

An alternative subtraction scheme for NLO QCD calculations

Tania Robens

based on

C.H.Chung, M. Krämer, TR (JHEP 1106:144,2011)

C.H.Chung, TR (Phys.Rev. D87 (2013))

TR (Mod. Phys. Lett. A, Vol. 28, No. 23 (2013))

M. Bach, TR (work in progress)

IKTP, TU Dresden

Radcor 2013

Lumley Castle, Chester le Street

NLO corrections: general structure

Masterformula

for m particles in the final state

$$\sigma_{\text{NLO,tot}} = \sigma_{\text{LO}} + \sigma_{\text{NLO}},$$

$$\sigma_{\text{LO}} = \int d\Gamma_m |\mathcal{M}_{\text{Born}}^{(m)}|^2(s) \quad \text{leading order contribution}$$

$$\sigma_{\text{NLO}} = \sigma_{\text{real}} + \sigma_{\text{virt}},$$

$$\sigma_{\text{real}} = \int d\Gamma_{m+1} |\mathcal{M}^{(m+1)}|^2 \quad \text{real emission}$$

$$\sigma_{\text{virt}} = \int d\Gamma_m 2 \operatorname{Re}(\mathcal{M}_{\text{Born}}^{(m)} (\mathcal{M}_{\text{virt}}^{(m)})^*) \quad \text{virtual contribution}$$

with $d\Gamma$: phase space integral, $\mathcal{M}$ matrix elements
(here: flux factors etc implicit)

Infrared divergencies and NLO subtraction schemes: ingredients

$$\sigma_{\text{NLO,tot}} = \underbrace{\int d\Gamma_m |\mathcal{M}_{\text{Born}}^{(m)}|^2}_{\sigma_{\text{LO}}} + \underbrace{2 \int d\Gamma_m \text{Re}(\mathcal{M}_{\text{Born}}^{(m)} (\mathcal{M}_{\text{virt}}^{(m)})^*)}_{\sigma_{\text{virt}}(\varepsilon)} + \underbrace{\int d\Gamma_{m+1} |\mathcal{M}^{(m+1)}|^2}_{\sigma_{\text{real}}(\varepsilon)}$$

- **infrared poles** $1/\varepsilon$, $1/\varepsilon^2$ cancel in $\sum \sigma_{\text{real}} + \sigma_{\text{virt}}$
- **matrix elements factorize in singular limits, unique behaviour** (depending on nature of splitting)

$$|\mathcal{M}^{(m+1)}|^2 \longrightarrow D_{ij}(p_i, p_j) |\mathcal{M}^{(m)}|^2, \quad D_{ij} \sim \frac{1}{p_i p_j}$$

- D_{ij} : **dipoles**, contain complete singularity structure

$$\implies \int d\Gamma_{m+1} \left(|\mathcal{M}^{(m+1)}|^2 - \sum_{ij} D_{ij} |\mathcal{M}^{(m)}|^2 \right) = \text{finite}$$

- general idea of dipole subtraction:

shift singular parts from $m+1$ to m particle phase space

Dipole subtraction for total cross sections

Master formula

$$\begin{aligned}
 \sigma &= \sigma^{LO} + \sigma^{NLO} \\
 \sigma^{NLO} &= \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C \\
 &= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m (d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C),
 \end{aligned}$$

⇒ effectively added "0"; both integrals finite

$$\begin{aligned}
 \sigma_m^{NLO}(s) &= \int_m \left\{ |\widetilde{\mathcal{M}}_{\text{virt}}(s; \varepsilon)|^2 + \mathbf{I}(\varepsilon) |\mathcal{M}_{\text{Born}}(s)|^2 \right. \\
 &\quad \left. + \int_0^1 dx (\mathbf{K}(x) + \mathbf{P}(x; \mu_F)) |\mathcal{M}_{\text{Born}}(x, s)|^2 \right\}
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 \end{aligned}$$

Ingredient for subtraction schemes: momentum mapping

- previous slide: add and subtract "0" in terms of

$$\int d\Gamma_m \tilde{F}_{\text{sing}} |\mathcal{M}_{\text{Born}}^{(m)}|^2 - \int d\Gamma_{m+1} F_{\text{sing}} |\mathcal{M}_{\text{Born}}^{(m)}|^2$$

- addition and subtraction takes place in different phase spaces

$$p_{\tilde{a}}^{(m)} = F\left(p_a^{(m+1)}, p_b^{(m+1)}, \dots\right)$$

This function is highly scheme dependent !!!

requirements: keep total energy/ momentum conserved, all particles onshell

$$\sum_m p_{\tilde{a}} \stackrel{!}{=} \sum_{m+1} p_a, p_i^2 = \tilde{p}_i^2 = m_i^2$$

(sum over outgoing particles only)

Nagy Soper subtraction scheme

- many different subtraction schemes are around
(best known: Catani, Seymour, 1996)
- all schemes: poles have to be the same; finite parts can differ

Main motivation for new scheme

- proposal of **improved parton shower**: Nagy, Soper
(arXiv:0706.0017, 0801.1917, 0805.0216)
 - basic idea: can use the **splitting functions** of the new shower
as **dipole subtraction terms**
- ⇒ (cf Catani Seymour Showers in Sherpa (Schumann ea '07),
Herwig++ (Plätzer ea '11), ... (Winter ea, Dinsdale ea '07, ...))
- **introduce new mapping between m and $m + 1$ phase spaces**
- ⇒ **leads to a much smaller number of subtraction terms**
especially important for large number of external particles
(same dipoles in shower and subtraction scheme: facilitates
matching with NLO calculations)

Interim: Status of NLO tools

(I already apologize for non-completeness of the list)

- **two main ingredients**, living in **different phase spaces**:

$$\sigma_{\text{virt}} = \int d\Gamma_m 2 \operatorname{Re}(\mathcal{M}_{\text{Born}}^{(m)} (\mathcal{M}_{\text{virt}}^{(m)})^*); \quad \sigma_{\text{real}} = \int d\Gamma_{m+1} |\mathcal{M}^{(m+1)}|^2$$

- **virtual contribution**: **many many** [(new) (non) public] **tools on the market**, within MC or as standalone "virtual" generators (eg Openloops (Pozzorini ea, '12), Golem/Gosam (Cullen ea, '11), Blackhat(Bern ea, '08), aMC@NLO (Frixione ea, '11), and many more)

⇒ this part handled by above tools ("NLO revolution")

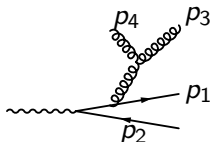
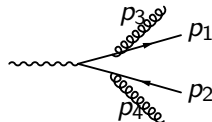
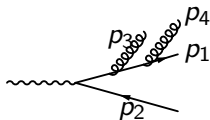
- **real emission**:
(typically) **handled by the interfacing Monte Carlo**

⇒ **this is where new scheme becomes important** ⇐

Shifting momenta: Example

$$\gamma^* \longrightarrow q(p_1) \bar{q}(p_2) g(p_3) \text{ (@ NLO)}$$

$q \bar{q} g g$ real emission contributions:



CS: 1 momentum shift/ spectator

p_2, p_3 : 2 transformations

NS: 1 total transformation

$\Rightarrow$ from simple counting: $(+(p_1 \leftrightarrow p_2))$

10 transformations using **CS** vs **5** using **NS** dipoles !!

Maximal number of transformations

Maximal number of **momentum mappings** using
Catani Seymour or Nagy Soper scheme
counting: consider gluon-splittings only
(maximal number of mappings)

emitter, spectator	CS	NS
$\sum \sim$	$N^3/2$	$N^2/2$

$\Rightarrow$ **each mapping requires reevaluation of $\mathcal{M}_{\text{Born}}$** $\Leftarrow$
large (computational) effects as N increases

Final state mapping: Catani Seymour vs Nagy Soper

- CS mapping (per spectator $\tilde{p}_k$)

$$\tilde{p}_i = p_i + p_j - \frac{y}{1-y} p_k, \quad \tilde{p}_k^\mu = \frac{1}{1-y} p_k^\mu$$

- NS mapping

$$\tilde{p}_i = \frac{1}{\lambda} (p_i + p_j) - \frac{1-\lambda+y}{2\lambda a} Q, \quad \tilde{p}_k^\mu = \Lambda^\mu_\nu p_k^\nu \quad \text{all fs particles}$$

$$\Lambda^{\mu\nu} = g^{\mu\nu} - \frac{2(K+\tilde{K})^\mu (K+\tilde{K})^\nu}{(K+\tilde{K})^2} + \frac{2K^\mu \tilde{K}^\nu}{K^2}, \quad K=Q-p_i-p_j, \quad \tilde{K}=Q-\tilde{p}_i$$

- integration measure (identical, same pole structure)

$$[dp_j]_{\text{CS}} = \frac{(2\tilde{p}_i\tilde{p}_k)^{1-\epsilon}}{16\pi^2} \frac{d\Omega_{d-3}}{(2\pi)^{1-\epsilon}} dz dy (1-y)^{1-2\epsilon} y^{-\epsilon} [z(1-z)]^{-\epsilon},$$

$$[dp_j]_{\text{NS}} = \frac{(2\tilde{p}_i Q)^{1-\epsilon}}{16\pi^2} \frac{d\Omega_{d-3}}{(2\pi)^{1-\epsilon}} dz dy \lambda^{1-2\epsilon} y^{-\epsilon} [z(1-z)]^{-\epsilon}, \quad [\lambda(p_i, Q)]$$

Scheme validation (C.H.Chung, M. Krämer, TR, JHEP 1106 (2011) 144; C.H.Chung, TR, Phys.Rev. D87 (2013))

- **main reason** for improved scaling: **different mapping**
- leads to **more complicated integrated subtraction terms**
- ✓ **all done and verified:** (final result independent of subtraction scheme)

Test-processes

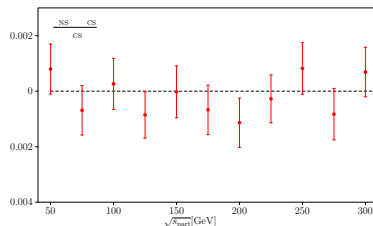
- single W at hadron colliders
- Dijet production at lepton colliders
- $p\bar{p} \rightarrow H$ and $H \rightarrow g g$
- DIS
- $e e \rightarrow 3 \text{ jets}$

DIS: Catani Seymour vs Nagy Soper - numerical result

considered process:

$$e(p_{in}) q(p_1) \longrightarrow e(p_{out}) q(p_4) [g(p_3)]$$

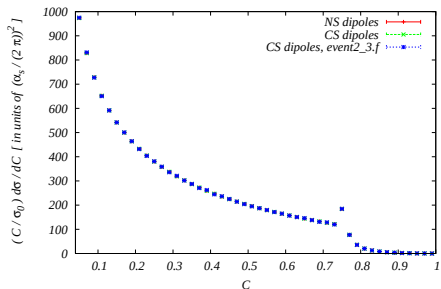
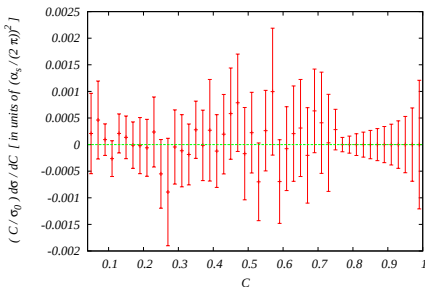
apply both schemes: get the same result



relative difference between CS and NS: $\frac{\sigma_{CS} - \sigma_{NS}}{\sigma_{CS}}$

agree on the sub-permill level ✓

Applications

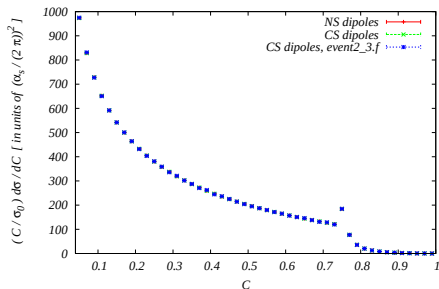
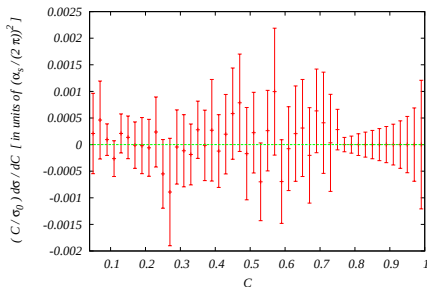
 $e^+ e^- \longrightarrow 3 \text{ jets } (N_C^2 C_F \text{ component}) - \text{numerical result}$ C - distribution: $(C/\sigma_0) d\sigma/dC$ C - distribution: $(C/\sigma_0) d\sigma/dC$, single contributions

Comparison between NS and CS
implementation as well as
event2_3.f from M. Seymour

Relative difference between NS and
CS implementation

- results agree on the permill level, compatible with 0 ✓
- remark: for $C > 0.75$, only real emission contributes $\Rightarrow$ difference exactly 0 (when using the same setup)

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Nagy Soper - subtraction in the virtual contribution (here:DIS)

⇒ **major difference in integrated subtraction terms**

$$\int_0^1 dx |\mathcal{M}|_2^2 = \int_0^1 dx \left\{ \frac{\alpha_s}{2\pi} C_F \delta(1-x) \left[-9 + \frac{1}{3}\pi^2 - \frac{1}{2}\text{Li}_2[(1-\tilde{z}_0)^2] \right. \right. \\ \left. \left. + 2 \ln 2 \ln \tilde{z}_0 + 3 \ln \tilde{z}_0 + 3 \text{Li}_2(1-\tilde{z}_0) + \mathbf{l}_{\text{fin}}^{\text{tot},0}(\tilde{\mathbf{z}}_0) + \mathbf{l}_{\text{fin}}^1(\tilde{\mathbf{a}}) \right] \right. \\ \left. + K_{\text{fin}}^{\text{tot}}(x; \tilde{\mathbf{z}}) + P_{\text{fin}}^{\text{tot}}(x; \mu_F^2) \right\} |\mathcal{M}|_{\text{Born}}^2(x, p_1),$$

$$\mathbf{K}_{\text{fin}}^{\text{tot}}(\mathbf{x}; \tilde{\mathbf{z}}) = \frac{\alpha_s}{2\pi} C_F \left\{ \frac{1}{x} \left[2(1-x) \ln(1-x) - \left(\frac{1+x^2}{1-x} \right)_+ \ln x \right. \right. \\ \left. \left. + 4x \left(\frac{\ln(1-x)}{1-x} \right)_+ \right] + \mathbf{l}_{\text{fin}}^1(\tilde{\mathbf{z}}, \mathbf{x}) \right\},$$

⇒ **contains integrals which need to be evaluated numerically** ⇐

Nagy Soper - integrals to be evaluated numerically

⇒ **Integrals** contain **nontrivial functions** depending on m and $m + 1$ four-momenta $\Leftarrow$

$$\begin{aligned}
 I_{\text{fin}}^{\text{tot},0}(\tilde{z}_0) &= 2 \int_0^1 \frac{dy}{y} \left\{ \frac{\tilde{z}_0}{\sqrt{4y^2(1-\tilde{z}_0) + \tilde{z}_0^2}} \right. \\
 &\quad \times \ln \left[\frac{2z \sqrt{4y^2(1-\tilde{z}_0) + \tilde{z}_0^2}(1-y)}{\left(2y + \tilde{z}_0 - 2y\tilde{z}_0 + \sqrt{4y^2(1-\tilde{z}_0) + \tilde{z}_0^2}\right)^2} \right] + \ln 2 \left. \right\}. \\
 I_{\text{fin}}^1(\tilde{a}) &= 2 \int_0^1 \frac{du}{u} \int_0^1 \frac{dx}{x} \\
 &\quad \times \left[\frac{\mathbf{x}(1-x+ux[(1-ux)\tilde{a}+2])}{\mathbf{k}(\mathbf{u}, \mathbf{x}, \tilde{a})} - \frac{1}{\sqrt{1+4\tilde{a}_0 u^2(1+\tilde{a}_0)}} \right]. \\
 I_{\text{fin}}^1(\tilde{z}, \mathbf{x}) &= \frac{2}{(1-x)_+} \frac{1}{\pi} \int_0^1 \frac{dy'}{y'} \left[\int_0^1 \frac{dv}{\sqrt{v(1-v)}} \frac{\tilde{z}}{\mathbf{N}(\mathbf{x}, \mathbf{y}', \tilde{z}, \mathbf{v})} - 1 \right],
 \end{aligned}$$

DIS: Nagy Soper - variables in integrals to be evaluated numerically

for some integrals, $m + 1$ variables have to be reconstructed
 $\Rightarrow$ difference wrt standard scheme(s) $\Leftarrow$

in initial state subtraction terms

$$\mathbf{N} = \frac{\hat{p}_3 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}} \frac{1}{1-x} + y', \tilde{z} = \frac{1}{x} \frac{p_1 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}}$$

$$\hat{p}_3 = \underbrace{\frac{(1-x)(1-y')}{x}}_{\alpha} p_1 + \underbrace{(1-x)y'}_{\beta} p_i - k_{\perp}, \hat{p}_4^{\mu} = \Lambda^{\mu}_{\nu}(\hat{K}, K) \hat{p}_4^{\nu}$$

$$k_{\perp}^2 = -2\alpha\beta p_1 \cdot p_i, k_{\perp} = -|k_{\perp}| \begin{pmatrix} 0 \\ 2\sqrt{\frac{1-2v}{v(1-v)}} \\ 0 \end{pmatrix},$$

in final state subtraction terms

$$k^2(x, u, \tilde{a}) = \left[(1+ux-x)(z-z') + ux((1-ux)\tilde{a}+1) \right]^2 \\ + 4uxz'(1-z)(1+ux-x)((1-ux)\tilde{a}+1)$$

$$\tilde{a} = \frac{p_1 \cdot p_o}{p_1 \cdot (p_i - (1-y)p_o)}$$

Alternative subtraction scheme

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Alternative subtraction scheme

Numerical integrals - approximations through grids/ polynomials (1) (work done by M. Bach)

status as on arXiv:

scheme has in total **7** integrals which need to be evaluated numerically

- **5** depend on **1** external parameter
(leftover finite terms from singular regions, à la $\varepsilon \times \frac{1}{\varepsilon}$)

$$I_3(a), I_{\text{fin}}(a), I_{\text{fin}}^{(b)}(a), I_{\text{fin}}^{(e)}(a), I_{\text{fin}}(\tilde{z}_0)$$

- **2** depend on **2** external parameters
(finite terms for interference integrals in non-singular regions)

$$I_{\text{fin}}^{(d)}(\tilde{a}, a), I_{\text{fin}}(\tilde{z}_0, x)$$

- (not really a problem though...)

Numerical integrals - approximations through grids/ polynomials (2) (work done by M. Bach)

- **work in progress:** approximate these using polynomials and/or grids
- all 'one-parameter' integrals: **approximated by polynomials, implemented and checked** (apart from $I_{\text{fin}}(\tilde{z}_0)$)
- agreement for approximation: typically $\mathcal{O}(10^{-6})$ **or better**
- set up for **interpolating grids for others:** on the way

⇒ **no additional integrations needed** ⇐

⇒ **implementation 'like' Catani Seymour scheme** ⇐

(however: the hard part are always the real emissions...)

What about constant scaling ?? (for the experts...)

(this is typically a question from an FKS user/ author...)

- aMC@NLO (Frixione ea): constant scaling for certain processes
- makes use of symmetries in matrix elements and phase space

⇒ **also possible here** ⇐

- need to **partition**, but not to **parametrize** á la FKS
($\hat{=}$ each partition contains at most one soft/ soft and collinear divergence)
- then dipoles in our scheme which reflect this singularity structure obey **single mapping** ⇒ **constant scaling**

!! very preliminary, no implementation yet...

- details on constant scaling in **aMC@NLO**: Frederix ea, JHEP 0910 (2009) 003
- details on constant scaling in **our scheme**: C.H.Chung, TR, Phys.Rev. D87 (2013); TR, Mod. Phys. Lett. A, Vol. 28, No. 23 (2013)

Summary and Outlook

Summary

- scheme works, no phase space reparametrization needed, scaling $\leq N^2$

Outlook

- finish interpolation of finite integrals
- **implementation in current NLO tool(s)**
- **make available as code-independent library**
- (further down the road: combine w parton shower, extend the approach to massive scheme)

! Thanks for listening !

Appendix

Difference 2: Combining showers and NLO (1)

Very very short...

- double counting: hard real emissions are described in both shower and "real emission" matrix element
- want: hard: matrix element, soft: shower (always talk about 1 jet)
- can be achieved by adding and subtracting a counterterm

$$- \int_{m+1} d\sigma^{\text{PS}}|_{m+1} + \int_{m+1} d\sigma^{\text{PS}}|_m$$

details eg in hep-ph/0204244: "Matching NLO QCD computations and parton shower simulations" (Frixione, Webber), MC@NLO

Difference 2: Combining showers and NLO (2)

- important: have new terms in $m + 1$ phase space

$$\int_{m+1} \left(d\sigma^R - \underbrace{d\sigma^A + d\sigma^{PS}|_m}_{=0} - d\sigma^{PS}|_{m+1} \right)$$

- same splitting functions: second and third term cancel !!
left with

$$\int_{m+1} \left(d\sigma^R - d\sigma^{PS}|_{m+1} \right)$$

⇒ improves numerical efficiency

- [more details on this](#), also for MC@NLO vs Powheg:
S. Hoeche et al, "A critical appraisal of NLO+PS matching methods", arXiv:1111.1220

Processes at hadron colliders: general

- hadron colliders (as Tevatron, LHC) collide **hadrons**
- QCD: talks about **partons**
- transition: parton distribution functions (PDFs) $f_l(x, \mu_F)$;
 $l = q, \bar{q}, g$ flavour, x momentum fraction, (μ_F factorization scale)

masterformula

$$\sigma_{\text{hadr}}(p \bar{p} \rightarrow X) = \sum_{l_1, l_2} \int dx_1 \int dx_2 f_{l_1}(x_1) f_{l_2}(x_2) \sigma_{\text{part}}(x_1, x_2; l_1 l_2 \rightarrow X)$$

- **perturbative**, **nonperturbative** part

Infrared divergences in NLO corrections

- source of infrared divergence: integration over phase space of emitted massless particles in real and virtual contribution
- KNL theorem: infrared poles cancel in $\sigma_{\text{real}} + \sigma_{\text{virt}}$
- appear in matrix elements as terms

$$\frac{1}{p_i p_j} = \frac{1}{E_i E_j (1 - \cos \theta_{ij})}$$

$E_j \rightarrow 0$: soft divergence, $\cos \theta_{ij} \rightarrow 1$: collinear divergence

- matrix element level: in the singular limits,

$$|\mathcal{M}^{(m+1)}|^2 \longrightarrow D_{ij}(p_i, p_j) |\mathcal{M}^{(m)}|^2, \quad D_{ij} \sim \frac{1}{p_i p_j} \quad (1)$$

- D_{ij} : **dipoles**, contain complete singularity structure
 $\Rightarrow$ **need to have a good (analytical) parametrization of the singularity structure**

Dipole subtraction: general idea

- here: go from $D = 4$ to $D = 4 - 2\varepsilon$ dimensions in integration over phase space
- poles then appear as

$$\sigma_{\text{real}} \sim \frac{A}{\varepsilon^2} + \frac{B}{\varepsilon} + \dots$$

- A, B depend on the splitting process, but are independent of the rest of reaction = **general**

$\Rightarrow$ underlying idea of dipole subtraction schemes $\Leftarrow$

- add and subtract a function in the m (=Born, virtual corrections) and $m + 1$ (real emission) phase space which mimicks singular behaviour in singular regions
- also implies "smoothing" of the integrand

Dipole subtraction: Real master formula

Real Masterformula ($s = (p_a + p_b)^2$)

$$\begin{aligned}
 \sigma(s) = & \int_m d\Phi^{(m)}(s) \frac{1}{n_c(a)n_c(b)} |\mathcal{M}^{(m)}|^2(s) F_J^{(m)} \\
 & + \int d\Phi^{(m+1)}(s) \left\{ \frac{1}{n_c(a)n_c(b)} |\mathcal{M}^{(m+1)}|^2(s) F_J^{(m+1)} - \sum_{\text{dipoles}} (\mathcal{D} \cdot F_J^{(m)}) \right\} \\
 & + \int d\Phi^{(m)}(s) \left\{ \frac{1}{n_c(a)n_c(b)} |\mathcal{M}^{(m)}|^2_{1\text{ loop}}(p_a, p_b) + \mathbf{I}(\varepsilon) |\mathcal{M}^{(m)}|^2(s) \right\}_{\varepsilon=0} F_J^{(m)} \\
 & + \left\{ \int dx_a dx_b \delta(x - x_a) \delta(x_b - 1) \int d\Phi^{(m)}(x_a p_a, x_b p_b) |\mathcal{M}^{(m)}|^2(x_a p_a, x_b p_b) \right. \\
 & \quad \times \left(\mathbf{K}^{a,a'}(x) + \mathbf{P}^{a,a'}(x_a p_a, x_b p_b, x; \mu_F^2) \right) \Big\} + (a \leftrightarrow b)
 \end{aligned}$$

where all colour/ phase space factors have been accounted for

Integrated Dipoles in more details: I, K, P (1)

$m + 1$ phase space: in principle easy

$$\int d\Gamma_{m+1} \left(|\mathcal{M}_{\text{real}}|^2 - \sum D \right), \text{ finite}$$

m particle phase space: more complicated

need integration variables (emission from p_1):

$$x = 1 - \frac{p_4(p_1 + p_2)}{p_1 p_2} \text{ softness, } \tilde{v} = \frac{p_1 p_4}{p_1 p_2} \text{ collinearity}$$

Integrated Dipoles in more details: I, K, P (2)

- in principle, obtain $\int d\Gamma_1 D = \int_0^1 dx \left(I(\varepsilon) + \tilde{\mathbf{K}}(x, \varepsilon) \right)$
- $I(\varepsilon) \propto \delta(1-x)$: corresponds to loop part
- $\tilde{\mathbf{K}}(x, \varepsilon)$ contains finite parts as well as **collinear singularities**
- latter need to be cancelled by adding **collinear counterterm**

$$\frac{1}{\epsilon} \left(\frac{4\pi\mu^2}{\mu_F^2} \right)^\epsilon P^{qq}(x)$$

depends on factorization scale μ_F ($P^{qq}(x)$ splitting function)

- PDFs come in again: term already accounted for by folding w PDF, needs to be subtracted
- for $qg \rightarrow Wq$ like processes, only singularity which appears

Second ingredient: Parametrization of integration variables

- again: remember you have

$$F_{\text{sing}} \propto D_{ij}, \quad \tilde{F}_{\text{sing}} = \int d\Gamma_1 D_{ij}, \quad d\Gamma_1 \propto d^4 p_j \delta(p_j^2)$$

$$\implies \tilde{F}_{\text{sing}} \propto \int d^4 p_j \delta(p_j^2) D_{ij}$$

- 3 free variables (in D dimensions: $D - 1$)
!! need to be written in terms of m particle variables !!
- now all ingredients:
total energy momentum conservation, onshellness of external particles, need for integration variables

Catani Seymour vs Nagy Soper: Shifting momenta

- matching between m and $m + 1$ particle spaces requires reshuffling of momenta

- for

$$p_{\text{mother}}^{(m)} = p_{\text{daughter},1}^{(m+1)} + p_{\text{daughter},2}^{(m+1)}$$

not all particles can be onshell simultaneously

⇒ need additional spectators to take over additional momenta

- Catani Seymour: define emitter-spectator pair, momentum goes to 1 additional particle only

⇒ quite easy integrations; however, for increasing number of particles, huge number of transformations necessary

- Nagy Soper:

shift momenta to **all** non-emitting external particles

- number of transformations = number of emitters
- leads to more complicated integrals during framework setup
- in general: # of transformations: CS $\sim N_{\text{jets}}^3/2$, NS $\sim N_{\text{jets}}^2/2$

Maximal number of transformations

Maximal number of **momentum mappings** using
Catani Seymour or Nagy Soper scheme
counting: consider gluon-splittings only
(maximal number of mappings)

emitter, spectator	CS, (ij)	CS, k	NS, (ij)
fin,fin	$\binom{N'}{2}$	$(N' - 2)$	$\binom{N'}{2}$
fin,ini	$\binom{N'}{2}$	2	—
ini,fin	$2 N'$	$(N' - 1)$	$2 N'$
ini,ini	$2 N'$	1	—
total	$N'^2(N' + 3)/2 =$		$N'(N' + 3)/2 =$
$(\sum_{\text{comb's}}(ij) \times (k))$	$(N + 1)^2(N + 4)/2$		$(N + 1)(N + 4)/2$
$\sim$	$N^3/2$		$N^2/2$

(N' number of real emission, N number of Born type final state particles)

NS integration measures (1)

Initial state

$$d\xi_p = dx \int_0^1 dy' \int_0^1 dv \frac{(2p_a \cdot p_b)^{1-\varepsilon} x^{\varepsilon-1}}{(4\pi)^2} \frac{\pi^{\varepsilon-\frac{1}{2}}}{\Gamma\left(\frac{1-2\varepsilon}{2}\right)} \\ \times [y'(1-y')]^{-\varepsilon} [v(1-v)]^{-\frac{1+2\varepsilon}{2}} \Theta[(1-x)x]$$

Final state, $2 \rightarrow 2$ processes

$$d\xi_p = \frac{(2p_\ell \cdot Q)^{1-\varepsilon}}{16} \frac{\pi^{-\frac{5}{2}+\varepsilon}}{\Gamma\left(\frac{1}{2}-\varepsilon\right)} \\ \times \int_0^1 du u^{-\varepsilon} (1-u)^{-\varepsilon} \int_0^1 dx x^{1-2\varepsilon} (1-x)^{-\varepsilon} \int_0^1 dv [v(1-v)]^{-\frac{1+2\varepsilon}{2}}$$

NS integration measures (2)

Final state, $2 \rightarrow n$ processes

$$d\xi_p = \frac{(2 p_i Q)^{1-\varepsilon}}{16} \frac{\pi^{-\frac{5}{2}+\varepsilon}}{\Gamma(\frac{1}{2}-\varepsilon)} \int_0^1 du u^{-\varepsilon} \int_0^1 dx \delta^{1-\varepsilon} \gamma^{1-2\varepsilon} [(1-x)(x-x_0)]^{-\varepsilon} \int_0^1 dv [v(1-v)]^{-\frac{1+2\varepsilon}{2}}.$$

Variables

$$\lambda = \sqrt{(1+y)^2 - 4ay}, \quad a = \frac{Q^2}{2 p_i Q}, \quad y = \frac{\hat{p}_i \hat{p}_j}{p_i Q}$$
$$\gamma = \frac{1}{2} (1+y+\lambda), \quad x_0(y, a) = \frac{1-\lambda+y}{1+\lambda+y},$$

$q \rightarrow qg$ for initial state quarks: Catani Seymour (1)

- $q(\tilde{p}_1) \rightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\pi\mu^2\alpha_s C_F}{s+t+u} \left(\frac{2s(s+t+u)}{t(t+u)} + (1-\varepsilon) \frac{t+u}{t} \right)$$

- matching ($\tilde{p}_2 = p_2$)

$$\tilde{p}_1 = x p_1, \quad x = 1 - \frac{p_4(p_1 + p_2)}{(p_1 p_2)}$$

$$\tilde{p}_k^\mu = \Lambda^\mu{}_\nu p_k^\nu, \quad (k: \text{final state particles})$$

$$\Lambda^{\mu\nu} = -g^{\mu\nu} - \frac{2(K + \tilde{K})^\mu(K + \tilde{K})^\nu}{(K + \tilde{K})^2} + \frac{2K^\mu \tilde{K}^\nu}{K^2}$$

$$K = p_1 + p_2 - p_4, \quad \tilde{K} = \tilde{p}_1 + p_2$$

$q \rightarrow qg$ for initial state quarks: Catani Seymour (2)

- integration variables:

$$v = \frac{p_1 p_4}{p_1 p_2}, \quad x = 1 - \frac{p_4 (p_1 + p_2)}{(p_1 p_2)}$$

- in p_1, p_2 cm system: $E_4 \rightarrow 0 \Rightarrow x \rightarrow 1$ (softness)
 $\cos \theta_{14} \rightarrow 1 \Rightarrow v \rightarrow 0$ (collinearity)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\pi\alpha_s C_F}{v x s} \left(\frac{1+x^2}{1-x} - \varepsilon(1-x) \right)$$

- integration measure

$$[dp_j] = \frac{(2p_1 p_2)^{1-\varepsilon}}{16\pi^2} \frac{d\Omega_{d-3}}{(2\pi)^{1-\varepsilon}} dv dx (1-x)^{-2\varepsilon} \left[\frac{v}{1-x} \left(1 - \frac{v}{1-x} \right) \right]^{-\varepsilon}$$

where $v \leq 1-x$ and all integrals between 0 and 1

$q \rightarrow qg$ for initial state quarks: Catani Seymour (3)

- result

$$\mu^{2\varepsilon} \int [dp_j] D^{14,2} = \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\varepsilon)} C_F \left(\frac{2\mu^2\pi}{p_1 p_2} \right)^\varepsilon$$

$$\times \int_0^1 dx \left(\mathbf{I}(\varepsilon) \delta(1-x) + \tilde{\mathbf{K}}(x, \varepsilon) - \underbrace{\frac{1}{\varepsilon} P^{qq}(x)}_{\text{killed by coll CT}} \right)$$

with

$$\mathbf{I}(\varepsilon) = \frac{1}{\varepsilon^2} + \frac{3}{2\varepsilon} - \frac{\pi^2}{6}$$

$$\mathbf{K}(x) = (1-x) - 2(1+x) \ln(1-x) + \left(\frac{4}{1-x} \ln(1-x) \right)_+$$

$$P^{qq}(x) = \left(\frac{1+x^2}{1-x} \right)_+ \quad \text{regularized splitting function}$$

$q \rightarrow qg$ for initial state quarks: Nagy Soper (1)

- $q(\tilde{p}_1) \rightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\pi\mu^2\alpha_s C_F}{s+t+u} \left(\frac{2su(s+t+u)}{t(t^2+u^2)} + (1-\varepsilon)\frac{u}{t} \right)$$

as CS, same pole structure as CS

- matching, integration variables, integration measure:
as Catani Seymour ($v \leftrightarrow y$)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\pi\alpha_s C_F}{xs} \times \left(\frac{1-x-y}{y}(1-\varepsilon) + \frac{2x}{y(1-x)} - \frac{2x[2y-(1-x)]}{(1-x)[y^2+(1-x-y)^2]} \right)$$

$q \rightarrow qg$ for initial state quarks: Nagy Soper (2)

- result

$$\mu^{2\varepsilon} \int [dp_j] D^{14,2} = \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\varepsilon)} C_F \left(\frac{2\mu^2\pi}{p_1 p_2} \right)^\varepsilon$$

$$\times \int_0^1 dx \left(\textcolor{red}{\mathbf{I}}(\varepsilon) \delta(1-x) + \textcolor{green}{\tilde{\mathbf{K}}}(\textcolor{green}{x}, \varepsilon) - \underbrace{\frac{1}{\varepsilon} P^{qq}(x)}_{\text{killed by coll CT}} \right)$$

with

$$\textcolor{red}{\mathbf{K}}(x) =$$

$$(1-x) - 2(1+x) \ln(1-x) + \left(\frac{4}{1-x} \ln(1-x) \right)_+ - \textcolor{green}{(1-x)}$$

- equivalence of dipoles schemes checked analytically

Final state $g \rightarrow q \bar{q}$: Catani Seymour vs Nagy Soper (1)

- $g(\tilde{p}_i) \rightarrow q(p_i) + \bar{q}(p_j)$,
spectator: any other final state parton, p_k
- Dipole (in terms of integration variables):

$$D_{\text{NS, CS}}^{ij,k} \propto \underbrace{\frac{1}{y}}_{\text{sing}} \left[1 - \frac{z(1-z)}{1-\varepsilon} \right]$$

- NS definitions

$$y_{\text{NS}} = \frac{p_i p_j}{(p_i + p_j)Q - p_i p_j}, \quad z_{\text{NS}} = \frac{p_j \tilde{n}}{p_i \tilde{n} + p_j \tilde{n}}$$

$$\tilde{n} = \frac{1+y+\lambda}{2\lambda} Q - \frac{a}{\lambda} (p_i + p_j), \quad \lambda = \sqrt{(1+y)^2 - 4ay}, \quad a = \frac{Q^2}{(p_i + p_j)Q - p_i p_j}$$

- CS definitions:

$$y_{\text{CS}} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}, \quad z_{\text{CS}} = \frac{p_i p_k}{p_i p_k + p_j p_k}$$

Final state $g \rightarrow q \bar{q}$: Catani Seymour vs Nagy Soper (2)

- CS matching (all other final state particles untouched)

$$\tilde{p}_i = p_i + p_j - \frac{y}{1-y} p_k, \quad \tilde{p}_k^\mu = \frac{1}{1-y} p_k^\mu$$

- NS matching

$$\tilde{p}_i = \frac{1}{\lambda} (p_i + p_j) - \frac{1 - \lambda + y}{2 \lambda a} Q, \quad \tilde{p}_k^\mu = \Lambda^\mu_\nu p_k^\nu \quad \text{all fs particles}$$

$$\Lambda^{\mu\nu} = g^{\mu\nu} - \frac{2(K+\tilde{K})^\mu(K+\tilde{K})^\nu}{(K+\tilde{K})^2} + \frac{2K^\mu\tilde{K}^\nu}{K^2}, \quad K=Q-p_i-p_j, \quad \tilde{K}=Q-\tilde{p}_i$$

- integration measure (identical, same pole structure)

$$[dp_j]_{\text{CS}} = \frac{(2 \tilde{p}_i \tilde{p}_k)^{1-\epsilon}}{16 \pi^2} \frac{d\Omega_{d-3}}{(2 \pi)^{1-\epsilon}} dz dy (1-y)^{1-2\epsilon} y^{-\epsilon} [z(1-z)]^{-\epsilon},$$

$$[dp_j]_{\text{NS}} = \frac{(2 \tilde{p}_i Q)^{1-\epsilon}}{16 \pi^2} \frac{d\Omega_{d-3}}{(2 \pi)^{1-\epsilon}} dz dy \lambda^{1-2\epsilon} y^{-\epsilon} [z(1-z)]^{-\epsilon}$$

Final state $g \rightarrow q \bar{q}$: Catani Seymour vs Nagy Soper (3)

- result CS

$$\mu^{2\varepsilon} \int [dp_j] D^{ij,k} = \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\varepsilon)} T_R \left(\frac{2\mu^2\pi}{\tilde{p}_i \tilde{p}_k} \right)^\varepsilon \left[-\frac{2}{3\varepsilon} - \frac{16}{9} \right]$$

- result NS

$$\mu^{2\varepsilon} \int [dp_j] D^{ij} = T_R \frac{\alpha_s}{2\pi} \frac{\alpha_s}{\Gamma(1-\varepsilon)} \left(\frac{2\pi\mu^2}{p_i Q} \right)^\varepsilon \times \left[-\frac{2}{3\varepsilon} - \frac{16}{9} + \frac{2}{3} [(a-1) \ln(a-1) - a \ln a] \right],$$

- for $a = 1$, reduces completely to Catani Seymour result
- (reason: $a = 1$ implies only 2 particles in the final state, $\tilde{n} \rightarrow p_k$, $\Rightarrow$ complete equivalence)
- tradeoff: all final state particles get additional momenta: integral more complicated, but fewer transformations necessary

Single W production: Nagy Soper vs Catani Seymour subtraction (easy)

$$\text{NS, CS-NS, CS= NS+CS-NS}$$

- 2 particle phase space (real emission)

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \underbrace{\frac{1}{4} \frac{1}{9} \sum |\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9} g^2 \alpha_s \pi}_{\text{finite}}$$

- 1 particle phase space (virtual contribution)

$$\text{I}(\epsilon) |\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi} \frac{1}{\Gamma(1-\epsilon)} (-8 + \frac{2}{3}\pi^2) |\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

$$\begin{aligned} \mathcal{K}^a(xp_a) &= \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left[-(1-x) \ln x + 2(1-x) \ln(1-x) \right. \\ &\quad \left. + 4x \left(\frac{\ln(1-x)}{1-x} \right)_+ - \frac{2x \ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x} \right)_+ \ln \left(\frac{4\pi\mu^2}{2xp_a \cdot p_b} \right) \right. \\ &\quad \left. + (1-x) \right] \end{aligned}$$

compare to Nagy Soper :

pole structure the same, finite terms differ ✓

Single W production: Nagy Soper vs Catani Seymour subtraction (easy)

$$\text{NS, CS-NS, CS= NS+CS-NS}$$

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$$\begin{aligned} \mathbf{K}^a(xp_a) = & \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left[-(1-x) \ln x + 2(1-x) \ln(1-x) \right. \\ & + 4x \left(\frac{\ln(1-x)}{1-x} \right)_+ - \frac{2x \ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x} \right)_+ \ln \left(\frac{4\pi\mu^2}{2xp_a \cdot p_b} \right) \\ & \left. + (1-x) \right] \end{aligned}$$

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compare to Nagy Soper :

pole structure the same, finite terms differ ✓

Single W production: Nagy Soper vs Catani Seymour subtraction (easy)

NS, **CS-NS**, CS= NS+**CS-NS**

- 2 particle phase space (real emission)

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \underbrace{\frac{1}{4} \frac{1}{9} \sum |\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9} g^2 \alpha_s \pi}_{\text{finite}}$$

- 1 particle phase space (virtual contribution)

$$\mathbf{I}(\epsilon) |\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi} \frac{1}{\Gamma(1-\epsilon)} (-8 + \frac{2}{3}\pi^2) |\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

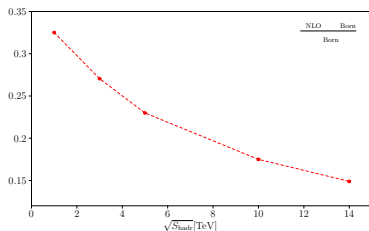
$$\begin{aligned} \mathbf{K}^a(xp_a) = & \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left[-(1-x) \ln x + 2(1-x) \ln(1-x) \right. \\ & + 4x \left(\frac{\ln(1-x)}{1-x} \right)_+ - \frac{2x \ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x} \right)_+ \ln \left(\frac{4\pi\mu^2}{2xp_a \cdot p_b} \right) \\ & \left. + (1-x) \right] \end{aligned}$$

compare to Nagy Soper :

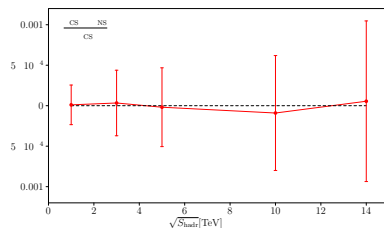
pole structure the same, finite terms differ ✓

Numerical results for single W (slide by C. Chung)

input: $M_W = 80.35$ GeV, PDF $\Rightarrow$ cteq6m, $\alpha_s(M_W) = 0.120299$

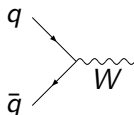


$\frac{\sigma_{NLO} - \sigma_{LO}}{\sigma_{LO}}$ as a function of $\sqrt{S_{hadr}}$
corrections up to 30%

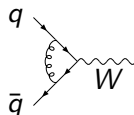


relative difference between CS and NS: $\frac{\sigma_{CS} - \sigma_{NS}}{\sigma_{CS}}$
agree on the sub-permill level ✓

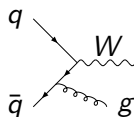
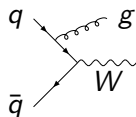
Single W production (slide by C.H. Chung)



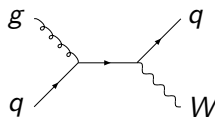
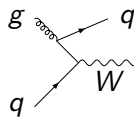
Tree level: $q\bar{q} \rightarrow W$



Virtual corrections: $q\bar{q} \rightarrow W$



Real corrections: $q\bar{q} \rightarrow Wg$



$gq \rightarrow Wq$ (+ 2 more diagrams)

$$\frac{1}{4} \frac{1}{9} |\mathcal{M}_B|^2 = \frac{g^2}{12} |V_{qq'}|^2 M_W^2, \quad \frac{1}{4} \frac{1}{9} \sum |\mathcal{M}_R|^2 = \frac{8g^2 \pi \alpha_s}{9} |V_{qq'}|^2 \frac{\hat{t}^2 + \hat{u}^2 + 2M_W^2 \hat{s}}{\hat{t}\hat{u}}$$

$$|\mathcal{M}_V|^2 = |\mathcal{M}_B|^2 \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right\}$$

Deep inelastic scattering (subprocess of...) (hard)

$$e(p_{in}) q(p_1) \longrightarrow e(p_{out}) q(p_4) [g(p_3)]$$

- **CS**: spectator for final state gluon emission:

initial state quark

- **NS**: spectator for final state gluon emission:

final state lepton

- (“spectator” = spectator in momentum mapping)

$\Rightarrow$ first nontrivial check of NS scheme $\Leftarrow$

DIS: Catani Seymour

Real emission subtraction terms

$$\mathbf{D}_{43,1} = \frac{4\pi\alpha_s}{p_3 p_4} \frac{1}{x_{43,1}} C_F \left[\frac{2}{1 - \tilde{z}_4 + (1 - x_{43,1})} - (1 + \tilde{z}_4) \right] |\mathcal{M}|_{\text{Born}}^2(\tilde{p}_1, \tilde{p}_4)$$

$$\mathbf{D}_{13,4} = \frac{4\pi\alpha_s}{p_1 p_3} \frac{1}{x_{34,1}} C_F \left[\frac{2}{1 - x_{34,1} + u_3} - (1 + x_{34,1}) \right] |\mathcal{M}|_{\text{Born}}^2(\tilde{p}_1, \tilde{p}_4)$$

$$\tilde{z}_4 = \frac{p_1 p_4}{(p_3 + p_4) p_1}, \quad x_{43,1} = x_{34,1} = \frac{p_i p_o}{p_1 p_4 + p_1 p_3}, \quad u_3 = \frac{p_1 p_3}{(p_3 + p_4) p_1}$$

Mapping

$$\tilde{\mathbf{p}}_1 = x_{43,1} p_1, \quad \tilde{\mathbf{p}}_4 = p_3 + p_4 - (1 - x_{43,1}) p_1$$

Integrated subtraction terms

$$\int_0^1 dx |\mathcal{M}|_{2,\text{tot}}^2 = \int_0^1 \frac{dx}{x} \left\{ -\frac{9}{2} \frac{\alpha_s}{2\pi} C_F \delta(1-x) + K_{\text{fin}}^{\text{eff}}(x) + P_{\text{fin}}^{\text{eff}}(x; \mu_F^2) \right\} |\mathcal{M}|_{\text{Born}}^2(x p_1)$$

$$\mathbf{K}^{\text{eff}}(x) = \frac{\alpha_s}{2\pi} C_F \left\{ \left(\frac{1+x^2}{1-x} \ln \frac{1-x}{x} \right)_+ + \frac{1}{2} \delta(1-x) + (1-x) - \frac{3}{2} \frac{1}{(1-x)_+} \right\}$$

DIS: Nagy Soper - real emission terms

Initial state real emission subtraction

$$\mathbf{D}^{1,3} = \frac{4\pi\alpha_s}{xy\hat{p}_1 \cdot \hat{p}_i} C_F \left(1 - x - y + \frac{2\tilde{z}x}{v(1-x)+y} \right) |\mathcal{M}_{\text{Born}}(p)|^2$$

$$x = \frac{\hat{p}_o \cdot \hat{p}_4}{\hat{p}_i \cdot \hat{p}_1}, y = \frac{\hat{p}_1 \cdot \hat{p}_3}{\hat{p}_1 \cdot \hat{p}_i}, \tilde{z} = \frac{\hat{p}_1 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}}, v = \frac{(\hat{p}_1 \cdot \hat{p}_i)(\hat{p}_3 \cdot \hat{p}_4)}{(\hat{p}_4 \cdot \hat{Q})(\hat{p}_3 \cdot \hat{Q})}.$$

Initial state: mapping

$$\mathbf{p}_1 = x\hat{p}_1, \mathbf{p}_i = \hat{p}_i, \mathbf{p}_{o,4}^\mu = \mathbf{L}^\mu_{\nu}(\mathbf{K}, \hat{\mathbf{K}})\hat{p}_{o,4}^\nu, K = x\hat{p}_1 + \hat{p}_i, \hat{K} = \hat{p}_1 + \hat{p}_i - \hat{p}_3.$$

Final state real emission subtraction

$$\mathbf{D}^{4,3} = \frac{4\pi\alpha_s C_F}{y(\hat{p}_i \cdot \hat{p}_1)} \left[\frac{y}{1-y} F_{\text{eik}} + z + 2 \frac{(1-v)(1-z(1-y))}{v[1-z(1-y)] + y[(1-y)\tilde{a} + 1]} \right] |\mathcal{M}_{\text{Born}}(p)|^2$$

$$y = \frac{\hat{p}_3 \cdot \hat{p}_4}{\hat{p}_1 \cdot \hat{p}_i}, z = \frac{\hat{p}_3 \cdot \hat{p}_o}{\hat{p}_3 \cdot \hat{p}_o + \hat{p}_4 \cdot \hat{p}_o}, v = \frac{\hat{p}_1 \cdot \hat{p}_3}{\hat{p}_1 \cdot \hat{p}_3 + \hat{p}_1 \cdot \hat{p}_4}, F_{\text{eik}} = 2 \frac{(\hat{p}_3 \cdot \hat{p}_o)(\hat{p}_4 \cdot \hat{p}_o)}{(\hat{p}_3 \cdot \hat{Q})^2}.$$

Final state: mapping

$$\mathbf{p}_i = \hat{p}_i, \mathbf{p}_1 = \hat{p}_1, \mathbf{p}_4 = \frac{1}{1-y} [\hat{p}_3 + \hat{p}_4 - y(\hat{p}_1 + \hat{p}_i)], \mathbf{p}_o = \frac{\hat{p}_o}{1-y}.$$

$e^+ e^- \longrightarrow 3 \text{ jets } (1)$

- consider process

$$e^+ e^- \longrightarrow q \bar{q} g$$

at NLO

- real emission contributions:

$$e^+ e^- \longrightarrow q \bar{q} q \bar{q}, q \bar{q} g g$$

- number of necessary mappings (in total):

$$(8 + 10)_{CS} \text{ vs } (4 + 5)_{NS}$$

- 3 different color structures: $C_A C_F^2$, $C_A C_F n_f T_R$, $C_A^2 C_F$
- singular parts: $C_A C_F n_f T_R$: $q \bar{q} q \bar{q}$ only,
 $C_A^2 C_F$, $C_A C_F^2$: $q \bar{q} g g$ only
- result known for a long time: Ellis ea 1980
 (also Kuijf 1991, Giele ea 1992)

$e^+ e^- \longrightarrow 3 \text{ jets } (2)$

- singularity structure in integrated dipoles for all color configurations: **done** ✓
- $q\bar{q}q\bar{q}$ real emission terms and all finite contributions: **done** ✓
- $q\bar{q}gg$ real emission terms and all finite contributions: **done**(✓)
- current work: **improve numerics**, especially for $g \rightarrow g g$ splittings
- infrared safe observable: C-distribution (Ellis ea 1980),

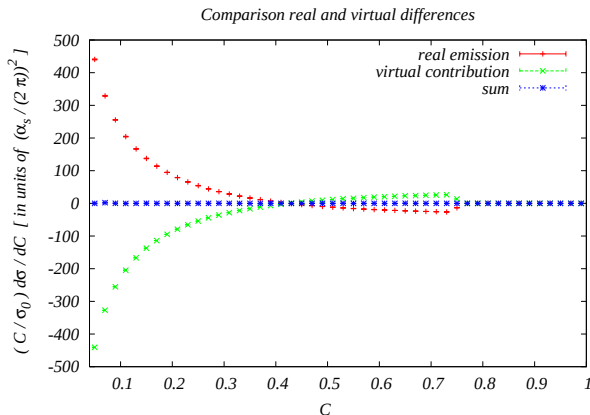
$$C^{(n)} = 3 \left\{ 1 - \sum_{i,j=1, i < j}^n \frac{s_{ij}^2}{(2 p_i \cdot Q)(2 p_j \cdot Q)} \right\}$$

$e^+ e^- \longrightarrow 3 \text{ jets } (2)$

- singularity structure in integrated dipoles for all color configurations: **done** ✓
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$e e \longrightarrow 3\text{jets}$: single components



differences between real and virtual contributions from CS and NS dipoles respectively