UNIVERSITY OF Southampton
School of Physics and Astronomy

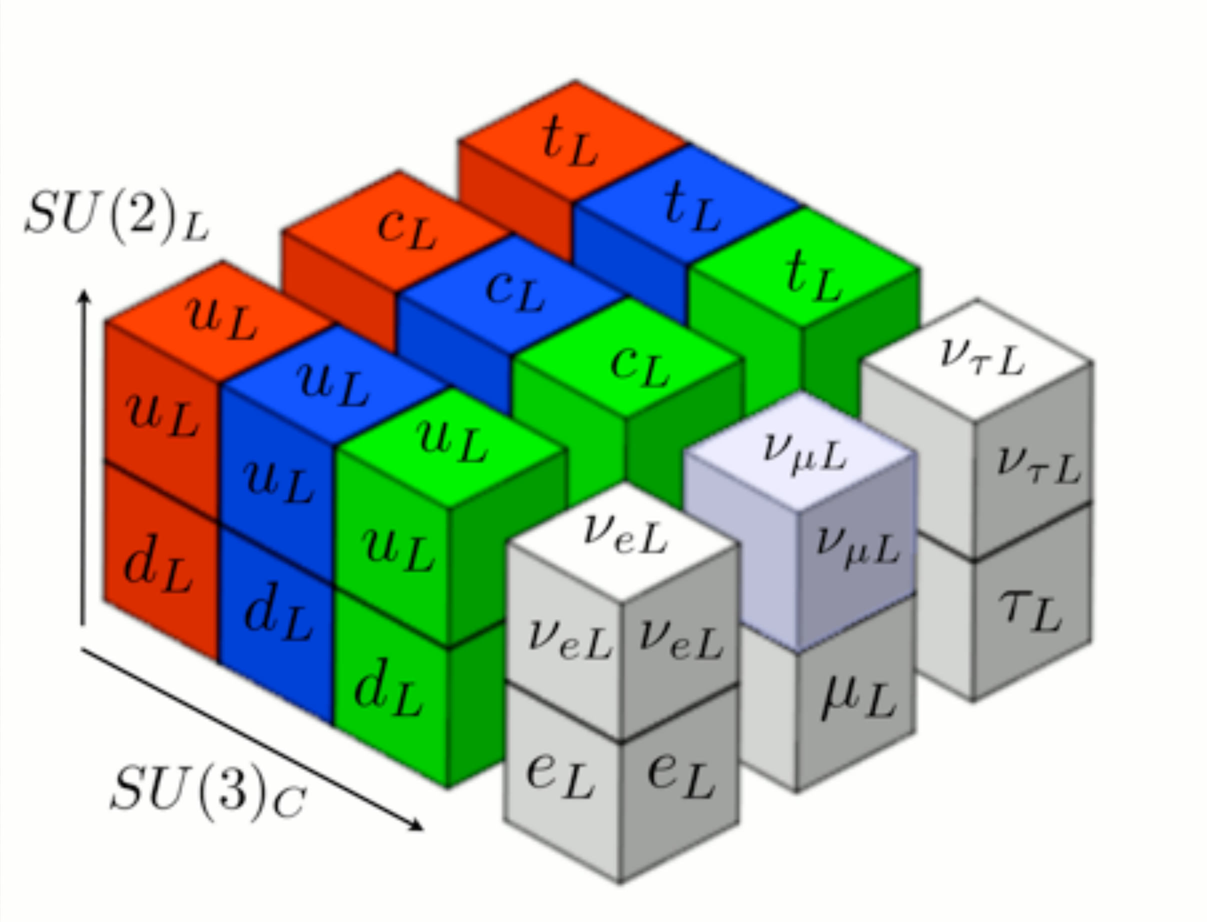
Models of mixing in the leptonic (and quark) sector

Steve King, Cosener's House, Abingdon, UK HEP forum on "Quarks and Leptons" 14–15 November 2013

Standard Model of Quarks and Leptons

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Left-handed
quarks and leptons



Right-handed
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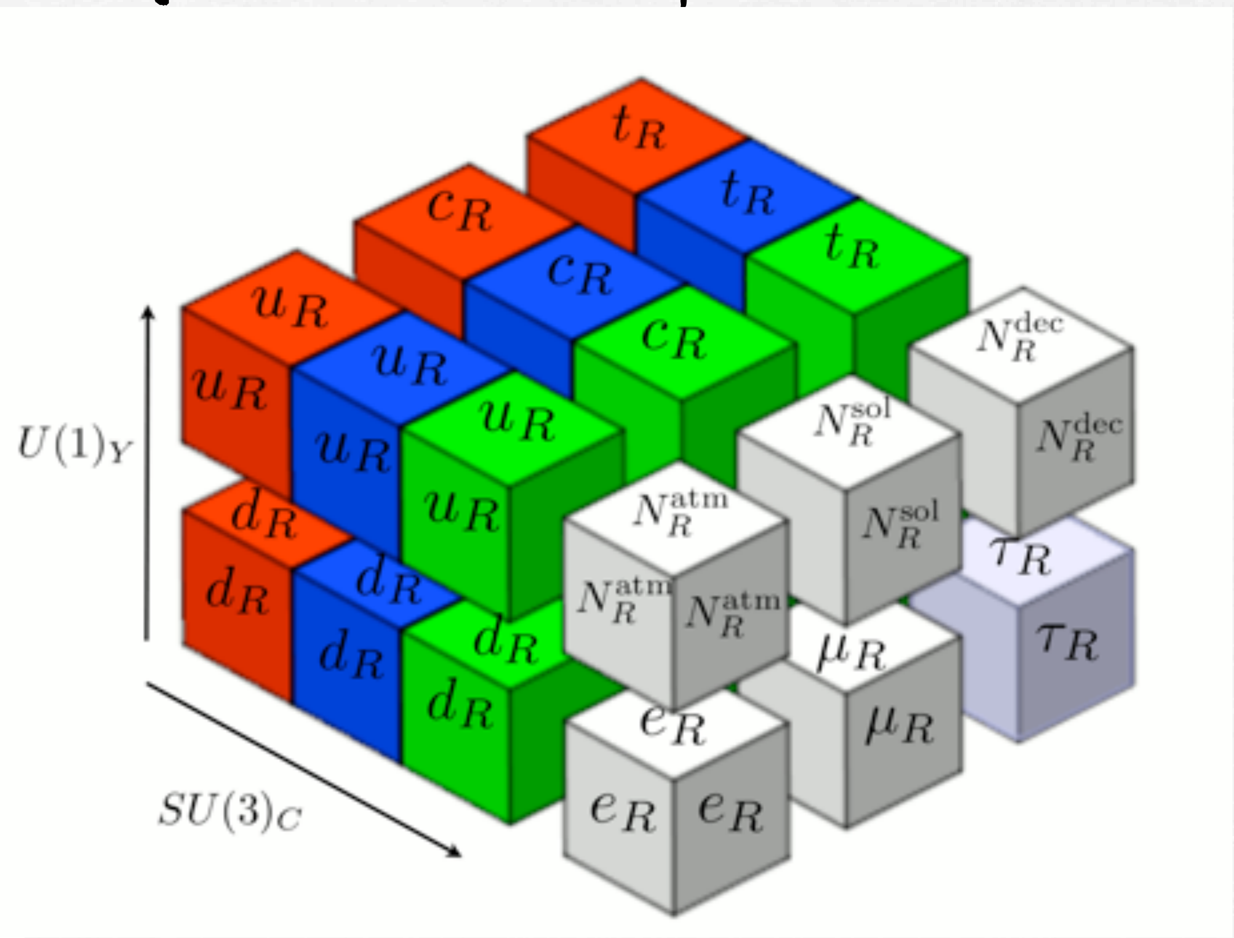
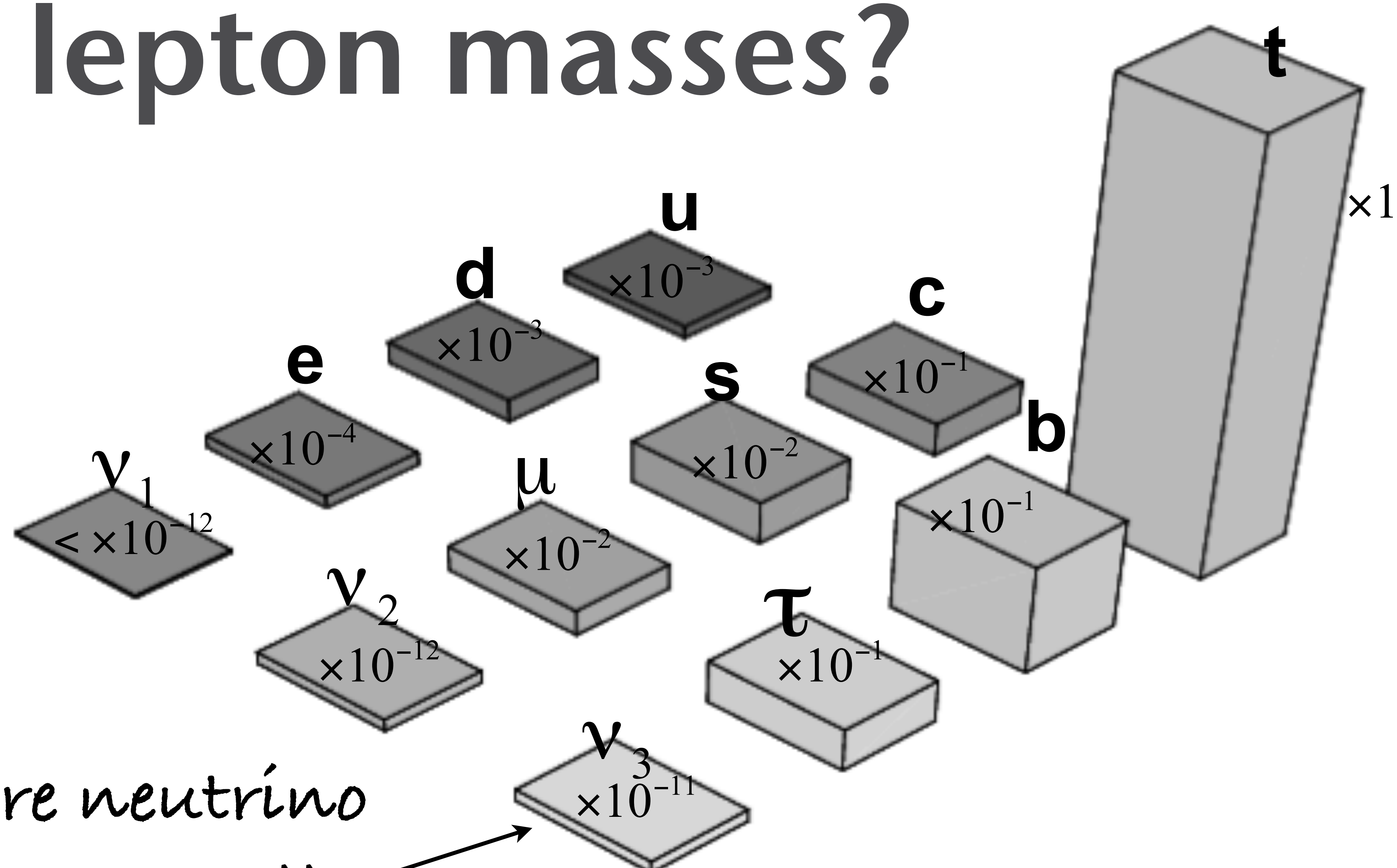


FermionMasses

The Flavour Problem

What is the origin of quark and lepton masses?



Why are neutrino masses so small?

The Flavour Problem

Why is quark mixing so small?

Cabibbo
Kobayashi
Maskawa

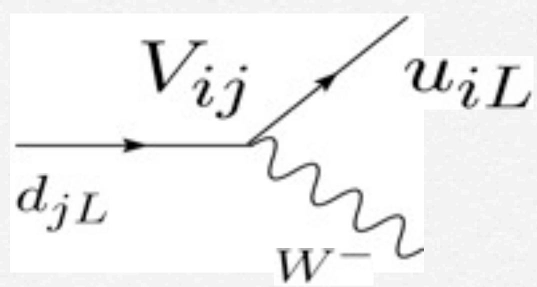
$$U_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

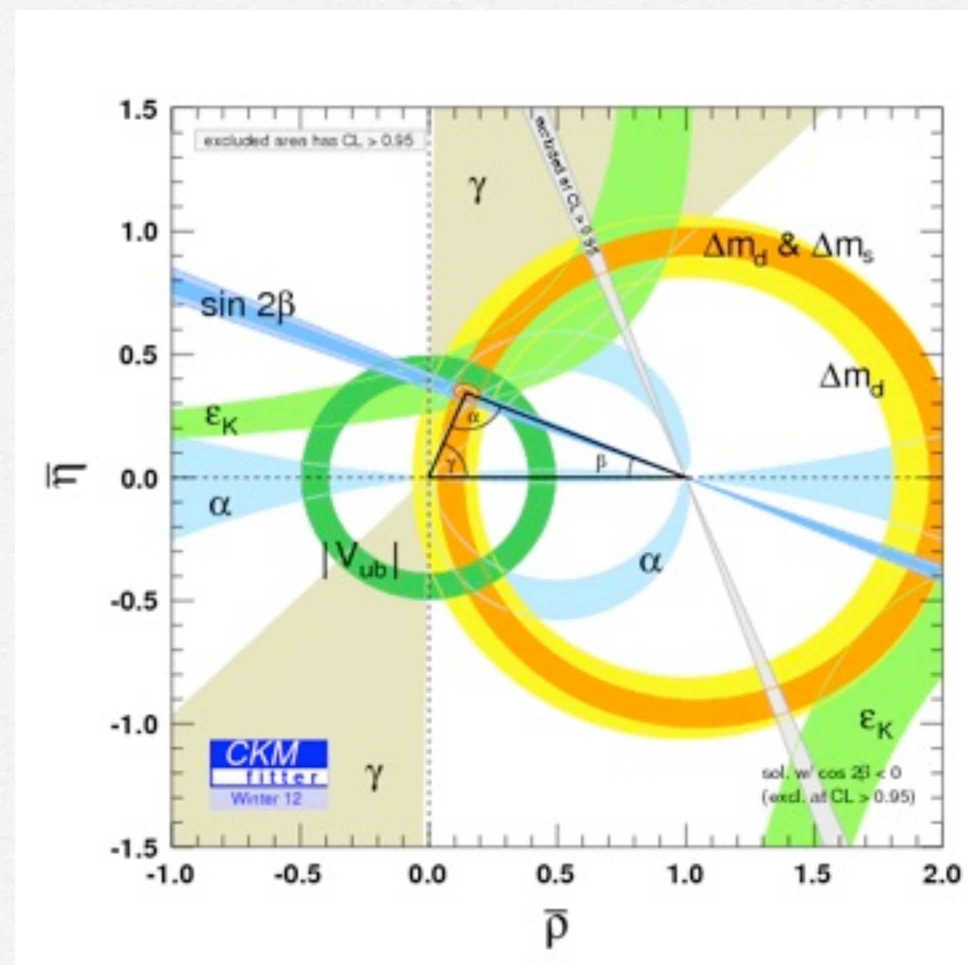
$\theta_{12} = 13^\circ \pm 0.1^\circ$

$\theta_{23} = 2.4^\circ \pm 0.1^\circ$

$\theta_{13} = 0.20^\circ \pm 0.05^\circ$

$\delta_{CP} \approx 70^\circ \pm 5^\circ$





The Flavour Problem

Why is lepton mixing so large?

Standard Model states

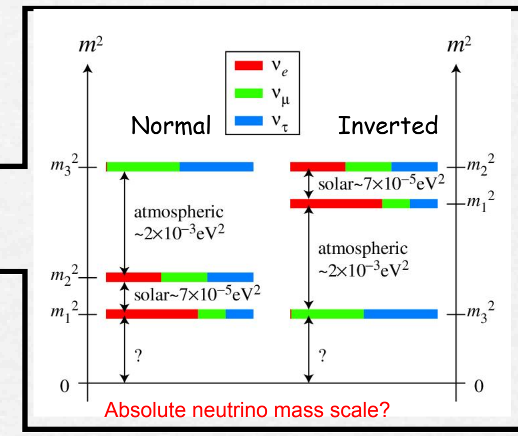
$$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau \\ \tau^-\end{pmatrix}_L$$

PMNS matrix

Neutrino mass states

Pontecorvo
Maki
Nakagawa
Sakata

$$\begin{pmatrix}\nu_e \\ \nu_\mu \\ \nu_\tau\end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix} \begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3\end{pmatrix}$$



$$U_{PMNS} = \begin{pmatrix}1 & 0 & 0 \\ 0 & c^l_{23} & s^l_{23} \\ 0 & -s^l_{23} & c^l_{23}\end{pmatrix} \begin{pmatrix} c^l_{13} & 0 & s^l_{13}e^{-i\delta^l} \\ 0 & 1 & 0 \\ -s^l_{13}e^{i\delta^l} & 0 & c^l_{13}\end{pmatrix} \begin{pmatrix} c^l_{12} & s^l_{12} & 0 \\ -s^l_{12} & c^l_{12} & 0 \\ 0 & 0 & 1\end{pmatrix} \begin{pmatrix}1 & 0 & 0 \\ 0 & \frac{\alpha_{21}}{2} & 0 \\ 0 & 0 & \frac{\alpha_{31}}{2}\end{pmatrix}$$

$s^l_{ij} = \sin(\theta^l_{ij})$

$c^l_{ij} = \cos(\theta^l_{ij})$

Atmospheric | Reactor | Solar | Majorana

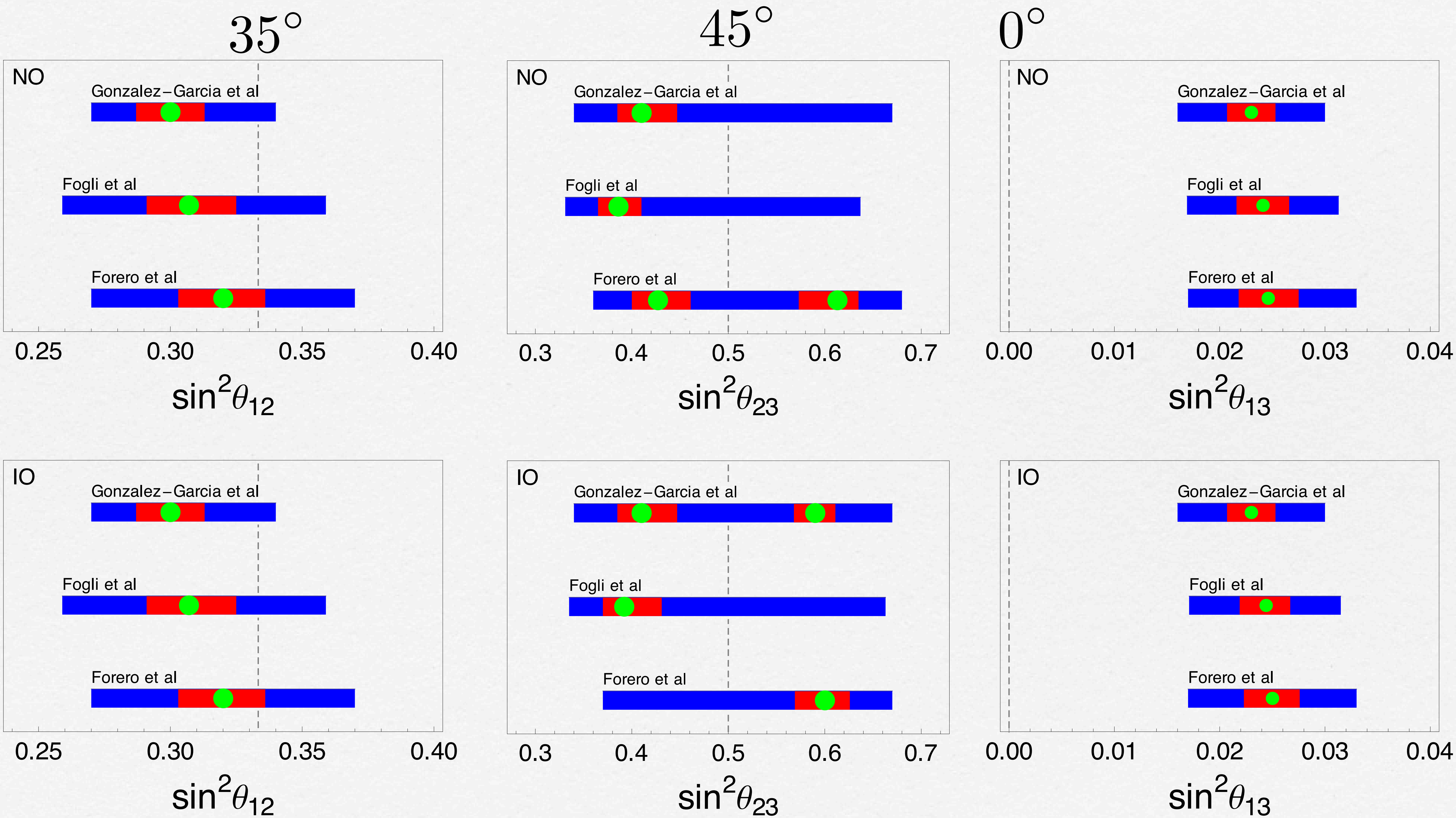
Oscillation phase δ^l

Majorana phases α_{21}, α_{31}

3 masses + 3 angles + 1 (or 3) phase(s)
= 7 (or 9) new parameters for SM

Global Fits 2012

SFK and C.Luhn,
``Neutrino Mass and Mixing with Discrete Symmetry,"
arXiv:1301.1340

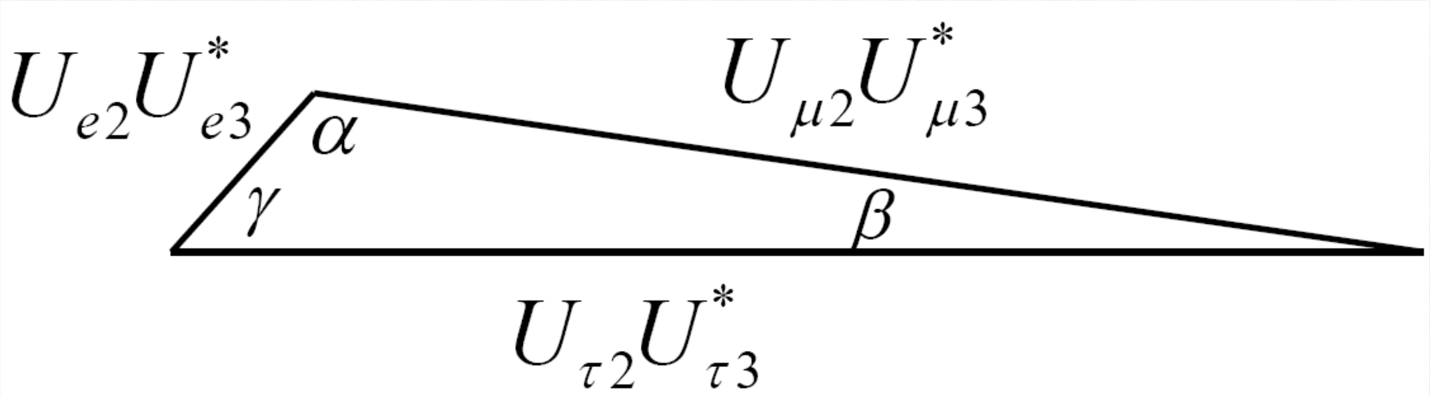


2013 Update from NuFIT

NuFIT 1.2 (2013)

	Free Fluxes + RSBL		Huber Fluxes, no RSBL	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.346$	$0.313^{+0.013}_{-0.012}$	$0.277 \to 0.355$
$\theta_{12}/^\circ$	$33.57^{+0.77}_{-0.75}$	$31.38 \to 36.01$	$34.02^{+0.80}_{-0.76}$	$31.78 \to 36.55$
$\sin^2\theta_{23}$	$0.446^{+0.007}_{-0.007} \oplus 0.587^{+0.032}_{-0.037}$	$0.366 \to 0.663$	$0.444^{+0.036}_{-0.031} \oplus 0.592^{+0.028}_{-0.042}$	$0.361 \to 0.665$
$\theta_{23}/^\circ$	$41.9^{+0.4}_{-0.4} \oplus 50.0^{+1.9}_{-2.2}$	$37.2 \to 54.5$	$41.8^{+2.1}_{-1.8} \oplus 50.3^{+1.7}_{-2.4}$	$36.9 \to 54.6$
$\sin^2\theta_{13}$	$0.0229^{+0.0020}_{-0.0019}$	$0.0170 \to 0.0288$	$0.0244^{+0.0020}_{-0.0019}$	$0.0184 \to 0.0305$
$\theta_{13}/^\circ$	$8.71^{+0.37}_{-0.38}$	$7.50 \to 9.78$	$8.99^{+0.36}_{-0.37}$	$7.80 \to 10.05$
$\delta_{\mathrm{CP}}/^\circ$	265^{+56}_{-61}	$0 \to 360$	270^{+77}_{-67}	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.45^{+0.19}_{-0.16}$	$6.98 \to 8.05$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.08$
$\frac{\Delta m^2_{31}}{10^{-3}\ \mathrm{eV}^2}$ (N)	$+2.417^{+0.013}_{-0.013}$	$+2.247 \to +2.623$	$+2.429^{+0.055}_{-0.054}$	$+2.249 \to +2.639$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$ (I)	$-2.410^{+0.062}_{-0.062}$	$-2.602 \to -2.226$	$-2.421^{+0.063}_{-0.061}$	$-2.614 \to -2.235$

Possible leptonic unitarity triangles

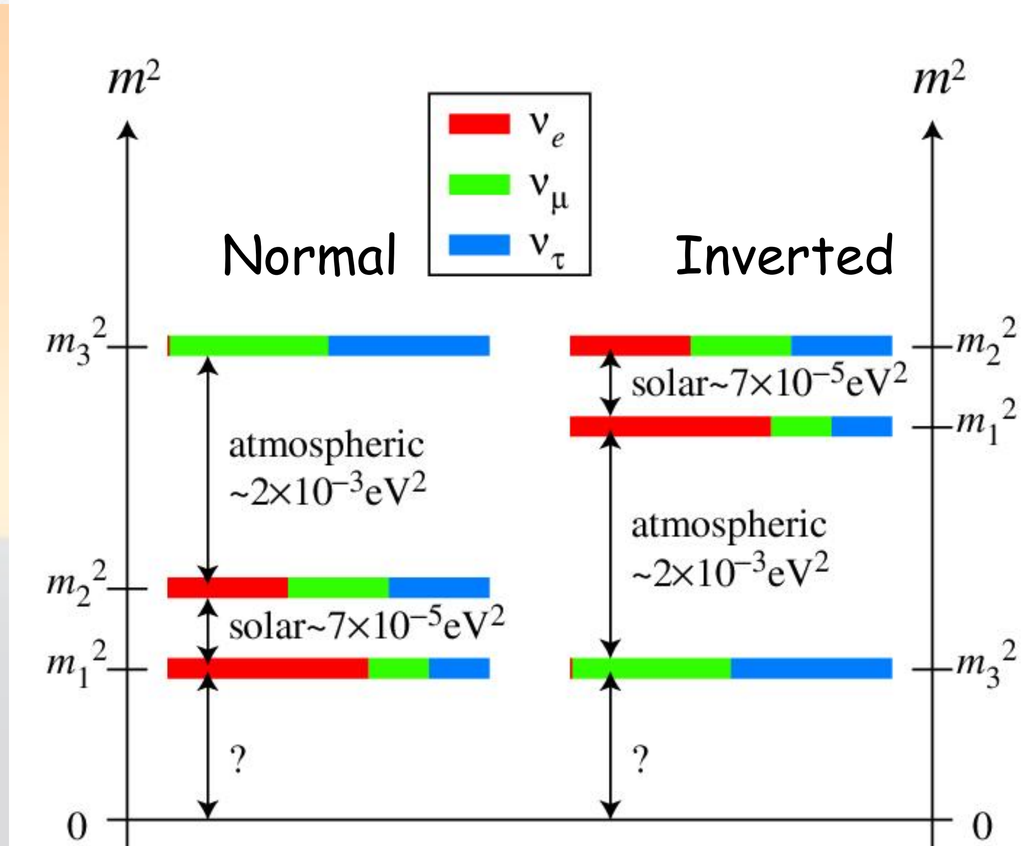


or

?!

We simply have no idea...

Neutrino Mass Questions



Absolute neutrino mass scale?

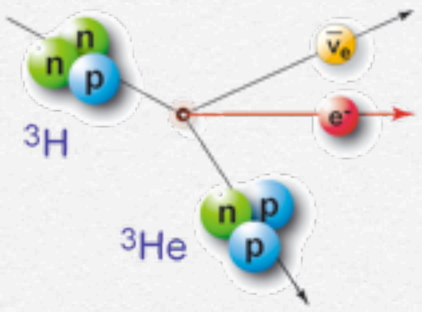
- ☐ What is the mass squared ordering (normal or inverted) ?
- ☐ What is the neutrino mass scale (mass of lightest neutrino)?
- ☐ What is the nature of neutrino mass (i.e. Dirac or Majorana)?

⟷ **Origin of neutrino mass?**

How we can learn about neutrino mass

$\beta\beta_{0\nu}$	Δm^2_{13}	KATRIN	Conclusion
yes	> 0	yes	Degenerate, Majorana
yes	> 0	No	Degenerate, Majorana
			or normal, Majorana with heavy particle contribution
yes	< 0	no	Inverted, Majorana
yes	< 0	yes	Degenerate, Majorana
no	> 0	no	Normal, Dirac or Majorana
no	< 0	no	Dirac
no	< 0	yes	Dirac
no	> 0	yes	Dirac

Tritium beta decay



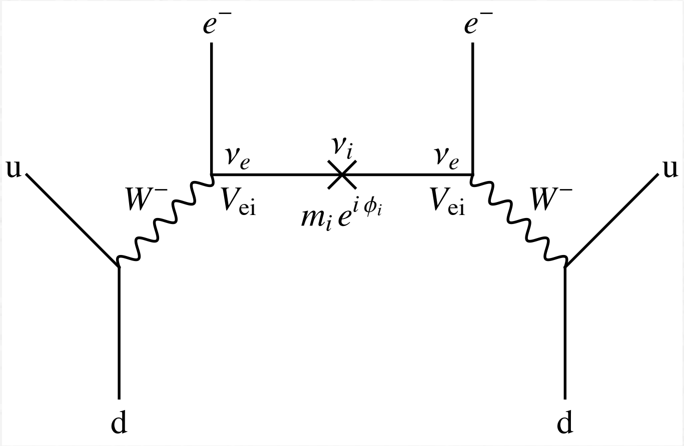
$$|m_{\nu_e}|^2 = \sum_i |U_{ei}|^2 |m_i|^2$$

Present Mainz	< 2.2 eV
KATRIN	~0.35eV

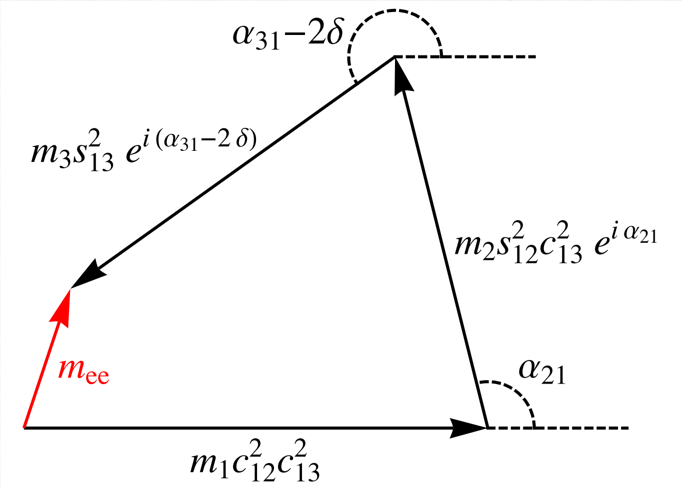
Neutrinoless double beta decay

Majorana only (no signal if Dirac)

$$|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$$



$$|m_{ee}|_{\rm PDG} = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31}-2\delta)}|$$



Neutrino Mass Sum Rules

SFK, Merle, Stuart

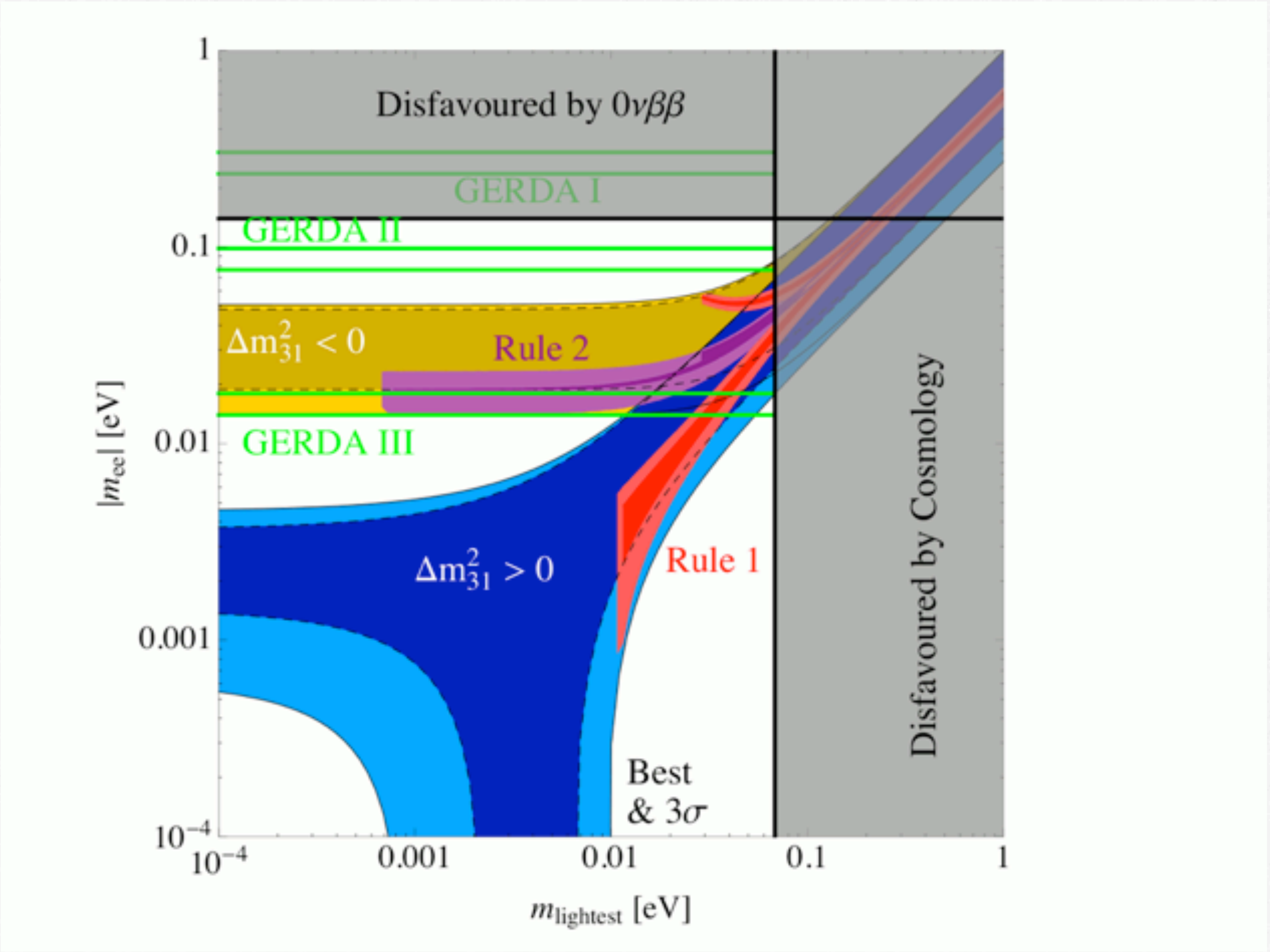
Rule 1

$$\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m_3}$$

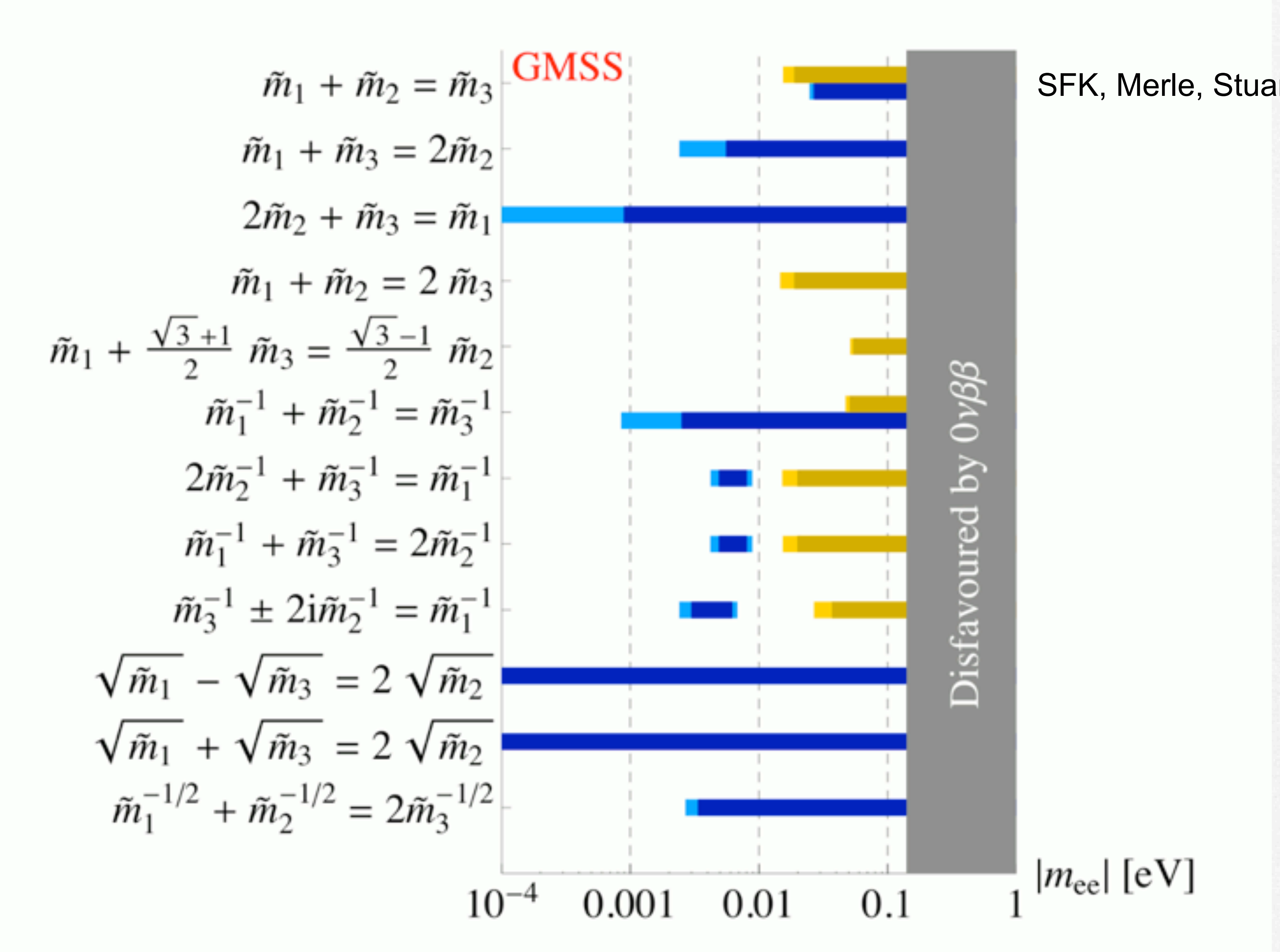
Rule 2

$$m_1 + m_2 = m_3$$

Give restricted regions

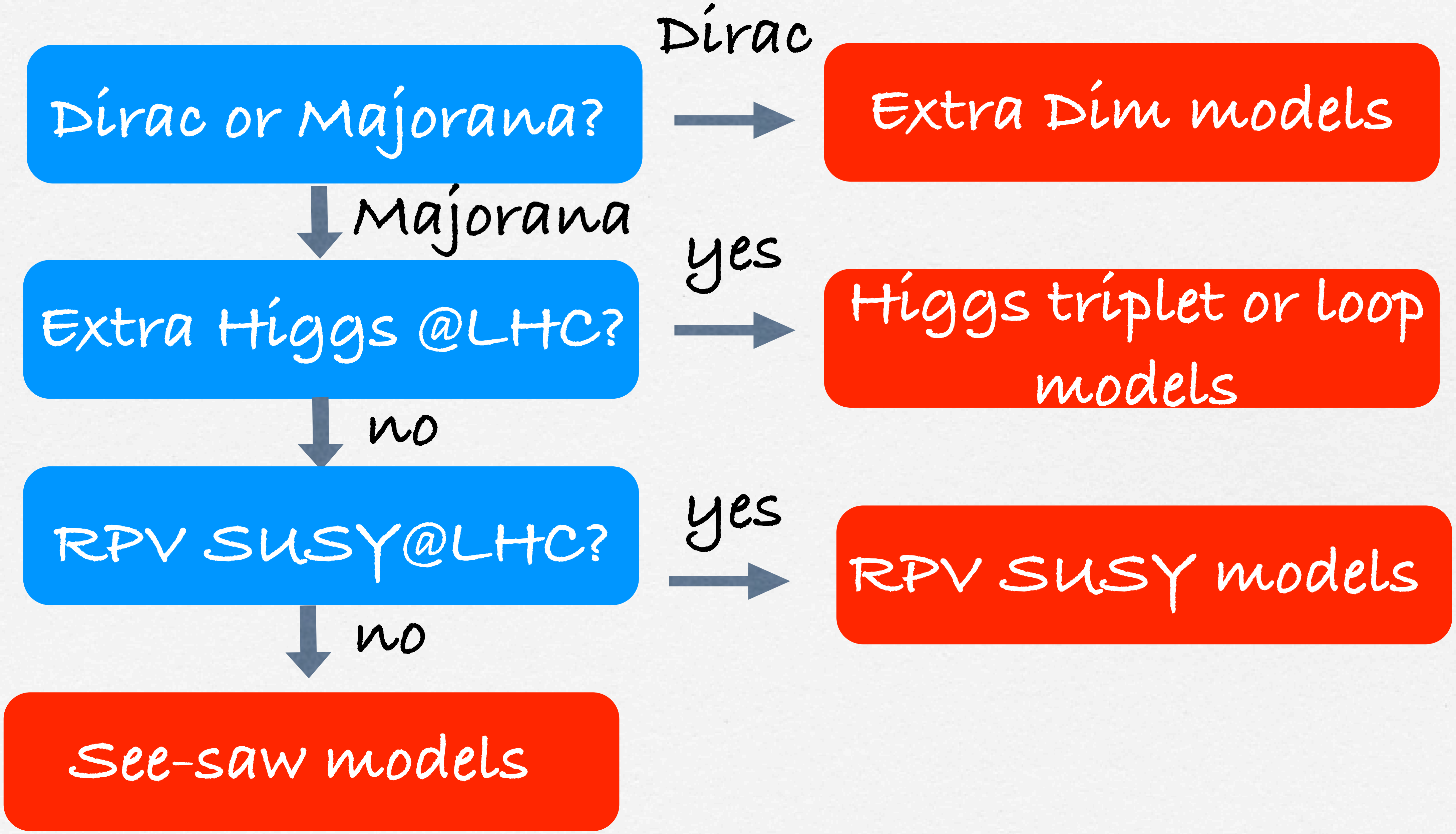


Predictions of sum rules



SFK, Merle, Stuart

Neutrino mass roadmap



See-saw mechanisms

Possible type II contribution

Dirac matrix

$$\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\begin{pmatrix}0 & m_{LR}\\ m_{LR}^T & M_{RR}\end{pmatrix}\begin{pmatrix}\nu_L^c\\ \nu_R\end{pmatrix}$$

$$M^{\nu} = m_{LR}.\frac{1}{M_{RR}}.m_{LR}^T$$

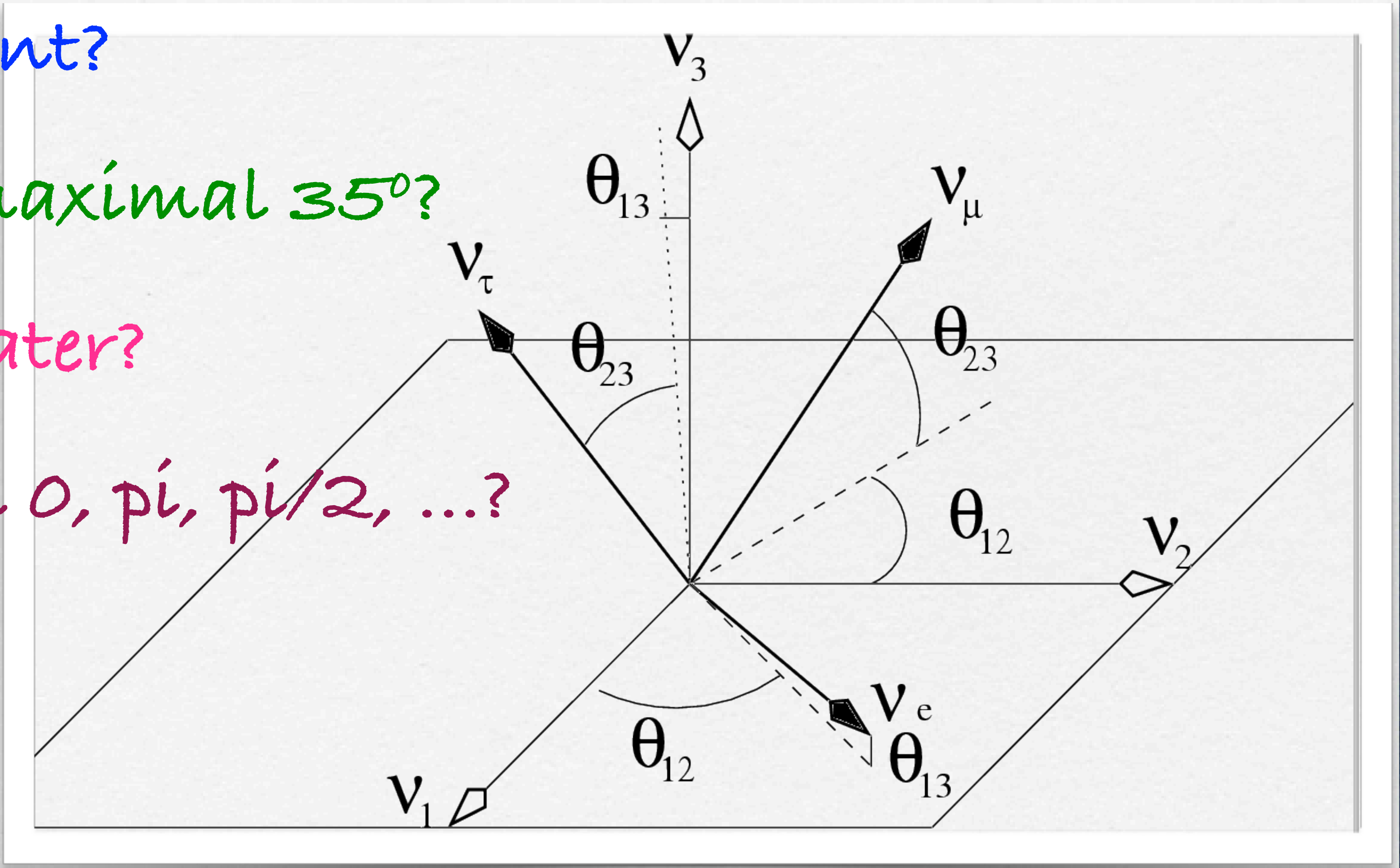
Light Majorana matrix

Heavy Majorana matrix

- Neutrinos are light because RH neutrinos are heavy
- No explanation of neutrino mixing without further ingredients

Neutrino Mixing Questions

- ☐ Is the atmospheric angle maximal 45°?
- ☐ If not then which octant?
- ☐ Is the solar angle trimaximal 35°?
- ☐ If not then less or greater?
- ☐ Is the CP phase special 0, pi, pi/2, ...?
- ☐ If not then what is it?



↔ Origin of neutrino mixing?

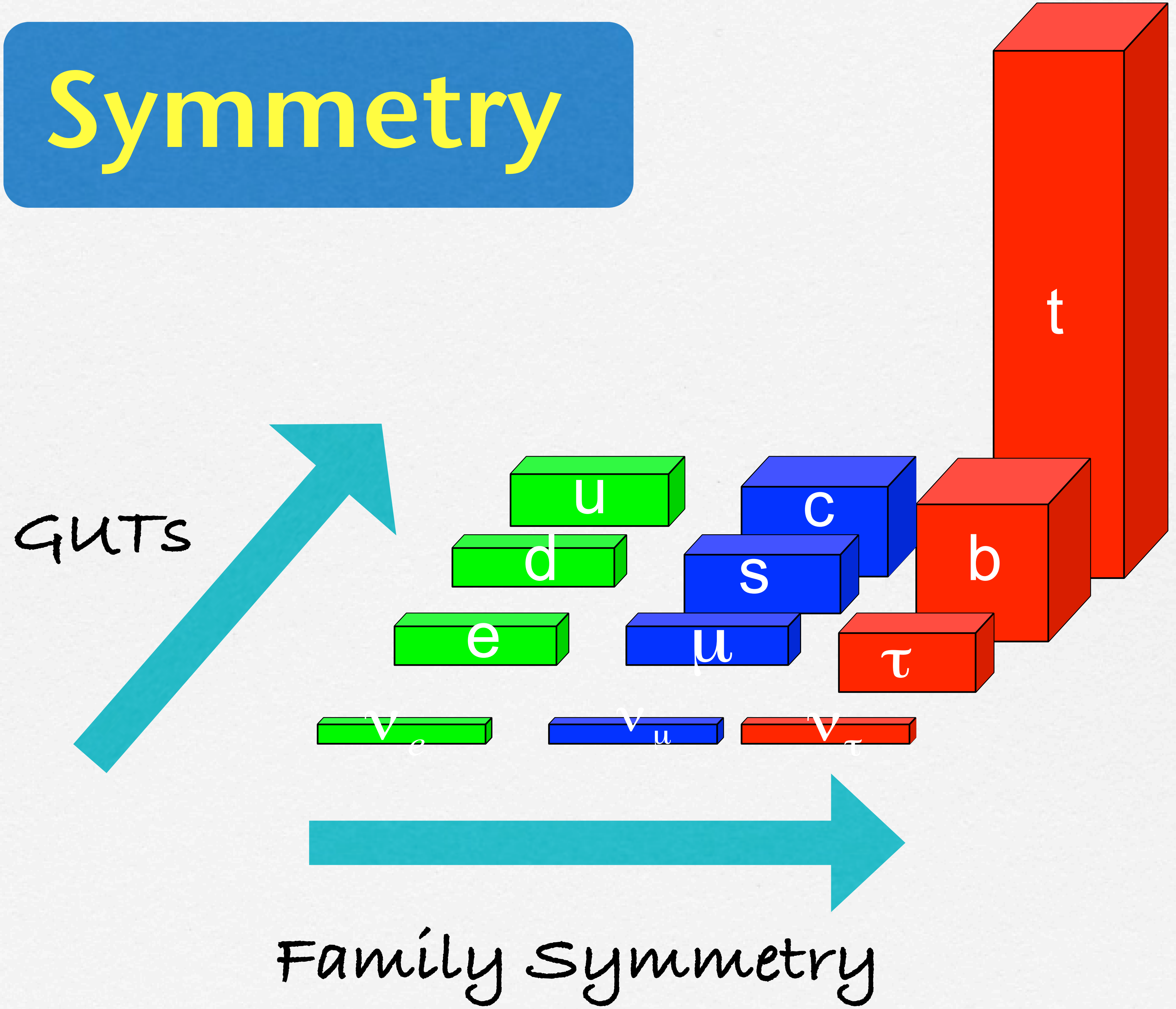
Origin of neutrino mixing

Daya Bay/RENO

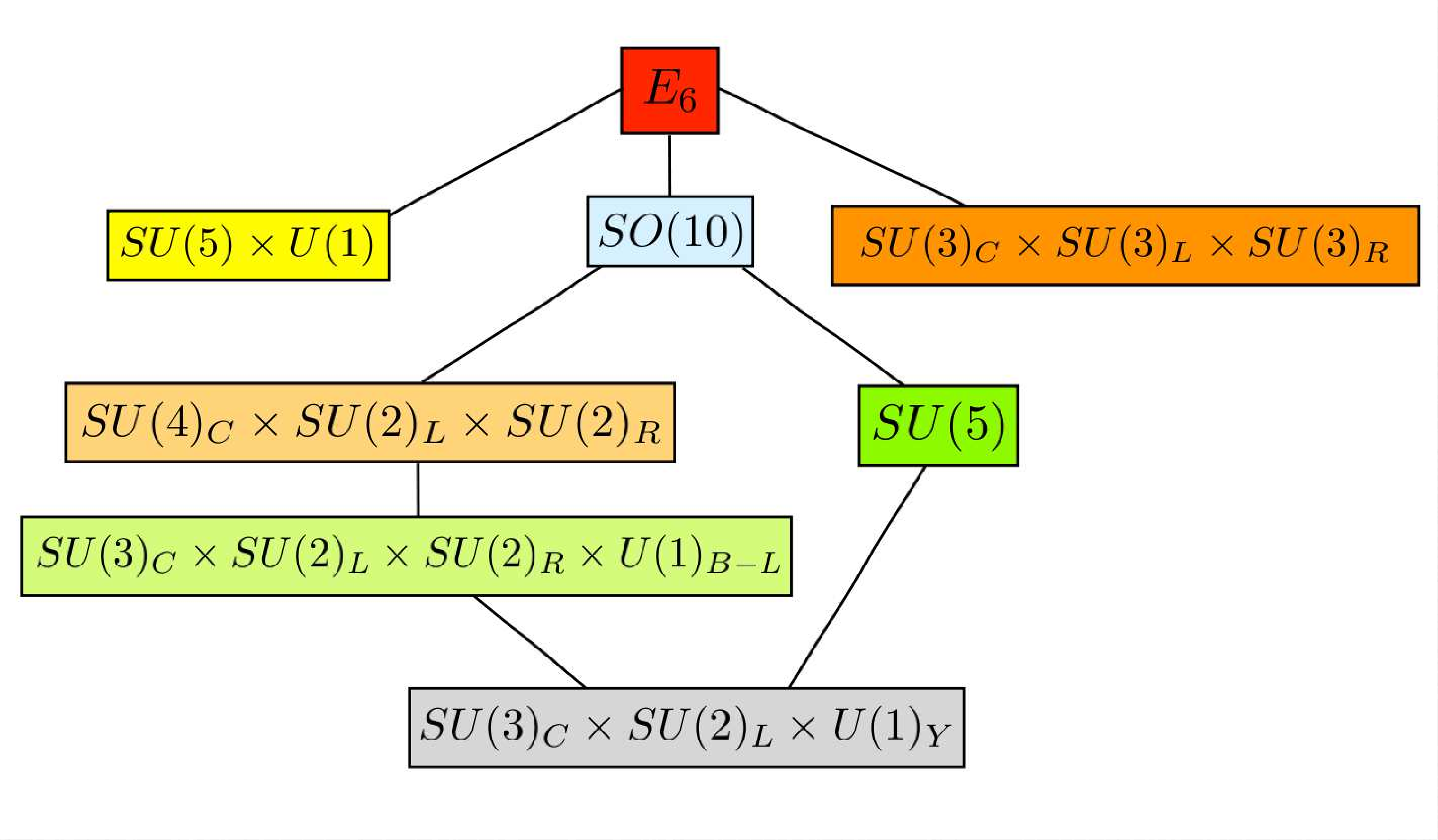
↙ ↘

Symmetry | **Anarchy**

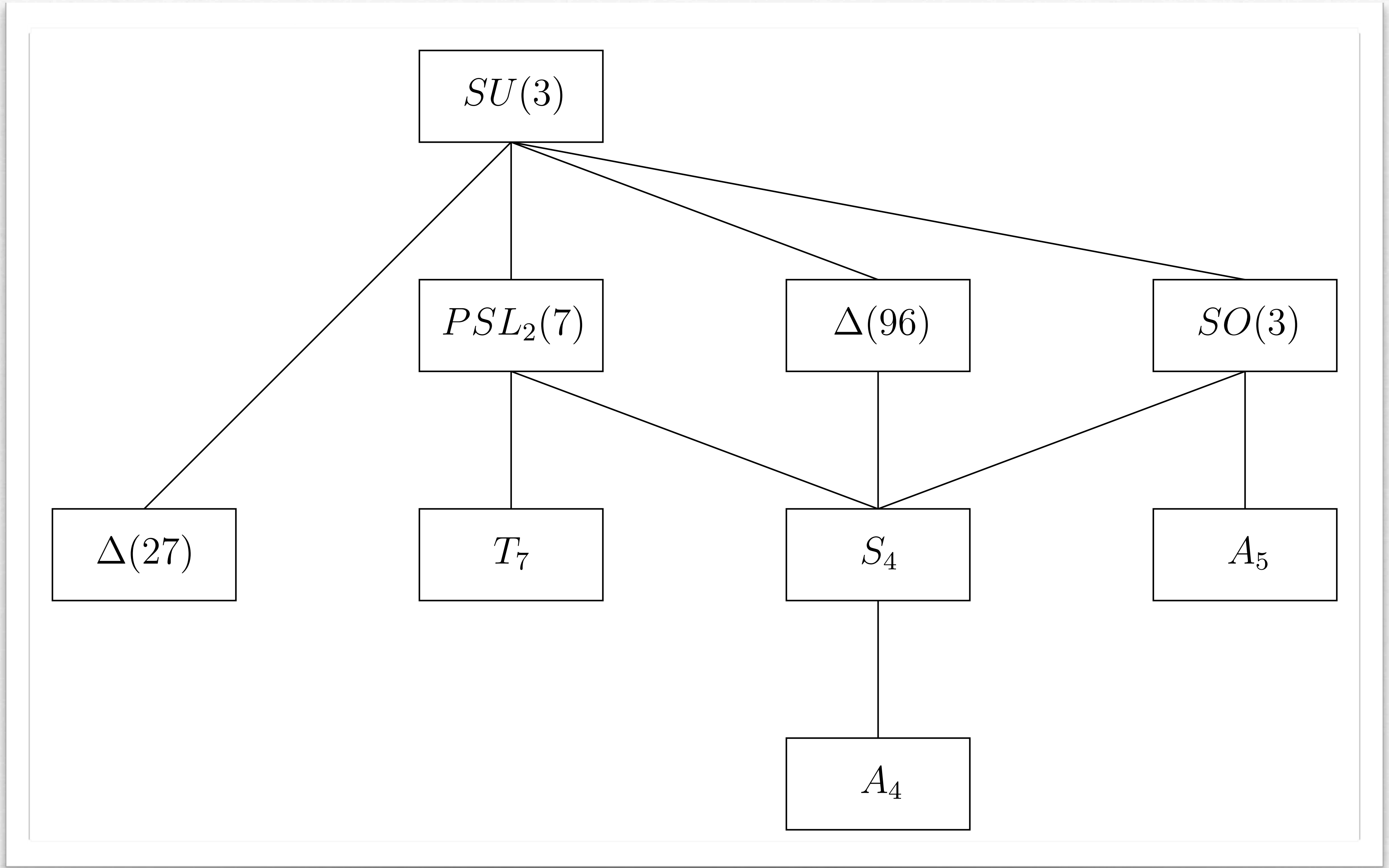
Symmetry



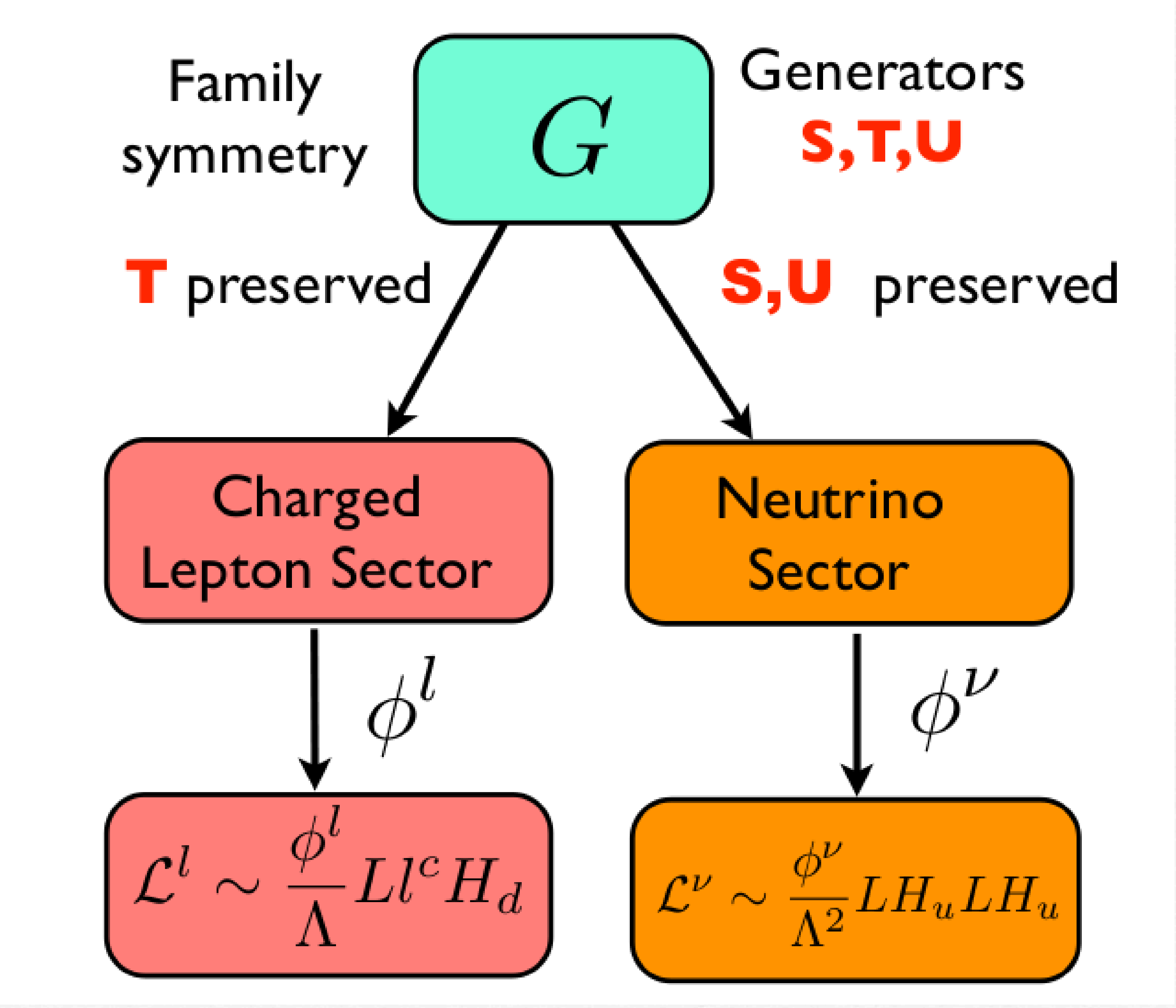
GUTs are based on continuous gauge groups



Family Symmetry may be continous or discrete



Direct Models



Direct models can give simple mixing patterns (now excluded)

- Bimaximal

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 45^o$$

- Tri-bimaximal

Harrison, Perkins, Scott

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 35.26^o$$

- Golden ratio

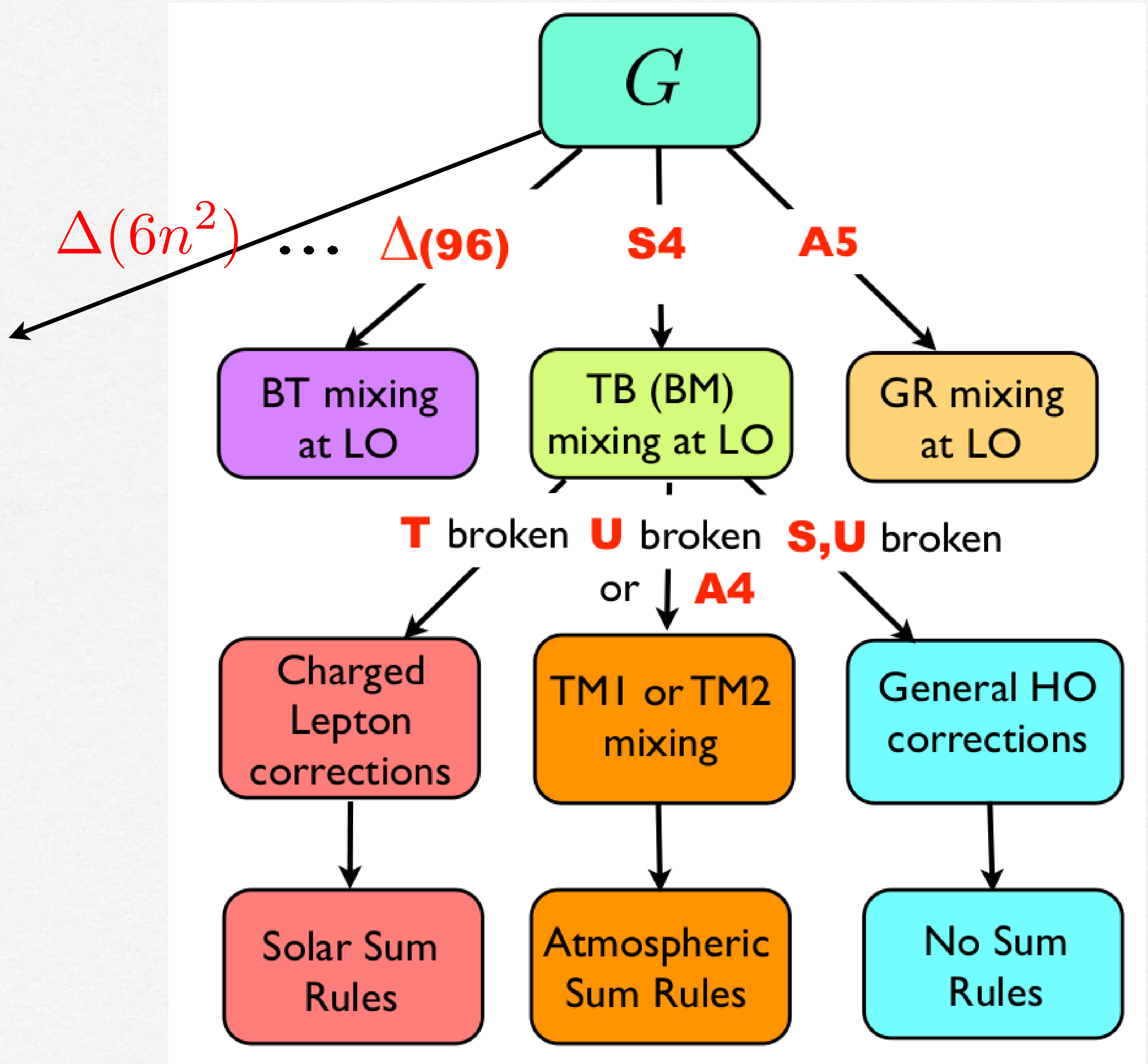
Kajirama, Raidal, Strumia; Everett, Stuart

$$\phi = \frac{1+\sqrt{5}}{2}$$

$$U_{BM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

$$\tan\theta_{12} = \frac{1}{\phi} \qquad \theta_{12} = 31.7^o$$

Direct Model Building



Example of TM2 mixing

$\Delta(6n^2)$

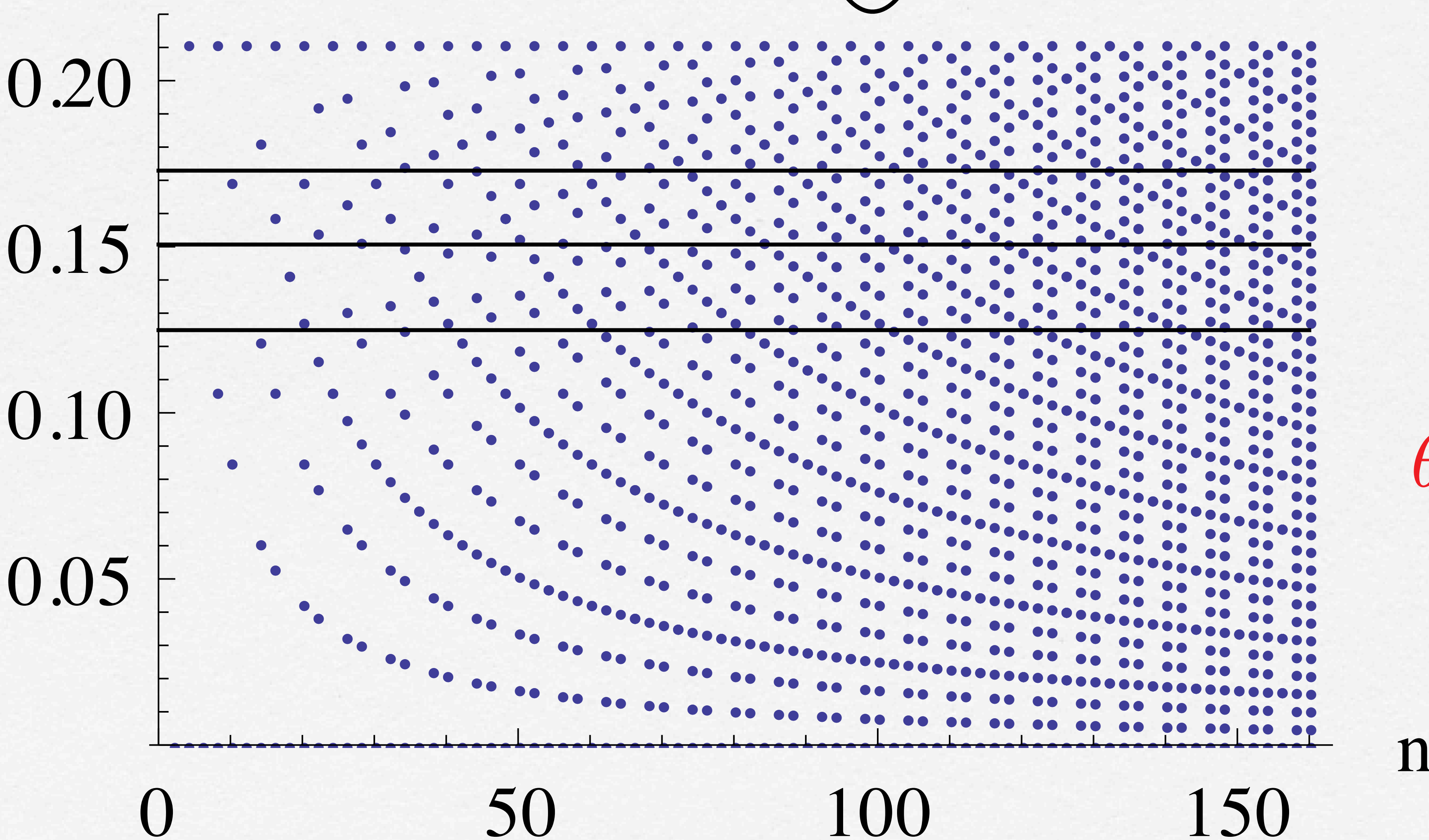
SK, Neder, Stuart

$$V=\begin{pmatrix} \sqrt{\frac{2}{3}}\cos(\vartheta) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin(\vartheta) \\ -\sqrt{\frac{2}{3}}\sin\left(\frac{\pi}{6}+\vartheta\right) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\frac{\pi}{6}+\vartheta\right) \\ \sqrt{\frac{2}{3}}\sin\left(\frac{\pi}{6}-\vartheta\right) & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\frac{\pi}{6}-\vartheta\right) \end{pmatrix}$$

$$\vartheta = \pi\gamma'/n$$

$$\gamma' = 1,\ldots,n/2.$$

$|V_{13}|$



Predictions:

$$\delta = 0, \pi$$

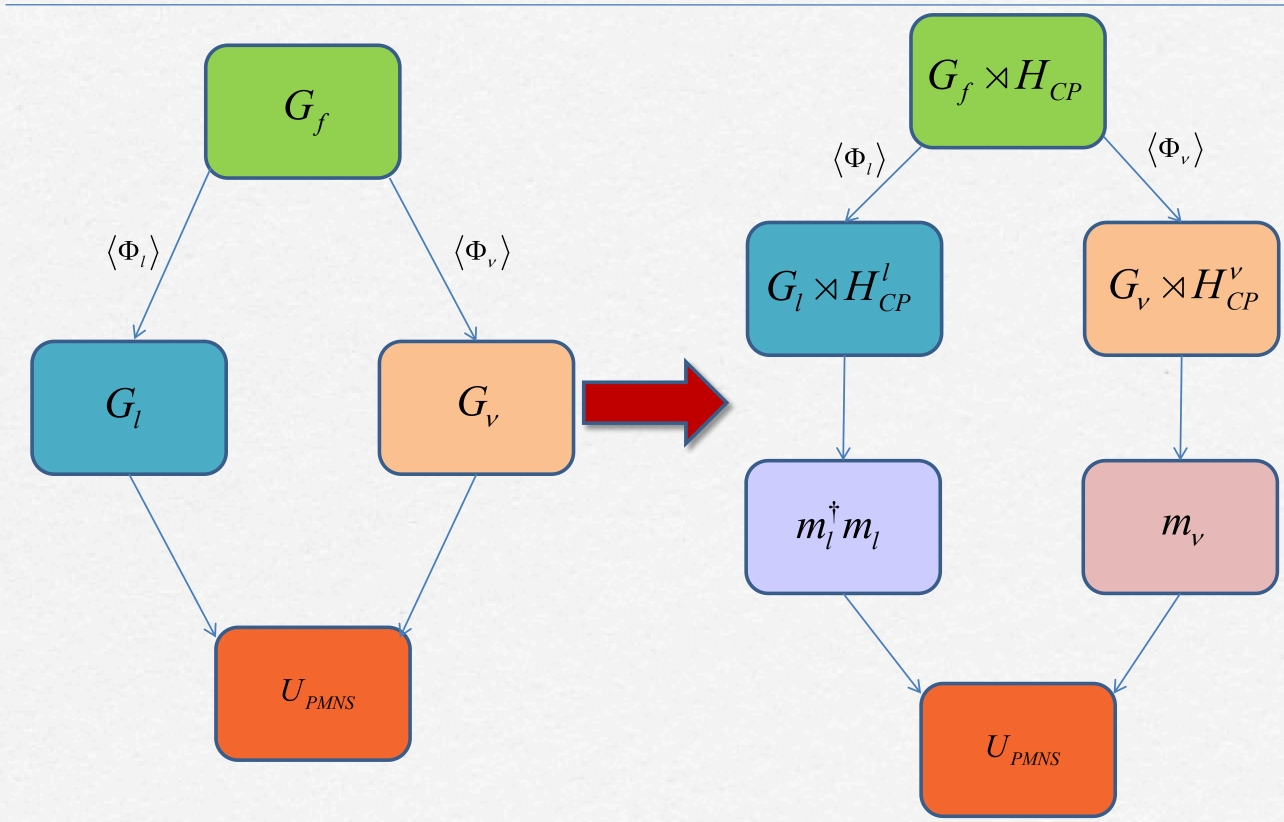
$$\theta_{23} = 45^\circ \mp \theta_{13}/\sqrt{2}.$$

Example of atmospheric sum rule

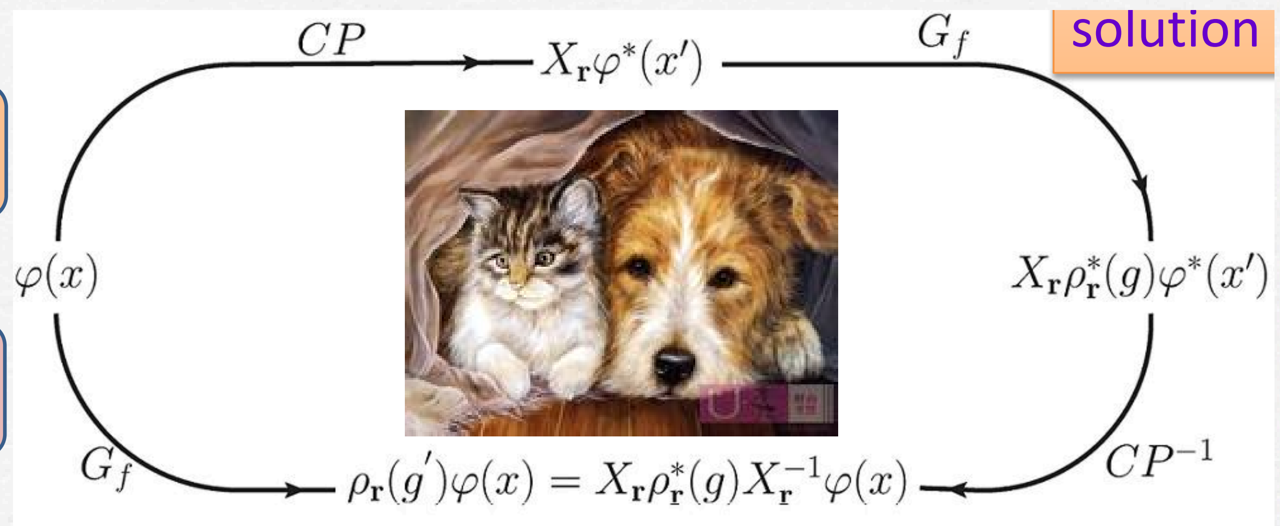
Spontaneous CP violation in direct models

Feruglio, Hagedorn;
Holthausen, Lindner, Schmidt;
Ding, SK, Luhn, Stuart;
Nishi, Xing

Predicting CP: Flavour symmetry⇒ Flavour symmetry "+" Generalised CP symmetry

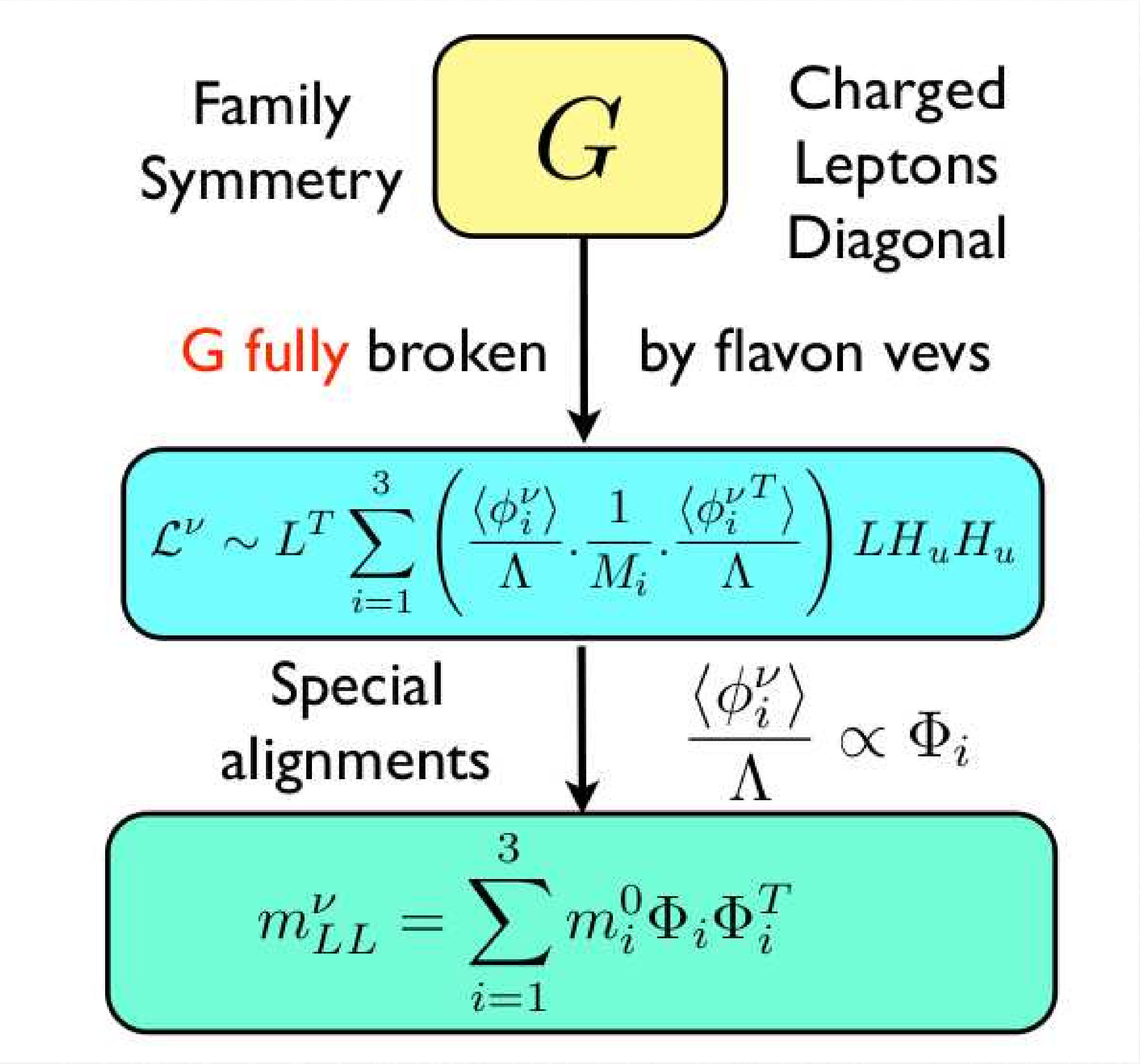


Both mixing angles and CP phases take definite values!



●S_4 and A_4 models with CP symmetry are constructed,all the possible cases following from the model-independent analysis can be realized. **Dirac CP phase is predicted to be trivial or maximal**.

Indirect Models



SK

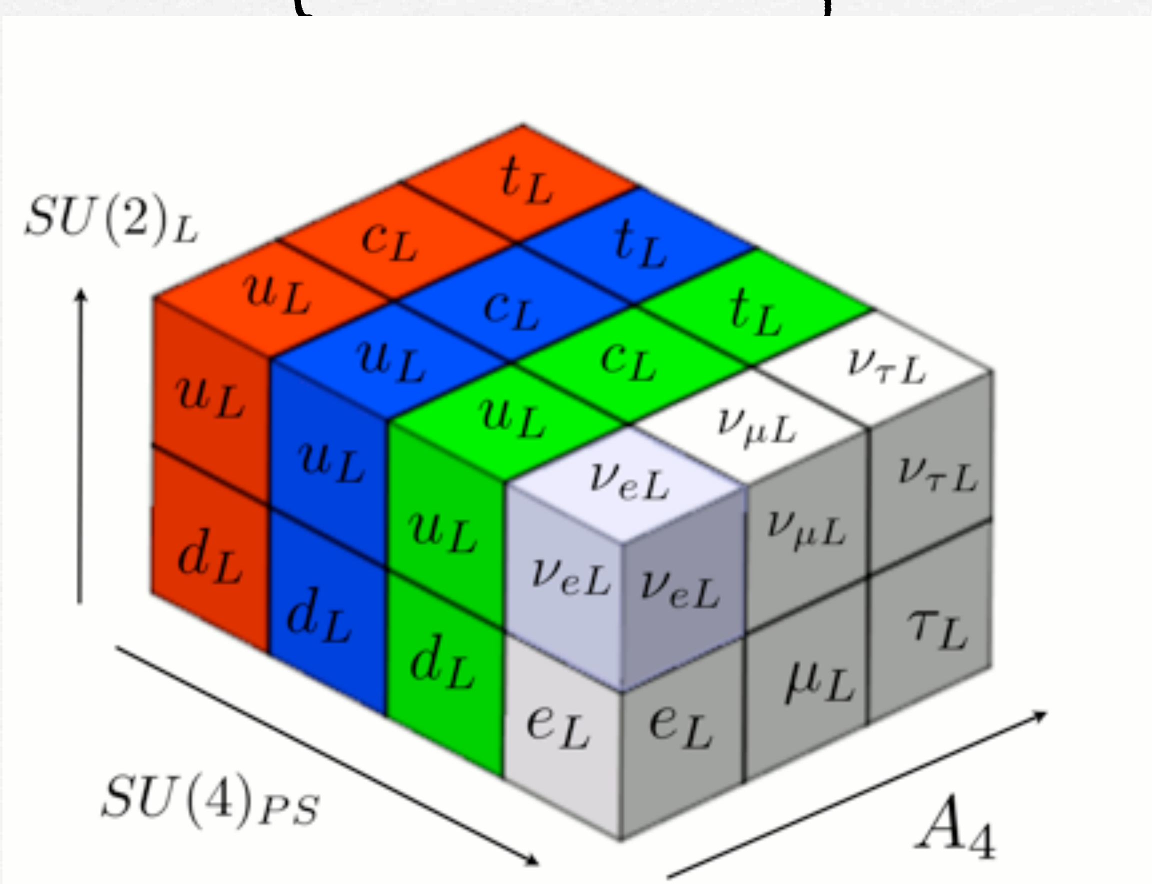
A model of quark and lepton mixing

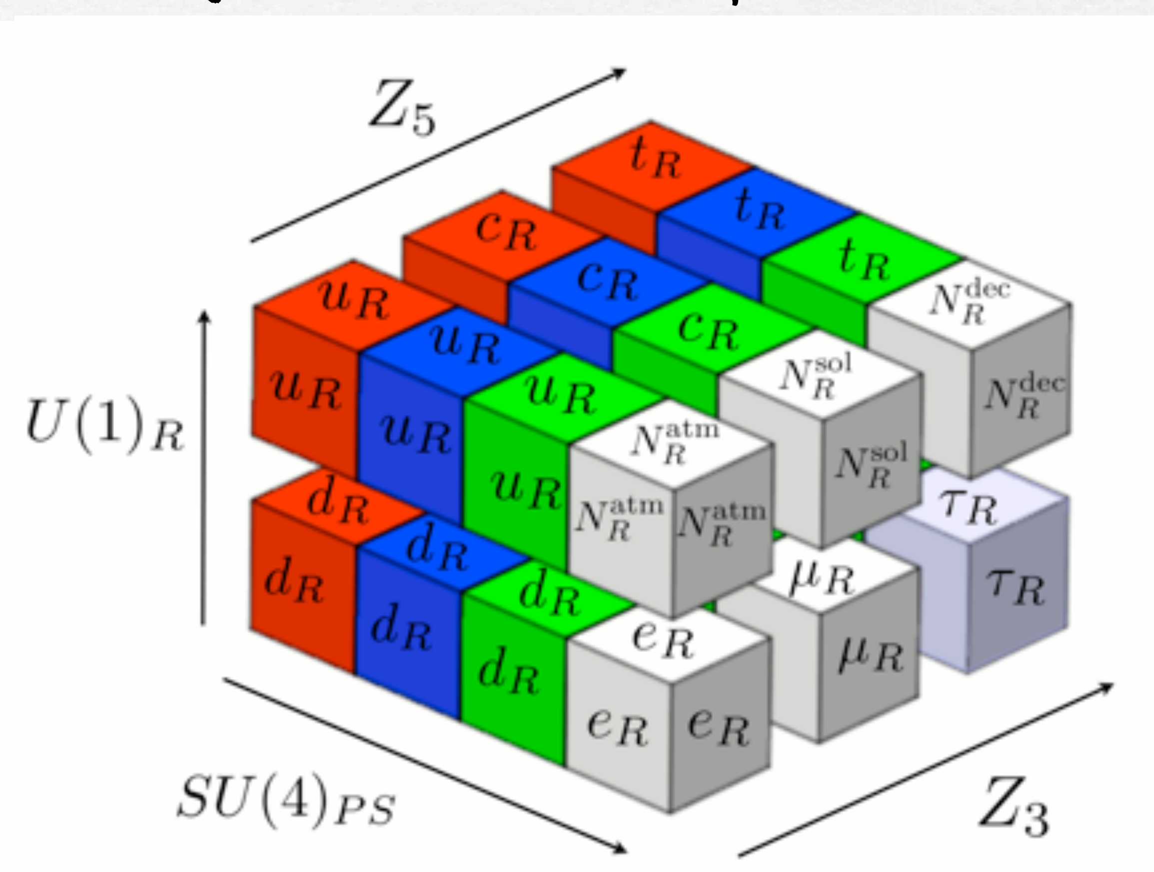
$$A_4 \times SU(4)_{PS} \times SU(2)_L \times U(1)_R$$

Tetra-model

Left-handed quarks and leptons

Right-handed quarks and leptons





	$\mathcal{Q}$	$\mathcal{U}_i^c$	$\mathcal{D}_i^c$	$\phi_{\mathcal{U}_i^c}$	$\phi_{\mathcal{D}_i^c}$	$\mathcal{H}_\mathcal{U}$	$\overline{\mathcal{H}}_\mathcal{U}$	h_u	h_d	$h_\mathcal{D}$
A_4	3	1	1	3	3	1	1	1	1	1
$SU(4)_{PS}$	4	$\overline{4}$	$\overline{4}$	1	1	$\overline{4}$	4	1	1	15
$SU(2)_L$	2	1	1	1	1	1	1	2	2	2
$U(1)_R$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

Vacuum alignment

Approximate Yukawas

$$\langle\phi_{\mathcal{U}_1^c}\rangle = \frac{v_{\mathcal{U}_1^c}}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \langle\phi_{\mathcal{U}_2^c}\rangle = \frac{v_{\mathcal{U}_2^c}}{\sqrt{21}}\begin{pmatrix}1\\4\\2\end{pmatrix}, \quad \langle\phi_{\mathcal{U}_3^c}\rangle = v_{\mathcal{U}_3^c}\begin{pmatrix}0\\0\\1\end{pmatrix}, \quad (\phi_{\mathcal{U}_i^c}.\mathcal{Q})\mathcal{U}_i^c \longrightarrow Y^\nu \sim Y^u \sim \begin{pmatrix}0 & b & 0\\ a & 4b & 0\\ a & 2b & c\end{pmatrix}$$

and

$$\langle\phi_{\mathcal{D}_1^c}\rangle = v_{\mathcal{D}_1^c}\begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \langle\phi_{\mathcal{D}_2^c}\rangle = v_{\mathcal{D}_2^c}\begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \langle\phi_{\mathcal{D}_3^c}\rangle = v_{\mathcal{D}_3^c}\begin{pmatrix}0\\0\\1\end{pmatrix}. \quad (\phi_{\mathcal{D}_i^c}.\mathcal{Q})\mathcal{D}_i^c \longrightarrow Y^d \sim Y^e \sim \begin{pmatrix}y_d & 0 & 0\\ 0 & y_s & 0\\ 0 & 0 & y_b\end{pmatrix}$$

Leading Order Yukawas

$$Y^d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y^e = \begin{pmatrix} y_d/3 & 0 & 0 \\ 0 & 3y_s & 0 \\ 0 & 0 & y_b \end{pmatrix},$$

The Cabibbo connection

$$\theta_C \approx 1/4 \text{ or } \theta_C \approx 14^\circ$$

$$Y^u = \begin{pmatrix} 0 & b\epsilon & 0 \\ a\epsilon^2 & 4b\epsilon & 0 \\ a\epsilon^2 & 2b\epsilon & c \end{pmatrix}, \quad Y^\nu = \begin{pmatrix} 0 & b\epsilon & 0 \\ a\epsilon^2 & 4b\epsilon & 0 \\ a\epsilon^2 & 2b\epsilon & c/3 \end{pmatrix}, \quad M_R = \begin{pmatrix} \epsilon^4 \tilde{M}_1 & 0 & 0 \\ 0 & \epsilon^2 \tilde{M}_2 & 0 \\ 0 & 0 & \tilde{M}_3 \end{pmatrix}$$

See-saw $\quad m^\nu = -v_u^2 Y^\nu M_{\rm R}^{-1} Y^{\nu T}$ $\qquad \eta = 2\pi/5$

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{2i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 1 \end{pmatrix} + m_c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Dominant $\qquad$ Sub-dominant $\qquad$ Decoupled

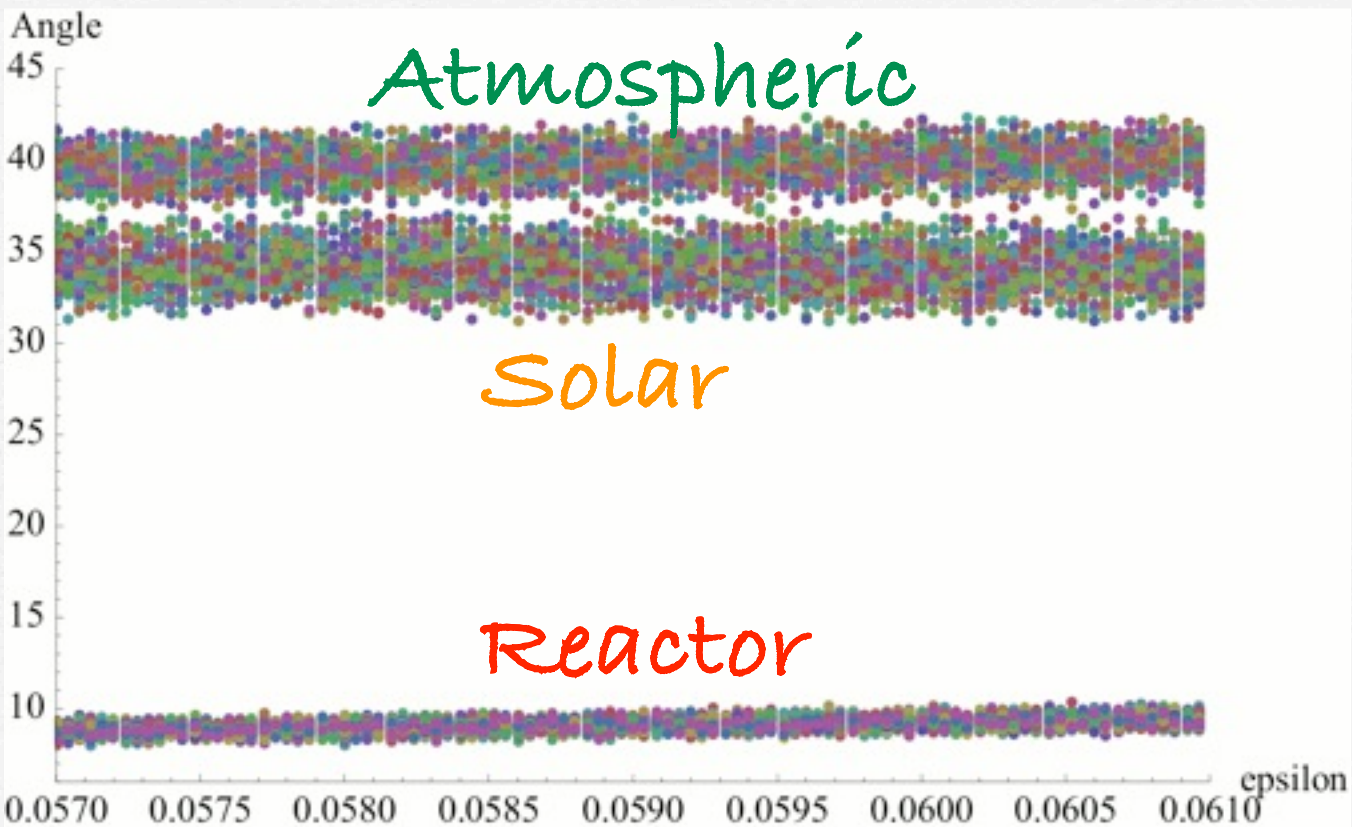
Higher Order Yukawas

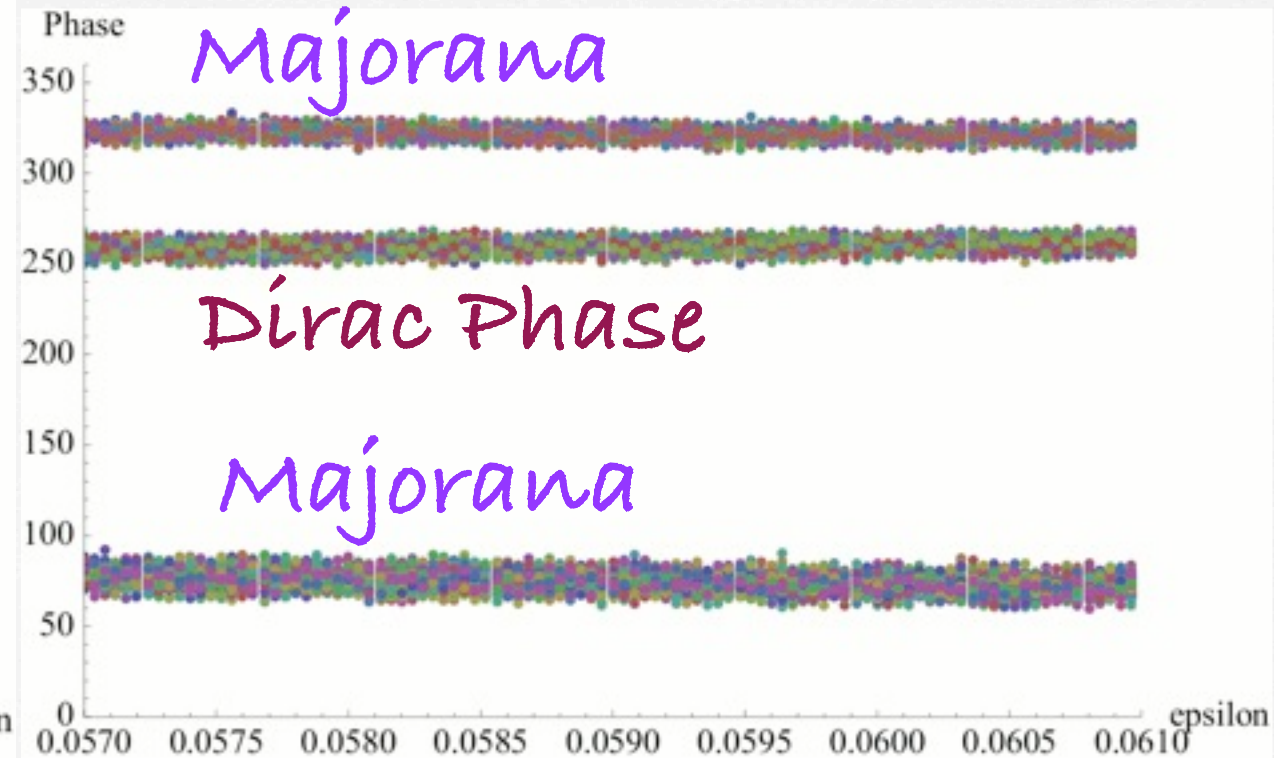
$$Y^u = \begin{pmatrix} \varepsilon_{11}\epsilon^2 & b\epsilon(1+\varepsilon_{12}) & \varepsilon_{13}c \\ a\epsilon^2(1+\varepsilon_{21}) & 4b\epsilon(1+\varepsilon_{22}) & \varepsilon_{23}c \\ a\epsilon^2(1+\varepsilon_{31}) & 2b\epsilon(1+\varepsilon_{32}) & c(1+\varepsilon_{33}) \end{pmatrix}$$

$$\begin{aligned} \theta^q_{12} &\approx \frac{1}{4}|1+\varepsilon_{12}-\varepsilon_{22}| \\ \theta^q_{23} &\approx |\varepsilon_{23}|\,, \\ \theta^q_{13} &\approx |\varepsilon_{23}/4-\varepsilon_{13}|\,, \end{aligned}$$

Small quark mixing angles entirely from HO corrections

Lepton Mixing predictions





$$Y^{\nu} = \begin{pmatrix} \varepsilon_{11}\epsilon^2 & b\epsilon(1+\varepsilon_{12}) & \varepsilon_{13} \\ a\epsilon^2(1+\varepsilon_{21}) & 4b\epsilon(1+\varepsilon_{22}) & \varepsilon_{23} \\ a\epsilon^2(1+\varepsilon_{31}) & 2b\epsilon(1+\varepsilon_{32}) & c/3(1+\varepsilon_{33}) \end{pmatrix}$$

random scans over complex $|\varepsilon_{ij}| < 0.03$

Conclusions

- The Origin of Neutrino Mass is unknown but see-saw most likely option if no new physics found at LHC
- But see-saw does not explain lepton mixing, we need symmetry (or anarchy)
- Direct models preserve part of the family symmetry and tend to give simple patterns (excluded by data)
- Many strategies e.g. large groups Delta $(6n^2)$
- Or use indirect models which completely break the family symmetry
- We have considered a model of quark and lepton mixing - the tetra-model which at leading has 10 predictions including all 6 PMNS parameters, the three down-quark masses and the Cabibbo angle (the "Cabibbo connection")
- At higher order the predictions become blurred but still predicts: a normal neutrino mass hierarchy, atmospheric angle in first octant 40+/- 1 degree, solar angle 34+/- 1 degree, reactor angle 9.0+/- 0.5 degree, Dirac oscillation phase 260+/-5 degrees