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NU FLAVOURS

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BSM neutrino physics

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pedagogical review

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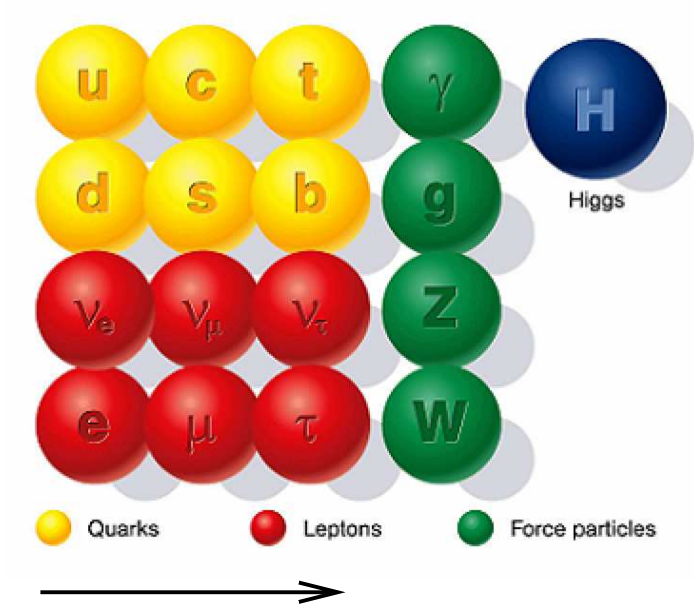
Outline

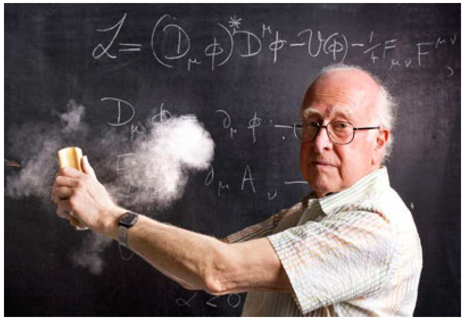
- massive neutrinos require physics beyond the Standard Model
- neutrino mass
  - nature and origin
  - seesaw mechanisms
- neutrino mixing
  - simple patterns
  - family symmetries
- some finite group theory
- family symmetries in model building
  - spontaneous breaking via Higgs-type fields (flavons)
  - direct and indirect implementation
  - deviations from tri-bimaximal mixing

Standard Model (of particle physics)



families or generations

- highly successful theory
- based on gauge symmetry
  $SU(3)_C \times SU(2)_W \times U(1)_Y$
- broken by Higgs vacuum



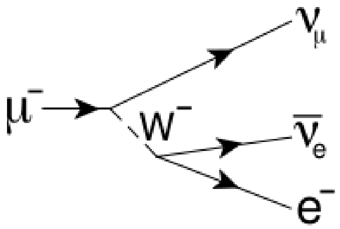
open questions/problems

- stability of electroweak scale
- baryon asymmetry of universe
- nature of dark matter & dark energy
- origin of three families of quarks & leptons
- neutrino masses and mixing

Elusive neutrinos

Standard Model picture

- belong to $SU(2)_W$ doublet
- three flavour states $\nu_e \quad \nu_\mu \quad \nu_\tau$
- couple only weakly
- massless
- individual lepton numbers L_e, L_μ, L_τ are conserved



in reality

- oscillate from one flavour to another (Teppei Katori's lecture)
  - $\rightarrow$ neutrino mass differences
  - $\rightarrow$ neutrino mixing
- extremely light ($\lesssim$ eV)
- individual lepton numbers L_e, L_μ, L_τ are violated

Fermion mixings

- mismatch of flavour (weak) and mass eigenstates

$$\Psi_{\text{flavour}} \;=\; V^{\dagger}\,\Psi_{\text{mass}}$$

- quark sector: V_L^u and V_L^d

$$U_{\text{CKM}} \;=\; V_L^u\, V_L^{d\,\dagger} \;\approx\; \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim 0.22$$

- lepton sector: V_L^e and V_L^ν

$$U_{\text{PMNS}} \;=\; V_L^e\, V_L^{\nu\,\dagger} \;\approx\; \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

Gonzalez-Garcia et al. (2012)

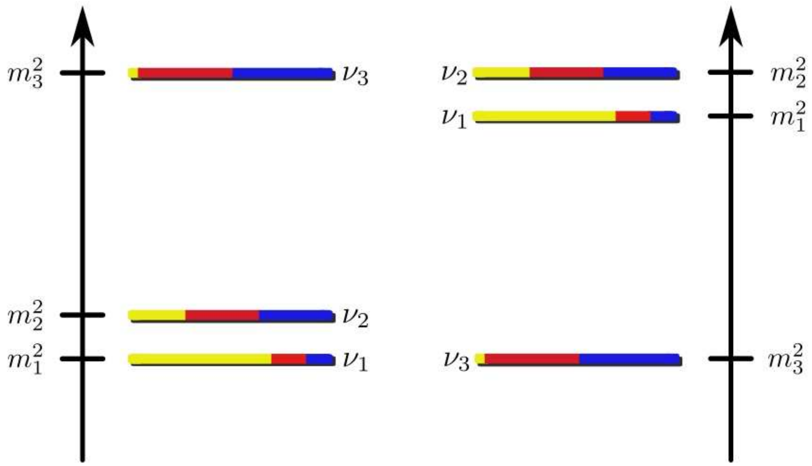
mixing $\iff$ each family knows of the existence of the others!

Three neutrino flavour mixing

(in diagonal charged lepton basis)

flavour — PMNS mixing — mass

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$



atmospheric — reactor + Dirac — solar — Majorana

$$U_{\text{PMNS}} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & e^{\frac{\alpha_2}{2}} & 0\\ 0 & 0 & e^{\frac{\alpha_3}{2}}\end{pmatrix}$$

$\theta_{23} \approx 45^\circ$ $\quad\theta_{13} \approx 9^\circ$ $\quad\theta_{12} \approx 33^\circ$

Known unknowns

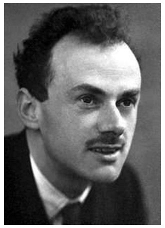
massive neutrinos $\rightarrow$ 7 (9) new parameters

3 masses – $m_1 \quad m_2 \quad m_3$

3 mixing angles – $\theta_{23} \quad \theta_{12} \quad \theta_{13}$

1 Dirac CP phase – δ

(2 Majorana CP phases – $\alpha_2 \quad \alpha_3$)



Dirac

or

Majorana

measured	unmeasured
$\Delta m^2_{21} \quad \lvert\Delta m^2_{32}\rvert$	sign of Δm^2_{32} (mass ordering)
	m_{lightest} (absolute mass scale)
$\theta_{23} \quad \theta_{12} \quad \theta_{13}$	
	Dirac CP phase
	(Majorana CP phases)

Dirac / Majorana masses

(cf. lectures by Pilar Hernandez & Frank Deppisch)

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad \text{in basis with} \quad \boxed{\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}}$$

$$\sigma^\mu = (1, \vec{\sigma})$$

$$\bar{\sigma}^\mu = (1, -\vec{\sigma})$$

$$P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \qquad P_R = \frac{1+\gamma^5}{2} = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix} \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$P_L\Psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} \qquad P_R\Psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

mass terms in Weyl spinor notation

$$m_\mathrm{D} \overline{\Psi}\Psi = m_\mathrm{D} \Psi^\dagger \gamma^0 \Psi = m_\mathrm{D} (\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L)$$

couple left- & right-chiral degrees of freedom

ψ_R = charge conjugate of ψ_L

$$(P_L\Psi)^c = C\overline{(P_L\Psi)}^T = \begin{pmatrix} 0 \\ -i\sigma^2\psi_L^* \end{pmatrix} \qquad C = i\gamma^2\gamma^0$$

$$m_\mathrm{M} \left(\psi_L^\dagger [-i\sigma^2] \psi_L^* + \psi_L^T [i\sigma^2] \psi_L \right) = m_\mathrm{M} \sum_\alpha (\psi_L)^\alpha (\psi_L)_\alpha + h.c.$$

Origin of neutrino mass

Standard Model predicts massless neutrinos because

1. there are no right-chiral neutrinos ν_R
2. Higgs H transforms as a doublet of $SU(2)_W$
3. there are only renormalisable terms

remedy

1. introduce right-chiral neutrinos ν_R

   $\rightarrow$ Dirac mass term $\langle H^0 \rangle \, \nu_L^\dagger \, \nu_R$ [ν_R completely neutral]

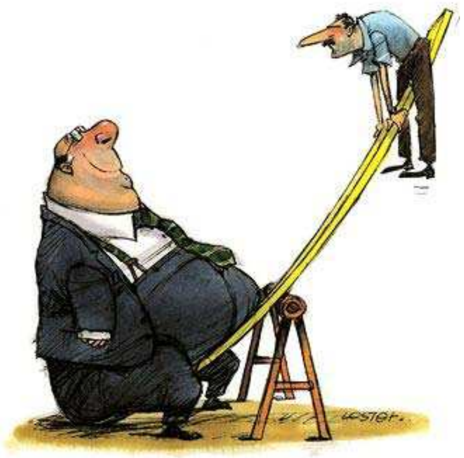
   $\rightarrow$ Majorana mass term $M_R \, \nu_R \, \nu_R$
2. introduce Higgs triplet Δ

   $\rightarrow$ $\langle \Delta^0 \rangle \, \nu_L \, \nu_L$
3. effective dimension-five operator (Weinberg operator)

   $\rightarrow$ $\frac{1}{\Lambda} \langle H^0 \rangle^2 \, \nu_L \, \nu_L$ [$\Lambda \sim 10^{14}$ GeV]

Seesaw mechanism – type I

- add right-chiral neutrinos ν_R
- neutral under SM gauge symmetry

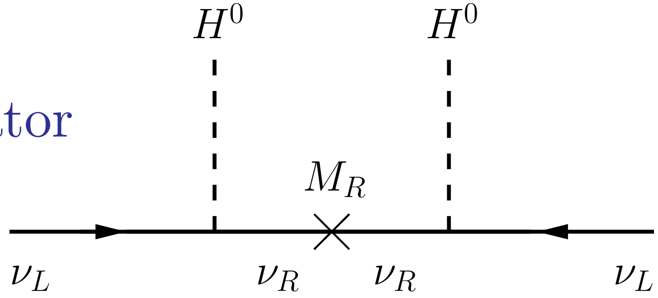


Dirac	$\langle H^0 \rangle \, \nu_L^\dagger \, \nu_R$	$\rightarrow$	m_{LR}
heavy Majorana	$M_R \, \nu_R \, \nu_R$	$\rightarrow$	M_{RR}

- diagonalise mass matrix $\begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix}$ for $m_{LR} \ll M_{RR}$

seesaw formula $\rightarrow$ $\begin{cases} m_{\text{light}} = - \, m_{LR} \, M_{RR}^{-1} \, m_{LR}^T \\ M_{\text{heavy}} = M_{RR} \end{cases}$

- diagrammatic generation of Weinberg operator



Seesaw mechanism – type III

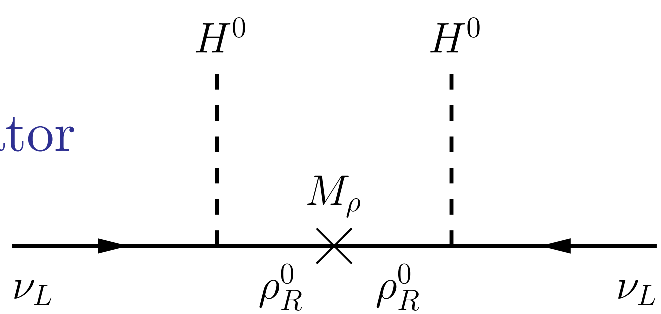
- add right-chiral $U(1)_Y$ neutral fields ρ_R
- $SU(2)_W$ triplet $(\rho_R^-, \rho_R^0, \rho_R^+)$

Dirac	$\langle H^0 \rangle \, \nu_L^\dagger \, \rho_R^0$	$\rightarrow \quad m_{LR}$
heavy Majorana	$M_\rho \, \rho_R^0 \, \rho_R^0$	$\rightarrow \quad M_{RR}$

- diagonalise mass matrix $\begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix}$ for $\quad m_{LR} \ll M_{RR}$

seesaw formula $\quad \rightarrow \quad \begin{cases} m_{\text{light}} = - \, m_{LR} \, M_{RR}^{-1} \, m_{LR}^T \\ M_{\text{heavy}} = M_{RR} \end{cases}$

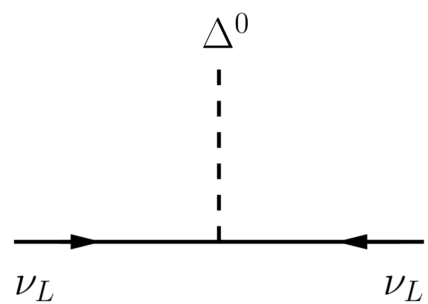
- diagrammatic generation of Weinberg operator



Seesaw mechanism – type II

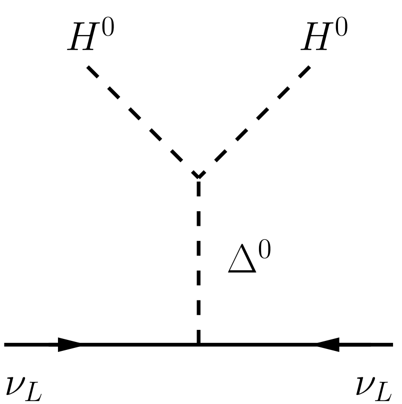
- add scalar $U(1)_Y$ charged field Δ
- $SU(2)_W$ triplet $(\Delta^0, \Delta^+, \Delta^{++})$

light Majorana $\langle \Delta^0 \rangle \, \nu_L \, \nu_L \;\; \rightarrow \;\; m_{LL}$



note scalar coupling $\mu_\Delta \, \Delta^0 \, H^0 \, H^0$

$$m_{LL} \sim \frac{\mu_\Delta \langle H^0 \rangle^2}{M_\Delta^2}$$



Seesaw mechanisms with extra singlets S

$$\begin{pmatrix} 0 & m_{LR} & M_{LS} \\ m_{LR}^T & 0 & M_{RS} \\ M_{LS}^T & M_{RS}^T & M_{SS} \end{pmatrix}$$

double seesaw $\quad M_{LS} = 0 \quad M_{RS} \ll M_{SS}$

$$M_{RR} = M_{RS}\, M_{SS}^{-1}\, M_{RS}^T \quad \longrightarrow \quad m_{LL} = m_{LR}\, M_{RR}^{-1}\, m_{LR}^T$$

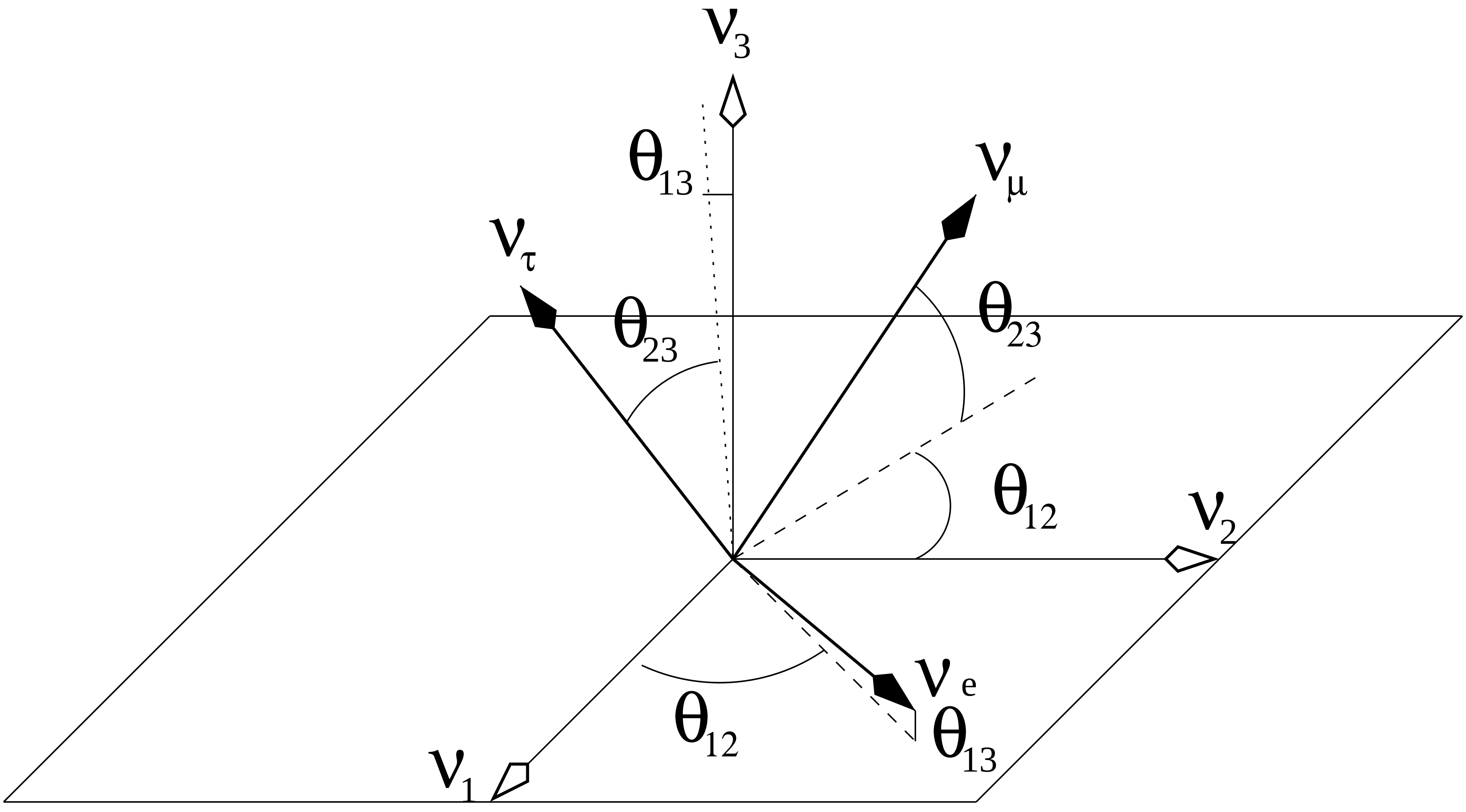
inverse seesaw $\quad M_{LS} = 0 \quad M_{SS} \ll M_{RS}$

$$m_{LL} = m_{LR}\, M_{RS}^{T\,-1}\, M_{SS}\, M_{RS}^{-1}\, m_{LR}^T$$

linear seesaw $\quad M_{SS} = 0$

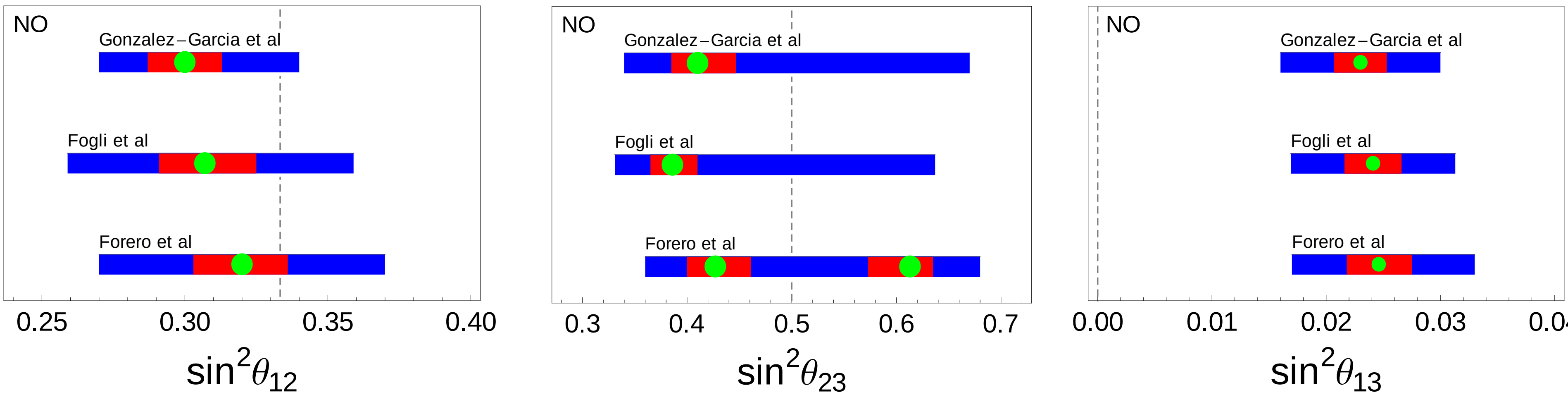
$$m_{LL} = m_{LR}\, M_{RS}^{T\,-1}\, M_{LS}^T \;+\; M_{LS}\, M_{RS}^{-1}\, m_{LR}^T$$

Neutrino mixing

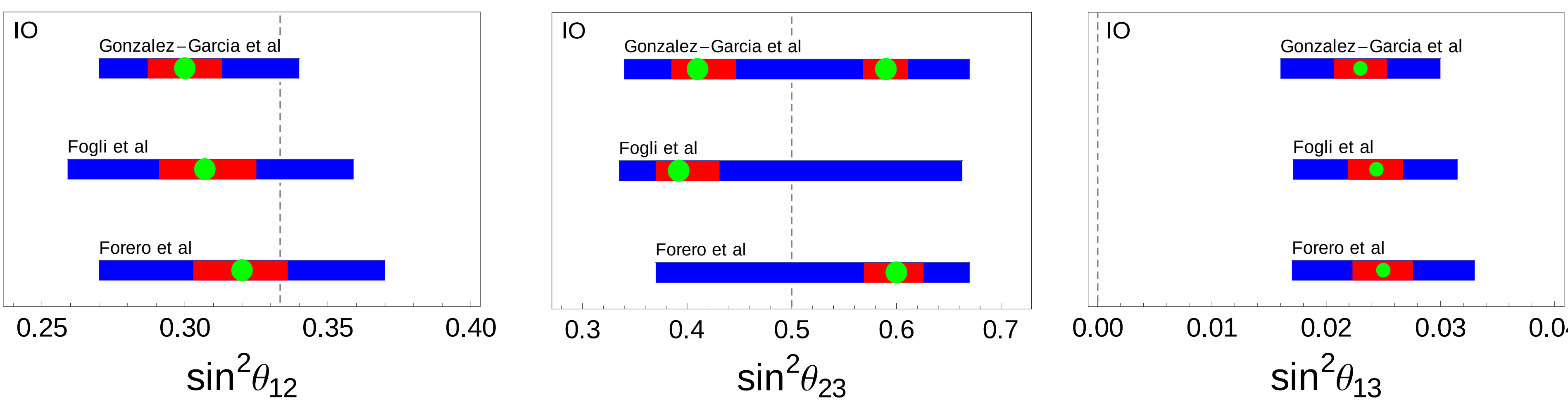


Global neutrino fits

normal mass ordering



inverted mass ordering

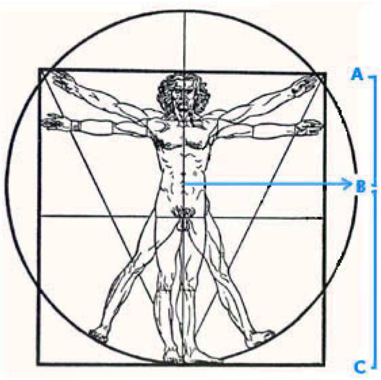


Simple mixing patterns – tri-bimaximal

$$U_{\rm PMNS} \approx U_{\rm TB} \equiv \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12} \approx 35.3^\circ \qquad \theta_{23} = 45^\circ \qquad \theta_{13} = 0^\circ$$

Simple mixing patterns – golden ratio



$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

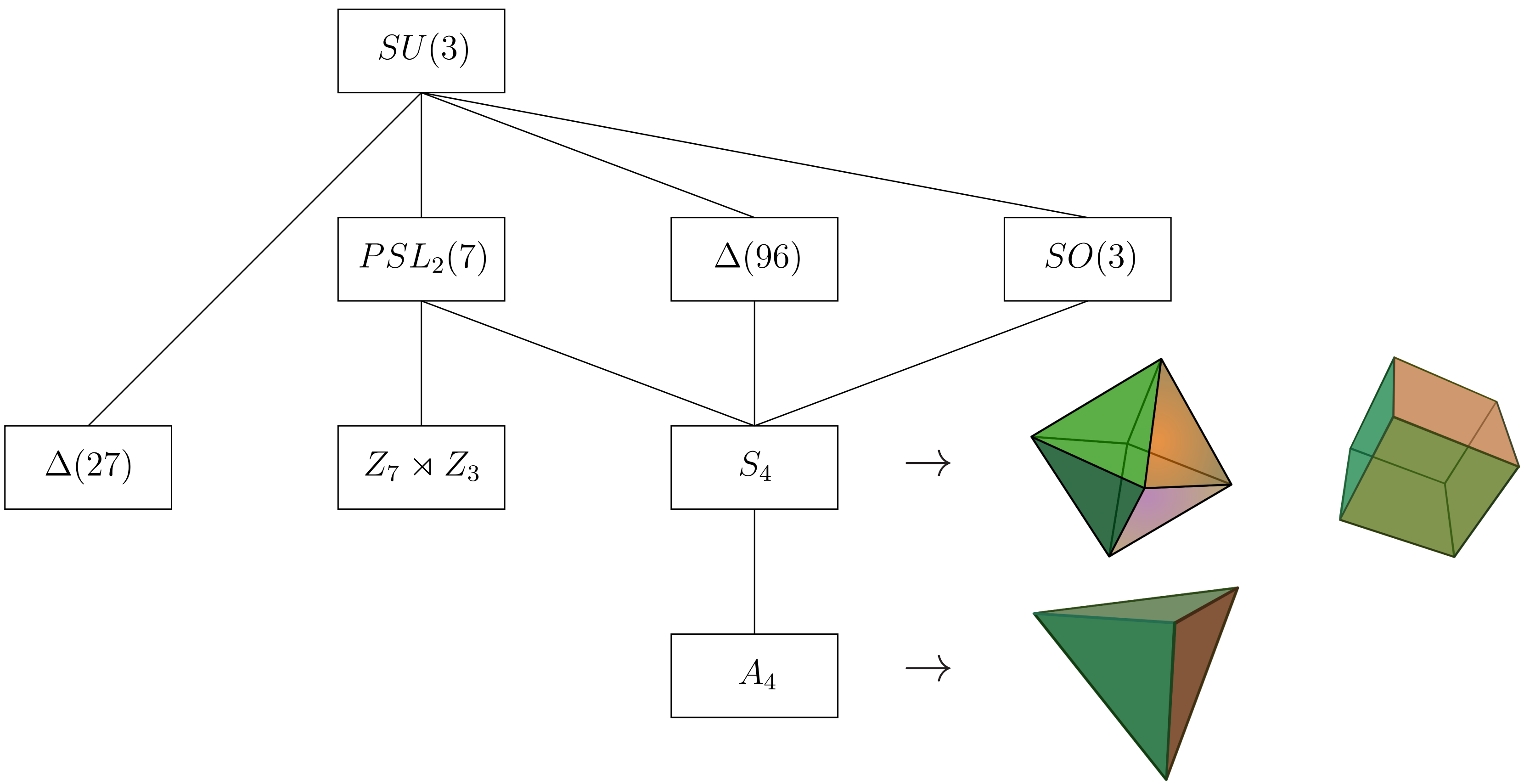
$$\tan\theta_{12} = \frac{1}{\varphi}$$

$$U_{\rm PMNS} \approx U_{\rm GR} \equiv \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12} \approx 31.7^\circ \qquad \theta_{23} = 45^\circ \qquad \theta_{13} = 0^\circ$$

Family symmetries

- horizontal unification
- symmetry group must be non-Abelian
- underlying group should have two- or <u>three-dimensional</u> representations



Group theory with finite groups, e.g. $\mathcal{S}_3$

group multiplication table

	1	a_1	a_2	b_1	b_2	b_3
1	1	a_1	a_2	b_1	b_2	b_3
a_1	a_1	a_2	1	b_2	b_3	b_1
a_2	a_2	1	a_1	b_3	b_1	b_2
b_1	b_1	b_3	b_2	1	a_2	a_1
b_2	b_2	b_1	b_3	a_1	1	a_2
b_3	b_3	b_2	b_1	a_2	a_1	1

generators and the presentation

choose generators $T \equiv a_1$ and $U \equiv b_1 \quad \Rightarrow \quad a_2 = T^2,\ b_2 = TU,\ b_3 = UT$

$< T\,,\, U \mid T^3 = U^2 = (TU)^2 = 1 >$ defines the group uniquely

Classes and representations

conjugacy classes

$$\begin{aligned} 1C_1(1) &= \{g\,1\,g^{-1}\,|\,g\in\mathcal{S}_3\} = \{1\} \\ 2C_2(a) &= \{g\,T\,g^{-1}\,|\,g\in\mathcal{S}_3\} = \{T,\,T^2\} \\ 3C_3(b) &= \{g\,U\,g^{-1}\,|\,g\in\mathcal{S}_3\} = \{U,\,TU,\,UT\} \end{aligned}$$

irreducible representations

$$\mathbf{1}: \quad T=1, \quad U=1$$

$$\mathbf{1}': \quad T=1, \quad U=-1$$

$$\mathbf{2}: \quad T=\begin{pmatrix} e^{2\pi i/3} & 0 \\ 0 & e^{-2\pi i/3}\end{pmatrix}, \quad U=\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}$$

depend on basis

number of classes = number of irreps

Character table

character χ = trace of the matrix representation of element g

all elements within one conjugacy class have the same character

$\mathcal{S}_3$	$1C_1(1)$	$2C_2(T)$	$3C_3(U)$
$\chi^{[\mathbf{1}]}$	1	1	1
$\chi^{[\mathbf{1}']}$	1	1	-1
$\chi^{[\mathbf{2}]}$	2	-1	0

$\Longrightarrow$ Kronecker products $\quad \mathbf{r} \otimes \mathbf{s} = \sum_{\mathbf{t}} \underbrace{d(\mathbf{r}, \mathbf{s}, \mathbf{t})} \; \mathbf{t}$

$$d(\mathbf{r}, \mathbf{s}, \mathbf{t}) = \frac{1}{N} \sum_i N_i \cdot \chi_i^{[\mathbf{r}]} \chi_i^{[\mathbf{s}]} \chi_i^{[\mathbf{t}]\,*}$$

$$\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}$$

$$\mathbf{1}' \otimes \mathbf{2} = \mathbf{2}$$

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} + \mathbf{1}' + \mathbf{2}$$

i denotes class with N_i elements

N is number of group elements

Clebsch-Gordan coefficients

consider product $\boldsymbol{\alpha} \otimes \boldsymbol{\beta} \;\rightarrow\; \boldsymbol{\gamma}$

$$\gamma_k = \textstyle\sum_{i,j} c_{ij}^k \; \alpha_i \, \beta_j$$

basis dependent ! ! !

$$\mathbf{1}' \otimes \mathbf{1}' \;\rightarrow\; \mathbf{1} \qquad \alpha\beta$$

$$\mathbf{1}' \otimes \mathbf{2} \;\rightarrow\; \mathbf{2} \qquad \alpha \begin{pmatrix} \beta_1 \\ -\beta_2 \end{pmatrix}$$

$$\mathbf{2} \otimes \mathbf{2} \;\rightarrow\; \mathbf{1} \qquad \alpha_1\beta_2 + \alpha_2\beta_1$$

$$\mathbf{2} \otimes \mathbf{2} \;\rightarrow\; \mathbf{1}' \qquad \alpha_1\beta_2 - \alpha_2\beta_1$$

$$\mathbf{2} \otimes \mathbf{2} \;\rightarrow\; \mathbf{2} \qquad \begin{pmatrix} \alpha_2\beta_2 \\ \alpha_1\beta_1 \end{pmatrix}$$

$\mathcal{S}_4$ symmetry

presentation
$$S^2 = T^3 = U^2 = 1$$
$$(ST)^3 = (SU)^2 = (TU)^2 = (STU)^4 = 1$$

irreducible representations

	S	T	U
$\mathbf{1}$	1	1	1
$\mathbf{1}'$	1	1	-1
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\mathbf{3}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^{-1} & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$-\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathbf{3}'$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^{-1} & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$\mathcal{A}_4$ symmetry

presentation $\quad S^2 = T^3 = 1$

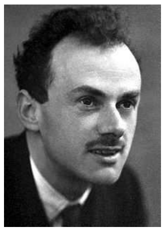
$(ST)^3 = 1$

irreducible representations

	S	T	
$\mathbf{1}$	1	1	
$\begin{pmatrix} \mathbf{1}' \\ \mathbf{1}'' \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix}$	
$\mathbf{3}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^{-1} & 0 \\ 0 & 0 & \omega \end{pmatrix}$	

Symmetries of the mass matrices (in flavour basis)

charged leptons



Dirac

$$M_\ell \;=\; \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

symmetric under diagonal phase transformation T

$$\boxed{(M_\ell M_\ell^\dagger) \;=\; T^T (M_\ell M_\ell^\dagger)\, T^*} \qquad \text{e.g.} \quad T = \mathrm{diag}\,(1, e^{\frac{4\pi i}{3}}, e^{\frac{2\pi i}{3}})$$

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neutrinos

Majorana

$$M_\nu \;=\; U_{\mathrm{PMNS}}\, \mathrm{diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\; U_{\mathrm{PMNS}}^T$$

symmetry of M_ν depends on U_{PMNS}

$$\boxed{M_\nu \;=\; k^T\, M_\nu\, k} \qquad k = U_{\mathrm{PMNS}}^*\, \mathrm{diag}\,(+1, -1, -1)\; U_{\mathrm{PMNS}}^T$$

four different k $\rightarrow$ generate $\mathcal{Z}_2 \times \mathcal{Z}_2$ symmetry group

Klein symmetry: $\mathcal{K} = \{1\,,\, S\,,\, U\,,\, SU\}$

for $U_{\mathrm{PMNS}} = U_{\mathrm{TB}}$:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Family symmetry breaking

- different symmetries in charged lepton and neutrino sector
- family symmetry broken spontaneously by so-called flavon fields ϕ

M_{Pl}

flavon VEV $\langle\phi\rangle$ breaks family symmetry

M_{GUT}

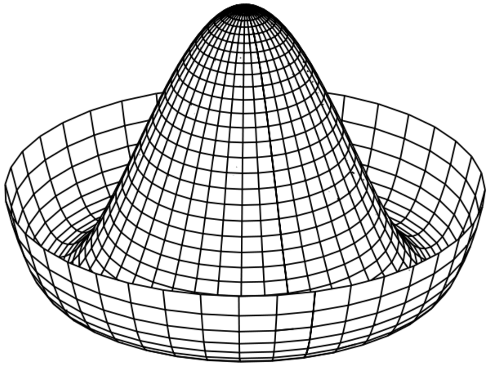
$\rightarrow$ Yukawas Y_{ij}

M_{seesaw}

Higgs VEV $\langle H\rangle$ breaks electroweak symmetry

M_{EW}

$\rightarrow$ fermion masses $M_{ij} = Y_{ij}\langle H\rangle$



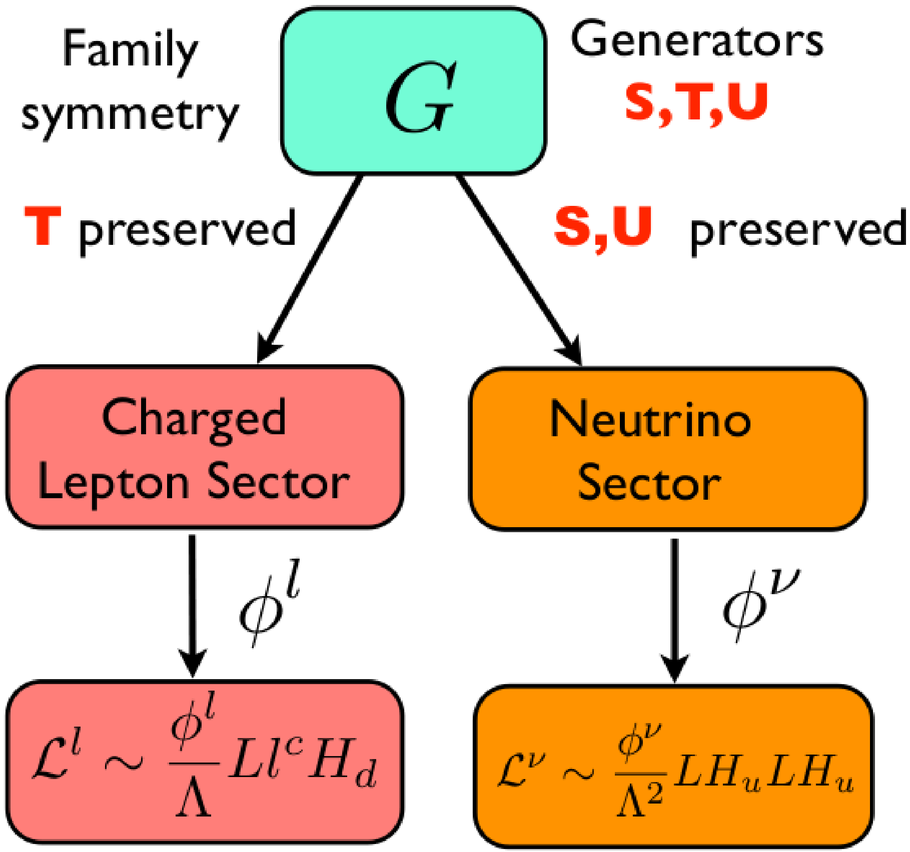
Origin of the Klein symmetry $\mathcal{K}$

- direct models
  - $\boxed{\text{Klein symmetry } \mathcal{K} \;\subset\; \text{family symmetry } \mathcal{G}}$
  - flavons ϕ are multiplets of $\mathcal{G}$
  - their VEVs $\langle\phi\rangle$ break $\mathcal{G}$ down to $\mathcal{K}$ in neutrino sector
  - for TB mixing (S, U, T) generate permutation group $\mathcal{S}_4$
- indirect models
  - $\boxed{\text{Klein symmetry } \mathcal{K} \;\not\subset\; \text{family symmetry } \mathcal{G}}$
  - $\mathcal{G}$ responsible for generating particular flavon VEV configurations $\langle\phi\rangle$
  - for TB mixing – from e.g. $\Delta(27)$, $\mathcal{Z}_7 \rtimes \mathcal{Z}_3$

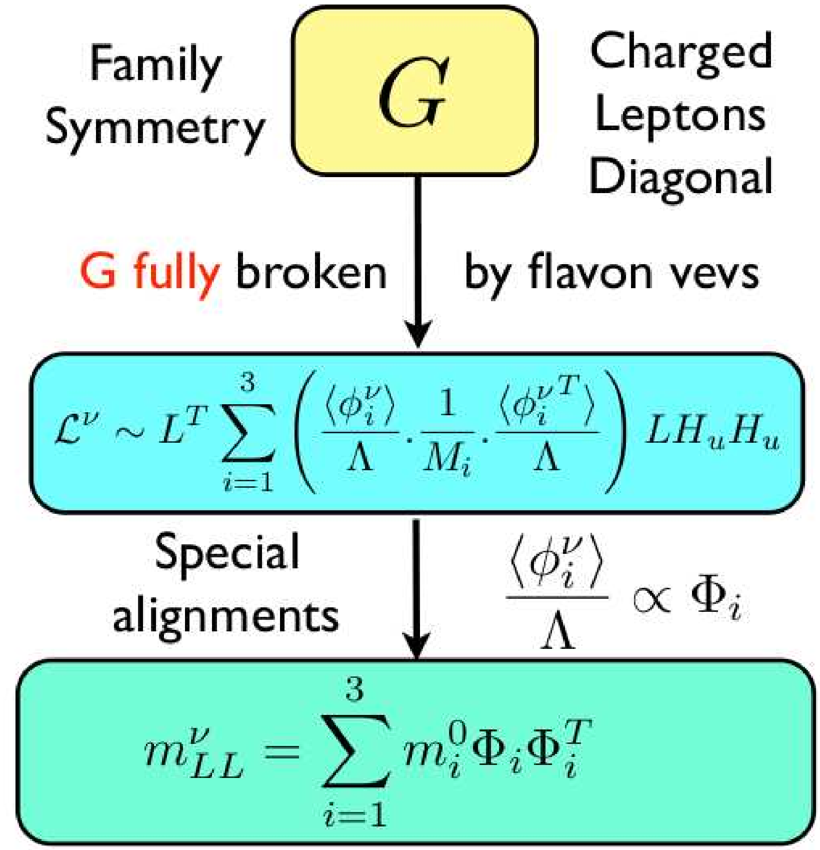
$$\langle\phi_1\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_2\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_3\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{L}_\nu \quad \sim \quad \nu\,(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\,\nu\; H\, H$$

Two model building approaches



(direct models)



(indirect models)

Building a direct model with tri-bimaximal mixing

- choose family symmetry group – $\mathcal{S}_4$
- identify suitable flavon VEV configurations

$$S\langle\phi^\nu\rangle = U\langle\phi^\nu\rangle = \langle\phi^\nu\rangle \qquad T\langle\phi^\ell\rangle = \langle\phi^\ell\rangle$$

$\mathcal{S}_4$	S	U	T	$\langle\phi^\nu\rangle$	$\langle\phi^\ell\rangle$
$\mathbf{1},\mathbf{1}'$	1	± 1	1	$\mathbf{1}$	$\mathbf{1},\mathbf{1}'$
$\mathbf{2}$	$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	$\begin{pmatrix}\omega & 0\\ 0 & \omega^2\end{pmatrix}$	$\mathbf{2} \propto \begin{pmatrix}1\\ 1\end{pmatrix}$	$-$
$\mathbf{3},\mathbf{3}'$	$\frac{1}{3}\begin{pmatrix}-1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1\end{pmatrix}$	$\mp\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$	$\begin{pmatrix}1 & 0 & 0\\ 0 & \omega^2 & 0\\ 0 & 0 & \omega\end{pmatrix}$	$\mathbf{3}' \propto \begin{pmatrix}1\\ 1\\ 1\end{pmatrix}$	$\mathbf{3},\mathbf{3}' \propto \begin{pmatrix}1\\ 0\\ 0\end{pmatrix}$

- control coupling of flavons to fermions by extra $\mathcal{Z}_N$ or $U(1)$ symmetry

$$\frac{\phi^\nu}{\Lambda^2} L H_u L H_u \qquad \frac{\phi^\ell}{\Lambda} L\,\ell^c H_d$$

- with type I seesaw

$$\mathcal{L}_\nu \sim L\,H_u\,\nu^c + \phi^\nu\nu^c\nu^c$$

Building an indirect model with tri-bimaximal mixing

- family symmetry $\mathcal{G} \subset SU(3)$
- diagonal charged leptons
- type I seesaw with 2 or 3 ν_a^c in singlet representation of $\mathcal{G}$
- diagonal right-handed neutrino mass matrix (e.g. due to $\mathcal{Z}_2$ symmetry)

$$\mathcal{L}_\nu \sim \sum_a \frac{\phi_a^\nu}{\Lambda} L\, H_u\, \nu_a^c + M_a \nu_a^c \nu_a^c$$

- $\phi_a^\nu \sim \overline{\mathbf{3}}$ and $L \sim \mathbf{3}$ of $\mathcal{G}$
- $\mathcal{G}$ or $SU(3)$ invariant $\rightarrow \phi_{a\,1}^\nu L_1 + \phi_{a\,2}^\nu L_2 + \phi_{a\,3}^\nu L_3 = \phi_a^{\nu\,T} L$
- integrate out ν_a^c (seesaw formula)

$$\mathcal{L}_\nu \sim L^T \sum_{a=1} \left(\frac{\langle \phi_a^\nu \rangle}{\Lambda} \cdot \frac{1}{M_a} \cdot \frac{\langle \phi_a^\nu \rangle^T}{\Lambda} \right) L H_u H_u$$

- tri-bimaximal if $\left[\langle \phi_1^\nu \rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \right] \quad \langle \phi_2^\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \langle \phi_3^\nu \rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

Aligning triplet flavons in $\Delta(27)$, $\mathcal{Z}_7 \rtimes \mathcal{Z}_3$, $\mathcal{A}_4$

$$V(\phi) \;=\; -\,m^2 \sum_i {\phi_i}^\dagger \phi_i \;+\; \lambda \left(\sum_i {\phi_i}^\dagger \phi_i\right)^2 \;+\; \Delta V$$

central terms in ΔV

(i) $\quad \kappa \sum_i \phi_i^\dagger \phi_i \phi_i^\dagger \phi_i$

$$\kappa > 0 \;\rightarrow\; \langle\phi\rangle \;\propto\; \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\kappa < 0 \;\rightarrow\; \langle\phi\rangle \;\propto\; \begin{pmatrix}1\\0\\0\end{pmatrix}$$

(ii) $\quad \tilde\kappa \sum_{i,j} (\phi_i^\dagger \tilde\phi_i)(\tilde\phi_j^\dagger \phi_j)$

$\tilde\kappa > 0 \;\rightarrow\;$ orthogonality condition $\langle\phi\rangle \perp \langle\tilde\phi\rangle$

e.g. $\langle\tilde\phi\rangle \;\propto\; \begin{pmatrix}0\\1\\-1\end{pmatrix}$

... $\langle\widetilde{\tilde\phi}\rangle \;\propto\; \begin{pmatrix}-2\\1\\1\end{pmatrix}$

Flavon alignment in supersymmetry

- SUSY unbroken at scale of family symmetry breaking
- introduce so-called driving fields X which couple to flavons
- flavon superpotential W^{flavon} linear in X due to $U(1)_R$ symmetry
- F-terms of driving fields need to vanish

$$\boxed{F^*_{X_i} = -\frac{\partial W^{\text{flavon}}}{\partial X_i} = 0}$$

- two examples in $\mathcal{S}_4$

$$W^{\text{flavon}} \sim X_{\mathbf{1}}\phi_{\mathbf{2}}\phi_{\mathbf{2}} = X_{\mathbf{1}}(\phi_{\mathbf{2},1}\phi_{\mathbf{2},2} + \phi_{\mathbf{2},2}\phi_{\mathbf{2},1}) = 2X_{\mathbf{1}}\phi_{\mathbf{2},1}\phi_{\mathbf{2},2}$$

$$\longrightarrow \langle\phi_{\mathbf{2}}\rangle \propto \begin{pmatrix}1\\0\end{pmatrix} \quad \text{or} \quad \langle\phi_{\mathbf{2}}\rangle \propto \begin{pmatrix}0\\1\end{pmatrix}$$

$$W^{\text{flavon}} = g_0 X_{\mathbf{3}}\phi^\nu_{\mathbf{3}'}\phi^\nu_{\mathbf{2}} + X_{\mathbf{3}'}\left(g_1\phi^\nu_{\mathbf{3}'}\phi^\nu_{\mathbf{3}'} + g_2\phi^\nu_{\mathbf{3}'}\phi^\nu_{\mathbf{2}} + g_3\phi^\nu_{\mathbf{3}'}\phi^\nu_{\mathbf{1}}\right)$$

$$\longrightarrow \langle\phi^\nu_{\mathbf{3}'}\rangle = \varphi^\nu_{\mathbf{3}'}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \langle\phi^\nu_{\mathbf{2}}\rangle = \varphi^\nu_{\mathbf{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \varphi^\nu_{\mathbf{2}} = -\tfrac{g_3}{2g_2}\varphi^\nu_{\mathbf{1}}$$

- flavon alignments independent of g_i

(exact) Tri-bimaximal mixing is ruled out!

T2K [arXiv:1106.2822]

- $\theta_{13} = 0$ disfavoured at $\sim 2.5\sigma$
- $5° \lesssim \theta_{13} \lesssim 18°$ at 90% C.L.

Double Chooz [arXiv:1207.6632]

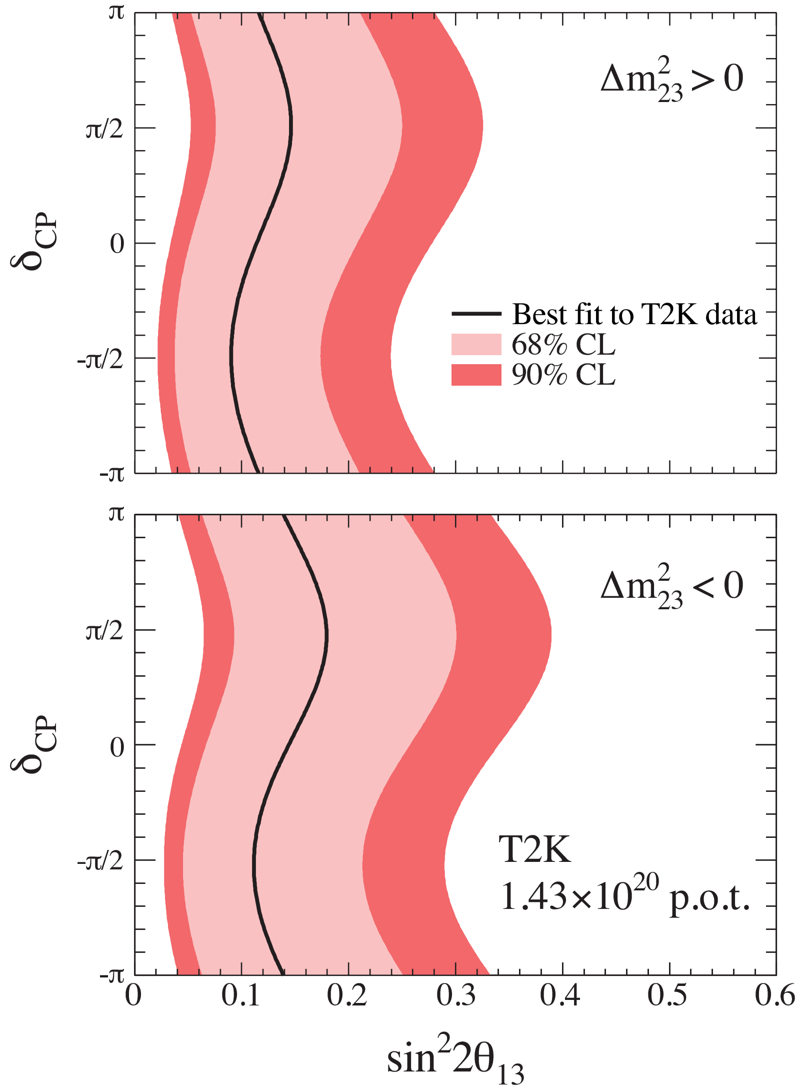
- $\theta_{13} = 0$ disfavoured at $\sim 2.9\sigma$
- $6° \lesssim \theta_{13} \lesssim 12°$ at 90% C.L.

RENO [arXiv:1204.0626]

- $\theta_{13} = 0$ disfavoured at $\sim 4.9\sigma$
- $8.0° \lesssim \theta_{13} \lesssim 11.4°$ at 90% C.L.

Daya Bay [arXiv:1210.6327]

- $\theta_{13} = 0$ disfavoured at $\sim 7.7\sigma$
- $7.7° \lesssim \theta_{13} \lesssim 9.6°$ at 90% C.L.



Requires new model building strategies

<u>direct models</u>

TB plus corrections

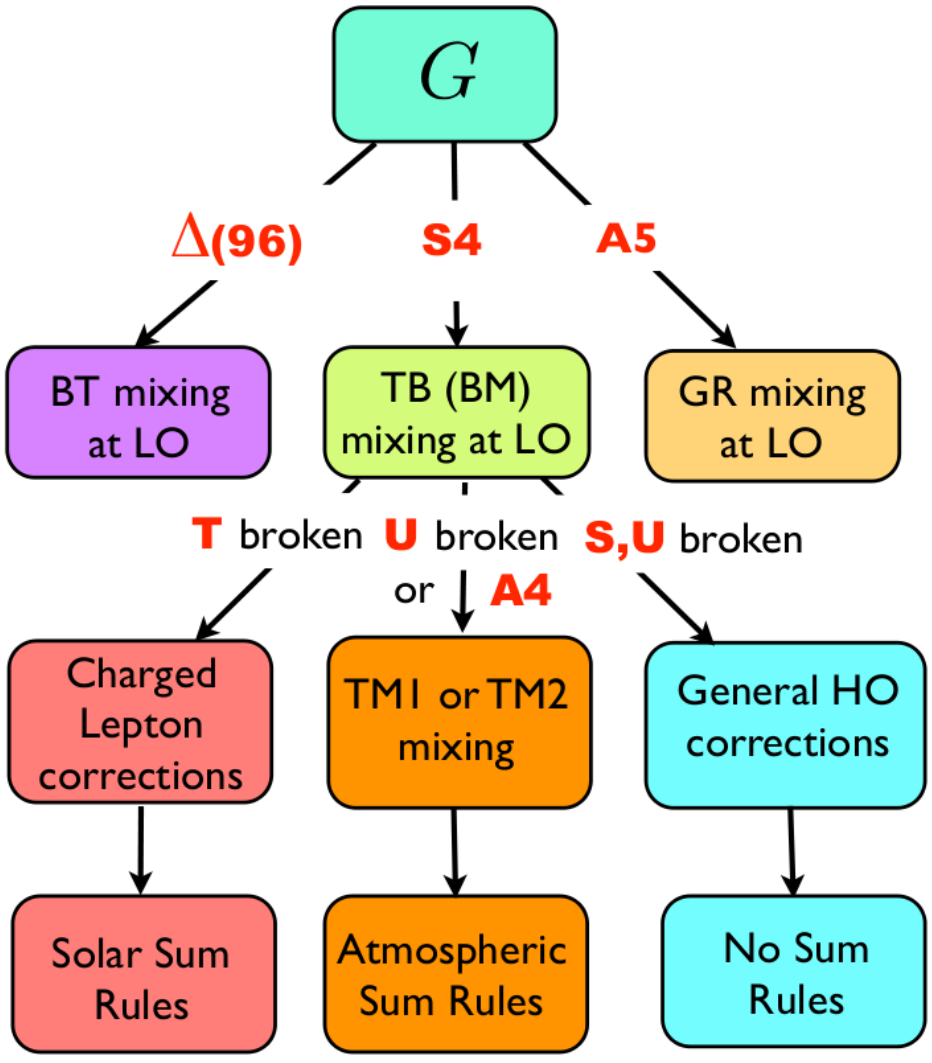
other family symmetries
with non-standard $\mathcal{K}$

<u>indirect models</u>

TB plus corrections

non-standard flavon
VEV configurations

Direct models after Daya Bay and RENO



mixing patterns:

	θ_{13}	θ_{23}	θ_{12}
TB	$0°$	$45°$	$35.3°$
BM	$0°$	$45°$	$45°$
GR	$0°$	$45°$	$31.7°$
BT	$12.2°$	$36.2°$	$36.2°$
TM	$\neq 0°$	$\neq 45°$	$35.3°$

TB = tri-bimaximal
BM = bimaximal
GR = golden ratio
BT = bi-trimaximal
TM = trimaximal

Charged lepton corrections to TB mixing

- charged lepton mass matrix might not be diagonal (GUTs)
- $U_{\mathrm{PMNS}} = V_{\ell_L} V_{\nu_L}^\dagger$ and $V_{\nu_L}^\dagger = U_{\mathrm{TB}}$

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23} \\ 0 & -\hat{s}_{23}^* & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}_{13}^* & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & \hat{s}_{12} & 0 \\ -\hat{s}_{12}^* & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{12}\, e^{i\delta_{12}} \approx \frac{1}{\sqrt{3}} \left(e^{i\delta_{12}^\nu} - \theta_{12}^\ell\, e^{i\delta_{12}^\ell} + \theta_{13}^\ell\, e^{i(\delta_{13}^\ell - \delta_{23}^\nu)} \right)$$

$$s_{23}\, e^{i\delta_{23}} \approx \frac{1}{\sqrt{2}} \left(e^{i\delta_{23}^\nu} - \theta_{23}^\ell\, e^{i\delta_{23}^\ell} \right)$$

$$s_{13}\, e^{i\delta_{13}} \approx \frac{1}{\sqrt{2}} \left(-\theta_{12}^\ell\, e^{i(\delta_{12}^\ell + \delta_{23}^\nu)} - \theta_{13}^\ell\, e^{i\delta_{13}^\ell} \right)$$

$$c_{ij} = \cos\theta_{ij}$$
$$\hat{s}_{ij} = \sin\theta_{ij}\, e^{-i\delta_{ij}}$$

- $\theta_{12}^\ell \sim \theta_C \sim 0.22 \quad \rightarrow \quad \theta_{13} \sim 9^\circ$
- solar sum rule $\quad \rightarrow \quad \theta_{12} \approx 35.3^\circ + \theta_{13} \cos\delta$

Breakdown of the TB Klein symmetry

- add higher order term which breaks U generator of $\mathcal{S}_4$

$$\boxed{\mathcal{L}_\nu \sim LH_u\nu^c + (\phi^\nu_{\mathbf{3}'} + \phi^\nu_{\mathbf{2}} + \phi^\nu_{\mathbf{1}})\nu^c\nu^c + \frac{1}{M}\widetilde{\phi}^\nu_{\mathbf{1}'}\phi^\nu_{\mathbf{2}}\nu^c\nu^c}$$

$\mathcal{S}_4$	S	U	VEV alignment
$\mathbf{3}'$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\langle\phi^\nu_{\mathbf{3}'}\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\langle\phi^\nu_{\mathbf{2}}\rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{1}$	1	1	$\langle\phi^\nu_{\mathbf{1}}\rangle \propto 1$
$\mathbf{1}'$	1	-1	$\langle\widetilde{\phi}^\nu_{\mathbf{1}'}\rangle \propto 1$

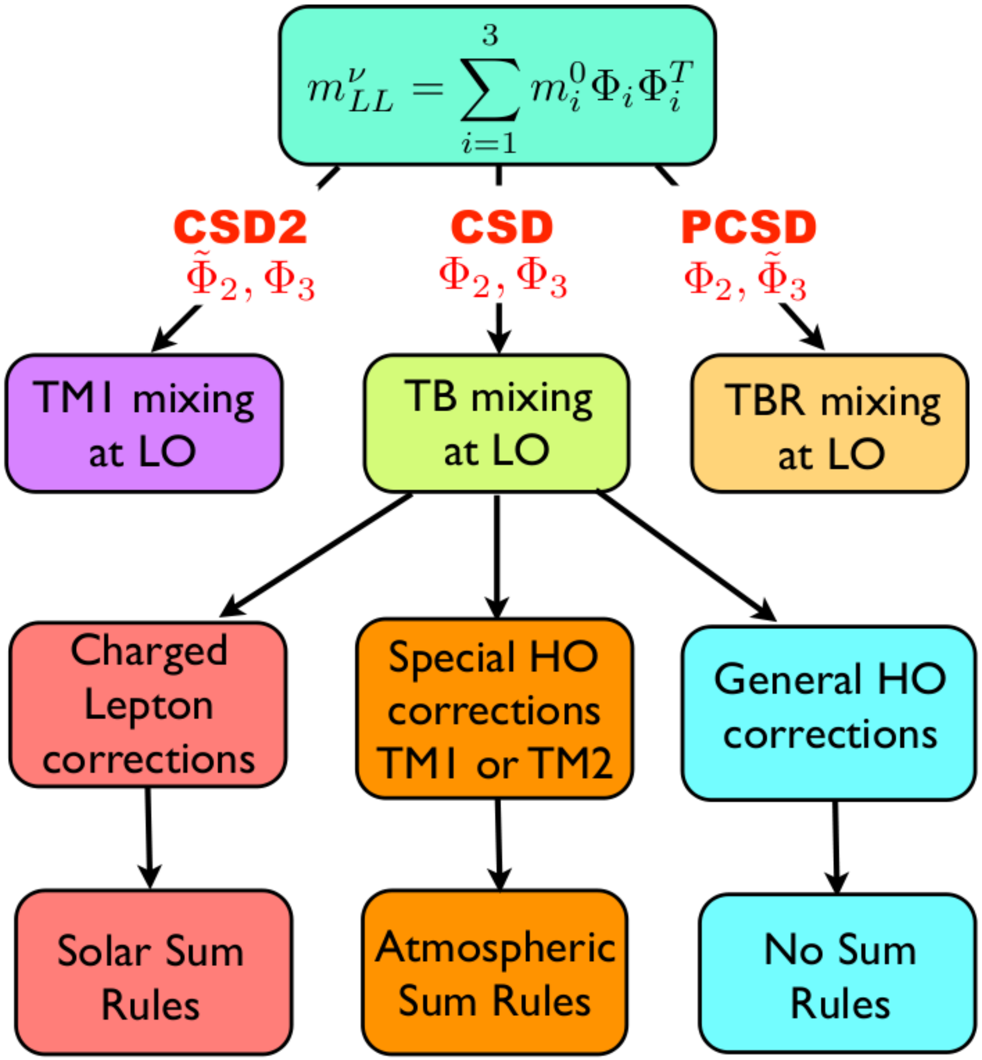
- resulting mixing

$$U_{\mathrm{PMNS}} \approx \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \end{pmatrix}$$

TM_2 mixing due to S symmetry

- atmospheric sum rule $\rightarrow$ $\theta_{23} \approx 45^\circ - \frac{1}{\sqrt{2}}\,\theta_{13}\cos\delta$

Indirect models after Daya Bay and RENO



flavon alignments:

	$\langle \Phi_2 \rangle$	$\langle \Phi_3 \rangle$
CSD	$\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$
CSD2	$\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\\0\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$
PCSD	$\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}\epsilon\\1\\-1\end{pmatrix}$

Variations of constrained sequential dominance (CSD)

$$\boxed{m_\nu \;=\; m_2^0\,\Phi_2\Phi_2^T \;+\; m_3^0\,\Phi_3\Phi_3^T} \qquad m_2^0 \ll m_3^0$$

- CSD

tri-bimaximal

$$\frac{m_2^0}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix}0&0&0\\0&1&-1\\0&-1&1\end{pmatrix} \Longrightarrow \boxed{\begin{array}{ll}\theta_{13}=0 & m_1^\nu=0\\ \theta_{23}=45^\circ & m_2^\nu=m_2^0\\ \theta_{12}=35.3^\circ & m_3^\nu=m_3^0\end{array}}$$

- CSD2

trimaximal 1 (TM_1)

$$\frac{m_2^0}{5}\begin{pmatrix}1&2&0\\2&4&0\\0&0&0\end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix}0&0&0\\0&1&-1\\0&-1&1\end{pmatrix} \qquad \boxed{\begin{array}{ll}\theta_{13}\approx\frac{\sqrt{2}}{3}\frac{m_2^\nu}{m_3^\nu} & m_1^\nu=0\\ \theta_{23}\approx 45^\circ+\sqrt{2}\theta_{13}\cos\delta & m_2^\nu\approx\frac{3}{5}m_2^0\\ \theta_{12}\approx 35.3^\circ & m_3^\nu\approx m_3^0\end{array}}$$

- PCSD (partially CSD)

tri-bimaximal-reactor

$$\frac{m_2^0}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix}\epsilon^2&\epsilon&-\epsilon\\\epsilon&1&-1\\-\epsilon&-1&1\end{pmatrix} \Longrightarrow \boxed{\begin{array}{ll}\theta_{13}\approx\frac{\epsilon}{\sqrt{2}} & m_1^\nu=0\\ \theta_{23}\approx 45^\circ & m_2^\nu\approx m_2^0\\ \theta_{12}\approx 35.3^\circ & m_3^\nu\approx m_3^0\end{array}}$$

Summary

- neutrinos have mass and they mix
- various seesaw mechanisms
- horizontal unification
- discrete family symmetries
  - origin of simple mixing patterns
  - direct vs indirect models
- non-zero reactor angle θ_{13}
  - deviations from tri-bimaximal mixing
  - mixing sum rules
  - predictions for CP violation
- precision neutrino experiments can/will test family symmetry models
- family symmetries can control flavour in BSM scenarios such as SUSY

Epilogue

The order that our mind imagines is like a net, or like a ladder,
built to attain something.
But afterward you must throw the ladder away,
because you discover that, even if it was useful, it was meaningless.

William of Baskerville, The Name of the Rose