EW and QCD One-Loop Amplitudes with RECOLA

Sandro Uccirati

Torino University and INFN

Istituto Nazionale di Fisica Nucleare

In collaboration with S. Actis, A. Denner, R. Feger, L. Hofer, A. Scharf

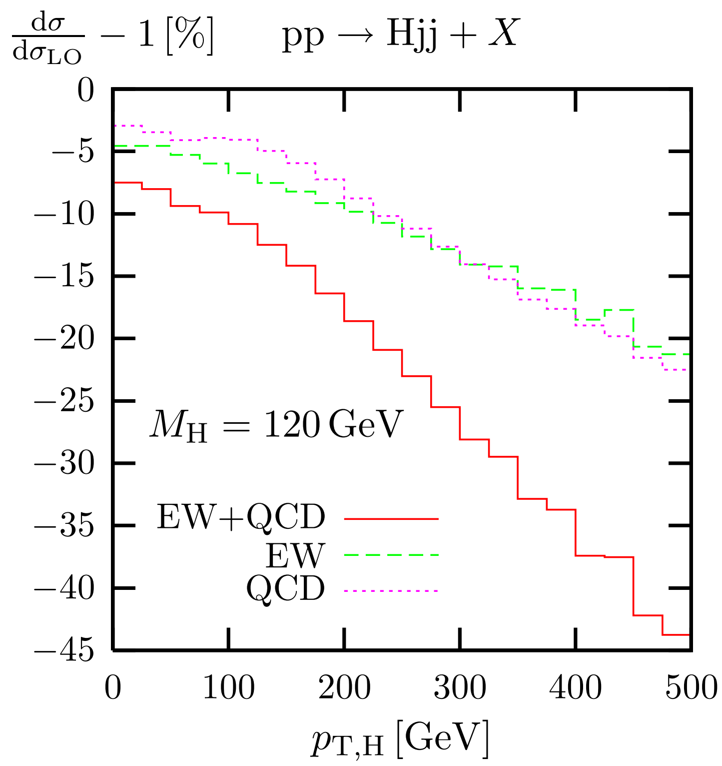
RADCOR 2013, 22-27 September 2013

After the discovery of the Higgs boson:

- Precise investigation of the Standard Model and beyond
- Need to have under control potential large corrections for several processes

After the discovery of the Higgs boson:

- Precise investigation of the Standard Model and beyond
- Need to have under control potential large corrections for several processes
- QCD corrections are known to be large
- EW corrections can be enhanced:
    - in high energy regions (Sudakov log's)
    - in Higgs physics
    - by photon emission (mass-singular log's)



Let's concentrate on one loop corrections

Les Houches wishlist 2013 at one loop

- **QCD:**

  $pp \to t\bar{t}H, \qquad pp \to t\bar{t} + j \qquad$ (top decays)

- **EW:**

  $pp \to 3j,$

  $pp \to t\bar{t}, \qquad pp \to t\bar{t}H, \qquad pp \to t\bar{t} + j \qquad$ (top decays)

  $pp \to V + 2j, \qquad pp \to VV', \qquad pp \to VV + j,$

  $pp \to VV + 2j \qquad pp \to VV'\gamma, \qquad pp \to VV'V'',$

  $(V, V', V'' = W, Z$ decay leptonically)

- Many issues at hadronic level:

  Multi-channel MCs, Real emission, PDFs, Parton Shower, ...

- At least the partonic processes should be **automatized**

Many codes have been produced:

MCFM	Campbell, Ellis
BlackHat	Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maître
VBFNLO	Arnold, Bähr, Bozzi, Campanario, Englert, Figy, Greiner, Hackstein, Hankele, Jäger, Klämke, Kubocz, Oleari, Plätzer, Prestel, Worek, Zeppenfeld
HELAC-NLO	Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek
GoSam	Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
NJet	Badger, Biedermann, Uwer, Yundin
AMC@NLO	Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau
OpenLoops	Cascioli, Maierh”ofer, Pozzorini

Most of them are efficient codes for **QCD**

Many codes have been produced:

MCFM	Campbell, Ellis
BlackHat	Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maître
VBFNLO	Arnold, Bähr, Bozzi, Campanario, Englert, Figy, Greiner, Hackstein, Hankele, Jäger, Klämke, Kubocz, Oleari, Plätzer, Prestel, Worek, Zeppenfeld
HELAC-NLO	Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek
GoSam	Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
NJet	Badger, Biedermann, Uwer, Yundin
AMC@NLO	Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau
OpenLoops	Cascioli, Maierh”ofer, Pozzorini

Most of them are efficient codes for **QCD**

REcursive Computation of One Loop Amplitudes

- In the **full Standard Model**
- Based on **recursive relations** for **off-shell currents**

Off-shell tree currents

Given a process with L external legs:

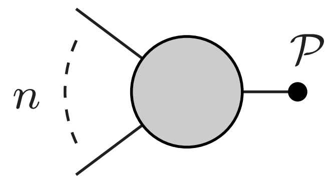
$$\underbrace{\mathcal{P}_1 + \ldots + \mathcal{P}_{L-1}}_{\text{primary}} + \underbrace{\mathcal{P}_L}_{\text{last}} \to 0$$

Off-shell current of a particle $\mathcal{P}$ with n primary legs:

Def: Amplitude made of n primary on-sheel particles and the off-sheel particle $\mathcal{P}$

color

$$w(\mathcal{P}, \mathcal{C}, \{l_1, \ldots, l_n\}) =$$



particle: e^-, e^+, γ, ...

List of primary legs

- w is a scalar, spinor or vector
- The off-shell currents for external legs are the wave functions:

- Binary notation for $\{l_1, \ldots, l_n\}$ (HELAC):

  Binary numbers $1, 2, 4, 8, \ldots, 2^{L-1}$ for the primary legs

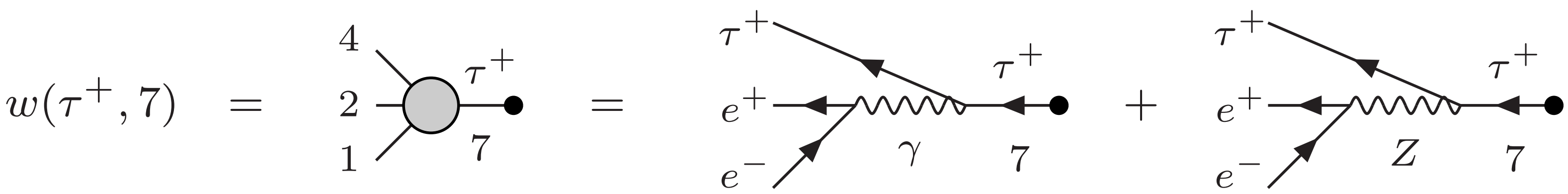
  $\{l_1, \ldots, l_n\}$ can be expressed by $\mathcal{B}_n =$ sum of the n binaries

Example: $\{1, 2, 8\} \quad \to \quad \mathcal{B}_3 = 1 + 2 + 8 = 11$

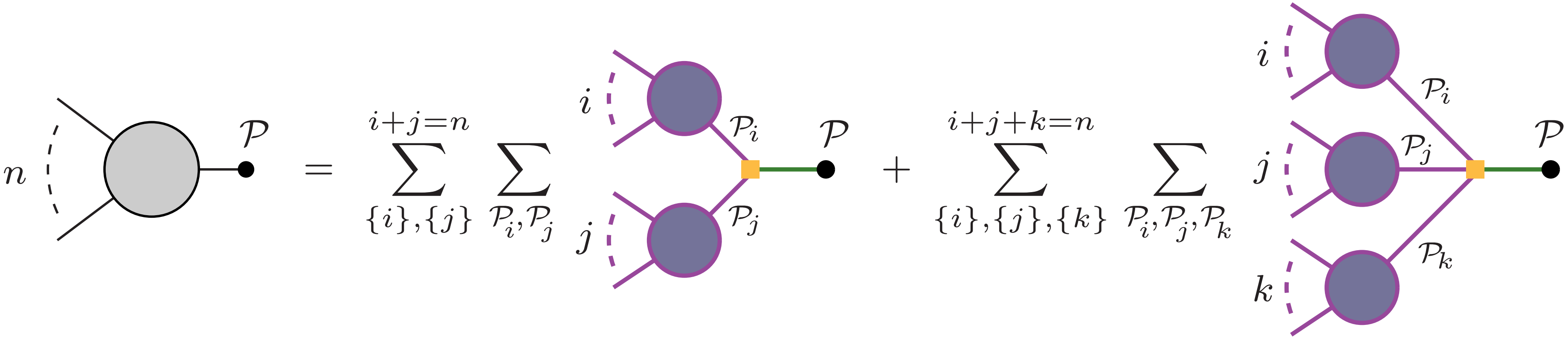
Note: The off-shell currents just keep trace of the primary legs used to build them, not the way it has been done.

Example: Process $e^- + e^+ + \tau^+ + \tau^- \to 0$

1 2 4

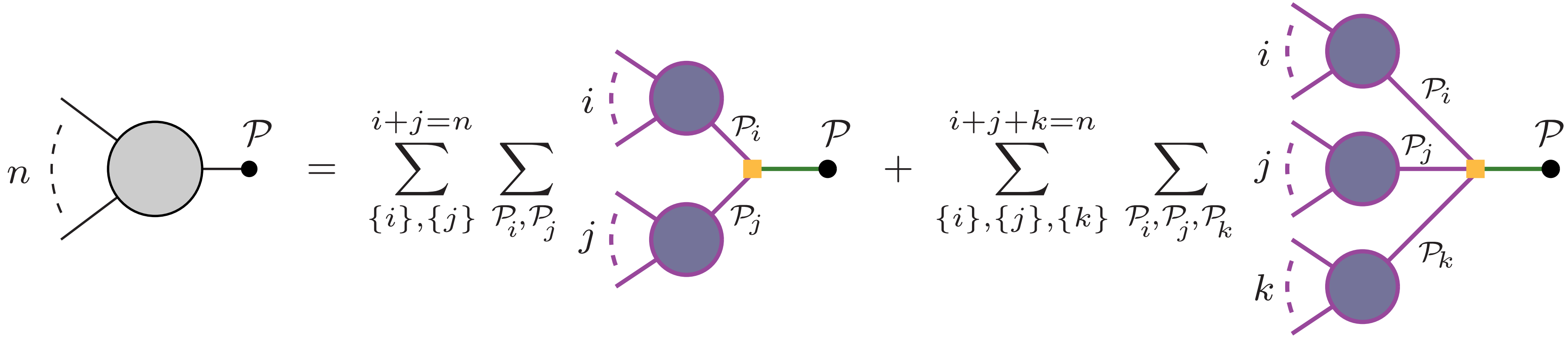


Recursion relation for tree amplitudes



(**incoming currents**) $\times$ (**coupling**) $\times$ (**propagator**)

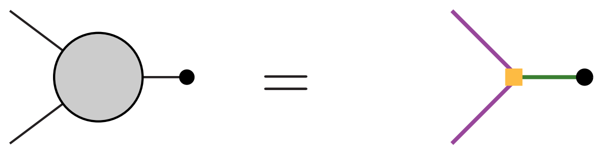
Recursion relation for tree amplitudes



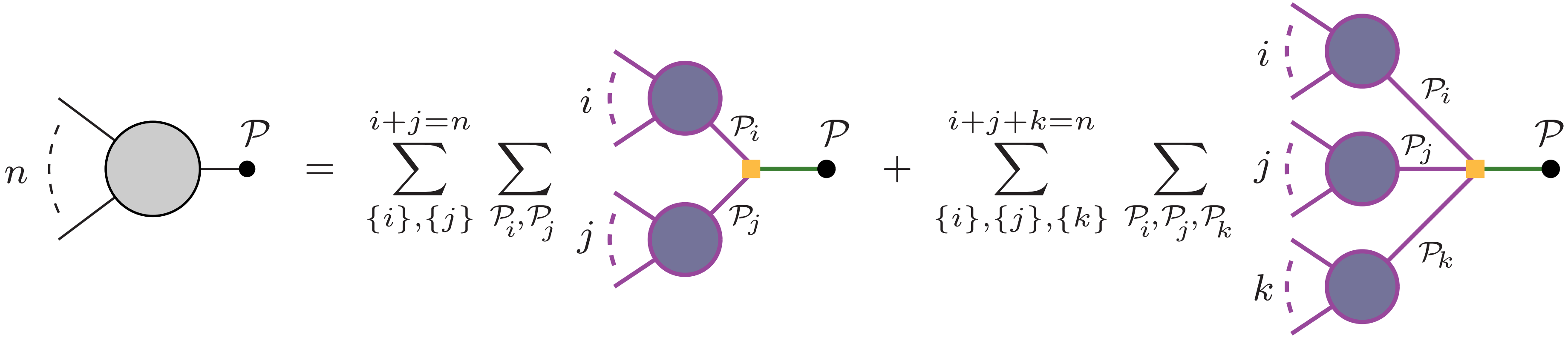
(**incoming currents**) $\times$ (**coupling**) $\times$ (**propagator**)

- Recursive procedure:

2-leg currents:

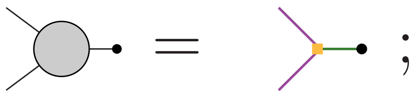


Recursion relation for tree amplitudes

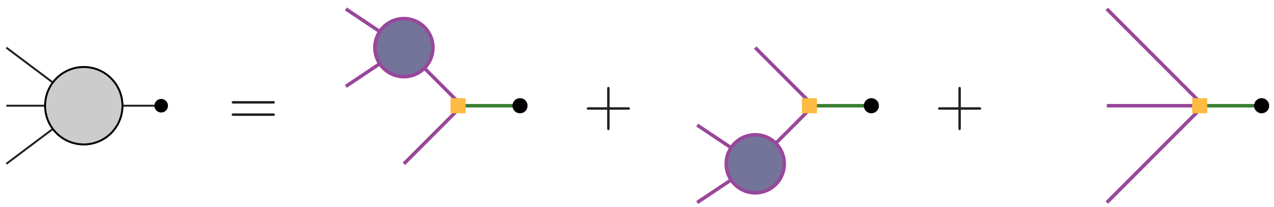


(**incoming currents**) × (**coupling**) × (**propagator**)

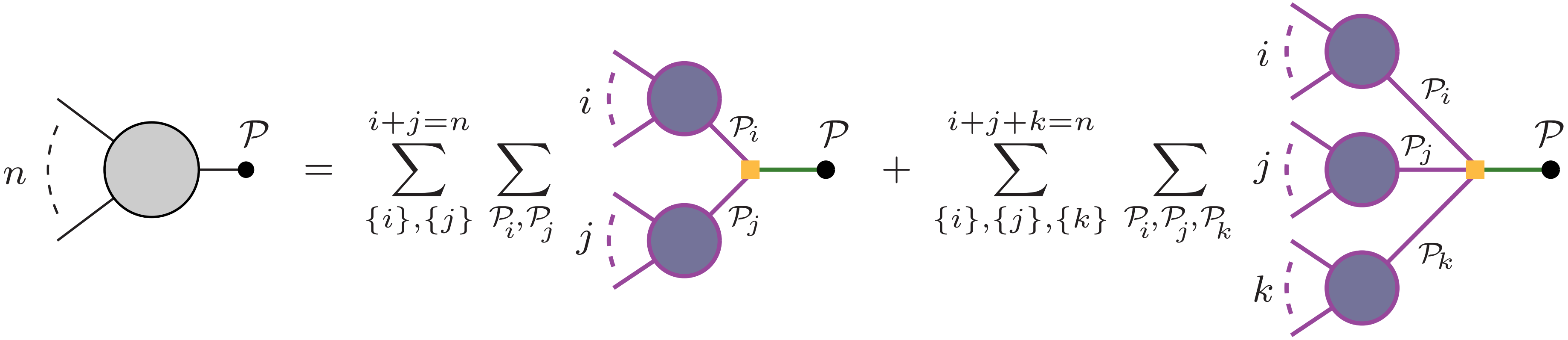
- Recursive procedure:



3-leg currents:

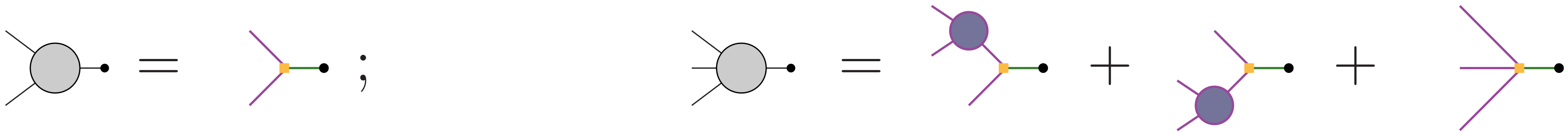


Recursion relation for tree amplitudes

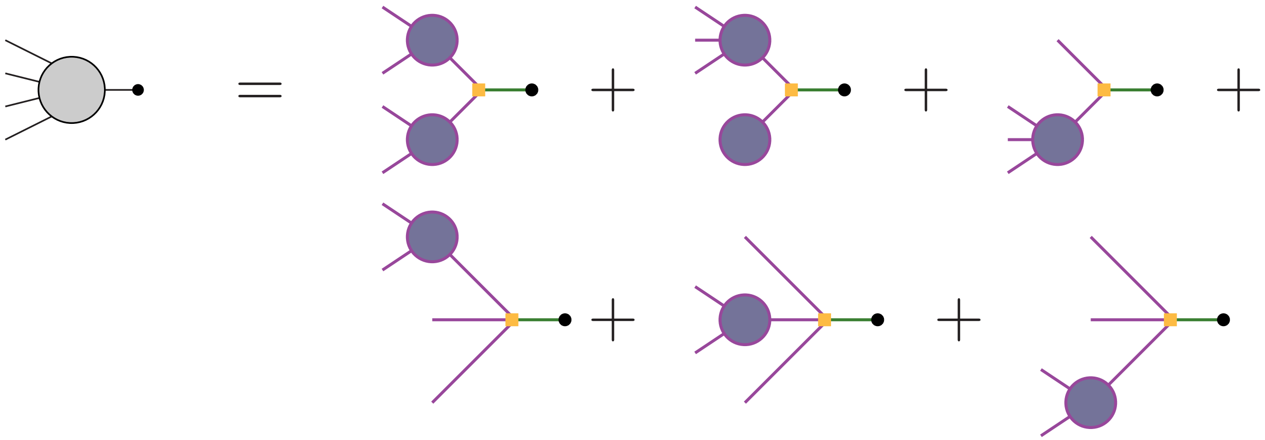


(**incoming currents**) × (**coupling**) × (**propagator**)

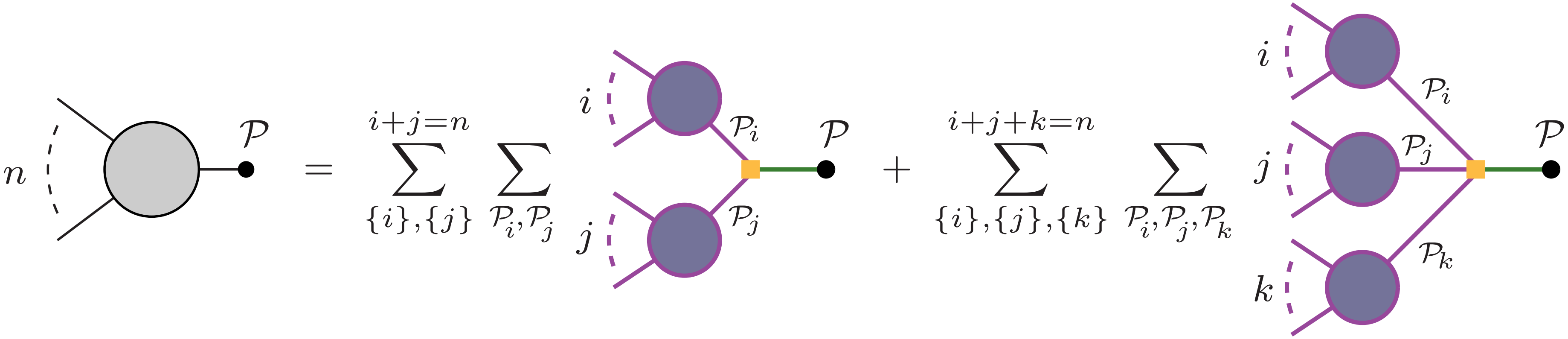
- Recursive procedure:



4-leg currents:

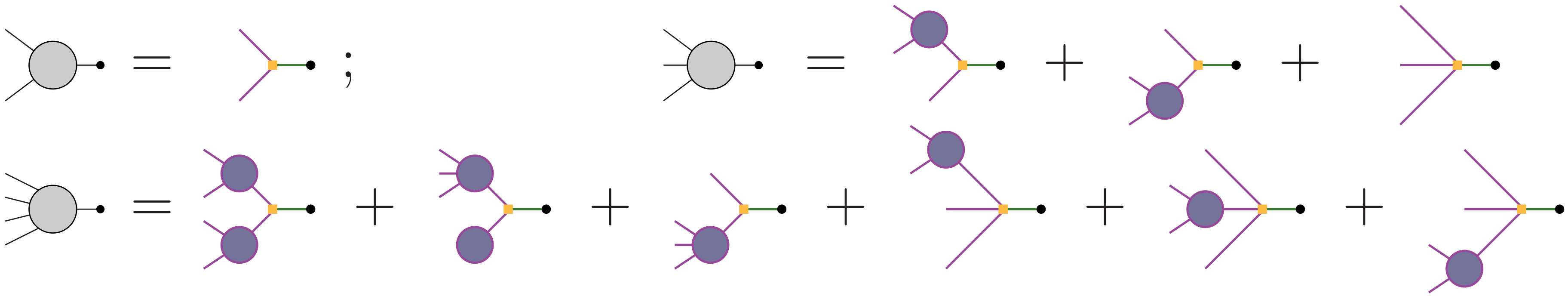


Recursion relation for tree amplitudes



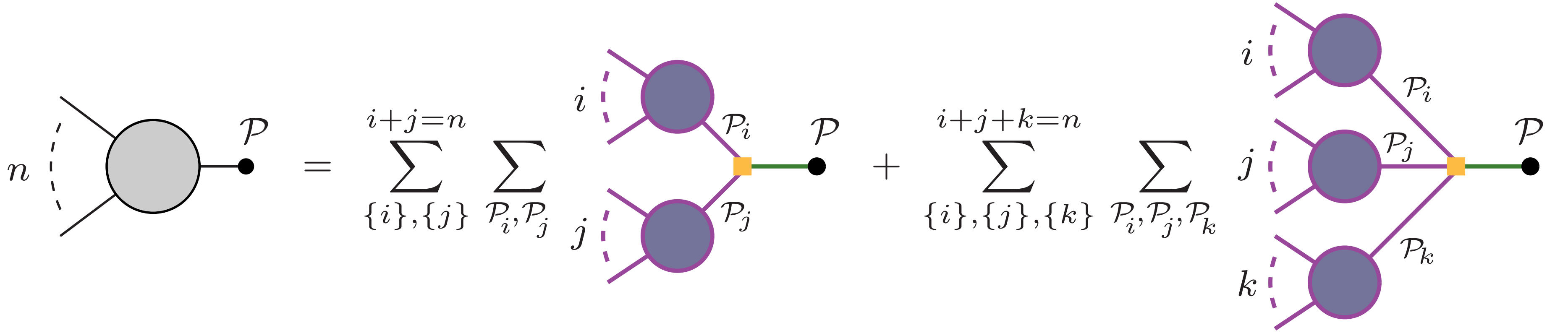
(**incoming currents**) × (**coupling**) × (**propagator**)

- Recursive procedure:



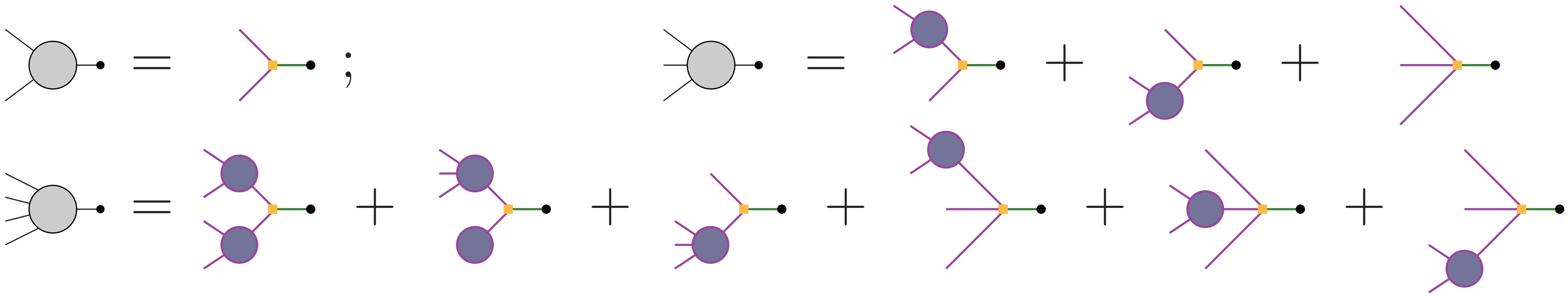
etc. . . .

Recursion relation for tree amplitudes



(**incoming currents**) $\times$ (**coupling**) $\times$ (**propagator**)

- Recursive procedure:



etc. . . .

- Amplitude: $\mathcal{A} = w(\mathcal{P}_L, 2^{L-1}-1) \times$ (**propagator**)$^{-1} \times w(\mathcal{P}_L, 2^{L-1})$

Recursion relation for loop amplitudes

General form of the amplitude:

$$\mathcal{A} = \sum_t c^{(t)}_{\mu_1 \dots \mu_{r_t}} \, T^{\mu_1 \dots \mu_{r_t}}_{(t)}$$

$c^{(t)}_{\mu_1 \dots \mu_{r_t}}$ → Tensor Coefficients (TCs)

$T^{\mu_1 \dots \mu_{r_t}}_{(t)}$ → Tensor Integrals (TIs)

$$T^{\mu_1 \cdots \mu_{r_t}}_{(t)} = \int \frac{d^n q \, q^{\mu_1} \cdots q^{\mu_{r_t}}}{D^{(t)}_0 \cdots D^{(t)}_{k_t}} \qquad D^{(t)}_{k_t} = (q + p^{(t)}_{k_t})^2 - (m^{(t)}_{k_t})^2$$

Remark: Indices $\mu_1, \dots, \mu_{r_t}$ are computed numerically in D=4 dimensions.

⇝ The rational part R2 is computed separatly:
Inclusion of effective tree-level Feynman rules (as for the counterterms)
[Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau ’09-’10]

Recursion relation for loop amplitudes

General form of the amplitude:

$$\mathcal{A} = \sum_t c^{(t)}_{\mu_1 \dots \mu_{r_t}} \, T^{\mu_1 \dots \mu_{r_t}}_{(t)}$$

$c^{(t)}_{\mu_1 \dots \mu_{r_t}}$ → Tensor Coefficients (TCs)

$T^{\mu_1 \dots \mu_{r_t}}_{(t)}$ → Tensor Integrals (TIs)

$$T^{\mu_1 \cdots \mu_{r_t}}_{(t)} = \int \frac{d^n q \, q^{\mu_1} \cdots q^{\mu_{r_t}}}{D^{(t)}_0 \cdots D^{(t)}_{k_t}} \qquad D^{(t)}_{k_t} = (q + p^{(t)}_{k_t})^2 - (m^{(t)}_{k_t})^2$$

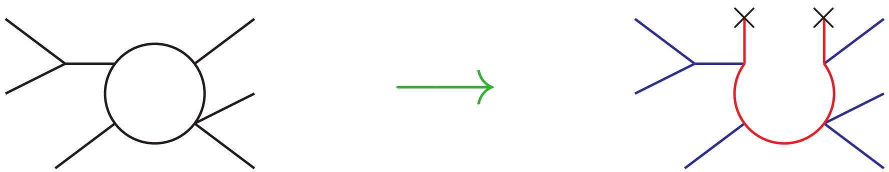
Remark: Indices $\mu_1, \dots, \mu_{r_t}$ are computed numerically in D=4 dimensions.

⇝ The rational part R2 is computed separatly:
Inclusion of effective tree-level Feynman rules (as for the counterterms)

[Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau '09-'10]

Basic idea: Cut the loop line and consider tree diagrams with two more legs.

[A. van Hameren, JHEP 0907 (2009) 088]



Given the loop process

$$\mathcal{P}_1 + \ldots + \mathcal{P}_L \to 0$$

we consider the tree processes

$$\underbrace{\mathcal{P}_1 + \ldots + \mathcal{P}_L + \mathcal{P}}_{\text{primary}} + \underbrace{\bar{\mathcal{P}}}_{\text{last}} \to 0 \qquad \forall\, \mathcal{P} \in \{\text{Particle of the SM}\}$$

Given the loop process

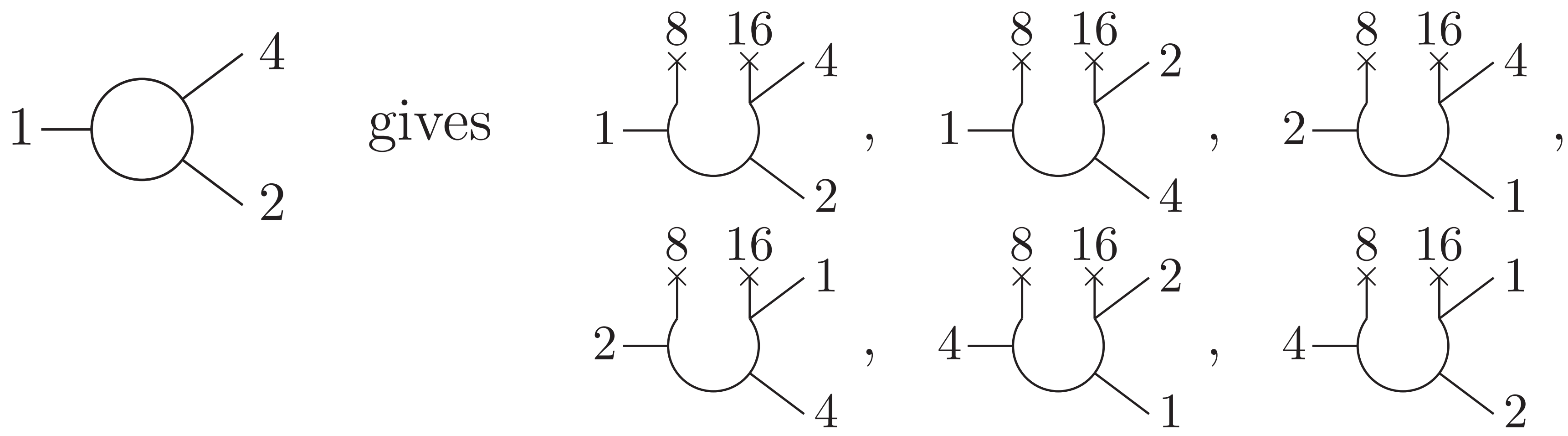
$$\mathcal{P}_1 + \ldots + \mathcal{P}_L \to 0$$

we consider the tree processes

$$\underbrace{\mathcal{P}_1 + \ldots + \mathcal{P}_L + \mathcal{P}}_{\text{primary}} + \underbrace{\bar{\mathcal{P}}}_{\text{last}} \to 0 \qquad \forall\, \mathcal{P} \in \{\text{Particle of the SM}\}$$

Problem:

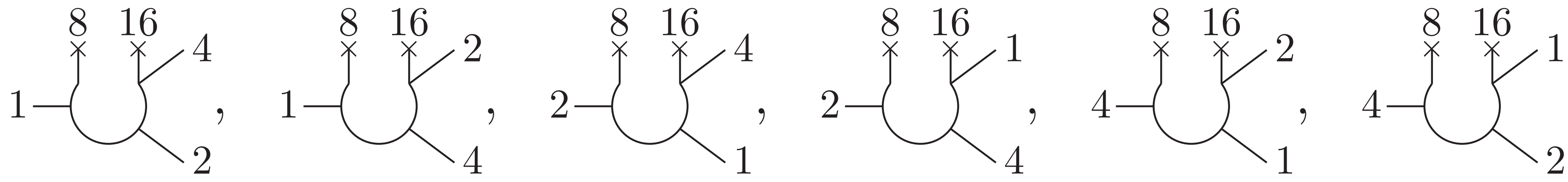
Associated tree diagrams are more than the original loop diagrams:



Rules to avoid double counting of the associated trees:

Rule 1: $\rightarrow$ Fix starting point of loop flow

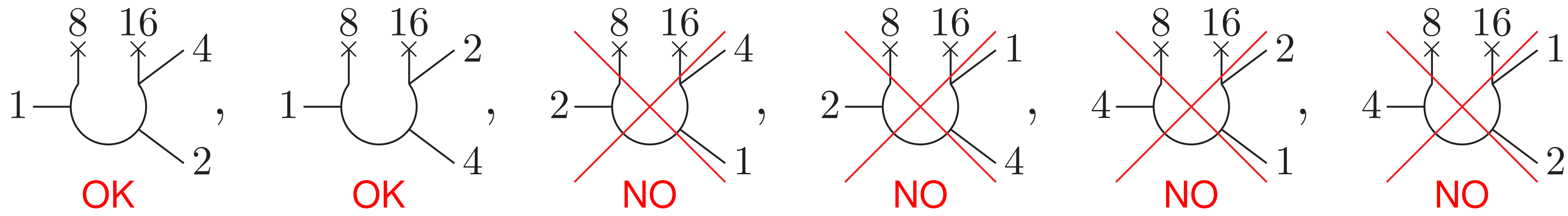
The current containing the first external line enters the loop flow first



Rules to avoid double counting of the associated trees:

Rule 1: → Fix starting point of loop flow

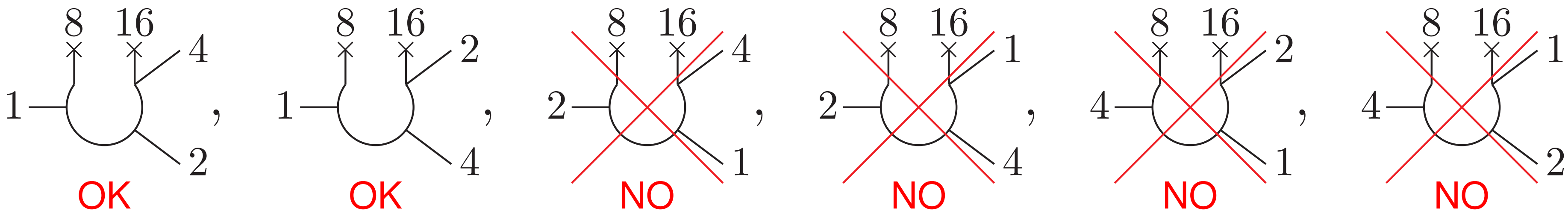
The current containing the first external line enters the loop flow first



Rules to avoid double counting of the associated trees:

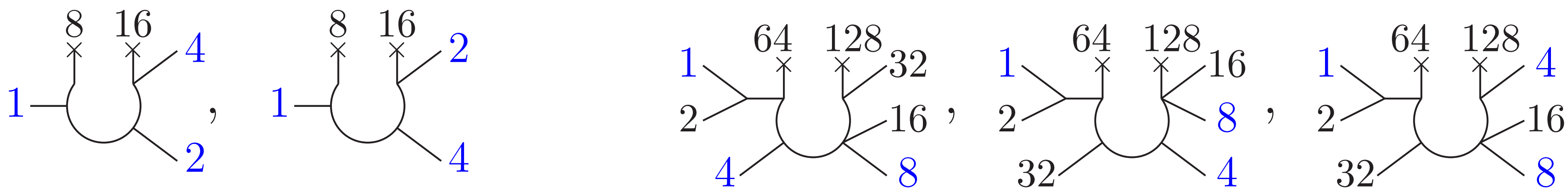
Rule 1: $\rightarrow$ Fix starting point of loop flow

The current containing the first external line enters the loop flow first



Rule 2: $\rightarrow$ Fix direction of loop flow

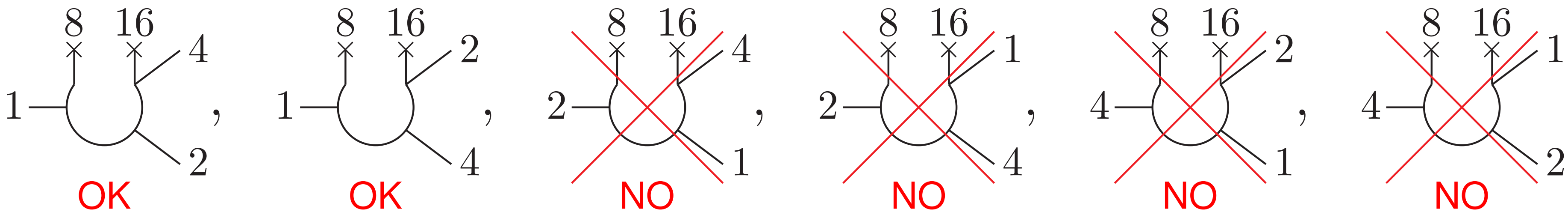
The 3 currents with the 3 smallest binaries enter the loop flow in fixed order



Rules to avoid double counting of the associated trees:

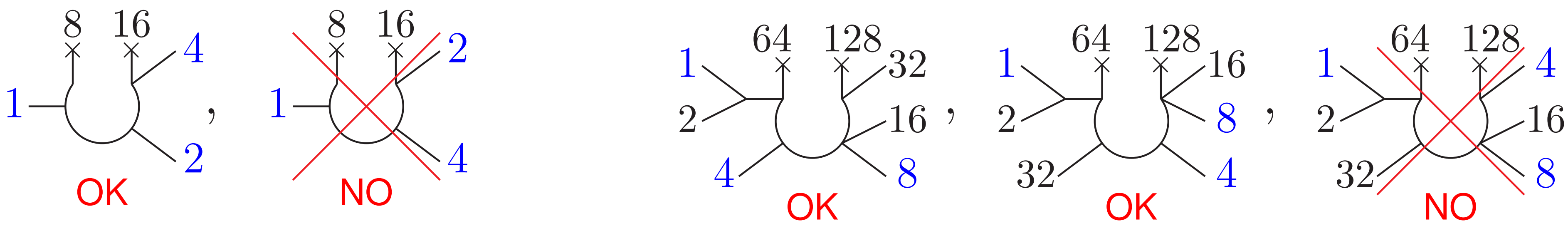
Rule 1: $\longrightarrow$ Fix starting point of loop flow

The current containing the first external line enters the loop flow first

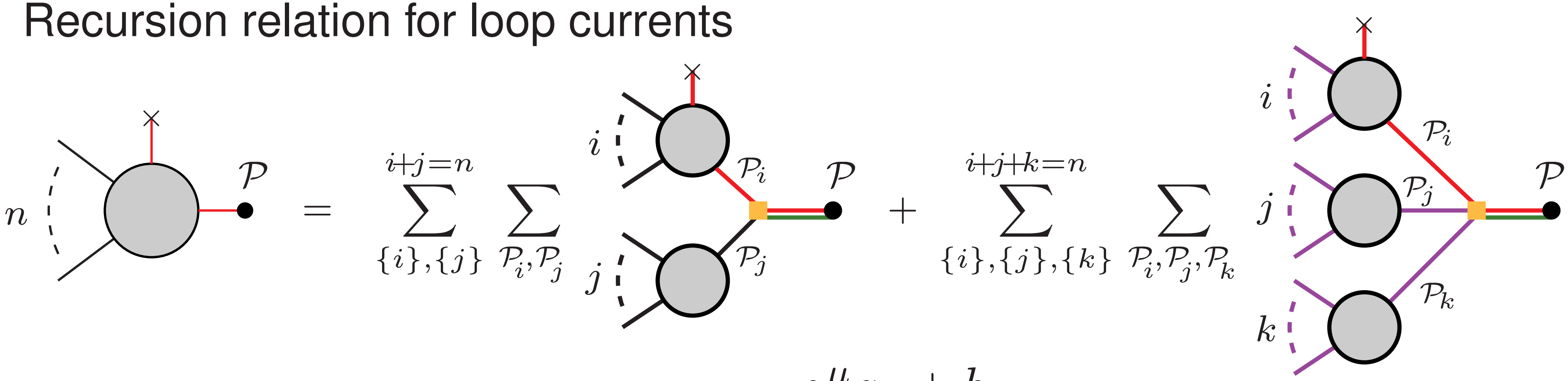


Rule 2: $\longrightarrow$ Fix direction of loop flow

The 3 currents with the 3 smallest binaries enter the loop flow in fixed order



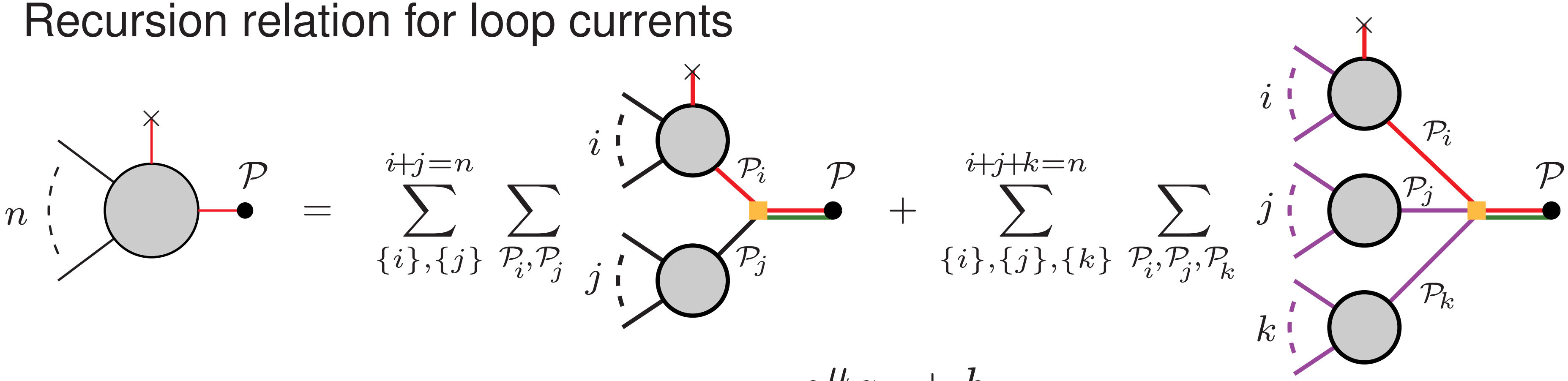
- Recursion relation for loop currents



$$(\textbf{coupling}) \times (\textbf{propagator}) = \frac{a^\mu q_\mu + b}{(q+p)^2 - m^2}$$

$q =$ loop momentum

- Recursion relation for loop currents



$$(\text{coupling}) \times (\text{propagator}) = \frac{a^\mu q_\mu + b}{(q+p)^2 - m^2}$$

$q =$ loop momentum

$$\text{loop current (q)} = \sum_{r=0}^{k} a_{k,r}^{\mu_1 \cdots \mu_r} \frac{q_{\mu_1} \cdots q_{\mu_r}}{\prod_{h=0}^{k}[(q+p_h)^2 - m_h^2]}$$

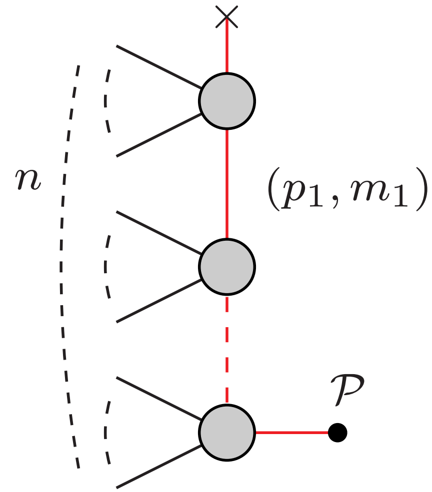
- k: number of propagators
- r: rank
- $a_{k,r}^{\mu_1 \cdots \mu_r}$: computed in the recursion relation
- $\frac{q_{\mu_1} \cdots q_{\mu_r}}{\prod_{h=0}^{k}[(q+p_h)^2 - m_h^2]}$: goes in the TIs

Remark: Indices $\mu_1, \ldots, \mu_r$ are symmetrized at each step

- The coefficients $a_{k,r}^{\mu_1 \cdots \mu_r}$ of the last current give the TCs $c^{(t)}_{\mu_1 \ldots \mu_{r_t}}$

Loop off-shell currents

$$w_{i_k}(\,\mathcal{P},\,\mathcal{C},\,\mathcal{B}_n,\,\{p_1,\ldots,p_k\},\,\{m_1,\ldots,m_k\}) =$$



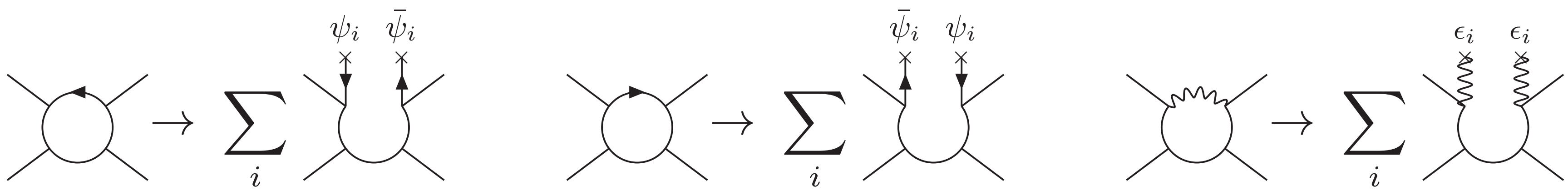
Sequence of momenta in loop propagators

Sequence of masses in loop propagators

- i_k is the tensorial index:

$$\begin{aligned} i_k &= 0 && \rightarrow && w_{i_k} = a_{k,0} \\ i_k &= 1,\ldots,4 && \rightarrow && w_{i_k} = a_{k,1}^{\mu_1} \\ i_k &= 5,\ldots,14 && \rightarrow && w_{i_k} = a_{k,2}^{\mu_1\mu_2} \\ &\ldots \end{aligned}$$

- Special wave functions for the cutted line:



where the components are $(\psi_i)_j = (\bar{\psi}_i)_j = \delta_{ij}$, $\epsilon_i^\mu = \delta_{i\mu}$.

Treatment of the colour

Color-flow representation [Maltoni, Paul, Stelzer, Willenbrock '02]:

$$\text{Gluon field} \; : \quad \sqrt{2}\, A^a_\mu \, (\lambda^a)^i_j \; = \; (\mathcal{A}_\mu)^i_j$$

"usual" gluon with color index $a = 1, \ldots, 8$

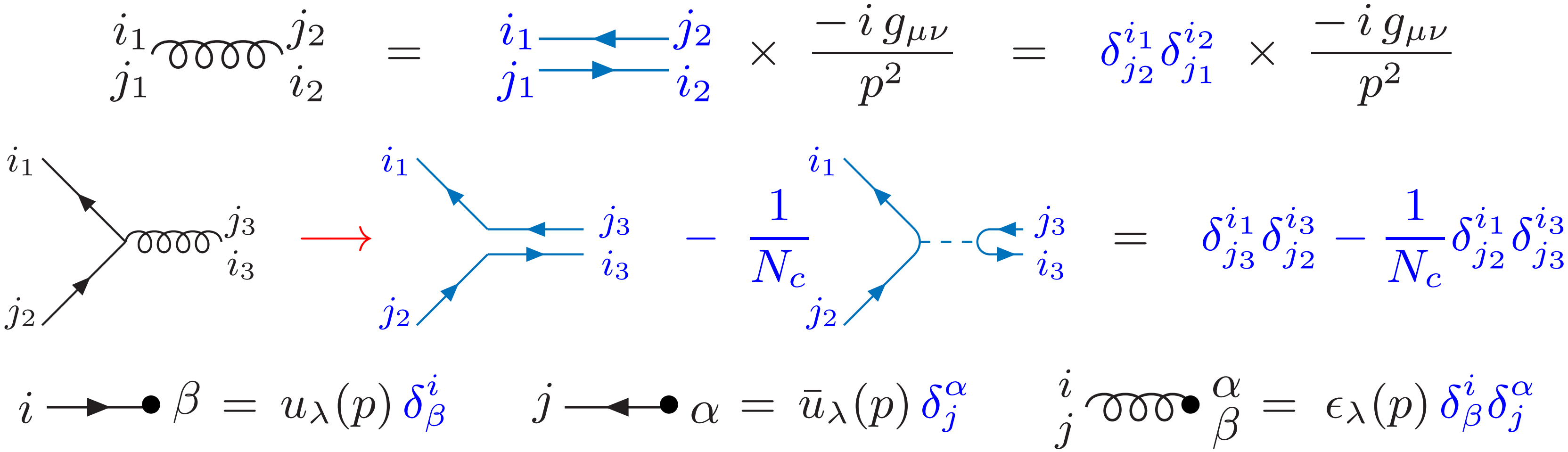
gluon with color-flow ${}^i_j$

$i, j = 1, 2, 3$

$\sum_i (A_\mu)^i_i = 0$

Feynman rules:

- Multiply gluon fields A^a_μ by $(\lambda^a)^i_j/\sqrt{2}$ and use properties of $(\lambda^a)^i_j$
- The color part of the Feynman rules is just product of deltas:



Structure of amplitude: $$\mathcal{A}^{\alpha_1\cdots\alpha_n}_{\beta_1\cdots\beta_n} = \sum_{P(\beta_1,\ldots,\beta_n)} \delta^{\alpha_1}_{\beta_1}\cdots\delta^{\alpha_n}_{\beta_n}\,\mathcal{A}_P$$

- Colour-dressed amplitudes:

  → Compute $\mathcal{A}^{\alpha_1\cdots\alpha_n}_{\beta_1\cdots\beta_n}$ for all possible colours $(N_c^{2\,n})$

  Squared amplitude: $$\overline{\mathcal{M}^2} = \sum_{\alpha_1\ldots\alpha_n,\beta_1,\ldots,\beta_n} (\mathcal{A}^{\alpha_1\cdots\alpha_n}_{\beta_1\cdots\beta_n})^*\mathcal{A}^{\alpha_1\cdots\alpha_n}_{\beta_1\cdots\beta_n}$$

  It requires colour-dressed currents

- Structure-dressed (or colour-ordered) amplitudes:

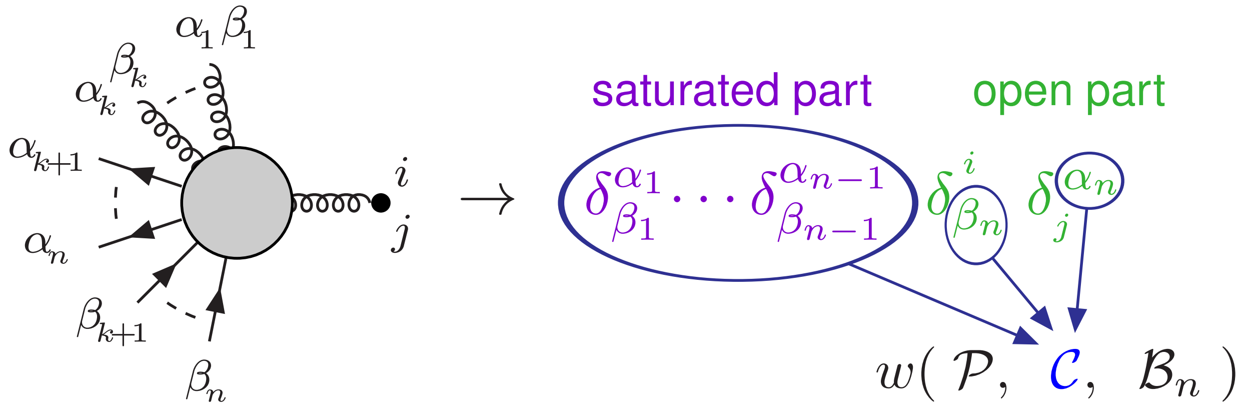
  → Compute $\mathcal{A}_P$ for all possible P $(n!)$

  Squared amplitude: $$\overline{\mathcal{M}^2} = \sum_{P,P'} \mathcal{A}^*_P\, C_{PP'}\, \mathcal{A}_{P'}$$

  It requires structure-dressed currents

Structure-dressed off-shell currents

Colour structure of off-shell current:



with all possible permutations of $\beta_1, \ldots \beta_n, j$

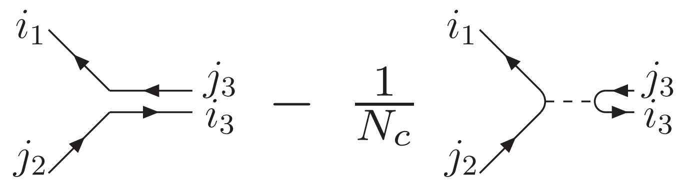
In the recursion procedure:

- Saturated parts of incoming currents multiply
- Open parts of incoming currents are contracted

Optimization: Compute once currents differing just by the colour structure

⇝ Overcome lack of colour factorization

Example:



The code RECOLA is structured in two parts:

- Generation of the recursion procedure (to be run once)
  - A current-index is given to all currents of the recursion procedure
  - A branch-index is given to each step (branch) of the resursion procedure
  - Identify currents differing just by the colour structure
  - To each branch are associated the relevant indices
  - The list of all needed TIs is generated

The code RECOLA is structured in two parts:

- Generation of the recursion procedure (to be run once)
  - A current-index is given to all currents of the recursion procedure
  - A branch-index is given to each step (branch) of the resursion procedure
  - Identify currents differing just by the colour structure
  - To each branch are associated the relevant indices
  - The list of all needed TIs is generated
- Computation of the currents (to be run at each PS point)
  - All needed TIs are computed
  - The currents for all polarizations and colors are computed
  - The last currents are contracted with the TIs to give the amplitude

Features of RECOLA (fortran 95)

- Full Standard Model in the complex mass scheme with:
  - Feynman rules for rational parts
  - Feynman rules for on-shell Counterterms
  - Selection of resonant contributions
- Need external libraries for TIs (link to the COLLIER library implemented) — Talk by Ansgar Denner
- Numerical check of cancellation of UV divergences
- Mass and dimensional regularization for collinear and soft singularities
- Select/unselect powers of α_s in the amplitude
- Computation of Colour- and Spin-correlations
- Optimizations:
  - Helicity sum avoids recalculation of currents
  - Use conservation of helicity for massless fermions
  - Use partial factorization of colour structure

Performances

- Memory needed for executables, object files and libraries: negligible
- RAM needed: less than 2 Gbyte also for complicated processes
- **CPU time** (processor Intel(R) Core(TM) i5-2400 CPU @ 3.10GHz):

Process	t_{TIs} (COLLIER)	t_{gen} t_{TCs} (single helicity)	t_{gen} t_{TCs} (partial hel. sum)	t_{gen} t_{TCs} (helicity sum)
$u\bar{u} \to W^+W^-g$ (QCD)	2.8 ms	0.3 s 0.6 ms (hel: - + - + -)	0.4 s 1.3 ms (hel: S S - + S)	1.6 s 9.8 ms (hel: S S S S S)
$u\bar{d} \to W^+g\,g\,g$ (QCD)	130 ms	14 s 14 ms (hel: - + - - - -)	25 s 76 ms (hel: S S - S S S)	52 s 221 ms (hel: S S S S S S)
$ug \to u\,g\,Z$ (EW)	8.2 ms	0.5 s 1.4 ms (hel: - - - - -)	1.0 s 6.7 ms (hel: S S S S -)	2.2 s 20.2 ms (hel: S S S S S)
$ug \to u\,g\,\tau^-\tau^+$ (EW)	28 ms	1.3 s 2.5 ms (hel: - - - - - +)	2.0 s 14.2 ms (hel: S S S S - +)	3.8 s 29.0 ms (hel: S S S S S S)

S = sum over helicity

Summary

- Efficient automatization for elementary EW and QCD processes at NLO
- Recursion relations $\rightarrow$ good tool also in the EW sector
- **used for EW corrections to** $pp \rightarrow Z + 2j$ $\rightarrow$ Talk by Ansgar Denner

Outlook

- Publication of the code
  - $\rightarrow$ Robust checks
  - $\rightarrow$ Implement dynamical running of α_s
  - $\rightarrow$ Allow extensions to other Models
  - $\rightarrow$ Prepare for MC over polarizations and colours
- Let's compute other LHC processes