A fresh look at (non)renormalizable QFTs

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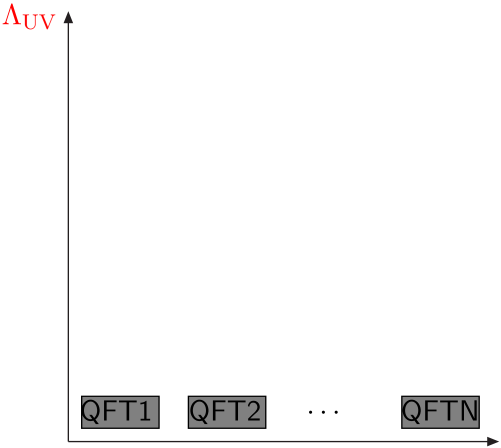
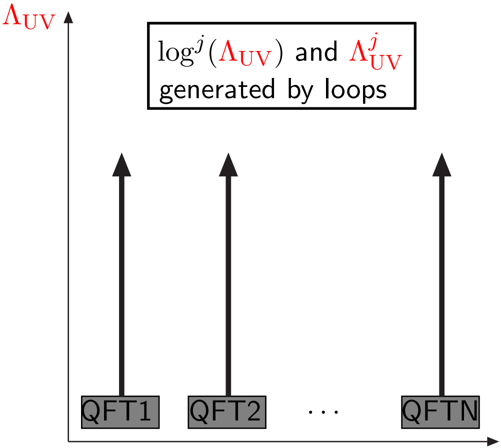
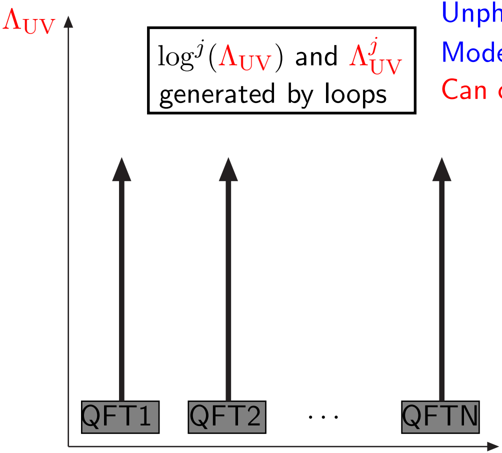
QFTs vs UV cutoff (I)



QFTs vs UV cutoff (II)



QFTs vs UV cutoff (III)

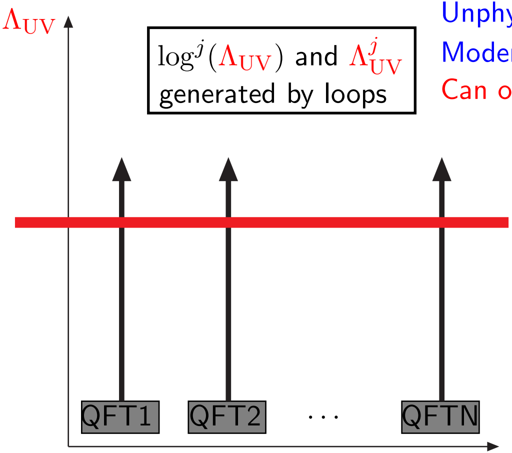


Unphysical if $\Lambda_{UV} \to \infty$

Modern version of *aether*?

Can one just **ignore** them?

QFTs vs UV cutoff (IV)



Unphysical if $\Lambda_{\rm UV} \to \infty$

Modern version of *aether*?

Can one just **ignore** them?

This talk

For this idea to have success:

1. Physics of Renormalizable theories should be reproduced (Bottom-up approach)
2. Non Renormalizable theories should be given a sensible meaning (Top-down approach)

In this talk:

1. I mostly elaborate on the Bottom-up approach, using its outcome as a guideline in the Top-down direction
2. **Ignoring** UV effects has also a great potential in simplifying loop-calculations:

working in the physical four-dimensional Minkowsky space suitable for fully exploiting the potential of numerical approaches

The Four Dimensional Regularization/Renormalization approach (FDR)

R. P., **arXiv:1208.5457**

A. M. Donati and R. P., **arXiv:1302.5668**

R. P., **arXiv:1305.0419**

R. P., **arXiv:1307.0705**

FDR vs UV Infinities (a 1-loop example)

- Consider

$$\int d^4q \frac{q^\alpha q^\beta}{D_0 D_1} \qquad \begin{cases} D_0 = q^2 - M_0^2 \\ D_1 = (q+p_1)^2 - M_1^2 \end{cases}$$

$$D_i \;=\; q^2 - d_i\,, \quad d_i \;=\; M_i^2 - p_i^2 - 2(q\cdot p_i)\,, \quad p_0 = 0$$

- UV convergence *"improved"* by $\mathbf{D_i} \to \mathbf{\bar{D}_i} \;=\; \mathbf{D_i} - \mu^2$ $(*)$ (with $\mu \to 0$) and partial fraction

$$\frac{1}{\bar{D}_i} = \frac{1}{\bar{q}^2} + \frac{d_i}{\bar{q}^2 \bar{D}_i}, \qquad \bar{q}^2 \;=\; q^2 - \mu^2$$

$(*)$ $-\mu^2$ can be identified with the $+i\epsilon$ **propagator prescription!**

- The *integrand* becomes

$$\frac{q^\alpha q^\beta}{\bar{D}_0\bar{D}_1} = \left[\frac{q^\alpha q^\beta}{\bar{q}^4}\right] + \left[\frac{q^\alpha q^\beta (d_0+d_1)}{\bar{q}^6}\right] + \left[\frac{4 q^\alpha q^\beta (q\cdot p_1)^2}{\bar{q}^8}\right] + J_F^{\alpha\beta}(q)$$

$$J_F^{\alpha\beta}(q) = q^\alpha q^\beta \left(\frac{4(q\cdot p_1)^2 d_1}{\bar{q}^8\,\bar{D}_1} + (M_1^2 - p_1^2)\frac{d_0+d_1-2(q\cdot p_1)}{\bar{q}^6\,\bar{D}_1} -2d_0\frac{(q\cdot p_1)}{\bar{q}^6\,\bar{D}_1} + \frac{d_0^2}{\bar{q}^4\,\bar{D}_0\bar{D}_1}\right)$$

$q^2 \to 0$ behavior of $J_F^{\alpha\beta}(q)$ regulated by μ^2

- **No** physical information in the *brown* terms (vacuum integs)

$$\frac{q^\alpha q^\beta}{\bar{D}_0 \bar{D}_1} = \left[\frac{q^\alpha q^\beta}{\bar{q}^4}\right] + \left[\frac{q^\alpha q^\beta (d_0 + d_1)}{\bar{q}^6}\right] + \left[\frac{4 q^\alpha q^\beta (q \cdot p_1)^2}{\bar{q}^8}\right] + J_F^{\alpha\beta}(q)$$

$$\Uparrow \qquad \Uparrow \qquad \Uparrow$$

CO: Λ^2_{UV} $\quad \ln \frac{\Lambda^2_{\mathrm{UV}}}{\mu^2}$ $\quad \ln \frac{\Lambda^2_{\mathrm{UV}}}{\mu^2}$

DR: 0 $\quad \ln \frac{\mu_R^2}{\mu^2}$ $\quad \ln \frac{\mu_R^2}{\mu^2}$

- **Ignoring** the *brown* terms allows one to define

$$B^{\alpha\beta}(p_1^2, M_0^2, M_1^2) =$$

$$\int [d^4q] \frac{q^\alpha q^\beta}{\bar{D}_0 \bar{D}_1} \equiv \lim_{\mu \to 0} \int d^4q \, J_F^{\alpha\beta}(q)$$

What have we done?

- UV divergences **subtracted** before integration

What about gauge invariance?

- One has to be consistent . . .

"Gauge invariance implies a tight interplay between the numerator of an integrand and its denominator. Changing either of the two will generally destroy gauge invariance."

Veltman (1974)

The global treatment of $\bar{q}^2$

- If a q^2 from Feynman rules appears in the numerator it should also be *"deformed"*: $q^2 \to \bar{q}^2 = q^2 - \mu^2$

- The generated extra integrals e.g.

$$\mathfrak{I}^{\rm FDR}(\mu^2) = \int [d^4q] \frac{\mu^2}{\bar{D}_0 \bar{D}_1}$$

require the same denominator expansion of $\int [d^4q] \frac{q^\alpha q^\beta}{\bar{D}_0 \bar{D}_1}$

$$\begin{aligned} \mathfrak{I}^{\rm FDR}(\mu^2) &= \lim_{\mu \to 0} \int d^4q \, \mu^2 \left(\frac{4(q \cdot p_1)^2 d_1}{\bar{q}^8 \bar{D}_1} + \cdots \right) \\ &= \frac{i\pi^2}{2} \left(M_0^2 + M_1^2 - \frac{p_1^2}{3} \right) \end{aligned}$$

- Cancellations ensured between numerators and denominators in *divergent* integrals: usual manipulations hold **at the integrand level**

$$\int [d^4q] \frac{\bar{q}^2}{\bar{D}_0 \bar{D}_1} = \int [d^4q] \frac{1}{\bar{D}_1} + \int [d^4q] \frac{M_0^2}{\bar{D}_0 \bar{D}_1}$$

- One also proves shift invariance properties for the $\int [d^4q]$ integral

Getting rid of the cutoff μ^2

What is the cost of throwing away infinities?

- No cost for polynomially divergent infinities (decoupling)
- Logarithmic infinities leave a $\ln \mu^2$ such that $\mu \to 0$ *cannot be taken*

$$B(p_1^2, M_0^2, M_1^2) = \int [d^4q] \frac{1}{\bar{D}_0 \bar{D}_1} =$$

$$-i\pi^2 \lim_{\mu\to 0} \int_0^1 dx \, \ln\left(\frac{\mu^2 + M_0^2 x + M_1^2(1-x) - p_1^2 x(1-x)}{\mu^2}\right)$$

- Fully subtracting logarithmic infinities is too much

$$\frac{1}{\bar{D}_0 \bar{D}_1} = \left[\frac{1}{\bar{q}^4}\right] + \frac{d_1}{\bar{q}^4 \bar{D}_1} + \frac{d_0}{\bar{q}^2 \bar{D}_0 \bar{D}_1}$$

$$\lim_{\mu \to 0} \int_\Lambda d^4q \left[\frac{1}{\bar{q}^4}\right] = \lim_{\mu \to 0} 2i\pi^2 \left(\int_0^{\mu_R} dq + \int_{\mu_R}^{\Lambda} dq \right) \frac{q^3}{(q^2+\mu^2)^2}$$

$$\Uparrow$$

$$-i\pi^2 \left(1 + \ln \frac{\mu^2}{\mu_R^2}\right)$$

- μ_R is an arbitrary separation scale from the UV regime (Renormalization Scale)
- Summing this $\ln \frac{\mu^2}{\mu_R^2}$ to the previous result, $\ln \mu^2$ is replaced by $\ln \mu_R^2$ and the limit $\mu \to 0$ *can be taken* (this mechanism can be proven to be valid at all orders!)

$$\int [d^4q] \frac{1}{\bar{D}_0 \bar{D}_1} = -i\pi^2 \int_0^1 dx \, \ln\left(\frac{M_0^2 x + M_1^2 (1-x) - p^2 x (1-x)}{\mu_R^2} \right)$$

Result cutoff independent!

The symbol $\int [d^4q]$ means

1. Use partial fraction to move all divergences in vacuum integrands treating $\bar{q}^2$ globally
2. Drop all divergent vacuum terms from the integrand
3. Integrate over d^4q
4. Take $\mu \to 0$ until a logarithmic dependence on μ is reached
5. Compute the result in $\mu = \mu_R$ ($\mu \to \mu_R$ in $[d^4q]$ definition)

Intermezzo . . .

- Only logarithmic infinities influence the physical spectrum (μ_R pops up in physical observables when separating them)
- Physics at $\Lambda_{\rm UV}$ scale manifests itself only logarithmically at lower energies

$$\ln(M_{\rm Higgs}/{\rm GeV}) \sim 5$$
$$\ln(M_{\rm Plank}/{\rm GeV}) \sim 44$$

Hierarchy problem?

With more loops

$$J = [J_V] + J_F$$

$$\mathfrak{I}_\ell^{\mathrm{FDR}} = \int \prod_{i=1}^{\ell} [d^4 q_i] \; J(\{\bar{q}^2\}) \equiv \lim_{\mu \to 0} \int \prod_{i=1}^{\ell} d^4 q_i \; J_F(\{\bar{q}^2\}) \Big|_{\mu = \mu_R}$$

A two-loop example

$$\mathfrak{I}_2^{\mathrm{FDR}} = \int [d^4 q_1][d^4 q_2] \frac{1}{\bar{D}_1 \bar{D}_2 \bar{D}_{12}}$$

$$\bar{D}_1 = \bar{q}_1^2 - m_1^2 \,, \quad \bar{D}_2 = \bar{q}_2^2 - m_2^2 \,, \quad \bar{D}_{12} = \bar{q}_{12}^2 - m_{12}^2$$

$$
\begin{aligned}
J = \frac{1}{\bar{D}_1 \bar{D}_2 \bar{D}_{12}} &= \left[\frac{1}{\bar{q}_1^2 \bar{q}_2^2 \bar{q}_{12}^2}\right] \\
+ \quad & m_1^2 \left[\frac{1}{\bar{q}_1^4 \bar{q}_2^2 \bar{q}_{12}^2}\right] + \frac{m_1^4}{(\bar{D}_1 \bar{q}_1^4)} \left[\frac{1}{\bar{q}_2^4}\right] - m_1^4 \frac{q_1^2 + 2(q_1 \cdot q_2)}{(\bar{D}_1 \bar{q}_1^4) \bar{q}_2^4 \bar{q}_{12}^2} \\
+ \quad & m_2^2 \left[\frac{1}{\bar{q}_1^2 \bar{q}_2^4 \bar{q}_{12}^2}\right] + \frac{m_2^4}{(\bar{D}_2 \bar{q}_2^4)} \left[\frac{1}{\bar{q}_1^4}\right] - m_2^4 \frac{q_2^2 + 2(q_1 \cdot q_2)}{\bar{q}_1^4 (\bar{D}_2 \bar{q}_2^4) \bar{q}_{12}^2} \\
+ \quad & m_{12}^2 \left[\frac{1}{\bar{q}_1^2 \bar{q}_2^2 \bar{q}_{12}^4}\right] + \frac{m_{12}^4}{(\bar{D}_{12} \bar{q}_{12}^4)} \left[\frac{1}{\bar{q}_1^4}\right] - m_{12}^4 \frac{q_{12}^2 - 2(q_1 \cdot q_{12})}{\bar{q}_1^4 \bar{q}_2^2 (\bar{D}_{12} \bar{q}_{12}^4)} \\
+ \quad & \frac{m_1^2 m_2^2}{(\bar{D}_1 \bar{q}_1^2)(\bar{D}_2 \bar{q}_2^2) \bar{q}_{12}^2} + \frac{m_1^2 m_{12}^2}{(\bar{D}_1 \bar{q}_1^2) \bar{q}_2^2 (\bar{D}_{12} \bar{q}_{12}^2)} + \frac{m_2^2 m_{12}^2}{\bar{q}_1^2 (\bar{D}_2 \bar{q}_2^2)(\bar{D}_{12} \bar{q}_{12}^2)} \\
+ \quad & \frac{m_1^2 m_2^2 m_{12}^2}{(\bar{D}_1 \bar{q}_1^2)(\bar{D}_2 \bar{q}_2^2)(\bar{D}_{12} \bar{q}_{12}^2)}
\end{aligned}
$$

FDR Renormalization

- Only *finite* $\ln^j(\mu_R)$ remain

  (generated when subtracting log divergent vacuum integs)

- A *finite renormalization* reabsorbes them into the physical parameters of the theory

- At 1-loop equivalent to *Dimensional Reduction* in the $\overline{\mathrm{MS}}$ scheme

Physical interpretation

Classification (independent of the number of external legs!)

1. $\left[\frac{1}{\bar{q}^4}\right]$ is the only possible **subtracted** 1-loop log divergent vacuum integrand

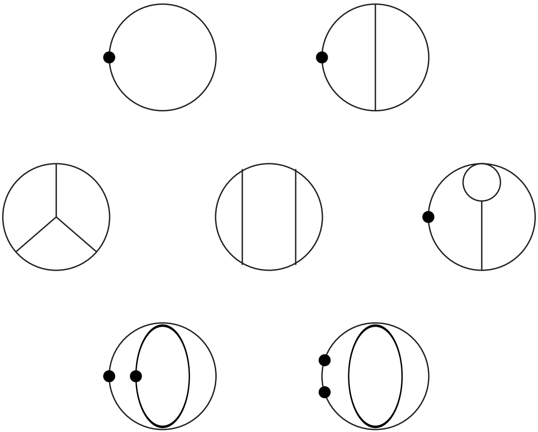
2. At 2 loops $\left[\frac{1}{\bar{q}_1^4\bar{q}_2^2\bar{q}_{12}^2}\right]$

3. Five additional log divergent vacuum integrands at 3 loops

$$\left[\frac{1}{\bar{q}_1^2\bar{q}_2^2\bar{q}_3^2\bar{q}_{12}^2\bar{q}_{13}^2((q_2-q_3)^2-\mu^2)}\right] \quad \left[\frac{1}{\bar{q}_1^2\bar{q}_3^2\bar{q}_2^4\bar{q}_{12}^2\bar{q}_{23}^2}\right]$$

$$\left[\frac{1}{\bar{q}_1^4\bar{q}_2^2\bar{q}_3^2\bar{q}_{12}^2\bar{q}_{123}^2}\right] \quad \left[\frac{1}{\bar{q}_1^4\bar{q}_2^4\bar{q}_3^2\bar{q}_{123}^2}\right] \quad \left[\frac{1}{\bar{q}_1^6\bar{q}_2^2\bar{q}_3^2\bar{q}_{123}^2}\right]$$

Corresponding 1-, 2- and 3-loop log topologies



Divergent tensor integrands are reducible to combinations of those topologies plus finite constants

- Infinities are directly put into the vacuum, rather than in the parameter of the Lagrangian

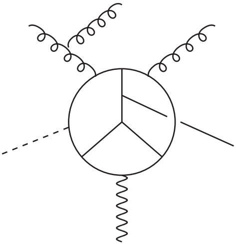
(**Order by order vacuum redefinition** similar to the denominator of the Gell-Mall-Low formula)

- The vacuum back-reacts by trading the cutoff μ for μ_R, which, however, drops after a **finite** renormalization

- This procedure is equivalent to the standard renormalization program. However, it could provide an extra handle when interpreting the non-renormalizable case

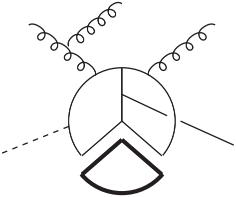
The vacuum is by far more efficient in accommodating infinities than the Lagrangian

Vacuum inside loops (pictorially)



(a)

(b)



(c)

(b) *and* (c) *are* **Vacuum Bubbles** *generated by the generic diagram* (a) *contributing to the interaction*

Why μ_R drops?

- Consider the Lagrangian of a **renormalizable** QFT dependent on m parameters p_i $(i = 1 : m)$

$$\mathcal{L}(p_1, \ldots, p_m)$$

- Before an observable $\mathcal{O}^{\rm TH}_{m+1}$ can be calculated, p_i must be fixed by means of m measurements

$$\mathcal{O}^{\rm TH}_i(p_1, \ldots, p_m) = \mathcal{O}^{\rm EXP}_i$$

which determine p_i in terms of observables $\mathcal{O}^{\rm EXP}_i$ and corrections computed at the loop level ℓ one is working:

$$p_i = p_i^{\ell-loop}(\mathcal{O}^{\rm EXP}_1, \ldots, \mathcal{O}^{\rm EXP}_m) \equiv \bar{p}_i$$

$$\mathcal{O}^{\rm TH}_{m+1}(\bar{p}_1, \ldots, \bar{p}_m)$$

is then a **finite** prediction of the QFT

- The divergent scalar integrands are *linearly independent* $\Rightarrow$ *must* cancel out *separately*. For instance, up to two loops

$$J(q_1,q_2) = a_0(q_1,q_2) + a_1(q_1)\left[\frac{1}{\bar{q}_2^4}\right] + a_2\left[\frac{1}{\bar{q}_1^4}\right]\left[\frac{1}{\bar{q}_2^4}\right] + a_3\left[\frac{1}{\bar{q}_1^4\bar{q}_2^2\bar{q}_{12}^2}\right]$$

with a_1, a_2, a_3 vanishing *independently*

- No need to compute a regulated version of the integrals: a subtraction before integration *à la FDR* is all one has to do

- The p_i remain finite and, since the μ dependence of the divergent contribution also drops *at the perturbative order one is working*, the same happens in the physical contribution (where $\mu = \mu_R$)

$$\mathcal{O}^{\rm TH}_{m+1}(\bar{p}_1,\ldots,\bar{p}_m) = \lim_{\mu\to 0}\int d^4q_1 d^4q_2\, a_0(q_1,q_2)$$

FDR vs CL/IR Virtual Infinities

- CL/IR singularities also regulated by μ^2, e.g.

$$B^{\alpha\beta}(0,0,0) = \lim_{\mu\to 0}\int d^4q\,\frac{q^\alpha q^\beta d_1^3}{\bar{q}^8\bar{D}_1} = -8p_1^\rho p_1^\sigma p_1^\tau \lim_{\mu\to 0}\int d^4q\,\frac{q^\alpha q^\beta q_\rho q_\sigma q_\tau}{\bar{q}^8\bar{D}_1} = \mathbf{0!}$$

Analogously $B^\alpha(0,0,0) = B(0,0,0) = 0$

- **Due to a cancellation between UV and CL regulators**

$$B(p_1^2,0,0) = -i\pi^2 \lim_{\mu\to 0}\int_0^1 dx\,\left[\ln(\mu^2 - p_1^2x(1-x)) - \ln(\mu^2)\right]$$

- **Should be matched in the treatment of the Reals**

TEST1: $H \to \gamma(k_1^\mu)\,\gamma(k_2^\nu)$ (generic R_ξ gauge)

Alice M. Donati and R.P., arXiv:1302.5668 [hep-ph]

$$\begin{aligned}
\mathcal{M}^{\mu\nu}(\beta,\eta) &= \Big(\widetilde{\mathcal{M}}_W(\beta) + \sum_f N_c Q_f^2\, \widetilde{\mathcal{M}}_f(\eta)\Big)\, T^{\mu\nu}\,, \\
T^{\mu\nu} &= k_1^\nu k_2^\mu - (k_1\cdot k_2)\, g^{\mu\nu}\,, \\
\widetilde{\mathcal{M}}_W(\beta) &= \frac{i\,e^3}{(4\pi)^2 s_W M_W}\Big[2 + 3\beta + 3\beta(2-\beta)f(\beta)\Big]\,, \\
\widetilde{\mathcal{M}}_f(\eta) &= \frac{-i\,e^3}{(4\pi)^2 s_W M_W}\, 2\eta\Big[1 + (1-\eta)f(\eta)\Big]
\end{aligned}$$

$$\beta = \frac{4\,M_W^2}{M_H^2}\,, \qquad \eta = \frac{4\,m_f^2}{M_H^2}\,, \qquad f(x) = -\frac{1}{4}\ln^2\left(\frac{1+\sqrt{1-x+i\varepsilon}}{-1+\sqrt{1-x+i\varepsilon}}\right)$$

NOTE : $$\int [d^4q]\frac{\bar{q}^2 g_{\mu\nu} - 4q_\mu q_\nu}{(\bar{q}^2-M^2)^3} = \int [d^4q]\frac{-\mu^2}{(\bar{q}^2-M^2)^3}g_{\mu\nu} = -\frac{i\pi^2}{2}g_{\mu\nu}$$

TEST2: $\Gamma(\mathbf{H} \to \mathbf{gg})$

R. P., arXiv:1307.0705 [hep-ph]

- FDR is used to compute the NLO QCD corrections to $\mathbf{H} \to \mathbf{gg}$ in the large top mass limit
- The well known fully inclusive result

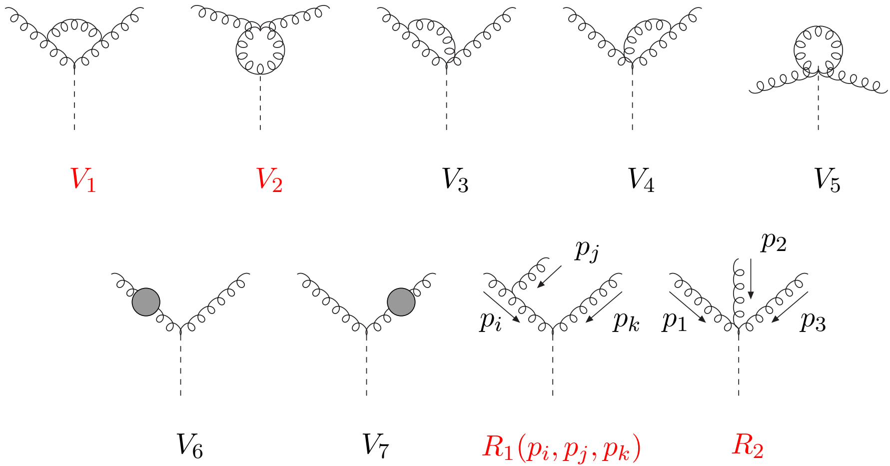
$$\Gamma(\mathbf{H} \to \mathbf{gg}) = \Gamma^{(0)}(\alpha_S(M_H^2)) \left[1 + \frac{95}{4}\frac{\alpha_S}{\pi}\right]$$

is re-derived, where

$$\Gamma^{(0)}(\alpha_S(M_H^2)) = \frac{G_F \alpha_S^2(M_H^2)}{36\sqrt{2}\pi^3} M_H^3$$

- **UV**, **IR** and **CL** divergences, besides α_S **renormalization**

Contributing Diagrams



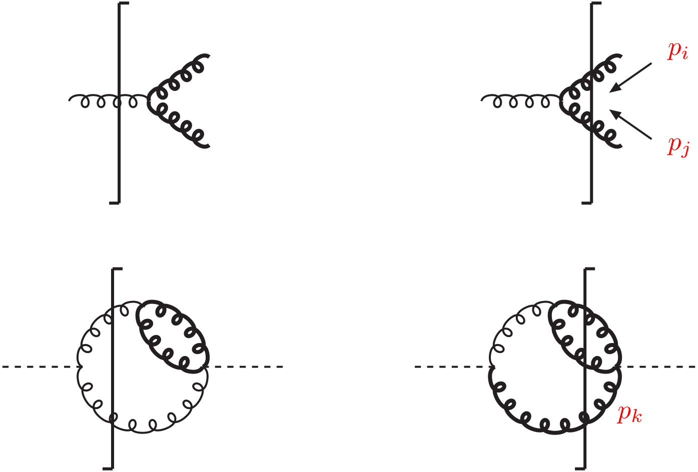
The Virtual Part

- Overlapping CL/IR infinities **regulated by** μ^2

$$
\begin{aligned}
C(s) &= \int [d^4q] \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2} = \lim_{\mu \to 0} \int d^4q \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2} \\
&= \frac{i\pi^2}{s} \left[\frac{\ln^2(\mu_0) - \pi^2}{2} + i\,\pi \ln(\mu_0) \right] \\
s &= M_H^2 = -2(p_1 \cdot p_2) \quad \text{with} \quad (\mu_0 = \mu^2/s)
\end{aligned}
$$

$$
\Gamma_V(\mathbf{H} \to \mathbf{gg}) = -3 \frac{\alpha_S}{\pi} \Gamma^{(0)}(\alpha_S)\, M_H^2\, \mathcal{R}e \left[\frac{C(M_H^2)}{i\pi^2} \right]
$$

The Real Part



$$\frac{1}{2(p_i \cdot p_j)} \to \frac{1}{(p_i + p_j)^2} = \frac{1}{s_{ij}}$$ with $p^2_{i,j,k} = \mu^2 \to 0$ ($\mu-$massive PS)

- The matrix element squared reads (diagrams R_1 and R_2)

$$|M|^2 = 192\,\pi\alpha_S A^2 \Bigg[\frac{s_{23}^3}{s_{12}s_{13}} + \frac{s_{13}^3}{s_{12}s_{23}} + \frac{s_{12}^3}{s_{13}s_{23}} + \frac{2(s_{13}^2+s_{23}^2)+3s_{13}s_{23}}{s_{12}} + \frac{2(s_{12}^2+s_{23}^2)+3s_{12}s_{23}}{s_{13}} + \frac{2(s_{12}^2+s_{13}^2)+3s_{12}s_{13}}{s_{23}} + 6(s_{12}+s_{13}+s_{23}) \Bigg]$$

- To be integrated over the μ-massive 3-body PS

$$\int d\Phi_3 = \frac{\pi^2}{4s} \int ds_{12}ds_{13}ds_{23}\,\delta(s-s_{12}-s_{13}-s_{23}+3\mu^2)$$

- $\frac{1}{s_{ij}s_{jk}}$ generate $\ln^2(\mu^2)$ terms of IR/CL origin
  $\frac{1}{s_{ij}}$ collinear $\ln(\mu^2)$s

- By introducing the dimensionless variables $(x+y+z=1)$

$$x = \frac{s_{12}}{s} - \mu_0\,, \quad y = \frac{s_{13}}{s} - \mu_0\,, \quad z = \frac{s_{23}}{s} - \mu_0$$

$$I(s) = \int_R dxdy\, \frac{1}{(x+\mu_0)(y+\mu_0)}\,, \quad J_p(s) = \int_R dxdy\, \frac{x^p}{(y+\mu_0)}$$

- Then $(\mu_0 = \mu^2/s)$

$$I(s) \sim \frac{\ln^2(\mu_0) - \pi^2}{2}$$

$$J_p(s) \sim -\frac{1}{p+1}\ln(\mu_0) - \frac{1}{p+1}\left[\frac{1}{p+1} + 2\sum_{n=1}^{p+1}\frac{1}{n}\right]$$

- Finally

$$\Gamma_R(\mathbf{H} \to \mathbf{ggg}) = 3\frac{\alpha_S}{\pi}\,\Gamma^{(0)}(\alpha_S) \times \left[\frac{1}{4} + I(M_H^2) - \frac{3}{2}J_0(M_H^2) - J_2(M_H^2)\right]$$

and

$$\Gamma(\mathbf{H} \to \mathbf{gg}) = \Gamma_V(\mathbf{H} \to \mathbf{gg}) + \Gamma_R(\mathbf{H} \to \mathbf{ggg})$$

$$= \Gamma^{(0)}(\alpha_S)\left[1 + \frac{\alpha_S}{\pi}\left(\frac{95}{4} - \frac{11}{2}\ln\frac{M_H^2}{\mu^2}\right)\right]$$

α_S Renormalization

- The residual μ^2 is a universal dependence on the renormalization scale $(\mu = \mu_R)$
- $\ln(\mu_R^2)$ can be reabsorbed in the gluonic running of the strong coupling constant **(Finite Renormalization)**

$$\Gamma^{(0)}(\alpha_S) \rightarrow \Gamma^{(0)}(\alpha_S(\mu_R^2))$$
$$\alpha_S(M_H^2) = \frac{\alpha_S(\mu_R^2)}{1 + \frac{\alpha_S}{2\pi}\frac{11}{2}\ln\frac{M_H^2}{\mu_R^2}}$$

$$\Gamma(\mathbf{H} \rightarrow \mathbf{gg}) = \Gamma^{(0)}(\alpha_S(M_H^2))\left[1 + \frac{95}{4}\frac{\alpha_S}{\pi}\right]$$

quod erat demostrandum

An attempt at the Top-down direction

By *copying* the FDR approach for a non-renormalizable QFT

1. $\mathcal{O}^{\mathrm{TH}}_{m+1}(\bar{p}_1, \ldots, \bar{p}_m, \log(\mu_R))$
2. At worst $\mu_R \sim$ typical scale of the Theory $\Rightarrow$ Effective QFT
3. Can *just one* additional measurement fix μ_R and restore *predictivity?* (without changing $\mathcal{L}$)

A possible way to determine μ_R

$$\mathcal{O}^{\rm TH}_{m+2}(\bar{p}_1,\ldots,\bar{p}_m,\log(\mu_R)) = \mathcal{O}^{\rm EXP}_{m+2}$$

Computed with the same FDR approach used for $\mathcal{O}^{\rm TH}_{m+1}$

- Does this μ_R render the calculation of $\mathcal{O}^{\rm TH}_{m+1}$ predictive at any order?
- For this to happen choosing $\mathcal{O}^{\rm TH}_{m+3}(\bar{p}_1,\ldots,\bar{p}_m,\log(\mu_R))$ should give the same result (universality)

- After all, subtracted topologies *mimic* the UV completion

- FDR respects the original symmetries of the Lagrangian, in particular the coefficients of $\log(\mu_R)$ in different Green's functions are linked by Slavnov-Taylor identities (if any)

- More investigation ($\equiv$ explicit calculations in concrete theories) needed

Summary

1. Based on the FDR classification of the UV infinities a new interpretation of the renormalization procedure is possible
2. One subtracts the divergences directly at the level of the *integrand* (order by order re-definition of the vacuum)
3. Equivalence with the standard renormalization procedure for renormalizable QFTs (only finite renormalization left)
4. It is postulated that in (some?) non-renormalizable QFTs ONE additional measurement could completely fix the theory, which could becomes predictive *without modifying the original Lagrangian/Symmetries*
5. Focus moved from occurrence of UV infinities to the consistency of the QFT at hand

Thank you!

Backup slides

Shift invariance

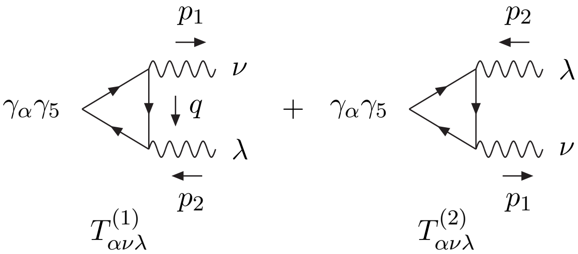
It is guaranteed since the FDR integral is a difference between a DR integral, $\mathfrak{I}_\ell^{\rm DR}$, and its vacuum configurations $(J = [J_V] + J_F)$

$$\mathfrak{I}_\ell^{\rm FDR} = \mathfrak{I}_\ell^{\rm DR} - \lim_{\mu \to 0} \mu_R^{-\ell\epsilon} \int \prod_{i=1}^{\ell} d^n q_i \, [J_V(\{\bar{q}^2\})] \Big|_{\mu=\mu_R}$$

$$\Longrightarrow$$

This - together with the *global treatment of* $\bar{q}^2$ - ensures that FDR preserves the original symmetries of the QFT

The ABJ anomaly



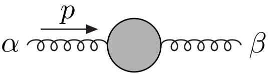
$$p^\alpha T_{\alpha\nu\lambda} = -i\frac{e^2}{4\pi^4}\mathrm{Tr}[\gamma_5 \not{p}_2\gamma_\lambda\gamma_\nu\not{p}_1]\int [d^4q]\,\mu^2\frac{1}{\bar{D}_0\bar{D}_1\bar{D}_2}$$

$$p^\alpha T_{\alpha\nu\lambda} = \frac{e^2}{8\pi^2}\mathrm{Tr}[\gamma_5 \not{p}_2\gamma_\lambda\gamma_\nu\not{p}_1]$$

Potential ambiguity from Schouten identity:

$$\begin{aligned}\epsilon(p_1,p_2,p_3,p_4)\,q^2 &= \epsilon(q,p_2,p_3,p_4)\,(q\cdot p_1)+\epsilon(p_1,q,p_3,p_4)\,(q\cdot p_2)\\ &+ \epsilon(p_1,p_2,q,p_4)\,(q\cdot p_3)+\epsilon(p_1,p_2,p_3,q)\,(q\cdot p_4)\end{aligned}$$

Gluon self-energy at one-loop (arbitrary gauge, n_f=0)



$$= i\left(g_{\alpha\beta} - p_\alpha p_\beta/p^2\right)\, \Pi(p^2)$$

$$\Pi(p^2)^{\rm FDR} = \Pi(p^2)^{\rm DR}\Big|_{\overline{\rm MS}} = N_{col}\left(\frac{\alpha_s}{4\pi}\right) p^2 \left[\left(-\frac{13}{6} + \frac{\xi}{2}\right) \ln\left(-\frac{p^2}{\mu_R}\right) + \left(\frac{85}{36} + \frac{\xi}{2} + \frac{\xi^2}{4}\right)\right]$$

In Conventional DR $\quad \frac{85}{36} \to \frac{97}{36}$

The dependence on μ

- $$\mu_R^{-\epsilon}\int d^nq\,\frac{1}{D_0D_1} = \lim_{\mu\to 0}\,\mu_R^{-\epsilon}\int d^nq\,\frac{1}{\bar{D}_0\bar{D}_1} \tag{1}$$

$$\frac{1}{\bar{D}_0\bar{D}_1} \;=\; \left[\frac{1}{\bar{q}^4}\right] + \frac{d_1}{\bar{q}^4\bar{D}_1} + \frac{d_0}{\bar{q}^2\bar{D}_0\bar{D}_1} \tag{2}$$

$$\int d^nq\left[\frac{1}{\bar{q}^4}\right] = -i\pi^2\left[\Delta + 2\ln\left(\frac{\mu}{\mu_R}\right)\right] \tag{3}$$

- Since the l.h.s. of (1) does not depend on μ, $\ln(\mu)$ in (3) gets compensated by and infrared $-\ln(\mu)$ obtained when integrating the Physical terms in (2)