

# Electroweak corrections to $Z + 2$ jets production at the LHC

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- Motivation
- COLLIER: a Fortran library for tensor integrals
- Electroweak corrections to  $pp \rightarrow Z + 2$  jets
- Conclusion

- **Discovery of Higgs boson:** spectacular success of Standard Model
- tasks for the future:
  - ▶ precise investigation of Higgs boson
  - ▶ precise study of other Standard Model processes
  - ▶ search for physics beyond Standard Model
- **QCD corrections are indispensable for decent predictions**
- **automation of NLO QCD corrections performed by many groups, e.g.**
  - BLACKHAT Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maître
  - NJET Badger, Biedermann, Uwer, Yundin
  - HELACNLO Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek
  - AMC@NLO Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau
  - GoSAM Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
  - OPENLOOPS Cascioli, Maierhöfer, Pozzorini

- generically:  $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2) \sim \text{few } \%$
- Electroweak (EW) corrections can be enhanced
  - ▶ high energy scales,  $E \gg M_W \Rightarrow$  Sudakov logarithms  $\ln^2 \left( \frac{E^2}{M_W^2} \right)$   
 $\Rightarrow$  corrections of several 10% for  $E \sim 1 \text{ TeV}$   
 tails of distributions  
 M. Ciafaloni, P. Ciafaloni, Comelli; Beccaria, Renard, Verzegnassi; Beenakker, Werthenbach; Denner, Pozzorini; Melles; Fadin, Lipatov, Martin; Hori, Kawamura, Kodaira; Jantzen, Kühn, Penin, Smirnov; Chiu, Fuhrer, Golf, Kelley, Manohar, ...
  - ▶ kinematic effects, e.g. photonic corrections near resonances  
 $\Rightarrow$  radiative tails
  - ▶ Higgs production in vector-boson fusion:  
 EW and QCD corrections have same order of magnitude ( $\sim 5\%$ )  
 Ciccolini, Denner, Dittmaier '07
- Les Houches wishlist 2013:  
 NNLO QCD and NLO EW for various processes desired

## Vector-boson + jets production

- important for tests of QCD and Standard Model
- important backgrounds for Higgs and new physics searches (Higgs, supersymmetry, ...)
- NLO QCD corrections exist for  $Z + 4j$ ,  $W + 5j$  Blackhat

## Z + j, W + j production

- electroweak corrections available Denner, Dittmaier, Kasprzik, Mück '09, '11, '12

## Z + 2 jets production

- background to Higgs production in vector-boson fusion  
⇒ study of systematics for  $H + jj$  final state
- nontrivial study case for (automatized) calculation of electroweak NLO corrections
- part of Les Houches wish list 2013
- electroweak corrections for  $\nu\bar{\nu} + 2$  jets in Sudakov limit Chiesa et al. '13

# COLLIER

## a Fortran library for tensor integrals

General form of one-loop amplitudes (free of unphysical singularities)

$$\delta\mathcal{M} = \sum_j \sum_{R_j} c_{\mu_1 \dots \mu_{R_j}}^{(j, R_j, N_j)} T_{(j, R_j, N_j)}^{\mu_1 \dots \mu_{R_j}} = \sum_j d^{(j, N_j)} T_{(j, 0, N_j)}$$

tensor integrals

$$T_{(j, R_j, N_j)}^{\mu_1 \dots \mu_{R_j}} = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{q^{\mu_1} \dots q^{\mu_{R_j}}}{D_{j,0} \dots D_{j, N_j-1}}, \quad D_{j,a} = (q + p_{j,a})^2 - m_{j,a}^2$$

$c_{\mu_1 \dots \mu_{R_j}}^{(j, R_j, N_j)}$  free of,  $d^{(j, N_j)}$  involve unphysical singularities

proposal of van Hameren '09:

calculate  $c_{\mu_1 \dots \mu_{R_j}}^{(j, R_j, N_j)}$  numerically in a recursive way

implemented for full Standard Model in RECOLA  $\Rightarrow$  talk by Sandro Uccirati  
(Recursive calculation of one-loop amplitudes)

evaluation of tensor integrals by COLLIER Denner, Dittmaier, Hofer, in preparation  
(Complex one loop library in extended regularizations)

Different methods used depending on number  $N$  of propagators

- $N = 1, 2$ : explicit analytical expressions (no reduction, numerically stable)
- $N = 3, 4$ : exploit Lorentz covariance  
different methods used depending on kinematics

- ▶ standard Passarino–Veltman (PV) reduction

Passarino, Veltman '79

- ▶ stable expansions in exceptional phase-space regions  
(small Gram determinants)

Denner, Dittmaier '05 (see also R.K.Ellis et al. '05; Binoth et al. '05; Ferroglia et al. '02)

- $N \geq 5$ : exploit 4-dimensionality of space-time  
direct reduction of  $T^{N,R}$  to  $T^{N-1,R-1}$  (free of inverse Gram determinants)

Melrose '65; Denner, Dittmaier '02,'05; Binoth et al. '05; Diakonidis et al. '08,'09

⇒ fast and stable numerical reduction algorithm

Basic scalar integrals  $A_0, B_0, C_0, D_0$  from explicit analytical expressions

't Hooft, Veltman '79; Beenakker, Denner '90; Denner, Nierste, Scharf '91; Ellis, Zanderighi '08;

Denner, Dittmaier '11

## Passarino–Veltman reduction

covariant decomposition of tensor integrals:

$$T^{N, \mu_1 \dots \mu_R} = \sum_{i_1, \dots, i_k} \underbrace{T_{0 \dots 0}^{N, R}}_{R-k}{}_{i_1 \dots i_k} \underbrace{\{g \dots g\}}_{(R-k)/2} p_{i_1} \dots p_{i_k} \}^{\mu_1 \dots \mu_R} \sim \int d^D q \frac{q^{\mu_1} \dots q^{\mu_R}}{D_{j,0} \dots D_{j,N_j-1}}$$

contract tensor integral with external momenta  $p_i^\mu$  and metric tensor  $g^{\mu\nu}$  and use

$$2p_i^\mu q_\mu = -f_i + D_i - D_0, \quad f_i = p_i^2 - m_i^2 + m_0^2$$

$$g^{\mu\nu} q_\mu q_\nu = m_0^2 + D_0$$

cancel denominators and insert covariant decomposition

⇒ recursive solution for tensor coefficients:

$$\Delta T^{N, R} = \left[ T^{N, R-1}, T^{N, R-2}, T^{N-1} \right]$$

reduction to **lower-rank** and **lower-point** integrals

Gram determinant:  $\Delta = \det(Z)$  with  $Z_{ij} = 2p_i p_j$ ,  $Z^{-1} = \tilde{Z} / \Delta$



$$2(D + R - N - 1) \underbrace{T_{00i_3 \dots i_R}^N}_{\text{rank } R} = 2m_0^2 \underbrace{T_{i_3 \dots i_R}^N}_{\text{rank } R-2} + \sum_{n=1}^{N-1} f_n \underbrace{T_{ni_3 \dots i_R}^N}_{\text{rank } R-1} + (T^{N-1} \text{ terms})$$

$$\underbrace{\Delta T_{i_1 \dots i_R}^N}_{\text{rank } R} = \sum_{n=1}^{N-1} \tilde{Z}_{i_1 n} \left[ -f_n \underbrace{T_{i_2 \dots i_R}^N}_{\text{rank } R-1} - 2 \sum_{r=2}^R \delta_{ni_r} \underbrace{T_{00i_2 \dots \hat{i}_r \dots i_R}^N}_{\text{rank } R} + (T^{N-1} \text{ terms}) \right]$$

$i_1 \neq 0$

↪ recursive calculation of  $T_{i_1 \dots i_R}^N$  from scalar integral  $T_0^N$  and  $T_{i_2 \dots i_R}^{N-1}$ :

$$T_0^N = \text{basis integral} \quad \rightarrow \quad T_{i_1}^N \quad \rightarrow \quad T_{i_1 i_2}^N \quad \rightarrow \quad T_{i_1 i_2 i_3}^N \quad \rightarrow \quad \dots$$

- explicit  $D$  requires expansion of  $T_{00 \dots}^N$  around  $D = 4$ 
  - ▶ UV-poles produce finite polynomial terms (rational terms) [ $\mathcal{O}(D - 4)/(D - 4)$ ] easily obtained from recursion relations (no  $\Delta^{-1}$  since  $T_{i_1 \dots i_R}^N$  finite for  $i_j \neq 0$ )
  - ▶  $T_{00 \dots}^N$  do not involve IR poles, reduction valid for any IR regularization
- appearance of inverse Gram determinant  $\Delta$ 
  - ⇒ potential instabilities for  $\Delta \rightarrow 0$  in exceptional points

$$\text{PV: } \Delta T^{N,R} = [T^{N,R-1}, T^{N,R-2}, T^{N-1}]$$

small Gram determinant:  $\Delta \rightarrow 0$

- finite  $T^{N,R}$  as sum of  $1/\Delta$ -singular terms
  - ▶ spurious singularities cancel to give  $\mathcal{O}(\Delta)/\Delta$ -result
  - ▶ **numerical determination of  $T^{N,R}$  becomes unstable**
- $T^{N,R-1}, T^{N,R-2}, T^{N-1}$  become linearly dependent
  - ⇒ scalar integrals  $D_0, C_0, B_0, A_0$  become linearly dependent
  - ⇒  $\mathcal{O}(\Delta)/\Delta$ -instabilities intrinsic to all methods relying on the full set of basis integrals  $D_0, C_0, B_0, A_0$
- solution: choose appropriate set of base functions depending on phase-space point

$$\Delta T^{N,R+1} = [T^{N,R}, T^{N,R-1}, T^{N-1}] \quad (1)$$

- exploit linear dependence of  $T^{N,R}, T^{N,R-1}, T^{N-1}$  for  $\Delta = 0$  to determine  $T^{N,R}$  up to terms of  $\mathcal{O}(\Delta)$
- calculate  $T^{N,R+1}$  from  $\Delta T^{N,R+2} = [T^{N,R+1}, T^{N,R}, T^{N-1}]$  in the same way
- use  $T^{N,R+1}$  in (1) to compute  $\mathcal{O}(\Delta)$  in  $T^{N,R}$
- higher orders in  $\Delta$  iteratively:  
 $\mathcal{O}(\Delta^k)$  of  $T^{N,R}$  requires lower-point  $T^{N-1}$  up to rank  $R + k$
- basis of scalar integrals effectively reduced  
(e.g.  $D_0$  expressed by  $C_0$ 's)

Expansion breaks down in certain regions of phase space

⇒ alternative expansions

- implementation of **tensor integrals** for (in principle) **arbitrary** number of external momenta  $N$   
(tested in physical processes up to  $N = 6$ )
- various **expansion methods** for exceptional phase-space points  
(to **arbitrary order** in expansion parameter)
- **mass- and dimensional regularization** supported for IR singularities
- **complex masses** supported (unstable particles)
- **cache-system** to avoid recalculation of identical integrals
- output: coefficients  $T_{0\dots 0i_1\dots i_k}^N$  or tensors  $T^{N,\mu_1\dots\mu_R}$
- two independent implementations  $\Rightarrow$  checks during run possible
- COLLIER contains **complete set of one-loop scalar integrals**
- used in RECOLA and OPENLOOPS **Cascioli, Maierhöfer, Pozzorini**

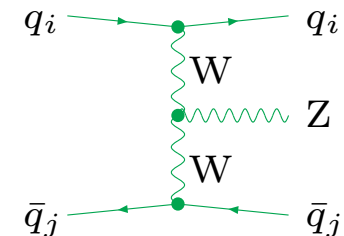
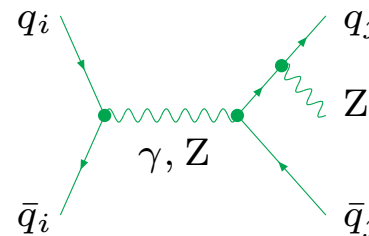
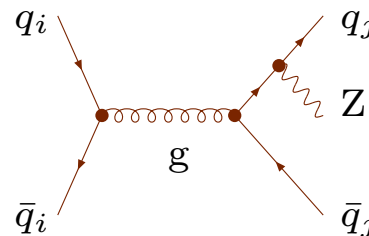
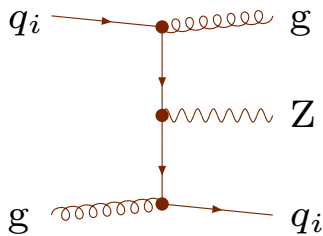
$pp \rightarrow Z + 2 \text{ jets production}$

## Partonic subprocesses + crossings

- gluon:  $qg \rightarrow qgZ: \sigma \sim \mathcal{O}(\alpha_s^2 \alpha)$
- four-quark:  $q\bar{q} \rightarrow q\bar{q}Z: \sigma \sim \mathcal{O}(\alpha_s^2 \alpha), \mathcal{O}(\alpha_s \alpha^2), \mathcal{O}(\alpha^3)$

## Sample diagrams

**QCD**,  $\mathcal{O}(g_s^2 e)$  and **EW**,  $\mathcal{O}(e^3)$  contributions



## Contributions (pp → Z + 2j)

Process class	$\sigma^{\text{LO}}$ [pb]	$\sigma^{\text{LO}} / \sigma_{\text{tot}}^{\text{LO}}$ [%]
gluon	1540.4(2)	80.02
four-quark	384.41(4)	19.98
sum	1924.8(2)	100.00

**Partonic channels:**  $gg, gq, g\bar{q}, q\bar{q}, qq, \bar{q}\bar{q}, \gamma q, \gamma\bar{q}$  (photon induced), contribution  $< 0.04\%$  for 8/13 TeV,  $q = u, c, d, s, b$

## Basic channels for $pp \rightarrow Zjj$

	$ug \rightarrow ugZ$ gluon	$us \rightarrow usZ$ 4-q NC	$us \rightarrow dcZ$ 4-q CC	
# LO diagrams	8	4 + 8	5	$\mathcal{O}(g_s^2 e), \mathcal{O}(e^3)$
# real photon diagrams	30	20 + 40	35	$\mathcal{O}(g_s^2 e^2), \mathcal{O}(e^4)$
# real gluon diagrams	50	24 + 40	24	$\mathcal{O}(g_s^3 e), \mathcal{O}(g_s e^3)$

## basic channels for $pp \rightarrow jjl^+l^-$

	$ug \rightarrow ugl^+l^-$	$us \rightarrow usl^+l^-$	$us \rightarrow dcl^+l^-$	
# LO diagrams	16	8 + 24	11	$\mathcal{O}(g_s^2 e^2), \mathcal{O}(e^4)$
# real photon diagrams	92	56 + 168	94	$\mathcal{O}(g_s^2 e^3), \mathcal{O}(e^5)$
# real gluon diagrams	100	48 + 112	52	$\mathcal{O}(g_s^3 e^2), \mathcal{O}(g_s e^4)$

## all channels can be constructed via

- replacement of quark pairs by different flavour, e.g.  $u \rightarrow d$
- combination of basic channels, e.g.  $ud \rightarrow udZ$  or  $uu \rightarrow uuZ$
- crossing of quarks, gluons and photons

Tower of contributions to  $\sigma$ : (additional  $\alpha$  for Z decay)

- $\mathcal{O}(\alpha_s^3\alpha)$ : QCD corrections to QCD diagrams Campbell, Ellis, Rainwater '02, '03
- $\mathcal{O}(\alpha_s^2\alpha^2)$ : this work
  - ▶ EW corrections to QCD diagrams
  - ▶ QCD corrections to EW–QCD interferences
- $\mathcal{O}(\alpha_s\alpha^3)$ :
  - ▶ EW corrections EW–QCD to interferences
  - ▶ QCD corrections to EW diagrams Oleari, Zeppenfeld '04
- $\mathcal{O}(\alpha^4)$ : EW corrections to EW diagrams

**first step:** EW corrections to gluon channels with stable Z  
(dominant contributions)

Actis, Denner, Hofer, Scharf, Uccirati '12

**preliminary:** complete  $\mathcal{O}(\alpha_s^2\alpha^2)$  corrections including leptonic Z decay



- $G_\mu$  scheme for electromagnetic coupling:

$$\alpha_{G_\mu} = \frac{\sqrt{2}G_\mu M_W^2}{\pi} \left( 1 - \frac{M_W^2}{M_Z^2} \right)$$

⇒ absorbs running of  $\alpha$  to EW scale and some universal  $m_t^2$  corrections

- 't Hooft–Feynman gauge
- complex-mass scheme for Z-boson resonances

Denner, Dittmaier, Roth, Wackerth, Wieders '99, '05

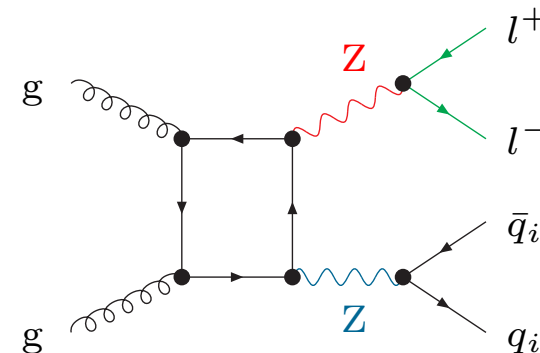
complex pole:  $\mu_Z^2 = M_Z^2 - iM_Z\Gamma_Z$

⇒ complex EW mixing angle

external Z boson treated as stable:

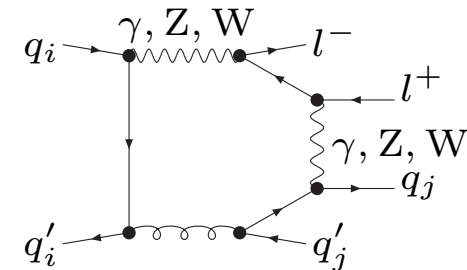
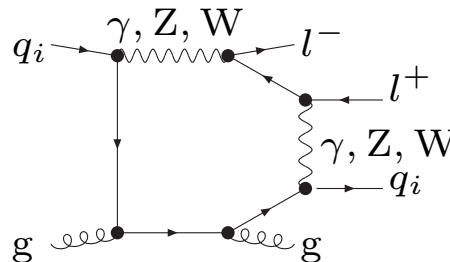
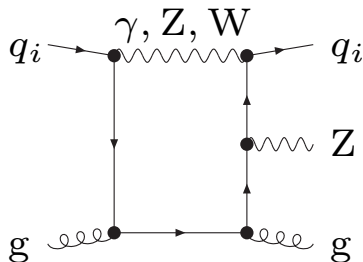
$$p^2 = M_Z^2$$

(not relevant for off-shell calculation)



# of loop diagrams contributing to  $\sigma$  in  $\mathcal{O}(\alpha_s^2\alpha^2)$  ( $\mathcal{O}(\alpha_s^2\alpha^3)$  including Z decay)

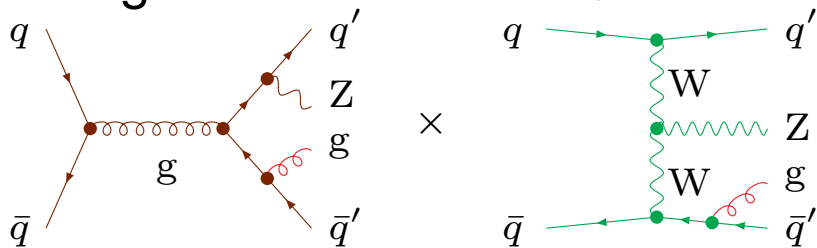
	$ug \rightarrow ugZ$	$ug \rightarrow ugl^+l^-$	$us \rightarrow usl^+l^-$	$us \rightarrow dcl^+l^-$
order	$\mathcal{O}(g_s^2e^3)$	$\mathcal{O}(g_s^2e^4)$	$\mathcal{O}(g_s^4e^2) + \mathcal{O}(g_s^2e^4)$	$\mathcal{O}(g_s^2e^4)$
loop diagrams	$\sim 300$	$\sim 1200$	$\sim 150 + 800$	$\sim 120$
hexagons	0	18	0 + 32	4
pentagons	20	85	8 + 50	24



- most complicated topology: **pentagon of rank 4** for on-shell Z  
**hexagon of rank 4** if Z decay included
- finite top-quark-mass effects:
  - fully included in closed fermion loops
  - neglected in contributions with external bottom quarks  
( $bg \rightarrow bgZ$ ,  $b\bar{b} \rightarrow ggZ$ ,  $gg \rightarrow b\bar{b}Z$ , LO contributions at per-cent level)

contributions to  $\sigma$  in  $\mathcal{O}(\alpha_s^2\alpha^2)$  ( $\mathcal{O}(\alpha_s^2\alpha^3)$  including Z decay)

- real photon emission from LO QCD contributions
- real gluon emission in **QCD–EW** interferences



soft and collinear singularities

- **Catani–Seymour dipole subtraction** Catani, Seymour '96
- initial-state collinear singularities cancelled by  $\overline{\text{MS}}$  redefinition of PDFs
- **recombination of collinear parton-photon pairs**  
 $\Rightarrow$  cancellation of singularities from collinear photon emission from quarks
- (soft-gluon) IR divergences in  $Z_{jj\gamma}$  related to virtual QCD corrections to  $Z_j\gamma$  (soft gluon recombined with hard photon):  
 eliminated via cut on photon energy fraction  $z_\gamma$  in jet and **photon fragmentation function contribution**  
 compare Denner, Dittmaier, Gehrmann, Kurz '10, Denner, Dittmaier, Kasprzik, Mück '09

## Setup for calculation with recursive method

- (tree-level and one-loop) matrix elements with [RECOLA](#)
- tensor integrals with [COLLIER](#)
- phase-space integration with in-house [multi-channel Monte Carlo](#)

## Check with independent calculation based on conventional methods (setup used for calculation of EW corrections to $Z + j$ , $W + j$

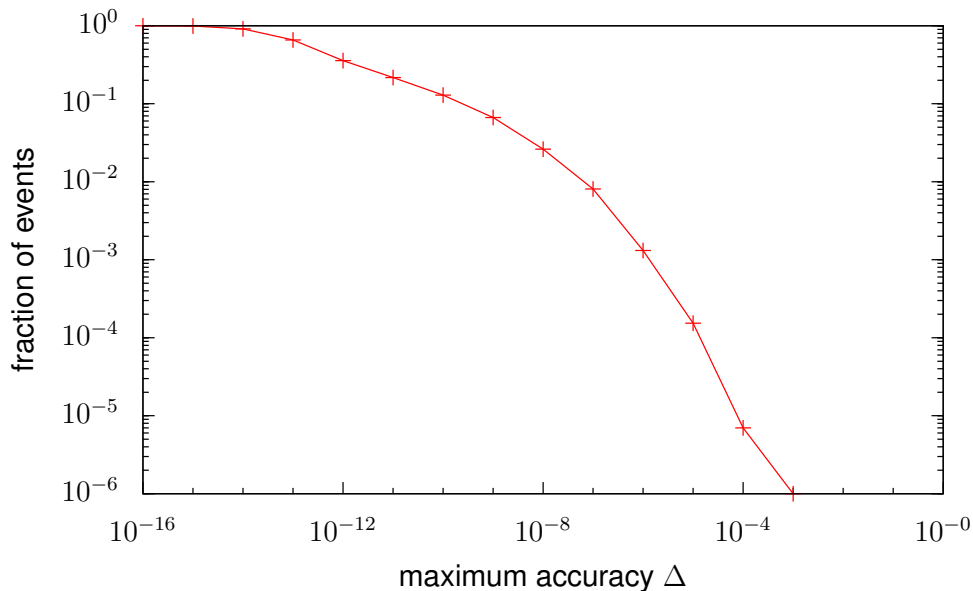
[Denner, Dittmaier, Kasprzik, Mück '09, '11, '12](#))

- matrix elements with [FEYNARTS/FORMCALC/POLE](#) [Hahn et al. '99, '01; Meier '06](#)
- tensor integrals with [COLLIER](#) (second independent implementation)
- phase-space integration with [LUSIFER](#) [Dittmaier, Roth '02](#)

comparison of results by **RECOLA** and **POLE**: ( $pp \rightarrow Zjj$ )  
 ( $5 \times 10^6/10^8$  and  $5 \times 10^5/10^7$  virtual/real events)

Process class	virtual [fb]	$ \mathbf{R}/\mathbf{P} - 1 $ [%]	real [fb]	$ \mathbf{R}/\mathbf{P} - 1 $ [%]
$qg \rightarrow qgZ,$ $\bar{q}g \rightarrow \bar{q}gZ$	$-14463 \pm 10$ $-14499 \pm 27$	$0.3 \pm 0.2$	$-825 \pm 9$ $-841 \pm 22$	$2 \pm 3$
$q\bar{q} \rightarrow ggZ$	$-1395 \pm 2$ $-1406 \pm 7$	$0.8 \pm 0.5$	$118 \pm 1$ $118 \pm 1$	$0.01 \pm 1$
$gg \rightarrow q\bar{q}Z$	$-1024 \pm 2$ $-1018 \pm 3$	$0.5 \pm 0.4$	$-186 \pm 1$ $-187 \pm 1$	$0.7 \pm 0.9$

comparison of virtual squared matrix element for  $10^6$  events:



- typical agreement:  
 $10^{-11} - 10^{-14}$
- less than **0.02%** of points with agreement worse than  $10^{-5}$

EW corrections small for total cross section:  $\sim -2.3\%$   
similar for all channels

## PRELIMINARY

process class	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{LO}} / \sigma_{\text{tot}}^{\text{LO}}$ [%]	$\sigma_{\text{EW}}^{\text{NLO}}$ [fb]	$\frac{\sigma_{\text{EW}}^{\text{NLO}}}{\sigma^{\text{LO}}} - 1$ [%]
gluonic	17948(4)	77.3	17534(4)	-2.31
four-quark	5270.0(5)	22.7	5139.4(7)	-2.48
sum	23218(4)	100	22674(4)	-2.34

## Setup

Jet clustering: anti- $k_T$  algorithm with  $\Delta R = 0.4$

cuts:

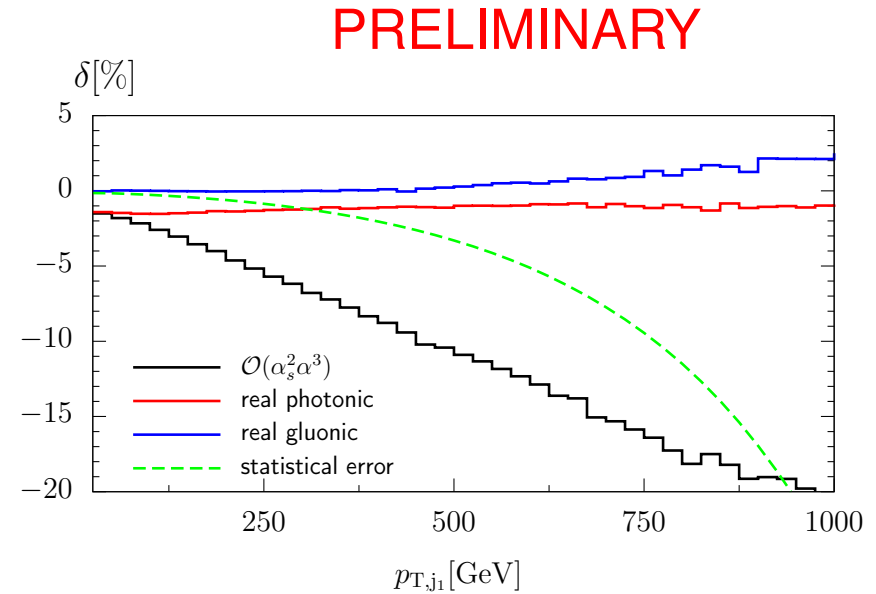
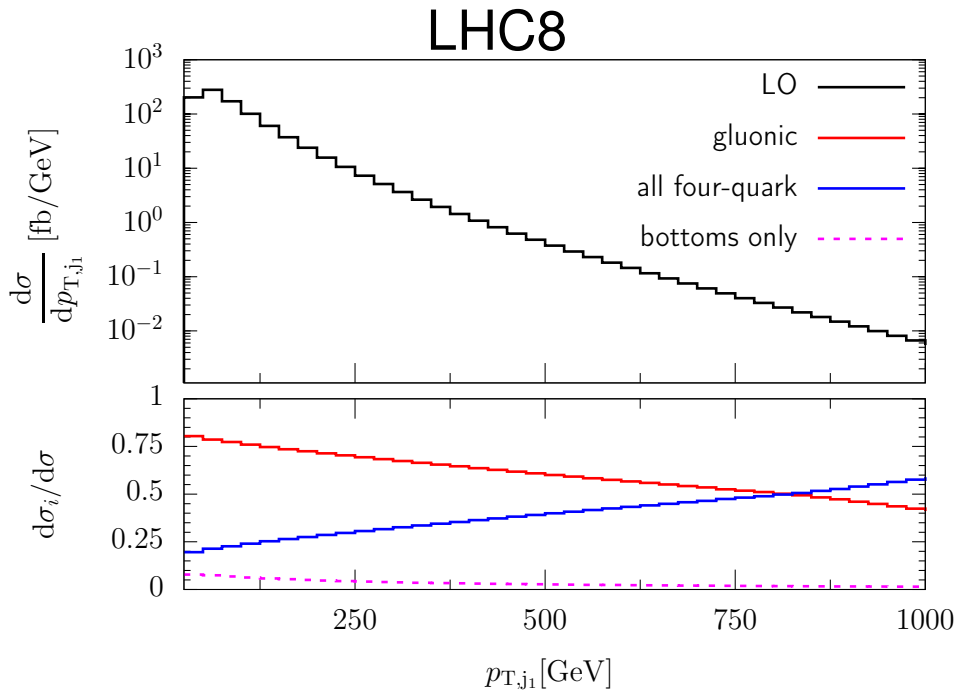
$$p_{T,j} > 30 \text{ GeV}, \quad |\eta_j| < 4.5$$

$$P_{T,\tau} > 20 \text{ GeV}, \quad |\eta_\tau| < 2.5$$

$$\Delta R_{\tau^+\tau^-} > 0.2, \quad 66 \text{ GeV} < M_{\tau^+\tau^-} < 116 \text{ GeV}$$

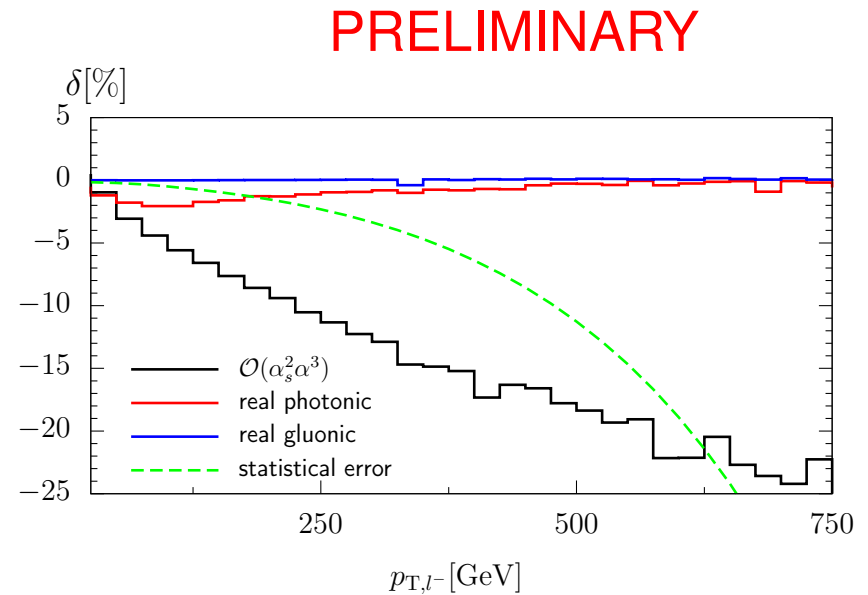
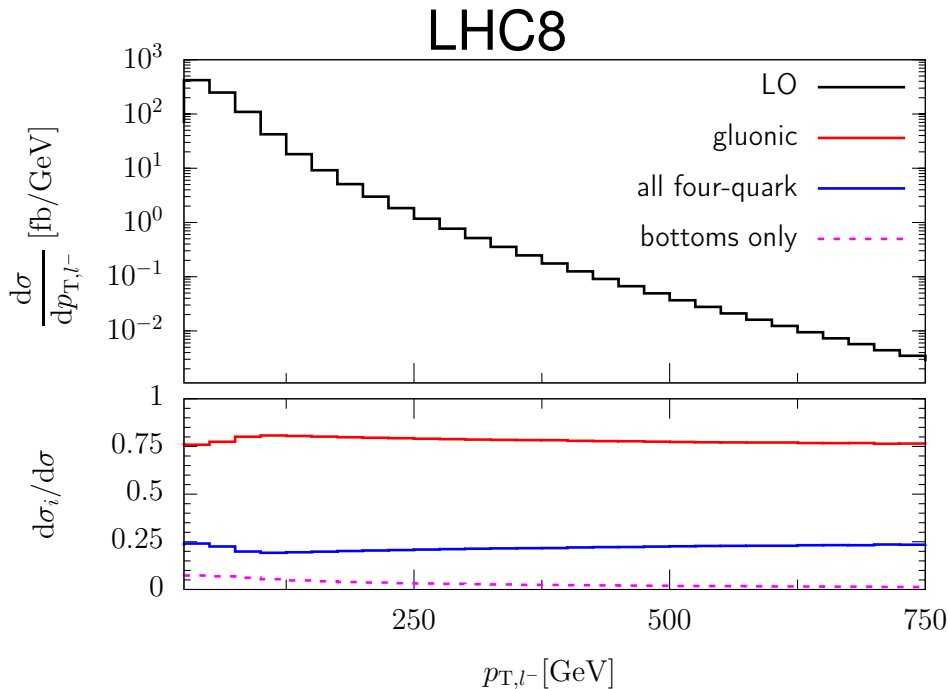
$$\Delta R_{j\tau^-} > 0.5, \quad \Delta R_{j\tau^+} > 0.5$$

$$\text{photon energy fraction in jet } z_\gamma < 0.7$$



statistical error based on  $20 \text{ fb}^{-1}$

- 4-quark channels dominate for high  $p_T$
- bottom contributions below 5%
- EW corrections sizeable for large  $p_T$ , dominated by virtual corrections (Sudakov logarithms)
- subtracted real corrections small ( $\lesssim 2\%$ )

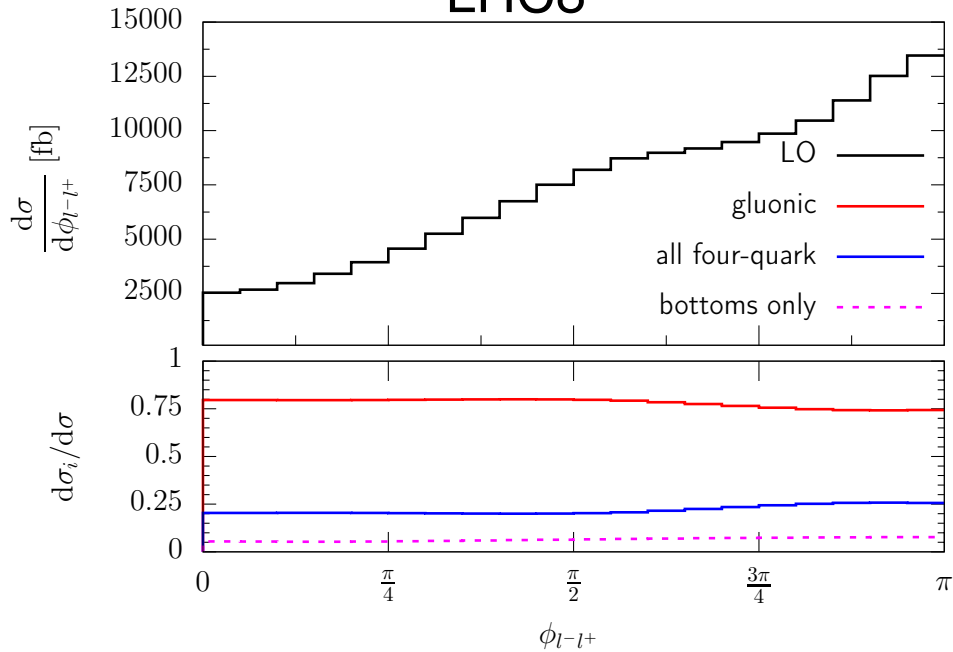


statistical error based on  $20 \text{ fb}^{-1}$

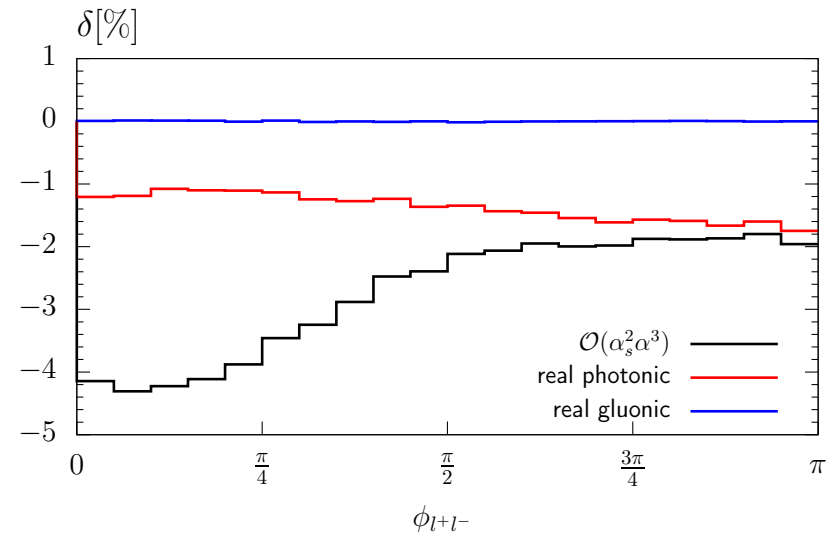
- gluon channels dominate for all  $p_{T,l}$
- bottom contributions below 5%
- EW corrections  $-25\%$  for  $p_{T,l^-} = 750 \text{ GeV}$   
dominated by virtual corrections (Sudakov logarithms)
- subtracted real corrections small ( $\lesssim 2\%$ )



LHC8



PRELIMINARY



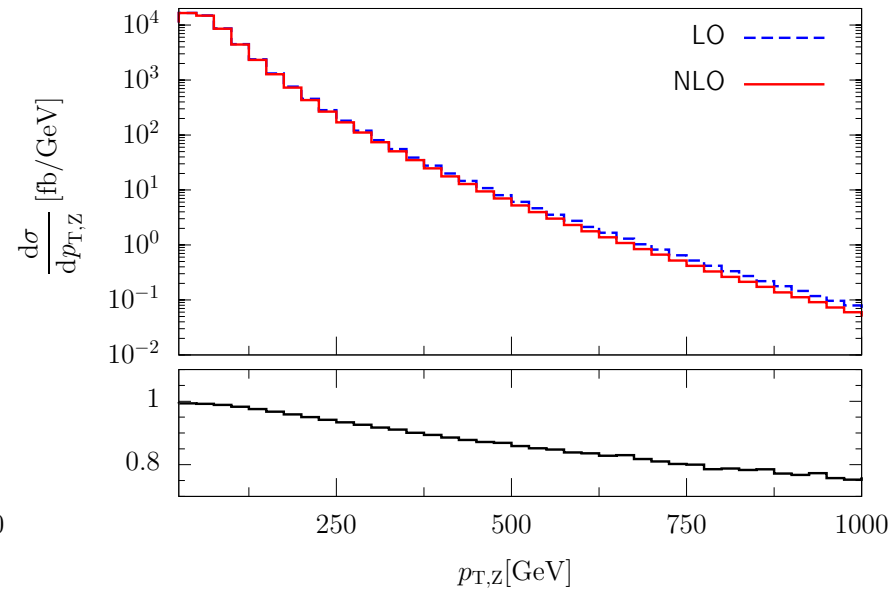
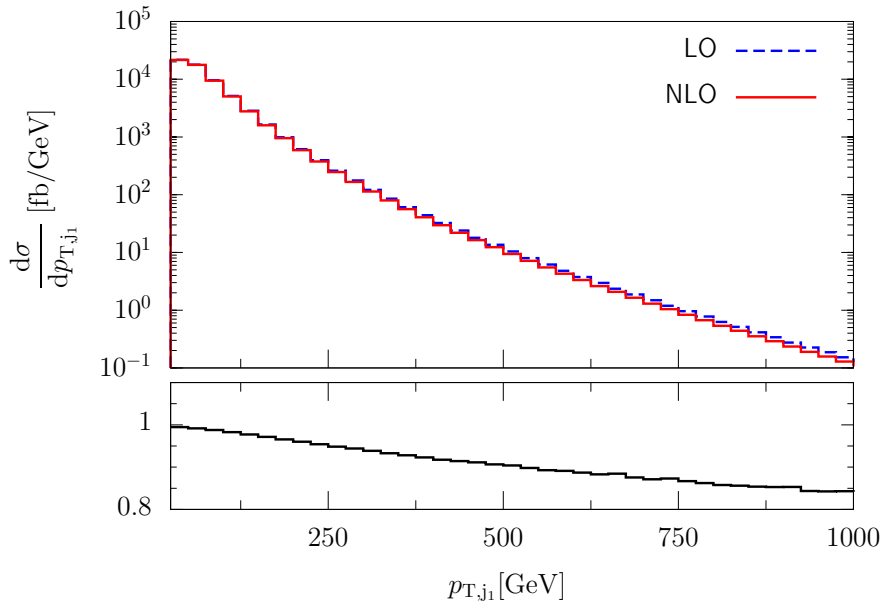
- distribution peaked in backward direction
- bottom contributions below 5%
- EW corrections distort distribution by 2%

- Electroweak corrections relevant for many LHC processes
- general tools for their calculation:
  - ▶ **COLLIER**: fast and numerically stable calculation of one-loop tensor integrals
  - ▶ **RECOLA**: recursive generator for tree-level and one-loop amplitudes in the full Standard Model (including EW corrections)
- Electroweak corrections to  $pp \rightarrow l^+l^- + 2 \text{ jets}$ 
  - ▶  $\mathcal{O}(\alpha^3\alpha_s^2)$  corrections calculated  
(EW corrections to LO QCD diagrams,  
QCD corrections to LO EW-QCD interferences)
  - ▶ corrections to total cross section at per-cent level
  - ▶ corrections of several ten per cent in high-energy tails of distributions from virtual Sudakov logarithms
  - ▶ real (photonic) corrections typically small

Backup

## On-shell Z boson:

- EW corrections small on total cross section:  $-1.2\%$   
similar for all gluon channels
- can be sizable in distributions where large energy scales are relevant  
(Sudakov logarithms)



**recombination:** photons and jets are recombined if  $R_{\gamma j} < 0.4$

**cuts:** two hard jets with  $p_{T,jet} > 25$  GeV,  $|y_{jet}| < 4.5$

photon energy fraction in jet  $z_\gamma < 0.7$