$\mathcal{B}(B_s \to \mu^+\mu^-)$ in the Standard Model and $\mathcal{B}(\overline{B} \to X_s\gamma)$ in Two Higgs Doublet Models to NNLO in QCD

Thomas Hermann[1], Mikołaj Misiak[2] and Matthias Steinhauser[1]

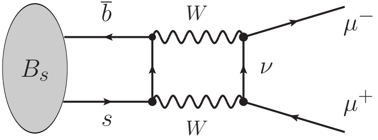
[1]Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology
[2]Institute of Theoretical Physics, Warsaw University

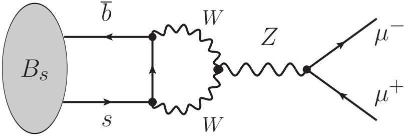
RADCOR 2013

Karlsruher Institut für Technologie

Outline

Introduction $\mathcal{B}(B_s \to \mu^+\mu^-)$





exp. results:

- until 2012 only upper limits [CDF, D0, Atlas, CMS, LHCb]
- first evidence 2012 [LHCb]
- latest combined results [LHCb and CMS at EPS 2013]

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}$$

strong reduction of error expected in the next years

$B_s \to \mu^+\mu^-$ very sensitive to physics beyond the SM, e.g. 2HDM, MSSM, ...

recent progress in determination of f_{B_s} from lattice calculations

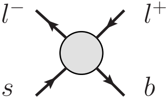
Effective theory approach

- B-meson decays occur at low energies $\mu \ll M_W$
- integrating out heavy particles: $W^\pm$, Z, top quark

$$\mathcal{L}_{full}$$
$$\Downarrow$$
$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED}(\text{leptons and five light quarks}) + N\sum_n C_n Q_n$$

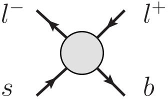
$$\begin{aligned} Q_A &= (\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu) \\ Q_S &= (\bar{b}\gamma_5 s)(\bar{\mu}\mu) \\ Q_P &= (\bar{b}\gamma_5 s)(\bar{\mu}\gamma_5\mu) \end{aligned}$$

- matching



full theory

$=$



effective theory

$$+ \quad \mathcal{O}\left(\frac{1}{M_W^4}\right)$$

Branching ratio

- instantaneous branching ratio

$$\mathcal{B}^{[t=0]}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi} \sqrt{1-r^2}\, \tau_{B_s} \left[|rC_A - uC_P|^2 + |uC_S|^2 (1-r^2) \right]$$

$$N = \frac{V_{tb}^* V_{ts} G_F^2 M_W^2}{\pi^2}, \qquad r = \frac{2m_\mu}{M_{B_s}}, \qquad u = \frac{M_{B_s}}{m_b + m_s}$$

- time-integrated branching ratio [De Bruyn et al. 2012]

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = \frac{\mathcal{B}^{[t=0]}(B_s \to \mu^+\mu^-)}{1 - \tau_{B_s} \Delta\Gamma_s/2}$$

- Standard Model:

$C_{S,P}$ are suppressed by $\frac{M_{B_s}^2}{M_W^2}$ $\Rightarrow$ $\mathcal{B}^{[t=0]}(B_s \to \mu^+\mu^-) \propto |C_A|^2$

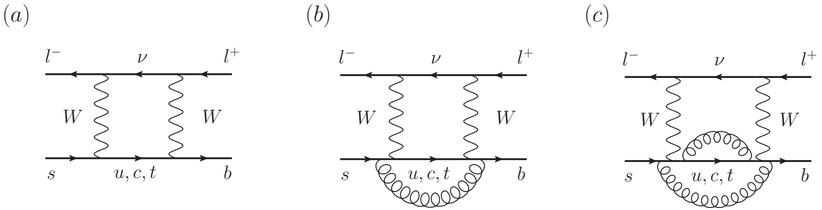
$$C_A = C_A^{(0)} + \frac{\alpha_s}{4\pi} C_A^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A^{(2)} + \ldots$$

LO: $C_A^{(0)} = \frac{1}{2} Y_0(x)$ [Inami and Lim 1981]

NLO: $C_A^{(1)}$ [Buchalla and Buras 1993, 1999; Misiak and Urban 1999]

$$C_A^{(n)} = C_A^{W,(n)} + C_A^{Z,(n)}$$

W-boson boxes



$$C_A^{W,(n)} = C_A^{W,t,(n)} - C_A^{W,c,(n)}$$

due to the unitartity of the CKM matrix

- off-shell 1LPI amplitudes in full and effective theory
- external momenta to zero $\rightarrow$ tadpole diagrams
- spurious IR-divergences on both sides
  a) matching in $d \neq 4$ dimensions $\rightarrow$ evanescent operators
  b) matching in $d = 4$ dimensions $\rightarrow$ light quark masses as IR regulators

W-boson boxes: Matching in d dimensions

- spurious IR-divergences regulated with dimensional regularisation
- IR-divergences cancel in matching procedure
  BUT: additional poles in ϵ at intermediate steps
- evanescent operator [Misiak and Urban 1999]

$$Q_A^E = (\bar{b}\gamma_{\alpha_1}\gamma_{\alpha_2}\gamma_{\alpha_3}\gamma_5 s)(\bar{\mu}\gamma^{\alpha_3}\gamma^{\alpha_2}\gamma^{\alpha_1}\gamma_5\mu) - 4\,Q_A$$

  $Q_A^E = 0$ in $d = 4$ dimensions
- limit $d \to 4$ after matching

W-boson boxes: Matching in d dimensions

calculation on full theory side

- project on Dirac structures of Q_A and Q_A^E in full theory diagrams

$$\Rightarrow C_{A,\text{bare}}^{W,(n)} \text{ and } C_{A,\text{bare}}^{E,(n)}$$

- three-loop vacuum integrals with two different mass scales: m_t and M_W
- 1L and 2L exact mass dependence
- 3L results for different expansions
  a) Taylor expansion in mass difference: $M_W \approx m_t$
     $w = 1 - \frac{M_W^2}{m_t^2}$ up to order w^{16}
  b) Asymptotic expansion: $M_W \ll m_t$
     $y = \frac{M_W}{m_t}$ up to order y^{12}

generate Feynman diagrams: QGRAF [Nogueira 1993]
asymptotic expansion: Q2E/EXP
[Harlander, Seidensticker, Steinhauser 1998 and Seidensticker 1999]

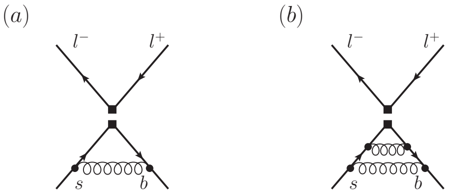
FORM package [Vermaseren 1991]
for 3L tadpoles with one mass scale: MATAD [Steinhauser 2001]

W-boson boxes: Matching in d dimensions

calculation on effective theory side

- in effective theory all loop diagrams vanish (massless tadpoles)
- but: UV counterterms are needed to get rid of IR-divergences on full theory side
- renormalization in effective theory

$$C_A Q_A + C_A^E Q_A^E \to Z_\psi \left(C_A Z_{NN} Q_A + C_A Z_{NE} Q_A^E + C_A^E Z_{EN} Q_A + C_A^E Z_{EE} Q_A^E \right)$$



- $Z_{NN} = 1$ (quark current conservation)
- $Z_{EN} = \alpha_s (\dots) + \alpha_s^2 (\dots) + \mathcal{O}(\alpha_s^3)$

W-boson boxes: Matching in d dimensions

matching formulae for Wilson coefficients: $(Q = c, t)$

$$C_A^{W,Q} = (1+\Delta Z_\psi) \sum_{n=0}^{2} \left(\frac{\alpha_s}{4\pi}\right)^n \left[\left(Z_g^{\mathsf{SM}}\right)^{2n} C_{A,\mathsf{bare}}^{W,Q,(n)} + \delta^{tQ} \Delta T^{W,t,(n)}\right]$$
$$-Z_{EN}\, C_A^{E,Q} + \mathcal{O}\left(\alpha_s^3\right)$$

- ΔZ_ψ: difference of wave-function renormalization constants in full and effective theory
- Z_g^{SM}: α_s renormalization constant
- ΔT: contributions from top-quark mass renormalization

$\alpha_s = \alpha_s^{(5)}$ is gauge coupling in 5-flavour QCD

W-boson boxes: Matching in $d = 4$ dimensions

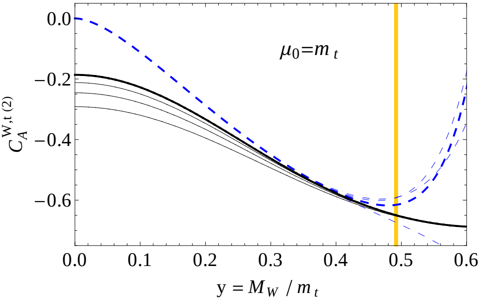
alternative approach for matching

- matching in $d = 4$ dimensions $\to$ no evanescent operator
- b- and s-quark masses as IR-regulators
- 1L and 2L loop amplitudes in effective theory
  asymptotic expansion: $m_s \ll m_b$
- in full theory two different expansions:
  $m_s \ll m_b \ll M_W \ll m_t$
  $m_s \ll m_b \ll M_W \approx m_t$
- renormalization of full theory and effective theory amplitudes
- matching of UV finite results in $d = 4$ dimensions
- after matching: $m_s \to 0$ and $m_b \to 0$

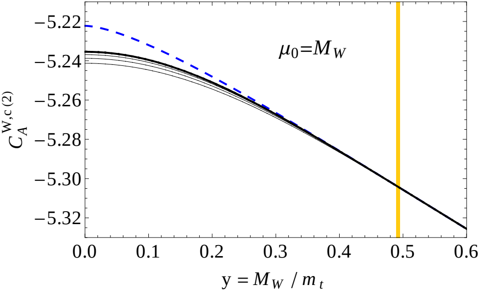
same results for matching in $d \neq 4$ and $d = 4$ dimensions

Results for $C_A^{W,(2)}$

top-quark contributions



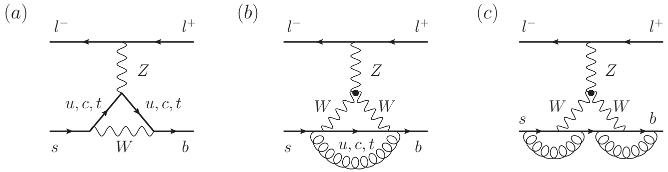
charm-quark contributions



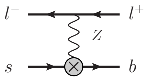
black solid lines:
$w = 1 - \frac{M_W^2}{m_t^2} \ll 1$
up to $\mathcal{O}\left(w^{16}\right)$

blue dashed lines:
$y = \frac{M_W}{m_t} \ll 1$
up to $\mathcal{O}\left(y^{12}\right)$

Z-boson penguins



- background field version of the 't Hooft-Feynman gauge for the electroweak bosons
- no contributions to evanescent Wilson coefficient C_A^E
- same expansions in masses as for W-boson boxes
- electroweak counterterm already at LO



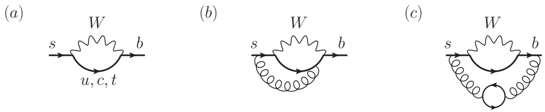
- quark triangle diagrams at three-loop level
  $\rightarrow$ special attention on γ_5

Z-boson penguins: Electroweak counterterm

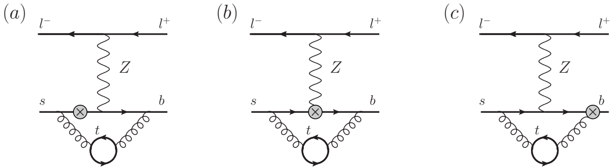
- electroweak counterterm Lagrangian

$$\mathcal{L}^{\rm ew}_{\rm counter} = i\,\frac{G_F M_W^2}{4\sqrt{2}\,\pi^2}\left(V_{cb}^* V_{cs}\, Z^c_{2,sb} + V_{tb}^* V_{ts}\, Z^t_{2,sb}\right)\bar{b}_L \not{D} s_L$$

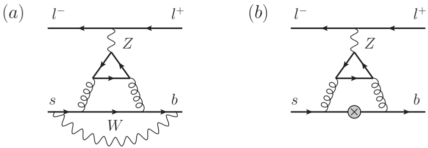
- determination of $Z^Q_{2,sb}$



- CT insertion sbZ into tree-level
- CT insertion sbZ, sbg and sb into 2L diagrams with top-quark loop



Z-boson penguins: Fermion triangle contributions



- γ_5 in axial-vector coupling in triangle loop
- trace evanescent operators [Gorbahn and Haisch 2005]
- no evaluation of trace in the triangle loop
  $\Rightarrow$ following structures in the results

$$\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5 \otimes \gamma_\mu\gamma_5 \ \mathrm{Tr}\,(\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\mu\gamma_5)$$
$$\gamma_\nu\gamma_\rho\gamma_\sigma \otimes \gamma_\mu\gamma_5 \ \mathrm{Tr}\,(\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\mu\gamma_5)$$

- trace evanescent operators: $Q^E_{1,2} = 0$ for $d = 4$

$$Q_1^E = (\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5 s)(\bar{\mu}\gamma_\mu\gamma_5\mu)\ \mathrm{Tr}\,(\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\mu\gamma_5) + 24\,(\bar{b}\gamma^\mu s)(\bar{\mu}\gamma_\mu\gamma_5\mu)$$
$$Q_2^E = (\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma s)(\bar{\mu}\gamma_\mu\gamma_5\mu)\ \mathrm{Tr}\,(\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\mu\gamma_5) + 24\,(\bar{b}\gamma^\mu\gamma_5 s)(\bar{\mu}\gamma_\mu\gamma_5\mu)$$

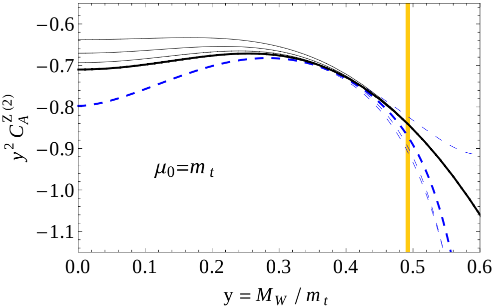
$\Rightarrow$ Wilson coefficients for Q_1^E, Q_2^E and Q_A

cross-check with Larin's method [Larin 1993]

Results for $C_A^{Z,(2)}$

$C_A^{Z,(2)}$ grows as m_t^2 for large values of m_t

$\Rightarrow y^2\, C_A^{Z,(2)}$ is plotted



black solid lines:

$w = 1 - \frac{M_W^2}{m_t^2} \ll 1$

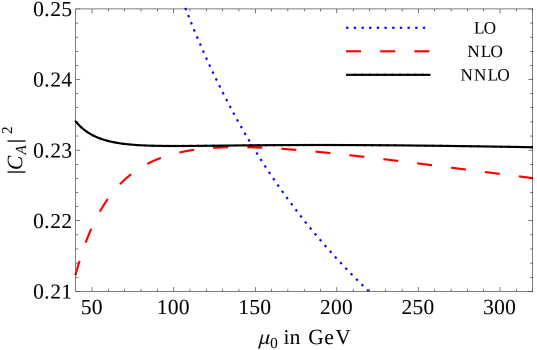
up to $\mathcal{O}\left(w^{16}\right)$

blue dashed lines:

$y = \frac{M_W}{m_t} \ll 1$

up to $\mathcal{O}\left(y^{12}\right)$

Matching scale dependence



$$\mathcal{B}|_{\text{NNLO}}(\mu_0 = m_t(m_t)) = 1.002 \cdot \mathcal{B}|_{\text{NLO}}(\mu_0 = m_t(m_t))$$
$$\mathcal{B}|_{\text{NNLO}}(\mu_0 = 80\,\text{GeV}) = 1.014 \cdot \mathcal{B}|_{\text{NLO}}(\mu_0 = 80\,\text{GeV})$$
$$\mathcal{B}|_{\text{NNLO}}(\mu_0 = 40\,\text{GeV}) = 1.10 \cdot \mathcal{B}|_{\text{NLO}}(\mu_0 = 40\,\text{GeV})$$

scale uncertainty for $\mu_0 \in [\frac{1}{2}m_t, 2m_t]$

NLO QCD $\Rightarrow 1.8\%$
NNLO QCD $\Rightarrow 0.2\%$

$\mathcal{B}(B_s \to \mu^+\mu^-)$ in the SM

$+$ known NLO EW corrections

[Buchalla and Buras 1998]
[Bobeth,Gambino,Gorbahn,Haisch 2004]
[Huber,Lunghi,Misiak,Wyler 2006]

missing full NLO EW corrections $\rightarrow$ $5\,\%$

some input parameters and uncertainties

- $f_{B_s} = (227.7 \pm 4.5)$ MeV $\rightarrow$ $4\,\%$ [FLAG]
- $|V_{tb}^* V_{ts}| = 0.0416 \pm 0.0009$ $\rightarrow$ $4\,\%$ [CKMfitter, Gambino and Schwanda 2013]
- $M_t = (173.1 \pm 0.9)$ GeV $\rightarrow$ $1.6\,\%$ [PDG]
- ...

adding errors in quadrature $\rightarrow$ $8\,\%$

$\mu_0 = m_t(m_t) = 163.5$ GeV

$$\mathcal{B}^{[t=0]}(B_s \to \mu^+\mu^-) = (\,3.62\ \pm 0.29\,) \times 10^{-9}$$

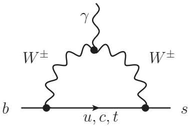
full NLO EW corrections
[C. Bobeth, M. Gorbahn, E. Stamou, to be published]

NNLO QCD corrections
[TH, M. Misiak, M. Steinhauser, to be published]

$\Bigg\}$ common update of BR in the SM

Introduction $\overline{B} \to X_s\gamma$

$$\Gamma\left(\overline{B} \to X_s\gamma\right)|_{E_\gamma > E_0} = \Gamma\left(b \to X_s^p\gamma\right)|_{E_\gamma > E_0} + \left(\begin{array}{c}\text{non-perturbative contributions}\\ \sim \pm 5\% \text{ [Benzke et al. 2010]}\end{array}\right)$$



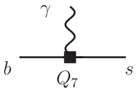
exp. world average: measured at CLEO, BELLE and BABAR

$$\mathcal{B}(\overline{B} \to X_s\gamma)|_{E_\gamma > 1.6\,\text{GeV}} = (3.43 \pm 0.22) \cdot 10^{-4}$$ [HFAG 2012]

SM NNLO prediction:

$$\mathcal{B}(\overline{B} \to X_s\gamma)|_{E_\gamma > 1.6\,\text{GeV}} = (3.15 \pm 0.23) \cdot 10^{-4}$$ [Misiak et al. 2006]

$$Q_7 = \frac{e}{16\pi^2} m_b \left(\bar{s}_L \sigma^{\mu\nu} b_R\right) F_{\mu\nu}$$
$$Q_8 = \frac{g}{16\pi^2} m_b \left(\bar{s}_L \sigma^{\mu\nu} T^a b_R\right) G^a_{\mu\nu}$$



Two Higgs Doublet Models

- SM + additional Higgs doublet (two Higgs doublets)
- physical basis: h, H, A and $H^\pm$
- interaction between charged Higgs $H^\pm$ and quarks:

$$\mathcal{L} = (2\sqrt{2}G_F)^{1/2} \sum_{i,j=1}^{3} \overline{u}_i \left(A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R \right) d_j H^+ + h.c.$$

2HDM Type II (e.g. MSSM):

$$A_u = -\frac{1}{A_d} = \frac{1}{\tan\beta} \quad \text{with} \quad \tan\beta = \frac{< \phi_2^0 >}{< \phi_1^0 >}$$

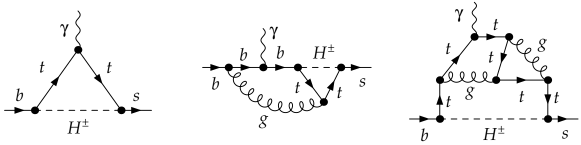
Wilson coefficients: $C_i = A_d\, A_u^\star \ldots + A_u\, A_u^\star \ldots$, terms with $A_d^\star$ are suppressed with m_s

1. matching: 2HDM
2. running/mixing: same as in SM [Misiak et al. 2006]
3. on-shell matrix elements: same as in SM

Calculation of C_7 and C_8 to 3L

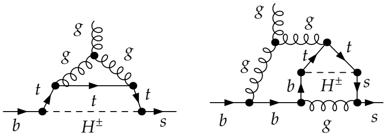
C_7: amputated 1LPI Green function $b \to s\,\gamma$ ~ 350 diagrams for 3L

(a) (b) (c)



C_8: amputated 1LPI Green function $b \to s\,g$ ~ 500 diagrams for 3L

(d) (e) (f)



Calculation of C_7 and C_8 to 3L

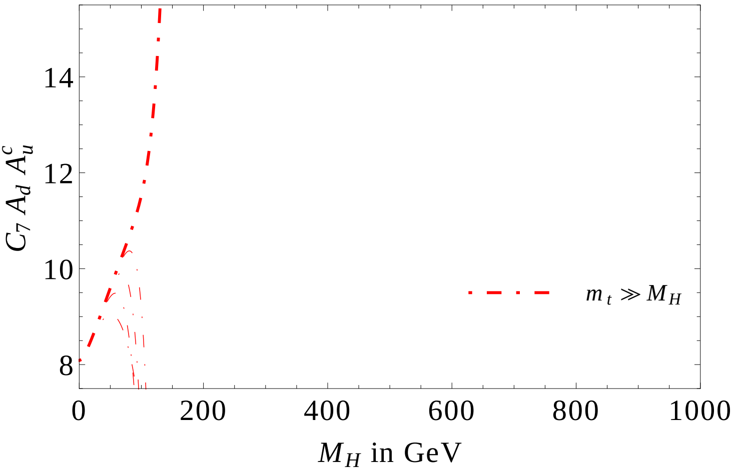
2 loops:
Wilson coefficients to 2 loops [Ciafaloni, Romanino and Strumia 1997] [Ciuchini, Degrassi, Gambino and Giudice 1997] [Borzumati and Greub 1998]

3 loops: [TH, Misiak, Steinhauser 2012]
three-loop vacuum integrals with two different mass scales m_t and $M_{H^\pm}$

1. $M_{H^\pm} \gg m_t$: asymptotic expansion $(m_t/M_{H^\pm})^{10}$
2. $M_{H^\pm} \approx m_t$: ordinary Taylor expansion $\left(M_{H^\pm}^2 - m_t^2\right)^{16}$
3. $M_{H^\pm} \ll m_t$: asymptotic expansion $(M_{H^\pm}/m_t)^{10}$

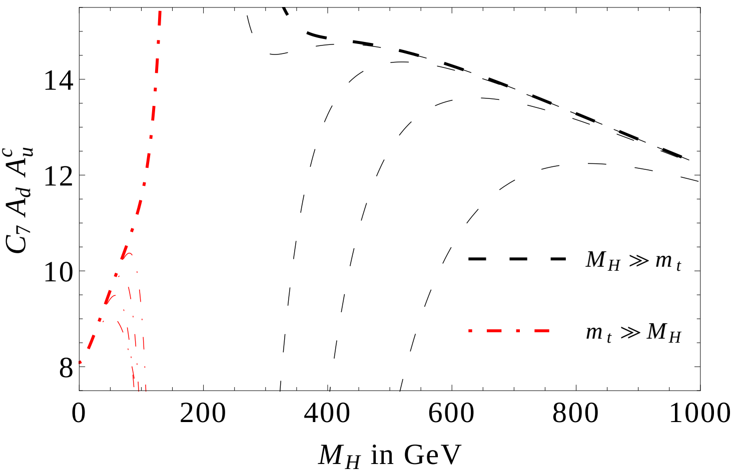
matching:
same matching formulae as in the SM [Misiak and Steinhauser 2004]
and similar to the C_A formulae

Results for C_7: $A_d A_u^\star$



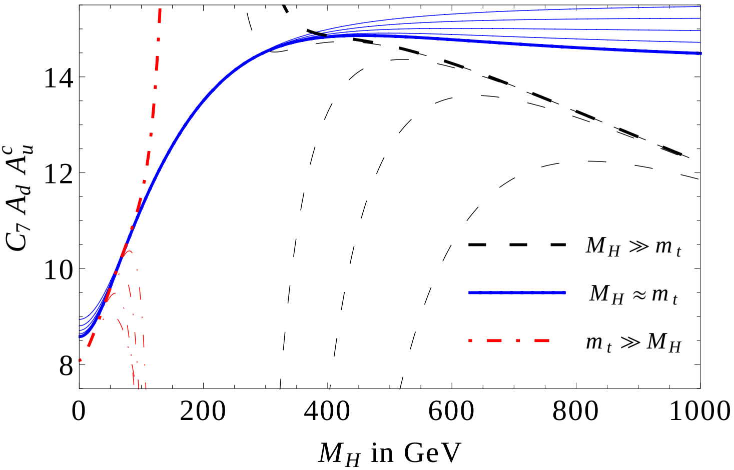
analog results for $C_{7,A_u A_u^\star}$, $C_{8,A_d A_u^\star}$ and $C_{8,A_u A_u^\star}$

Results for C_7: $A_d A_u^\star$



analog results for $C_{7,A_u A_u^\star}$, $C_{8,A_d A_u^\star}$ and $C_{8,A_u A_u^\star}$

Results for C_7: $A_d A_u^\star$

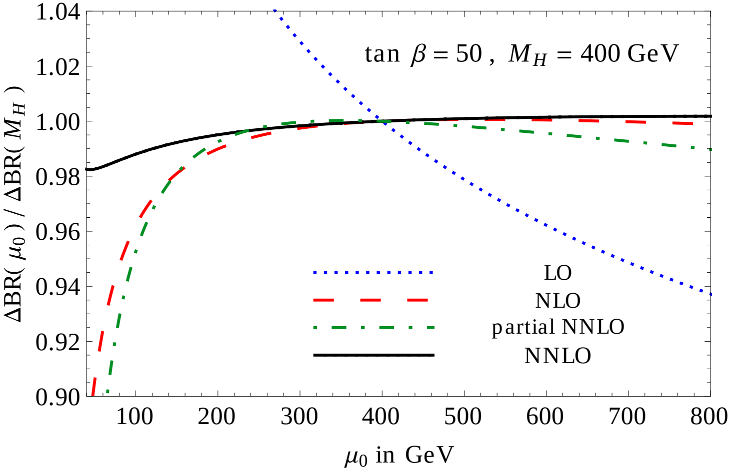


analog results for $C_{7,A_u A_u^\star}$, $C_{8,A_d A_u^\star}$ and $C_{8,A_u A_u^\star}$

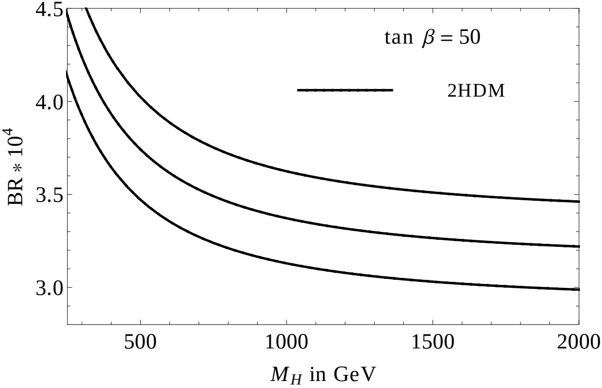
Matching scale dependence in 2HDM Type II

partial NNLO in 2HDM Type II without $C_{7,8}^{(2)\text{eff, 2HDM}}(\mu_0)$ [Misiak et al. 2006]

$\Delta\mathcal{B} \equiv \mathcal{B}_{\text{2HDM}} - \mathcal{B}_{\text{SM}}$

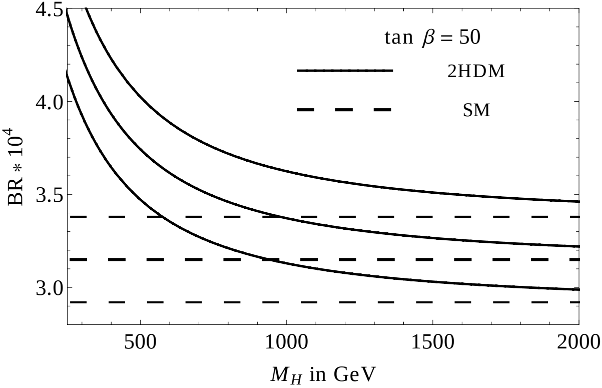


Uncertainty band in 2HDM Type II



$\tan\beta > 2$: $\mathcal{B}$ is almost independet of $\tan\beta$
$\tan\beta < 2$: $\mathcal{B}$ increases for smaller $\tan\beta \Rightarrow$ strengthen lower limit on M_{H^+}

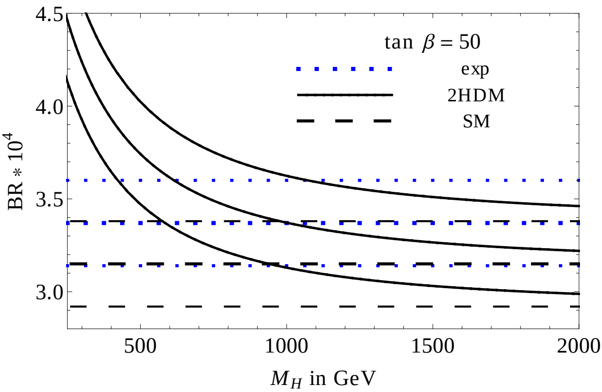
Uncertainty band in 2HDM Type II



$\tan\beta > 2$: $\mathcal{B}$ is almost independet of $\tan\beta$

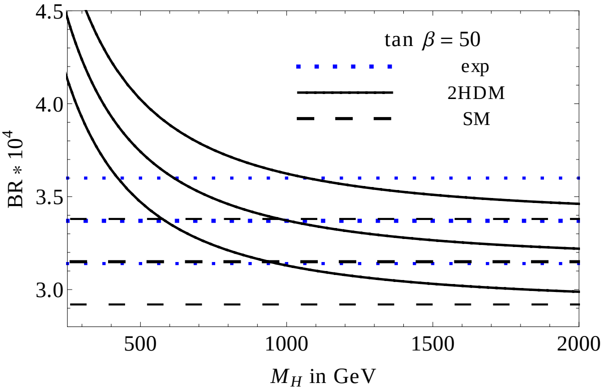
$\tan\beta < 2$: $\mathcal{B}$ increases for smaller $\tan\beta \Rightarrow$ strengthen lower limit on M_{H^+}

Uncertainty band in 2HDM Type II



$\tan\beta > 2$: $\mathcal{B}$ is almost independet of $\tan\beta$
$\tan\beta < 2$: $\mathcal{B}$ increases for smaller $\tan\beta \Rightarrow$ strengthen lower limit on M_{H^+}

Uncertainty band in 2HDM Type II



$\tan\beta > 2$: $\mathcal{B}$ is almost independet of $\tan\beta$
$\tan\beta < 2$: $\mathcal{B}$ increases for smaller $\tan\beta \Rightarrow$ strengthen lower limit on M_{H^+}

Lower bound

$$M_{H^+} \geq 360\,\text{GeV with } 95\% \text{ CL}$$

Conclusion

$\mathcal{B}(B_s \to \mu^+\mu^-)$ in the SM

- three-loop corrections to C_A
- $\mathcal{B}(B_s \to \mu^+\mu^-)$ to NNLO accuracy in QCD
  $\Rightarrow$ no remaining QCD scale uncertainty
- full NLO EW corrections [C. Bobeth, M. Gorbahn, E. Stamou, to be published]
  common update of branching ratio (work in progress)
  $\Rightarrow$ significant reduction of perturbative uncertainties

$\mathcal{B}(\overline{B} \to X_s\gamma)$ in 2HDMs

- C_7 and C_8 to three-loop order in Two Higgs Doublet Models
  $\Rightarrow$ consistent NNLO estimation in 2HDMs
- reduction of matching scale dependence
- lower bound on charged Higgs mass in 2HDM Type II:

$$M_{H^+} \geq 360\,\text{GeV with } 95\%\text{ CL}$$