve the current situation: More fixed order, or more logarithms

Powheg and MC@NLO have

N^{1}LO / N$^{0.5}$LL

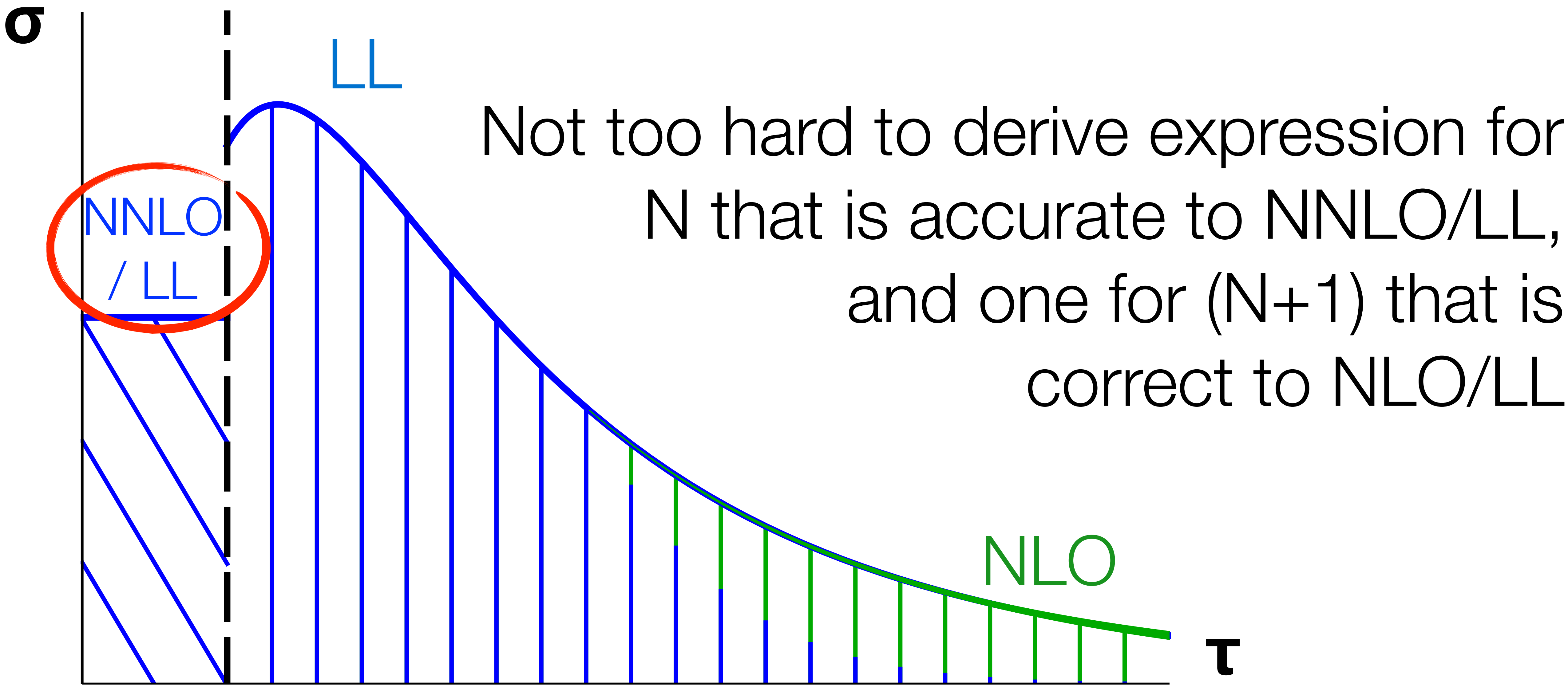
Two obvious ways to go beyond:

Go beyond (N)LL in resummation

GENEVA

Go beyond NLO in fixed order

Before one starts any calculations, a clear idea of the desired accuracy is required



But how to ensure inclusive remains correct?

"Area" determined by LL, no better than LO

Proceed in three steps to the final expression

1. Get an expression for $\sigma^{MC}_{N}(T_N^{cut})$ which is accurate to NNLO/LL
2. Get an expression for $\sigma^{MC}_{\geq N+1}$ that is correct an NLO/LL and integrates correctly
3. Divide the $\sigma^{MC}_{\geq N+1}$ into $\sigma^{MC}_{\geq N+1}$ and $\sigma^{MC}_{\geq N+1}$ correct to NLO/LL and LO/LL respectively

Let's do this one by one...

The N-jet cross-section needs to be correct to NNLO / LL

The simplest way to achieve this is by writing

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\overline{\sigma}_N^{\mathrm{NNLO}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\Delta(\mathcal{T}_N^{\mathrm{cut}})$$

The $\overline{\sigma}_N$ is defined such that the it removes the expansion of the Sudakov factor

$$\left[\frac{\mathrm{d}\overline{\sigma}_N^{\mathrm{NNLO}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\,\Delta_N(\mathcal{T}_N^{\mathrm{cut}})\right]_{\mathrm{NNLO}} = \frac{\mathrm{d}\sigma_N^{\mathrm{NNLO}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

Since we only require LL accuracy, any Sudakov factor will do

The ≥N+1-jet cross-section needs to be correct to NLO / LL, but also combine with N-jet cross section to ≥N inclusive

The following definition for the cross-section gives the required correlation

$$\frac{d\sigma^{\mathrm{MC}}_{\geq N+1}}{d\Phi_{N+1}} = \frac{d}{d\mathcal{T}_N}\frac{d\sigma^{\mathrm{MC}}_{N}}{d\Phi_{N}}(\mathcal{T}_N)D(\Phi_{N+1};\Phi_N,\mathcal{T}_N)$$

with

$$D(\Phi_{N+1};\Phi_N,\mathcal{T}_N) = \frac{\frac{d\sigma^{\mathrm{NLO}}_{\geq N+1}}{d\Phi_{N+1}}}{\left[\frac{d}{d\mathcal{T}_N}\frac{d\sigma^{\mathrm{MC}}_{N}}{d\Phi_{N}}(\mathcal{T}_N)\right]_{\mathrm{NLO}}}$$

Let’s explain why this is correct

The ≥N+1-jet cross-section needs to be correct to NLO / LL, but also combine with N-jet cross section to ≥N inclusive

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}} = \frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_N}\frac{\mathrm{d}\sigma^{\mathrm{MC}}_N}{\mathrm{d}\Phi_N}(\mathcal{T}_N) D(\Phi_{N+1};\Phi_N,\mathcal{T}_N)$$

First, expanding $\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+1}$ to NLO accuracy, one finds

$$\left[\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}\right]_{\mathrm{NLO}} = \left[\frac{\frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_N}\frac{\mathrm{d}\sigma^{\mathrm{MC}}_N}{\mathrm{d}\Phi_N}(\mathcal{T}_N)}{\left[\frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_N}\frac{\mathrm{d}\sigma^{\mathrm{MC}}_N}{\mathrm{d}\Phi_N}(\mathcal{T}_N)\right]_{\mathrm{NLO}}}\right]_{\mathrm{NLO}} \frac{\mathrm{d}\sigma^{\mathrm{NLO}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}$$

$$= \frac{\mathrm{d}\sigma^{\mathrm{NLO}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}$$

The ≥N+1-jet cross-section needs to be correct to NLO / LL, but also combine with N-jet cross section to ≥N inclusive

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}} = \frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_N}\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{N}}{\mathrm{d}\Phi_{N}}(\mathcal{T}_N) D(\Phi_{N+1};\Phi_N,\mathcal{T}_N)$$

Second, integrating over Φ_{N+1}

$$\int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}} = \int_{\mathcal{T}_N^{\mathrm{cut}}}^{\mathcal{T}_N^{\mathrm{max}}} \mathrm{d}\mathcal{T}_N \; \frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_N}\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{N}}{\mathrm{d}\Phi_{N}}(\mathcal{T}_N) \; \frac{\int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N \mathrm{d}\mathcal{T}_N}\frac{\mathrm{d}\sigma^{\mathrm{NLO}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}}{\left[\frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_N}\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{N}}{\mathrm{d}\Phi_{N}}(\mathcal{T}_N)\right]_{\mathrm{NLO}}}$$

$$= \int_{\mathcal{T}_N^{\mathrm{cut}}}^{\mathcal{T}_N^{\mathrm{max}}} \mathrm{d}\mathcal{T}_N \; \frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_N}\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{N}}{\mathrm{d}\Phi_{N}}(\mathcal{T}_N)$$

$$= \frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N}}{\mathrm{d}\Phi_N} - \frac{\mathrm{d}\sigma^{\mathrm{MC}}_{N}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

The ≥N+1-jet cross-section can now be split into an N+1-jet and ≥N+2-jet cross-section

Following the same logic as before, one can write

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}^{\mathrm{cut}}_{N+1}) = \frac{\mathrm{d}\overline{\sigma}^{\mathrm{NLO+LL}}_{N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}^{\mathrm{cut}}_{N+1})\,\Delta_{N+1}(\mathcal{T}^{\mathrm{cut}}_{N+1})$$

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+2}}{\mathrm{d}\Phi_{N+2}} = \frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_{N+1}}\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_{N+1}) D(\Phi_{N+2};\Phi_{N+1},\mathcal{T}_{N+1})$$

This again integrates to the correct inclusive (N+1)-jet cross-section derived on the last slide

In conclusion, the GENEVA framework allows to cast higher order perturbative calculations (fixed order and resummation) in terms of MC cross-sections. These can be interfaced with parton shower algorithms to obtain fully inclusive event generators.

Questions?