Colourful Antenna Subtraction

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Infrared structure in perturbation theory

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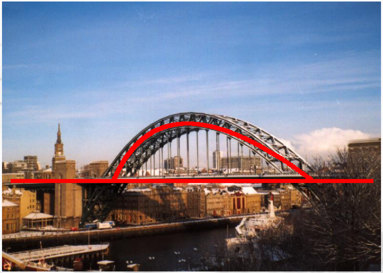
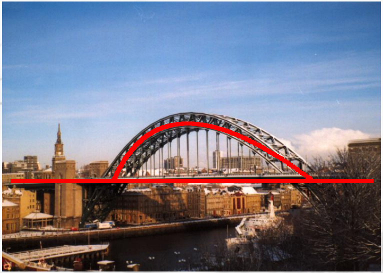


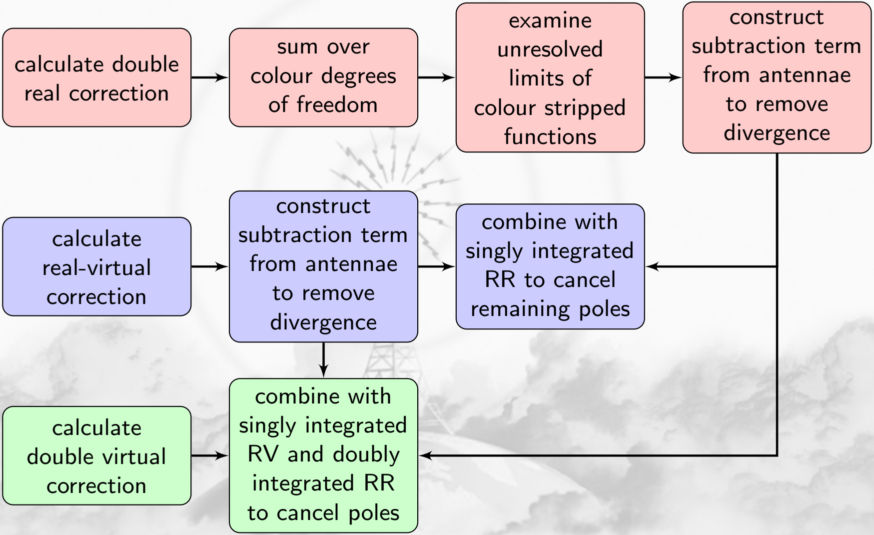
Infrared structure in perturbation theory



Useful facts about IR singularities

1. They cancel between real and virtual contributions
2. In unresolved limits, QCD amplitudes factorize
3. Real singularities described by universal functions
   - process dependent information in reduced matrix element
4. Virtual singularities described by universal functions
   - obey general colour space factorization formula [Catani '98]
5. Universality + factorization (of ME and PS) allows a finite cross section to be obtained using subtraction techniques

Traditional NNLO (antenna) subtraction



Pros and cons of traditional approach

Pros:

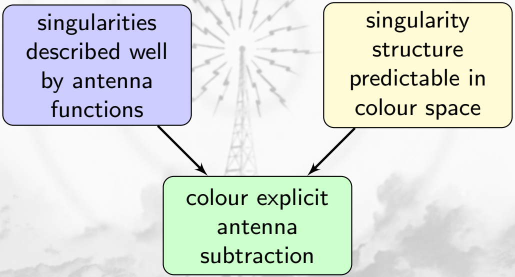
- complete analytic control of the singularities
- all integrated antennae are known
  - final-final [Gehrmann-De Ridder, Gehrmann, Glover, '05]
  - initial-final [Daleo, Gehrmann, Maître, '07],[Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, '10]
  - initial-initial [Boughezal, Gehrmann-De Ridder, Ritzmann, '11. Gehrmann, Ritzmann '12],[Gehrmann, Monni, '11]
- IR limits of squared partial amplitudes well understood
- antennae constructed from squared partial amplitudes

Cons:

- antennae constructed from squared partial amplitudes
- loses structural information in the sum over colour
- process-independent method can be difficult to follow

New approach: colourful antenna subtraction synthesis

- Take the best bits from traditional method. . . the antennae
- Match onto the predictable colour space singularity structure



- Resulting method needs no new integrals and applies to arbitrary processes

Notation and 1-loop recap

Move into colour space with set of abstract basis vectors $|\mathbf{c}\rangle$

$$\mathcal{M}_n = \langle \mathbf{c}|\mathcal{M}_n\rangle \quad \& \quad |\mathcal{M}_n|^2 = \langle \mathcal{M}_n|\mathcal{M}_n\rangle$$

One-loop amplitude pole structure governed by colour dipole insertion operator,

$$\mathbf{I}^{(1)}(\epsilon) \quad = \quad \sum_{\mathrm{pairs}(i,j)} \mathcal{I}^{(1)}_{ij}(\epsilon)\,(\mathbf{T}_i \cdot \mathbf{T}_j)$$

One-loop cross section pole structure given by,

$$2\mathrm{Re}\left[\langle \mathcal{M}^0_n|\mathcal{M}^1_n\rangle\right] \quad = \quad \sum_{(i,j)} 2\mathrm{Re}\left[\mathcal{I}^{(1)}_{ij}(\epsilon)\right]\langle \mathcal{M}^0_n|(\mathbf{T}_i \cdot \mathbf{T}_j)|\mathcal{M}^0_n\rangle + \mathcal{O}(\epsilon^0)$$

IR structure at 2 loops

Similar amplitude factorization formula at 2-loops

$$|\mathcal{M}_n^2\rangle = \mathbf{I}^{(1)}(\epsilon)|\mathcal{M}_n^1\rangle + \mathbf{I}^{(2)}(\epsilon)|\mathcal{M}_n^0\rangle + \mathcal{O}(\epsilon^0)$$

where,

$$\mathbf{I}^{(2)}(\epsilon) = -\frac{1}{2}\mathbf{I}^{(1)}(\epsilon)\mathbf{I}^{(1)}(\epsilon) - \frac{\beta_0}{\epsilon}\mathbf{I}^{(1)}(\epsilon) + \frac{e^{-\epsilon\gamma}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K\right)\mathbf{I}^{(1)}(2\epsilon) + \mathbf{H}^{(2)}(\epsilon)$$

Leads to the two-loop cross section pole structure,

$$\begin{aligned}
|M_n^2|^2 \quad = \quad & \langle\mathcal{M}_n^1|2\mathrm{Re}\left[\mathbf{I}^{(1)}(\epsilon)\right]|\mathcal{M}_n^0\rangle + \langle\mathcal{M}_n^0|2\mathrm{Re}\left[\mathbf{I}^{(1)}(\epsilon)\right]|\mathcal{M}_n^1\rangle \\
- \quad & \frac{1}{2}\langle\mathcal{M}_n^0|2\mathrm{Re}\left[\mathbf{I}^{(1)}(\epsilon)\right]^2|\mathcal{M}_n^0\rangle - \frac{\beta_0}{\epsilon}\langle\mathcal{M}_n^1|2\mathrm{Re}\left[\mathbf{I}^{(1)}(\epsilon)\right]|\mathcal{M}_n^0\rangle \\
+ \quad & e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K\right)\langle\mathcal{M}_n^0|2\mathrm{Re}\left[\mathbf{I}^{(1)}(2\epsilon)\right]|\mathcal{M}_n^0\rangle \\
+ \quad & \langle\mathcal{M}_n^0|2\mathrm{Re}\left[\mathbf{H}^{(2)}(\epsilon)\right]|\mathcal{M}_n^0\rangle + \mathcal{O}(\epsilon^0)
\end{aligned}$$

A dipole form for the Hard Function

Has the general form,

$$\mathbf{H}^{(2)}(\epsilon) = \sum_i C_i \boldsymbol{\mathcal{H}}_i^{(2)}(\epsilon) + \check{\mathbf{H}}^{(2)}(\epsilon) + \mathcal{O}(\epsilon)$$

$$\left.\begin{array}{l} \blacktriangleright\ \boldsymbol{\mathcal{H}}_i^{(2)}(\epsilon) \text{ diagonal in colour space} \\ \blacktriangleright\ \check{\mathbf{H}}^{(2)}(\epsilon) \text{ a non-dipole tensor in colour space} \end{array}\right\} \text{neither are colour dipoles}$$

However. . .

$$\langle \mathcal{M}_n^0 | \check{\mathbf{H}}^{(2)}(\epsilon) | \mathcal{M}_n^0 \rangle = 0$$

then using colour conservation,

$$\mathbf{H}^{(2)}(\epsilon) = -\sum_{(i,j)} \boldsymbol{\mathcal{H}}_{ij}^{(2)}(\epsilon)\ (\mathbf{T}_i \cdot \mathbf{T}_j) + \text{irrelevant terms}$$

2-loop antenna dipoles

Strategy: construct new functions from integrated antennae which reproduce poles of Catani operators

$$
\begin{aligned}
\mathcal{J}^{(1)}(\epsilon) &= \sum_{(i,j)} \mathcal{J}_2^{(1)}(i,j)\,(\mathbf{T}_i \cdot \mathbf{T}_j) \\
\mathcal{J}^{(2)}(\epsilon) &= \sum_{(i,j)} \mathcal{J}_2^{(2)}(i,j)\,(\mathbf{T}_i \cdot \mathbf{T}_j)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{J}_2^{(1)}(i,j) &= \mathbf{J}_2^{(1)}(i,j) + \frac{N_F}{N}\mathbf{J}_{2,N_F}^{(1)}(i,j), \\
\mathcal{J}_2^{(2)}(i,j) &= N\mathbf{J}_2^{(2)}(i,j) - \frac{1}{N}\widetilde{\mathbf{J}}_2^{(1)}(i,j) + N_F\mathbf{J}_{2,N_F}^{(1)}(i,j) + \ldots
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{J}_2^{(1)}(i,j) &\sim \mathcal{X}_3^0(s_{ij}) \\
\mathbf{J}_2^{(2)}(i,j) &\sim \mathcal{X}_4^0(s_{ij}) + \mathcal{X}_3^1(s_{ij}) + \frac{\beta_0}{\epsilon}\mathcal{X}_3^0(s_{ij})\left[\left(\frac{s_{ij}}{\mu^2}\right)^{-\epsilon} - 1\right] + \left[\mathcal{X}_3^0 \otimes \mathcal{X}_3^0\right](s_{ij})
\end{aligned}
$$

Double virtual subtraction term

Strategy: Match poles of integrated dipoles onto the known IR structure,

$$\mathrm{d}\hat{\sigma}^{U}_{NNLO} \sim$$
$$2\mathrm{Re}\left[\langle\mathcal{M}^0_n|\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)|\mathcal{M}^1_n\rangle\right] + \frac{1}{2}\langle\mathcal{M}^0_n|\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)^2|\mathcal{M}^0_n\rangle + \langle\mathcal{M}^0_n|\boldsymbol{\mathcal{J}}^{(2)}(\epsilon)|\mathcal{M}^0_n\rangle$$

- completely predictable for arbitrary NNLO processes
- generates both leading and sub-leading colour
- clear link with unintegrated subtraction terms

Cascading down the calculation: VV→RV→RR

- integrated antennae in $\mathbf{J}_2^{(1)}$ and $\mathbf{J}_2^{(2)}$ determine unintegrated antennae
- colour charge sandwiches determine structure of the subtraction term

e.g.

$$\mathbf{J}_2^{(2)}(i,j)(\mathbf{T}_i \cdot \mathbf{T}_j) \quad \sim \quad \mathcal{X}_4^0(s_{ij})(\mathbf{T}_i \cdot \mathbf{T}_j) + \mathcal{X}_3^1(s_{ij})(\mathbf{T}_i \cdot \mathbf{T}_j) + \cdots$$

$$\underbrace{X_4^0(i,k,l,j)(\mathbf{T}_{(\widetilde{ikl})} \cdot \mathbf{T}_{(\widetilde{klj})})}_{\subset \text{ RR}} \qquad \underbrace{X_3^1(i,k,j)(\mathbf{T}_{(\widetilde{ik})} \cdot \mathbf{T}_{(\widetilde{kj})})}_{\subset \text{ RV}}$$

- antennae fixed without considering unresolved limits of the RR or RV
- directly inherited terms indirectly fix remaining subtraction terms

RR, RV and VV subtraction terms fixed by colour algebra at LC and SLC

Advantage #1: economy

Many subtraction terms have a common origin.
e.g. $e^+e^- \to 3j$ one-loop antenna contribution:

Traditional approach:

1. decompose all four-parton one-loop ME into colour stripped functions
2. examine unresolved limits of the functions independently
3. subtract unresolved limits of each function using 1-loop antennae

Colourful antenna approach:

1. all one-loop antennae come from $\mathcal{J}^{(2)}(\epsilon)$ (predictable)
2. generates a single, completely fixed, insertion operator

$$\mathbb{X}_3^{(1)}(\epsilon) \quad = \sum_{i\neq j\neq k} X_3^{(1)}(i,j,k)(\mathbf{T}_I \cdot \mathbf{T}_K)$$

3. many terms originate in one predictable object

the fully colour summed result. . .

$$
\begin{aligned}
\langle \widetilde{\mathcal{M}}_3^0 | \mathbb{X}_3^{(1)}(\epsilon) | \widetilde{\mathcal{M}}_3^0 \rangle = & \sum_{(i,j)\in P(3,4)} \Bigg\{ \frac{N^2}{2} \Bigg[\; d_3^1(1,i,j) B_3^0((\widetilde{1i})_q, (\widetilde{ij})_g, 2_{\bar q}) J_3^{(3)}(\{p\}_3) \\
& \quad + d_3^1(2,i,j) B_3^0(1_q, (\widetilde{ij})_g, (\widetilde{2j})_{\bar q}) J_3^{(3)}(\{p\}_3) \\
& \quad - \tilde{A}_3^1(1,i,2) B_3^0((\widetilde{1i})_q, j_g, (\widetilde{2i})_{\bar q}) J_3^{(3)}(\{p\}_3) \Bigg] \\
& \quad - \Bigg[\; A_3^1(1,i,2) B_3^0((\widetilde{1i})_q, j_g, (\widetilde{2i})_{\bar q}) J_3^{(3)}(\{p\}_3) \\
& \quad + \tilde{A}_3^1(1,i,2) B_3^0((\widetilde{1i})_q, j_g, (\widetilde{2i})_{\bar q}) J_3^{(3)}(\{p\}_3) \Bigg] \\
& + \frac{1}{N^2} \Bigg[\; \tilde{A}_3^1(1,i,2) B_3^0((\widetilde{1i})_q, j_g, (\widetilde{2i})_{\bar q}) J_3^{(3)}(\{p\}_3) \Bigg] \Bigg\} \\
& + N_F N \Bigg[\sum_{(i,j)\in P(3,4)} \Big(\hat{d}_3^1(1,i,j) B_3^0((\widetilde{1i})_q, (\widetilde{ij})_g, 2_{\bar q}) J_3^{(3)}(\{p\}_3) \\
& \quad + \hat{d}_3^1(2,i,j) B_3^0(1_q, (\widetilde{ij})_g, (\widetilde{2j})_{\bar q}) J_3^{(3)}(\{p\}_3) \Big) \\
& \quad + E_3^1(1,3,4) B_3^0((\widetilde{13})_q, (\widetilde{34})_g, 2_{\bar q}) J_3^{(3)}(\{p\}_3) \\
& \quad + E_3^1(2,3,4) B_3^0(1_q, (\widetilde{34})_g, (\widetilde{24})_{\bar q}) J_3^{(3)}(\{p\}_3) \Bigg] \\
& - \frac{N_F}{N} \Bigg[\; \tilde{E}_3^1(1,3,4) B_3^0((\widetilde{13})_q, (\widetilde{34})_g, 2_{\bar q}) J_3^{(3)}(\{p\}_3) \\
& \quad + \tilde{E}_3^1(2,3,4) B_3^0(1_q, (\widetilde{34})_g, (\widetilde{24})_{\bar q}) J_3^{(3)}(\{p\}_3) \\
& \quad + \sum_{(i,j)\in P(3,4)} \hat{A}_3^1(1,i,2) B_3^0((\widetilde{1i})_q, j_g, (\widetilde{2i})_{\bar q}) J_3^{(3)}(\{p\}_3) \Bigg] \\
& + N_F^2 \Bigg[\; \hat{E}_3^1(1,3,4) B_3^0((\widetilde{13})_q, (\widetilde{34})_g, 2_{\bar q}) J_3^{(3)}(\{p\}_3) \\
& \quad + \hat{E}_3^1(2,3,4) B_3^0(1_q, (\widetilde{34})_g, (\widetilde{24})_{\bar q}) J_3^{(3)}(\{p\}_3) \Bigg] \Bigg\}
\end{aligned}
$$

Advantage #2: basis independence

Traditional approach requires explicit knowledge of ME IR behaviour:

- decomposition into colour ordered sub-amplitudes
- factorization of colour ordered functions dictated by colour connection

Colourful antenna approach formulated in terms of sandwiches of abstract vectors in colour space, e.g.,

$$\langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(1)}(\epsilon) | \mathcal{M}_n^0 \rangle$$

- IR colour structure independent of basis used
- related to traditional approach by choosing colour ordered basis
- free to pick the most convenient basis for explicit calculation

Advantage #3: Less thinking required

Traditional approach requires a good understanding of ME divergences

- squared partial amplitudes ✓
- general interferences of partial amplitudes ✗
  - additional relations sometimes required
  - single ordering can have intermediate (pseudo) divergences
- more work needed to understand IR behaviour of the full amplitude

Colourful antenna approach driven by IR pole structure of the full amplitude

- only knows about the divergent piece of the ME
- avoids pseudo divergences
- deals with LC and SLC on an equal footing

Example: NNLO correction to $gg \to gg$ at SLC

Interesting test because:

- RR and RV MEs not written in terms of squared partial amplitudes
- non-trivial single and double unresolved factorization pattern
- strategy not immediately obvious

Start with VV subtraction term:

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}^{U}_{NNLO} \sim \\
&\Bigg\{ \sum_{(i,j)} \mathbf{J}_2^{(1)}(i,j)\; 2\mathrm{Re}\langle \mathcal{A}_4^0 | \mathbf{T}_i \cdot \mathbf{T}_j | \mathcal{A}_4^1 \rangle \\
&+ \sum_{(i,j)} \sum_{(k,l)} \frac{1}{2} \left[\mathbf{J}_2^{(1)}(i,j) \otimes \mathbf{J}_2^{(1)}(k,l) \right] \langle \mathcal{A}_4^0 | (\mathbf{T}_i \cdot \mathbf{T}_j)(\mathbf{T}_k \cdot \mathbf{T}_l) | \mathcal{A}_4^0 \rangle \\
&+ \sum_{(i,j)} \mathbf{J}_2^{(2)}(i,j)\; \langle \mathcal{A}_4^0 | \mathbf{T}_i \cdot \mathbf{T}_j | \mathcal{A}_4^0 \rangle \Bigg\}\; J_2^{(2)}(p_3, p_4).
\end{aligned}
$$

Colour charge sandwiches (I)

In all-gluon approx, $M^{0,1,2}_{6,5,4} \sim (N^2-1)\Big[\mathcal{O}(N^4)+\mathcal{O}(N^2)\Big]$,

$$\underbrace{\mathbf{J}_2^{(2)}(i,j)}_{\mathcal{O}(N)}\ \underbrace{\langle \mathcal{A}_4^0|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{A}_4^0\rangle}_{(N^2-1)\mathcal{O}(N^3)} \sim (N^2-1)\mathcal{O}(N^4)$$

- $\mathbf{J}_2^{(2)}(i,j)$ term does not contribute to SLC subtraction
- no F_4^0 antennae in RR
- no F_3^1 antennae in RV
- only F_3^0 antennae needed to describe all singularities at SLC

Colour charge sandwiches (II)

$$\sum_{(i,j)}\sum_{(k,l)}\left[\mathbf{J}_2^{(1)}(i,j)\otimes\mathbf{J}_2^{(1)}(k,l)\right]\langle\mathcal{A}_4^0|(\mathbf{T}_i\cdot\mathbf{T}_j)(\mathbf{T}_k\cdot\mathbf{T}_l)|\mathcal{A}_4^0\rangle \sim \frac{1}{2}\sum_{(ij)}$$

$$\begin{aligned}
\Big\{ \quad & \left(\frac{1}{2}\mathcal{F}_3^0(s_{\bar{1}i})+\frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}j})-\frac{1}{2}\mathcal{F}_3^0(s_{\bar{1}j})-\frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}i})\right)\\
&\otimes\left(\frac{1}{2}\mathcal{F}_3^0(s_{\bar{1}i})+\frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}j})-\mathcal{F}_3^0(s_{\bar{1}\bar{2}})-\frac{1}{3}\mathcal{F}_3^0(s_{ij})\right)A_4^0(\hat{\bar{1}},\hat{\bar{2}},i,j)\\
+\quad & \frac{1}{2}\left(\mathcal{F}_3^0(s_{\bar{1}\bar{2}})+\frac{1}{3}\mathcal{F}_3^0(s_{ij})-\frac{1}{2}\mathcal{F}_3^0(s_{\bar{1}j})-\frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}i})\right)\\
&\otimes\left(\mathcal{F}_3^0(s_{\bar{1}\bar{2}})+\frac{1}{3}\mathcal{F}_3^0(s_{ij})-\frac{1}{2}\mathcal{F}_3^0(s_{\bar{1}i})-\frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}j})\right)A_4^0(\hat{\bar{1}},i,\hat{\bar{2}},j)\Big\}
\end{aligned}$$

- colour algebra dictates $(++--)$ pattern of antennae
- no collinear divergences at unintegrated level

Colour charge sandwiches (III)

$$\sum_{(i,j)} \mathbf{J}_2^{(1)}(i,j)\; 2\mathrm{Re}\langle \mathcal{A}_4^0 | \mathbf{T}_i \cdot \mathbf{T}_j | \mathcal{A}_4^1 \rangle \sim 2\mathrm{Re} \sum_{(i,j)}$$

$$\Big\{ \quad \left(\frac{1}{2}\mathcal{F}_3^0(s_{\bar{1}i}) + \frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}j}) - \frac{1}{2}\mathcal{F}_3^0(s_{\bar{1}j}) - \frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}i}) \right) \mathcal{A}_4^{0\dagger}(\hat{\bar{1}}, \hat{\bar{2}}, i, j) \mathcal{A}_{4,1}^1(\hat{\bar{1}}, \hat{\bar{2}}, j, i)$$

$$+ \quad \left(\frac{1}{2}\mathcal{F}_3^0(s_{\bar{1}i}) + \frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}j}) - \mathcal{F}_3^0(s_{\bar{1}\bar{2}}) - \frac{1}{3}\mathcal{F}_3^0(s_{ij}) \right) \mathcal{A}_4^{0\dagger}(\hat{\bar{1}}, \hat{\bar{2}}, i, j) \mathcal{A}_{4,1}^1(\hat{\bar{1}}, i, \hat{\bar{2}}, j)$$

$$+ \quad \left(\mathcal{F}_3^0(s_{\bar{1}\bar{2}}) + \frac{1}{3}\mathcal{F}_3^0(s_{ij}) - \frac{1}{2}\mathcal{F}_3^0(s_{\bar{1}i}) - \frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}j}) \right) \mathcal{A}_4^{0\dagger}(\hat{\bar{1}}, i, \hat{\bar{2}}, j) \mathcal{A}_{4,1}^1(\hat{\bar{1}}, \hat{\bar{2}}, i, j) \Big\}$$

When combined with (II):

- all initial-state collinear poles cancel without use of MF kernels
- leading singularity $\sim 1/\epsilon^2$ with coefficient $\sim \log(s_{ij})$
- exactly cancels the VV poles of the 2-loop matrix elements

RR and RV subtraction

$$\mathbf{J}_2^{(2)}\langle\mathcal{A}_4^0|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{A}_4^0\rangle = 0$$

$$\mathbf{J}_2^{(1)}\langle\mathcal{A}_4^0|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{A}_4^1\rangle$$

$$\mathbf{J}_2^{(1)}\mathbf{J}_2^{(1)}\langle\mathcal{A}_4^0|(\mathbf{T}_i\cdot\mathbf{T}_j)(\mathbf{T}_k\cdot\mathbf{T}_l)|\mathcal{A}_4^0\rangle$$

RR and RV subtraction

$$\mathbf{J}_2^{(2)}\langle\mathcal{A}_4^0|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{A}_4^0\rangle = 0$$

$$\mathbf{J}_2^{(1)}\langle\mathcal{A}_4^0|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{A}_4^1\rangle$$

$$\mathbf{J}_2^{(1)}\mathbf{J}_2^{(1)}\langle\mathcal{A}_4^0|(\mathbf{T}_i\cdot\mathbf{T}_j)(\mathbf{T}_k\cdot\mathbf{T}_l)|\mathcal{A}_4^0\rangle$$

$$\mathbf{J}_2^{(1)}\langle\mathcal{A}_5^0|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{A}_5^0\rangle$$

$$\mathbf{X}_3^0\langle\mathcal{A}_4^0|\mathbf{T}_I\cdot\mathbf{T}_J|\mathcal{A}_4^1\rangle$$

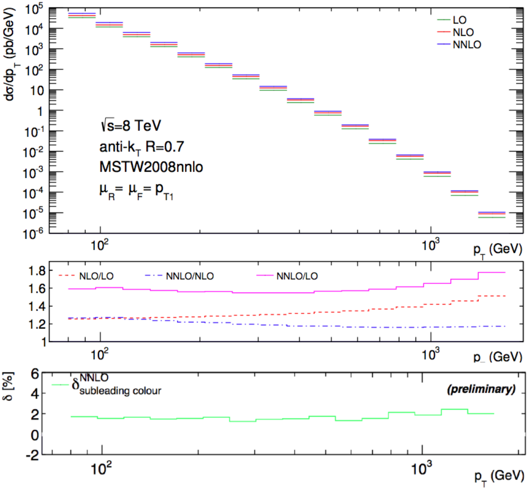
$$\mathbf{X}_3^0\mathbf{J}_2^{(1)}\langle\mathcal{A}_4^0|(\mathbf{T}_I\cdot\mathbf{T}_J)(\mathbf{T}_K\cdot\mathbf{T}_L)|\mathcal{A}_4^0\rangle$$

RR and RV subtraction

$$
\begin{tikzcd}
\mathbf{J}_2^{(2)}\langle\mathcal{A}_4^0|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{A}_4^0\rangle = 0 & \mathbf{J}_2^{(1)}\langle\mathcal{A}_4^0|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{A}_4^1\rangle \arrow[d] & \mathbf{J}_2^{(1)}\mathbf{J}_2^{(1)}\langle\mathcal{A}_4^0|(\mathbf{T}_i\cdot\mathbf{T}_j)(\mathbf{T}_k\cdot\mathbf{T}_l)|\mathcal{A}_4^0\rangle \arrow[d] \\
\mathbf{J}_2^{(1)}\langle\mathcal{A}_5^0|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{A}_5^0\rangle \arrow[d] & \mathbf{X}_3^0\langle\mathcal{A}_4^0|\mathbf{T}_I\cdot\mathbf{T}_J|\mathcal{A}_4^1\rangle & \mathbf{X}_3^0\mathbf{J}_2^{(1)}\langle\mathcal{A}_4^0|(\mathbf{T}_I\cdot\mathbf{T}_J)(\mathbf{T}_K\cdot\mathbf{T}_L)|\mathcal{A}_4^0\rangle \arrow[d] \\
\mathbf{X}_3^0\langle\mathcal{A}_5^0|\mathbf{T}_I\cdot\mathbf{T}_J|\mathcal{A}_5^0\rangle & & \mathbf{X}_3^0\mathbf{X}_3^0\langle\mathcal{A}_4^0|(\mathbf{T}_I\cdot\mathbf{T}_J)(\mathbf{T}_K\cdot\mathbf{T}_L)|\mathcal{A}_4^0\rangle
\end{tikzcd}
$$

Results. . . it worked!

- all VV and RV poles cancel. . . analytically
- all IR divergence in RR and RV subtracted



To take away...

Colourful antennae: a reformulation of antenna subtraction

- take the best bits from traditional method... no new integrals
- combine with known NNLO IR structure in colour space
- synthesise into colour explicit antenna subtraction
- IR pole structure fixes RR, RV and VV subtraction terms
- applicable to arbitrary processes inc. SLC
- we tried it alongside traditional method... it worked
- first steps towards a genuinely automatable NNLO method