Kaon Physics

UK hep Forum
14.11.2013

Martin Gorbahn
University of Liverpool

Main Characters

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Decay	SM	NP

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Heroes

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$K^+ \rightarrow \pi^+ \bar{\nu} \nu$	Z-Penguin & Box	~~MFV~~ 100 TeV
$K_L \rightarrow \pi^0 \bar{\nu} \nu$	@ 1-loop, $V_{ts} V^*_{td}$	~~CP~~ ~~MFV~~ 100 TeV
K_{l2} & K_{l3}	$W^\pm$ @ tree, V_{us}	EW precision 10 TeV

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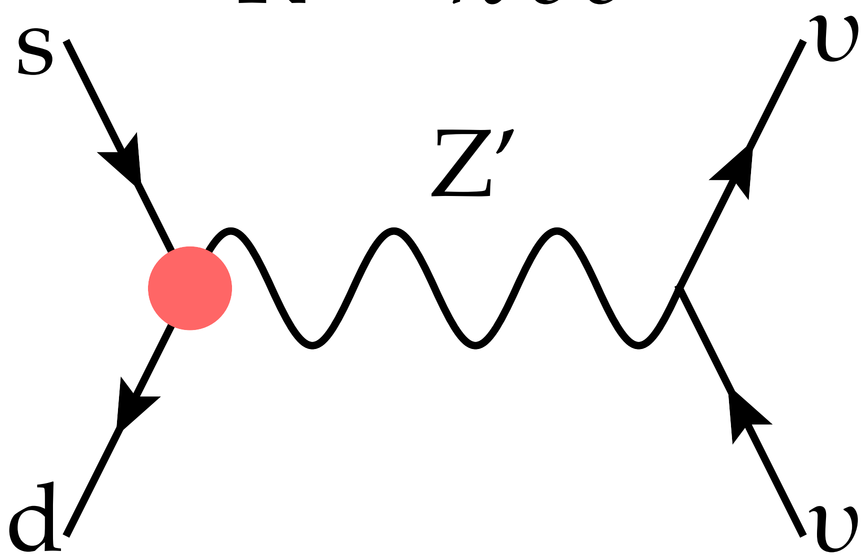
A simplified Z’ example

A simplified Z′ example

$K \to \pi \bar{\nu}\nu$



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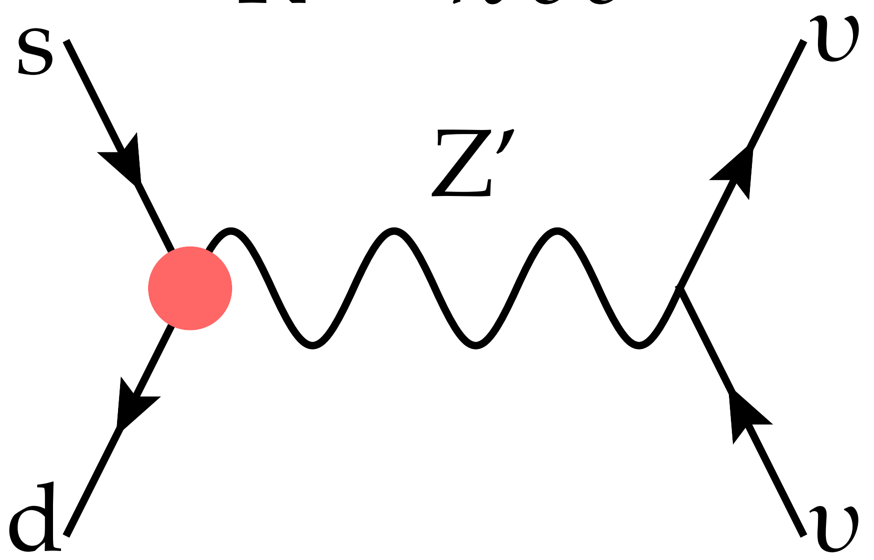

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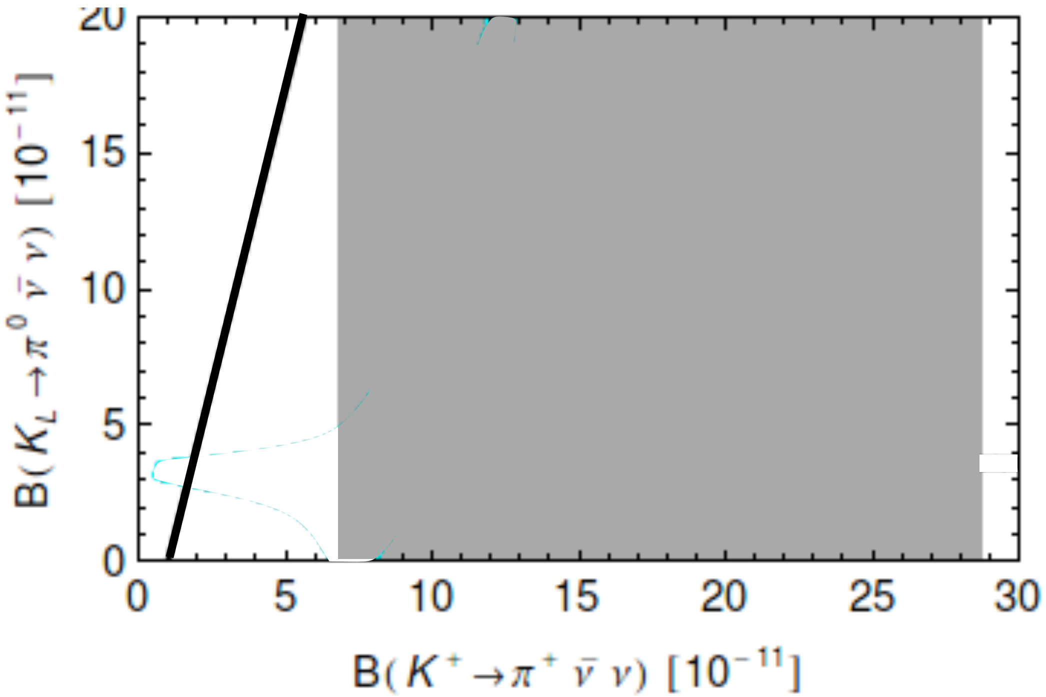

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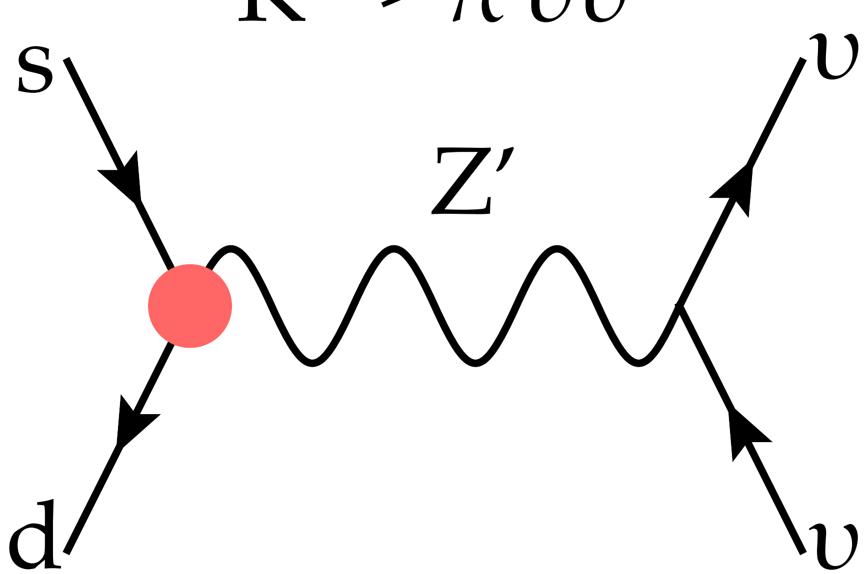
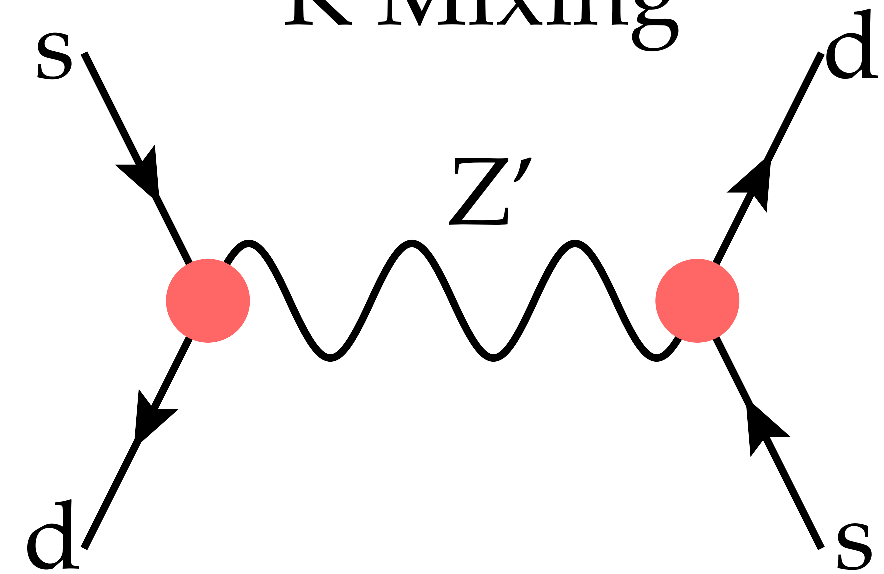


Only left-handed currents

K Mixing



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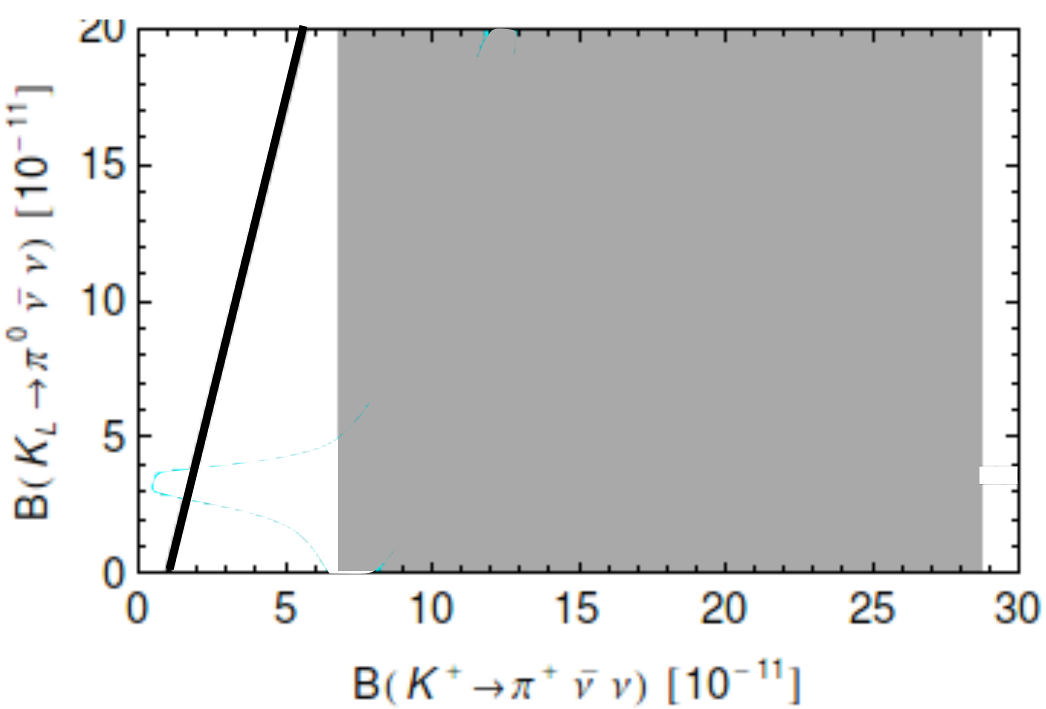

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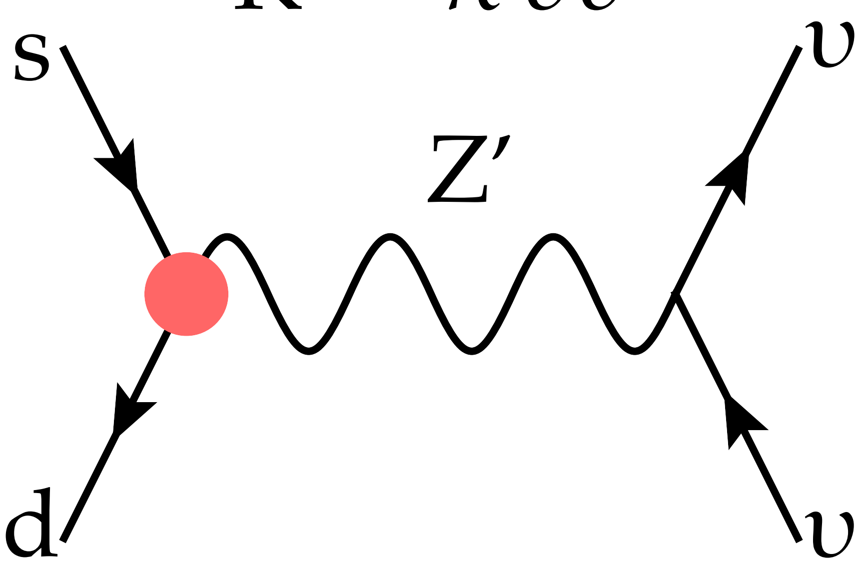

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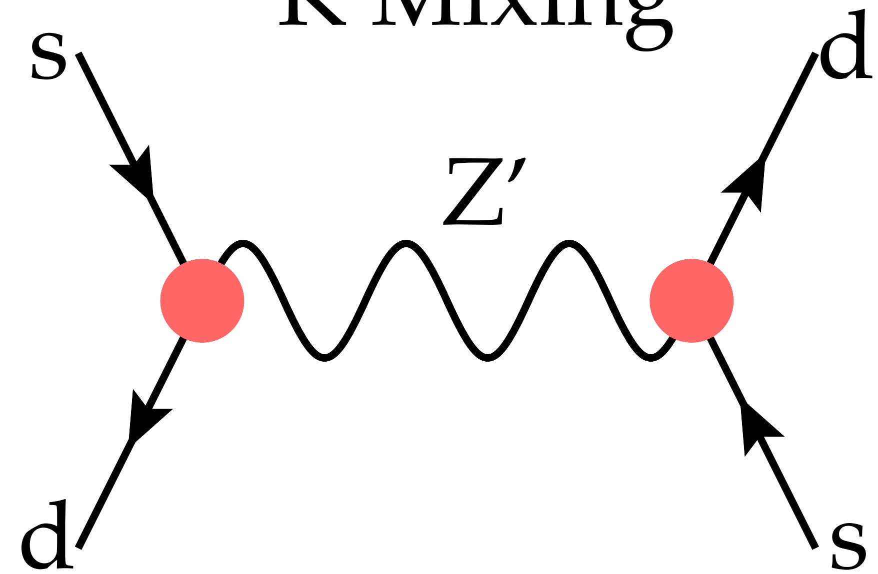

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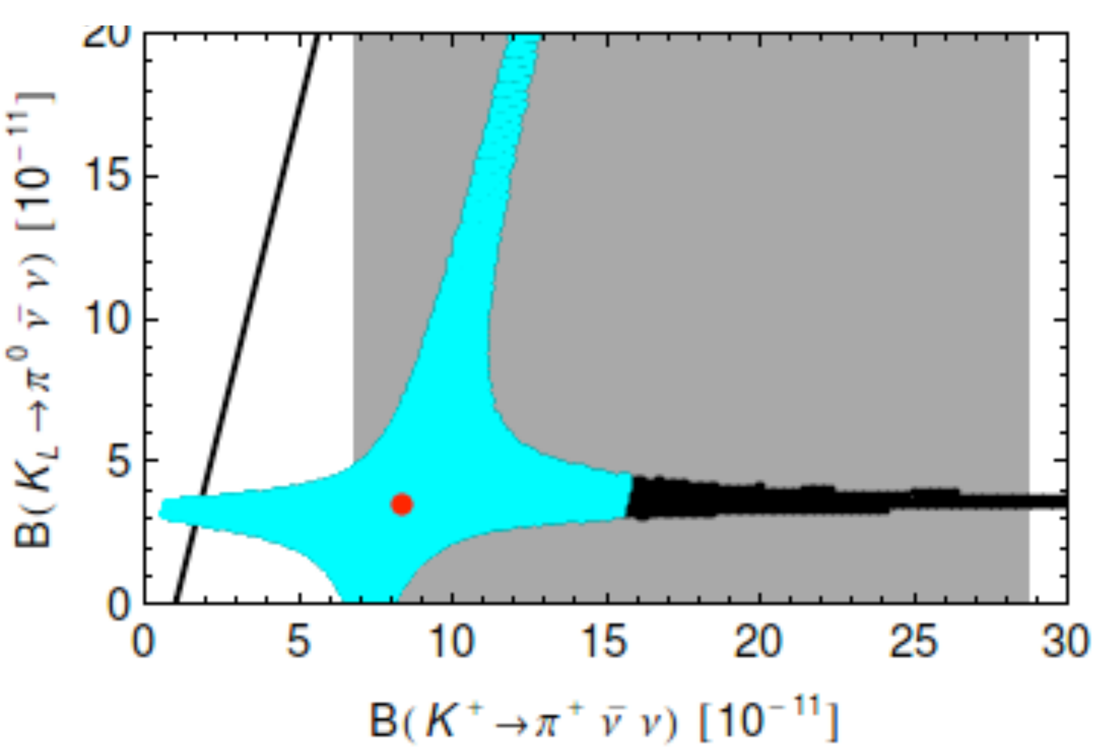

$M_{Z'}$ = 1TeV

$K_L \to \mu^+ \mu^-$

3 [Buras et. al. `13]

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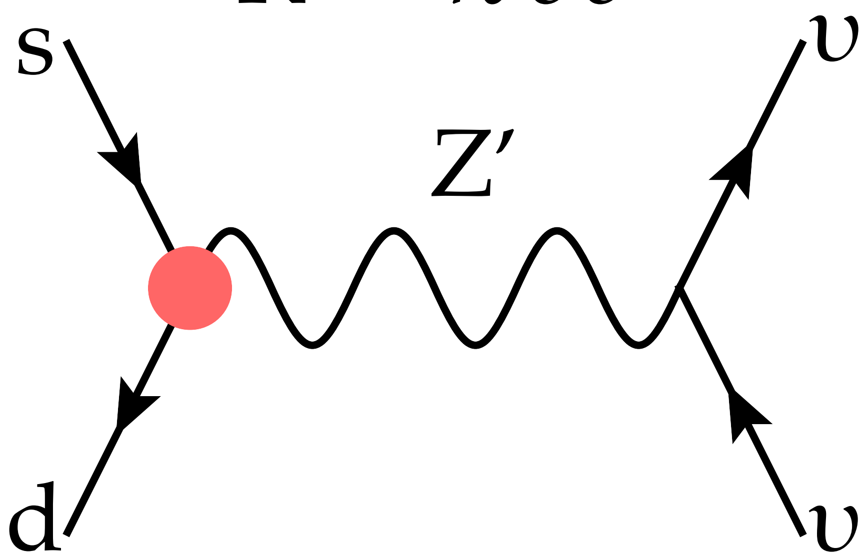

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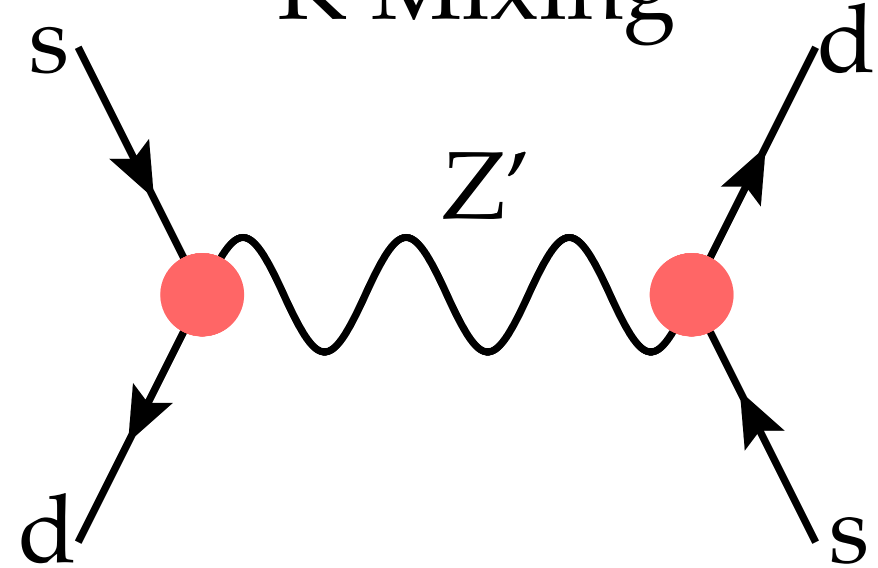

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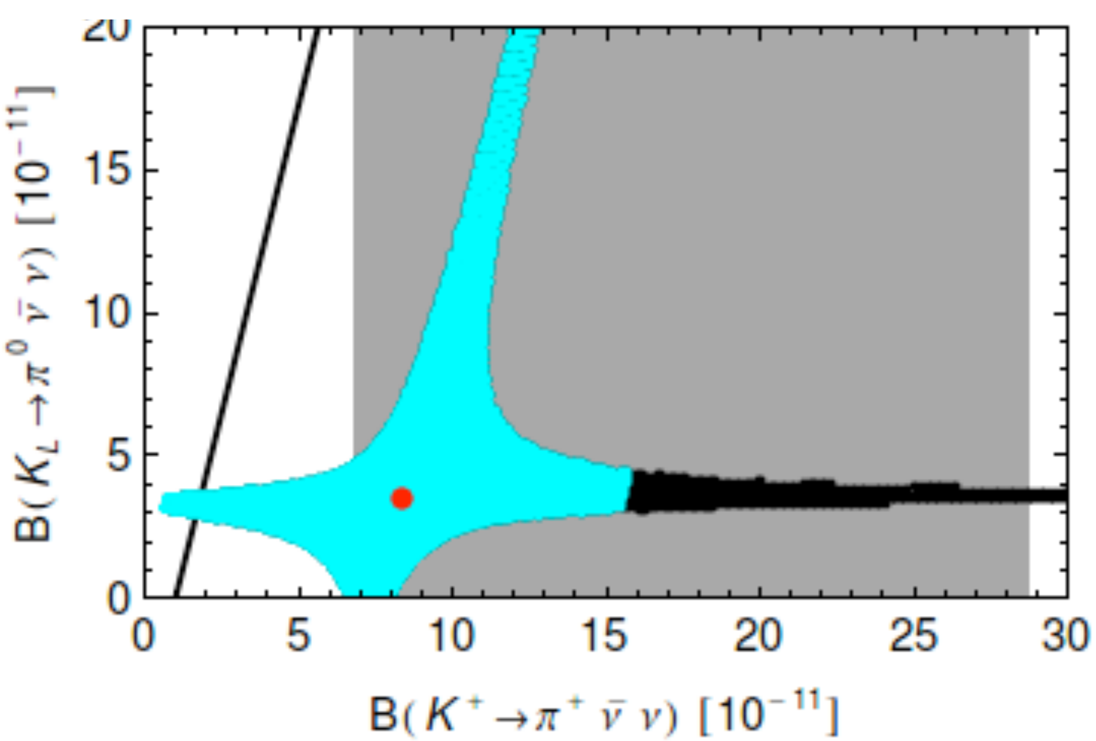

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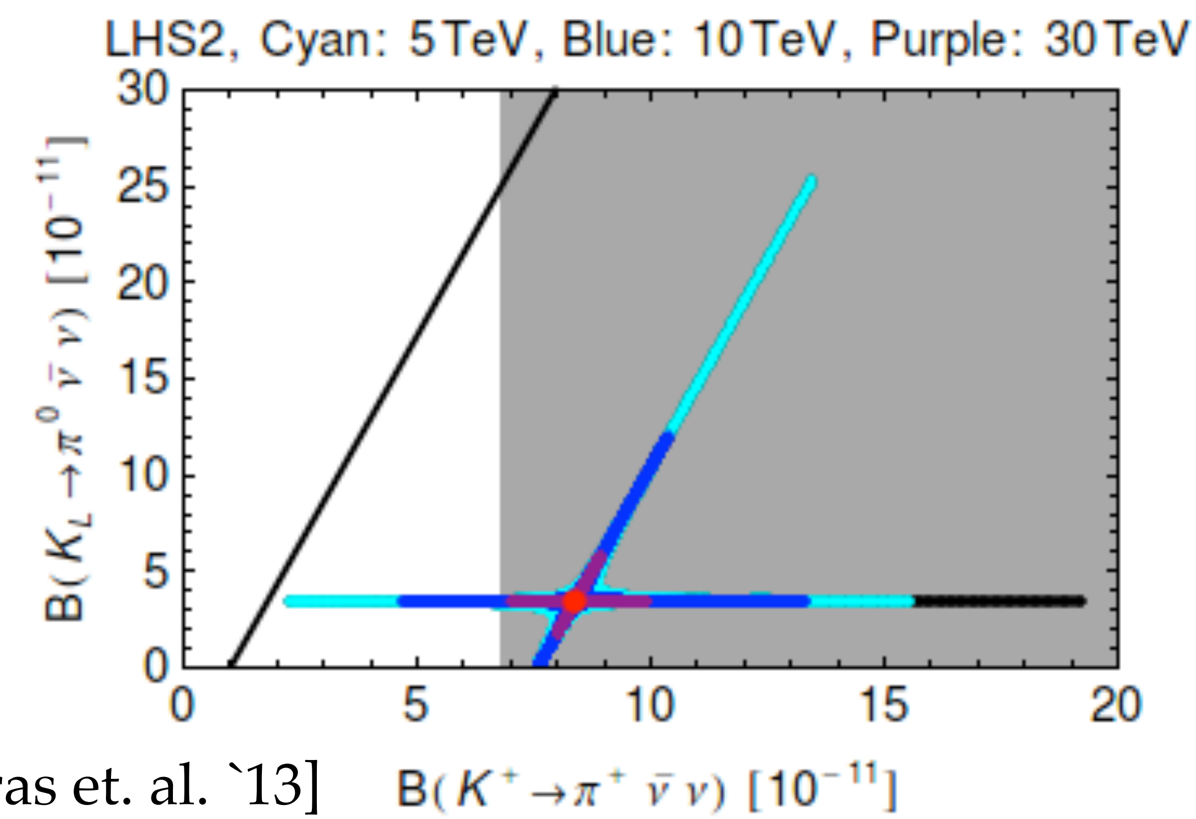

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$\rightarrow$ The suppression in the standard model

and the accuracy of the theory predictions

Why are Kaon Decays so rare?

Before the charm quark: why are the two Branching ratios

$$\mathcal{B}r(K_L \to \mu^+\mu^-) \simeq 6.84(11) \cdot 10^{-9} \qquad \mathcal{B}r(K_L \to \gamma\gamma) \simeq 5.47(4) \cdot 10^{-4}$$

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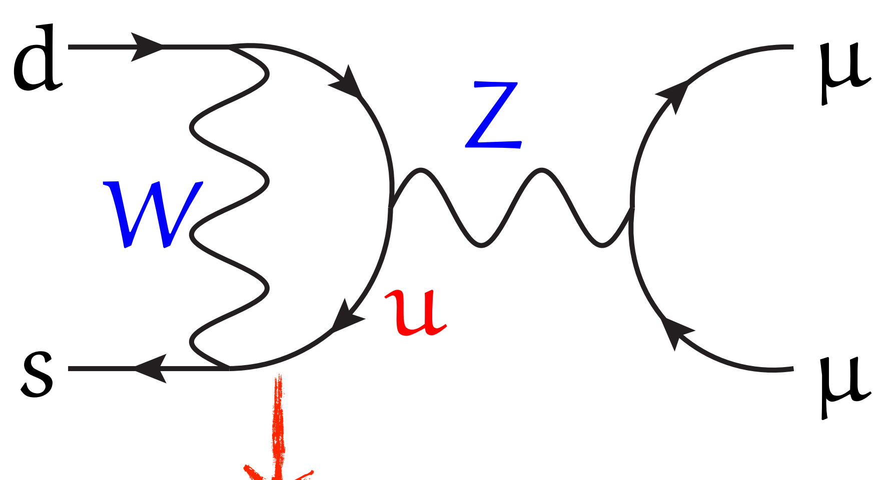
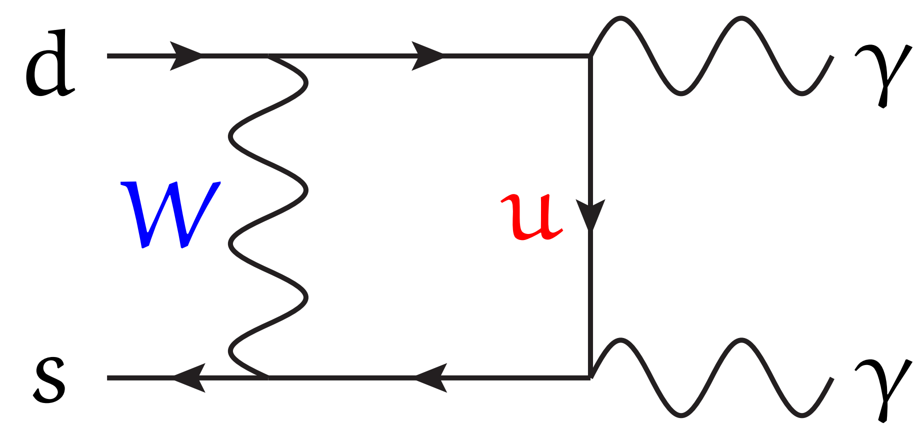
$K_L \to \mu^+ \mu^-$: The 2 μs are in J=0 state → no 1 γ coupling



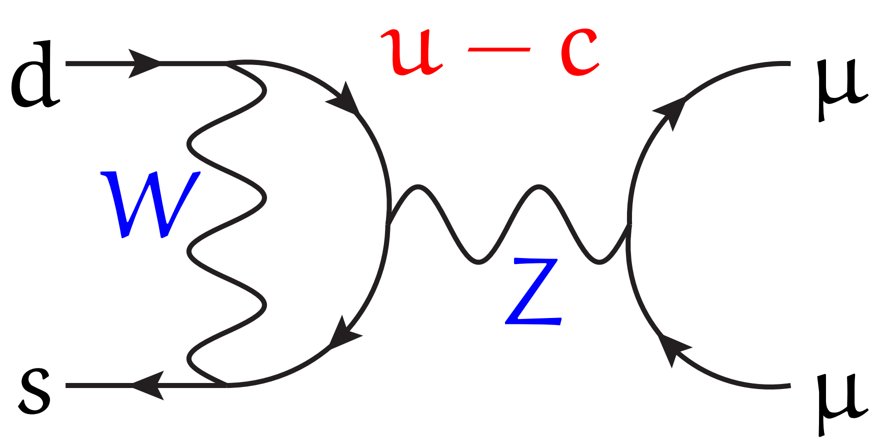
$\mathcal{O}(G_F^2 \Lambda_{\text{Fermi}}) = \mathcal{O}(G_F \alpha)$



$\mathcal{O}(G_F \alpha)$

The GIM Mechanism

GIM: charm quark to suppress neutral currents



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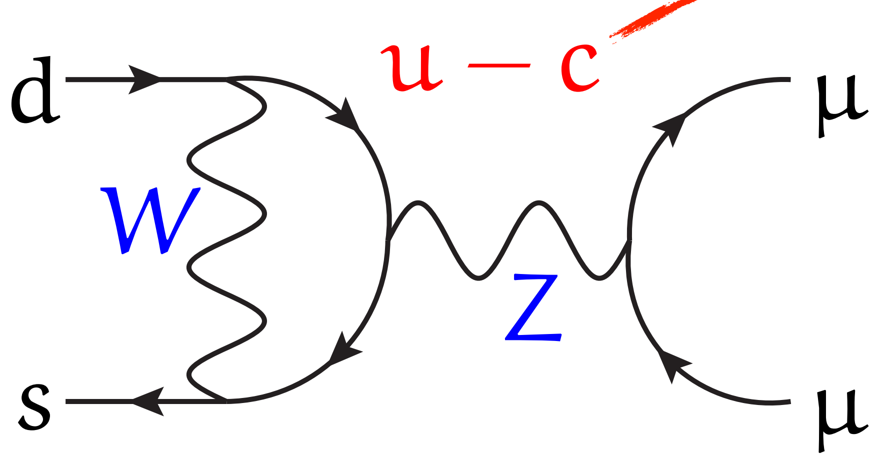

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Cutoff is then m_c^2

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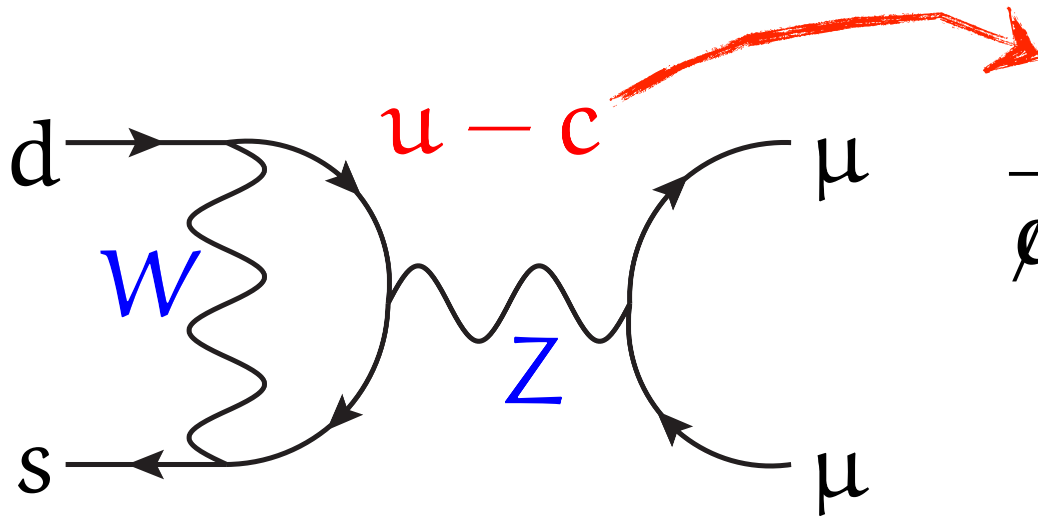

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$\frac{m_c^2}{M_W^2}$ suppression of light quark contributions

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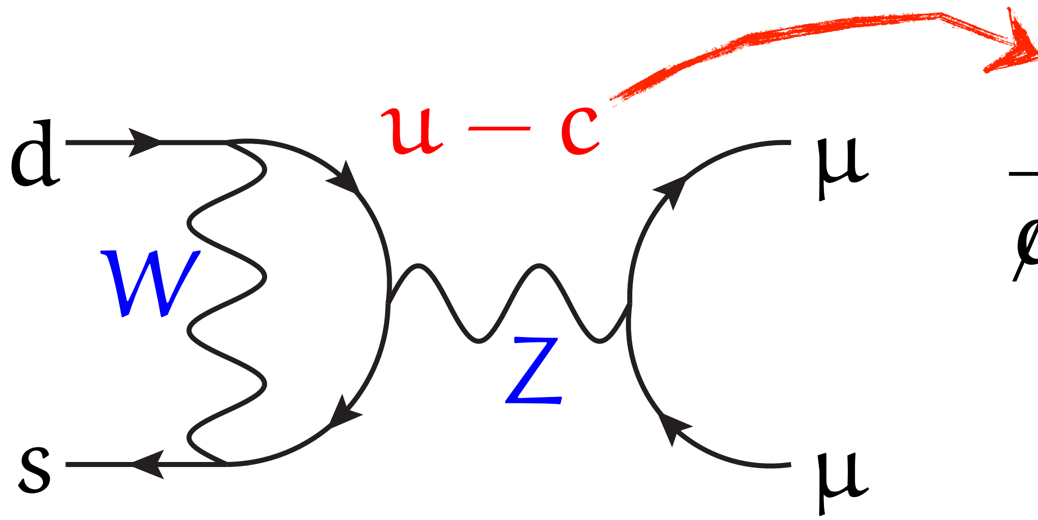

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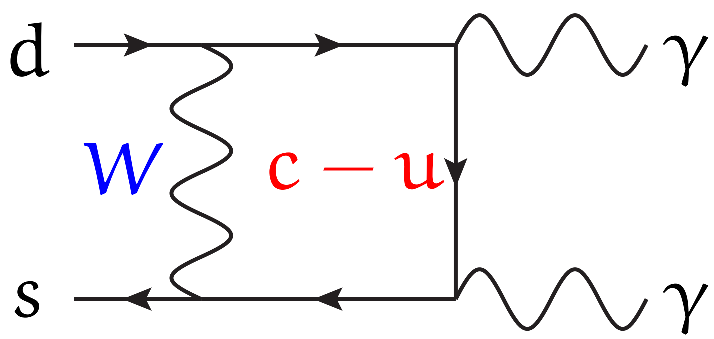
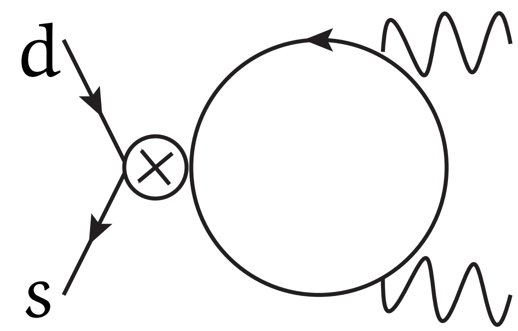
The resulting $m_c^2 G_F^2 \log \frac{m_c}{M_w}$ is known at NNLO [MG, Haisch `07]

Contributions to $K_L \to \mu^+\mu^-$

No quadratic suppression for $K_L \to \gamma\gamma$





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(same for photon penguin)

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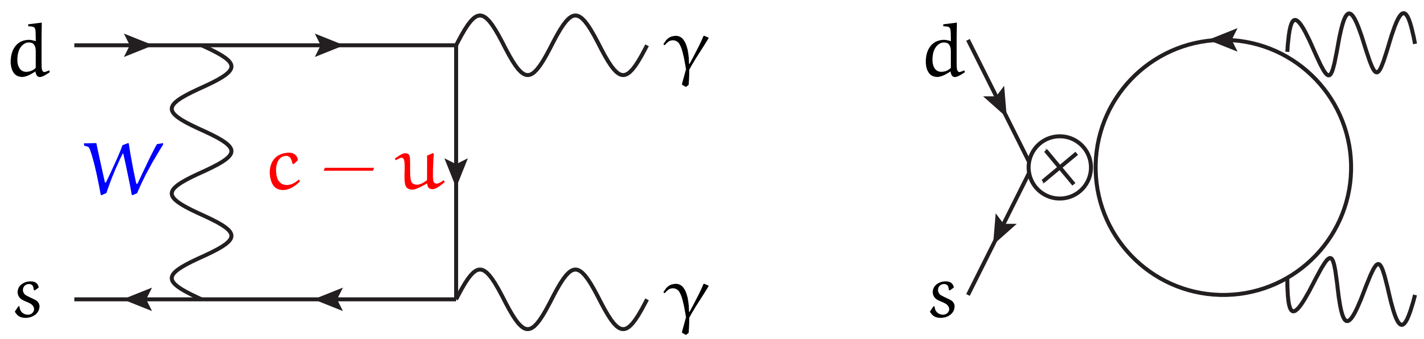

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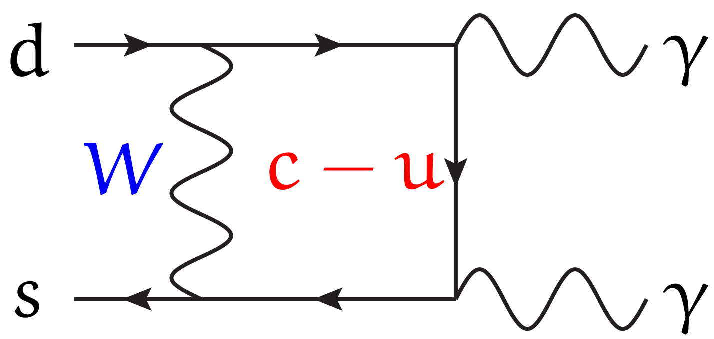

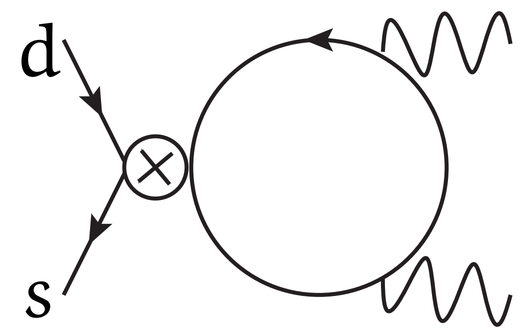

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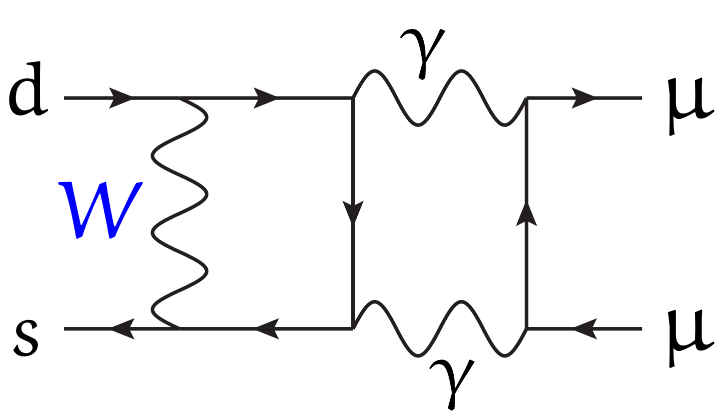
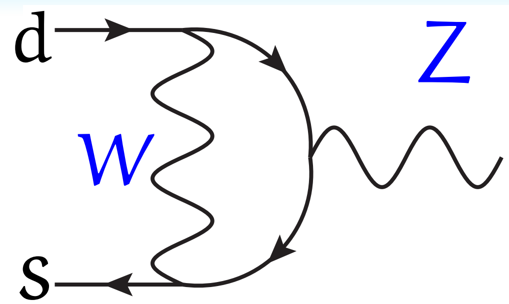
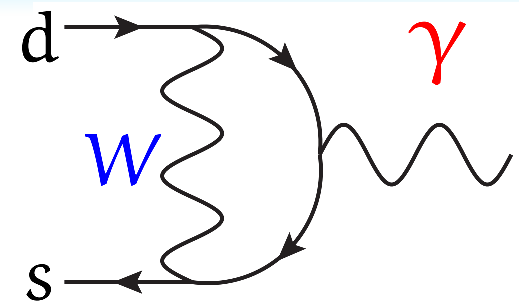
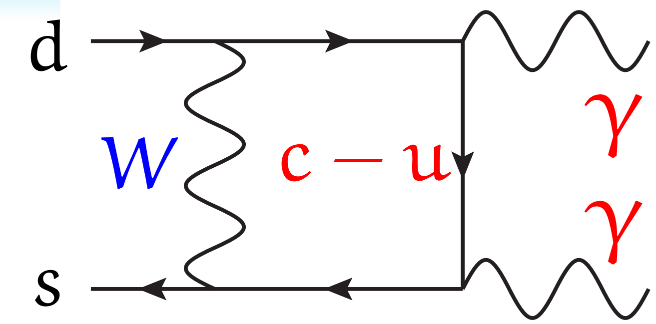


including top

$$\mathcal{B}r(K_L \to \mu^+\mu^-) \simeq \left((-0.95 \pm ???)^2 + 6.7\right) \cdot 10^{-9}$$

Dispersive Absorptive

Rare Kaon Decays

			
$K_L \to \mu^+\mu^-$	SD	–	α_e LD

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	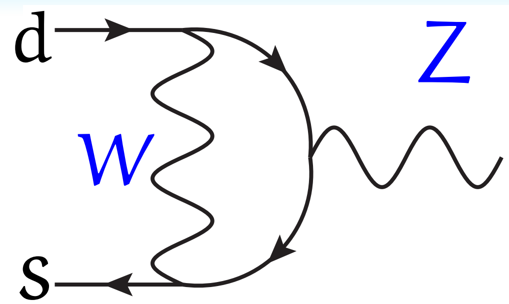	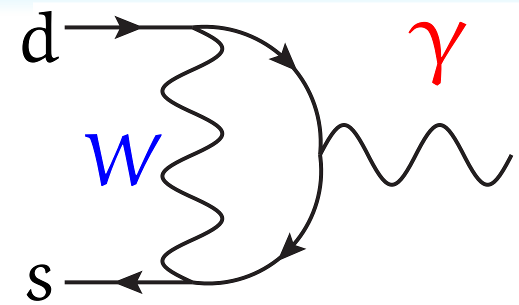	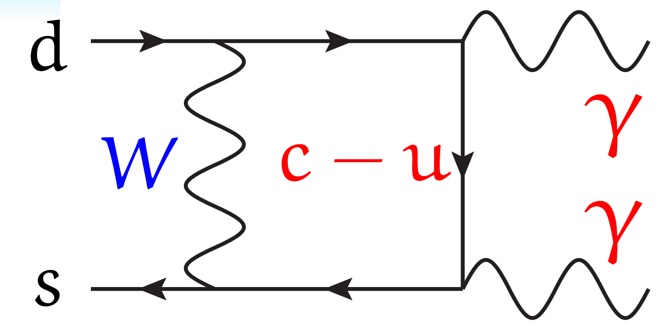
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$K \to \pi \nu \bar{\nu}$	SD	–	–

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	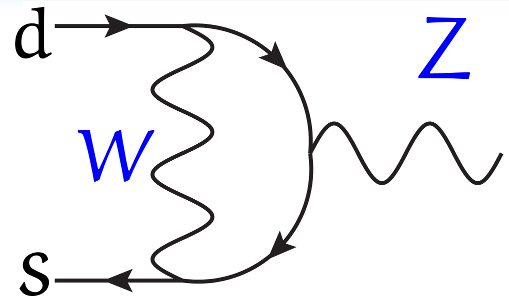	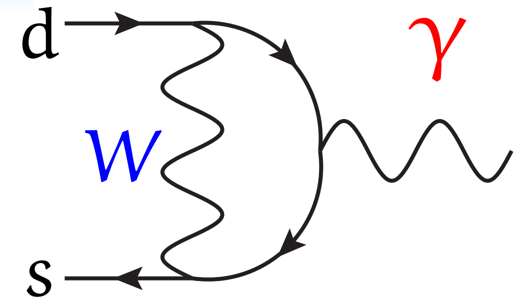	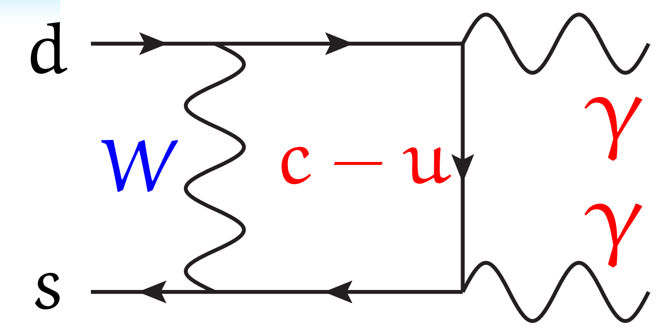
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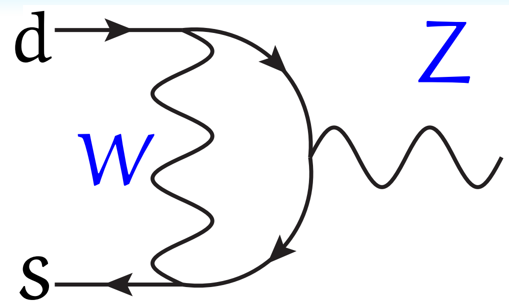
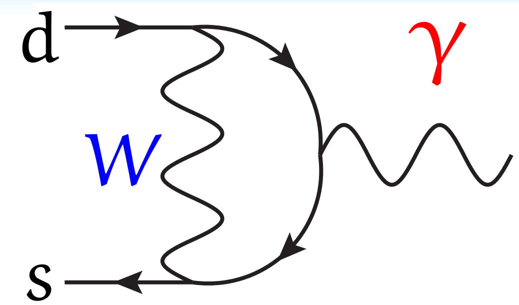
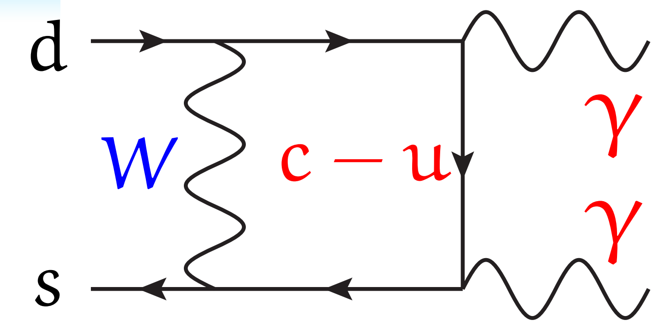
$K_S \to \pi l^+ l^-$	–	LD	–

Rare Kaon Decays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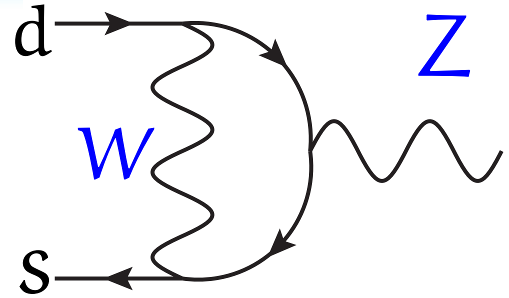

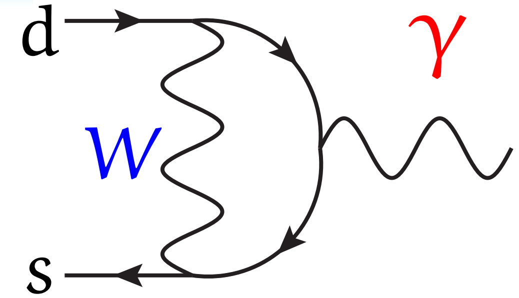

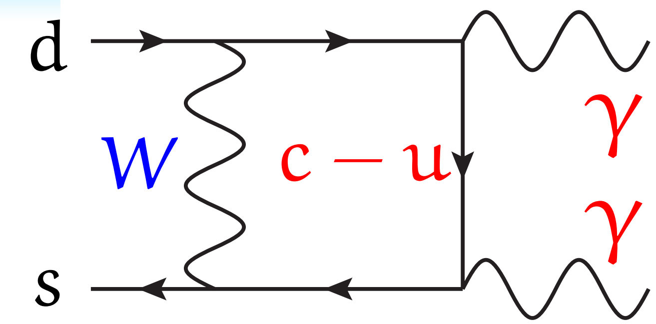

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$K_L \to \pi l^+ l^-$	SD	SD $+\epsilon_K$ LD	α_e LD
CP violating	NLO QCD [Buchalla et. al. \`95]	$K_L \to K_S$ & $K_S \to \pi^0 l^+ l^-$ [Mescia et. al. \`06]	Estimate from $K_L \to \pi^0\gamma\gamma$ [Isidori et. al. \`04]

Top quark

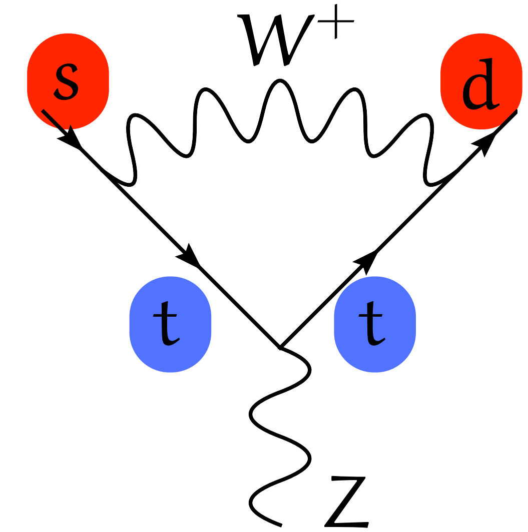
m_c^2 / M_W^2 suppression

$\rightarrow$ top-quark dominates

$K \rightarrow \pi \bar{\nu} \nu$

$$V_{ij} = \mathcal{O}\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\lambda = \mathcal{O}(0.2)$$



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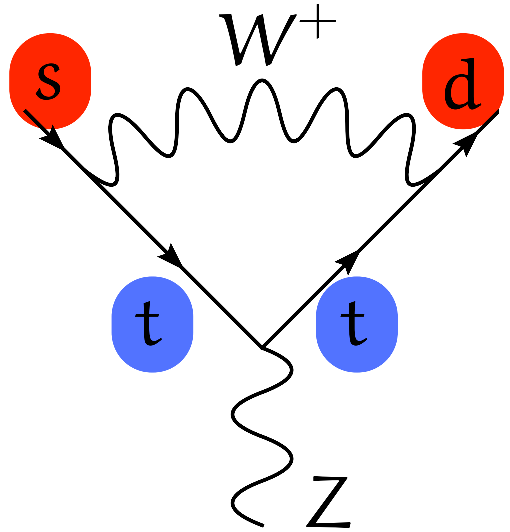

FCNCs which are dominated by top-quark loops:

$b \to s$:	$b \to d$:	$s \to d$:
$\lvert V_{tb}^* V_{ts} \rvert \propto \lambda^2$	$\lvert V_{tb}^* V_{td} \rvert \propto \lambda^3$	$\lvert V_{ts}^* V_{td} \rvert \propto \lambda^5$

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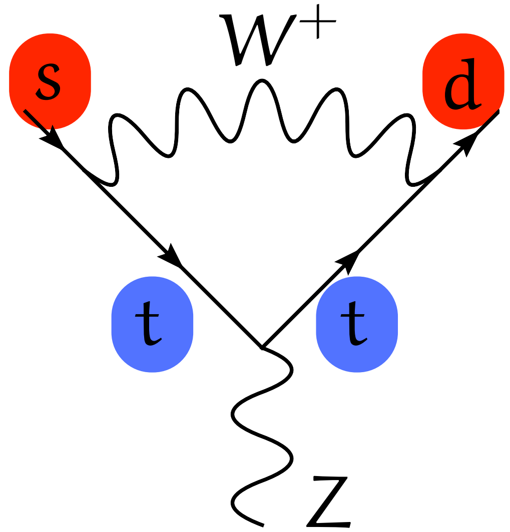

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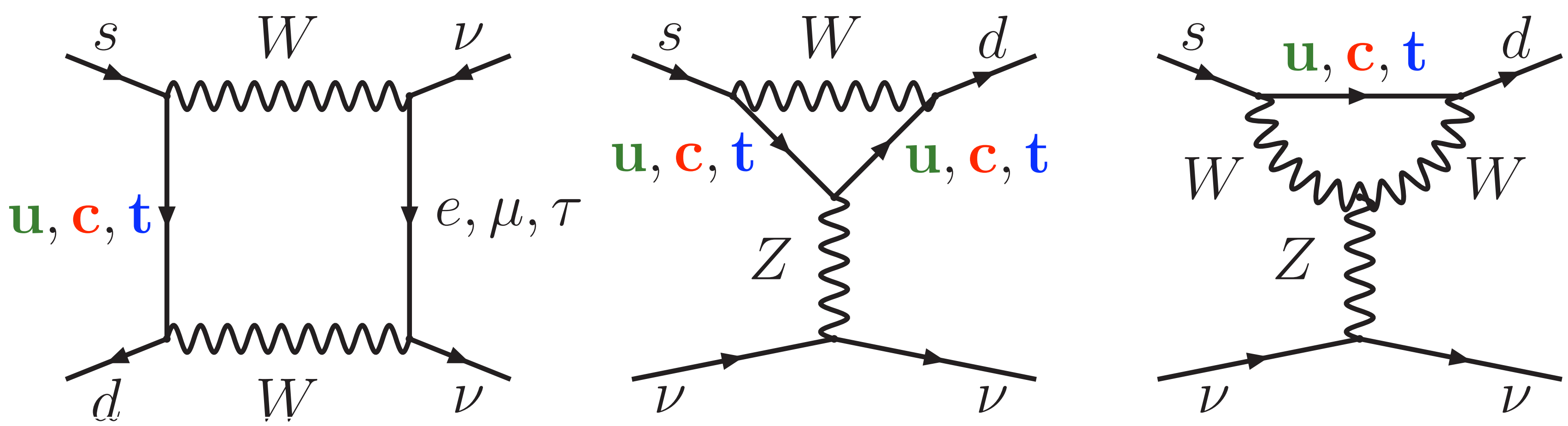
Kaons test new physics up to O(100) TeV

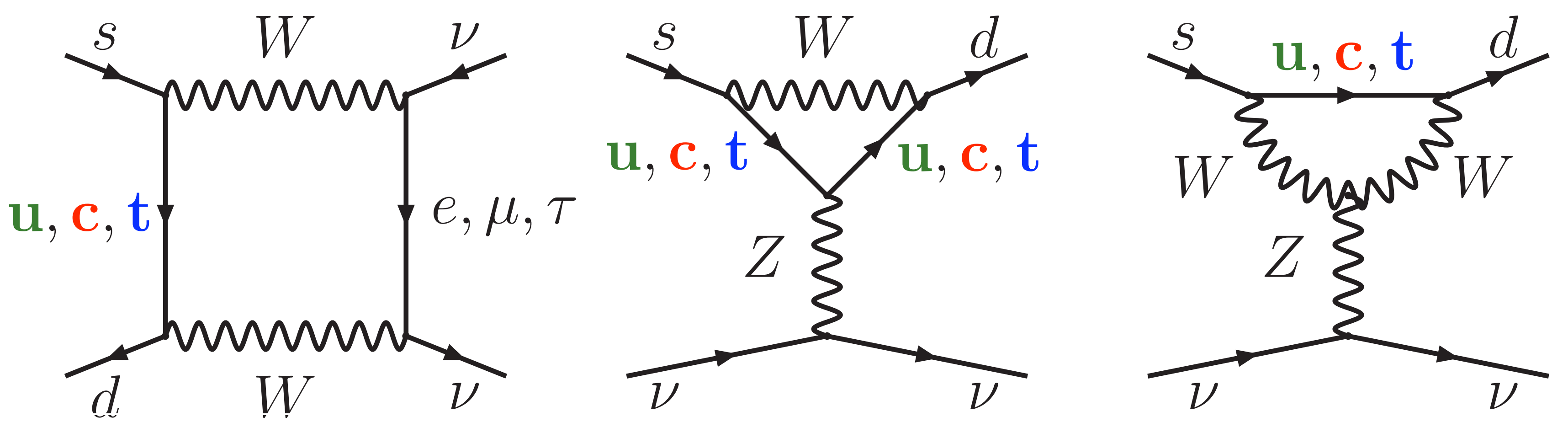
$K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ at M_W



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} \left(F(x_t) - F(x_u)\right) + V_{cs}^* V_{cd} \left(F(x_c) - F(x_u)\right)$$

$K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ at M_W



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$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$

Matching (NLO +EW):

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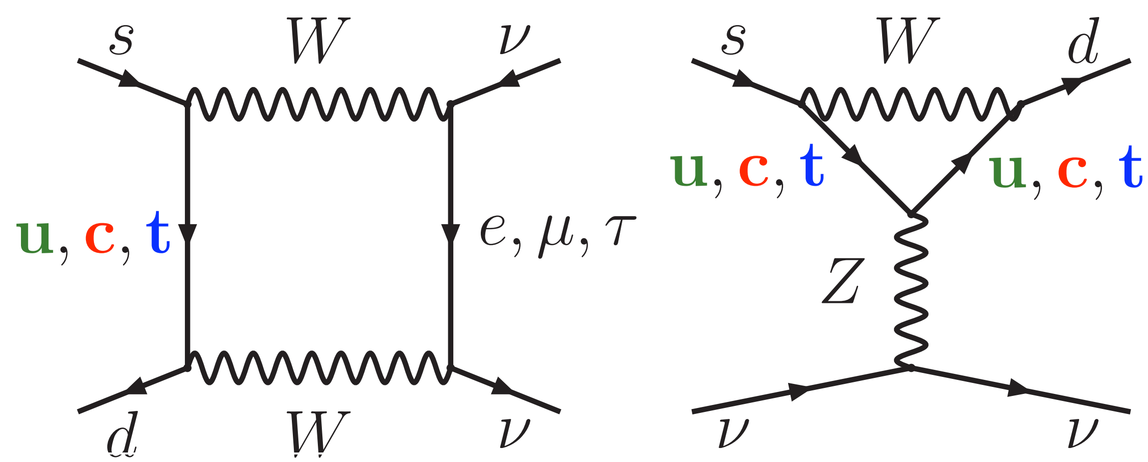

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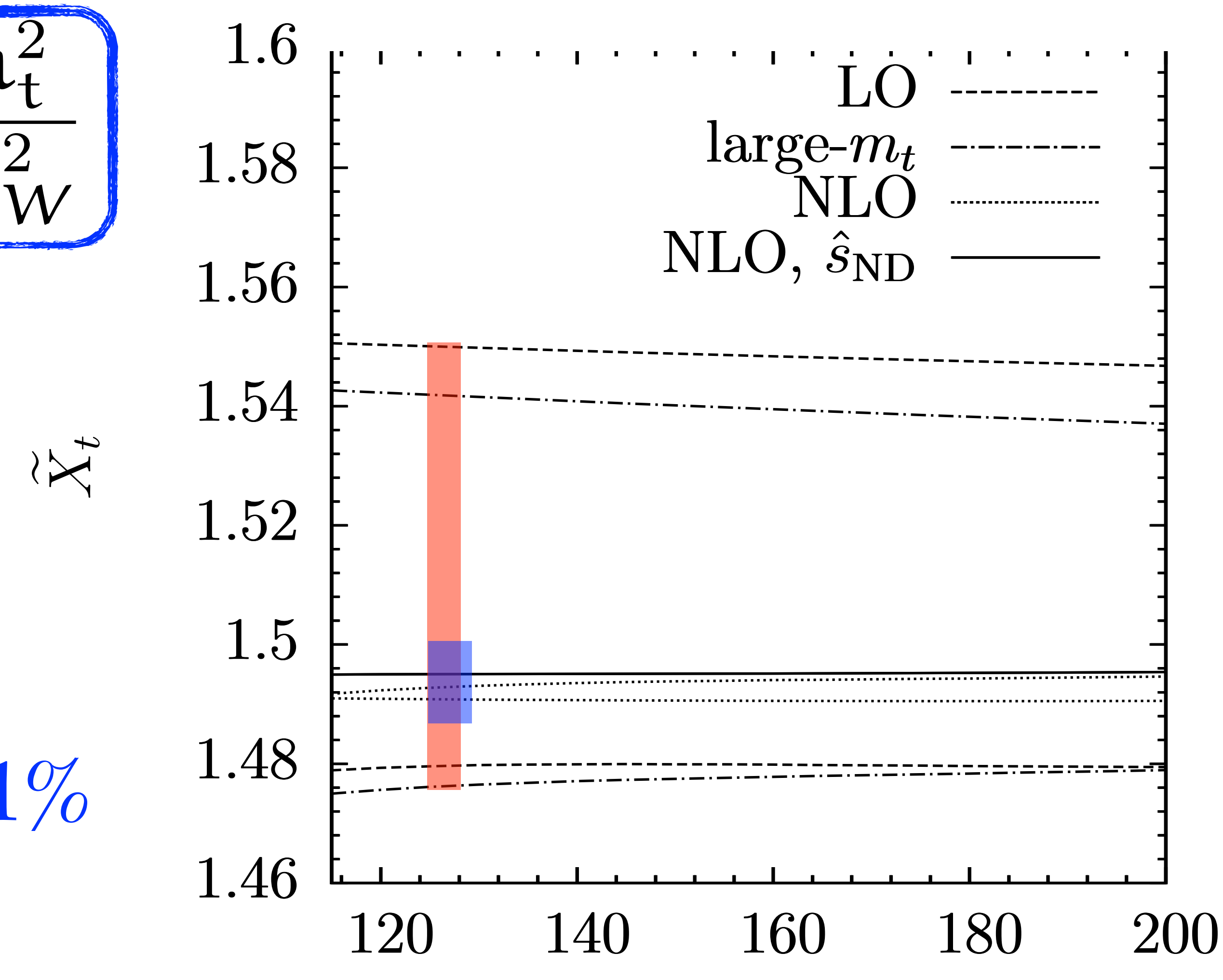
After 2011 uncertainty below 1%



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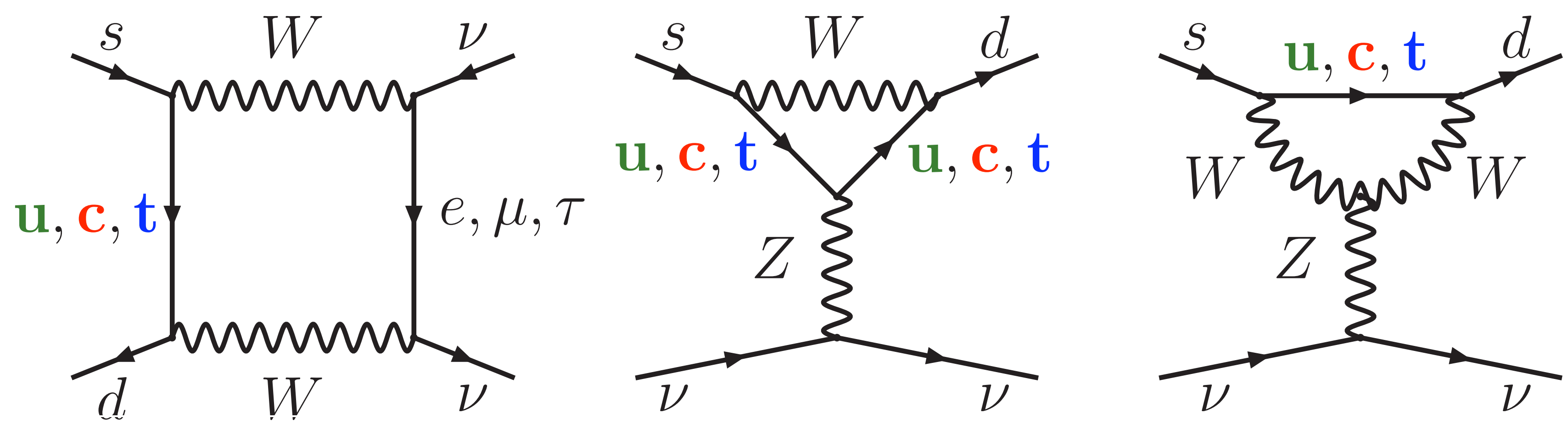

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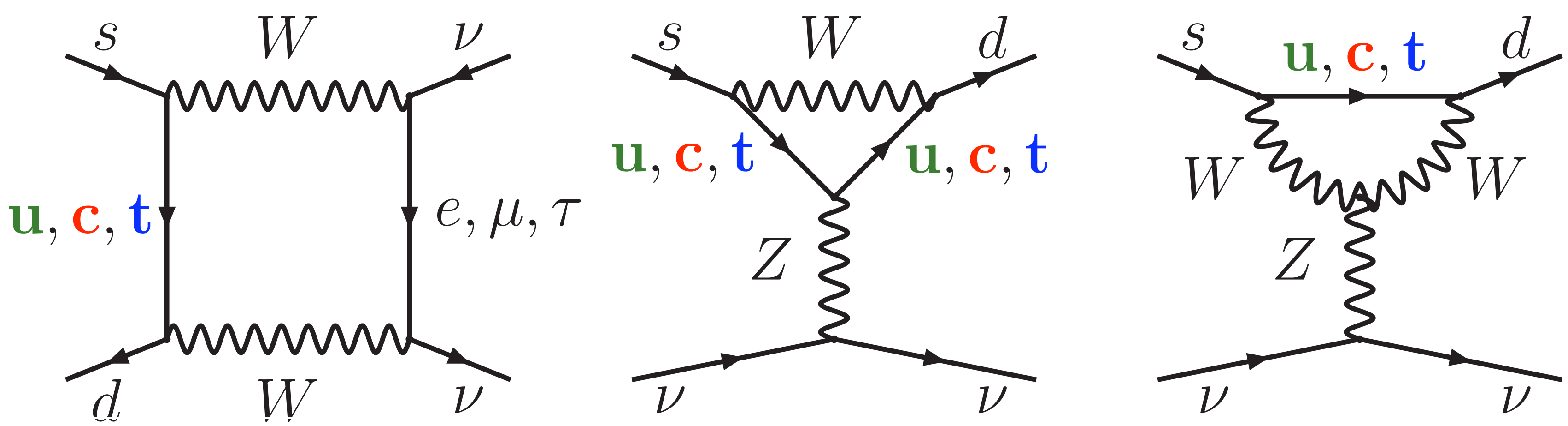

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Matrix element from K_{l3} decays
(Isospin symmetry: $K^+ \to \pi^0\, e^+\, \upsilon$)

[Mescia, Smith]

$K^+ \to \pi^+ \bar{\nu} \nu$ from M_W to m_c

P_c: charm quark contribution to $K^+ \to \pi^+ \bar{\nu} \nu$ (30% to BR)

Series converges very well (NNLO:10%→2.5% uncertainty)

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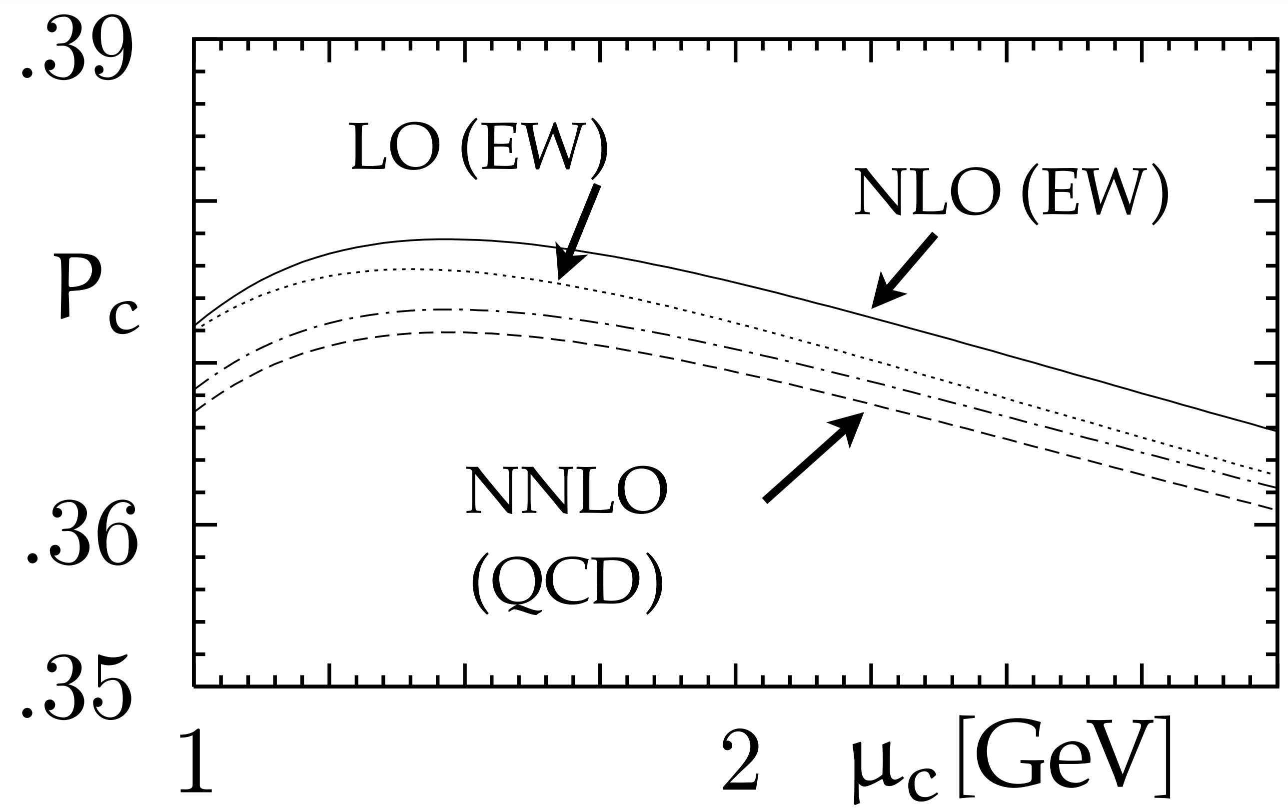

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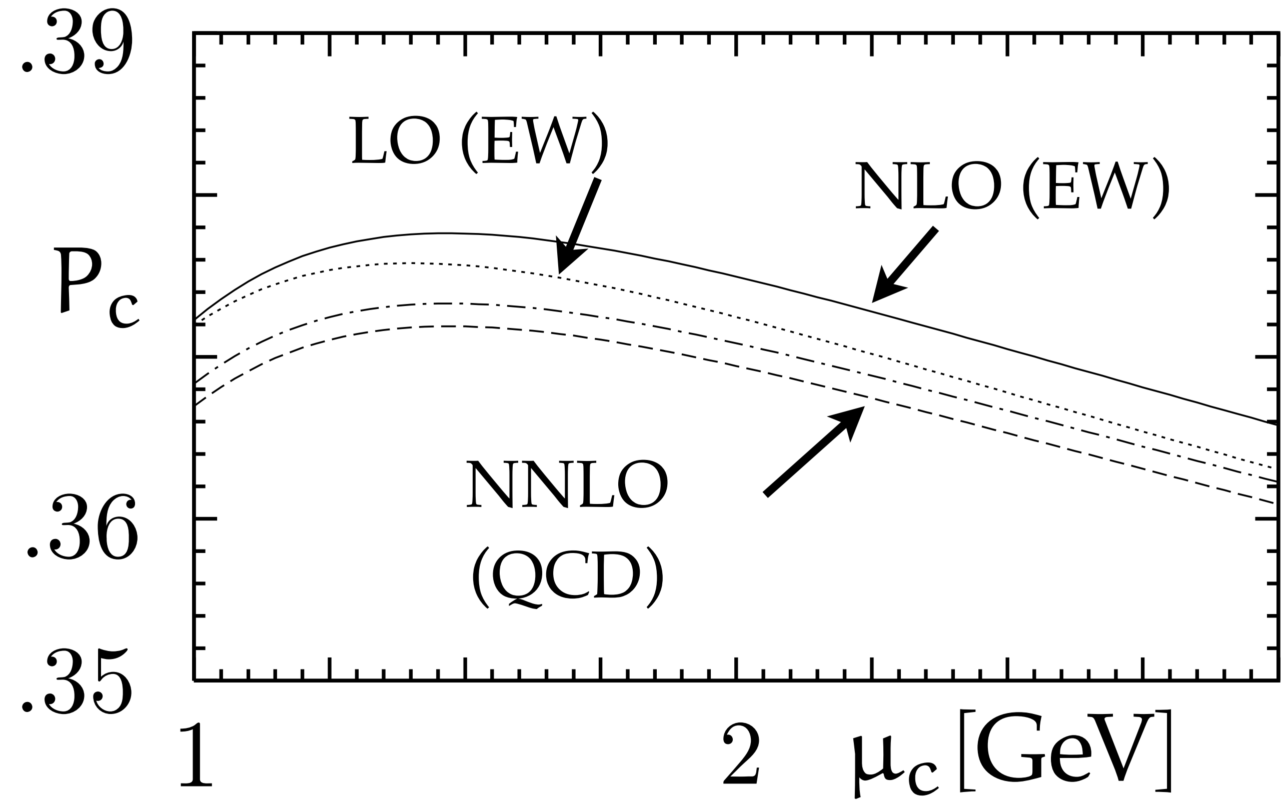

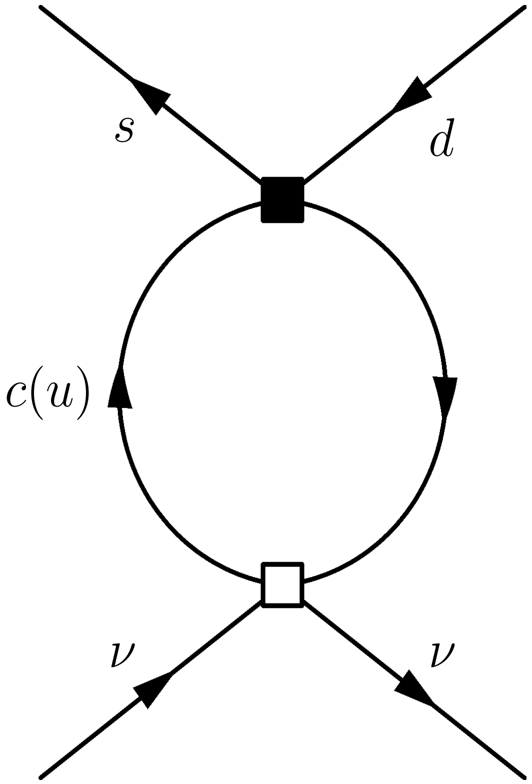

No GIM below the charm quark mass scale

higher dimensional operators UV scale dependent

One loop ChiPT calculation approximately cancels this scale dependence $\delta P_{c,u} = 0.04 \pm 0.02$

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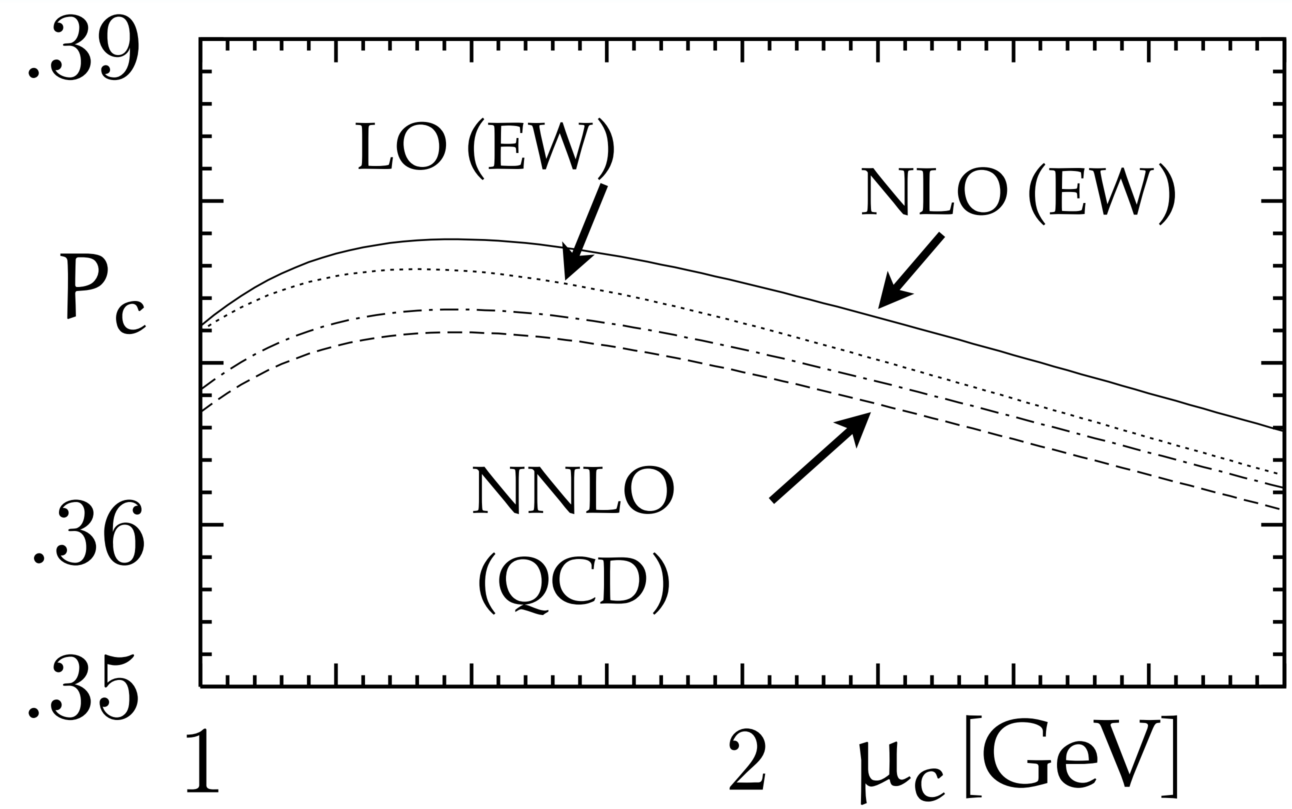

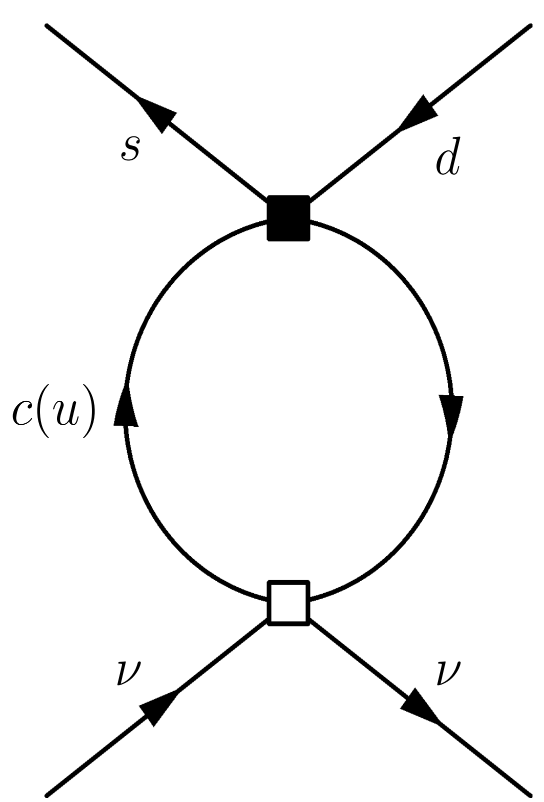

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Could be calculated on the lattice

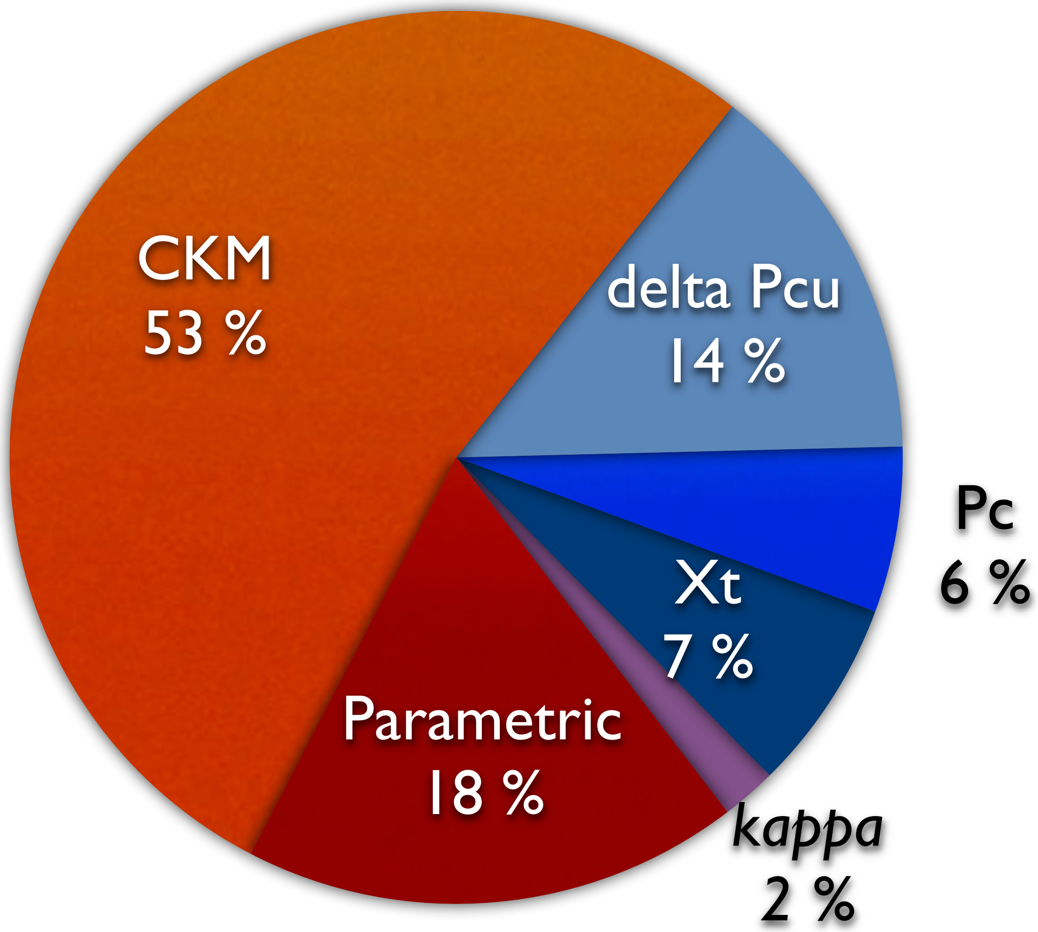
[Isidori, Martinelli, Turchetti `06]

$K \to \pi\, \bar{\upsilon}\, \upsilon$: Error Budget

$BR^{th}(K^+ \to \pi^+ \bar{\upsilon}\upsilon) = 8.2(3)(7) \cdot 10^{-11}$

$BR^{exp}(K^+ \to \pi^+ \bar{\upsilon}\upsilon) = 17(11) \cdot 10^{-11}$ [E787, E949 ´08]

NA62 aims at 10% accuracy



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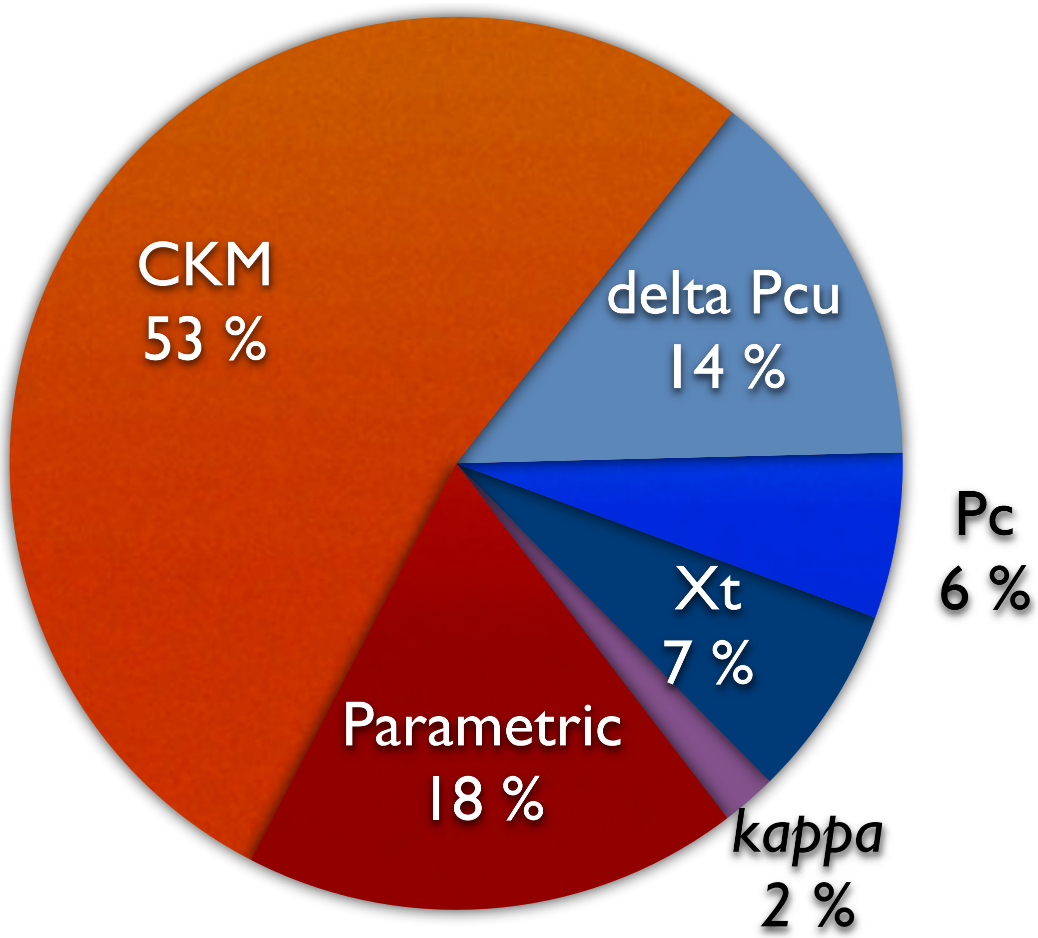
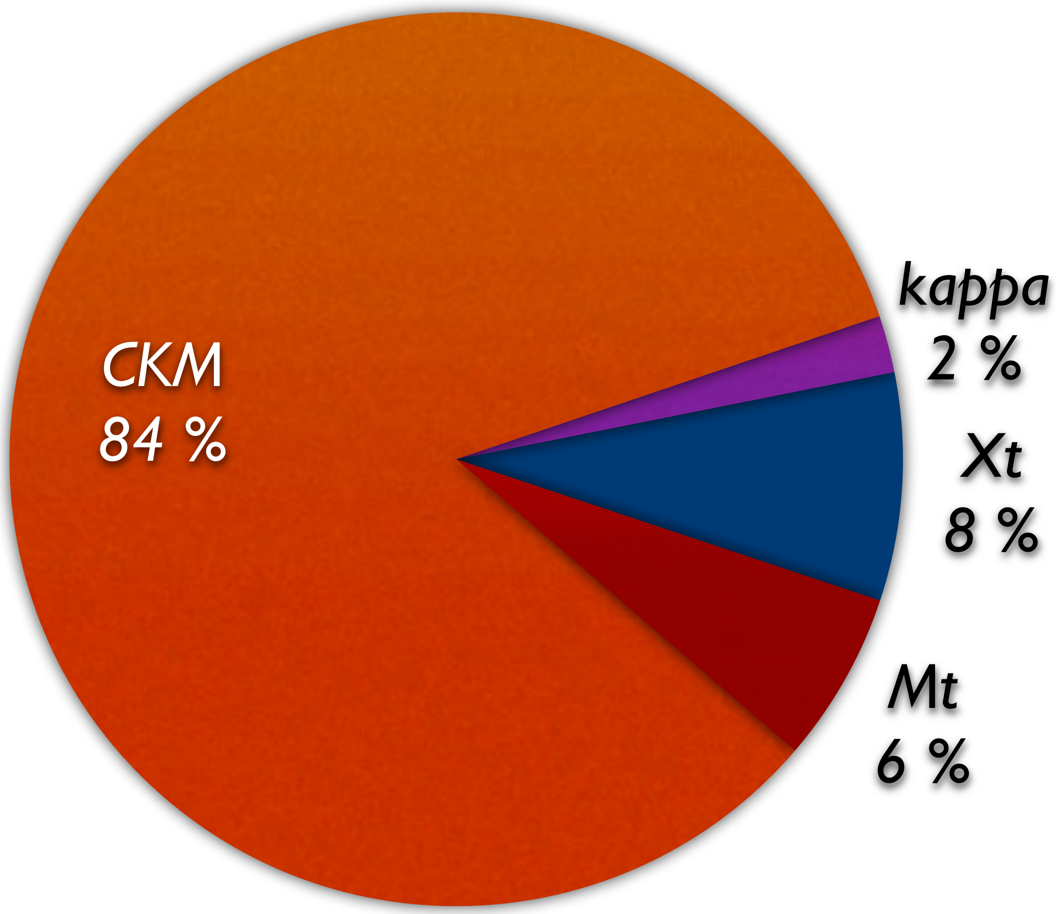
$BR^{exp}(K^+ \to \pi^+ \bar{\upsilon}\upsilon) < 6.7 \cdot 10^{-8}$ [E391a ´08]





εK: CP violation in Kaon Mixing

$$\epsilon_K \simeq \frac{\langle(\pi\pi)_{I=0}|K_L\rangle}{\langle(\pi\pi)_{I=0}|K_S\rangle}$$

$$\epsilon_K = e^{i\phi_\epsilon}\sin\phi_\epsilon\left(\frac{\mathrm{Im}(M_{12}^K)}{\Delta M_K} + \xi\right)$$

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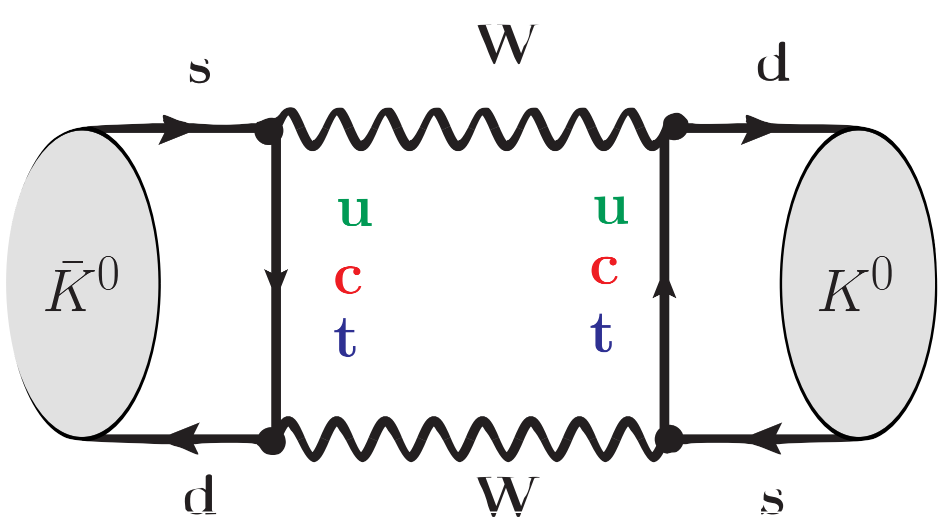
dispersive part



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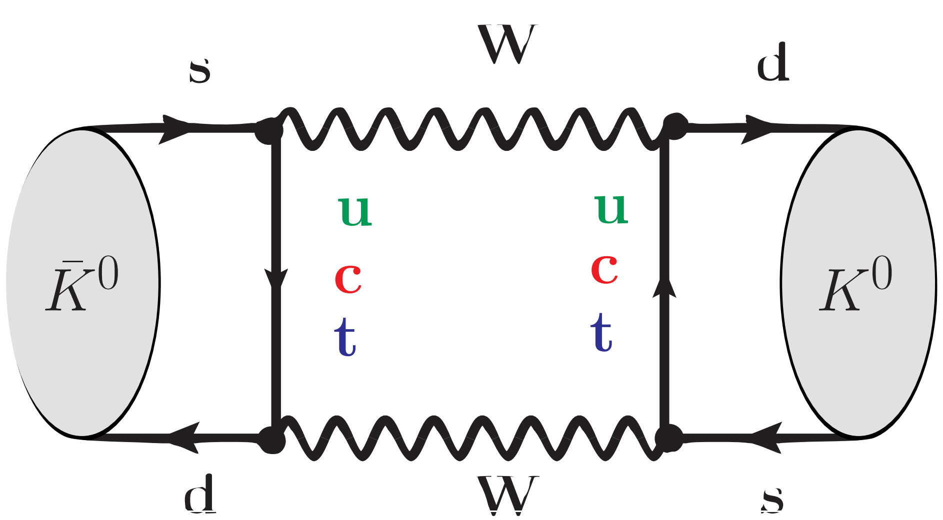

Local Interaction:

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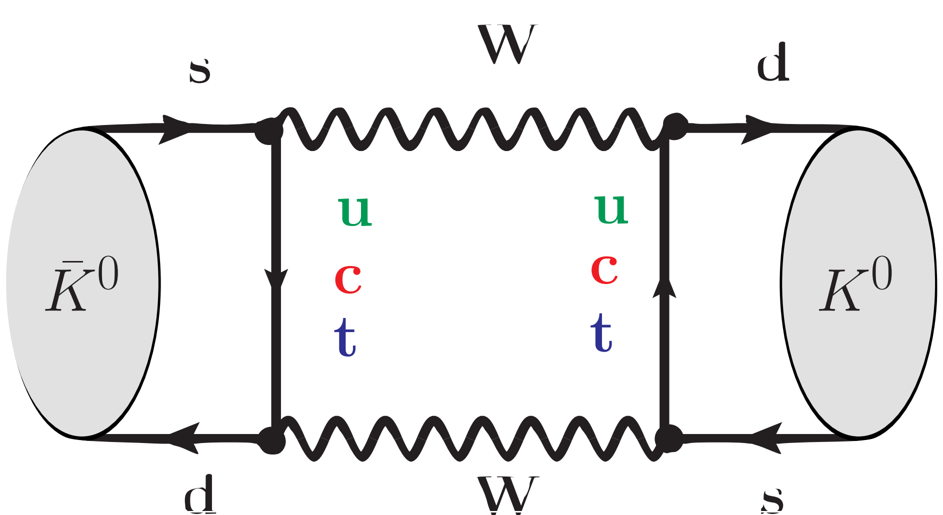

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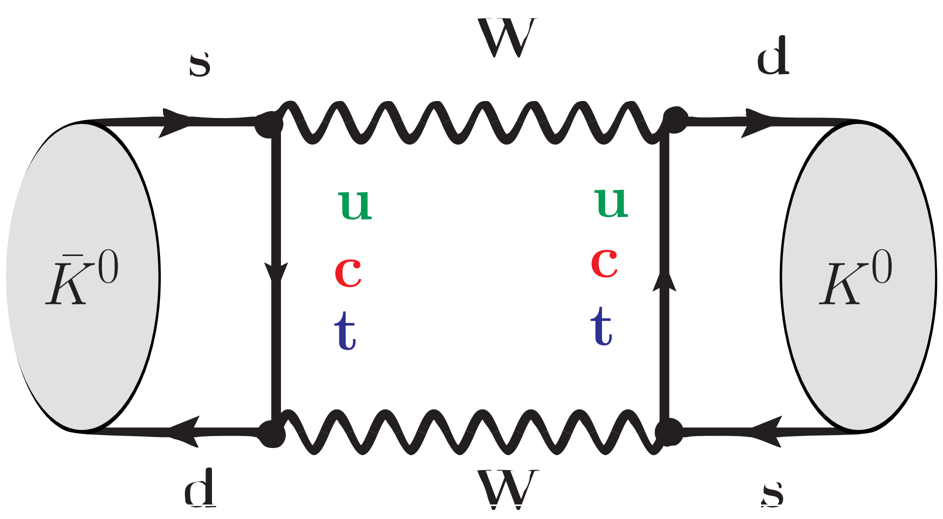

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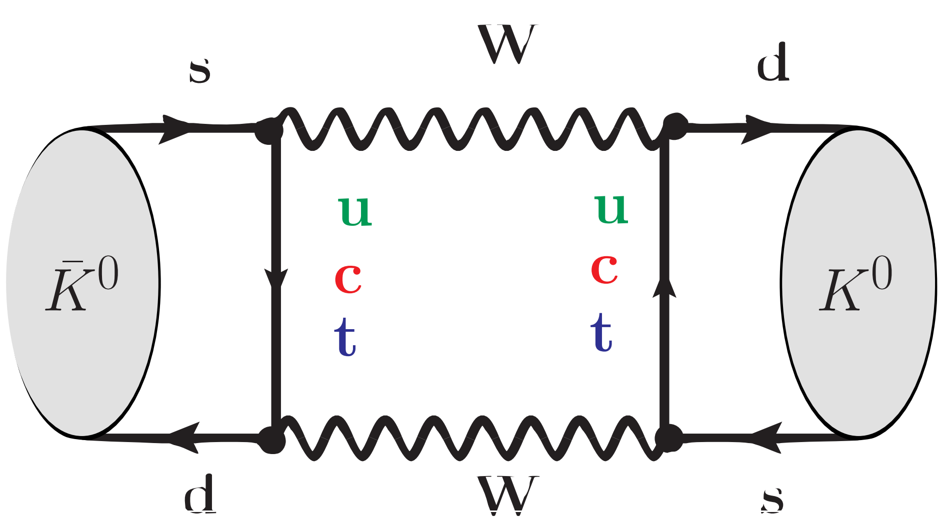

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(-15(6)%): $\lambda_c \lambda_c \, m_c^2 / M_W^2$

η_{ct}: 3-loop RGE, 2-loop Matching [Brod, MG `10]

η_{cc}: 3-loop RGE, 3-loop Matching [Brod, MG `12]

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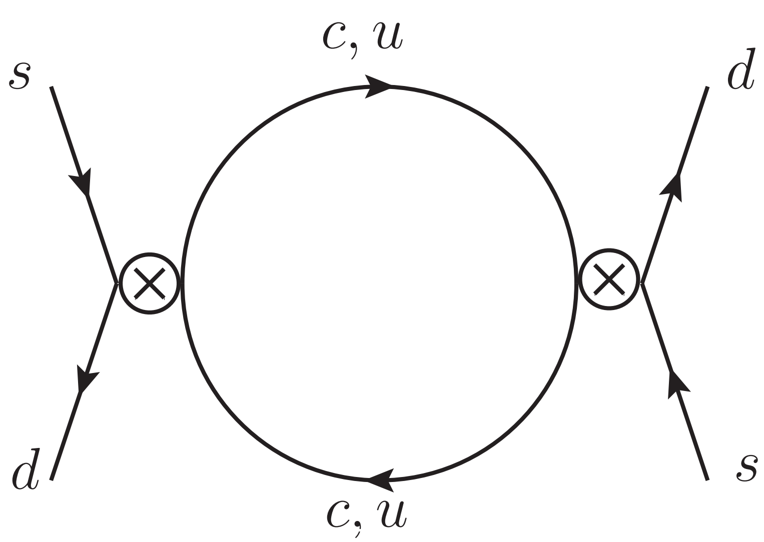
Long Distance ϵ_K



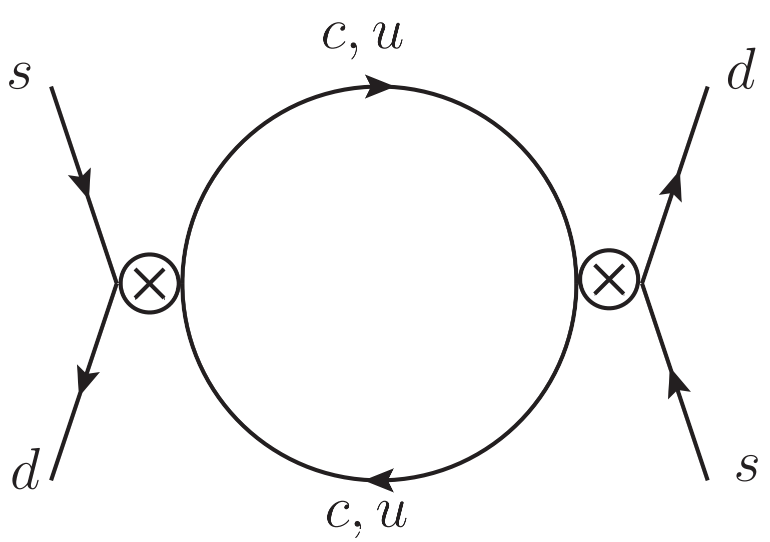
$$\int d^4x d^4y \, \langle K^0 | \, T\{H(x) \, H(y)\} \, | \bar{K}^0 \rangle$$

Integrate over $t_A < t_{x,y} < t_B$

[Christ et. al. 13]

Exploratory study for ΔM_K and ideas for ϵ_K

Long Distance ϵ_K



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Exploratory study for ΔM_K and ideas for ϵ_K

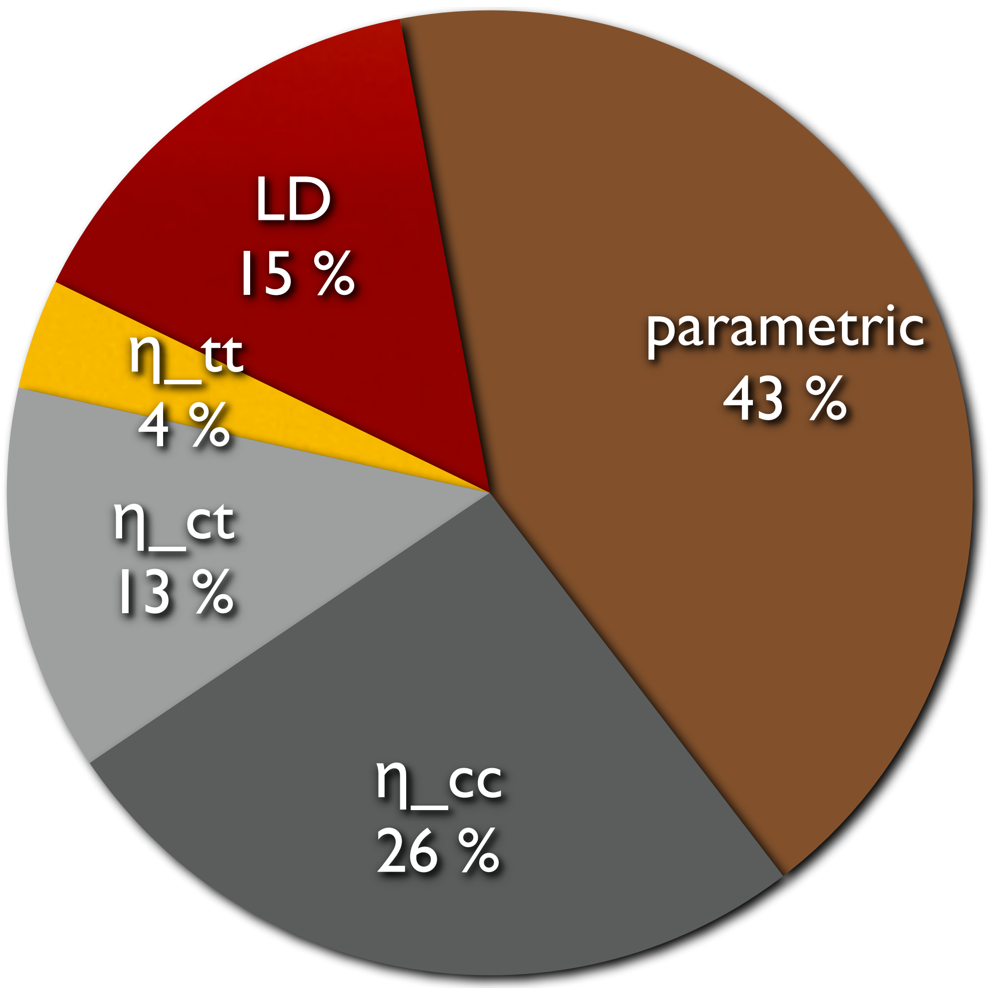
Use $\lambda_u \lambda_t$ instead of $\lambda_c \lambda_t$

$\lambda_u \lambda_u$ finite after GIM & charm – renormalise $\Delta S=1$ Operator

$\lambda_u \lambda_t$ log divergent – renormalise $\Delta S=1$ & $\Delta S=2$ Operator, i.e. match Lattice to continuum perturbation theory.

Residual Theory Uncertainty

After Lattice QCD & NNLO progress: η_{cc} dominant uncertainty



$$|\epsilon_K| = 1.81(28) \cdot 10^{-3}$$
$$\overset{\text{exp.}}{=} 2.23(1) \cdot 10^{-3}$$

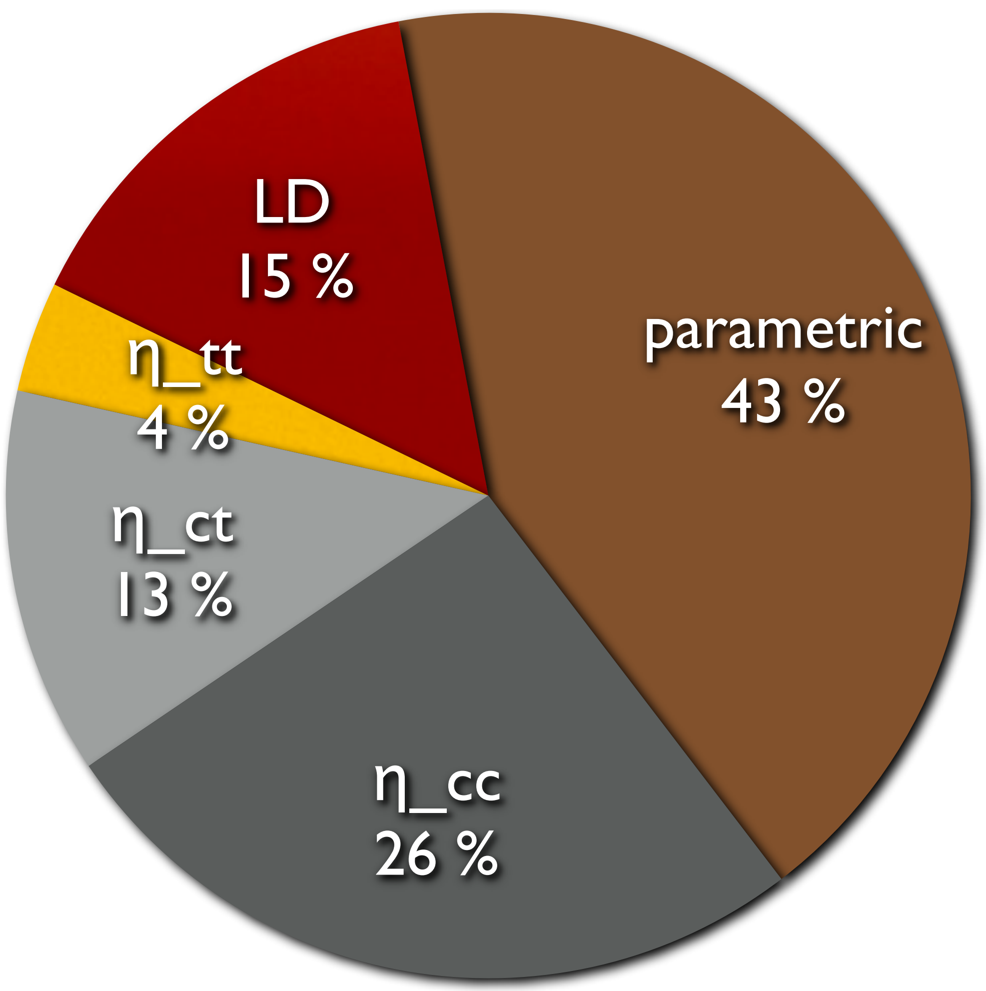
V_{cb} dominates parametric uncertainty

uncertainty in B_K sub-leading

Residual Theory Uncertainty

After Lattice QCD & NNLO progress: η_{cc} dominant uncertainty

ε_K is very important for phenomenology:
Future improvements are expected from Lattice QCD and interplay with perturbative QCD



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$K \to \pi \bar{\nu} \nu$ and ε_K in the MSSM

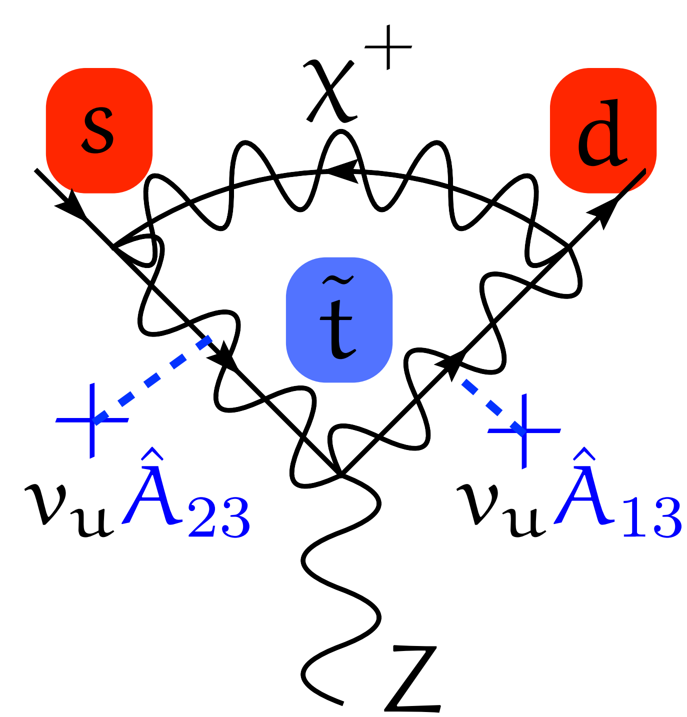
The MSSM has many sources of flavour violation encoded in the squark mass matrix

$$\hat{\mathcal{M}}^2_{\tilde{u}} = \begin{pmatrix} \hat{M}^2_{\tilde{u}_L} & v_u \hat{A}^\dagger_u - v_d \mu \hat{Y}^\dagger_u \\ v_u \hat{A}_u - v_d \mu^* \hat{Y}_u & \hat{M}^2_{\tilde{u}_R} \end{pmatrix}$$

In MFV no large effects are expected

Z Penguin sensitive
to up-type A-terms [Collangelo, Isidori `98]

The supersymmetry breaking A mass terms contribute to the numerator of the amplitude



Constraints on $K \rightarrow \pi \bar{\upsilon} \upsilon$

Decoupling property can be surprising in a specific models:

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For non-zero up-type A_{13} & A_{23}: $\varepsilon_K \propto M^{-4}$, $K \to \pi \bar{\upsilon} \upsilon \propto M^{-2}$

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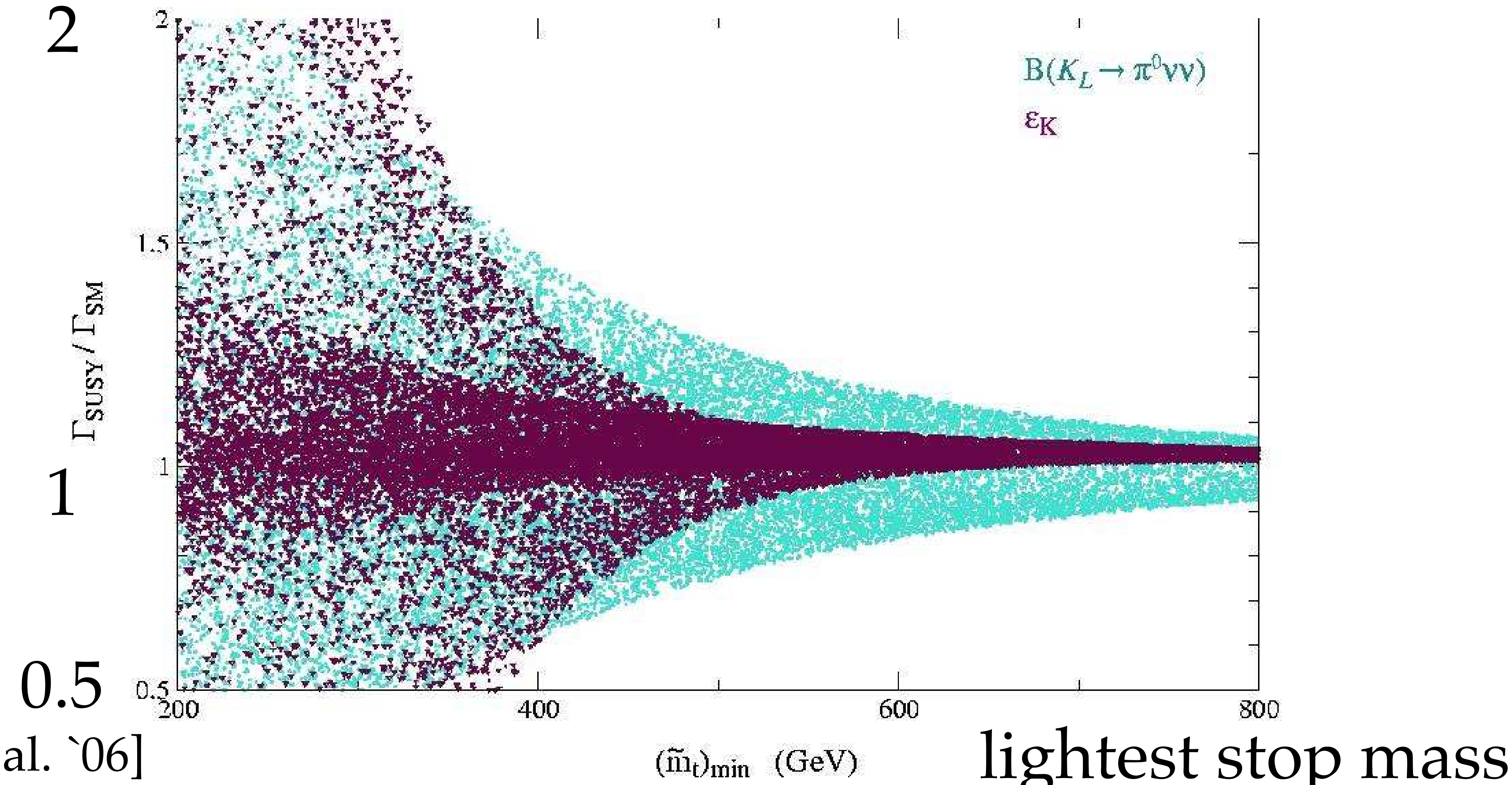

[Isidori et. al. `06]

Correlations with ε_K

The chiral enhancement of the scalar $(\bar{s}_R d_L)(\bar{d}_L s_R)$ operator breaks the ε_K & $K_L \to \pi^0 \bar{\upsilon} \upsilon$, but there is a correlation with $\varepsilon' / \varepsilon$

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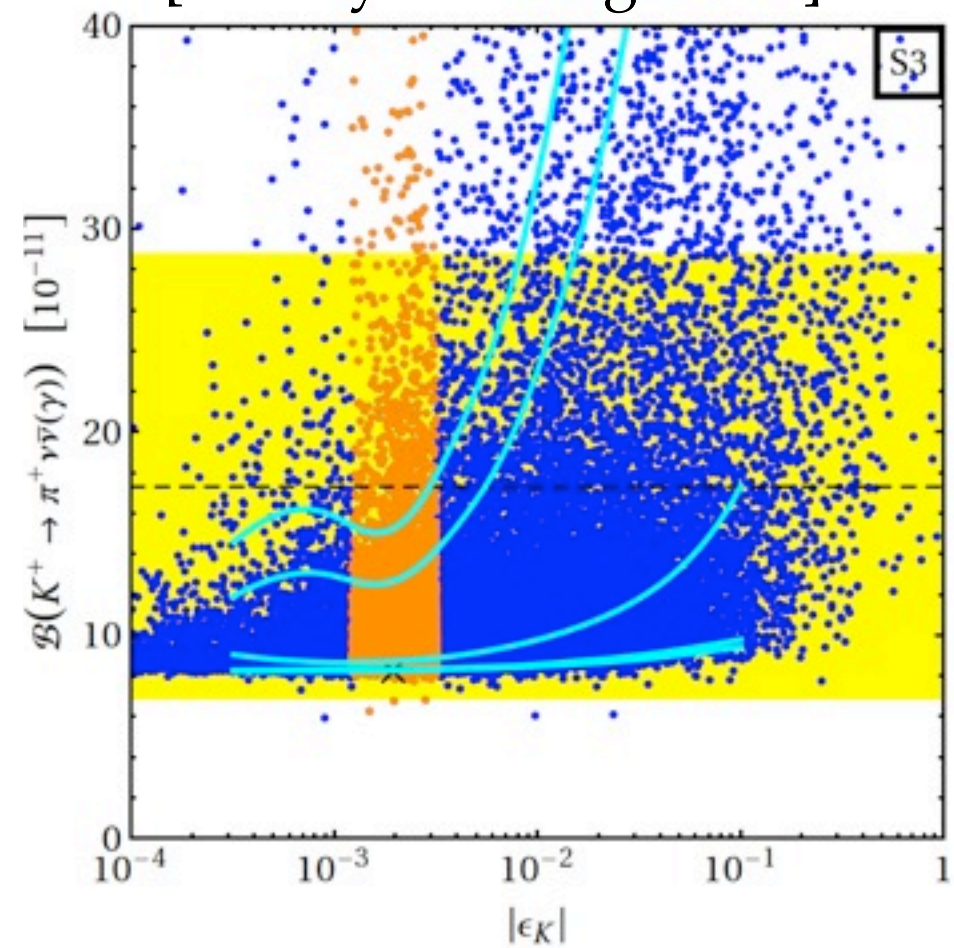
ε_K constraint can still lead to interesting restrictions of the model parameter space for $K^+ \to \pi^+ \bar{\upsilon} \upsilon$

The Higgs production channel puts severe constraints on these type of models

RS with common down-type bulk mass
[Plot by S. Casagrande]



Leptonic and Semileptonic

Observables: K(π) → l ῡ$_l$, K → π l ῡ$_l$

Observables: $K(\pi) \to l\bar{\nu}_l$ & $K \to \pi\, l\bar{\nu}_l$

$$\frac{\Gamma(K^{\pm}_{\ell 2(\gamma)})}{\Gamma(\pi^{\pm}_{\ell 2(\gamma)})} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K}{f_\pi^2 m_\pi} \left(\frac{1 - m_\ell^2/m_K^2}{1 - m_\ell^2/m_\pi^2}\right)^2 \times (1 + \delta_{\rm em})$$

[Marciano `04]

[Cirigliano, Giannotti, Neufeld `08]

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{G_F^2 m_K^5}{192\pi^3} C_K S_{\rm ew} |V_{us}|^2 f_+(0)^2 I_K^\ell(\lambda_{+,0}) \left(1 + \delta^K_{SU(2)} + \delta^{K\ell}_{\rm em}\right)^2$$

Isospin breaking effects: Flavianet `10

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Lattice

[Cirigliano, Giannotti, Neufeld `08]

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Isospin breaking effects:
Flavianet `10

CKM Unitarity

$\Gamma(K_{l3})$ $\quad |V_{us}|f_+(0) = 0.2163(5)$

$\frac{\Gamma(K_{l2})}{\Gamma(\pi_{l2})}$ $\quad \frac{|V_{us}|f_K}{|V_{ud}|f_\pi} = 0.2758(5)$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

3 Equations, 4 Unknowns (V_{us} , V_{us} , $f_+(0)$, f_K/f_π)

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3 Equations, 4 Unknowns (V_{us} , V_{us} , $f_+(0)$, f_K/f_π)

$f_+(0)$ from Lattice gives f_K/f_π V_{us} , V_{ud}

N_F =2+1
MILC 12
JLQCD 12
JLQCD 11
RBC/UKQCD 10
RBC/UKQCD 07

CKM Unitarity

$$\Gamma(K_{l3}) \qquad |V_{us}|f_+(0) = 0.2163(5)$$

$$\frac{\Gamma(K_{l2})}{\Gamma(\pi_{l2})} \qquad \frac{|V_{us}|f_K}{|V_{ud}|f_\pi} = 0.2758(5)$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

3 Equations, 4 Unknowns (V_{us} , V_{us} , $f_+(0)$, f_K/f_π)

$f_+(0)$ from Lattice gives f_K/f_π V_{us} , V_{ud}

N_F =2+1

MILC 12
JLQCD 12
JLQCD 11
RBC/UKQCD 10
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f_K/f_π from Lattice find $f_+(0)$ V_{us} , V_{ud}

N_F =2+1

RBC/UKQCD 12
Laiho 11
MILC 10
JLQCD/TWQCD 10
RBC/UKQCD 10A
.....

N_F =2+1+1

HPQCD 13A
MILC 13A
MILC 11
ETM 10E

CKM Unitarity Test

Test unitarity: $\Delta_{CKM} = |V_{ud}^2| + |V_{us}^2| + |V_{ub}^2| - 1$

Lattice for $N_F = 2+1$

$f_+(0)$ and $f_{K+}/f_{\pi+}$:

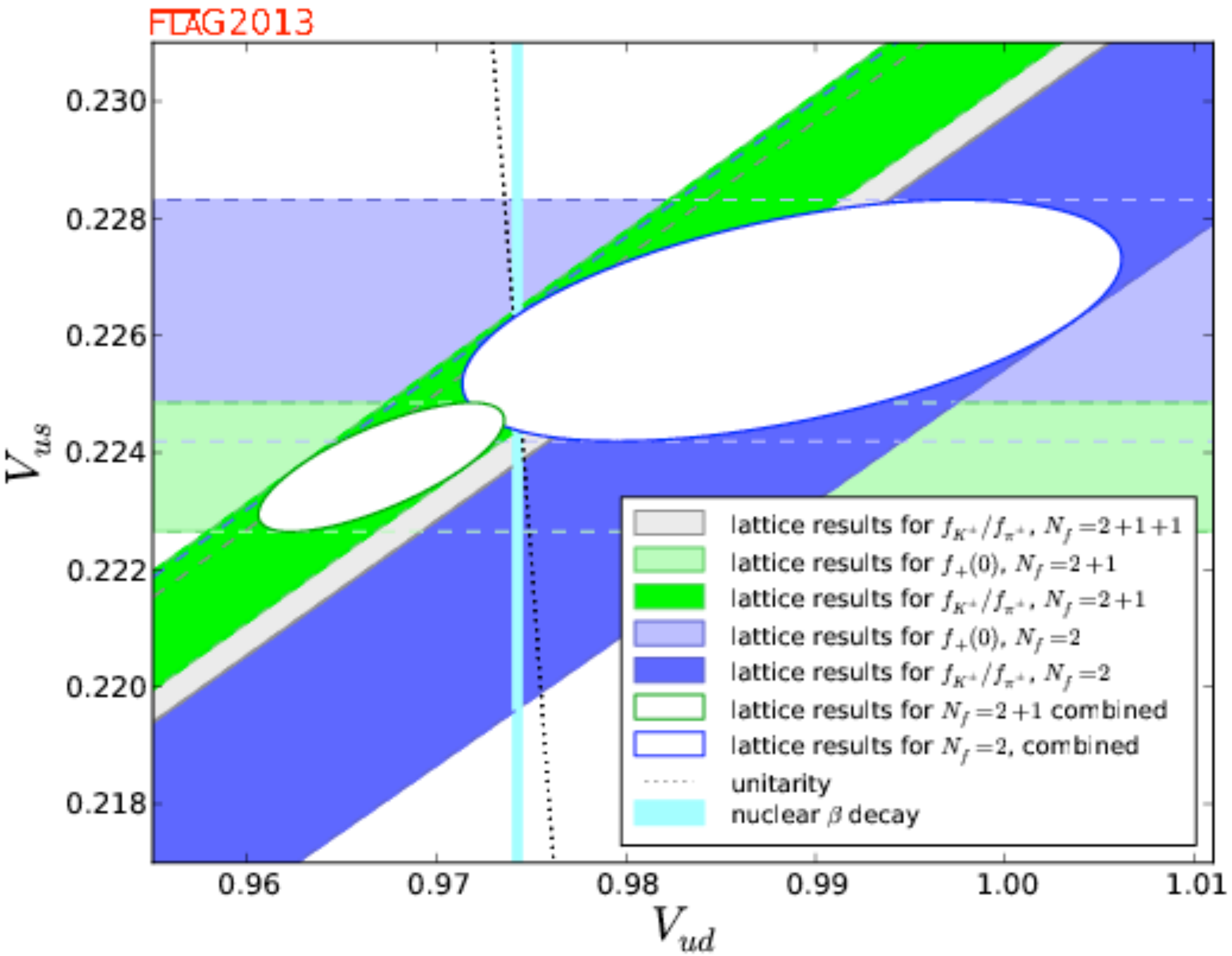
$\Delta_{CKM} = -13(15)\ 10^{-3}$

V_{ud} (n β decay) and $f_+(0)$:

$\Delta_{CKM} = -8(6)\ 10^{-4}$

V_{ud} (n β decay) and $f_{K+}/f_{\pi+}$:

$\Delta_{CKM} = 0(6)\ 10^{-4}$



CKM Unitarity (Model Independent)

[Cirigliano et. al. `09]

$$\Lambda_{NP} \gg M_W \qquad \text{Neglect} \quad \mathcal{O}\left(\frac{M_W}{\Lambda_{NP}}\right) \quad \text{corrections}$$

Use SU(2)⊗U(1) invariant operators [Buchmüller-Wyler `06]

(plus $U(3)^5$ flavour symmetry)

$$O_{lq}^{(3)} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q) \qquad O_{ll}^{(3)} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l)$$

Constrained from EW precision data [Han, Skiba `05]

Redefine

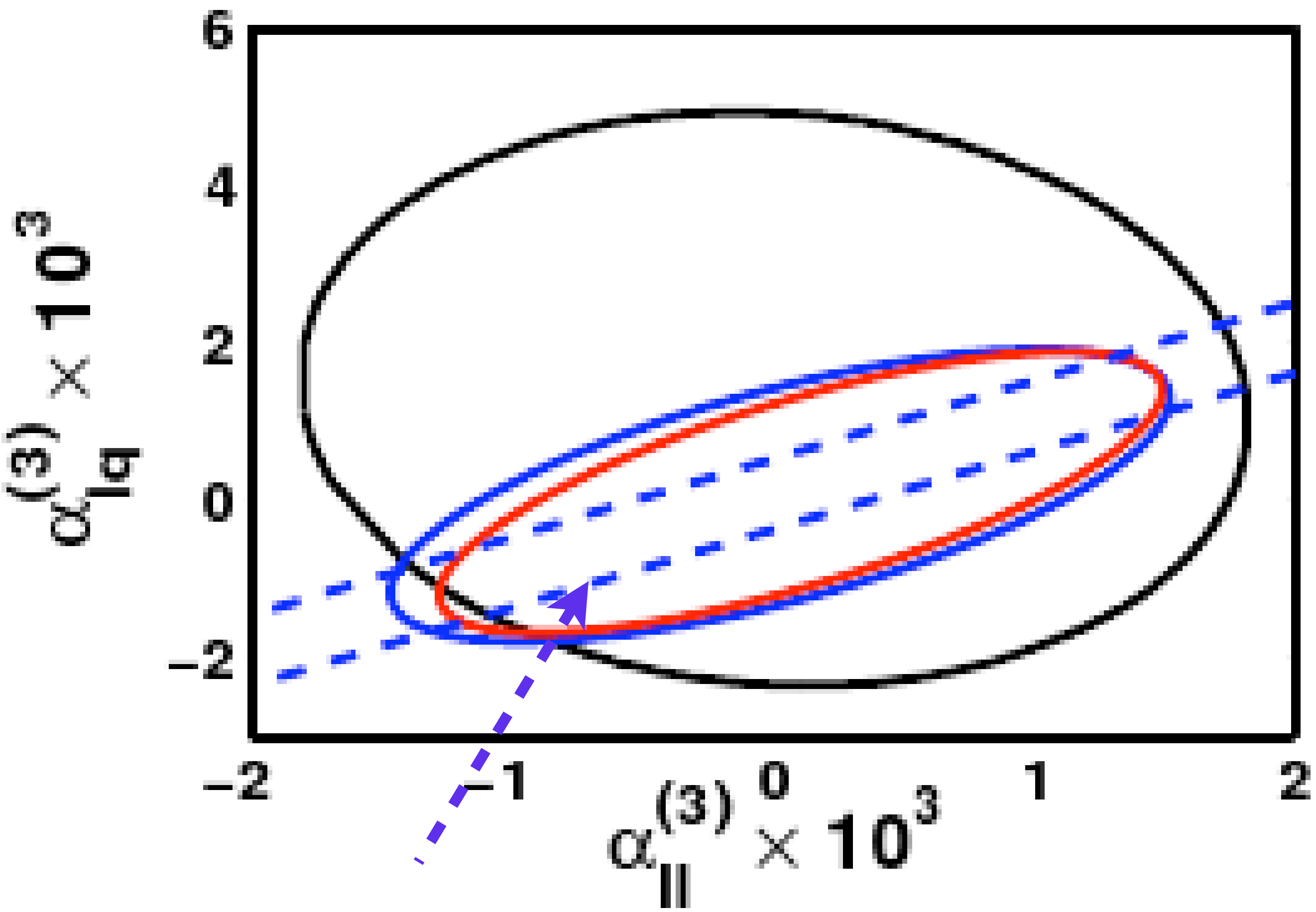
$$G_F(\mu \to e\nu\bar{\nu}) \to G_F(1 - 2\bar{\alpha}_{ll}^{(3)}) \longrightarrow G_F^{\mu}$$

$$G_F(d \to u e\bar{\nu}) \to G_F(1 - 2\bar{\alpha}_{lq}^{(3)}) \longrightarrow G_F^{SL}$$

CKM Unitarity (Model Independent)

$$V_{ud_i}^{PDG} = \frac{G_F^{SL}}{G_F^{\mu}} V_{ud_i} \longrightarrow \Delta_{CKM} = 4\left(\bar{\alpha}_{ll}^{(3)} - \bar{\alpha}_{lq}^{(3)} + \ldots\right)$$

[Cirigliano et. al. `09]



CKM
Unitarity

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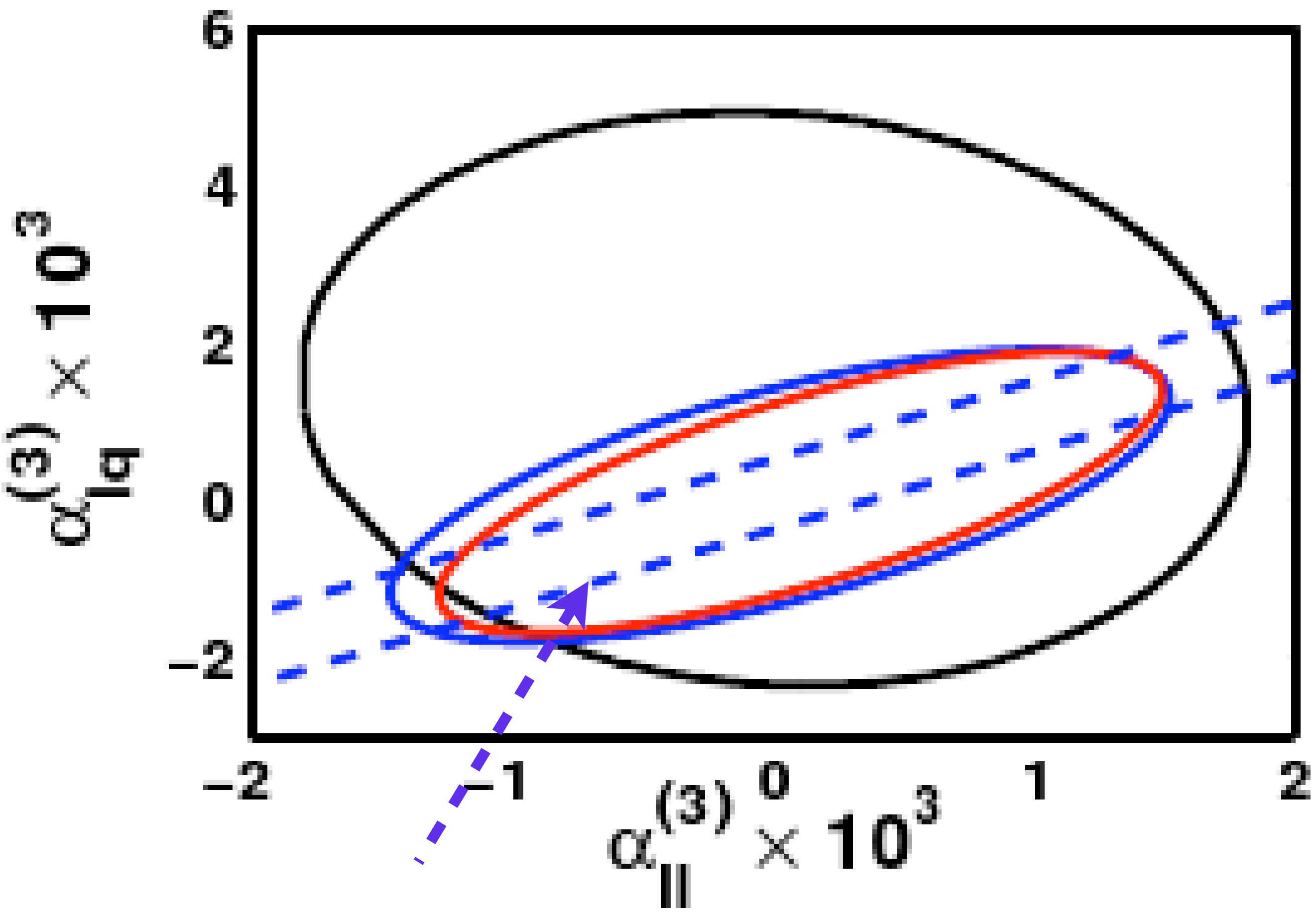

CKM Unitarity

$$G_F = \frac{2\sqrt{2}\pi\alpha}{M_Z^2 \sin^2 2\theta_W (1 - \Delta\hat{r})}$$

Before M_H measurement

Δr dominates uncertainties

CKM Unitarity (Model Independent)

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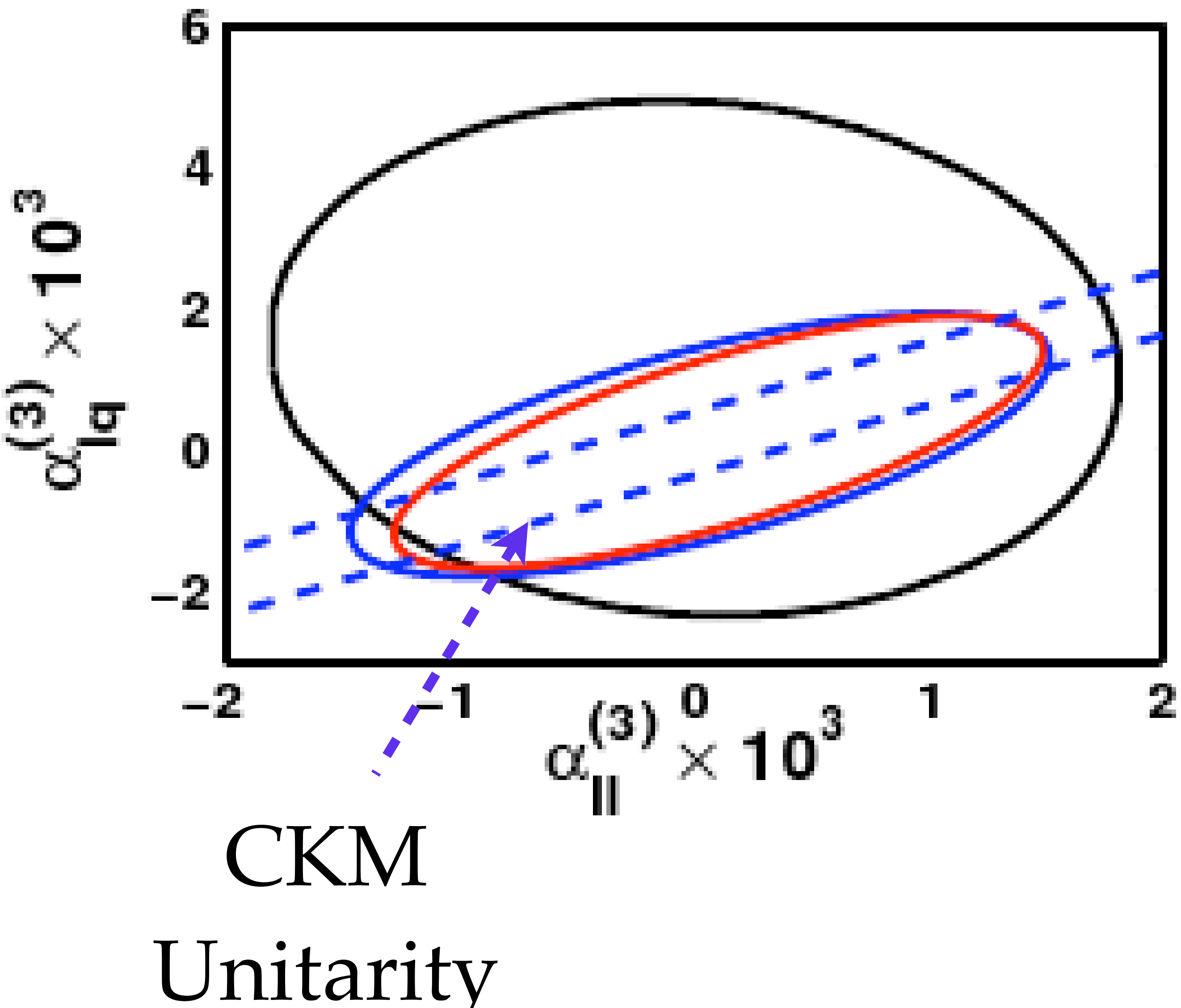

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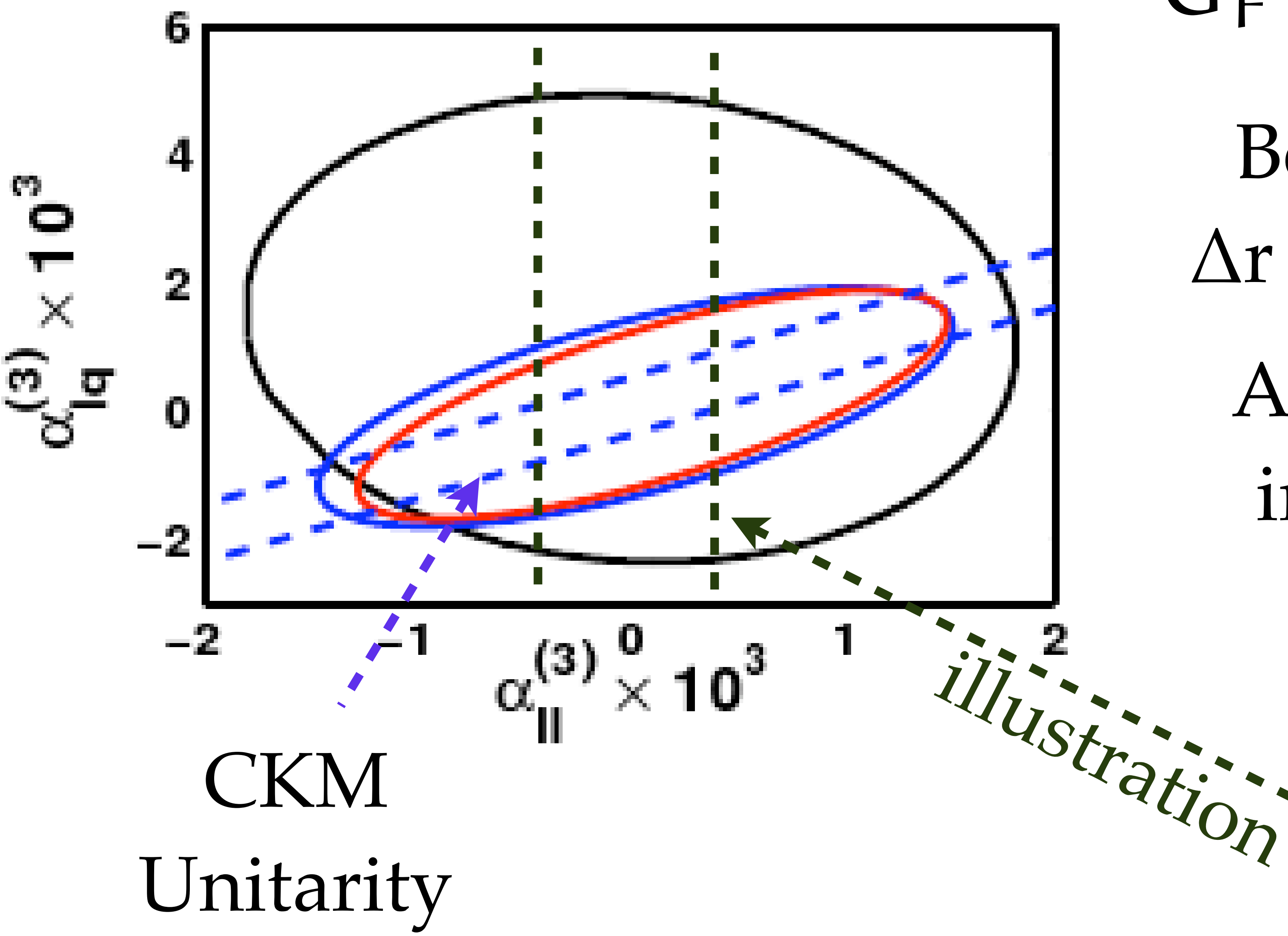

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Comparable to unitarity constraints

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On the theory side progress is expected from Lattice QCD on the calculation of previously sub-leading non-perturbative effects. This will also include a matching of the Lattice results to the continuum.

The upcoming searches for rare Kaon decays will test so far unconstrained parameter space of new physics.