Introduction to lattice field theory

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Outline

- ✤ Motivation
- ✤ The lattice
- ✤ Numerical methods
- ✤ Hadron $\Longleftrightarrow$ quark flavour
- ✤ Other applications

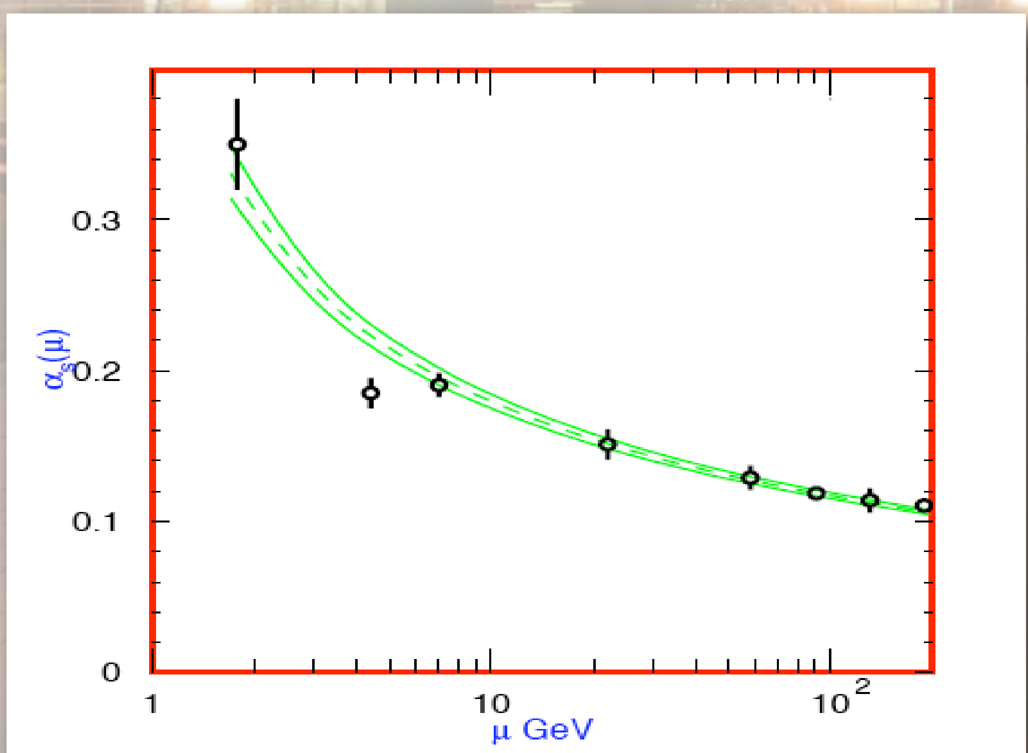
Motivation

QCD: Asymptotic freedom & confinement

q-*q*-*g* coupling becomes weaker at high energies

High-*E*: expand in number of gluon exchanges (Feynman diagrams)

Gross, Politzer, Wilczek



q-*q*-*g* coupling diverges around 300-500 MeV

Nonperturbative interactions

Only colourless states seen in nature

Role of Lattice QCD

Model builder:

Experimentalist:

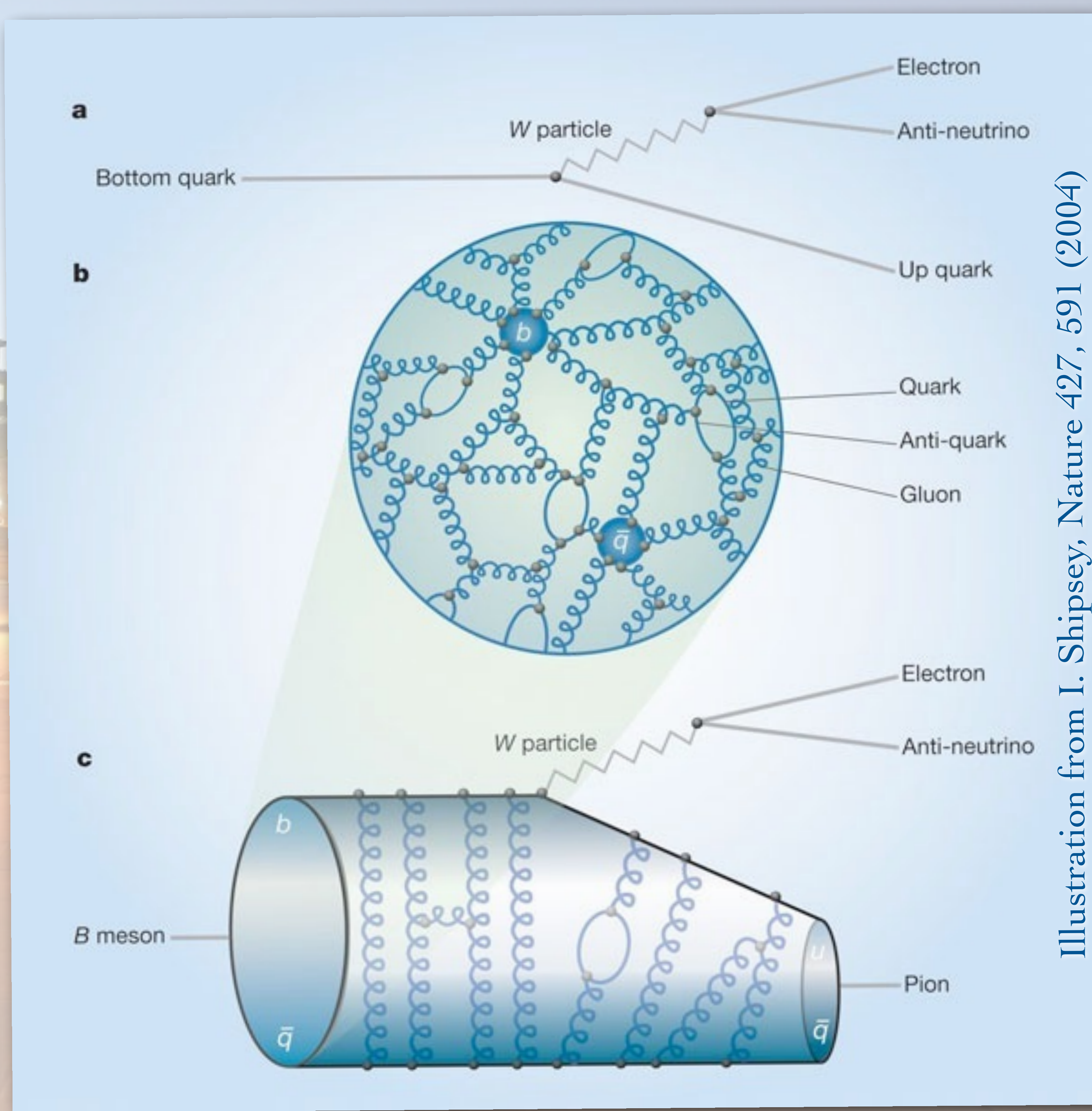


Illustration from I. Shipsey, Nature 427, 591 (2004)

Role of Lattice QCD

Model builder:

Lattice theorist

Experimentalist:

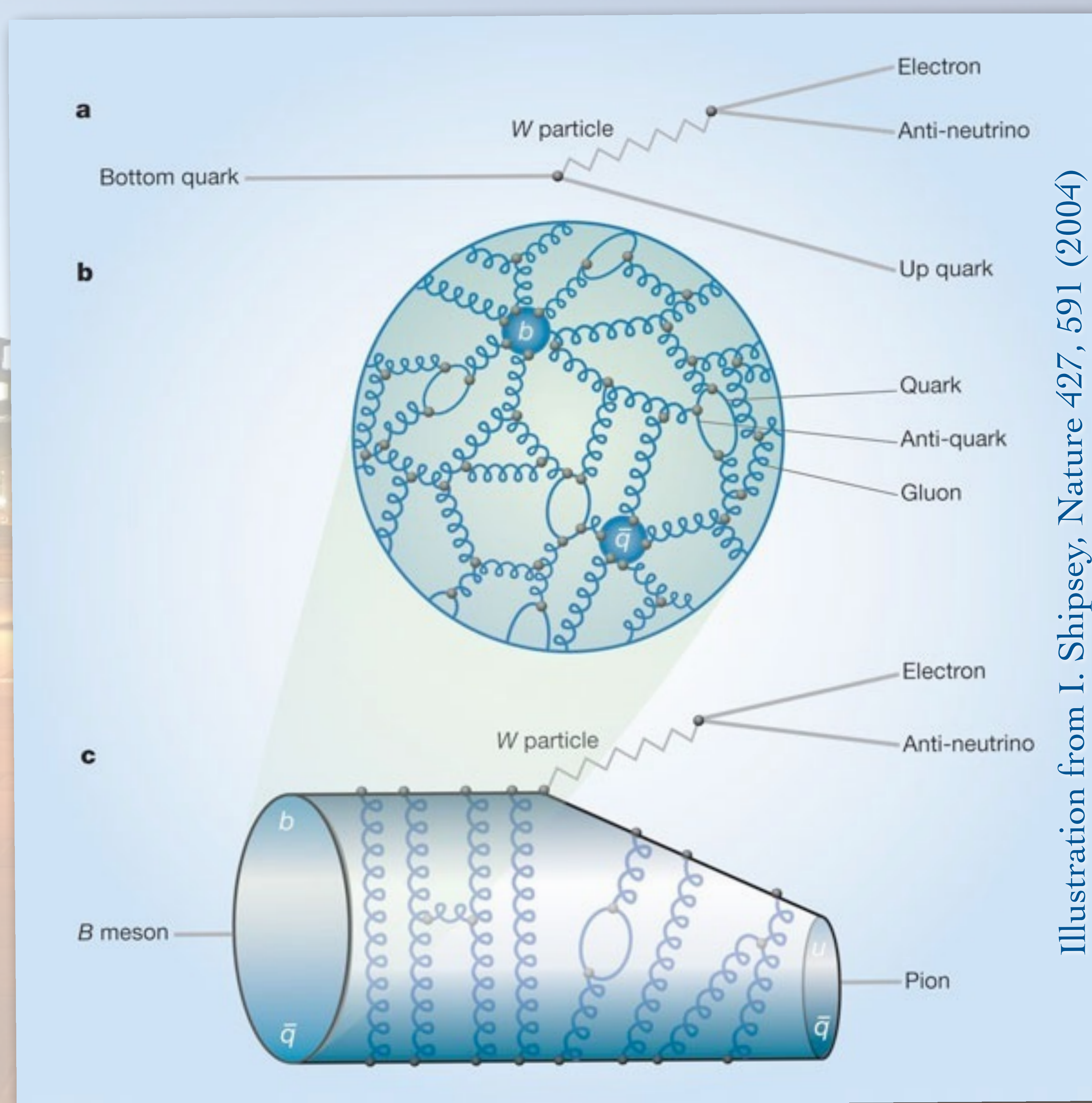


Illustration from I. Shipsey, Nature 427, 591 (2004)

Discretization

- QCD Lagrangian

$$\boldsymbol{\mathcal{L} = -\frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu} - \sum_q \overline{\psi}_q \left[\gamma^\mu(\partial_\mu - igA^a_\mu t^a) + m_q\right]\psi_q}$$

$$\boldsymbol{= \mathcal{L}_g - \overline{\psi}Q\psi}$$

- Break spacetime up into a grid
- Maintains gauge invariance
- Regulates the QFT nonperturbatively
- Breaking of Lorentz and translational symmetries

Discretization

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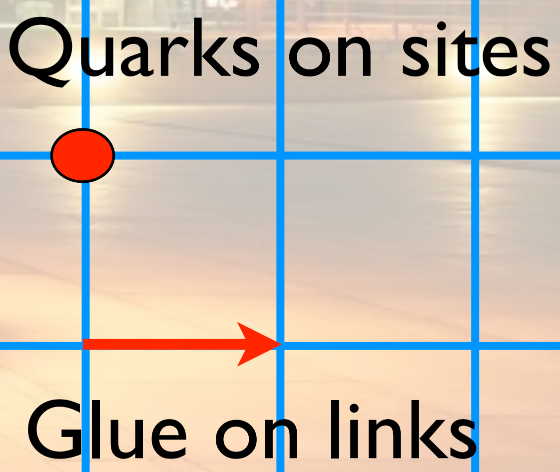
$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} - \sum_q \overline{\psi}_q \left[\gamma^\mu (\partial_\mu - i g A^a_\mu t^a) + m_q \right] \psi_q$$

$$= \mathcal{L}_g - \overline{\psi} Q \psi$$



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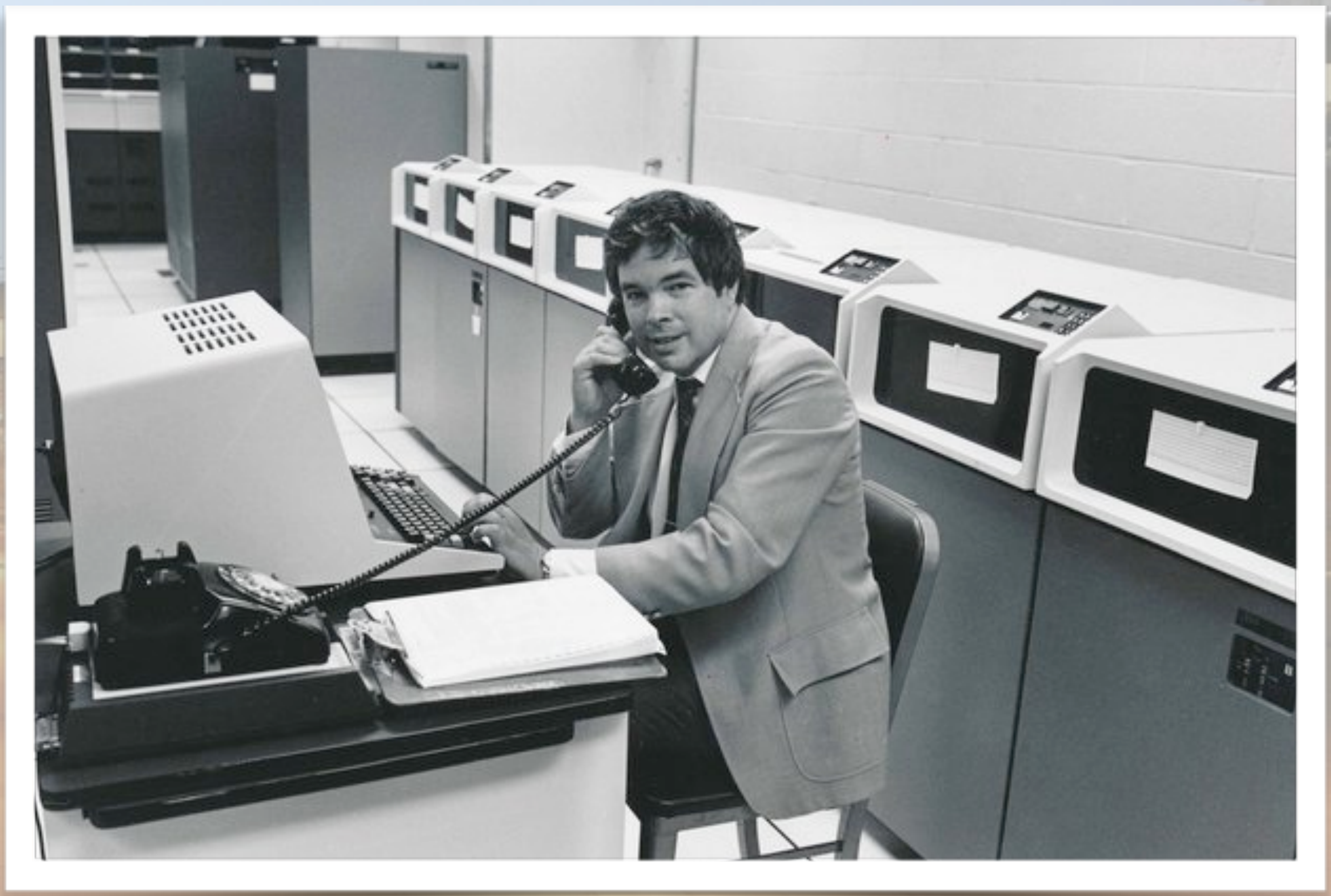
The Lattice

Ken Wilson

- 1936-2013
- Renormalization group
- Operator product expansion
- Lattice gauge theory



Remembrances: Kronfeld (arXiv:1312.6861), Jackiw (arXiv:1312.6634)

Scalar field

Transition amplitude in path integral representation

$$\langle \Phi_f | e^{-i\hat{H}t} | \Phi_i \rangle = \int_{\phi(0,\vec{x})=\Phi_i}^{\phi(t,\vec{x})=\Phi_f} \mathcal{D}\phi \, e^{i \int_0^t d\tilde{t} \, d^3\tilde{x} \, \mathcal{L}_M}$$

with Lagrangian

$$\mathcal{L}_M = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_i \phi)(\partial^i \phi) - V(\phi)$$

If $V(\varphi)$ is small, one Taylor-expands the exponential in a perturbative expansions, represented by Feynman diagrams

Integrand is a complex phase. Does the integral exist?

Imaginary time

Wick rotation: let $t = -i\,\tau$

$$\langle \Phi_f | e^{-\hat{H}\tau} | \Phi_i \rangle \; = \int_{\phi(0,\vec{x})=\Phi_i}^{\phi(\tau,\vec{x})=\Phi_f} \mathcal{D}\phi \, e^{-\int_0^\tau d\tilde{\tau}\, d^3\tilde{x}\, \mathcal{L}_E}$$

with

$$\mathcal{L}_E \; = \; \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\partial_i \phi)(\partial^i \phi) \, + V(\phi)$$

Integrand is now real and sharply peaked

Analytic continuation back to Minkowski straightforward for 0-1 particles in initial/final states; difficult otherwise.

Discretization

Define scalar field only on lattice points

$$x = a(n_1, n_2, n_3, n_4),\ n_\mu \in \mathbb{Z}$$

Replace derivative by finite difference

$$(\nabla_\mu^+ \phi)(x) = \frac{1}{a}[\phi(x + ae_\mu) - \phi(x)]$$

Path integral now ordinary integral

$$\langle \Phi_f | e^{-\hat{H}\tau} | \Phi_i \rangle \ = \int_{\phi(0,\vec{x})=\Phi_i}^{\phi(\tau,\vec{x})=\Phi_f} \left[\prod_{\tilde{x}} d\phi(\tilde{x}) \right] e^{-a^4 \sum_{\tilde{x}} \mathcal{L}}$$

$$\mathcal{L} \ = \ \frac{1}{2}(\nabla_\mu^+ \phi)(\nabla^{+\mu} \phi) \ + \ V(\phi)$$

Classical continuum limit

Expanding about a=0

$$(\nabla_\mu^+\phi)(x) = \frac{1}{a}[\phi(x+ae_\mu) - \phi(x)] \approx \partial_\mu\phi(x) + \frac{a}{2}\partial_\mu^2\phi(x) + \frac{a^2}{6}\partial_\mu^3\phi(x)$$

Equally good difference operator

$$(\nabla_\mu^-\phi)(x) = \frac{1}{a}[\phi(x) - \phi(x-ae_\mu)] \approx \partial_\mu\phi(x) - \frac{a}{2}\partial_\mu^2\phi(x) + \frac{a^2}{6}\partial_\mu^3\phi(x)$$

Combine to “improve” convergence to continuum limit

$$\nabla_\mu^\pm\phi = \frac{1}{2}(\nabla_\mu^+ + \nabla_\mu^-)\phi \approx \partial_\mu\phi + \frac{a^2}{6}\partial_\mu^3\phi$$

In practice, quantum corrections limit the precision of improvement

Gauge field

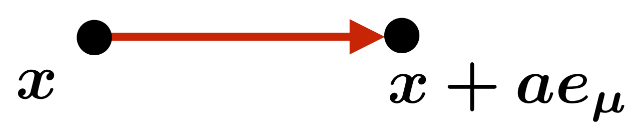
SU(N) gauge field

$$A_\mu(x) = A_\mu^a(x) T^a, \quad a = 1, \ldots, N^2 - 1$$

Under a gauge transformation

$$A_\mu(x) \mapsto \Lambda^{-1}(x) A_\mu(x) \Lambda(x) - \frac{i}{g} \Lambda^{-1}(x) \partial_\mu \Lambda(x)$$

Link (Wilson line)



$$U_\mu(x) = \exp\left[ig\mathcal{P} \int_x^{x+ae_\mu} dy\, A_\mu^a(y) T^a \right]$$

Under a gauge transformation

$$U_\mu(x) \mapsto \Lambda^{-1}(x + ae_\mu) U_\mu(x) \Lambda(x)$$

Gauge invariant action

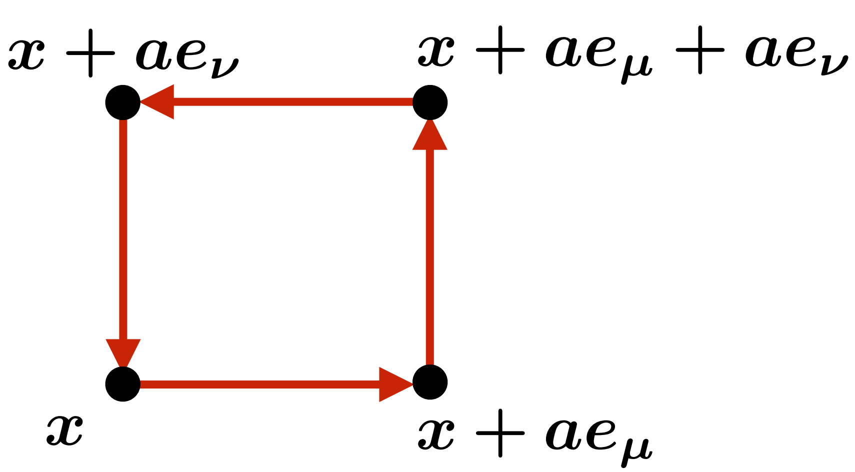
Traces of closed loops yield gauge invariant objects. E.g. the plaquette:

$$W_\Box(x,\mu,\nu) \;=\; U_\nu^\dagger(x)\,U_\mu^\dagger(x+ae_\nu)\,U_\nu(x+ae_\mu)\,U_\mu(x)$$

Plaquette, *aka* Wilson action (integral of Lagrangian)

$$S_\Box[U] \;=\; \beta \sum_{x,\mu,\nu>\mu}\left[1-\frac{1}{2N}\mathrm{Tr}\left[W_\Box(x,\mu,\nu)\;+\;W_\Box^\dagger(x,\mu,\nu)\right]\right]$$

In the classical continuum limit, with $U_\mu(x) = e^{igaA_\mu(x+\frac{1}{2}ae_\mu)}$



$$S_\Box \approx \frac{a^4}{4}\sum_x \mathrm{Tr}F_{\mu\nu}F^{\mu\nu} \qquad\qquad \beta=\frac{2N}{g^2}$$

Continuum action at LO. Rotationally & translationally invariant!

Fermion field

Naive discretization of Dirac action

$$S_f[\psi, \bar{\psi}, U] = a^4 \sum_x \bar{\psi}(x)[(m + \gamma \cdot \nabla^{\pm})\psi](x)$$

$$= a^4 \sum_x \bar{\psi}(x) \Big\{ m\,\psi(x)$$

$$+ \frac{1}{2a} \sum_{\mu=1}^{4} \gamma_\mu \Big[U_\mu^\dagger(x)\,\psi(x + ae_\mu) - U_\mu(x - ae_\mu)\psi(x - ae_\mu) \Big] \Big\}$$

Gauge invariant under

$$\psi(x) \mapsto \Lambda^{-1}(x)\psi(x)$$

$$U_\mu(x) \mapsto \Lambda^{-1}(x + ae_\mu)U_\mu(x)\Lambda(x)$$

Free lattice propagators

Momentum space, with lattice cutoff: $p_\mu \in \left(-\frac{\pi}{a}, \frac{\pi}{a}\right)$ (Brillouin zone)

Scalar

$$\phi(x) = \int \frac{d^4p}{(2\pi)^4}\, \tilde{\phi}(p)\, e^{ip\cdot x}$$

$$\Delta_\phi(p) = \frac{1}{m^2 + \hat{p}^2}$$

with $\hat{p}_\mu \equiv \frac{2}{a} \sin \frac{p_\mu a}{2}$

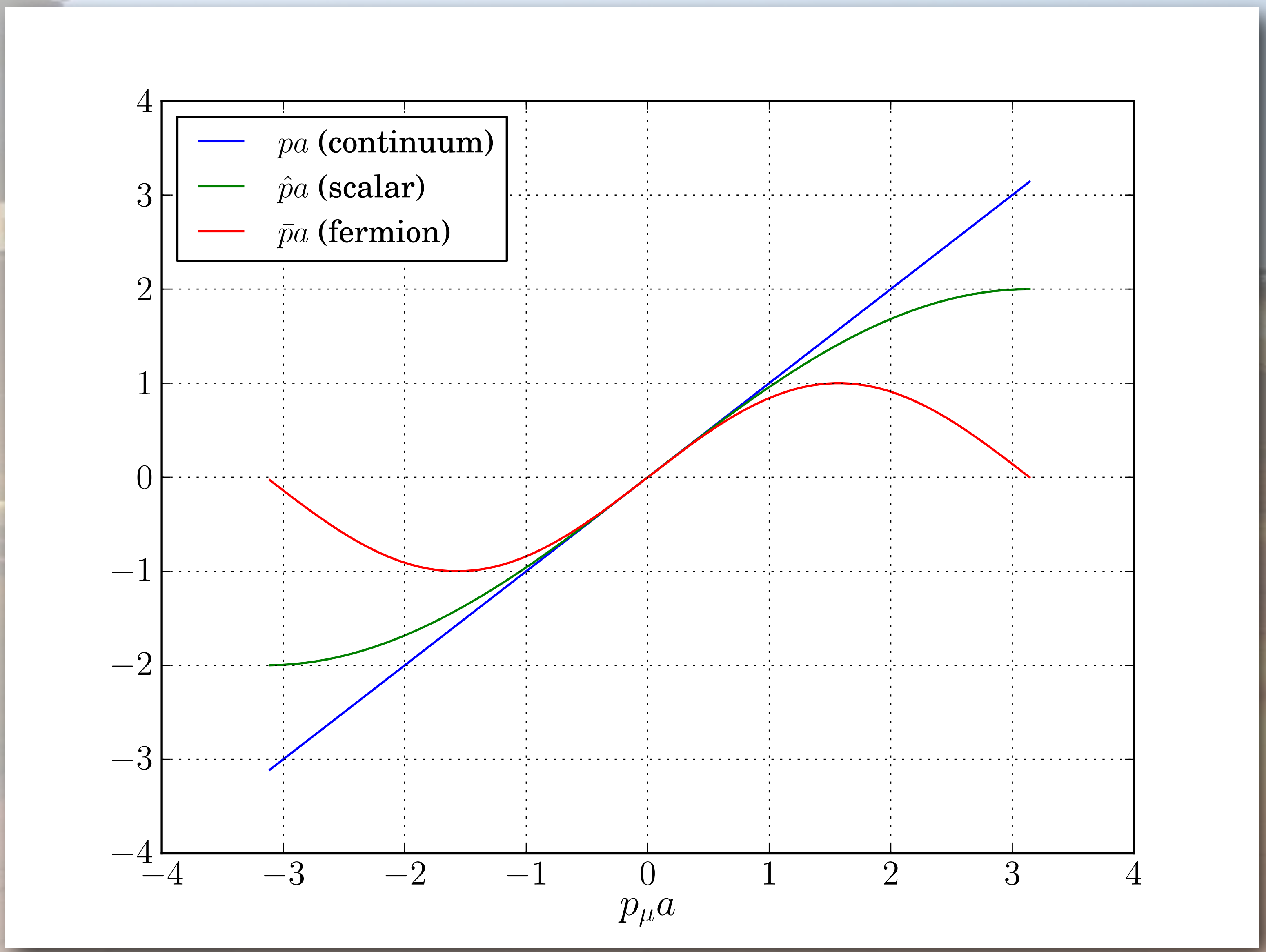
Fermion

$$\psi(x) = \int \frac{d^4p}{(2\pi)^4}\, \tilde{\psi}(p)\, e^{ip\cdot x}$$

$$\Delta_\psi(p) = \frac{m - i \sum_\mu \gamma_\mu \bar{p}_\mu}{m^2 + \bar{p}^2}$$

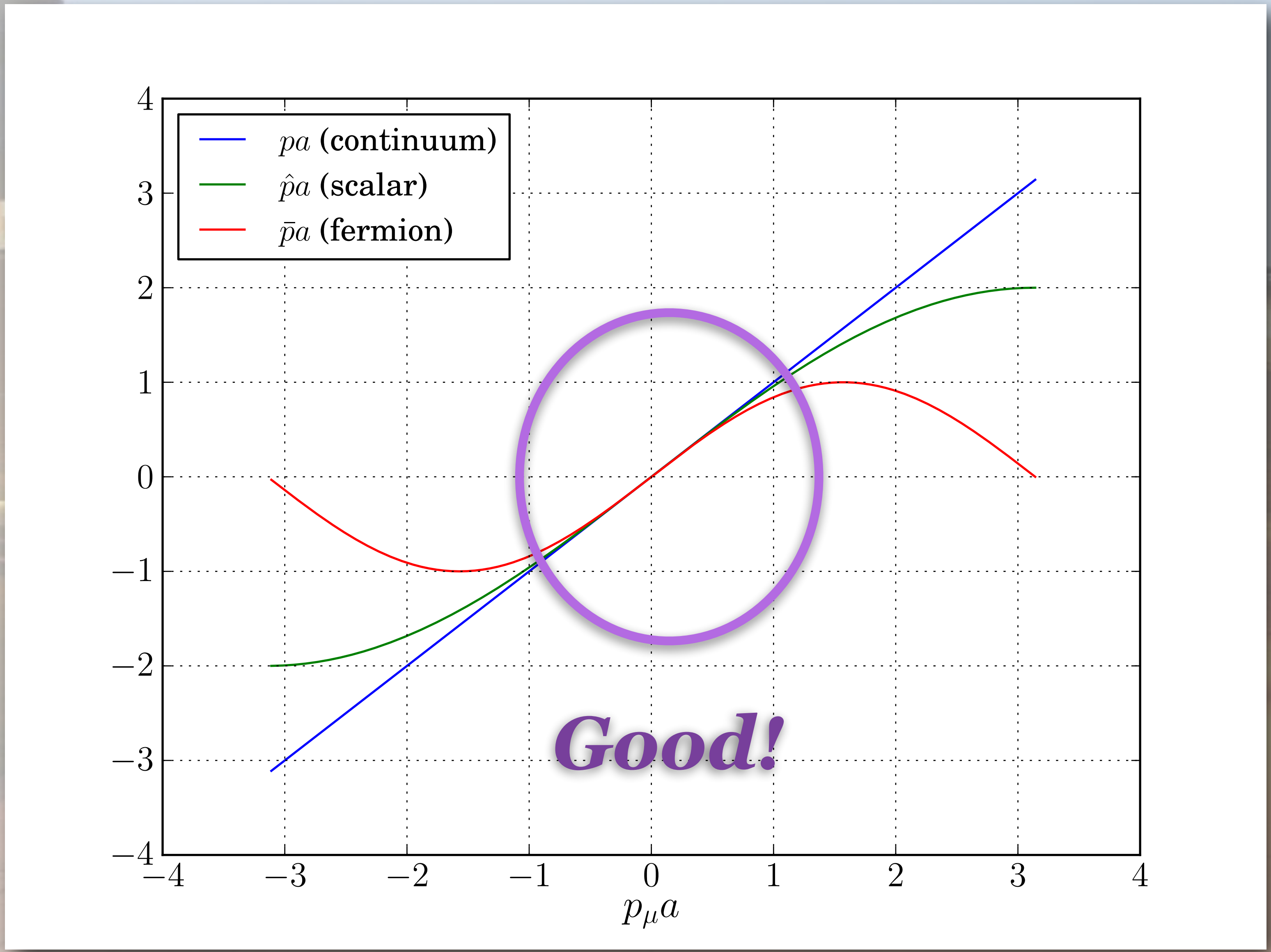
with $\bar{p}_\mu \equiv \frac{1}{a} \sin p_\mu a$

Lattice momenta



- Poles in propagators correspond to physical states
- Naive fermion has extra poles: *doublers*

Lattice momenta



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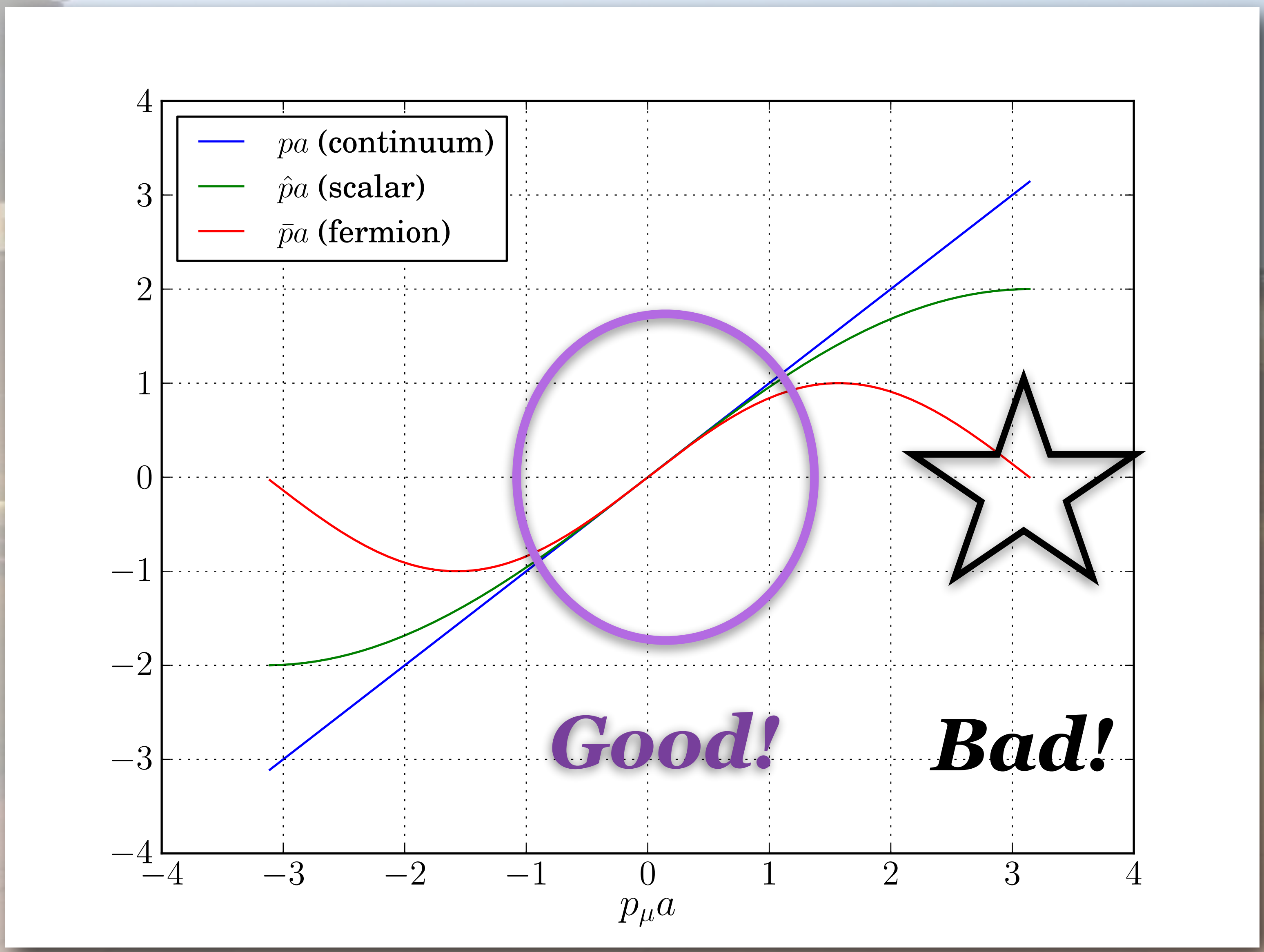
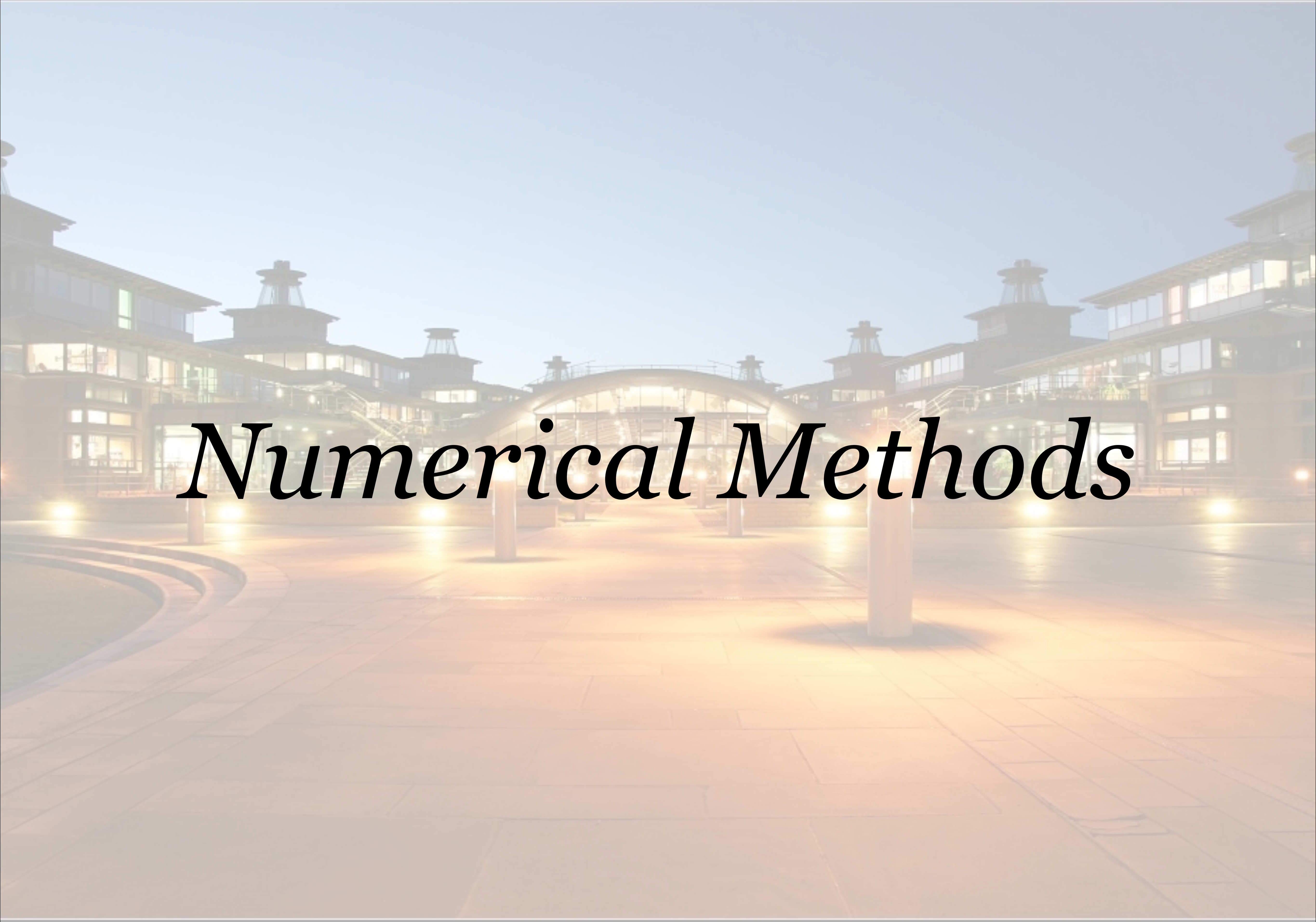


- Poles in propagators correspond to physical states
- Naive fermion has extra poles: *doublers*

Dealing with doubling

- Wilson fermions
  - Give doublers a mass, break chiral symmetry
- Staggered fermions
  - Reduce number of doublers to 4
- Overlap or domain wall fermions
  - Preserve a lattice version of chiral symmetry
- Others (twisted mass, minimally doubled, ...)
  - Might break flavour symmetries
  - Might break a discrete symmetry

Numerical Methods



Treat as statistical system

Quantum FT : Imaginary-time path integral

$$\langle J(z')J(z)\rangle = \frac{1}{Z}\int [d\psi][d\bar{\psi}][dU]\, J(z')J(z)\, e^{-S_E}$$

Statistical FT : Sum over all microstates

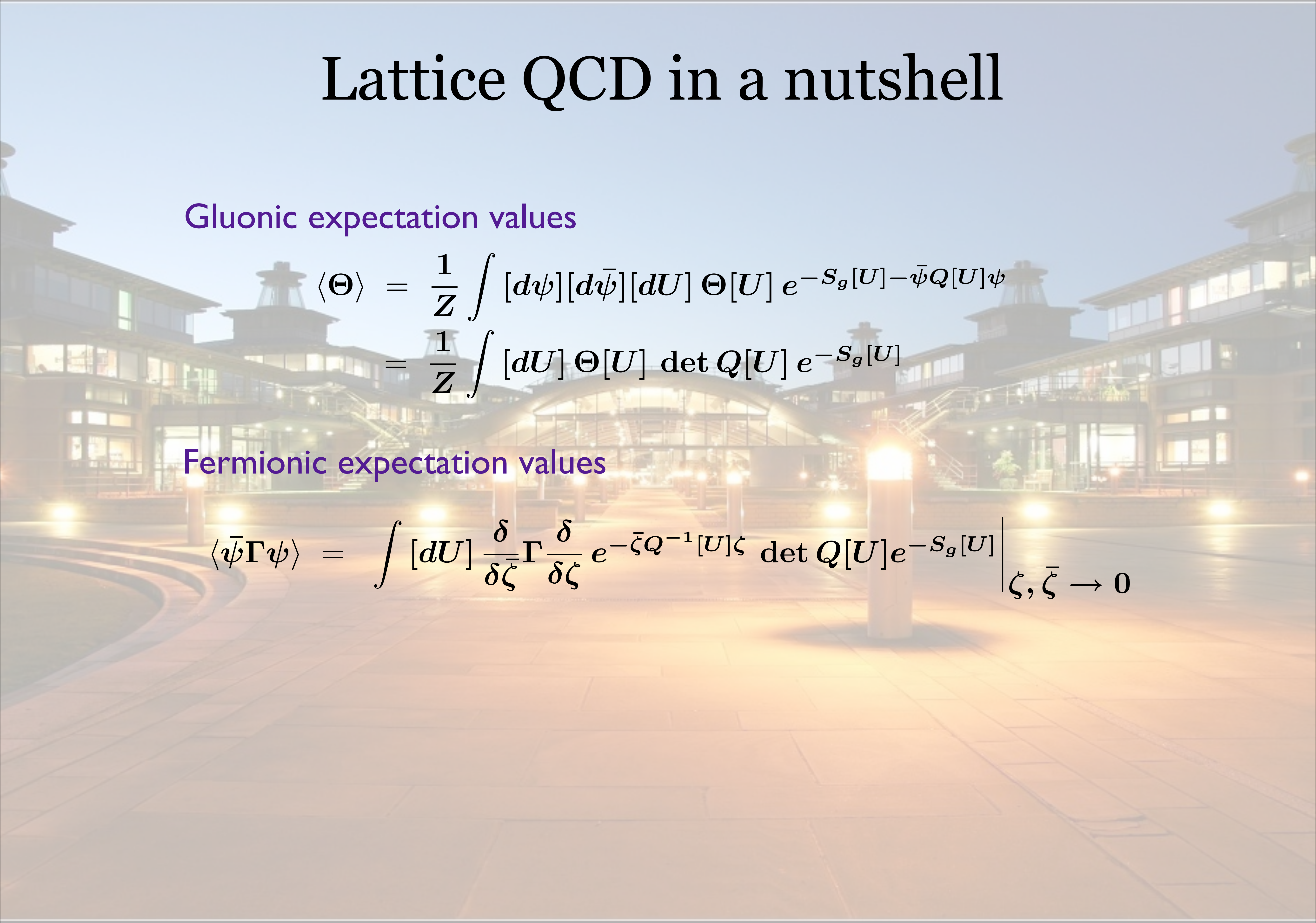
$$\langle J(z')J(z)\rangle = \frac{1}{Z}\,\mathrm{Tr}\,\left[J(z')J(z)\, e^{-\beta H}\right]$$

Use the same numerical methods!

Monte Carlo Calculation : Find and use field configurations which dominate the integral/sum

Markov chain : Initial configuration, algorithm for suggesting updates, accept/reject step

Lattice QCD in a nutshell



Gluonic expectation values

$$\langle \Theta \rangle = \frac{1}{Z}\int [d\psi][d\bar{\psi}][dU]\,\Theta[U]\,e^{-S_g[U]-\bar{\psi}Q[U]\psi}$$
$$= \frac{1}{Z}\int [dU]\,\Theta[U]\,\det Q[U]\,e^{-S_g[U]}$$

Fermionic expectation values

$$\langle \bar{\psi}\Gamma\psi \rangle = \int [dU]\,\frac{\delta}{\delta\bar{\zeta}}\Gamma\frac{\delta}{\delta\zeta}\,e^{-\bar{\zeta}Q^{-1}[U]\zeta}\,\det Q[U]e^{-S_g[U]}\Big|_{\zeta,\bar{\zeta}\to 0}$$

Lattice QCD in a nutshell

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Probability weight

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Determinant in probability weight difficult

1) Requires nonlocal updating; 2) Matrix becomes singular

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Partial quenching =

different mass for valence Q^{-1} than for sea $\det Q$

Lattice QCD in a nutshell

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Determinant in probability weight difficult

1) Requires nonlocal updating; 2) Matrix

Quenched approximation

Set $\det Q = 1$

Partial quenching =

different mass for valence Q^{-1} than for sea $\det Q$

Lattice QCD in a nutshell

Gluonic expectation values

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Fermionic expectation values

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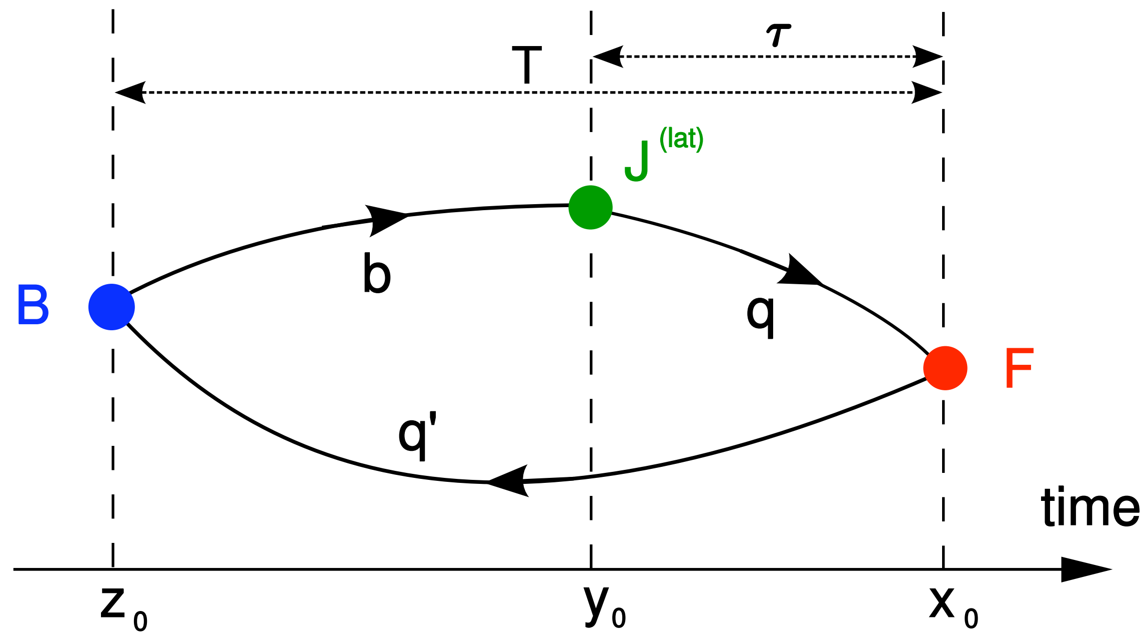
Correlation functions

3-point function

$$C_{FJB}(\mathbf{p}',\, \mathbf{p},\, x_0,\, y_0,\, z_0) = \sum_{\mathbf{y}} \sum_{\mathbf{z}} \left\langle \Phi_F(x)\, J(y)\, \Phi_B^\dagger(z) \right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})} e^{-i\mathbf{p}\cdot(\mathbf{y}-\mathbf{z})}$$

2-point functions

$$\begin{aligned} C_{BB}(\mathbf{p},\, x_0,\, y_0) &= \sum_{\mathbf{x}} \left\langle \Phi_B(x)\, \Phi_B^\dagger(y) \right\rangle e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}, \\ C_{FF}(\mathbf{p}',\, x_0,\, y_0) &= \sum_{\mathbf{x}} \left\langle \Phi_F(x)\, \Phi_F^\dagger(y) \right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})}. \end{aligned}$$

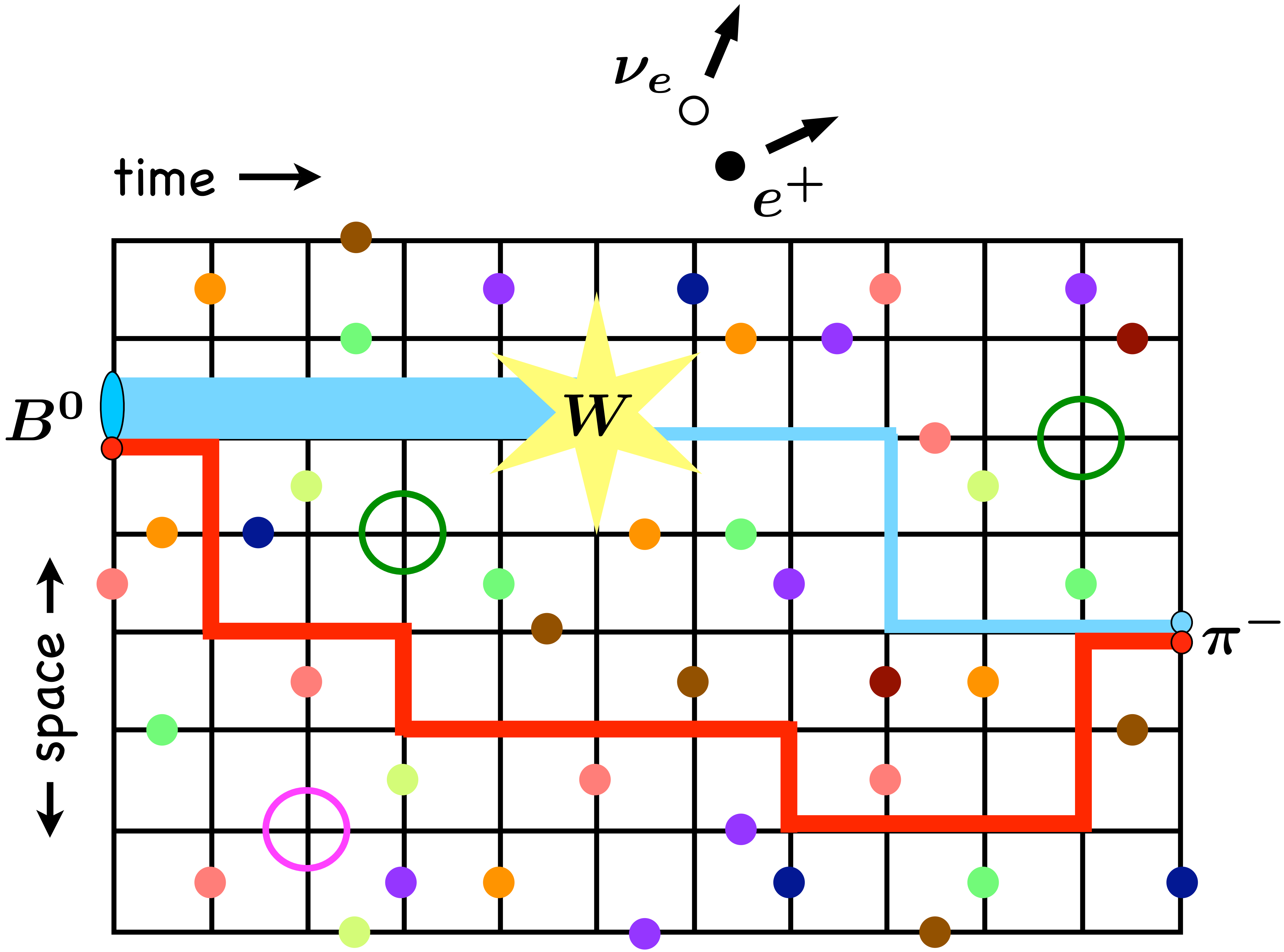


Interpolating operators

$$\boldsymbol{\Phi_V = \bar{u}\gamma_j s}$$

$$\boldsymbol{\Phi_B = \bar{u}\gamma_5 b}$$

$B \to \pi\, l\, \nu$ on the lattice



Correlation functions

Large Euclidean-time behavior

$$
\begin{aligned}
C_{FJB}(\mathbf{p}',\, \mathbf{p},\, \tau,\, T) &\rightarrow A^{(FJB)} e^{-E_F\tau}\, e^{-E_B(T-\tau)},\\
C_{FF}(\mathbf{p},\, \tau) &\rightarrow A^{(FF)}\, e^{-E_F\tau},\\
C_{BB}(\mathbf{p},\, \tau) &\rightarrow A^{(BB)}\, e^{-E_B\tau},
\end{aligned}
$$

$$
\begin{aligned}
A^{(FJB)} &= \frac{\sqrt{Z_V}}{2E_V}\frac{\sqrt{Z_B}}{2E_B} \boxed{\sum_s \varepsilon_j(p',s)\,\langle V\left(p',\varepsilon(p',s)\right)|\,J\,|B(p)\rangle,}\\
A^{(FF)} &= \sum_s \frac{Z_V}{2E_V}\,\varepsilon_j^*(p',s)\varepsilon_j(p',s)\\
A^{(BB)} &= \frac{Z_B}{2E_B},
\end{aligned}
$$

Systematic errors

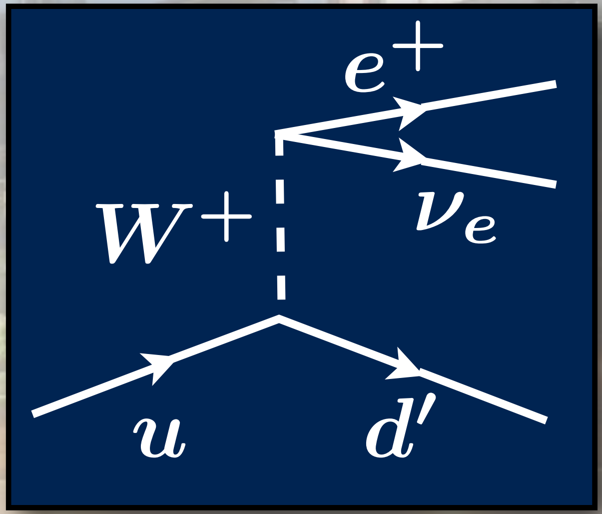
Source of error	Controllable limit	Theory
Lattice volume	$L \gg 1/m_\pi$	Chiral pert. th. Brute force
Lattice spacing	$a \ll 1/\Lambda_{\mathrm{QCD}}$	Symanzik EFT
Heavy quark mass	$m_Q \gg 1/a$ $m_Q < 1/a$ $m_Q \approx 1/a$	NRQCD, HQET Extra-fine, extra-improvement Fermilab
Light quark mass	$m_\pi \ll m_\rho, 4\pi f_\pi$	Chiral pert. th.

Flavour

Quark flavour in the SM

- Only charged weak interactions change quark flavour

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$$



- Flavour mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- *V* is the CKM matrix. Unitarity + "rephasing" implies 4 free SM parameters (one of them a CP-violating phase)

Quark flavour at the weak scale

- Heavy particles (gauge bosons, top, new physics particles) “integrated out”, replaced by local operators (just as in Fermi’s weak theory)
- Perturbative calculations in SM (or any other concrete theory) determine Wilson coefficients
- LQCD needed to determine matrix of the local operators, between hadronic initial & final states

Table of quantities

quantity	process	LQCD matrix el.
ε_K	K^0 - K^0-bar	B_K
$\Delta m_{d(s)}$	$B_{(s)}{}^0$ - $B_{(s)}{}^0$-bar	$f_{B(s)}{}^2\ B_{B(s)}$
$\|V_{ub}\|$	$B \rightarrow \pi\, l\, \nu$	$f_+(q^2)$
$\|V_{ub}\|$	$B \rightarrow \tau\, \nu$	f_B
$\|V_{cb}\|$	$B \rightarrow D\, l\, \nu$	$\mathcal{F}(w=1)$

Snapshot of recent work

f_B, f_{B_s}

ETM, PoS(LAT2009);
HPQCD, PRL 92 (2004);
FNAL/MILC, PoS(LAT2008);
HPQCD, PRD 80 (2009)

B_{B_d}, B_{B_s}

HPQCD, PRD 76 (2007);
RBC-UKQCD, PoS(LAT2007);
HPQCD, PRD 80 (2009);
RBC-UKQCD, PRD 82 (2010)

$f_+^{B\to\pi}(q^2)$

HPQCD, PRD 73 (2006);
FNAL/MILC, PRD 79 (2009) 054507;
FNAL/MILC, PRD 80 (2010)

$\mathcal{F}^{B\to D}(1)$

FNAL/MILC, NPB Proc Suppl (2005);
FNAL/MILC, PRD 85 (2012);
FNAL/MILC, PRL 109 (2012)

$\mathcal{F}^{B\to D^*}(1)$

FNAL/MILC, PRD 79 (2009) 014506;
FNAL/MILC, PoS(LAT2010)

$\hat{B}_K$

JLQCD, PRD 77 (2008);
HPQCD, PRD 73 (2006);
RBC-UKQCD, PRL 100 (2008);
Aubin et al., PRD 81 (2010);
BMW, PLB 705 (2011);
Bae et al, PRL 109 (2012)

$f_+^{K\to\pi}(0)$

RBC-UKQCD, PRL 100 (2008);
ETM, PRD 80 (2009);
RBC-UKQCD, EPJ C69 (2010)

f_π, f_K

NPLQCD, PRD 75 (2007);
HPQCD, PRL 100 (2008);
QCDSF, PoS(LAT2007);
PACS-CS, PoS(LAT2008);
PACS-CS, PRD 79 (2009);
RBC-UKQCD, PRD 78 (2008);
Aubin et al., PoS(LAT2008);
MILC, PoS(CD09);
MILC, RMP 82 (2010);
JLQCD/TWQCD, PoS(LAT2009);
ETM, JHEP 07 (2009);
BMW, PRD 82 (2010)

1995

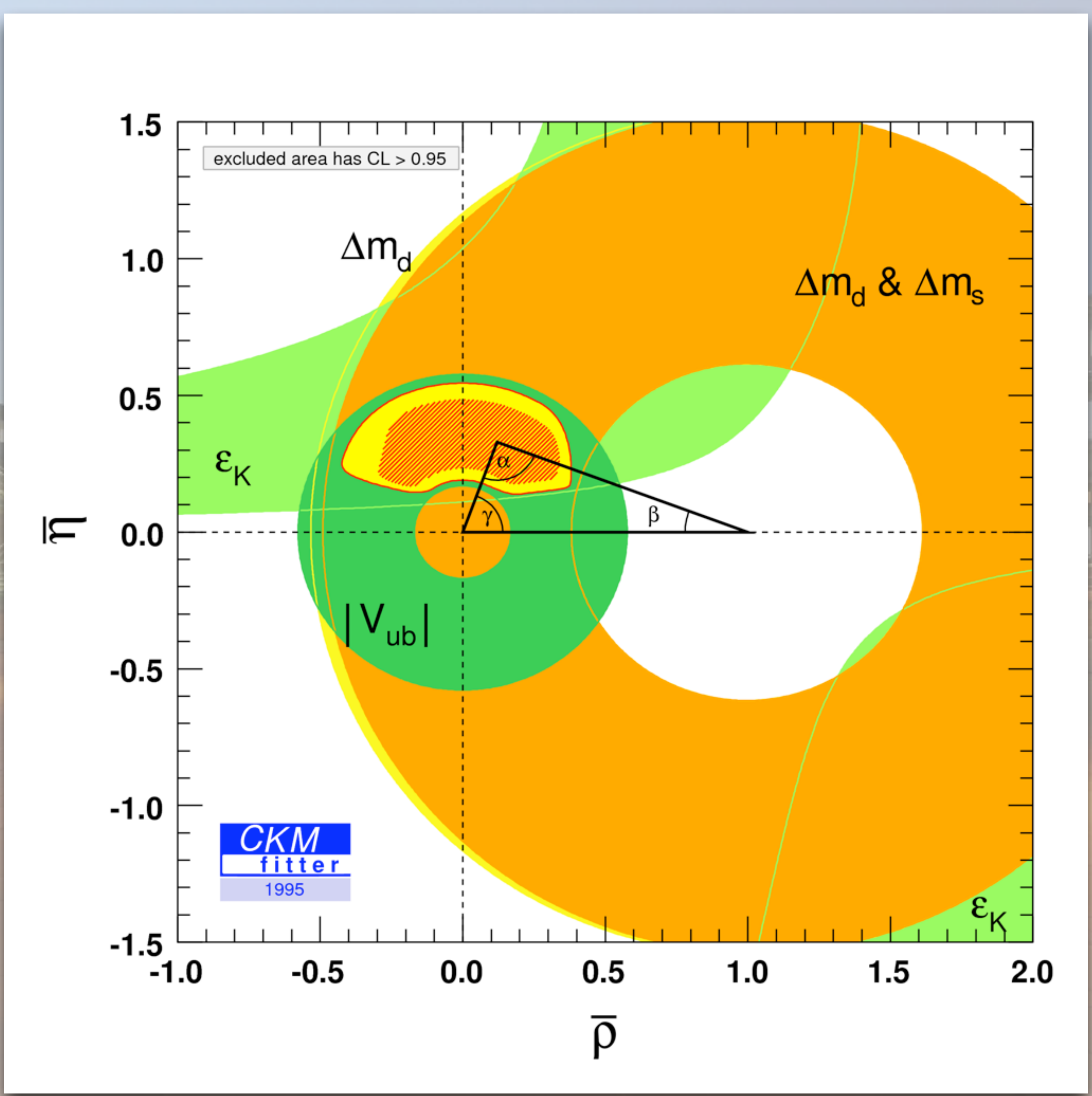
2001

2004

2006

2009

2012



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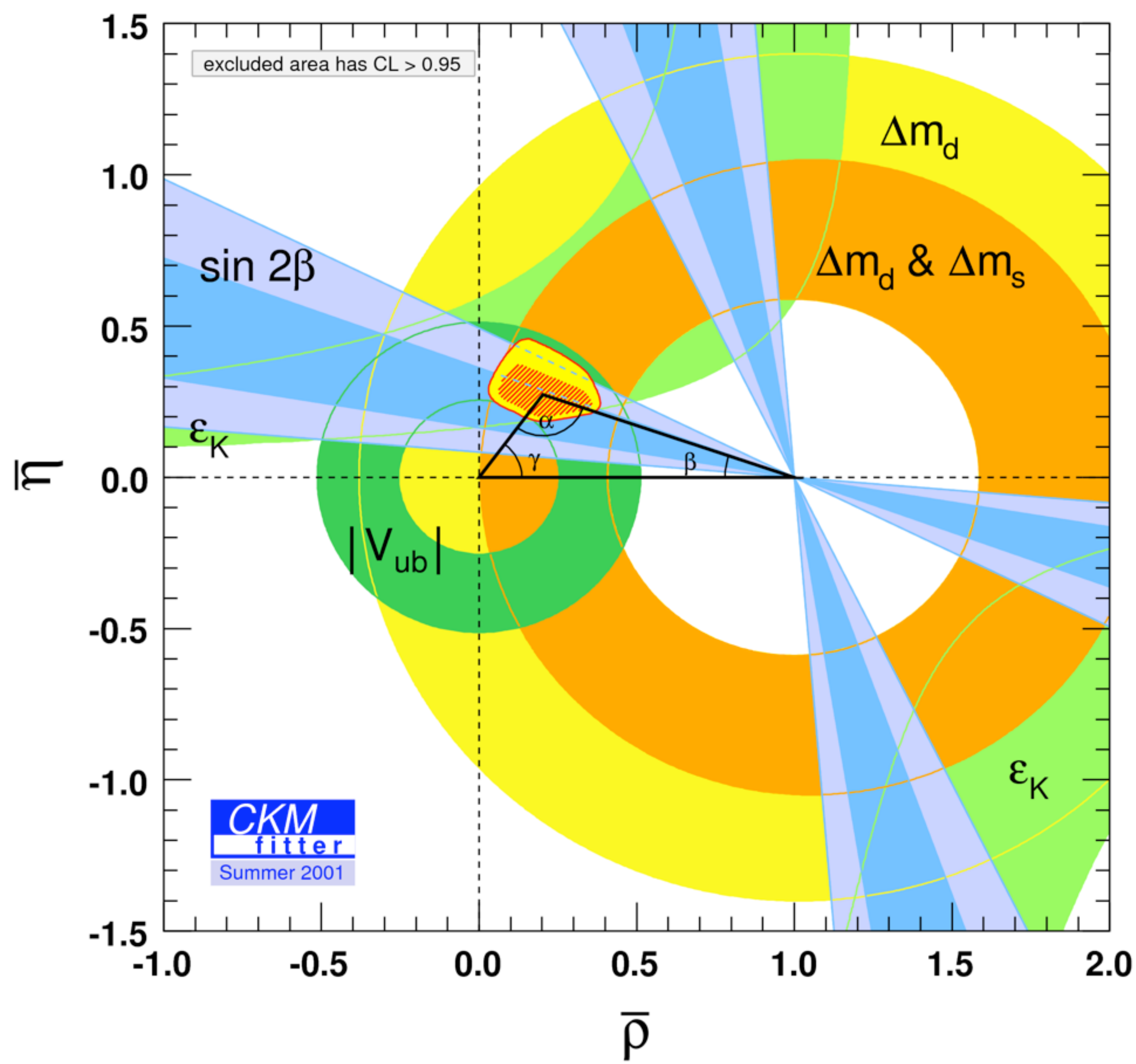

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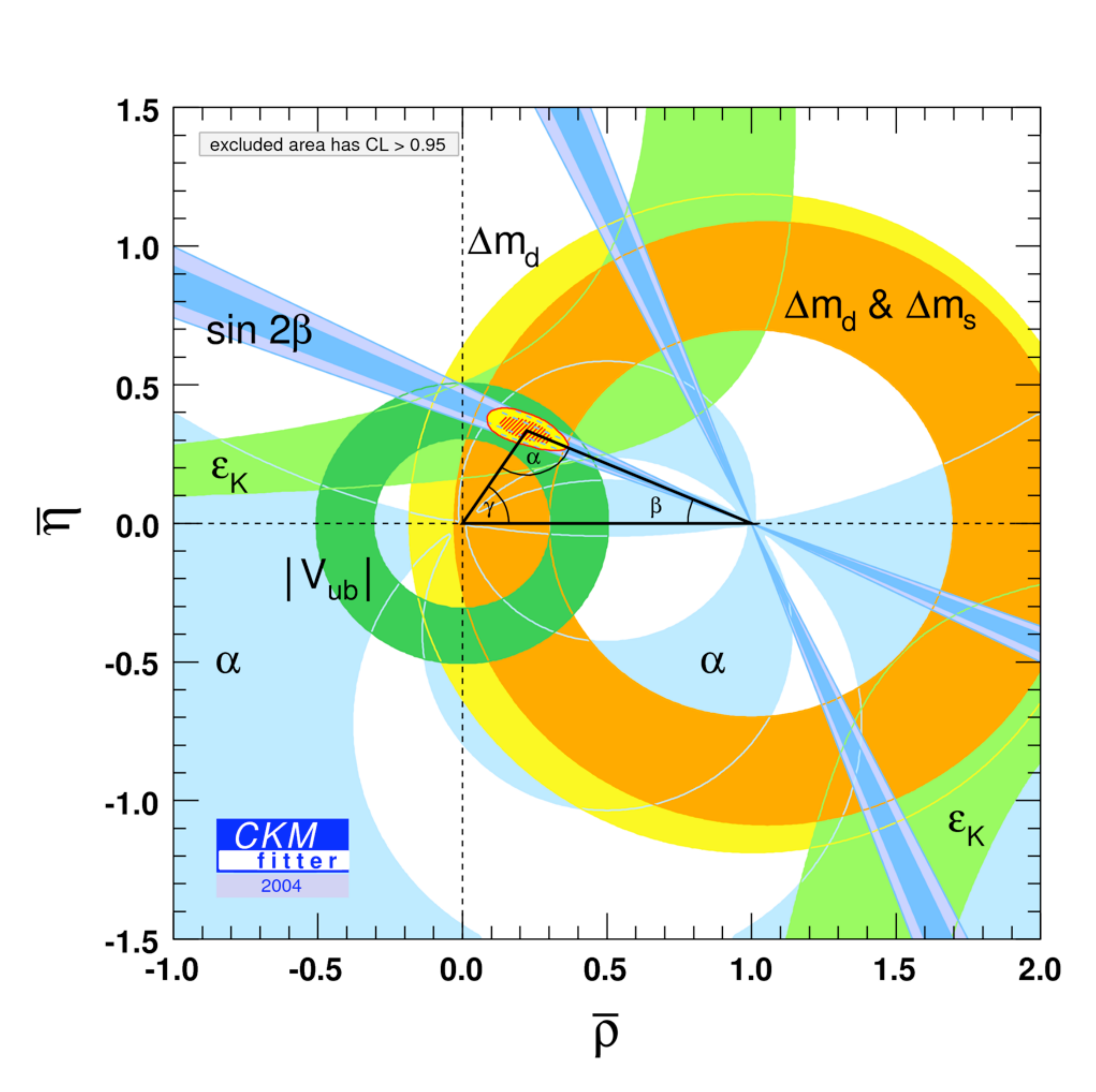

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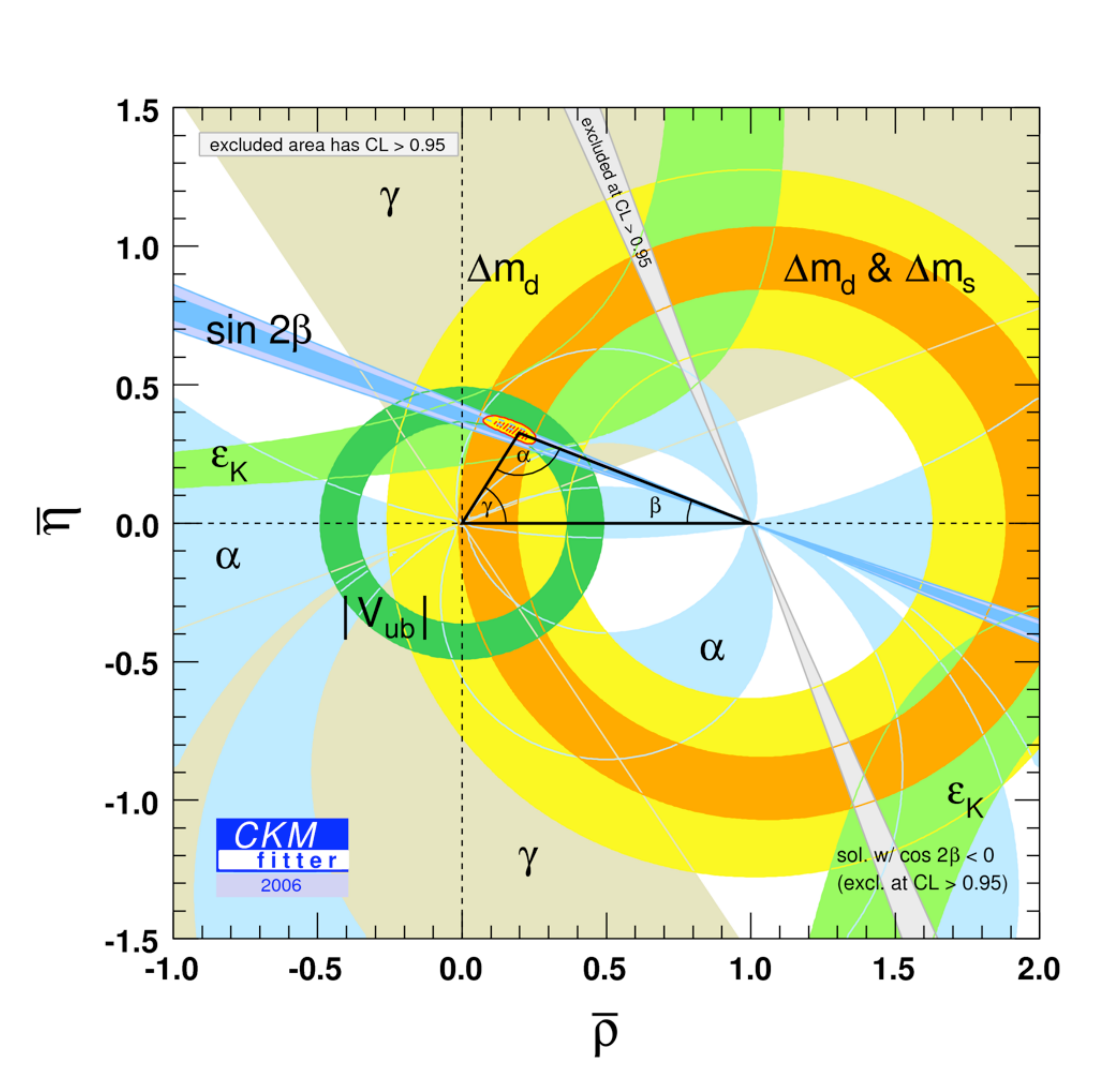
2006

2009

2012



http://ckmfitter.in2p3.fr/

1995

2001

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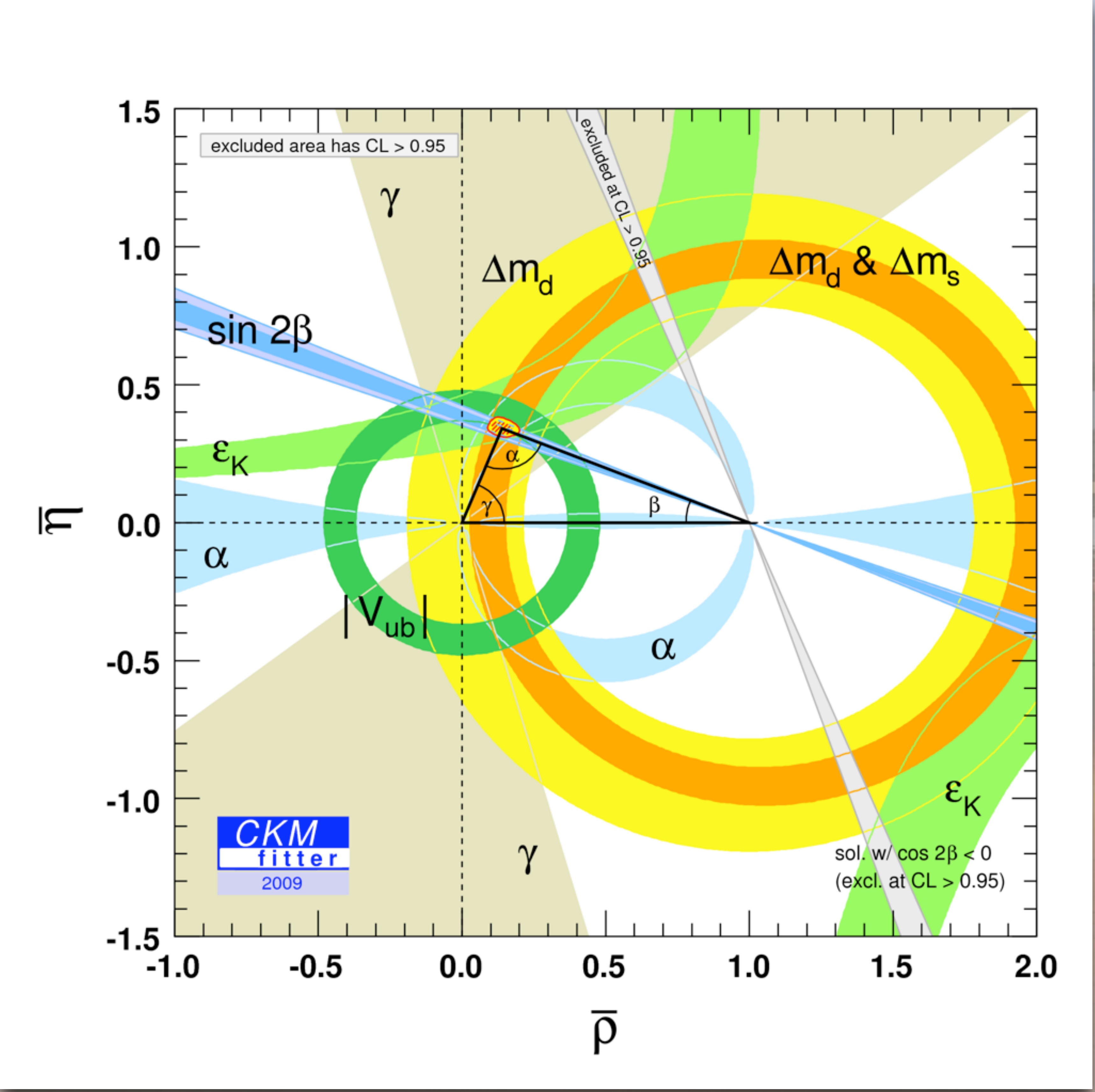

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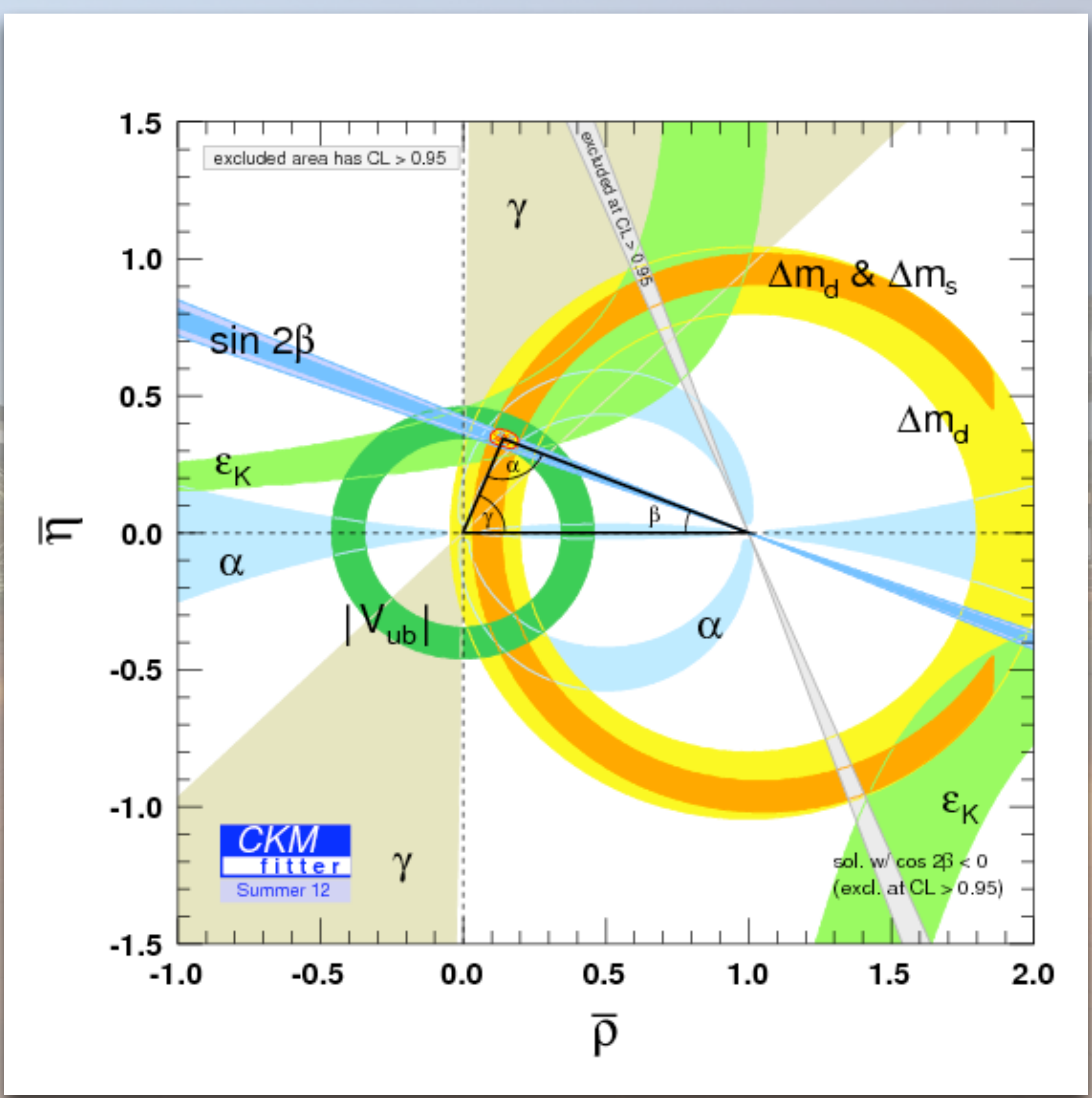
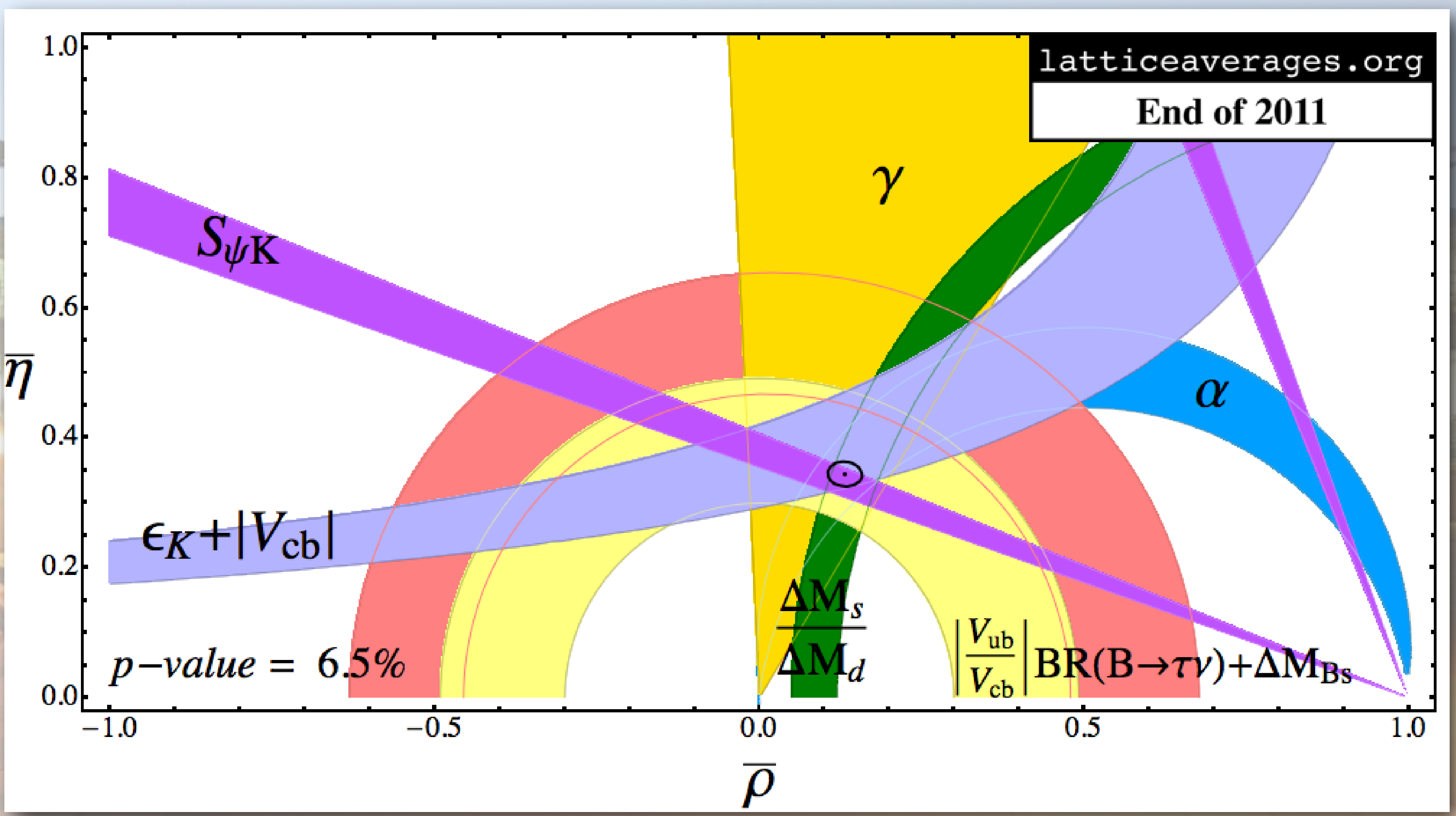


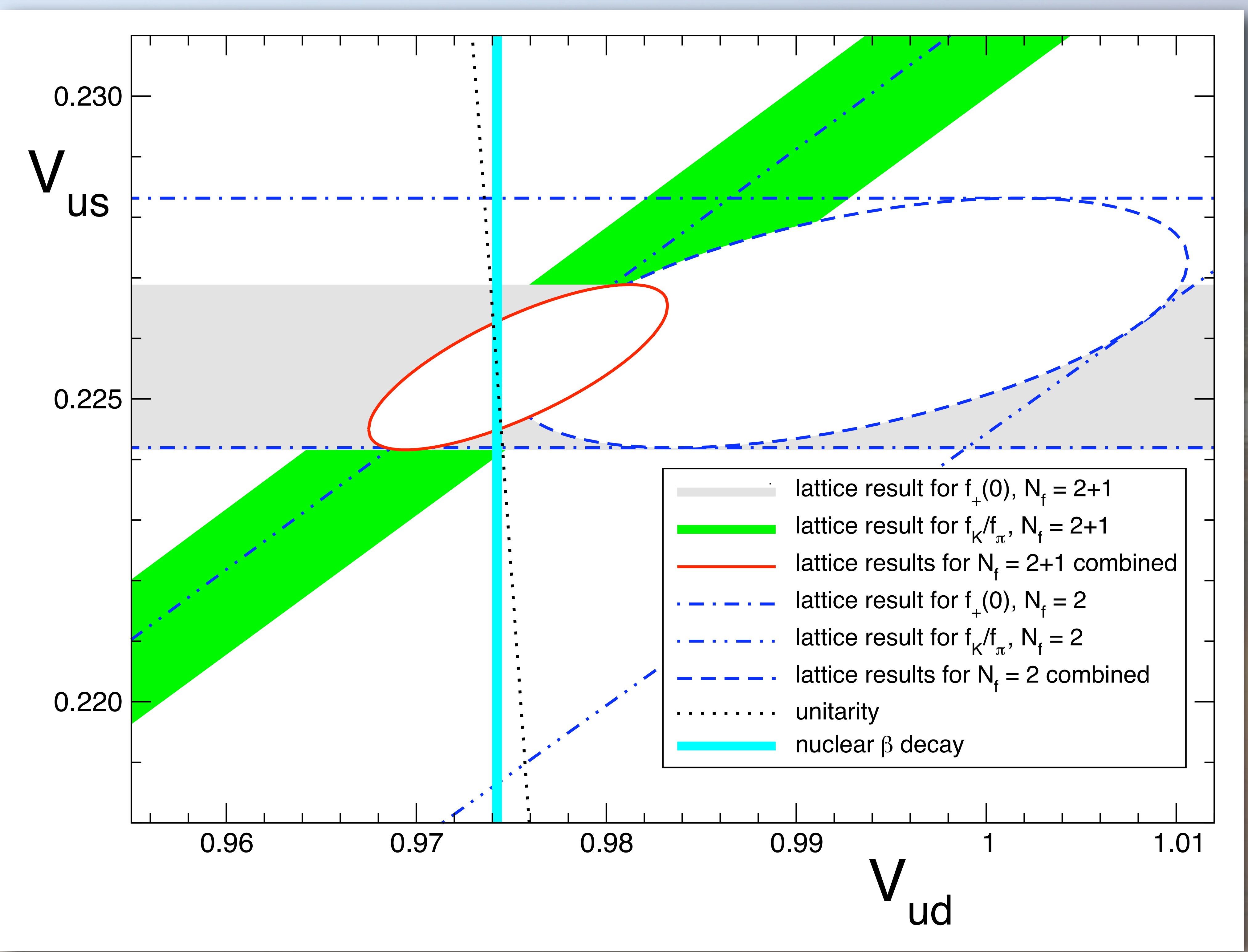
http://ckmfitter.in2p3.fr/

CKM unitarity triangle "tensions"



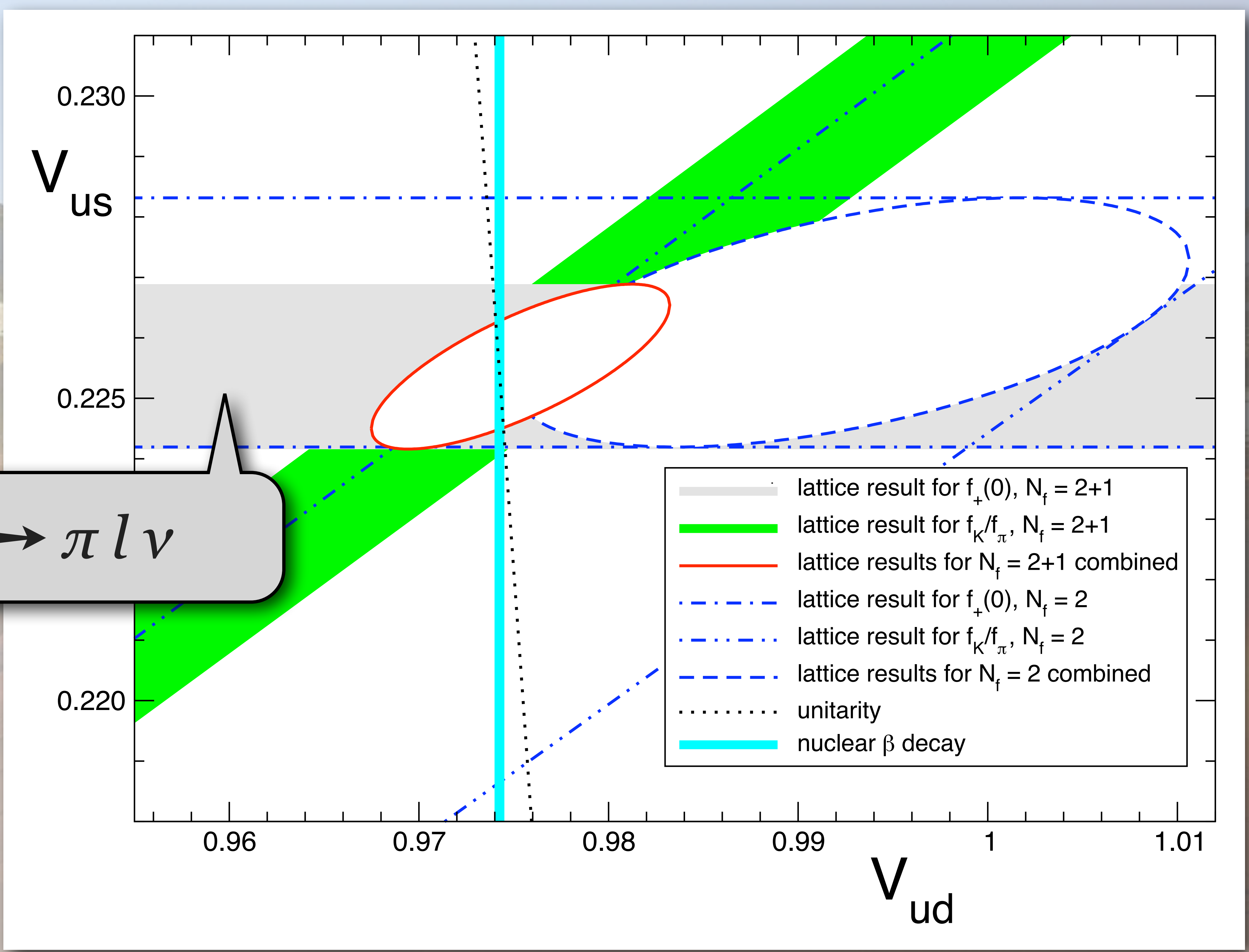
Laiho, Lunghi, Van de Water

V_{us} and top row unitarity



From, FLAG working group of FLAVIANET, arXiv:1011.4408

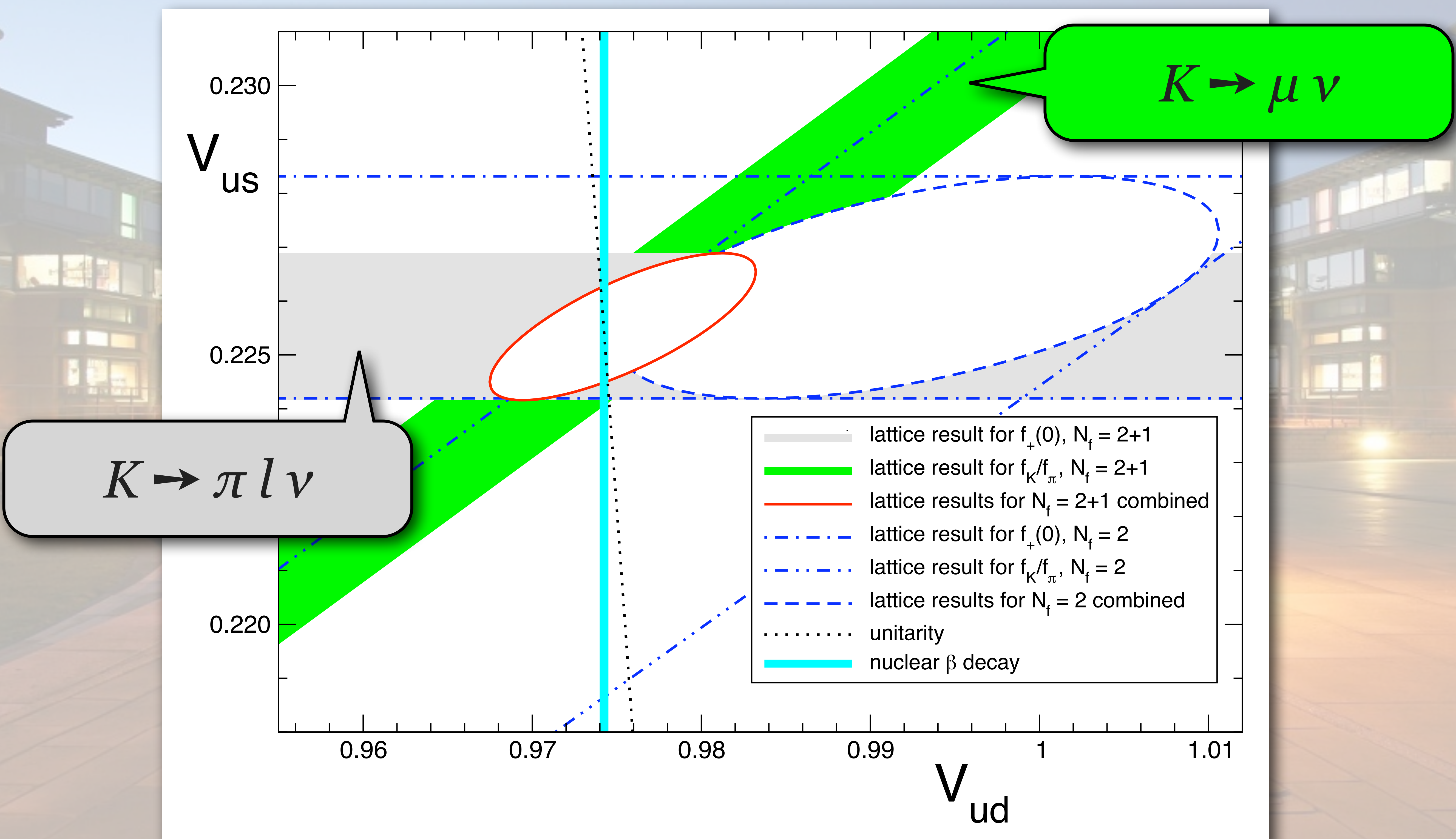
V_{us} and top row unitarity



$K \to \pi\, l\, \nu$

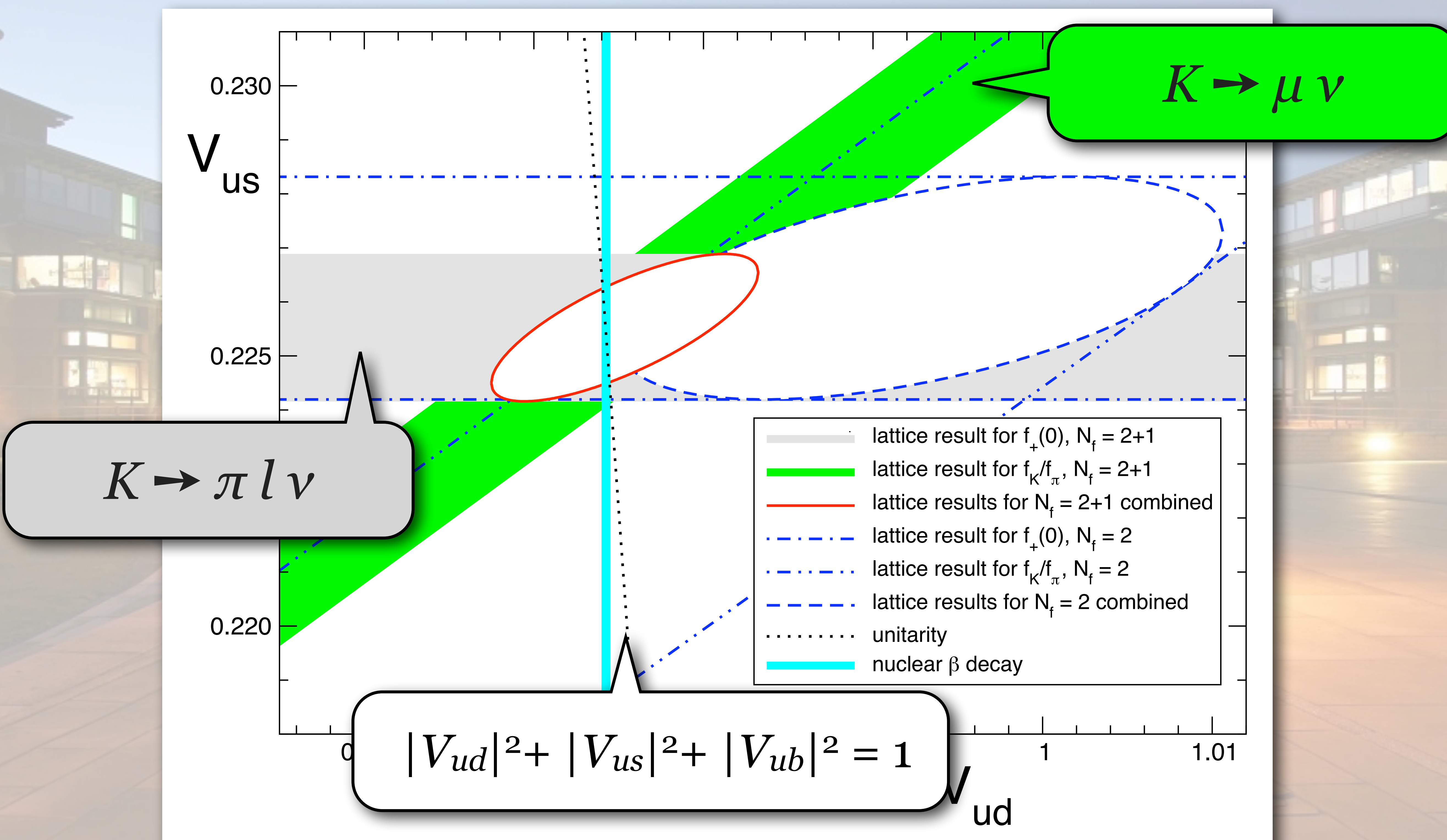
From, FLAG working group of FLAVIANET, arXiv:1011.4408

V_{us} and top row unitarity



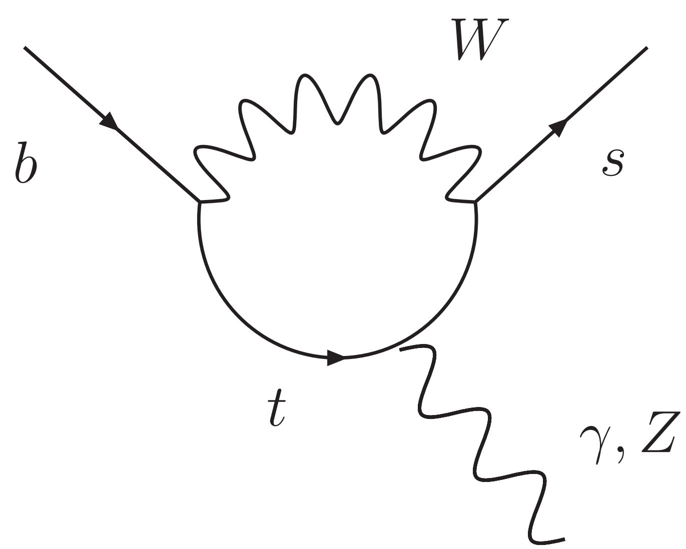
From, FLAG working group of FLAVIANET, arXiv:1011.4408

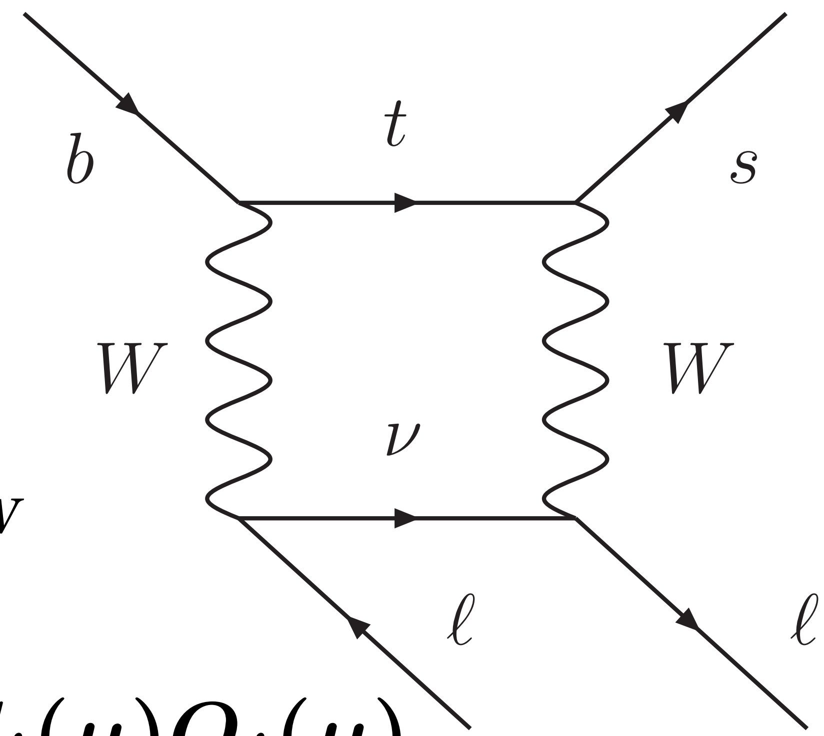
V_{us} and top row unitarity



From, FLAG working group of FLAVIANET, arXiv:1011.4408

$b \rightarrow s$ is rare in the SM



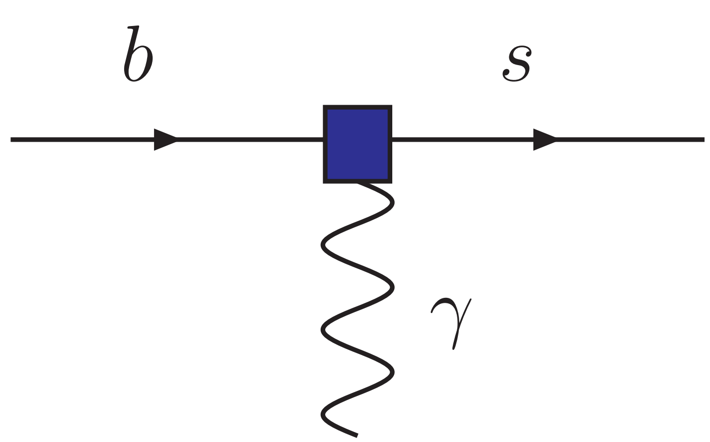


For energies $\ll m_W$

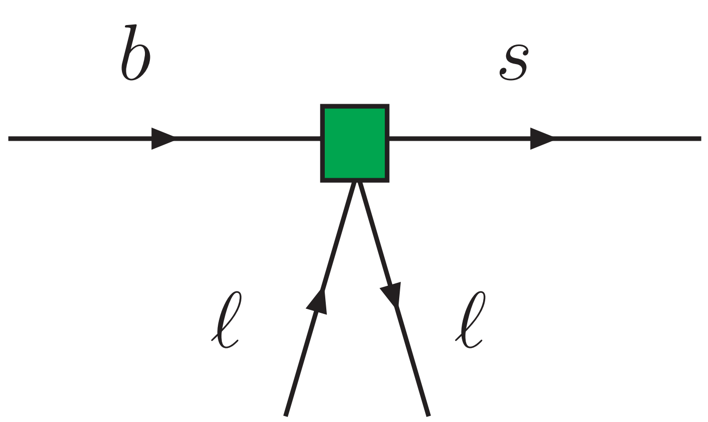
$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

Wilson coefficients

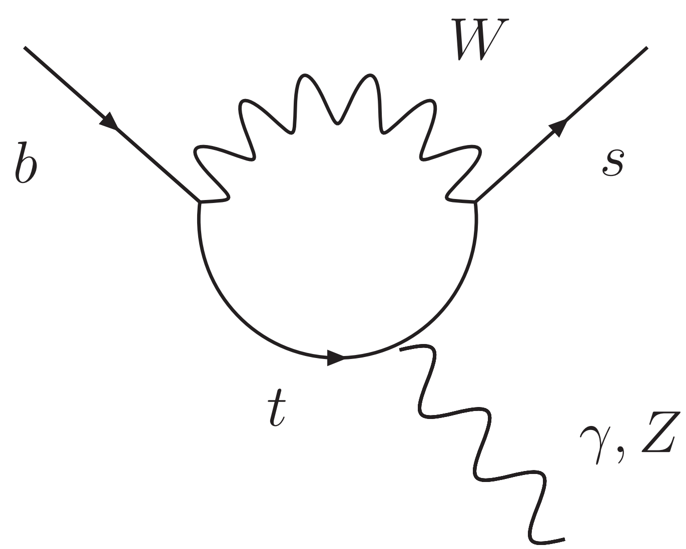
Local operators

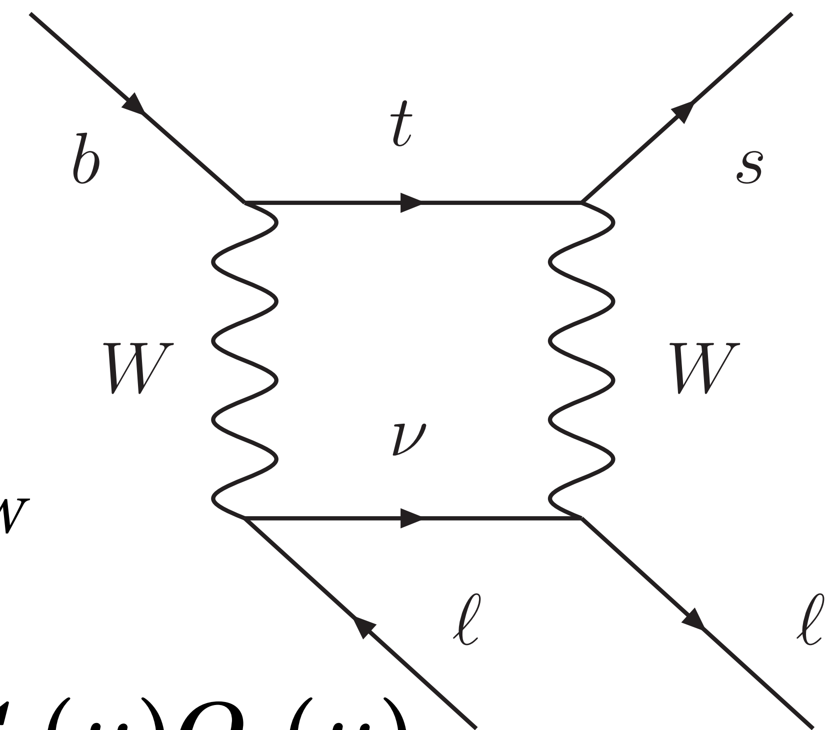


$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$



$b \to s$ is rare in the SM





For energies $\ll m_W$

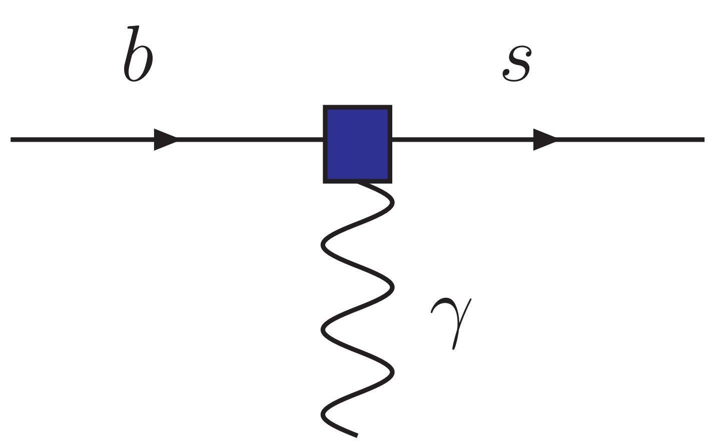
$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

SM or BSM pert.th.

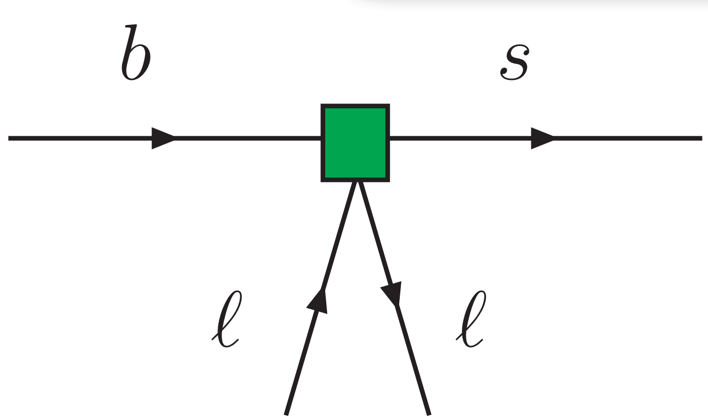
Wilson coefficients

Local operators

Non-pert. QCD



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$



Rare b decays

$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i (C_i \mathcal{O}_i + C_i' \mathcal{O}_i')$$

Most important short-distance effects in $b \to s\, ll$ come from:

$$\mathcal{O}_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b\, \bar{\ell}\gamma_\mu \ell \qquad \mathcal{O}_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b\, \bar{\ell}\gamma_\mu \gamma_5 \ell$$

$$\mathcal{O}_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b\, F_{\mu\nu}$$

Charmonium resonance effects arise from

$$\mathcal{O}_1 = \bar{c}^\alpha \gamma^\mu P_L b^\beta\, \bar{s}^\beta \gamma^\mu P_L c^\alpha \qquad \mathcal{O}_2 = \bar{c}^\alpha \gamma^\mu P_L b^\alpha\, \bar{s}^\beta \gamma^\mu P_L c^\beta$$

$$\mathcal{O}_{3..6} = \bar{s}^\alpha \gamma^\mu P_L b^{\alpha,\beta} \sum_q \bar{q}^\beta \gamma^\mu P_{L,R} q^{\beta,\alpha}$$

Form factors

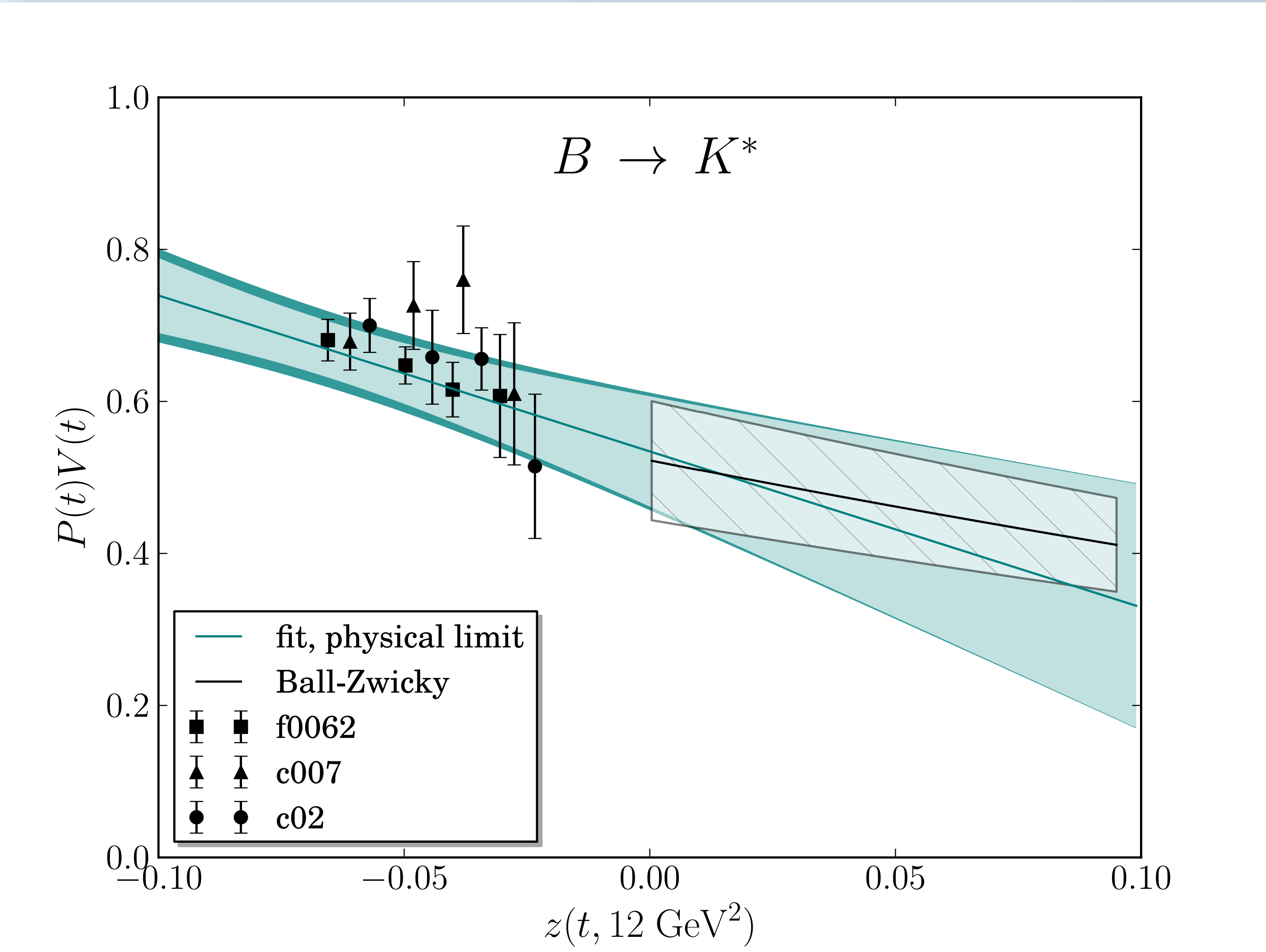
$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu b|B(p)\rangle = \frac{2i\boxed{V(q^2)}}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma$$

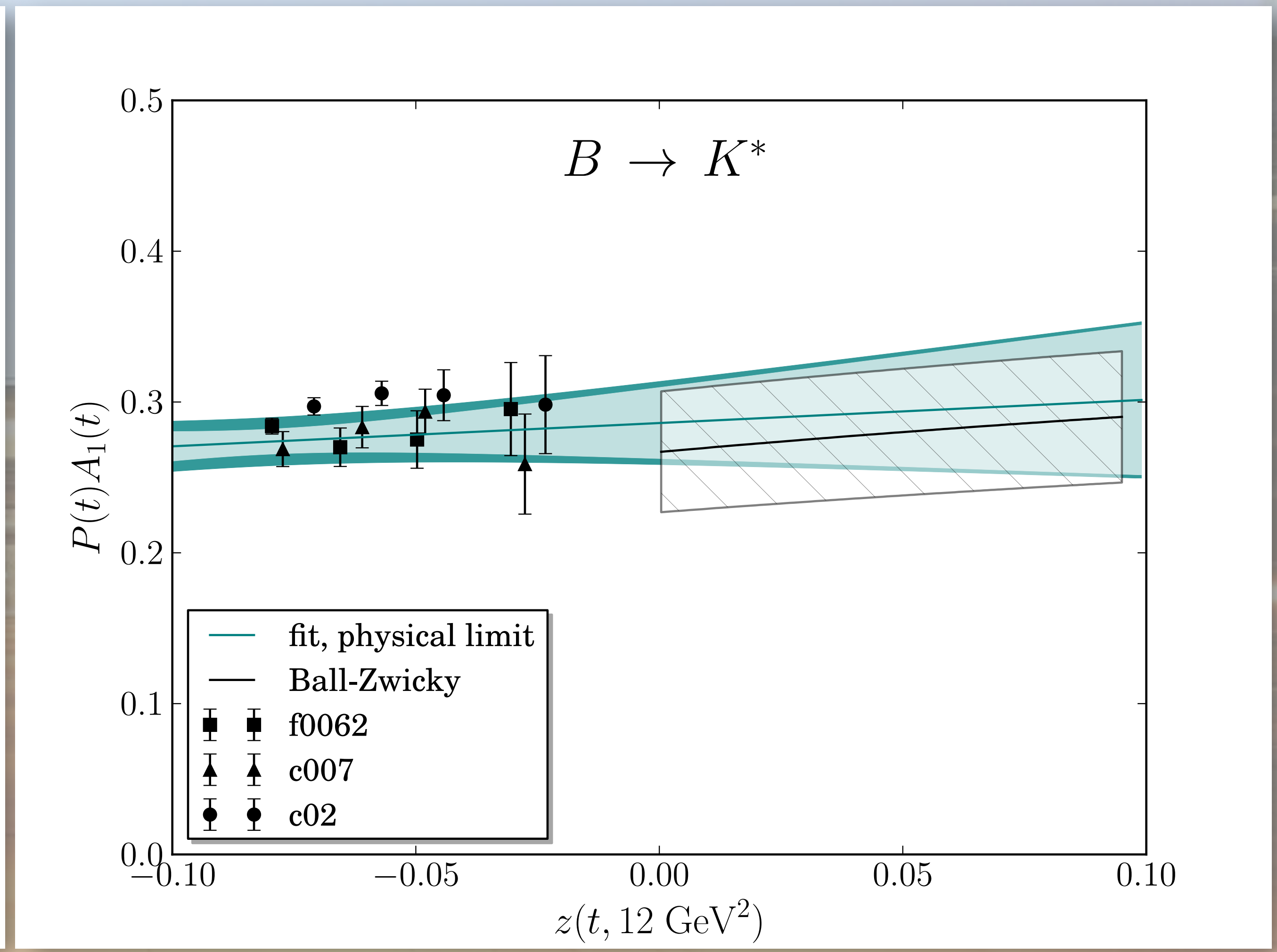
$$\begin{aligned}\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu\hat{\gamma}^5 b|B(p)\rangle &= 2m_V\boxed{A_0(q^2)}\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_B+m_V)\boxed{A_1(q^2)}\left(\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu\right) \\ &\quad - \boxed{A_2(q^2)}\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+k)^\mu - \frac{m_B^2-m_V^2}{q^2}q^\mu\right)\end{aligned}$$

$$q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle = 2\boxed{T_1(q^2)}\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau k^\sigma$$

$$\begin{aligned}-q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle &= i\boxed{T_2(q^2)}[\varepsilon^*_\mu(m_B^2-m_V^2) - (\varepsilon^*\cdot q)(p+k)_\mu] \\ &\quad + i\boxed{T_3(q^2)}(\varepsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]\end{aligned}$$

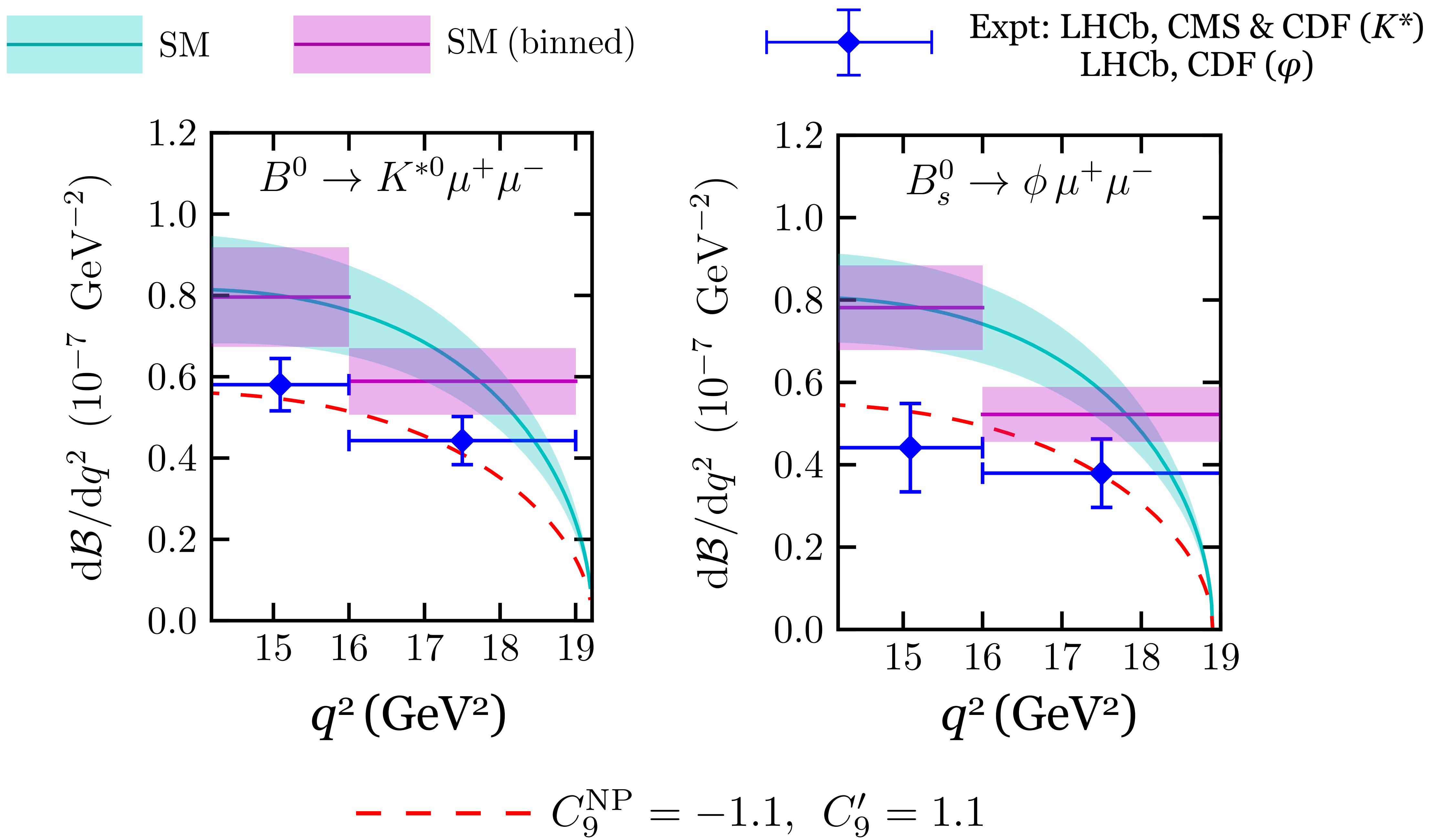
$B \to K^*$ form factors





Horgan, Liu, Meinel, Wingate, arXiv:1310.3722

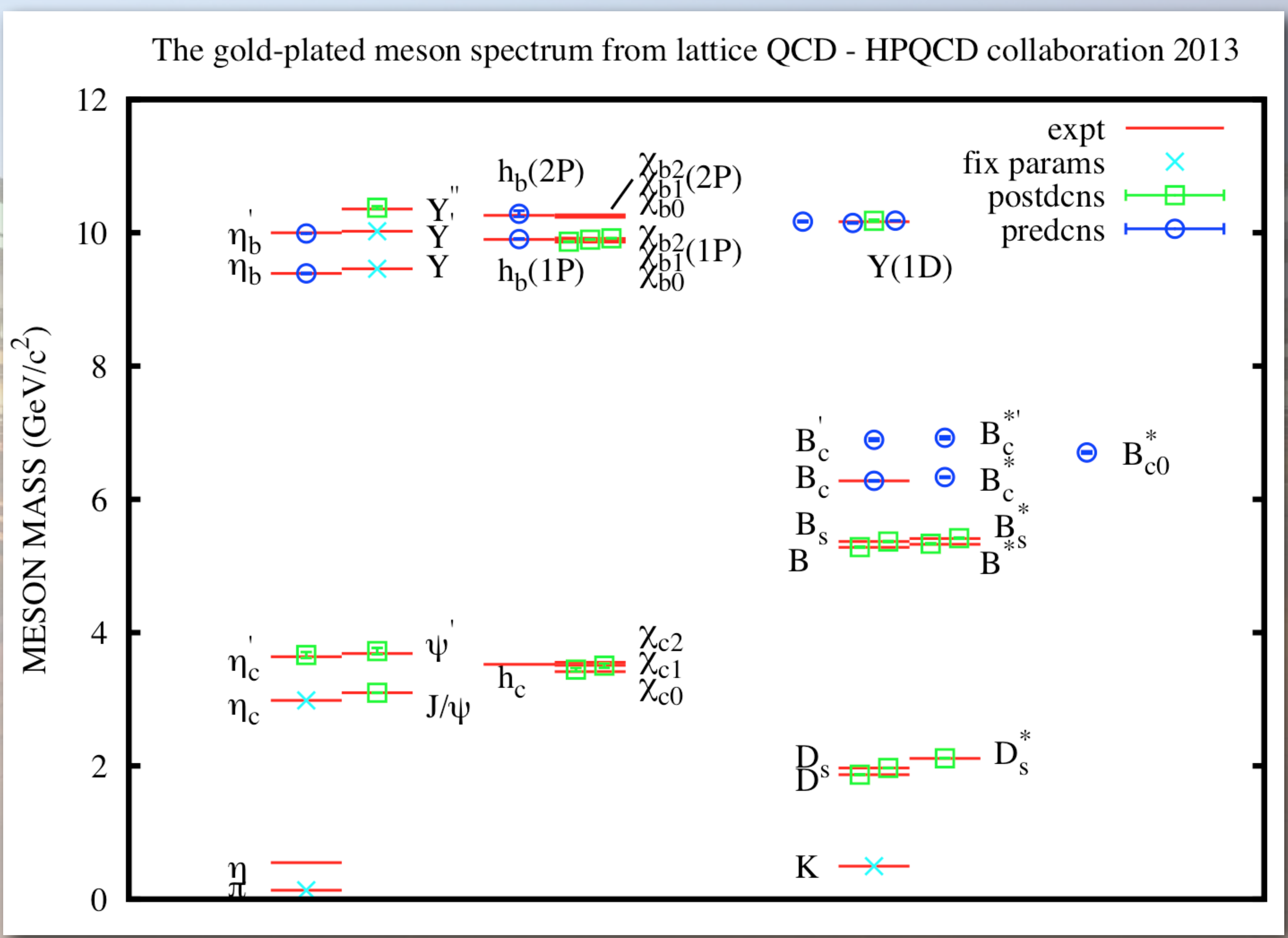
Branching fractions



Horgan, Liu, Meinel, Wingate, arXiv:1310.3887

Other applications

Hadron masses



More topics

- Excited state spectroscopy: exotics, hybrids, molecules
- Hadron-hadron scattering
- Hot QCD (& dense QCD)
- Strongly coupled gauge theories with $N_c \neq 3$, different fermion representations (BSM candidates, tests of theoretical ideas)
- Chiral gauge theories
- Sign problem

Annual conferences

Give me your up, your down, your strange
Yearning to be bound. — *R D Mawhinney*

Summary

- Lattice field theory
  - Nonperturbative regularisation
  - Interesting theoretical questions, esp. regarding fermions
- First-principles numerical calculations
  - Statistical uncertainties
  - Improvable systematic uncertainties
- Hadron matrix elements contribute to quark flavour
  - Global CKM fits
  - Rare decays
  - Contribute to SM and BSM theories at the weak scale
- Broadly applicable formulation
  - QCD applications
  - Other strongly interacting theories (technicolour, composite Higgs, theories with gravity duals