

# SM (quark) flavour theory



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# Outline

- Basics: quark flavour and CKM matrix
- Hadron flavour and QCD
- Selected observables

# Basics: flavour physics with quarks

# Flavours of quarks

$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$u_R$ $d_R$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$c_R$ $s_R$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$t_R$ $b_R$	$Q = +2/3$ $Q = -1/3$
$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	— $e_R$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	— $\mu_R$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	— $\tau_R$	$Q = 0$ $Q = -1$

Quarks are those elementary particles in the Standard Model that have spin 1/2 and strong interactions.

They come in 3 colours and six different mass eigenstates, called **flavours**: up, down, strange, charm, bottom, top in order of increasing mass. (The historical origin is ice-cream, which also carries colour and flavour, as observed by Fritzsche & Gell-Mann.)

3 of the quarks have charge +2/3 e, called “up-type” quarks

3 have charge -1/3 e, called “down-type” quarks

Only the left-chiral parts of the quark fields have SU(2) gauge interactions and couple to the W bosons

# Flavour symmetry

In modern parlance, the flavour symmetry group of a gauge theory is the largest global symmetry group (that commutes with the gauge group) of a gauge theory.

Gauge theory part of the SM Lagrangian

$$\mathcal{L}_{\text{gauge}} = \sum_f \bar{\psi}_f \gamma^\mu D_\mu \psi_f - \sum_{i,a} \frac{1}{4} g_i F_{\mu\nu}^{ia} F^{ia\mu\nu}$$

$f = Q_{Lj}, u_{Rj}, d_{Rj}, L_{Lj}, e_{Rj} \quad j = 1, 2, 3$

3 parameters only, no masses

invariant under transformations (3x3 matrices)

$$Q_L \rightarrow e^{i(b/3+a)} V_{Q_L} Q_L, \quad u_R \rightarrow e^{i(b/3-a)} V_{u_R} u_R, \quad d_R \rightarrow e^{i(b/3-a)} V_{d_R} d_R$$

$$G_{\text{flavor}} = SU(3)^5 \times U(1)_B \times U(1)_A \times U(1)_L \times U(1)_E$$

[Chivukula & Georgi 1987]

nb - anomalies reduce the 4 U(1) factors to U(1)<sub>B-L</sub>

broken by Yukawa couplings to Higgs field

$$\mathcal{L}_Y = -\bar{u}_R Y_U \phi^{c\dagger} Q_L - \bar{d}_R Y_D \phi^\dagger D_L - \bar{e}_R Y_E \phi^\dagger E_L$$

to

$$U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$$

(ignoring anomalies)

To go to mass eigenstates, need to diagonalise the Yukawa matrices. Achieved by simultaneous U(3) transformations:

$$u_L \rightarrow V_{u_L} u_L \quad u_R \rightarrow V_{u_R} u_R \quad Y_U \rightarrow V_{u_R} Y_U V_{u_L}^\dagger = Y_U^{\text{diag}}$$

$$d_L \rightarrow V_{d_L} d_L \quad d_R \rightarrow V_{d_R} d_R \quad Y_D \rightarrow V_{d_R} Y_D V_{d_L}^\dagger = Y_D^{\text{diag}}$$

If  $V_{u_L}$  and  $V_{d_L}$  are different, this is **not** a flavour symmetry transformation. W couplings become non-diagonal:

$$W_\mu^+ \bar{u}_L \gamma^\mu d_L \rightarrow W_\mu^+ \bar{u}_L \underbrace{V_{u_L}^\dagger V_{d_L}} \gamma^\mu d_L$$

CKM matrix

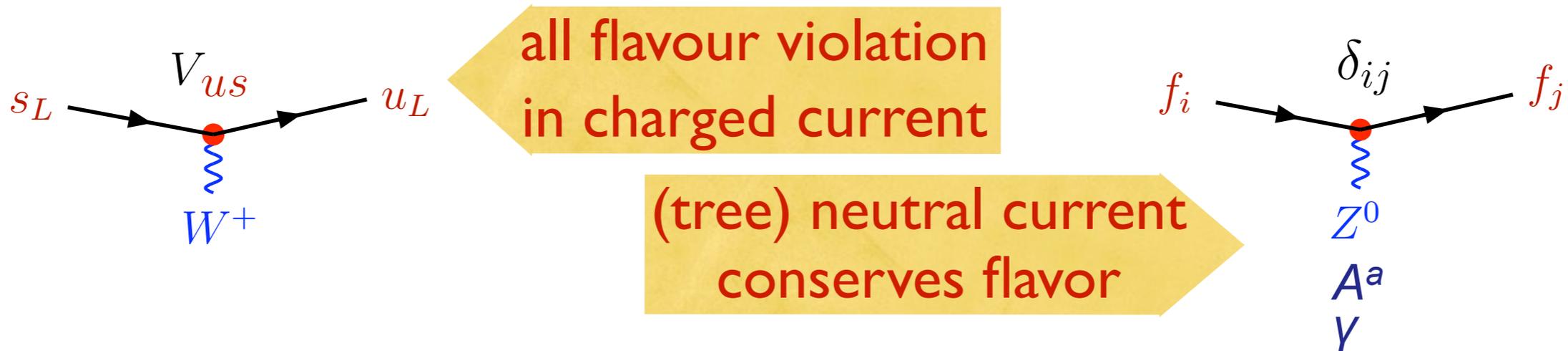
weak interactions violate flavour

# Quiz

1) Why do the Z couplings remain generation diagonal?

2) What would happen if there were only one quark SU(2) doublet and one further left-handed down-type singlet  $s'_L$  ? (Plus right-handed singlets)

# SM flavour: CKM matrix



$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \text{unitary matrix}$$

phase changes  $u_{Li} \rightarrow e^{i\alpha_i} u_{Li}, \quad u_{Ri} \rightarrow e^{i\alpha_i} u_{Ri}$

$$d_{Li} \rightarrow e^{i\beta_i} d_{Li}, \quad d_{Ri} \rightarrow e^{i\beta_i} d_{Ri}$$

change CKM (eg can use to make some CKM elements real+)

9 independent parameters in a unitary matrix  
 - (6 - 1) rephasings (1 universal rephasing is symmetry)  
 = 4 physical CKM parameters (3 angles, 1 phase)

weak interactions violate CP

# Quiz

3) Why can the work of Kobayashi and Maskawa (1972) be viewed as a prediction of a third SM generation?  
(CP violation was first observed in 1964.)

# Parameterisations

Standard parameterisation in terms of three angles  $\theta_{12}, \theta_{13}, \theta_{23}$  similarly to Euler angles, and a phase  $\delta$  (Particle data group)

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

empirically,  $s_{13} \ll s_{23} \ll s_{12} \ll 1$ ,  $V_{\text{CKM}}$  not far from diagonal

Wolfenstein parameters (phase convention independent)

$$\lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} \quad A = \frac{1}{\lambda} \frac{|V_{cb}|}{|V_{us}|} \quad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

[L Wolfenstein; Buras et al; PDG]

One can expand the CKM matrix in these:

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda = 0.22457^{+0.00186}_{-0.00014}, \quad A = 0.823^{+0.012}_{-0.033}, \quad \bar{\rho} = 0.1289^{+0.0176}_{-0.0094}, \quad \bar{\eta} = 0.348^{+0.012}_{-0.012}$$

nucl. beta decay, K decays, semileptonic b->c decays  
very precisely determined

[CKMfitter collaboration, FPCP13]

# Unitary triangle

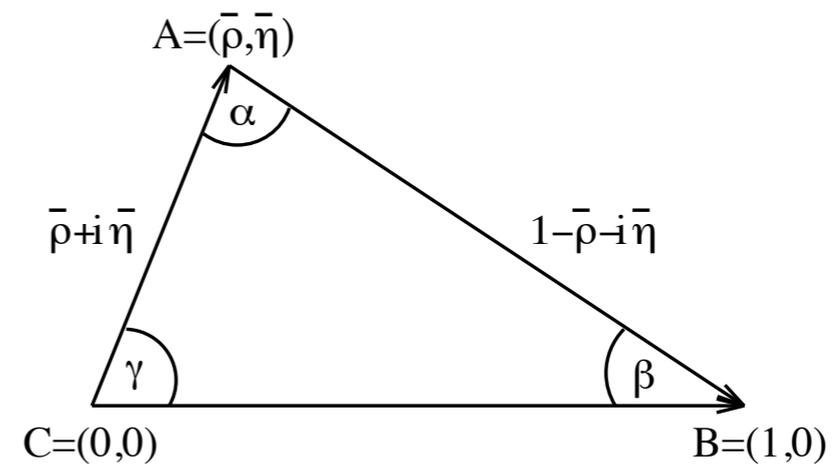
Unitarity implies orthogonality of any two rows or columns of CKM

Each such relation can be viewed as a closed triangle in the complex plane

$$V_{ud}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

$$-\frac{V_{ud}V_{ub}^*}{V_{cs}V_{cb}^*} - \frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} = 1$$

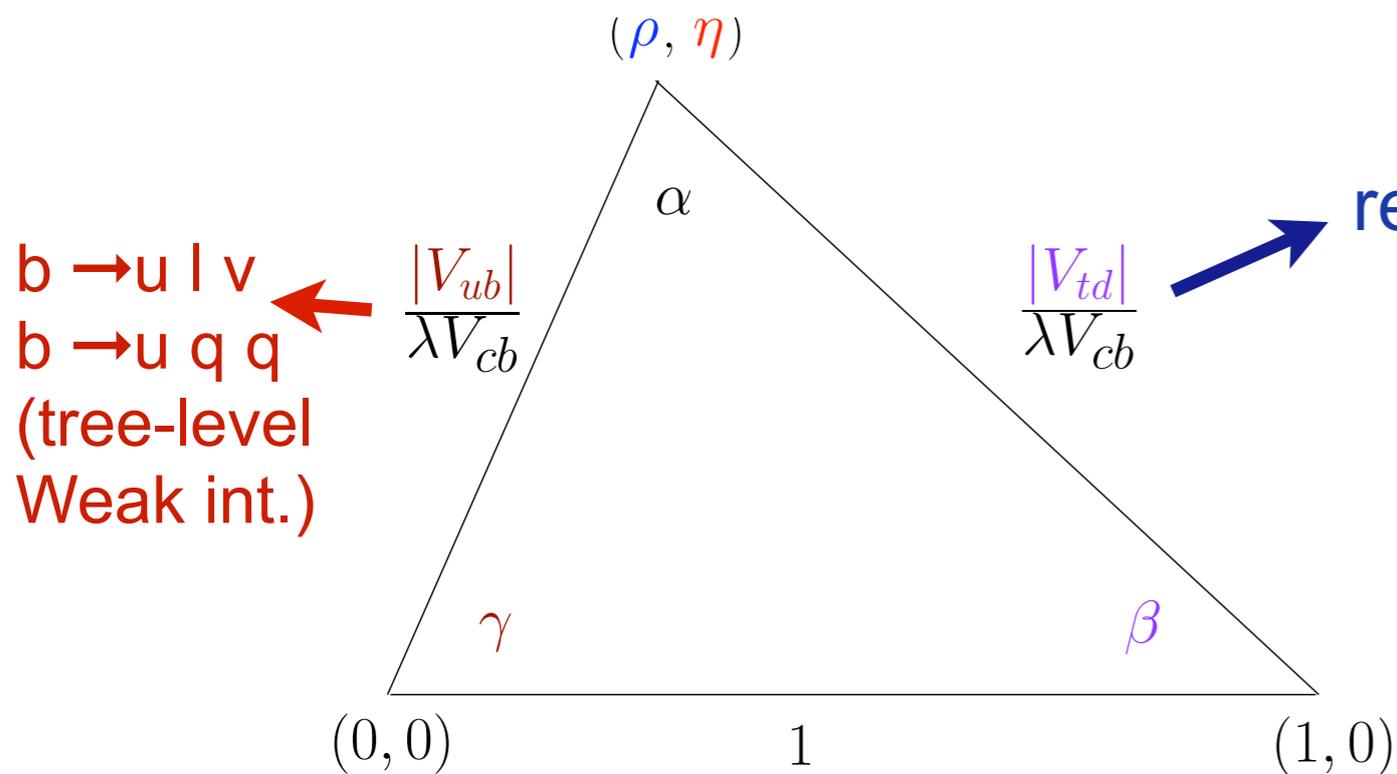
$$\bar{\rho} + i\bar{\eta} + (1 - \bar{\rho} - i\bar{\eta}) = 1$$



[fig.A Buras]

The form with one side normalised to 1 is most common and justified as the rescaling factor is quite precisely known (and largely insensitive to beyond-SM physics)

# Unitarity triangle determination

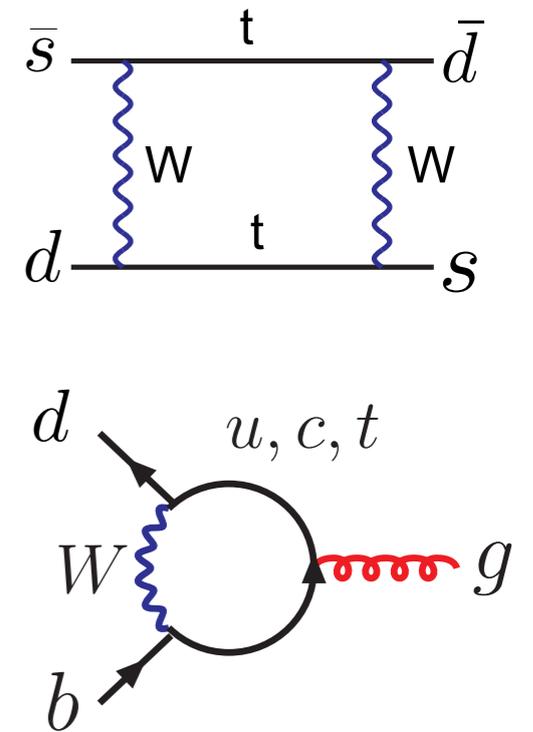


$b \rightarrow u \ell \nu$   
 $b \rightarrow u q \bar{q}$   
 (tree-level  
 Weak int.)

requires top loop

$$V_{ub} = |V_{ub}|e^{-i\gamma}$$

$$V_{td} = |V_{td}|e^{-i\beta}$$



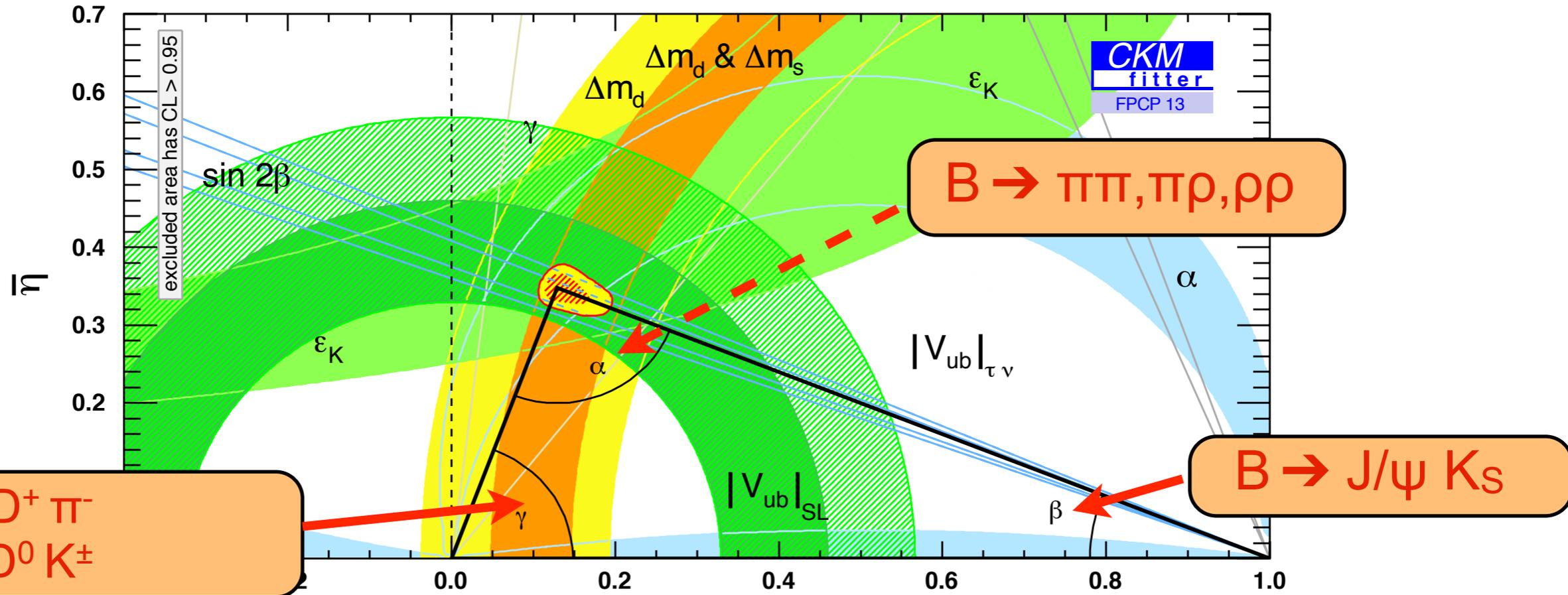
suppression of FCNC by loops and CKM hierarchy

This makes them sensitive to new physics!

# Global fits to the CKM matrix by two expert groups

CKMfitter <http://ckmfitter.in2p3.fr/>

UTfit <http://utfit.org/>



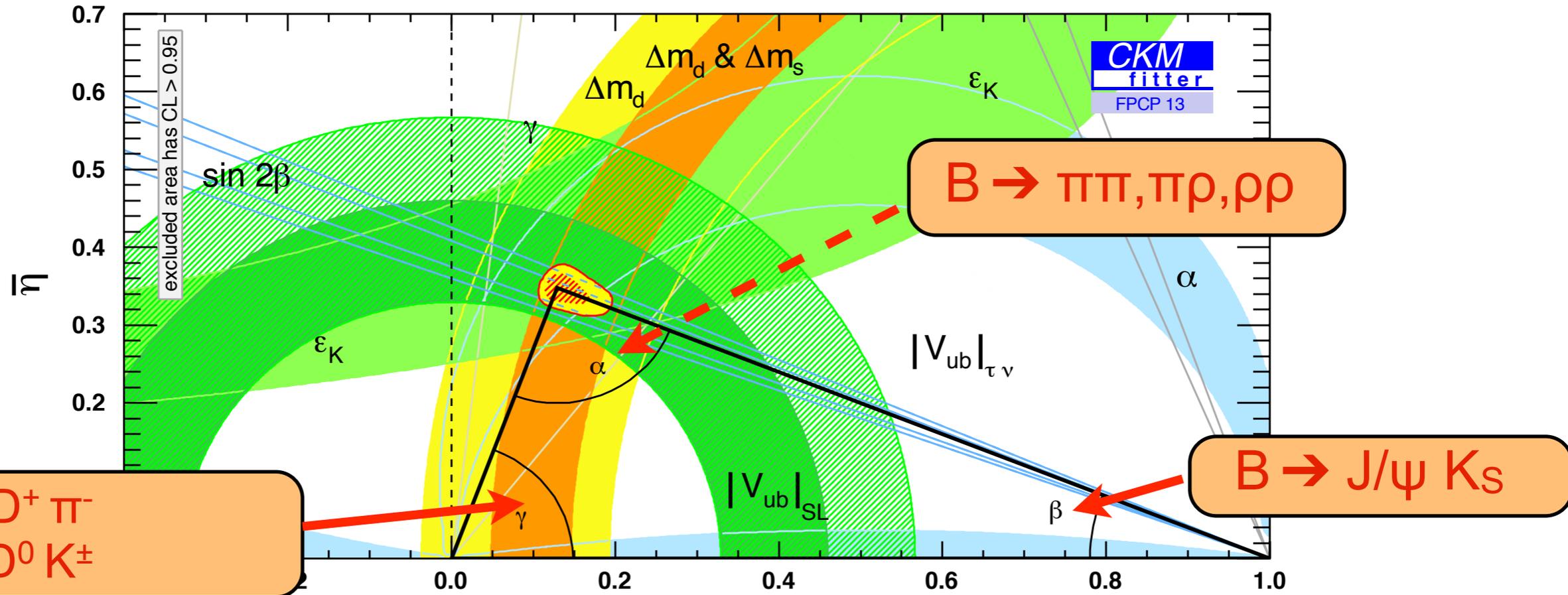
Each observable constrains  $\bar{\rho} + i\bar{\eta}$  to lie on a one-dimensional set (one or more lines). Bands due to uncertainties (theory & expt)

No apparent inconsistencies, CKM paradigm appears to work

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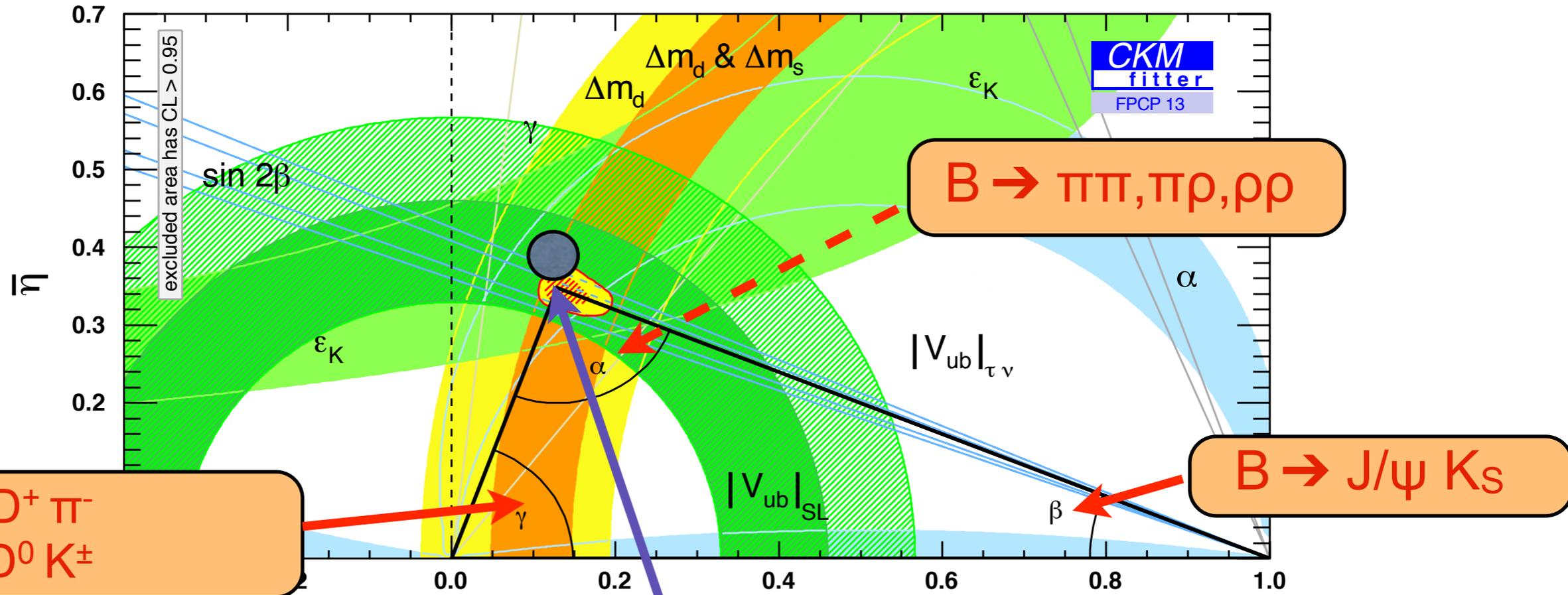
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Of all constraints, **only** the  $\gamma$  and  $|V_{ub}|$  determinations are robust against new physics as they do not involve loops.

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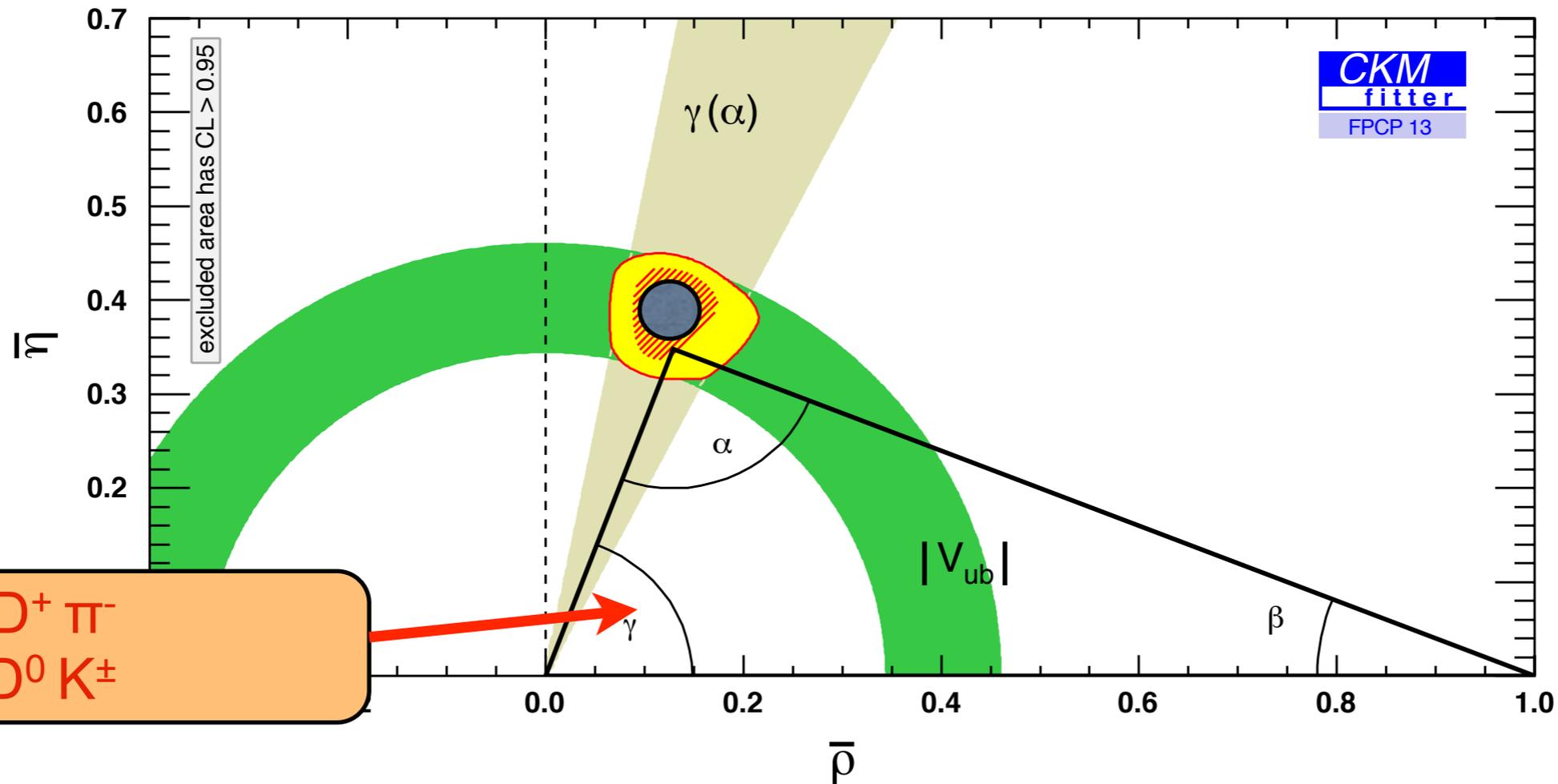
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No apparent inconsistencies, CKM paradigm appears to work

Of all constraints, **only** the  $\gamma$  and  $|V_{ub}|$  determinations are robust against new physics as they do not involve loops.

It is possible that the TRUE  $(\bar{\rho}, \bar{\eta})$  lies here (for example)

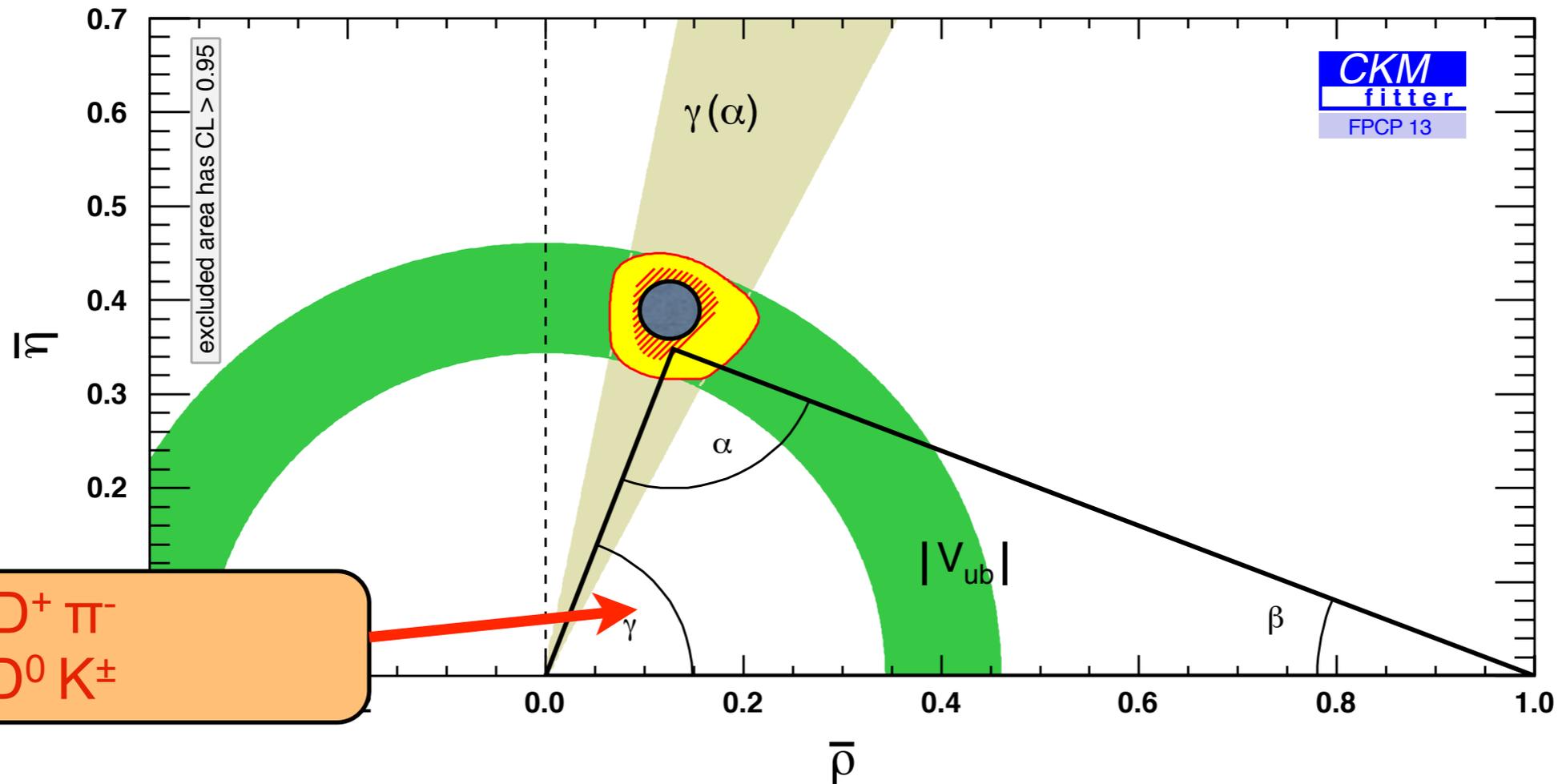
# “Tree” determinations



Plot showing only “NP-robust” measurements of  $\gamma$  and  $|V_{ub}|$ .

*Note: the  $\gamma(\alpha)$  constraint shown depends on assumptions (absence of BSM  $\Delta I=3/2$  contributions in  $B \rightarrow \pi\pi$ ); the “pure tree-level”  $\gamma$  determination (grey band) is more robust. Such determinations will be greatly improved by LHCb.*

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Certainly there is room for  $O(10\%)$  NP in loop processes as far as UT fits are concerned, moreover UT fit mainly constrains  $b \rightarrow d$

# Reality: flavour physics with hadrons

using mixing as example

observables

short and long distances

effective Hamiltonian

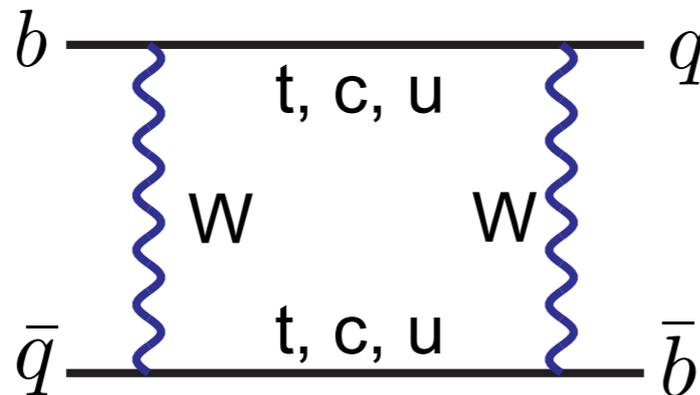
# Meson mixing

flavour violation implies particle-antiparticle mixing:

$$i \frac{\partial}{\partial t} \begin{pmatrix} B_q^0(t) \\ \bar{B}_q^0(t) \end{pmatrix} = \left( \hat{M}^q - \frac{i}{2} \hat{\Gamma}^q \right) \begin{pmatrix} B_q^0(t) \\ \bar{B}_q^0(t) \end{pmatrix}$$

$$\mathcal{A}(\bar{M}^0 \rightarrow M^0) \propto M_{12} - \frac{i}{2} \Gamma_{12} \neq 0$$

if we view a B meson as a quark-antiquark two-particle state:



similarly for  $K^0$ ,  $D^0$  mixing

The diagram is easy to calculate.

However, mesons are complicated QCD bound states, so QCD corrections must be understood

# Mixing: observables

$$\mathcal{A}(\bar{M}^0 \rightarrow M^0) \propto M_{12} - \frac{i}{2}\Gamma_{12} \neq 0$$

Three physical parameters (for each neutral meson system)

$$|M_{12}|, |\Gamma_{12}|, \phi \equiv \arg \frac{-M_{12}}{\Gamma_{12}}$$

give rise to three independent observables

$$\Delta M \approx 2|M_{12}|, \quad \Delta\Gamma \approx 2|\Gamma_{12}| \cos \phi, \quad a_{\text{fs}} = \frac{\Delta\Gamma}{\Delta M} \tan \phi$$

mass difference      width difference      flavour-specific  
(lifetime difference)      CP asymmetry

$a_{\text{fs}}$  can be measured from the CP asymmetry in any “flavour-specific” decay, ie one which vanishes in the absence of mixing.

$$B \rightarrow \bar{B} \rightarrow f$$

$$a_{\text{fs}}^q = \frac{\Gamma(\bar{B}_q^0(t) \rightarrow f) - \Gamma(B_q^0(t) \rightarrow \bar{f})}{\Gamma(\bar{B}_q^0(t) \rightarrow f) + \Gamma(B_q^0(t) \rightarrow \bar{f})} \quad (\text{nb - time dependence cancels between numerator and denominator})$$

Often  $f=X\mu^+$  - self-tagging. Hence “semileptonic CP as.”  $a_{\text{sl}} = a_{\text{fs}}$



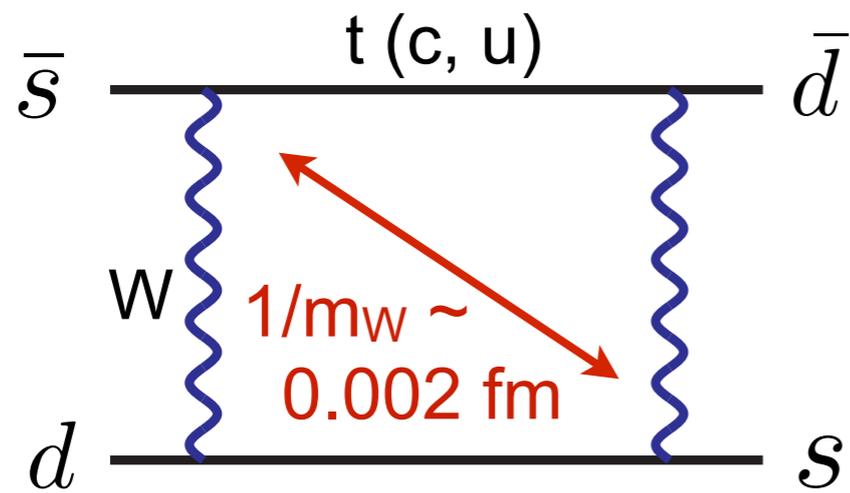
**short distance**

**VS**

**long distance**



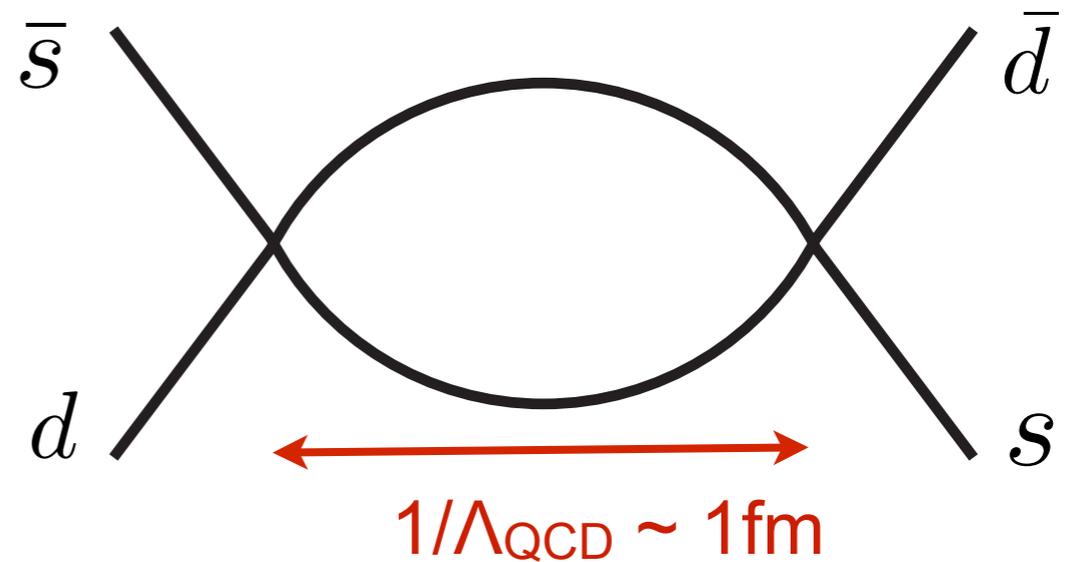
# Digression: $K^0 - \bar{K}^0$ mixing



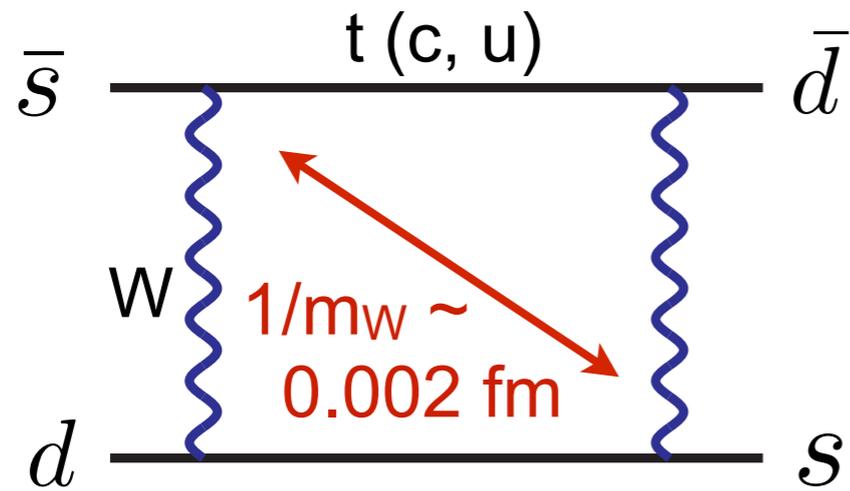
**short distance**  
perturbation theory applies

**VS**

**long distance**  
non-perturbative QCD essential



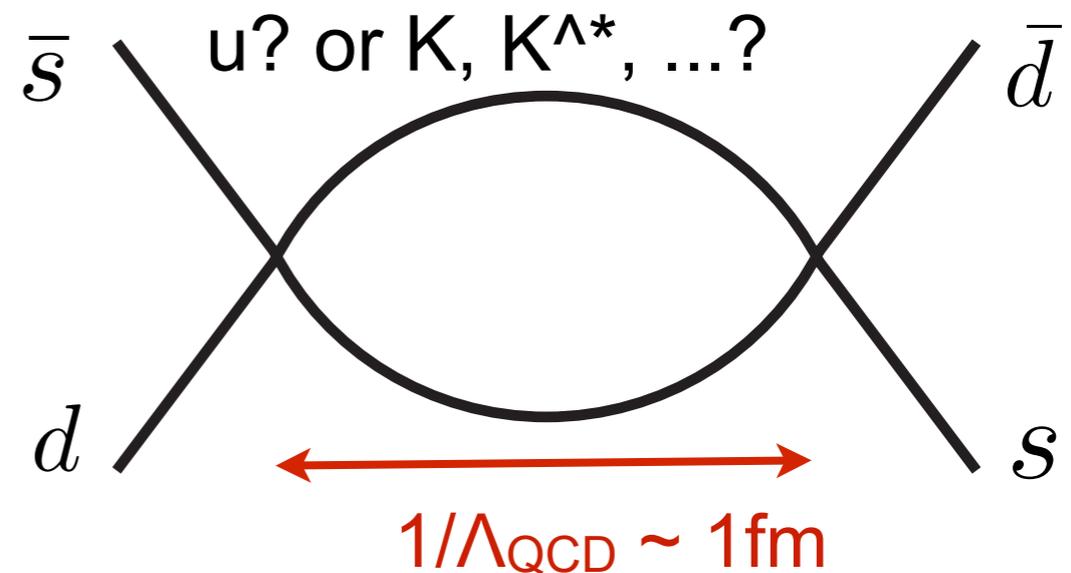
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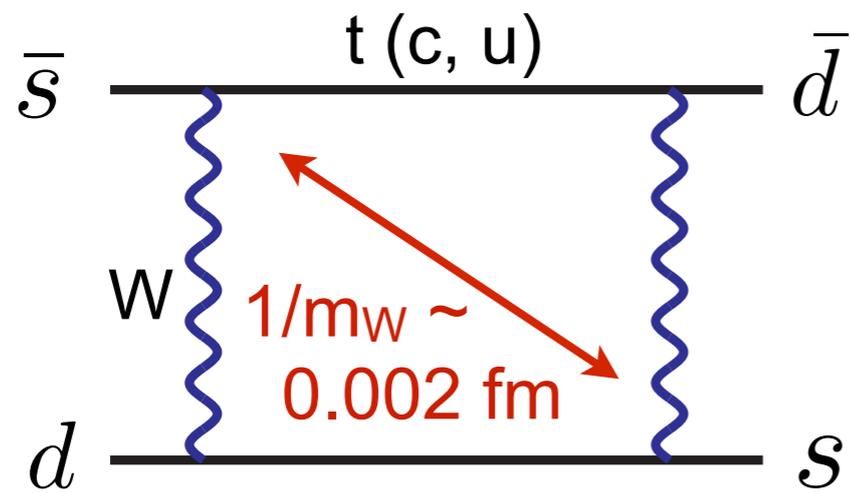
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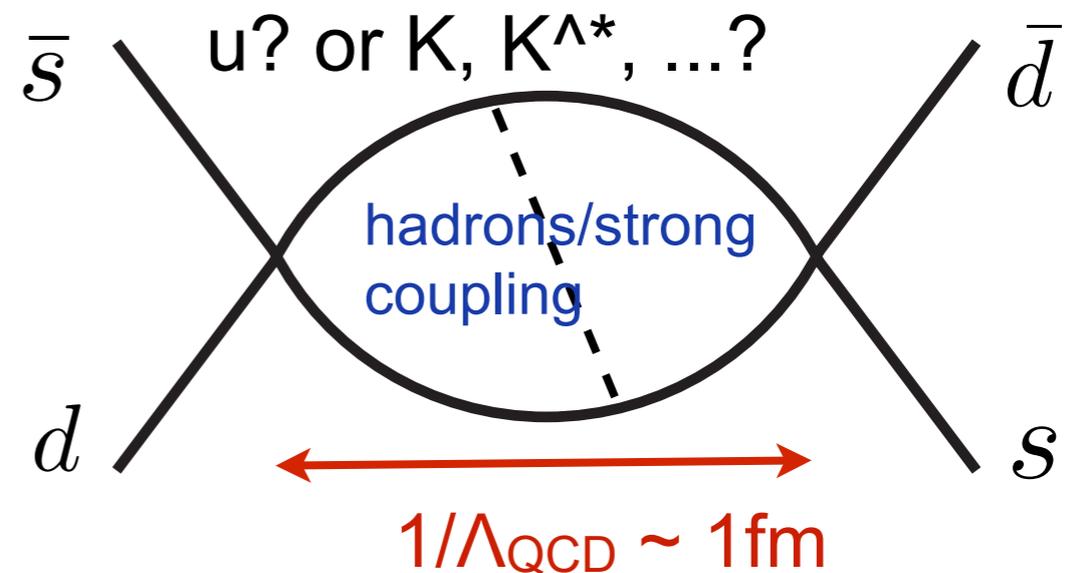
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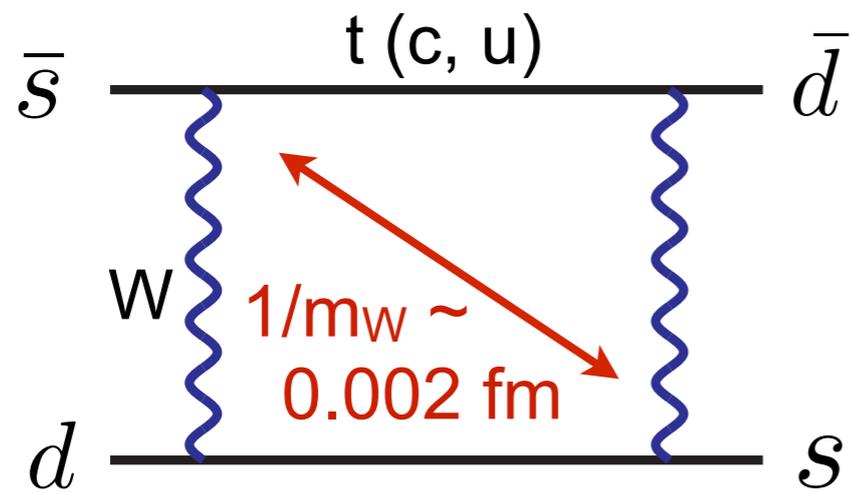
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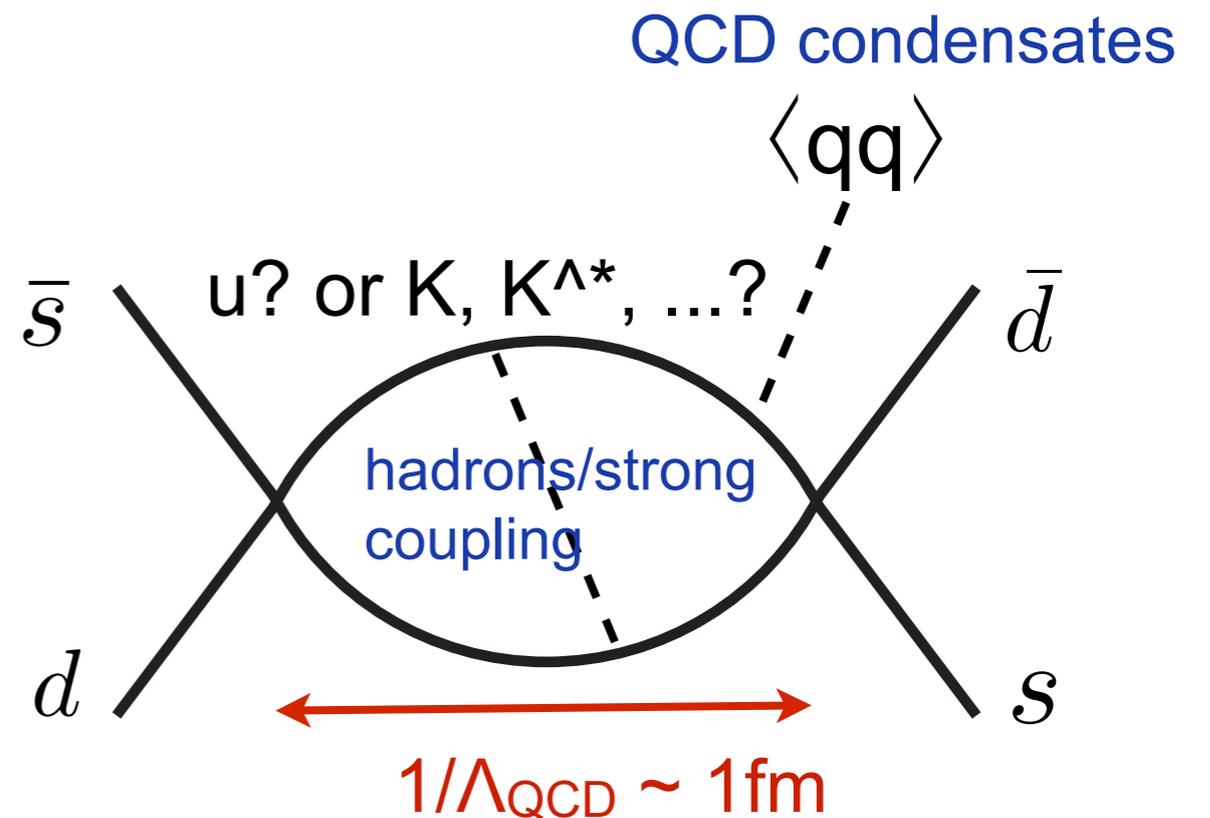
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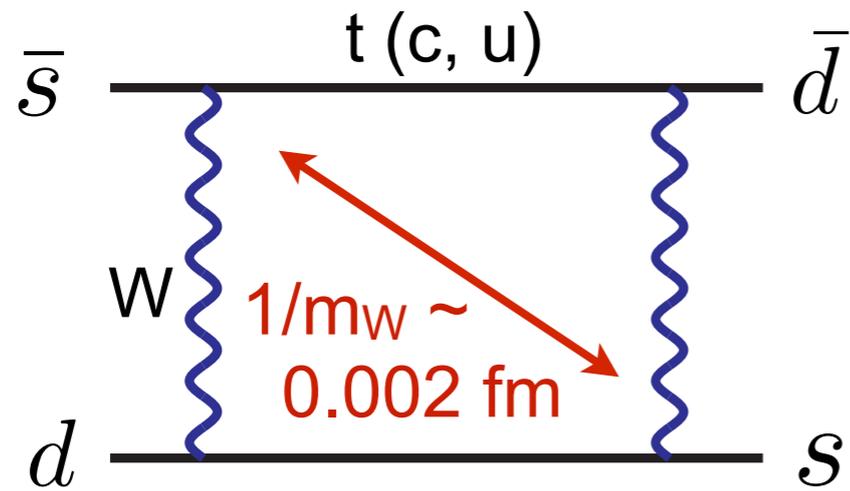
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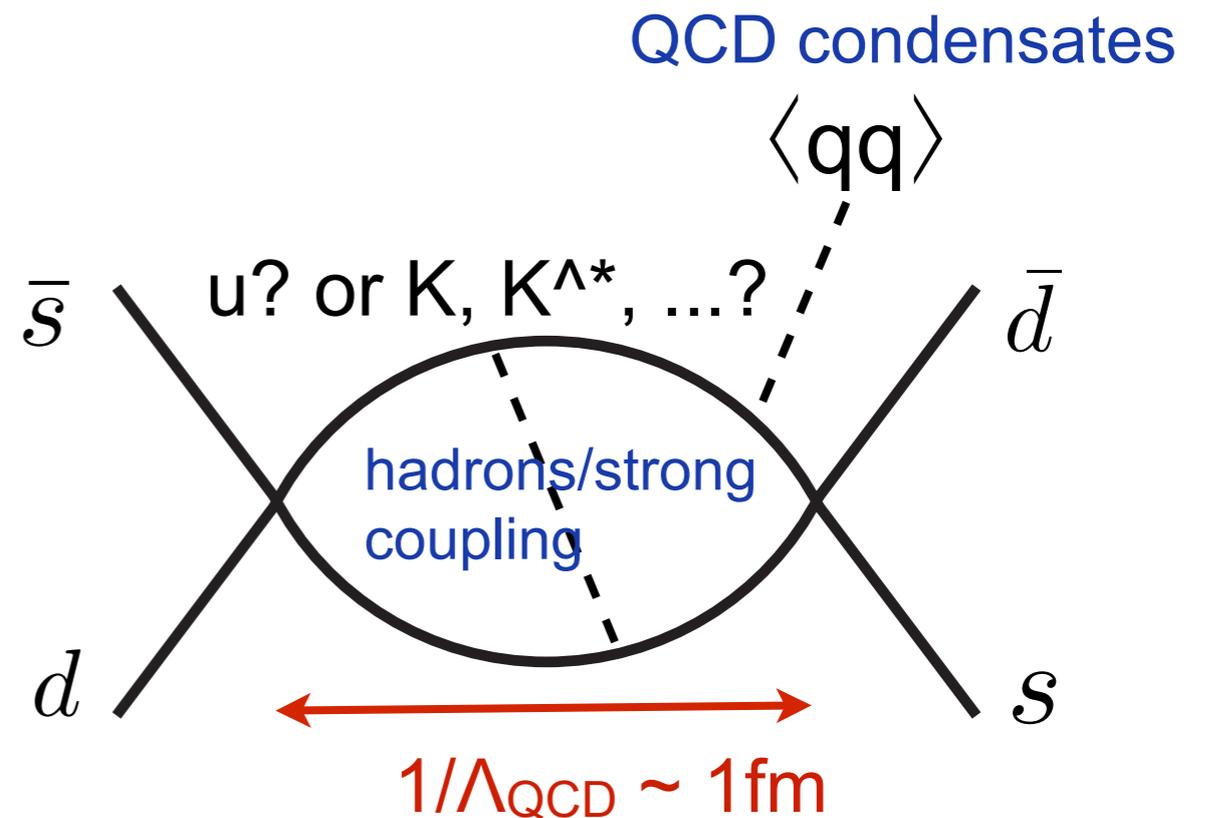
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**short distance**  
perturbation theory applies

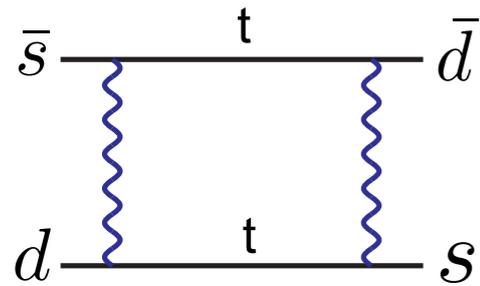
**VS**

**long distance**  
non-perturbative QCD essential

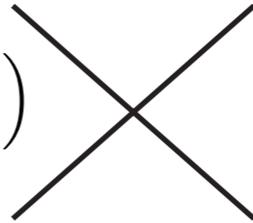


which dominates ?

# $K^0 - \bar{K}^0$ mixing

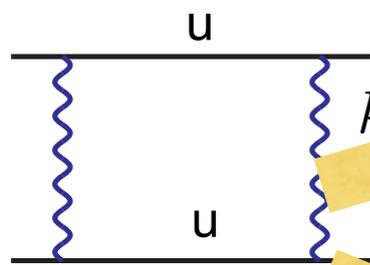


$$\propto (V_{ts}V_{td}^*)^2 \frac{1}{16\pi^2} \frac{1}{M_W^2} \left( \frac{m_t^2}{M_W^2} + \dots \right)$$



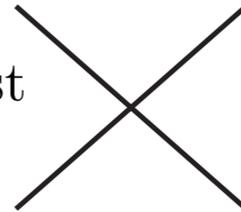
$$\sim \frac{10^{-6}}{M_W^2}$$

top quark loop  
CKM-suppressed



$k^2 \sim M_W^2$

$$\propto (V_{us}V_{ud}^*)^2 \frac{1}{16\pi^2} \frac{1}{M_W^2} \times \text{const}$$

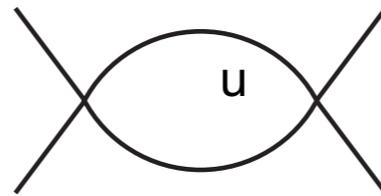


$$\sim \frac{10^{-4}}{M_W^2}$$

light quark loop  
CKM-enhanced

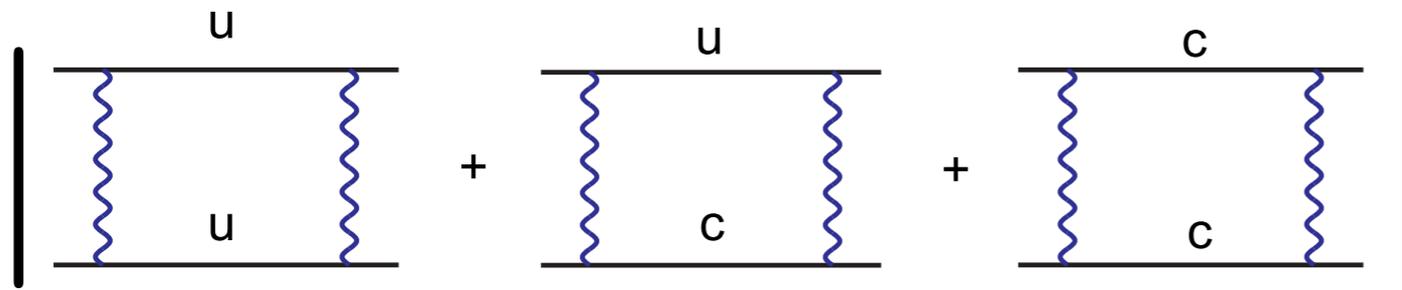
$k^2 \sim \Lambda_{\text{QCD}}^2$

$$\propto (V_{us}V_{ud}^*)^2 \frac{1}{M_W^4}$$



$$\propto (V_{us}V_{ud}^*)^2 \frac{\Lambda_{\text{QCD}}^2}{M_W^4}$$

long-distance  
power-suppressed  
but CKM-enhanced



currently incalculable

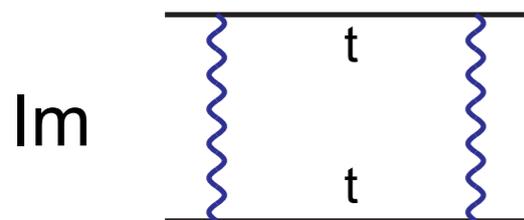
$$= \mathcal{O}(\Delta m_K^{\text{exp}}) \quad \text{if } m_c \lesssim \text{GeV}$$

$\Delta m_K$  long-distance dominated

$$(V_{us}V_{ud}^*)^2 + 2(V_{us}V_{ud}^*)(V_{cs}V_{cd}^*) + (V_{cs}V_{cd}^*)^2 = (V_{ts}V_{td}^*)^2$$

unitarity / GIM  
cancellation

CPV vanishes for 2 generations, so **must** involve top in the loop



**CPV in mixing short-distance dominated**

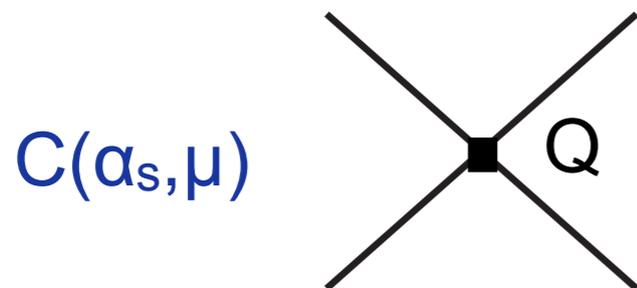
$$\Rightarrow \epsilon_K = \mathcal{O}(10^{-3})$$

constraint on  $V_{td}$



# Renormalisation group & hadronic matrix elements

The Wilson coefficients depend on a renormalisation scale  $\mu$  which enters in the course of renormalising divergences appearing from two loops. It has a physical, Wilsonian, interpretation as a cut-off on a low-energy effective theory, wherein a new coupling constant  $C(\mu)$  contains the physics from quantum fluctuations with  $k > \mu$ . We can lower  $\mu$  changing  $C(\mu)$  in a calculable manner (analogous to the running gauge couplings)



At  $\mu \sim 2$  GeV, you can now ask your lattice theory friend to calculate the matrix element  $\langle M | Q(\mu) | \bar{M} \rangle \equiv \frac{2}{3} f_M^2 m_M^2 B_M(\mu)$

and subsequently you can calculate the local part of the mixing amplitude as  $\mathcal{A}(\bar{M} \rightarrow M) = C(\mu) \langle M | Q(\mu) | \bar{M} \rangle$

This largely carries over to leptonic and semileptonic decays.

# CP violation in $K^0 - \bar{K}^0$ mixing

For CP violation in Kaon mixing,

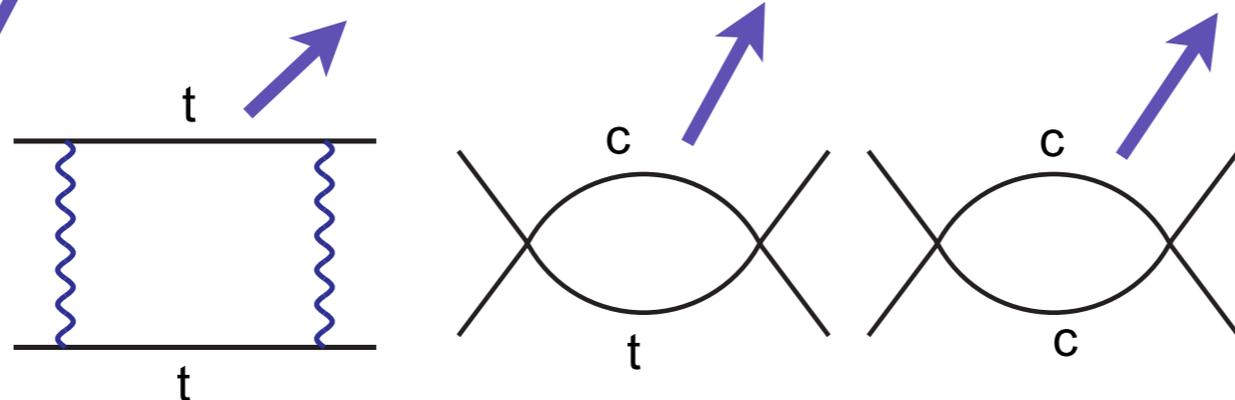
$$\epsilon_K = e^{i\phi_\epsilon} \sin \phi_\epsilon \left( \frac{\text{Im}(M_{12}^K)}{\Delta M_K} + \xi \right)$$

$$\hat{B}_K \propto \langle \bar{K} | \text{---} \times \text{---} | K \rangle$$

hadronic matrix element of the local operator Q (nonperturbative)

lattice calculation & (typically) perturbative continuum conversion

$$|\epsilon_K^{\text{SM}}| = \kappa_\epsilon C_\epsilon \hat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta} \times (|V_{ct}|^2 (1 - \bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c)$$



short-distance “local” contributions including higher-order perturbative QCD corrections

correction factor from non-local contributions

$$\kappa_\epsilon = 0.94 \pm 0.02 \quad \text{Buras, Guadagnoli, Isidori 10}$$

# B physics

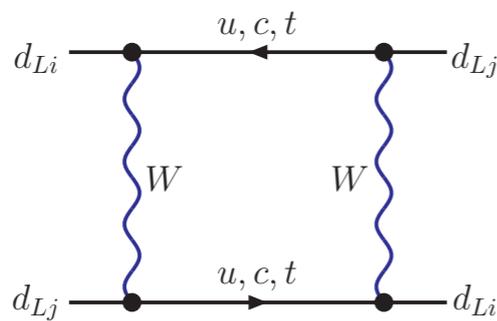
In  $B_d$  physics the CKM hierarchy is mild, and in  $B_s$  physics it is reversed. Hence, the nonlocal contributions are small enough to allow for a calculation of the mass differences.

Moreover, the b quark mass is large enough for an expansion in  $\Lambda/m_b$

This allows theoretical access to the lifetime difference, and is applicable to certain types of B decays with one or two hadrons in the final state.

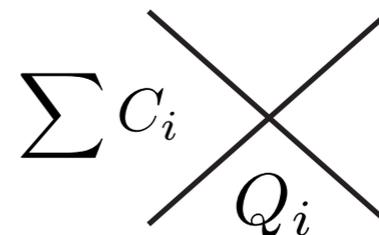
# $B_{(s)} - \bar{B}_{(s)}$ mixing

- flavour violation:  $\mathcal{A}(\bar{M}^0 \rightarrow M^0) \propto M_{12} - \frac{i}{2}\Gamma_{12} \neq 0$



+

OPE ( $m_B/m_W$ )



mixing-induced CP violation



$M_{12}$



$$\Delta M = 2|M_{12}|$$

$$Q_1 = (\bar{s}_L^a \gamma_\mu b_L^a)(\bar{s}_L^b \gamma^\mu b_L^b)$$

only operator present in SM

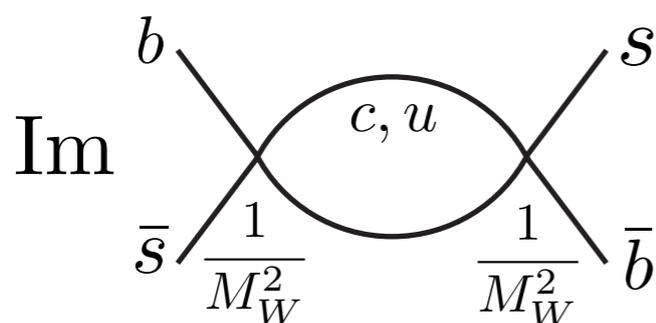
$$Q_2 = (\bar{s}_R^a b_L^a)(\bar{s}_R^b b_L^b),$$

$$Q_3 = (\bar{s}_R^a b_L^b)(\bar{s}_R^b b_L^a),$$

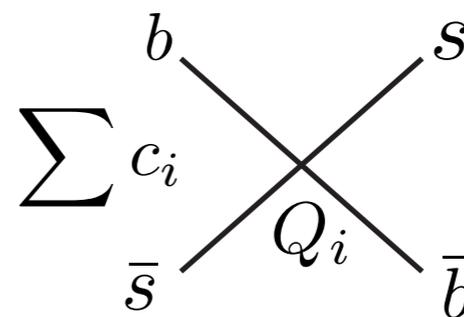
$$Q_4 = (\bar{s}_R^a b_L^a)(\bar{s}_L^b b_R^b),$$

$$Q_5 = (\bar{s}_R^a b_L^b)(\bar{s}_L^b b_R^a)$$

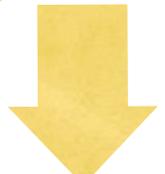
+ 3 more



OPE ( $\Lambda_{\text{QCD}}/m_B$ )



$\Gamma_{12}$

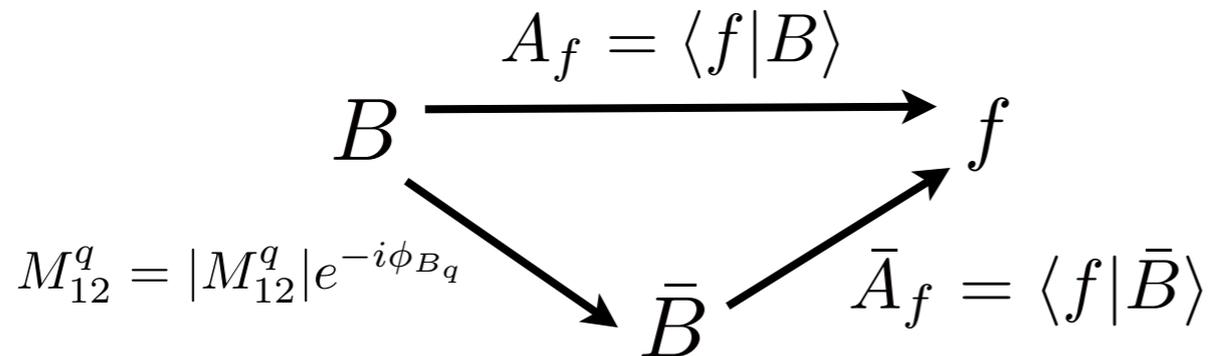


$\Delta\Gamma$

no NP contribution to  $\Gamma_{12}$  unless NP lighter than  $m_B$  or NP significantly affects  $b \rightarrow c$  decays (which are tree-level size in the SM)

# Time-dependent CP asymmetry

decay into CP eigenstate:



$$\lambda_f = e^{i\phi_{B_q}} \frac{\langle f | \bar{B}_q^0 \rangle}{\langle f | B_q^0 \rangle}$$

CP-violation parameter

$$A_f^{\text{CP}}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow f)}{\Gamma(\bar{B}^0(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow f)} = S_f \sin(\Delta M t) - C_f \cos(\Delta M t)$$

$$S_f = \frac{2 \text{Im } \lambda_f}{1 + |\lambda_f|^2}$$

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}$$

if only one decay amplitude:

$$A_f = A e^{i\theta} \quad \bar{A}_f = A e^{-i\theta} \quad C_f = 0 \quad -\eta_{\text{CP}}(f) S_f = \sin(\phi_{B_q} + 2\theta)$$

$$B_d^0 \rightarrow \psi K_S \quad S = \sin(\phi_{B_d}) = \sin(2\beta)$$

Beyond SM  $\phi_{B_d} \neq 2\beta$

$$B_d^0 \rightarrow \pi\pi, \pi\rho, \rho\rho \quad S = \sin(\phi_{B_d} + 2\gamma) = -\sin(2\alpha)$$

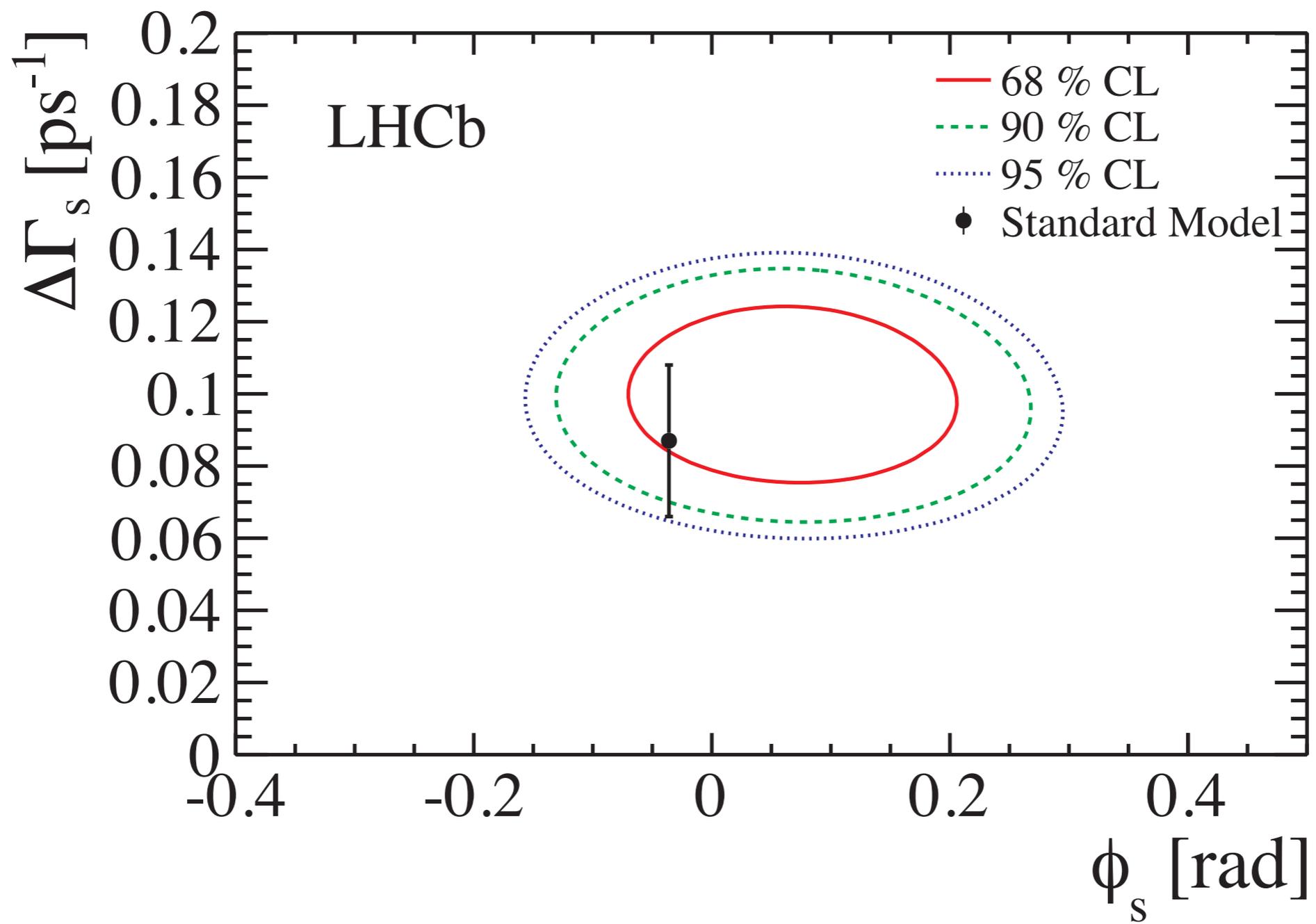
$$B_s^0 \rightarrow J/\psi \phi \quad \pm S = \sin \phi_{B_s} \approx 0$$

Beyond SM  $\phi_{B_s} \neq 0$

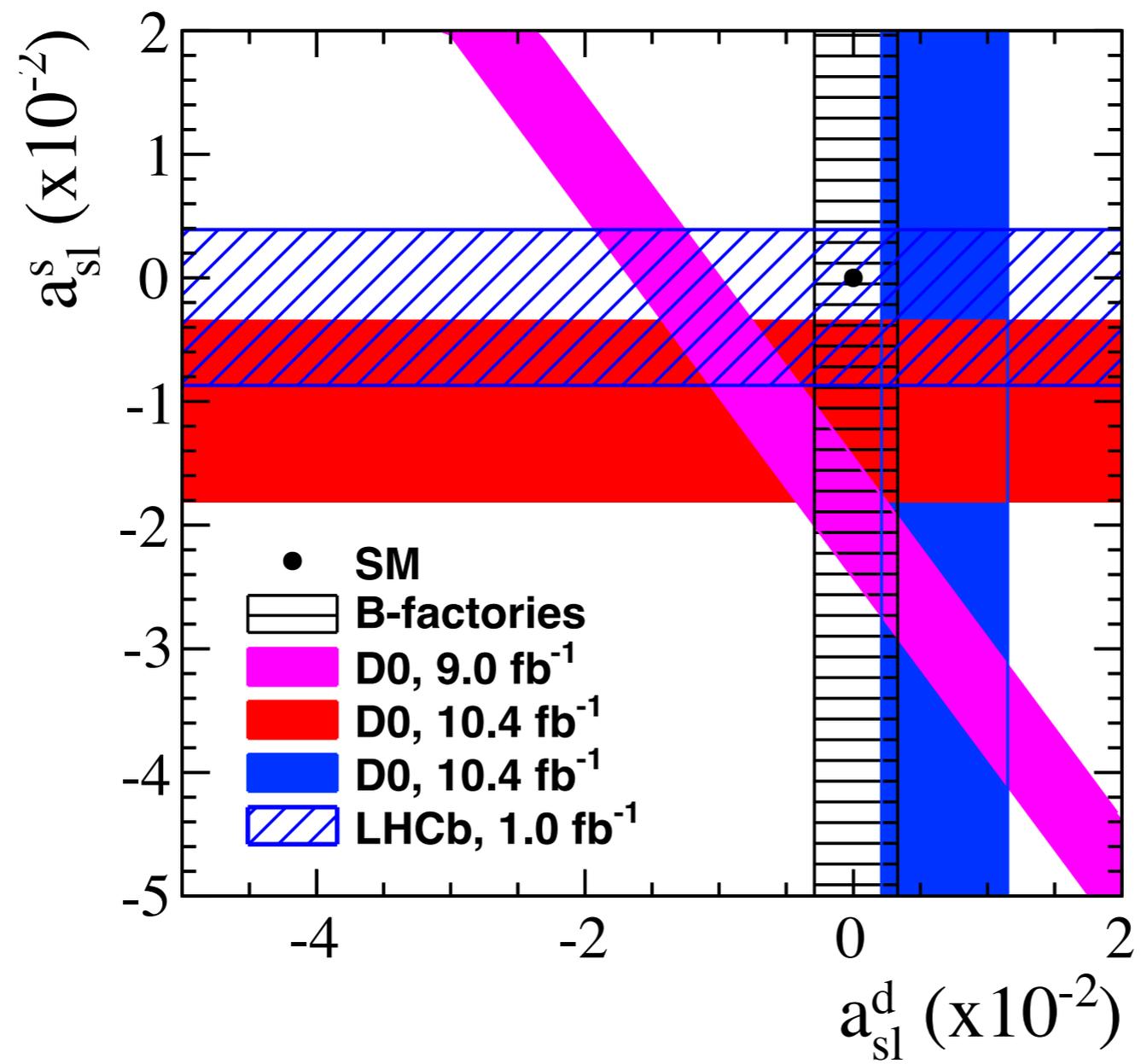
can be generalized to non-CP final states

$$\phi_{B_{d,s}} + \gamma \text{ from } B_{(s)}^0 \rightarrow D_{(s)} K$$

Note: The phase  $\phi_{B_q}$  on this slide is the phase of  $M_{12}$  in the standard parameterisation. For  $B_s$ , this is very close to  $\phi$  entering  $a_{fs}$  (also beyond SM)



[LHCb, arXiv:1304.2600v3]



[LHCb, arXiv:1306.0092v1]

# Outlook

This was but a brief introduction to a huge and active field.

On the technical side:

- BSM effects in Wilson coefficients (see David Straub's lecture)
- Factorisation methods (based on  $\Lambda/m_B$  expansions)
- Flavour symmetry ( $SU(3)_F$ ) methods

Key observables include

- many B decays accessible at LHCb, eg exclusive leptonic & semileptonic (e.g.  $B \rightarrow K^* \mu^+ \mu^-$ , which shows interesting features in exp. data - sadly no time to cover)
- CP violation in K decays, D decays

Another area that has seen (and is seeing) enormous theoretical activity are inclusive B decays such as  $B \rightarrow X_s \gamma$ . Accessible at lepton colliders (Babar, Belle, future Belle2)

These, as mixing, provide powerful constraints on, and vehicles to discovery of BSM physics (see David Straub's lecture)

# Reading

Here are a few examples out of many useful resources

Conventions and data:

PDG review “The CKM quark mixing matrix”, <http://pdg.lbl.gov/> (go to  
Reviews, Tables, Plots -> Standard Model and Related Topics)

CKM fitter site: <http://ckmfitter.in2p3.fr/>

UTfit web site: <http://utfit.org/>

Heavy flavour averaging group: <http://www.slac.stanford.edu/xorg/hfag/>

Technicalities of weak Hamiltonian, RGE, etc

A Buras, Les Houches lectures “Weak Hamiltonian, CP violation and Rare decays”, arXiv:hep-ph/984071, very detailed and pedagogical

G Buchalla, A Buras, M Lautenbacher, Rev Mod Phys 68 (1996) 1125

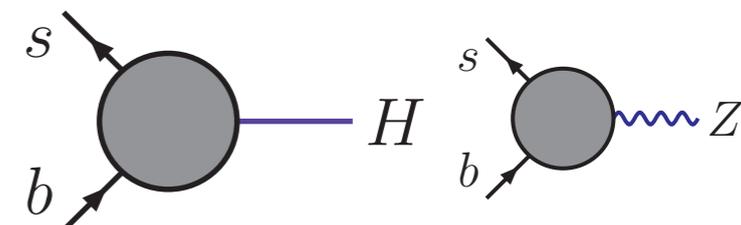
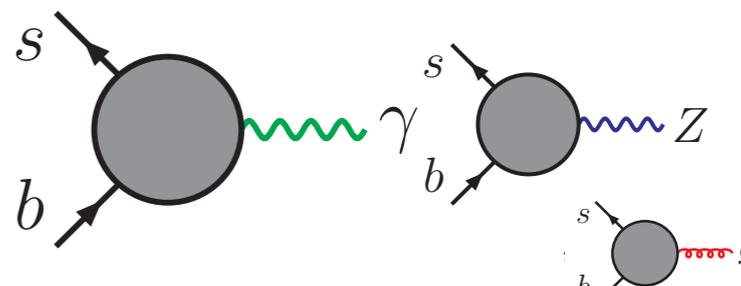
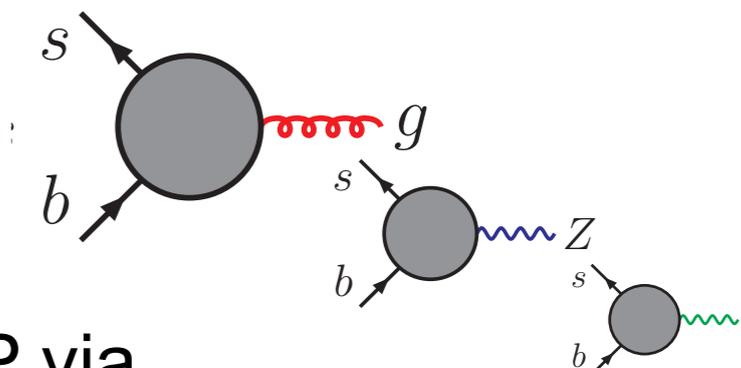
More recent, with more of a new-physics focus

Y Nir, lectures at the 2007 CERN summer school, arXiv:0708.1872

G Isidori, lectures at the 2011 CERN summer school, arXiv:1302.0661

Backup

# Exclusive decays at LHCb

final state	strong dynamics	#obs	NP enters through
Leptonic $B \rightarrow l^+ l^-$	decay constant $\langle 0   j^\mu   B \rangle \propto f_B$	$O(1)$	
semileptonic, radiative $B \rightarrow K^* l^+ l^-, K^* \gamma$	form factors $\langle \pi   j^\mu   B \rangle \propto f^{B\pi}(q^2)$	$O(10)$	
charmless hadronic $B \rightarrow \pi\pi, \pi K, \rho\rho, \dots$	matrix element $\langle \pi\pi   Q_i   B \rangle$	$O(100)$	

All non-radiative modes are also sensitive to NP via four-fermion operators

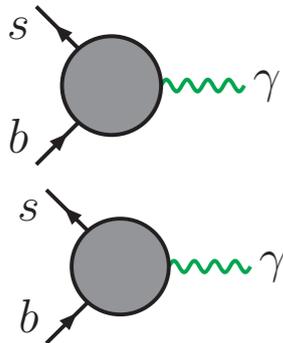
Decay constants and form factors are essential. Accessible by QCD sum rules and, increasingly, by lattice QCD.

# weak $\Delta B=\Delta S=1$ Hamiltonian

= EFT for  $\Delta B=\Delta S=1$  transitions (up to dimension six)

$$\mathcal{H}_{\text{eff}}^{\text{had}} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[ C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3\dots 6} C_i P_i + C_{8g} Q_{8g} \right] \quad C_i \sim g_{\text{NP}} \frac{m_W^2}{M_{\text{NP}}^2}$$

$$\mathcal{H}_{\text{eff}}^{\text{sl}} = -\frac{4G_F}{\sqrt{2}} \lambda_t \left[ C_7 Q_{7\gamma} + C'_7 Q'_{7\gamma} + C_9 Q_{9V} + C'_9 Q'_{9V} + C_{10} Q_{10A} + C'_{10} Q'_{10A} + C_S Q_S + C'_S Q'_S + C_P Q_P + C'_P Q'_P + C_T Q_T + C'_T Q'_T \right].$$

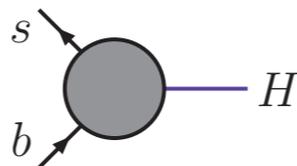


$$\mathcal{O}_7 = \frac{e}{16\pi^2} \hat{m}_b \bar{s} \sigma_{\mu\nu} P_R F^{\mu\nu} b,$$

$$\mathcal{O}_V = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{l} \gamma^\mu l),$$

$$\mathcal{O}_S = \frac{\alpha_{\text{em}}}{4\pi} \hat{m}_b (\bar{s} P_R b) (\bar{l} l),$$

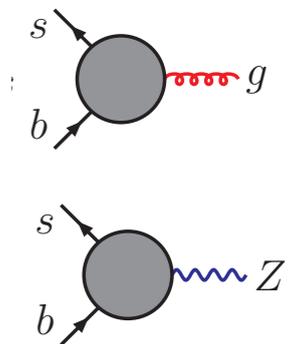
$$\mathcal{O}_T = \frac{\alpha_{\text{em}}}{4\pi} \hat{m}_b (\bar{s} \sigma_{\mu\nu} P_R b) (\bar{l} \sigma^{\mu\nu} P_R l),$$



$$\mathcal{O}_8 = \frac{g_s}{16\pi^2} \hat{m}_b \bar{s} \sigma_{\mu\nu} P_R G^{\mu\nu} b,$$

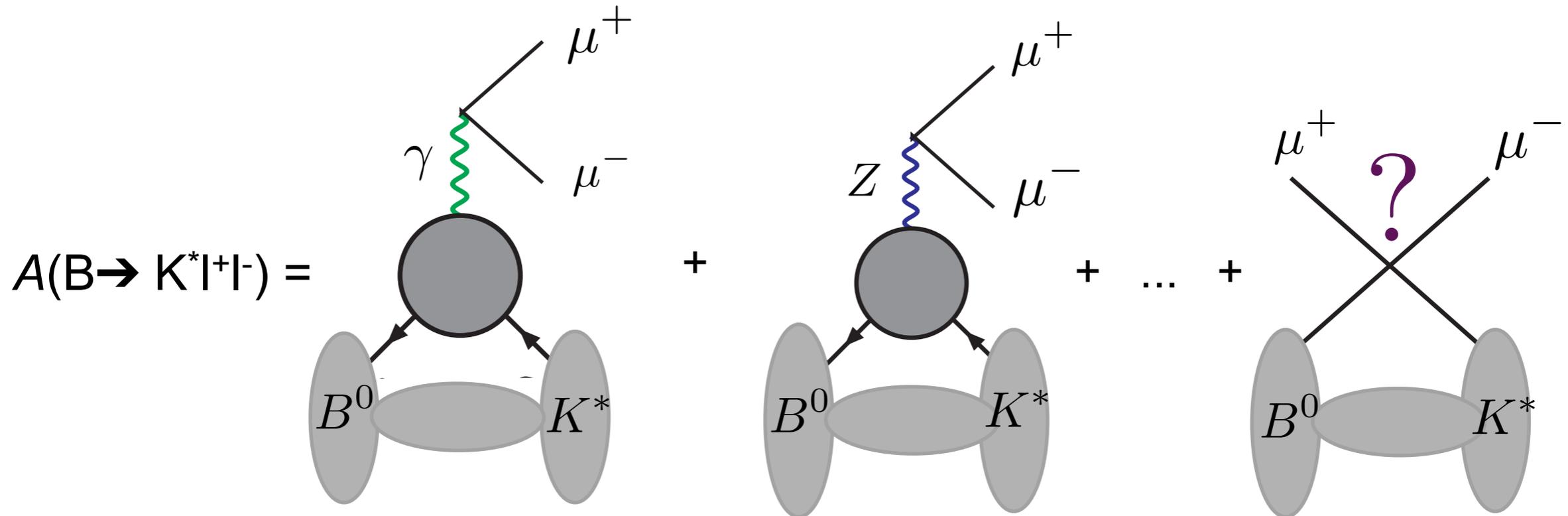
$$\mathcal{O}_A = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{l} \gamma^\mu \gamma^5 l)_A$$

$$\mathcal{O}_P = \frac{\alpha_{\text{em}}}{4\pi} \hat{m}_b (\bar{s} P_R b) (\bar{l} \gamma^5 l),$$



look for observables sensitive to  $C_i$ 's, specifically those that are suppressed in the SM

# Semileptonic decay

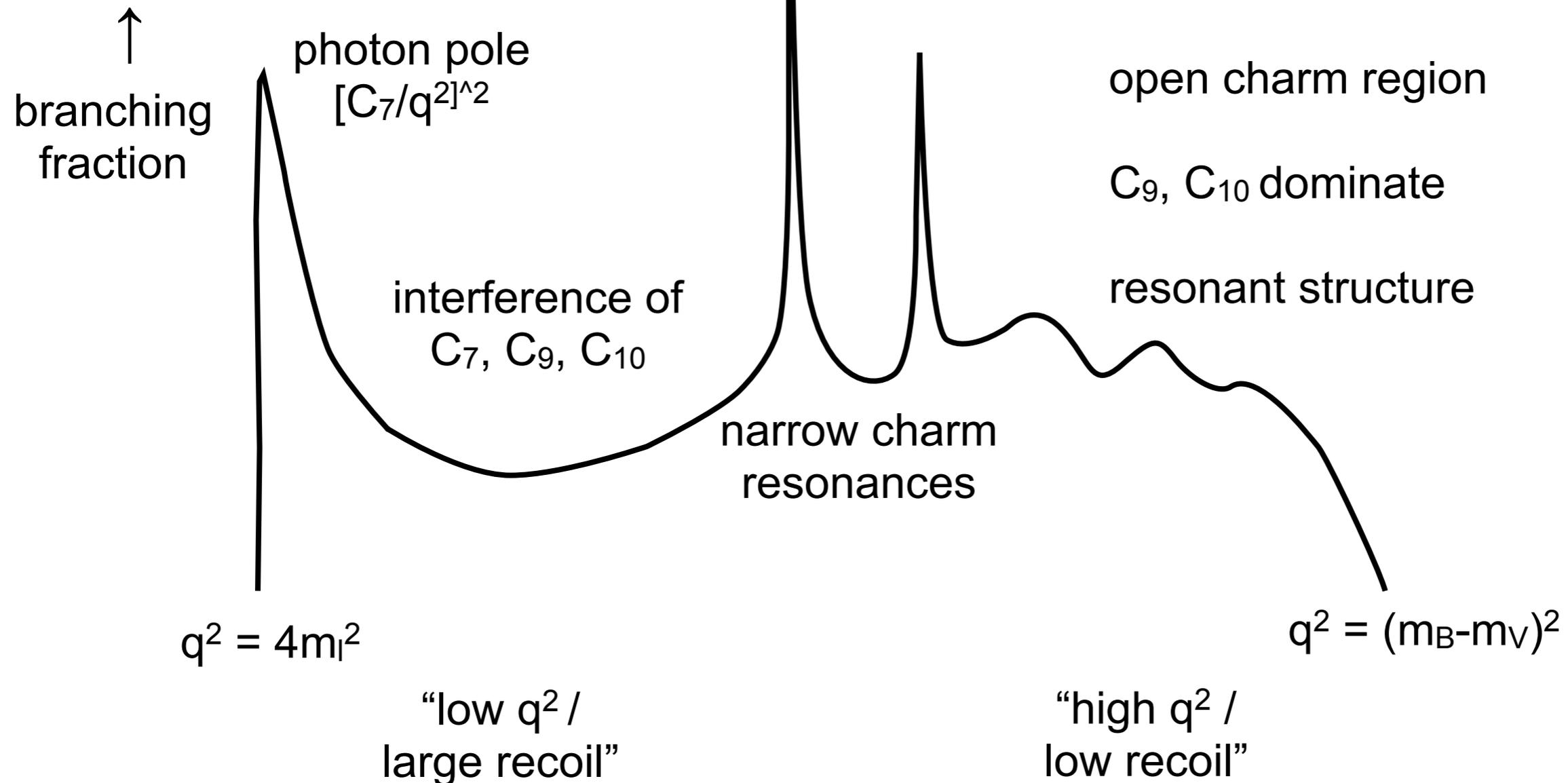


- kinematics described by dilepton invariant mass  $q^2$  and three angles

- Systematic theoretical description based on heavy-quark expansion ( $\Lambda/m_b$ ) for  $q^2 \ll m^2(J/\psi)$  (SCET) [Beneke, Feldmann, Seidel 01](#)  
also for  $q^2 \gg m^2(J/\psi)$  (OPE) [Grinstein et al; Beylich et al 2011](#)

Theoretical uncertainties on form factors, power corrections

# $q^2$ dependence (qualitative)

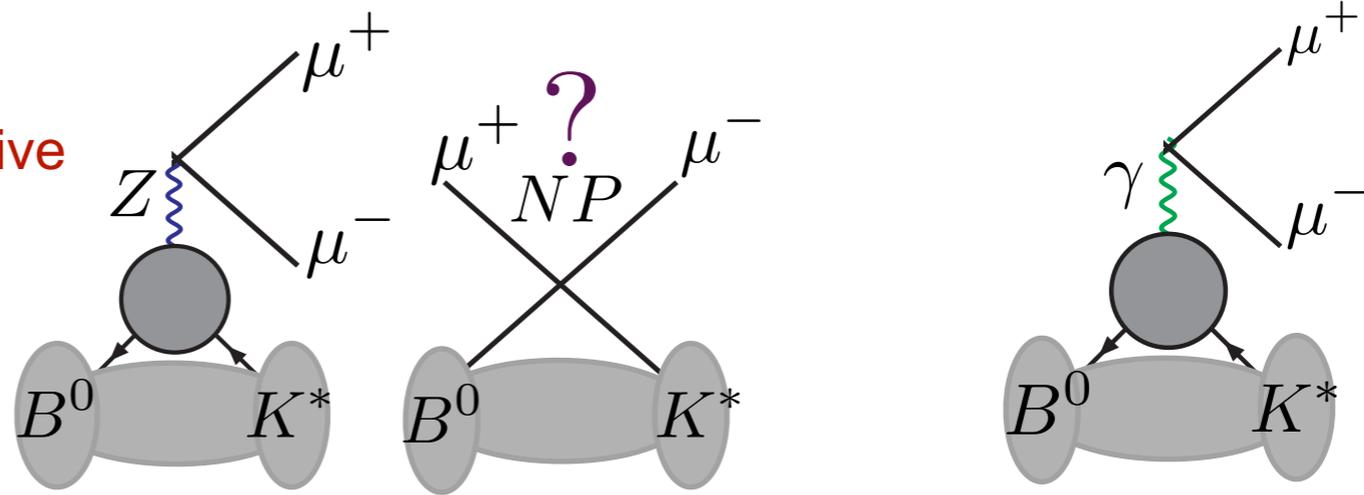


Note - artist's impression only.

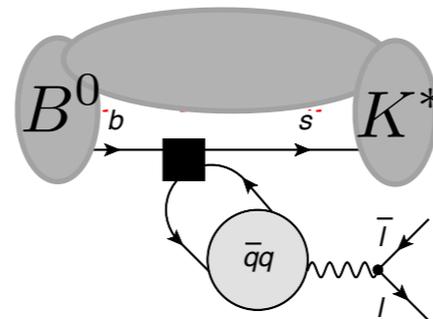
LHCb has not yet published sufficiently fine binning to show the resonant features [open charm resonances are however visible in published  $B \rightarrow K \ell \ell$  data]

# $B \rightarrow K^* l^+ l^-$ decay amplitude

matrix elements of semileptonic/radiative Hamiltonian factorize “naively”



$$\begin{aligned}
 \mathcal{A}(\bar{B} \rightarrow V l^- l^+) = & \sum_i C_i \langle l^- l^+ | \bar{l} \Gamma_i l | 0 \rangle \langle V | \bar{s} \Gamma'_i b | \bar{B} \rangle + C_7^{(\prime)} \frac{e^2}{q^2} \langle l^+ l^- | \bar{l} \gamma^\mu l | 0 \rangle \langle V | \bar{s} \sigma_{\mu\nu} P_{R(L)} b | \bar{B} \rangle \\
 & + \frac{e^2}{q^2} \langle l^- l^+ | \bar{l} \gamma^\mu l | 0 \rangle F.T. \langle V | T(j_{\mu,em}^{\text{had}}(x) \mathcal{H}_W^{\text{had}}(0)) | \bar{B} \rangle
 \end{aligned}$$



nonlocal “quark loops”  
do **not** factorize naively

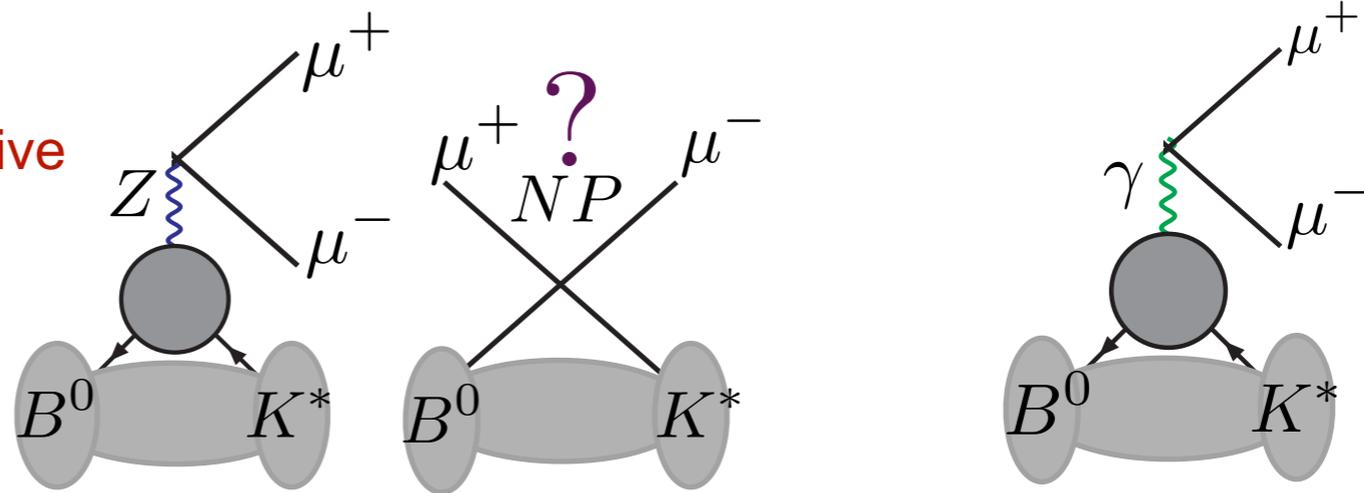
correct to lowest order in electromagnetism  
**exact** in QCD - no assumptions (yet)

three helicity states for  $V=K^*$   
 dilepton can have  $J=0$  or  $J=1$   
 several leptonic currents  
 photon couples only to **vector** leptonic current. At  $q^2 = 0$  photon pole

} 7 (14) helicity amplitudes in SM (BSM)

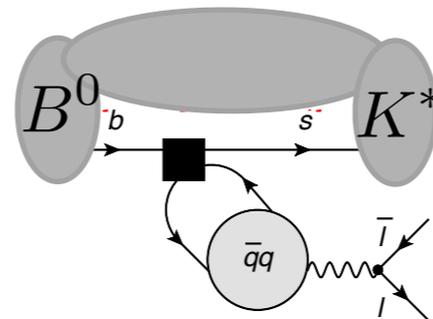
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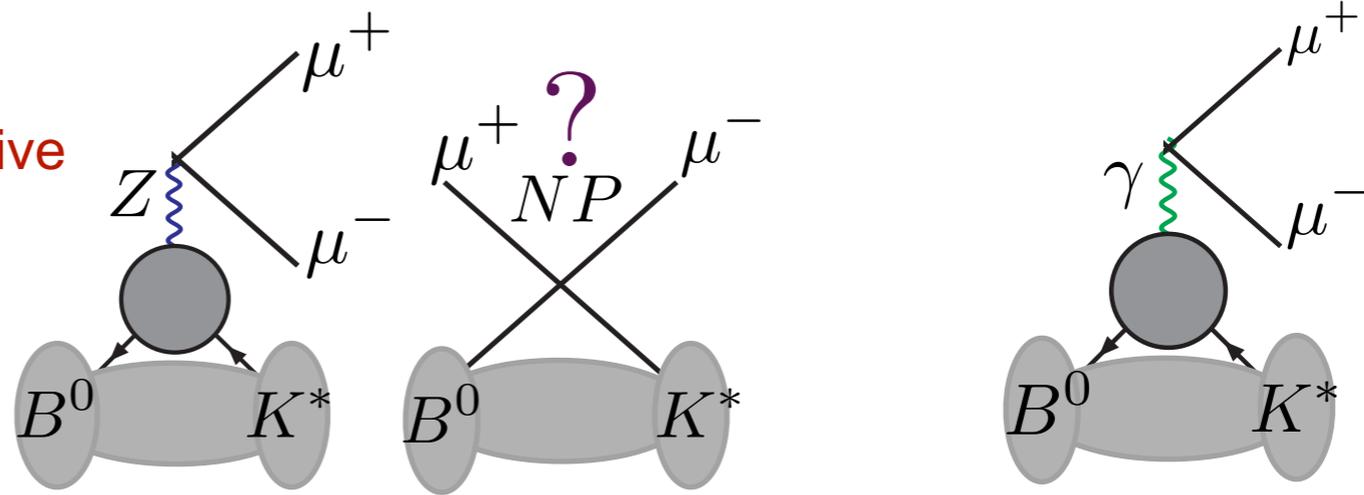
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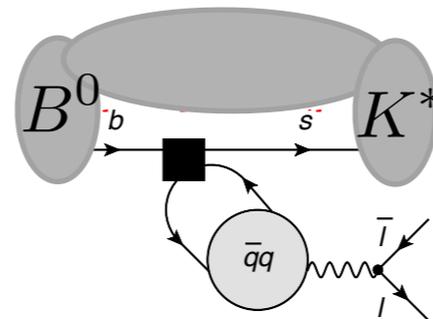
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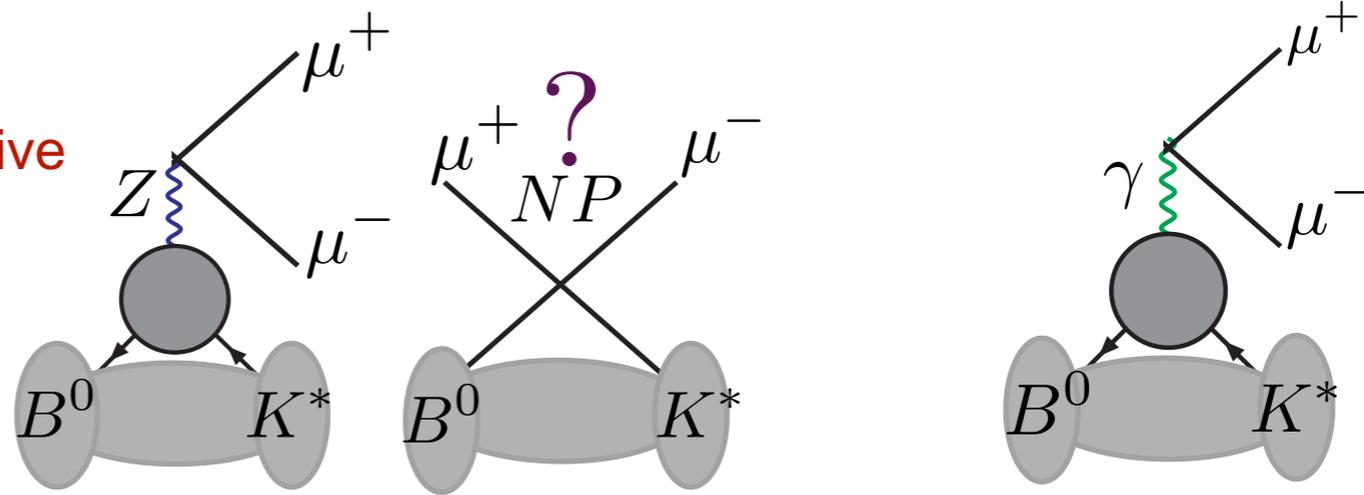
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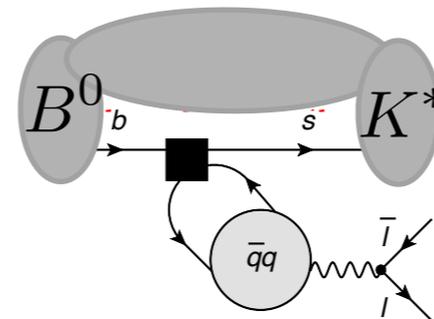
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# In the news

● PRL 111, 191801 (2013)

PHYSICAL REVIEW LETTERS

week ending  
8 NOVEMBER 2013



## Measurement of Form-Factor-Independent Observables in the Decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

R. Aaij *et al.*\*

(LHCb Collaboration)

(Received 9 August 2013; published 4 November 2013)

We present a measurement of form-factor-independent angular observables in the decay  $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ . The analysis is based on a data sample corresponding to an integrated luminosity of  $1.0 \text{ fb}^{-1}$ , collected by the LHCb experiment in  $pp$  collisions at a center-of-mass energy of 7 TeV. Four observables are measured in six bins of the dimuon invariant mass squared  $q^2$  in the range  $0.1 < q^2 < 19.0 \text{ GeV}^2/c^4$ . Agreement with recent theoretical predictions of the standard model is found for 23 of the 24 measurements. A local discrepancy, corresponding to 3.7 Gaussian standard deviations is observed in one  $q^2$  bin for one of the observables. Considering the 24 measurements as independent, the probability to observe such a discrepancy, or larger, in one is 0.5%.

DOI: [10.1103/PhysRevLett.111.191801](https://doi.org/10.1103/PhysRevLett.111.191801)

PACS numbers: 13.20.He, 11.30.Rd, 12.60.-i

Descotes-Genon, Matias, Virto e PRD 88,074002 claim 3.9 *global*

further model-independent fits: Altmannshofer&Straub; Beaujean, Bobeth, van Dyk

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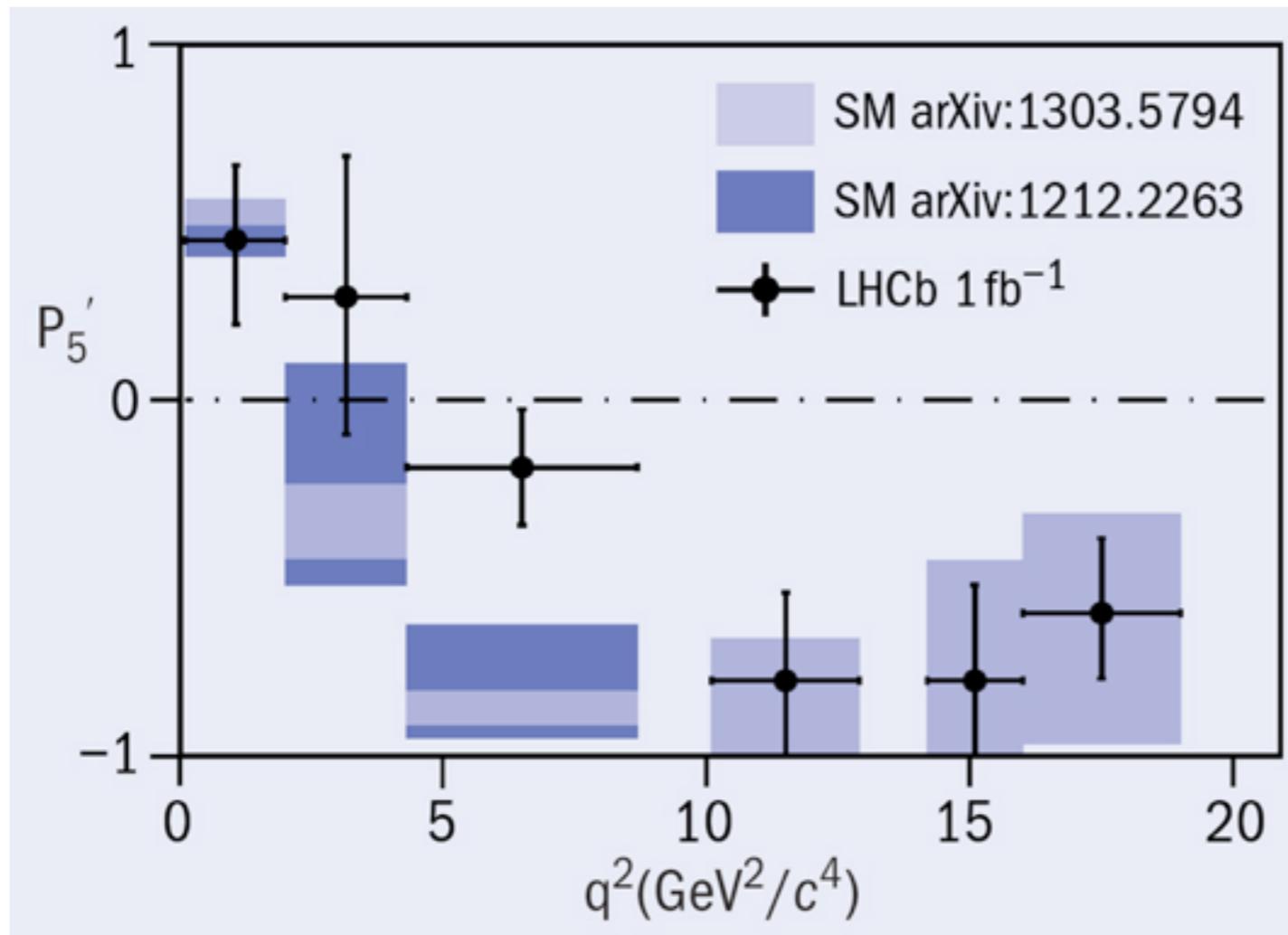
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# $P_5'$ “anomaly”

$$\langle P_5' \rangle = \frac{\langle \beta(\text{Re}[(H_V^- - H_V^+)H_A^{0*} + (H_A^- - H_A^+)H_V^{0*}]) \rangle}{\sqrt{\langle \beta^2 |H_V^0|^2 + |H_A^0|^2 \rangle \langle \beta^2 (|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2) \rangle}}$$

CERN Courier, December 2013



Descotes-Genon, Matias, Virto [DMV]

SJ, J Martin Camalich (4.3..8.68 bin actually a private update, not stated in paper)

$P_5'$  has strong sensitivity to long-distance power corrections.  
Ongoing discussion.

Flavour:  
the glorious past ...  
template for the future?

# A very brief history of flavour

1934 Fermi proposes Hamiltonian for beta decay

$$H_W = -G_F (\bar{p} \gamma^\mu n) (\bar{e} \gamma_\mu \nu)$$

1956-57 Lee&Yang propose parity violation to explain “ $\theta$ - $\tau$  paradox”.

Wu et al show **parity is violated** in  $\beta$  decay

Goldhaber et al show that the neutrinos produced in  $^{152}\text{Eu}$  K-capture always have **negative helicity**

1957 Gell-Mann & Feynman, Marshak & Sudarshan

$$H_W = -G_F (\bar{\nu}_\mu \gamma^\mu P_L \mu) (\bar{e} \gamma_\mu P_L \nu_e) - G (\bar{p} \gamma^\mu P_L n) (\bar{e} \gamma_\mu P_L \nu_e) + \dots$$

**V-A** current-current structure of weak interactions.

Conservation of vector current proposed

Experiments give  $G = 0.96 G_F$  (for the vector parts)

1960-63 To achieve a universal coupling, Gell-Mann&Levy and Cabibbo propose that a certain superposition of neutron and  $\Lambda$  particle enters the weak current.

**Flavour physics** begins!

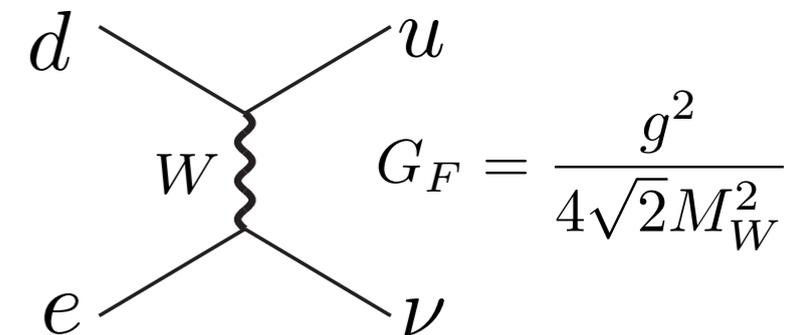
1964 Gell-Mann gives hadronic weak current in the quark model

$$H_W = -G_F J^\mu J_\mu^\dagger$$

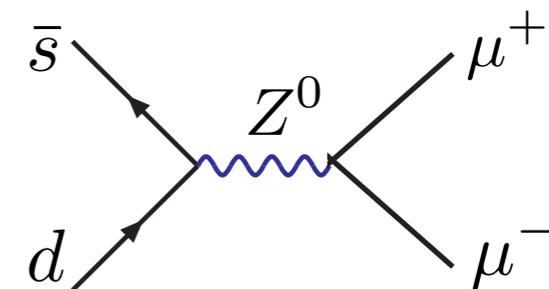
$$J^\mu = \bar{u}\gamma^\mu P_L(\cos\theta_c d + \sin\theta_c s) + \bar{\nu}_e\gamma^\mu P_L e + \bar{\nu}_\mu\gamma^\mu P_L \mu$$

1964 **CP violation** discovered in Kaon decays (Cronin&Fitch)

1960-1968  $J_\mu$  part of triplet of weak gauge currents. Neutral current interactions predicted and, later, observed at CERN.



However, the predicted **flavour-changing neutral current (FCNC)** processes such as  $K_L \rightarrow \mu^+\mu^-$  are *not* observed!



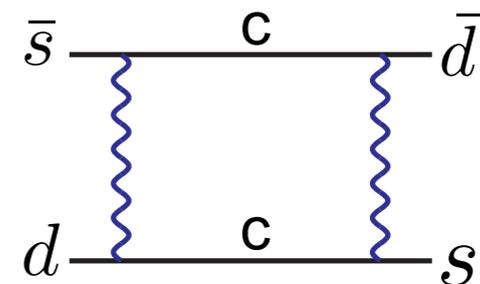
1970 To explain the absence of  $K_L \rightarrow \mu^+ \mu^-$ , Glashow, Iliopoulos & Maiani (GIM) couple a “charmed quark” to the formerly “sterile” linear combination  
 $-\sin \theta_c d_L + \cos \theta_c s_L$

The doublet structure eliminates the  $Zsd$  coupling!

1971 Weak interactions are renormalizable ('t Hooft)

1972 Kobayashi & Maskawa show that **CP violation requires extra particles, for example a third doublet.** CKM matrix

1974 Gaillard & Lee estimate loop contributions to the  $K_L$ - $K_S$  mass difference  
Bound  $m_c < 5 \text{ GeV}$

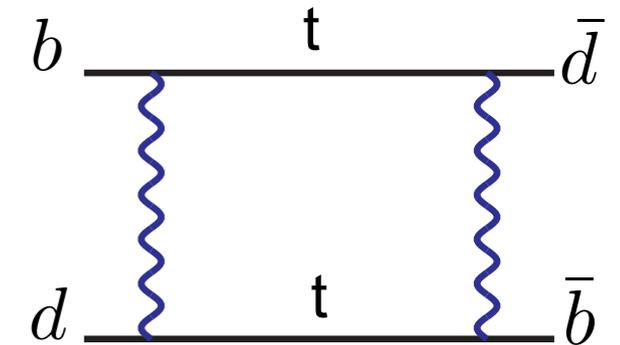


1974 Charm quark discovered

1977  $\tau$  lepton and bottom quark discovered

1983 W and Z bosons produced

1987 ARGUS measures  $B_d - \bar{B}_d$  mass difference  
 First indication of a heavy top



The diagram depends quadratically on  $m_t$

1995 top quark discovered at CDF & D0

$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$u_R$ $d_R$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$c_R$ $s_R$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$t_R$ $b_R$	$Q = +2/3$
$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	— $e_R$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	— $\mu_R$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	— $\tau_R$	$Q = -1$

Precision measurements: masses, running coupling, direct CP violation, B factories, determination of CKM elements, neutrino oscillations, search for electric dipole moments, proton decay, ...