Two-loop computations with SecDec 2.1 and their application to MSSM Higgs-boson masses

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Projects in collaboration with G. Heinrich & W. Hollik

Based on 1309.3492, Comput.Phys.Commun. 184 2552-2561, Comput.Phys.Commun. 184 396-408

RADCOR 2013, Lumley Castle, September 24th, 2013

http://secdec.hepforge.org/

Outline

- **The program** SecDec 2.1
- **Applications:**
  - Non-planar two-loop boxes entering the $t\bar{t}$ production @NNLO
  - Towards momentum dependent two-loop corrections to the MSSM neutral $\mathcal{CP}$-even Higgs-boson masses

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  - Extraction of IR and UV singularities
  - Numerical convergence in the presence of integrable singularities (e.g. thresholds)
  - Speed/accuracy

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  - Extraction of IR and UV singularities (solved with SecDec 1.0)
  - Numerical convergence in the presence of integrable singularities (e.g. thresholds) (solved with SecDec 2.0)
  - Speed/accuracy (improved with SecDec 2.1)

Public codes using the sector decomposition method

Idea and method of sector decomposition pioneered by
Hepp '66, Denner & Roth '96, Binoth & Heinrich '00

Public codes:

- sector_decomposition (uses GiNaC) C. Bogner & S. Weinzierl '07
  supplemented with `CSectors` Gluza, Kajda, Riemann, Yundin '10
  for construction of integrand in terms of Feynman parameters
- FIESTA (uses Mathematica, C) A.V. Smirnov, V.A. Smirnov, M. Tentyukov '08 '09
- SecDec (uses Mathematica, Fortran/C++) J. Carter & G. Heinrich '10; SB, J. Carter, G. Heinrich '12; SB & G. Heinrich '13

Many people are/have been working on PURELY numerical methods, e.g. Anastasiou et al., Weinzierl et al., Binoth/Heinrich et al., Boughezal/Melnikov/Petriello et al., Czakon et al., Freitas et al., Kurihara et al., Nagy/Soper et al., Passarino et al., ...

SecDec 2.1 can tackle ...

SecDec is a tool to numerically compute various sorts of integrals contributing to higher-order computations.

It can tackle:

- General Feynman integrals and more general parametric functions for arbitrary kinematics

Feynman graph | or | parametric function

General Feynman Integral

- **Generic Feynman integrals** in D dimensions at L loops with N propagators to power ν_j of rank R with $N_\nu = \sum_{j=1}^{N} \nu_j$, e.g. scalar multi-loop integral in Feynman parametrization

$$G = \frac{(-1)^{N_\nu}}{\prod_{j=1}^{N}\Gamma(\nu_j)}\Gamma(N_\nu - LD/2)\int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j-1}\,\delta(1-\sum_{l=1}^{N} x_l)\frac{\mathcal{U}^{N_\nu-(L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu-LD/2}(\vec{x})}$$

- Extension to physical kinematics including mass thresholds since SecDec 2.0: Limitation of multi-scale integrals to the Euclidean region lifted! SB, Carter, Heinrich '12

NEW in SecDec 2.1

- Computation of contracted **tensor** integrals at in principle arbitrary rank possible SB & Heinrich '13

$$T_{12345}^{\mathrm{Rank3}} = \iint \mathrm{d}^{\mathrm{D}}k_1\,\mathrm{d}^{\mathrm{D}}k_2\,\frac{p_{1\mu}k_1^{\mu}k_{1\nu}k_2^{\nu}}{D_1 D_2 D_3 D_4 D_5}$$

Parametric Functions

A general parametric function can be

- a phase space integral where IR divergences are regulated dimensionally
- functions similar to hypergeometric functions, e.g.

$$
{}_3F_2(a_1,\ldots,a_3;b_1,b_2;\beta) \propto
$$

$$
\iint_0^1 \mathrm{d}x\mathrm{d}y\; x^{a_1-1}(1-x)^{b_1-a_1-1}y^{a_2-1}(1-y)^{b_2-a_2-1}(1-\beta xy)^{-a_3}
$$

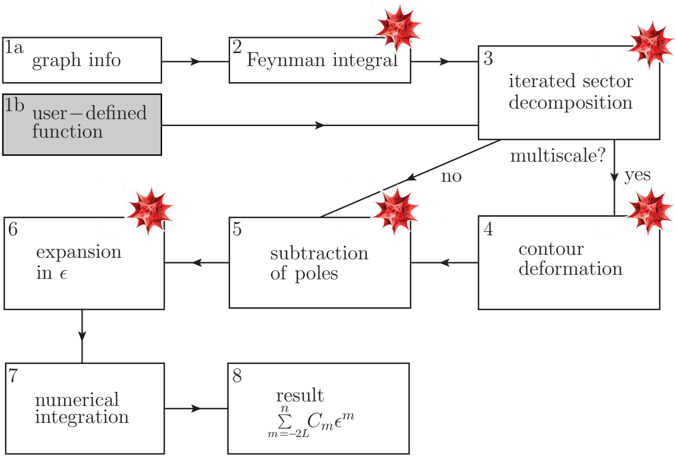
NEW in SecDec 2.1

- Computation of more general **user-defined polynomial integrals** matching the Feynman loop integral structure

  SB & Heinrich '12 '13

$$
G_{userdefined} = \frac{(-1)^{N_\nu}}{\prod_{j=1}^{N}\Gamma(\nu_j)}\Gamma(N_\nu - LD/2)\int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j\; x_j^{\nu_j-1}\,\delta(1-\sum_{l=1}^{N} x_l)\,\frac{\mathcal{U}^{N_\nu-(L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu-LD/2}(\vec{x})}
$$

Operational Sequence of the SecDec 2.1 Program



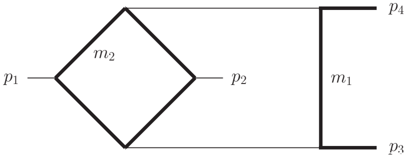
Numerical integration:

Cuba library Hahn et al. '04 '11 or Bases Kawabata '95

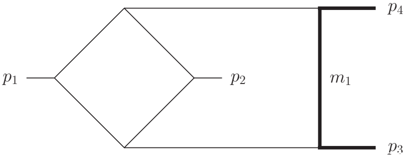
New features of the program SecDec Version 2.1

- Computation of contracted **tensor integrals** at in principle arbitrary rank possible
- **User-defined** functions amenable to contour deformation can be inserted and decomposed directly
- **User-friendliness** and **efficiency** improved (e.g. convergence behavior written to result files)

Application I:
Massive non-planar 2-loop diagrams for $t\bar{t}$@NNLO



(a) ggtt1



(b) ggtt2

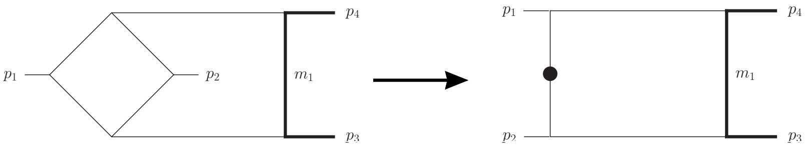
- Diagram *ggtt*1 entering heavy fermionic corrections:
  finite, **no** analytical results available
  $\rightarrow$ easily computable with SecDec
- Diagram *ggtt*2 entering light fermionic corrections:
  leading pole $\mathcal{O}(\epsilon^{-4})$, spurious divergence, analytic result by
  Manteuffel & Studerus '12 '13
  $\rightarrow$ many functions to integrate, cancellations
  $\Rightarrow$ **mixed approach:** analytical preparation beforehand

Analytical manipulations beforehand

Goals for better numerical convergence:

1) decrease number of numerical integration parameters
2) turn linear divergences $x^{-2-\epsilon}$ into logarithmic ones
3) decrease number of functions

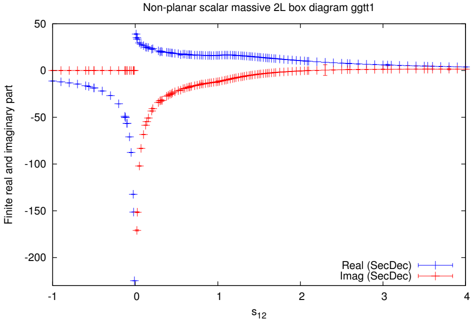
Achieving goal 1: Integrate out one loop first



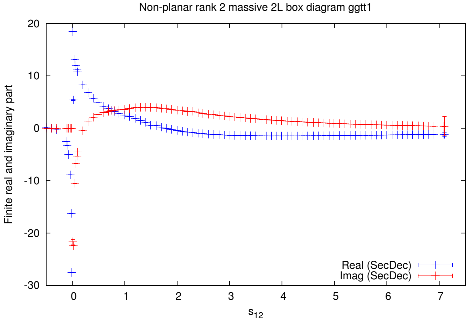
$\rightarrow$ analytical integration of one Feynman parameter is possible

Achieving goal 2 & 3: Using a new transformation, a more even distribution of divergences among Feynman parameters is possible

Results for the non-planar massive two loop diagram ggtt1



Scalar integral



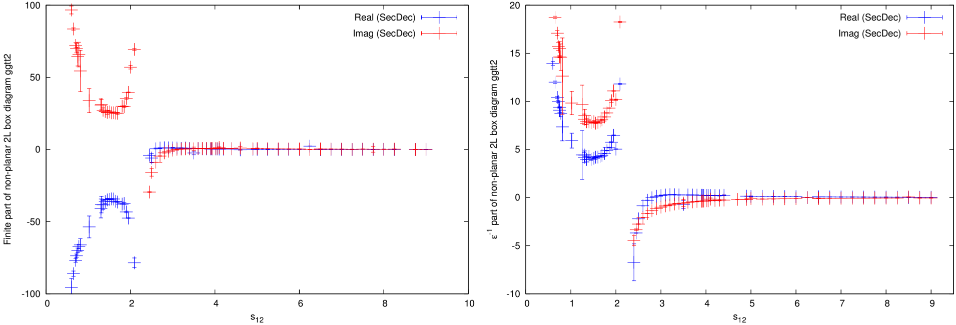
Rank 2

$m_1^2 = m_2^2 = 1$, $p_3^2 = p_4^2 = m_1^2$, $p_1^2 = p_2^2 = 0$, $s_{23} = -1.25$

timings: 11-60 secs (scalar) & 5-10 secs (rank 2) far from threshold (th.)
1600 secs (scalar) & max. 3600 secs (rank 2) very close to th.

rel. accuracy: 10^{-3}, abs. accuracy: 10^{-5}

Result for the non-planar massive two loop diagram ggtt2



Finite part $\qquad$ $1/\epsilon$ pole

$m_1^2 = 1,\ p_1^2 = p_2^2 = 0,\ p_3^2 = p_4^2 = m_1^2,\ s_{23} = -1.25$

timings: 250-4000 secs, rel. accuracy $5{\cdot}10^{-3}$, abs. accuracy: 10^{-5}

analytic results: Manteuffel & Studerus '13

Application II: Momentum dependent two-loop corrections to the neutral $\mathcal{CP}$-even Higgs-boson masses in the MSSM

- Minimal extension of the SM allowing for SUSY:
  2 Higgs doublets

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 + \mathrm{i}\chi_1^0) \\ -\phi_1^- \end{pmatrix} \qquad H_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + \mathrm{i}\chi_2^0) \end{pmatrix}$$

- MSSM Higgs potential (incl. soft SUSY breaking terms)

$$\begin{aligned} V =& m_1|H_1|^2 + m_2|H_2|^2 - m_{12}(\epsilon_{ab}H_1^a H_2^b + h.c.) \\ &+ \frac{1}{8}(g_1^2 + g_2^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g_2^2|H_1^\dagger H_2|^2 \end{aligned}$$

- MSSM Higgs potential fixed by g_1, g_2, the vevs in $\tan\beta \equiv \frac{v2}{v1}$ and soft SUSY breaking term in $m_A^2 = m_{12}^2(\tan\beta + \cot\beta)$

Public codes implementing the rMSSM corrections

FeynHiggs Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '00 '03 '07

SoftSusy Allanach '02 SPheno Porod '03

CPsuperH Carena, Choi, Drees, Ellis, Lee, Pilaftsis, Wagner '04 '09

Suspect Djouadi, Kneur, Moultaka '07

H3m Kant, Harlander, Mihaila, Steinhauser '10

Summary of the implemented rMSSM corrections:

- **1-loop** complete
- **2-loop** $\mathcal{O}(\alpha_s\alpha_t)$, $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_s\alpha_b)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$, gaugeless limit and $p^2 = 0$
- **3-loop** $\mathcal{O}(\alpha_s^2\alpha_t)$, gaugeless limit and $p^2 = 0$

dominant correction @ 2-loop: $\mathcal{O}(\alpha_s\alpha_t)$ $(p^2 = 0)$

⟶ **next improvement:** $\mathcal{O}(\alpha_s\alpha_t)$ for $p^2 \neq 0$

Analytical side: Two-loop renormalization for neutral $\mathcal{CP}$-even Higgs-boson self-energies

Feynman diagrammatic calculation performed in the gaugeless limit

- Renormalization corresponds to FeynHiggs renormalization
- Mass renormalization in the OS scheme:

$$\delta M_A^{2(2)},\ \delta t_1^{(2)},\ \delta t_2^{(2)},\ \delta m_{\tilde{t}_1}^{(1)},\ \delta m_{\tilde{t}_2}^{(1)},\ \delta m_t^{(1)}$$

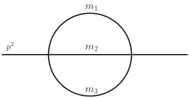
- Field renormalization in the $\overline{DR}$ scheme:

$$\delta Z_{H_1}^{(2)},\ \delta Z_{H_2}^{(2)},\ \delta \tan\beta^{(2)}$$

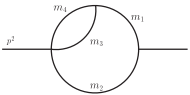
- Resulting input parameters: $m_t, \mu, X_t, M_{SUSY}, m_{\tilde{g}}, \tan\beta, m_A$

  $X_t = A_t - \mu \cot\beta$ and A_t the soft SUSY breaking parameters

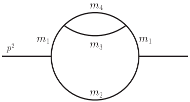
Numerical side: Integrals provided by SecDec



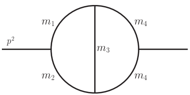
T_{234}



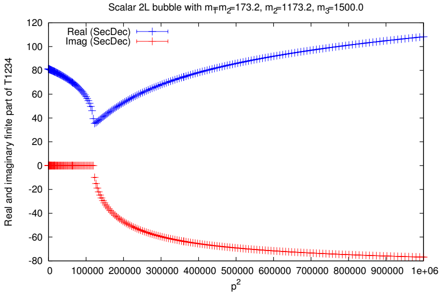
T_{1234}



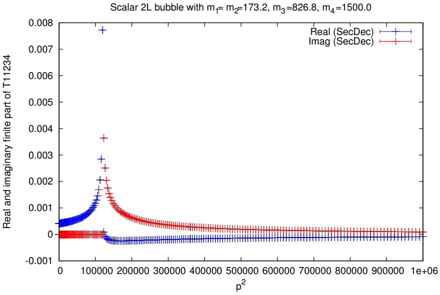
T_{11234}



T_{12345}



T_{1234}, finite part



T_{11234}, finite part

relative accuracy: 10^{-4}, max. 7.5 secs very close to threshold

Towards the neutral MSSM Higgs-boson masses

- Join analytical and numerical results to compute the neutral $\mathcal{CP}$-even Higgs-boson self-energies
- The new self-energy corrections are included in the inverse Higgs-boson propagator matrix

$$\Gamma \equiv \Delta_{\text{Higgs}}^{-1} = \begin{pmatrix} p^2 - m_{H,\text{tree}}^2 + \hat{\Sigma}_H(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_{h,\text{tree}}^2 + \hat{\Sigma}_h(p^2) \end{pmatrix}$$

  with renormalized self-energies $\hat{\Sigma}$ up to the two-loop level
- The propagator poles m_H^2 and m_h^2 are solutions to $\text{Det}(\Gamma) = 0$

Summary & Outlook

Summary

- SecDec 2.1 is a useful tool to compute various sorts of integrals: multi-loop integrals, contracted tensor integrals and user-defined polynomial integrals
- We computed non-planar 2-loop 4-point master integrals entering $t\bar{t}$@NNLO computations
- Momentum dependent 2-loop corrections to the MSSM Higgs masses are feasible using SecDec 2.1

Outlook

- Further applications to other massive two-loop amplitudes
- Combination with new unitarity inspired reduction of 2-loop amplitudes

Backup

Install SecDec 2.1

- **Download**:
  http://secdec.hepforge.org
- **Install**:
  tar xzvf SecDec.tar.gz
  cd SecDec-2.1
  ./install
- **Prerequisites**:
  Mathematica (version 6 or above), Perl, Fortran and/or C++ compiler

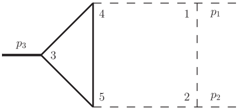
User Input I

- param.input: parameters for integrand specification and numerical integration

```
################ input parameters for sector decomposition ################
#-----------------------------------------------------------------------------------------------
# subdirectory for the mathematica output files (will be created if non-existent) :
# if not specified, a directory with the name of the graph given below will be created by default
subdir=2loop
#-----------------------------------------------------------------------------------------------
# if outputdir is not specified: default directory for
# the output will have integral name (given below) appended to directory above,
# otherwise specify full path for Mathematica output files here
outputdir=
#-----------------------------------------------------------------------------------------------
# graphname (can contain underscores, numbers, but should not contain commas)
graph=P126
#-----------------------------------------------------------------------------------------------
# number of propagators:
propagators=6
#-----------------------------------------------------------------------------------------------
# number of external legs:
legs=3
#-----------------------------------------------------------------------------------------------
# number of loops:
loops=2
#-----------------------------------------------------------------------------------------------
# construct integrand (F and U) via topological cuts (only possible for scalar integrals)
# default is 0 (no cut construction used)
cutconstruct=1
#########################################################
# parameters for subtractions and epsilon expansion
#########################################################
```

User Input II

- template.m: definition of the integrand (Mathematica syntax)




```
(******* USER INPUT for construction of integrand ***************************)

(******* Use with cutconstruct=1 *********************************************)

proplist={{ms[1],{3,4}},{ms[1],{4,5}},{ms[1],{5,3}},
          {0,{1,2}},{0,{1,4}},{0,{2,5}}};

(******* Use with cutconstruct=0 *********************************************)
(*
momlist={k1,k2};
proplist={k1^2-ms[1],(k1+p3)^2-ms[1],(k1-k2)^2-ms[1],
         (k2+p3)^2,(k2+p1+p3)^2,k2^2};
numerator={1};
*)

(******* Propagator powers (optional) ****************************************)

powerlist=Table[1,{i,Length[proplist]}];

(******* On-shell conditions (optional) **************************************)
onshell={ssp[1]->0,ssp[2]->0,ssp[3]->sp[1,2],sp[1,3]->0,sp[2,3]->0};

(******* Set Dimension *******************************************************)

Dim=4-2*eps;
(***************************************************************************)
```

Program Test Run

- ./launch -p param.input -t template.m

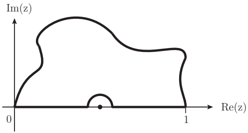
```
********** This is SecDec version 2.0 **********
Authors: Sophia Borowka, Jonathon Carter, Gudrun Heinrich
***********************************************
graph = P126
primary sectors 1,2,3,4,5,6, will be calculated
calculating F and U . . .
done
written to /home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126/FUN.m

results of the decomposition will be written to
/home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126
doing sector decomposition . . .
done

working on pole structure: 2 logarithmic poles, 0 linear poles, 0 higher poles
C++ functions created for pole structure 2l0h0
compiling 2l0h0/epstothe0 ...
doing numerical integrations in P126/2l0h0/epstothe0
compiling 2l0h0/epstothe-1 ...
doing numerical integrations in P126/2l0h0/epstothe-1
compiling 2l0h0/epstothe-2 ...
doing numerical integrations in P126/2l0h0/epstothe-2
working on pole structure: 1 logarithmic pole, 0 linear poles, 0 higher poles
C++ functions created for pole structure 1l0h0
compiling 1l0h0/epstothe0 ...
doing numerical integrations in P126/1l0h0/epstothe0
compiling 1l0h0/epstothe-1 ...
doing numerical integrations in P126/1l0h0/epstothe-1
working on pole structure: 0 logarithmic poles, 0 linear poles, 0 higher poles
C++ functions created for pole structure 0l0h0
compiling 1l0h0/epstothe0 ...
doing numerical integrations in P126/0l0h0/epstothe0
Output written to /home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126/P126_pfull.res

To remove intermediate files, execute the command /home/pcl335a/sborowka/Work/SecDecBeta/loop/launchcleanP126
```

Get the Result

- resultfile P126_full.res

```
************************************************
***OUTPUT: P126 p5 ***************
 point: 7.0
 ext. legs: 0.0 0.0 7.0
 prop. mass: 1.0 0. 0. 0. 0. 0.
Prefactor=-Exp[-2EulerGamma*eps]
************************************************
****** eps^-2 coeff ******

result       =0.07563683
             +0.1003924148 I
error        =0.000493522517701388
             + 0.00139691015080074 I
CPUtime (all eps^-2 subfunctions)  =0.04
CPUtime (longest eps^-2 subfunction)  =0.01
.
.
.

****** eps^0 coeff ******

result       =0.906978296750816
             -0.908781551612644 I
error        =0.00754504726896407
             + 0.0442867373250588 I
CPUtime (all eps^0 subfunctions)  =2.44
CPUtime (longest eps^0 subfunction)  =0.51

************************************************

Time taken for decomposition = 2.005725

Total time for subtraction and eps expansion = 41.5057 secs
Time taken for longest subtraction and eps expansion = 17.8613 secs
```

Deformation of the integration contour to integrate mass thresholds



- Integrand is analytically continued into the complex plane

$$\mathcal{F}(\vec{t}) \to \mathcal{F}(\vec{t} + \mathrm{i}\vec{y}(\vec{t})) = \mathcal{F}(\vec{t}) + \mathrm{i}\sum_j y_j(\vec{t})\frac{\partial \mathcal{F}(\vec{t})}{\partial t_j} + \mathcal{O}(y(\vec{t})^2)$$

- The integration contour is deformed by

$$\vec{t} \to \vec{z} = \vec{t} + \mathrm{i}\vec{y}\ ,$$

$$y_j(\vec{t}) = -\lambda t_j(1 - t_j)\frac{\partial \mathcal{F}(\vec{t})}{\partial t_j}$$

Soper '99

Soper, Nagy, Binoth; Kurihara et al., Anastasiou et al., Freitas et al., Becker et al.

Convert linear divergences into logarithmic ones

$\vec{x}_{jk}$ denotes all Feynman parameters excluding x_j and x_k

Assume $\alpha > 1$ and functions P, Q, R such that a linear divergence appears in x_j in Eq. (1) after sector decomposition

$$\prod_{i=1}^{N} \left\{ \int_0^1 \mathrm{d}x_i \right\} [x_j(P(\vec{x}_{jk}) + x_k Q(\vec{x}_{jk})) + R(\vec{x}_{jk})]^{-\alpha} \tag{1}$$

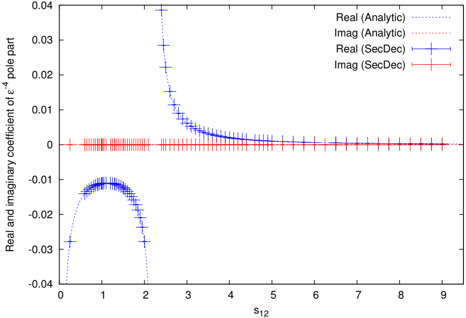
$$= \prod_{i=1}^{N} \left\{ \int_0^1 \mathrm{d}x_i \right\} \frac{1}{x_j}[x_j P(\vec{x}_{jk}) + x_k Q(\vec{x}_{jk}) + R(\vec{x}_{jk})]^{-\alpha}$$

$$- \prod_{i=1}^{N} \left\{ \int_0^1 \mathrm{d}x_i \right\} \frac{1}{x_j}[x_k(x_j P(\vec{x}_{jk}) + Q(\vec{x}_{jk})) + R(\vec{x}_{jk})]^{-\alpha}$$

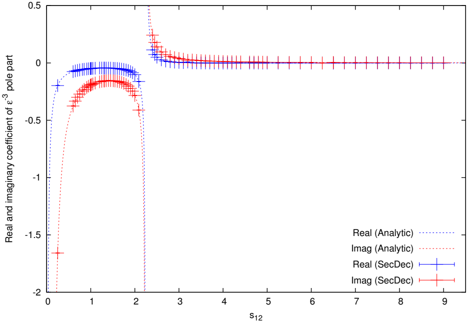
We rest with a logarithmic divergence in x_j.

- Linear divergences can be turned into logarithmic ones
- In the case of *ggtt*2 this leads to a total reduction of number of functions by 2/3

Result for the non-planar massive two loop diagram ggtt2



Leading pole



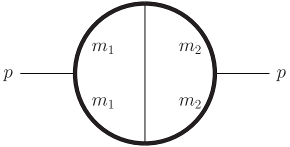
Sub-leading pole

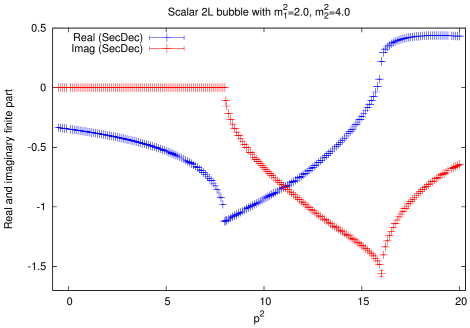
$$m_1^2 = 1,\ p_1^2 = p_2^2 = 0,\ p_3^2 = p_4^2 = m_1^2,\ s_{23} = -1.25$$

analytic results: Manteuffel & Studerus '12

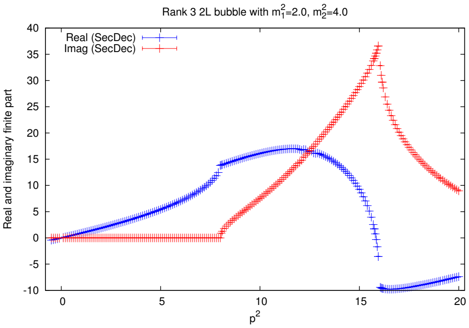
2-loop bubble with 2 mass scales - Results

thresholds at $4 \cdot m_1^2 = 8$ and $4 \cdot m_2^2 = 16$



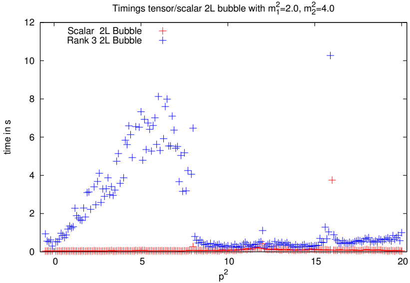


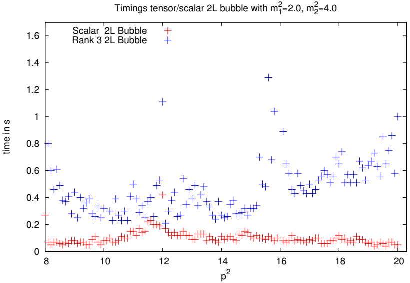
Scalar integral



Rank 3

2-loop bubble with 2 mass scales - Timings





- relative & absolute accuracy 0.1%
- Scalar integral is finite, rank 3 integral has $\mathcal{O}(\epsilon^{-2})$ poles
- Intel Core i7 Processor