Dipole subtraction with random polarisations

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Outline

- Numerical Polarisations
- Dipole Subtraction
- Random Polarisations @ NLO
- Summary

Part I

Numerical Polarisations

GOAL:

compute jet cross sections for unpolarised scattering using numerical methods; focus on

- general algorithm,
- high particle multiplicity (e.g. $2 \to 6, 7, 8$ particles) and
- speed

$$\sigma = \int_n \mathrm{d}\sigma \propto \int \mathrm{d}\phi_{n-2} \sum_{\lambda_1,\ldots,\lambda_n} |\mathcal{A}_{\lambda_1\ldots\lambda_n}|^2 F_J^{(n)}$$

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Perform integration with Monte Carlo methods!

$$I = \int_{[0,1)^d} \mathrm{d}^d x f(\vec{x}) \approx \frac{1}{N} \sum_{n=1}^{N} f(\vec{u}_n), \quad \vec{u}_n \in [0,1)^d$$

Error scales like $\frac{1}{\sqrt{N}}$, independent of d!

"Classical" Method: Helicity Amplitudes

$$\sum_{\lambda_1,\dots,\lambda_n} |\mathcal{A}_{\lambda_1\dots\lambda_n}|^2 = \sum_{\lambda_1=\pm} \cdots \sum_{\lambda_n=\pm} |\mathcal{A}_{\lambda_1\lambda_2\dots\lambda_n}|^2$$
$$= |\mathcal{A}_{+++\dots+}|^2 + |\mathcal{A}_{-++\dots+}|^2 + \cdots + |\mathcal{A}_{---\dots-}|^2$$

- compute all squared amplitudes, called helicity amplitudes
- 2^n amplitudes per phase space point

Possible speed-ups:

- use parity conservation $\rightarrow$ only 2^{n-1} amplitudes

$$\mathcal{A}_{\lambda_1\lambda_2\dots\lambda_n} = -\mathcal{A}^*_{-\lambda_1,-\lambda_2,\dots,-\lambda_n}$$

- recursive methods: store off-shell currents with same helicity configurations and reuse them

RANDOM POLARISATIONS

Replace helicity sum by MC integral:

$$\sum_{\lambda_1,\dots,\lambda_n} |\mathcal{A}_{\lambda_1\dots\lambda_n}|^2 = \int\limits_{[0,1)^n} \mathrm{d}^n\theta \, |\mathcal{A}_{\theta_1\dots\theta_n}|^2$$

where we use a different parametrisation [1] for particle polarisations:

$$\epsilon(p,\theta) = \mathrm{e}^{2\pi\mathrm{i}\theta}\,\epsilon_+(p) + \mathrm{e}^{-2\pi\mathrm{i}\theta}\,\epsilon_-(p)$$

[1] — *[Draggiotis, Kleiss, Papadopoulos; Phys. Lett.* ***B439****, 157 (1998)]*

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Why does this work?

Analytically, we can pull out polarisations of squared amplitude:

$$\sum_{\lambda_1,\dots,\lambda_n} |\mathcal{A}_{\lambda_1\dots\lambda_n}|^2 = \prod_{m=1}^{n} \left[\sum_{\lambda_m} \epsilon^{\mu_m}_{\lambda_m} \left(\epsilon^{\nu_m}_{\lambda_m}\right)^* \right] \times \mathcal{M}_{\mu_1,\dots\mu_n,\nu_1,\dots,\nu_n}$$

$$\epsilon^\mu(\theta)\left(\epsilon^\nu(\theta)\right)^* = \epsilon^\mu_+\left(\epsilon^\nu_+\right)^* + \epsilon^\mu_-\left(\epsilon^\nu_-\right)^* + \mathrm{e}^{4\pi\mathrm{i}\theta}\epsilon^\mu_+\left(\epsilon^\nu_-\right)^* + \mathrm{e}^{-4\pi\mathrm{i}\theta}\epsilon^\mu_-\left(\epsilon^\nu_+\right)^*$$

$$\int_0^1 \mathrm{d}\theta\, \epsilon^\mu(\theta)\left(\epsilon^\nu(\theta)\right)^* = \sum_{\lambda=\pm} \epsilon^\mu_\lambda\left(\epsilon^\nu_\lambda\right)^*$$

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Features:

- combine the integration with phase space integral by increasing dimension d!
- effectively only one squared amplitude per phase space point!
- similar for fermion spinors!
  (replace ϵ by $u/\bar{u}/v/\bar{v}$ and polarisation sum by spin sum)

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How does this work in practice?

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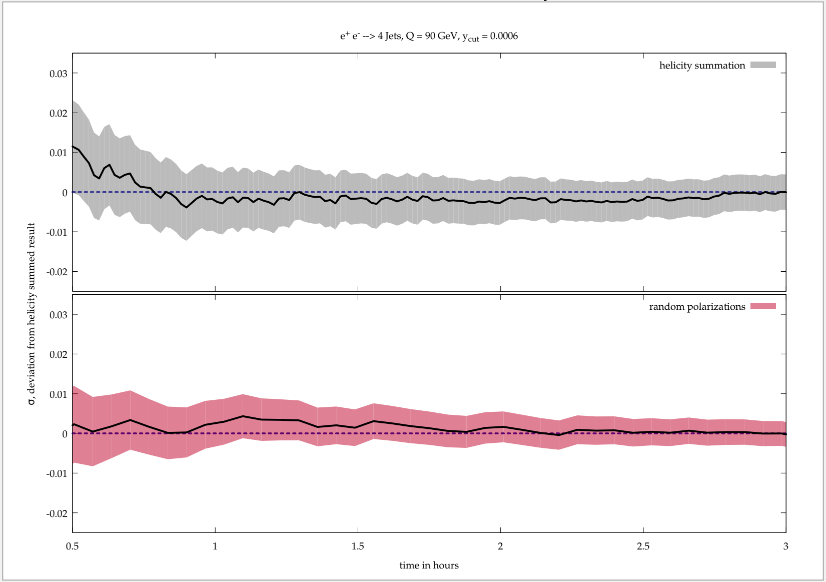
BORN-LEVEL EXAMPLE: $e^+e^- \rightarrow 4$ JETS



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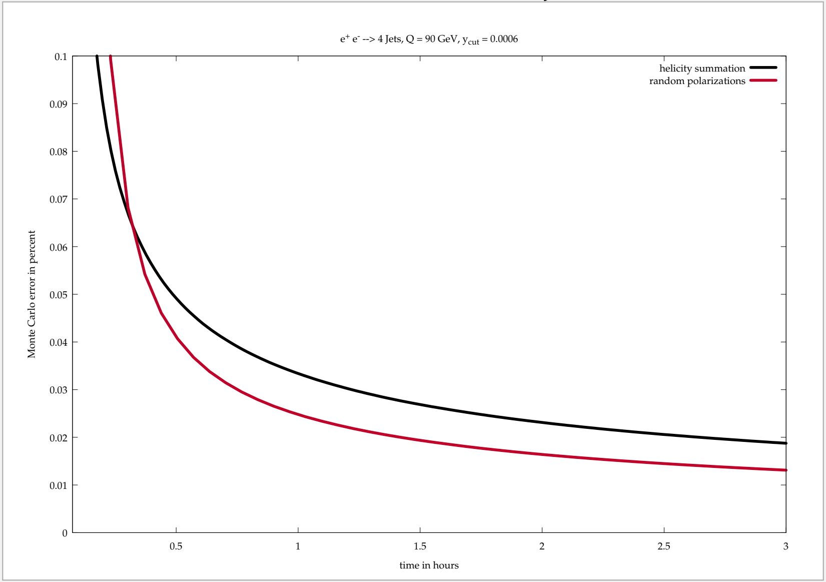
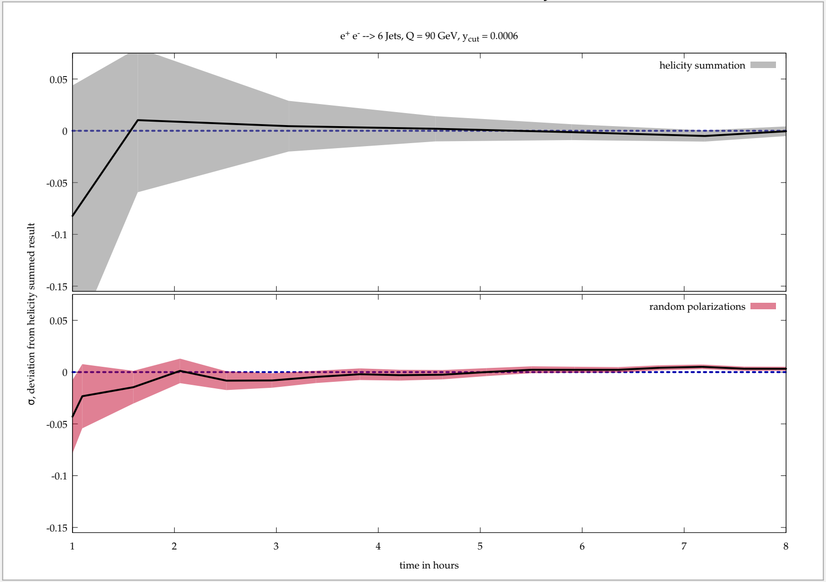


Born-Level Example: $e^+e^- \to 6$ jets



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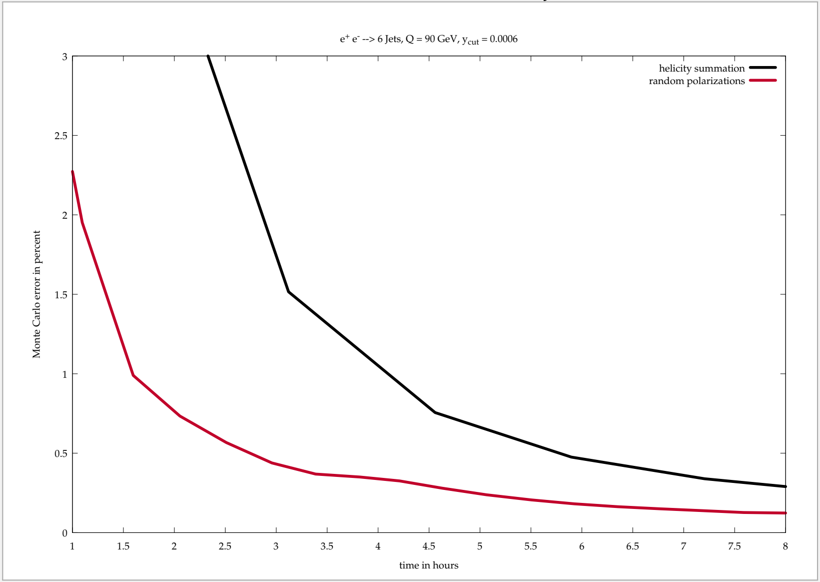

Part II

Dipole Subtraction

Jet Cross Sections @ NLO: Subtraction

$$\sigma^{\mathrm{NLO}} = \int_{n+1} \mathrm{d}\sigma^{\mathrm{R}} + \int_{n} \mathrm{d}\sigma^{\mathrm{V}}$$

Problem : terms separately IR divergent, only sum finite.

Numerically : need way to match singularities locally (i.e. per PS point)

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Subtraction using local counterterm $\mathrm{d}\sigma^{\mathrm{A}}$:

$$\sigma^{\mathrm{NLO}} = \int_{n+1} \left(\mathrm{d}\sigma^{\mathrm{R}} - \mathrm{d}\sigma^{\mathrm{A}}\right) + \int_{n} \left(\mathrm{d}\sigma^{\mathrm{V}} + \int_{1} \mathrm{d}\sigma^{\mathrm{A}}\right)$$

New $\mathrm{d}\sigma^{\mathrm{A}}$ must

- have same pointwise singular behavior as $\mathrm{d}\sigma^{\mathrm{R}}$,
- be integrable over the unresolved one-parton PS.

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How to construct $\mathrm{d}\sigma^{\mathrm{A}}$?

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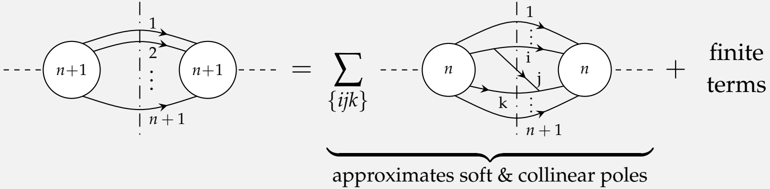
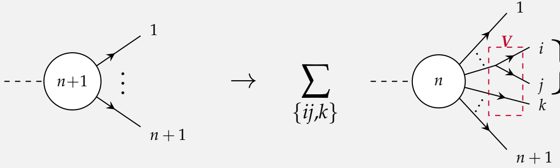
DIPOLE FORMALISM

formulated at level of squared, helicity summed amplitudes.

External-leg insertion rule: $|\mathcal{A}_{n+1}|^2 \rightarrow |\mathcal{A}_n|^2 \otimes \boldsymbol{V}_{ij,k}$



Factorisation in dipole formalism:



$\boldsymbol{V}_{ij,k}$ contains $\sum_{\lambda_i,\lambda_j}$!

Formalism & figures: *[Catani, Seymour — Nucl. Phys.* ***B485*** *(1997) 291–419]*

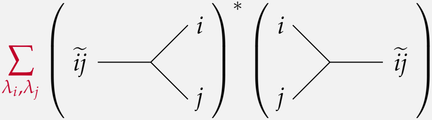
DIPOLE FORMALISM: SUBTRACTION TERM

$$\mathrm{d}\sigma^{\mathrm{A}} \propto \mathrm{d}\phi_{n-1} \left(\sum_{(i,j)} \sum_{k \neq i,j} \sum_{\substack{\lambda_1,\ldots,\not{\lambda}_i,\ldots,\\ \not{\lambda}_j,\ldots,\lambda_{n+1}}} \mathcal{D}_{ij,k} F_J^{(n)} + \text{ initial state dipoles} \ldots \right)$$

where

$$\mathcal{D}_{ij,k} = \frac{1}{2 p_i p_j} \langle \mathcal{A}_n(\widetilde{ij}, \widetilde{k}) | \frac{\mathbf{T}_k \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} \boldsymbol{V}_{ij,k} | \mathcal{A}_n(\widetilde{ij}, \widetilde{k}) \rangle$$

$\boldsymbol{V}_{ij,k}$ derived from singular part of squared, helicity summed splitting vertex:



use smooth momentum parametrisation:

$$\widetilde{p}_{ij} = p_i + p_j - \frac{y_{ij,k}}{1 - y_{ij,k}} p_k, \qquad y_{ij,k} = \frac{p_i p_k}{p_i p_j + p_j p_k + p_k p_i},$$

$$\widetilde{p}_k = \frac{1}{1 - y_{ij,k}} p_k, \qquad z_i = \frac{p_i p_k}{p_j p_k + p_i p_k} = 1 - z_j$$

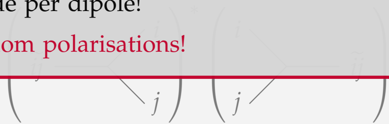
DIPOLE FORMALISM: SUBTRACTION TERM

$$d\sigma^{A} \propto d\phi_{n-1} \left(\sum_{(i,j)} \sum_{k \neq i,j} \sum_{\substack{\lambda_1,\ldots,\not\lambda_i,\ldots, \\ \not\lambda_j,\ldots,\lambda_{n+1}}} \mathcal{D}_{ij,k} F_J^{(n)} + \text{initial state dipoles} \ldots \right)$$

whe re

$V_{ij,k}$ deriv ... ex:

$$\sum_{\lambda_i,\lambda_j}$$



Note: each $\mathcal{D}$ has to be computed 2^{n-1} times,
once for every helicity amplitude $\mathcal{A}_n(\widetilde{ij},\widetilde{k})$!

$\Rightarrow$ reduce computation time by computing only one amplitude per dipole!

How? use random polarisations!

use smooth momentum parametrisation:

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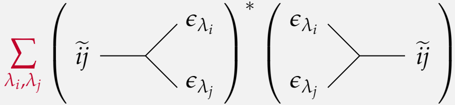
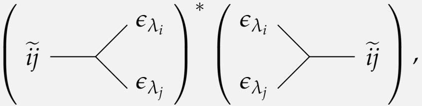
SUBTRACTION FOR RANDOM POLARISATIONS?

$\boldsymbol{V}_{ij,k}$ obtained from squared, helicity summed soft & collinear limits of splitting vertex:



Publication [1]: $\boldsymbol{V}_{ij,k}$ for helicity eigenstates, i.e. without sum



$$\lambda_i \in \{+,-\}, \quad \lambda_j \in \{+,-\}$$

1 — *[Czakon, Papadopoulos, Worek — JHEP 0908 (2009) 085]*

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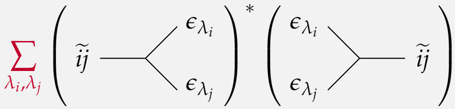

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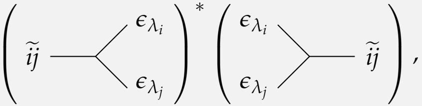


$$\lambda_i \in \{+,-\}, \quad \lambda_j \in \{+,-\}$$

Now: random polarisations (RP)
RP made up of helicity eigenstates

↪ why can't we use the terms in [1]?

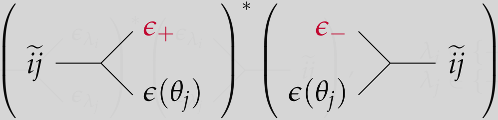
1 — *[Czakon, Papadopoulos, Worek — JHEP 0908 (2009) 085]*

Subtraction for Random Polarisations?

Recall: for both particles i and j, we have to use

$$\epsilon^\mu(\theta)\left(\epsilon^\nu(\theta)\right)^* = \sum_{\lambda=\pm} \epsilon^\mu_\lambda\left(\epsilon^\nu_\lambda\right)^* + e^{4\pi i\theta}\epsilon^\mu_+\left(\epsilon^\nu_-\right)^* + e^{-4\pi i\theta}\epsilon^\mu_-\left(\epsilon^\nu_+\right)^*$$

Right hand terms mix different helicities of same particle!
Example combination for particle i:



Not accounted for in [1]!

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Part III

Random Polarisations @ NLO

SUBTRACTION FOR RANDOM POLARISATIONS!

Take a second look at the product of RP:

$$\epsilon^{\mu}(\theta)\left(\epsilon^{\nu}(\theta)\right)^* = \sum_{\lambda=\pm} \epsilon^{\mu}_{\lambda}\left(\epsilon^{\nu}_{\lambda}\right)^* + \mathrm{e}^{4\pi\mathrm{i}\theta}\epsilon^{\mu}_{+}\left(\epsilon^{\nu}_{-}\right)^* + \mathrm{e}^{-4\pi\mathrm{i}\theta}\epsilon^{\mu}_{-}\left(\epsilon^{\nu}_{+}\right)^*$$

includes polarisation sum! ⇔ includes all terms from $\mathrm{d}\sigma^{\mathrm{A}}$!

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includes polarisation sum! ⇔ includes all terms from $\mathrm{d}\sigma^{\mathrm{A}}$!
⇒ we only need additional term taking care of helicity mixing terms!

$$\sigma^{\mathrm{NLO}} = \int_{n+1}\left(\mathrm{d}\sigma^{\mathrm{R}} - \left[\mathrm{d}\sigma^{\mathrm{A}} + \mathrm{d}\sigma^{\widetilde{\mathrm{A}}}\right]\right) + \int_{n}\left(\mathrm{d}\sigma^{\mathrm{V}} + \int_{1}\mathrm{d}\sigma^{\mathrm{A}}\right)$$

we need no integrated subtraction term because:

$$\int_0^1 \mathrm{d}\theta_i \int_0^1 \mathrm{d}\theta_j \, \mathrm{d}\sigma^{\widetilde{\mathrm{A}}} = 0$$

Subtraction for Random Polarisations!

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How to construct $d\sigma^{\widetilde{A}}$?

$$\sigma^{NLO} = \int_{n+1}\left(d\sigma^{R} - \left[d\sigma^{A} + d\sigma^{\widetilde{A}}\right]\right) + \int_{n}\left(d\sigma^{V} + \int_{1} d\sigma^{A}\right)$$

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COMPUTATION OF $\mathrm{d}\sigma^{\widetilde{\mathrm{A}}}$

Similar to dipole formalism, we have a sum over new dipoles:

$$\mathrm{d}\sigma^{\widetilde{\mathrm{A}}} \propto \mathrm{d}\phi_{n-1}\,\mathrm{d}\theta_{n+1} \left(\sum_{(i,j)} \sum_{k \neq i,j} \widetilde{\mathcal{D}}_{ij,k}\, F_J^{(n)} + \text{ initial state dipoles} \ldots \right)$$

where

$$\widetilde{\mathcal{D}}_{ij,k} = -4\pi\alpha_s\mu^{2\epsilon} \left(\mathcal{A}_n^*\right)^{\xi} \frac{\mathbf{T}_{ij} \cdot \mathbf{T}_k}{\mathbf{T}_{ij}^2} \left[\widetilde{\boldsymbol{V}}_{ij,k}(\widetilde{p}_{ij}, p_i, p_j, p_k, \theta_i, \theta_j) \right]_{\xi\xi'} \mathcal{A}_n^{\xi'}$$

$\mathcal{A}_n^{\xi}$: amplitude where polarisation of emitter particle (ij) is missing;
ξ, ξ': Lorentz index ($\widetilde{ij}$ gluon) or Dirac index ($\widetilde{ij}$ quark)

COMPUTATION OF $d\sigma^{\widetilde{A}}$

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We need only helicity mixing terms $\Rightarrow$ define operator $\mathcal{R}$

$$\mathcal{R} f(\theta_i, \theta_j) \equiv f(\theta_i, \theta_j) - \sum_{\lambda_i, \lambda_j} f(\lambda_i, \lambda_j)$$

Then:

$$\widetilde{\boldsymbol{V}}_{ij,k} = C_{(ij)\to i+j}\, \mathcal{R} \left[\widetilde{P}_{(ij)\to i+j} + \widetilde{S}_{(ij)\to i+j} \right]$$

where $C_{q\to qg} = C_F$, $C_{g\to gg} = C_A$ and $C_{g\to q\bar{q}} = T_R$.

COMPUTATION OF $\mathrm{d}\sigma^{\widetilde{\mathrm{A}}}$

Similar to dipole formalism, we have a sum over new dipoles:

$$\mathrm{d}\sigma^{\widetilde{\mathrm{A}}} \propto \mathrm{d}\phi_{n-1}\,\mathrm{d}\theta_{n+1}\left(\sum_{(i,j)}\sum_{k\neq i,j}\widetilde{\mathcal{D}}_{ij,k}\,F_J^{(n)} + \text{ initial state dipoles}\ldots\right)$$

where

To obtain $\widetilde{P}_{(ij)\to i+j}$ and $\widetilde{S}_{(ij)\to i+j}$, look at soft & collinear limits of $\mathcal{A}_{n+1}$ with random polarisations!

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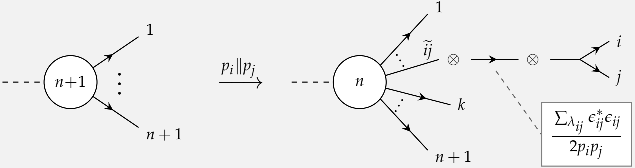
COLLINEAR LIMIT (I)



$$\lim_{p_i \| p_j} \mathcal{A}_{n+1} = g\mu^{\epsilon}\, \mathbf{T}_{(ij)\to i+j} \sum_{\lambda_{ij}} \text{Split}^{\lambda_{ij}}_{(ij)\to i+j}(\widetilde{p}_{ij}) \begin{pmatrix} u_{\lambda_{ij}}(\widetilde{p}_{ij}) \\ \epsilon_{\lambda_{ij}}(\widetilde{p}_{ij}) \end{pmatrix}_{\xi} \mathcal{A}^{\xi}_{n}$$

where Split^{λ} is made up of Feynman rules ($\epsilon_i \equiv \epsilon(p_i)$, etc.):

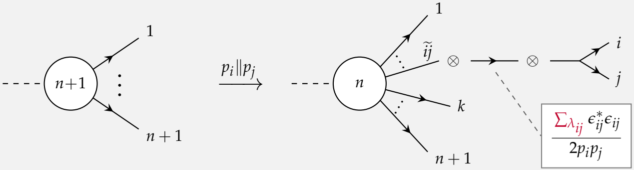
$$\text{Split}^{\lambda}_{q\to qg}(\widetilde{p}_{ij}) = \frac{1}{2p_i \cdot p_j} \bar{u}(p_i)\gamma^{\mu}\epsilon_{\mu}(p_j)u^{\lambda}(\widetilde{p}_{ij})$$

$$\text{Split}^{\lambda}_{g\to gg}(\widetilde{p}_{ij}) = \frac{2}{2p_i \cdot p_j}\Big[(\epsilon_i \cdot \epsilon_j)(p_i \cdot (\epsilon^{\lambda}_{ij})^*) + (\epsilon_j \cdot (\epsilon^{\lambda}_{ij})^*)(p_j \cdot \epsilon_i) - (\epsilon_i \cdot \epsilon^{\lambda}_{ij})^*)(p_i \cdot \epsilon_j)\Big]$$

$$\text{Split}^{\lambda}_{g\to q\bar{q}}(\widetilde{p}_{ij}) = \frac{1}{2p_i \cdot p_j} \bar{u}(p_i)\gamma^{\mu}\epsilon^{\lambda}_{\mu}(\widetilde{p}_{ij})^* v(p_j)$$

all external particle polarisations given by RP!

COLLINEAR LIMIT (I)



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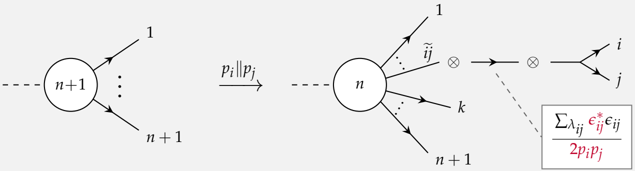
$$\mathrm{Split}^{\lambda}_{g \to gg}(\widetilde{p}_{ij}) = \frac{2}{2 p_i \cdot p_j} \left[(\epsilon_i \cdot \epsilon_j)(p_i \cdot (\epsilon^{\lambda}_{ij})^*) + (\epsilon_j \cdot (\epsilon^{\lambda}_{ij})^*)(p_j \cdot \epsilon_i) - (\epsilon_i \cdot \epsilon^{\lambda}_{ij})^*)(p_i \cdot \epsilon_j) \right]$$

$$\mathrm{Split}^{\lambda}_{g \to q\bar{q}}(\widetilde{p}_{ij}) = \frac{1}{2 p_i \cdot p_j} \bar{u}(p_i) \gamma^{\mu} \epsilon^{\lambda}_{\mu}(\widetilde{p}_{ij})^* v(p_j)$$

all external particle polarisations given by RP!

COLLINEAR LIMIT (I)



$$\lim_{p_i \| p_j} \mathcal{A}_{n+1} = g\mu^\epsilon \, \mathbf{T}_{(ij)\to i+j} \sum_{\lambda_{ij}} \text{Split}^{\lambda_{ij}}_{(ij)\to i+j}(\widetilde{p}_{ij}) \begin{pmatrix} u_{\lambda_{ij}}(\widetilde{p}_{ij}) \\ \epsilon_{\lambda_{ij}}(\widetilde{p}_{ij}) \end{pmatrix}_\xi \mathcal{A}^\xi_n$$

where Split^λ is made up of Feynman rules ($\epsilon_i \equiv \epsilon(p_i)$, etc.):

$$\text{Split}^\lambda_{q\to qg}(\widetilde{p}_{ij}) = \frac{1}{2p_i \cdot p_j} \bar{u}(p_i)\gamma^\mu \epsilon_\mu(p_j) u^\lambda(\widetilde{p}_{ij})$$

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all external particle polarisations given by RP!

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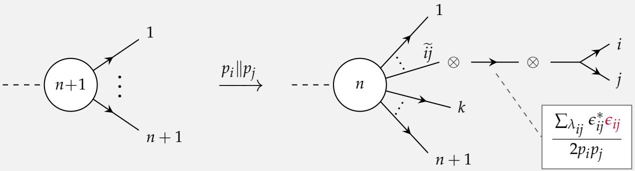

$$\lim_{p_i \| p_j} \mathcal{A}_{n+1} = g\mu^\epsilon \, \mathbf{T}_{(ij)\to i+j} \sum_{\lambda_{ij}} \text{Split}^{\lambda_{ij}}_{(ij)\to i+j}(\widetilde{p}_{ij}) \begin{pmatrix} u_{\lambda_{ij}}(\widetilde{p}_{ij}) \\ \epsilon_{\lambda_{ij}}(\widetilde{p}_{ij}) \end{pmatrix}_\xi \mathcal{A}^\xi_n$$

where Split^λ is made up of Feynman rules ($\epsilon_i \equiv \epsilon(p_i)$, etc.):

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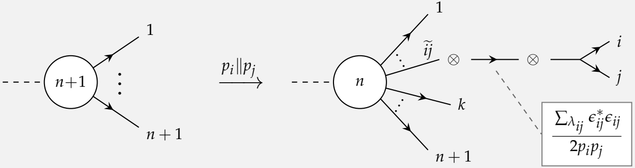
$$\text{Split}^\lambda_{g\to q\bar{q}}(\widetilde{p}_{ij}) = \frac{1}{2p_i \cdot p_j} \bar{u}(p_i)\gamma^\mu \epsilon^\lambda_\mu(\widetilde{p}_{ij})^* v(p_j)$$

all external particle polarisations given by RP!

Collinear limit (I)



$$\lim_{p_i \| p_j} \mathcal{A}_{n+1} = g\mu^{\epsilon}\, \mathbf{T}_{(ij)\to i+j} \sum_{\lambda_{ij}} \mathrm{Split}^{\lambda_{ij}}_{(ij)\to i+j}(\widetilde{p}_{ij}) \begin{pmatrix} u_{\lambda_{ij}}(\widetilde{p}_{ij}) \\ \epsilon_{\lambda_{ij}}(\widetilde{p}_{ij}) \end{pmatrix}_{\xi} \mathcal{A}^{\xi}_{n}$$

where Split^{λ} is made up of Feynman rules ($\epsilon_i \equiv \epsilon(p_i)$, etc.):

$$\mathrm{Split}^{\lambda}_{q\to qg}(\widetilde{p}_{ij}) = \frac{1}{2p_i \cdot p_j} \bar{u}(p_i)\gamma^{\mu}\epsilon_{\mu}(p_j)u^{\lambda}(\widetilde{p}_{ij})$$

$$\mathrm{Split}^{\lambda}_{g\to gg}(\widetilde{p}_{ij}) = \frac{2}{2p_i \cdot p_j}\left[(\epsilon_i \cdot \epsilon_j)(p_i \cdot (\epsilon^{\lambda}_{ij})^*) + (\epsilon_j \cdot (\epsilon^{\lambda}_{ij})^*)(p_j \cdot \epsilon_i) - (\epsilon_i \cdot \epsilon^{\lambda}_{ij})^*)(p_i \cdot \epsilon_j)\right]$$

$$\mathrm{Split}^{\lambda}_{g\to q\bar{q}}(\widetilde{p}_{ij}) = \frac{1}{2p_i \cdot p_j} \bar{u}(p_i)\gamma^{\mu}\epsilon^{\lambda}_{\mu}(\widetilde{p}_{ij})^* v(p_j)$$

all external particle polarisations given by RP!

COLLINEAR LIMIT (II)

$$\lim_{p_i \| p_j} \mathcal{A}_{n+1} = g\mu^\epsilon \, \mathbf{T}_{(ij)\to i+j} \sum_{\lambda_{ij}} \text{Split}^{\lambda_{ij}}_{(ij)\to i+j}(\widetilde{p}_{ij}) \begin{pmatrix} u_{\lambda_{ij}}(\widetilde{p}_{ij}) \\ \epsilon_{\lambda_{ij}}(\widetilde{p}_{ij}) \end{pmatrix}_\xi \mathcal{A}^\xi_n$$

Squared amplitude:

$$\lim_{p_i \| p_j} |\mathcal{A}_{n+1}|^2 = 4\pi\alpha_s\mu^{2\epsilon} \left(\mathcal{A}^*_n\right)^\xi \mathbf{T}^2_{(ij)\to i+j} \left[\widetilde{P}_{(ij)\to i+j}\right]_{\xi\xi'} \mathcal{A}^{\xi'}_n$$

where

$$\left[\widetilde{P}_{(ij)\to i+j}\right]_{\binom{\alpha\beta}{\mu\nu}} (\widetilde{p}_{ij}) = \sum_{\lambda,\lambda'} \begin{pmatrix} \bar{u}^\lambda_\alpha(\widetilde{p}_{ij}) \\ \epsilon^\lambda_\mu(\widetilde{p}_{ij})^* \end{pmatrix} \text{Split}^{\lambda\,*} \text{Split}^{\lambda'} \begin{pmatrix} u^{\lambda'}_\beta(\widetilde{p}_{ij}) \\ \epsilon^{\lambda'}_\nu(\widetilde{p}_{ij}) \end{pmatrix}$$

Product of Split-functions automatically generates correct product of polarisations!

SOFT LIMIT ($q \to qg$ AND $g \to gg$)

well-known:

$$\lim_{p_j \to 0} |\mathcal{A}_{n+1}|^2 = -4\pi\alpha_s \mu^{2\epsilon} \sum_{i=1}^{n} \sum_{\substack{k=1 \\ k \neq i}}^{n} \mathcal{A}_n^* \, \mathbf{T}_i \cdot \mathbf{T}_k \, S_{ij,k}(\epsilon_j) \, \mathcal{A}_n, \qquad \mathcal{A}_n = \begin{pmatrix} u(p_i) \\ \epsilon(p_i) \end{pmatrix}_\xi \mathcal{A}_n^\xi$$

where

$$S_{ij,k}(\epsilon_j) = \frac{(p_i \cdot \epsilon_j^*)(p_i \cdot \epsilon_j)}{(p_i \cdot p_j)^2} - \frac{(p_i \cdot \epsilon_j^*)(p_k \cdot \epsilon_j) + (p_k \cdot \epsilon_j^*)(p_i \cdot \epsilon_j)}{(p_i \cdot p_j)(p_i \cdot p_j + p_j \cdot p_k)}$$

SOFT LIMIT ($q \to qg$ AND $g \to gg$)

well-known:

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Beware of double counting!

collinear limit of $S_{ij,k}$:

$$\lim_{p_i \| p_j} S_{ij,k} = \frac{(p_i \cdot \epsilon_j^*)(p_i \cdot \epsilon_j)}{(p_i \cdot p_j)^2}$$

soft limit of $\widetilde{P}_{(ij)\to i+j}$:

$$\lim_{p_j \to 0} \left[\widetilde{P}_{(ij)\to i+j} \right]_{\binom{\alpha\beta}{\mu\nu}} = \frac{(p_i \cdot \epsilon_j^*)(p_i \cdot \epsilon_j)}{(p_i \cdot p_j)^2} \begin{pmatrix} u_\alpha(p_i)\bar{u}_\beta(p_i) \\ \epsilon_\mu(p_i)^* \epsilon_\nu(p_i) \end{pmatrix}$$

$\Rightarrow$ both limits agree! $\Rightarrow$ we need to supply $\widetilde{P}_{(ij)\to i+j}$ with missing term from $S_{ij,k}$!

SOFT LIMIT ($q \to qg$ AND $g \to gg$)

well-known:

where

Beware of double counting!

Additional soft terms:

$$\left[\widetilde{S}_{q\to qg}\right]_{\alpha\beta} = -\frac{(p_i\cdot\epsilon_j^*)(\widetilde{p}_k\cdot\epsilon_j) + (\widetilde{p}_k\cdot\epsilon_j^*)(p_i\cdot\epsilon_j)}{(p_i\cdot p_j)(p_i\cdot p_j + p_j\cdot\widetilde{p}_k)} u_\alpha(p_i)\bar{u}_\beta(p_i)$$

$$\left[\widetilde{S}_{g\to gg}\right]_{\mu\nu} = -\frac{(p_i\cdot\epsilon_j^*)(\widetilde{p}_k\cdot\epsilon_j) + (\widetilde{p}_k\cdot\epsilon_j^*)(p_i\cdot\epsilon_j)}{(p_i\cdot p_j)(p_i\cdot p_j + p_j\cdot\widetilde{p}_k)} \epsilon_\mu(p_i)^*\epsilon_\nu(p_i) - (i\leftrightarrow j)$$

$$\left[\widetilde{S}_{g\to q\bar{q}}\right]_{\mu\nu} = 0$$

collinear limit of $S_{ij,k}$: $\quad \lim_{p_i\parallel p_j} S_{ij,k} = \frac{(p_i\cdot\epsilon_j^*)(p_i\cdot\epsilon_j)}{(p_i\cdot p_j)^2}$

soft limit of $\widetilde{P}_{(ij)\to i+j}$: $\quad \lim_{p_j\to 0}\left[\widetilde{P}_{(ij)\to i+j}\right]_{\binom{\alpha\beta}{\mu\nu}} = \frac{(p_i\cdot\epsilon_j^*)(p_i\cdot\epsilon_j)}{(p_i\cdot p_j)^2}\begin{pmatrix} u_\alpha(p_i)\bar{u}_\beta(p_i) \\ \epsilon_\mu(p_i)^*\epsilon_\nu(p_i)\end{pmatrix}$

$\Rightarrow$ both limits agree! $\Rightarrow$ we need to supply $\widetilde{P}_{(ij)\to i+j}$ with missing term from $S_{ij,k}$!

FINALLY: DIPOLE SUBTRACTION WITH RP

$$\mathrm{d}\sigma^{\widetilde{A}} \propto \mathrm{d}\phi_{n-1}\,\mathrm{d}\theta_{n+1}\left(\sum_{(i,j)}\sum_{k\neq i,j}\widetilde{\mathcal{D}}_{ij,k}\,F_J^n + \text{IS}\ldots\right)$$

$$\widetilde{\mathcal{D}}_{ij,k} = -4\pi\alpha_s\mu^{2\epsilon}\left(\mathcal{A}_n^*\right)^{\xi}\frac{\mathbf{T}_{ij}\cdot\mathbf{T}_k}{\mathbf{T}_{ij}^2}\left[\widetilde{\boldsymbol{V}}_{ij,k}\right]_{\xi\xi'}\mathcal{A}_n^{\xi'}$$

$$\widetilde{\boldsymbol{V}}_{ij,k} = C_{(ij)\to i+j}\,\mathcal{R}\left[\widetilde{P}_{(ij)\to i+j} + \widetilde{S}_{(ij)\to i+j}\right]$$

$$\left[\widetilde{P}_{(ij)\to i+j}\right]_{\binom{\alpha\beta}{\mu\nu}} = \sum_{\lambda,\lambda'}\begin{pmatrix}u_\alpha^\lambda(\widetilde{p}_{ij})\\ \epsilon_\mu^\lambda(\widetilde{p}_{ij})^*\end{pmatrix}\text{Split}^{\lambda\,*}\text{Split}^{\lambda'}\begin{pmatrix}u_\beta^{\lambda'}(\widetilde{p}_{ij})\\ \epsilon_\nu^{\lambda'}(\widetilde{p}_{ij})\end{pmatrix}$$

$$\mathcal{R}f(\theta_i,\theta_j) \equiv f(\theta_i,\theta_j) - \sum_{\lambda_i,\lambda_j} f(\lambda_i,\lambda_j)$$

$$\widetilde{p}_{ij} = p_i + p_j - \frac{y_{ij,k}}{1-y_{ij,k}}p_k$$

$$\widetilde{p}_k = \frac{1}{1-y_{ij,k}}p_k$$

$$y_{ij,k} = \frac{p_i p_k}{p_i p_j + p_j p_k + p_k p_i}$$

	$q \to qg$	$g \to gg$	$g \to q\bar{q}$
$C_{(ij)\to i+j}$	C_F	C_A	T_R
$\text{Split}^\lambda_{(ij)\to i+j}$	$\frac{1}{2p_i\cdot p_j}\bar{u}(p_i)\gamma^\mu\epsilon_\mu(p_j)u^\lambda(\widetilde{p}_{ij})$	$\frac{2}{2p_i\cdot p_j}\Big[(\epsilon_i\cdot\epsilon_j)(p_i\cdot(\epsilon_{ij}^\lambda)^*)$ $+(\epsilon_j\cdot(\epsilon_{ij}^\lambda)^*)(p_j\cdot\epsilon_i)$ $-(\epsilon_i\cdot(\epsilon_{ij}^\lambda)^*)(p_i\cdot\epsilon_j)\Big]$	$\frac{1}{2p_i\cdot p_j}\bar{u}(p_i)\gamma^\mu\epsilon_\mu^\lambda(\widetilde{p}_{ij})^*v(p_j)$
$\left[\widetilde{S}_{(ij)\to i+j}\right]_{\xi\xi'}$	$-\frac{(p_i\cdot\epsilon_j^*)(\widetilde{p}_k\cdot\epsilon_j)+(\widetilde{p}_k\cdot\epsilon_j^*)(p_i\cdot\epsilon_j)}{(p_i\cdot p_j)(p_i\cdot p_j+p_i\cdot\widetilde{p}_k)}$ $\times u_\alpha(p_i)\bar{u}_\beta(p_i)$	$-\frac{(p_i\cdot\epsilon_j^*)(\widetilde{p}_k\cdot\epsilon_j)+(\widetilde{p}_k\cdot\epsilon_j^*)(p_i\cdot\epsilon_j)}{(p_i\cdot p_j)(p_i\cdot p_j+p_i\cdot\widetilde{p}_k)}$ $\times\epsilon_\mu(p_i)^*\epsilon_\nu(p_i) - (i\leftrightarrow j)$	0

ADDITIONAL REMARKS

Generality of the method

- previous slides: final-state emitter and spectator; using crossing symmetry one can easily derive other three cases (final-initial, initial-final, initial-initial)
  $\hookrightarrow$ see publication below!
- method also works with massive particles: need more complicated momentum parametrisations $\widetilde{p}_{ij}$ and $\widetilde{p}_k$
  $\rightarrow$ see publication below!

Feature of the method

- does not require knowledge of original subtraction term $d\sigma^{\mathrm{A}}$, only requires that it is formulated for helicity summed squared amplitudes
  $\hookrightarrow$ use with subtraction scheme of your choice!

[DG, Schwan, Weinzierl — Phys. Rev. D 85, 116011 (2012)]

SUMMARY

- RP provide great speed-up for numerical calculations!
- To use RP with subtraction method we have to extend existing subtraction methods due to helicity mixing terms
- There is no new integrated subtraction term!
- Formulate new terms using Feynman rules for splitting vertex plus a supplemental soft term.
- All cases covered: massless, massive, final & initial emitters/spectators

SUMMARY

- RP provide great speed-up for numerical calculations!
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- All cases covered: massless, massive, final & initial emitters/spectators

Thank you for your attention!

Backup Slides

RP FOR MASSIVE PARTICLES & CHANGES TO $\mathrm{d}\sigma^{\widetilde{\mathrm{A}}}$

We use light-cone parametrisation for massless fermion spinors.
To obtain a massive spinor with $P^2 = m^2$, use parametrisation

$$u_\pm(P,m) = \frac{1}{\langle P^\flat \pm |q\mp\rangle}(\not{P} + m)\, u_\mp(q,0)$$

where

- $P^\flat = P - \dfrac{P^2}{2P\cdot q}q$
- q is an arbitrary light-like reference momentum ($q^2 = 0$).

New momentum parametrisation for massive final-final $\mathrm{d}\sigma^{\widetilde{\mathrm{A}}}$:

$$\tilde{p}_{ij} = Q - p_k,$$

$$\tilde{p}_k = \frac{\sqrt{\lambda(Q^2, m_{ij}^2, m_k^2)}}{\sqrt{\lambda(Q^2, (p_i+p_j)^2, m_k^2)}}\left(p_k - \frac{Q\cdot p_k}{Q^2}Q\right) + \frac{Q^2 + m_k^2 - m_{ij}^2}{2Q^2}Q$$

where

- $Q = p_i + p_j + p_k,$
- $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx.$

RP & Dimensional Regularisation

Three different variants of DR wrt. spin/helicity treatment:

1. Conventional DR: all spin dof extended to d dimensions,
2. 't Hooft-Veltman: only spin dofs of unobserved particles extended to d dimensions,
3. four-dimensional helicity scheme: all spin dof 4 dimensional

There may be additional spin degrees of freedom for 1. and 2.

$\hookrightarrow$ ε-helicities denoted by $\epsilon_\mu^{(-2\varepsilon)}$

Polarisation sum given by

$$\epsilon_\mu^{+*}\epsilon_\nu^{+} + \epsilon_\mu^{-*}\epsilon_\nu^{-} + \epsilon_\mu^{(-2\varepsilon)*}\epsilon_\nu^{(-2\varepsilon)}$$

Let RP $\epsilon_\mu(\theta)$ act in four-dimensional subspace. We then find

$$\epsilon_\mu(\theta)^*\epsilon_\nu(\theta) + \epsilon_\mu^{(-2\varepsilon)*}\epsilon_\nu^{(-2\varepsilon)} = \left(\epsilon_\mu^{+*}\epsilon_\nu^{+} + \epsilon_\mu^{-*}\epsilon_\nu^{-} + \epsilon_\mu^{(-2\varepsilon)*}\epsilon_\nu^{(-2\varepsilon)}\right) + \mathrm{e}^{4\pi \mathrm{i}\theta}\epsilon_+^{\mu *}\epsilon_-^{\nu} + \mathrm{e}^{-4\pi \mathrm{i}\theta}\epsilon_-^{\mu *}\epsilon_+^{\nu}$$

$\Rightarrow$ RHS again polarisation sum plus helicity mixing terms!

ALTERNATIVE METHOD: HELICITY SAMPLING

Replace sum by integral:

$$\sum_{\Lambda_n} |\mathcal{A}_{\Lambda_n}|^2 = 2^n \int_{[0,1)^n} \mathrm{d}^n u \, |\mathcal{A}_{u_1 u_2 \dots u_n}|^2$$

with

$$\epsilon(p_i, u_i) = \begin{cases} \epsilon_-(p_i) & \text{for } 0 \leq u_i < 0.5 \\ \epsilon_+(p_i) & \text{for } 0.5 \leq u_i < 1 \end{cases}$$

(analogously for fermion spinors)

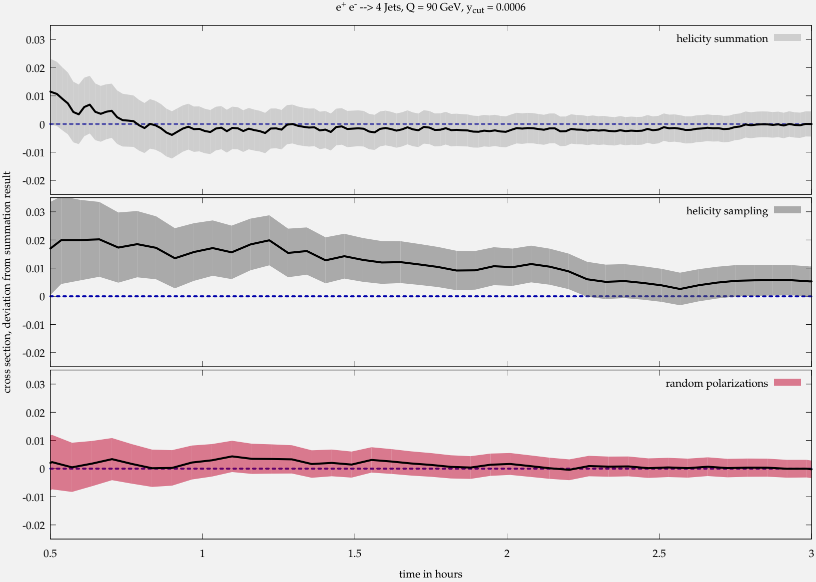
$\Rightarrow$ perform integral via MC alongside PS integral (increase d!)

- only one helicity amplitude per PS point!

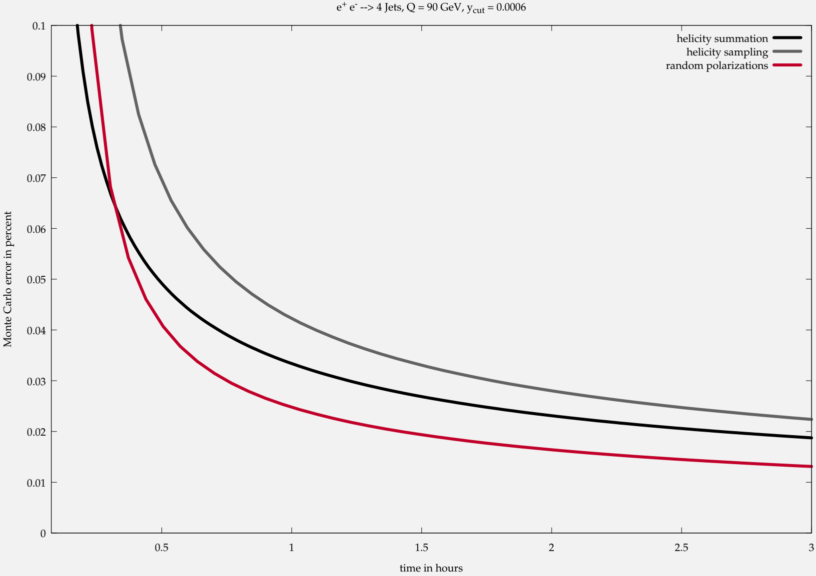
but: not ideal for VEGAS algorithm

- mapping $u_i \rightarrow \epsilon(p_i, u_i)$ is not a continuous function,
- no factorisation as required by VEGAS!

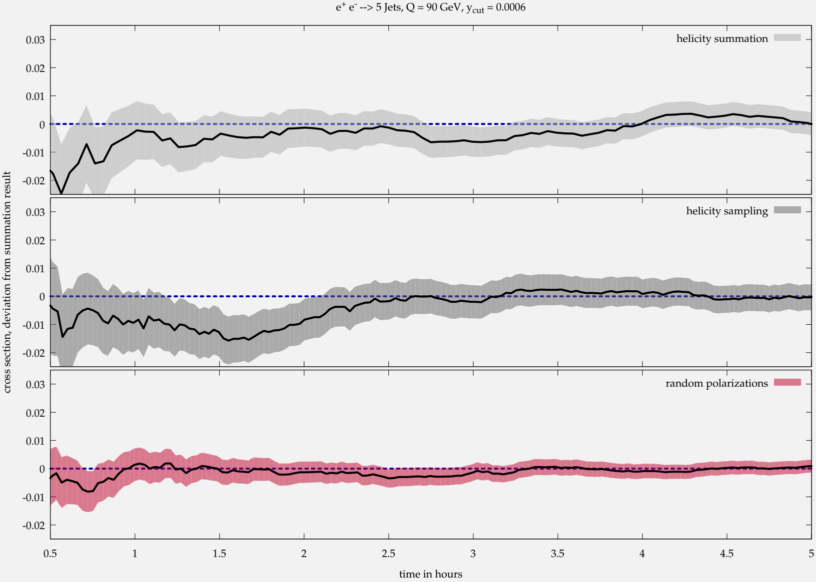
BORN-LEVEL EXAMPLE: $e^+e^- \rightarrow$ 4 JETS



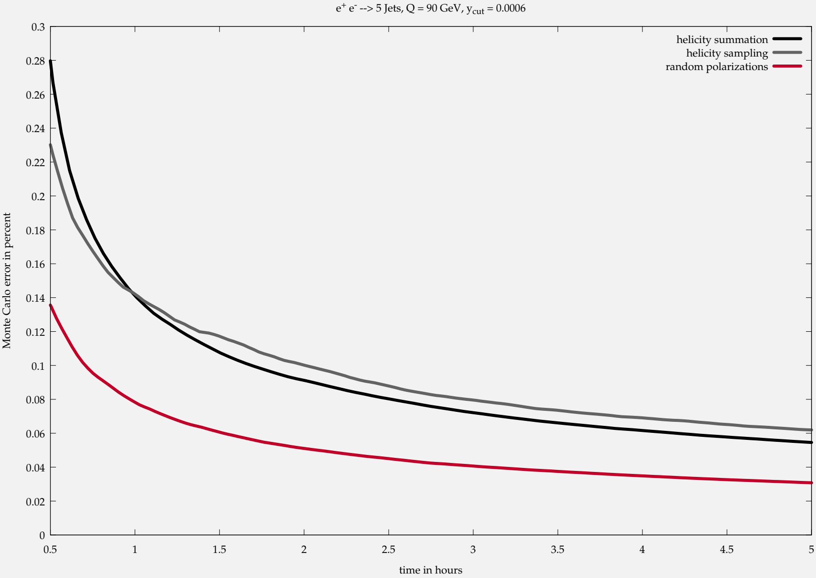
BORN-LEVEL EXAMPLE: $e^+e^- \to 4$ JETS



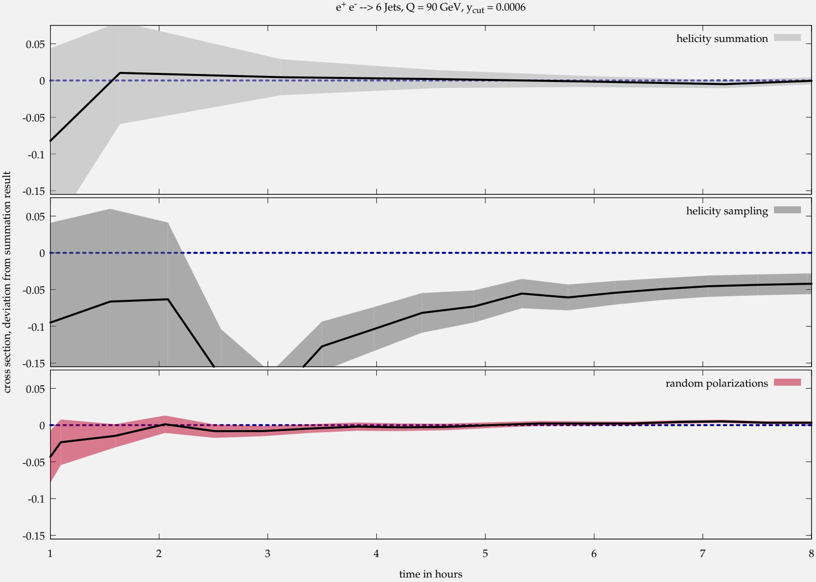
BORN-LEVEL EXAMPLE: $e^+e^- \to$ 5 JETS



BORN-LEVEL EXAMPLE: $e^+e^- \to 5$ JETS



BORN-LEVEL EXAMPLE: $e^+e^- \to 6$ JETS



BORN-LEVEL EXAMPLE: $e^+e^- \to 6$ JETS